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Optimal recharging scheduling for urban electric buses: A case study in Davis

Yusheng Wang [a,b], Yongxi Huang [b,1], Jiuping Xu [a,*], Nicole Barclay [c]

[a] Uncertainty Decision-Making Laboratory, Sichuan University, Chengdu 610064, PR China
[b] Glenn Department of Civil Engineering, Clemson University, Clemson, SC 29634, USA
[c] Department of Engineering Technology and Construction Management, University of North Carolina at Charlotte, Charlotte, NC 28223, USA

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ABSTRACT

In this paper, a modeling framework to optimize electric bus recharging schedules is developed, which determines both the planning and operational decisions while minimizing total annual costs. The model is demonstrated using a real-world transit network based in Davis, California. The results showed that range anxiety can be eliminated by adopting certain recharging strategies. Sensitivity analyses revealed that the model could provide transit agencies with comprehensive guidance on the utilization of electric buses and development of a fast charging system. The comparative analyses showed that it was more economical and environmentally friendly to utilize electric buses than diesel buses.

1. Introduction

In most metropolitan areas, public transit buses are often considered to be a “greener” alternative to private vehicles because of their potential to reduce per passenger greenhouse gas emissions (Eudy et al., 2014; Mahmoud et al., 2016). However, depending on operations, technology, age and fuel type, public transit buses can cause as much per passenger pollution as private vehicles (Alam and Hatzopoulou, 2014). According to Neff and Dickens (2016) diesel buses still accounted for about 51% of the U.S. transit bus fleet in 2015. Despite the development of increasingly stringent emissions standards, diesel buses remain a serious air pollution concern in many urban areas (Miles and Potter, 2014; Wang and Rakha, 2016).

Electric buses, in contrast, have many advantages, such as low or no environmental emissions, energy conservation and quiet operations, making them an ideal technology for urban areas (Jerram and Gartner, 2012; Lajunen and Lipman, 2016). Further, electric power is particularly well suited to transit buses which operate at low speeds and have frequent stops. In the prototype development phase, the U.S. Environmental Protection Agency (EPA) claimed that using electric buses could see a potential fuel consumption decrease of as much as 50% (Davis et al., 2016). As public transport networks have fixed routes and schedules, it is much easier to plan routes for electric buses and install charging stations where they are needed (Sebastiani et al., 2016). Over the past 20 years, the light-duty private vehicle and light truck market has led in the adoption of hybrid and battery electric technology and within the medium to heavy duty category, electric buses have also been growing in popularity (Daina et al., 2017). For instance, in the United States, the share of electric buses in the transit bus market increased rapidly from 2% in 2007 to nearly 20% in 2015 (Neff and Dickens, 2016).

* Corresponding author.
E-mail address: xujiuping@scu.edu.cn (J. Xu).

[1] Present address: Research Scientist, Amazon, 345 N Boren Ave, Seattle, WA 98109, USA.

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Although electric buses have many advantages, a major concern for electric bus consumers is range anxiety. In general, the maximum driving range of most diesel buses in urban conditions is greater than 300 km (Soylu, 2015); however, the maximum driving range for most electric buses currently on the market varies from 70 km to 200 km, 25–65% less than that of diesel buses, which means that it is difficult to operate them continuously without recharging (Stockes and Poger, 2013; Mahmoud et al., 2016). To ensure normal operations, the energy consumed must be replenished quickly using either battery swapping or fast charging technology (Marlay, 2013; Wang et al., 2014). For the battery swapping mode, decision-makers need to schedule the battery changes for the electric buses, determine the minimum quantity of spare batteries to stock and schedule recharging for the spare batteries, areas for which there has already been significant research progress (Choi and Kim, 2014; Dai et al., 2014; Kang et al., 2016; You et al., 2016). For the fast charging mode, decision-makers need to schedule the electric buses to be recharged, determine the location of the charging stations and decide on the number of needed chargers. This paper develops an optimization framework to provide concurrent decision support for these identified issues; or what is known as the electric bus recharging scheduling problem.

Compared to the research into electric vehicles, there has been much less research focused on electric buses, and of these few papers, most have discussed the introduction of electric buses (Kühne, 2010; Hua et al., 2014; Mahmoud et al., 2016), provided energy consumption assessments and management (Lajunen, 2014; Li et al., 2015; Ding et al., 2015; Li et al., 2016; Lajunen and Lipman, 2016), or conducted feasibility analyses for electric bus implementation (Sebastiani et al., 2016; Mohamed et al., 2017). There also have been a few recent papers focused on electric bus recharging scheduling problems. Paul and Yamada (2014) proposed a *k*-Greedy Algorithm-based approach for the recharging scheduling of electric buses and performed simulations using real bus diagram data for a transit network in Japan. Ke et al. (2016) developed a framework for simulating the operation and battery charging schedule of electric buses in Penghu, Taiwan, with the aim of minimizing electric bus transportation system construction costs. Qin et al. (2016) simulated daily recharging patterns and demand for an electric bus fleet in Tallahassee, Florida and identified an optimal recharging strategy to minimize demand. However, these papers have either focused only on the development of recharging schedule diagrams on the basis of the predetermined location and capacity of charging stations, or, have been concerned with the effect of recharging strategies on electricity costs. Little research has focused on solving concurrent transit network planning and recharging scheduling problems.

To best utilize a fast recharging system, it is crucial to develop a cost-effective decision-making framework that can provide optimal strategies for planning and operational decisions while satisfying the recharging demand of all electric buses without delays or congestion. Operational decisions regarding the recharging schedule or the assignment of electric buses to chargers relies on comprehensive planning decisions regarding the location and capacity of the charging stations. In an integrated modeling framework, therefore, both the planning and operational decisions have to be made concurrently to achieve overall cost effectiveness.

Distinct from prior research, this paper develops an optimization framework which allows concurrent planning decisions to be made for charging station locations, the number of chargers to be installed and recharging scheduling decisions. The proposed optimization framework is implemented in a real-world case based in Davis, California. The results showed that this optimization framework can help transit agencies develop low cost electric bus recharging systems. Sensitivity analyses examine various recharging strategies and a comparison with diesel buses is also given. The proposed framework is also generalized so it can be applied to different transit networks. The remainder of this paper is organized as follows. Detailed background on electric buses and fast charging systems is provided in Section 2. The optimization framework is presented in Section 3, and the results of a real-world transit network in Davis are presented with sensitivity analyses in Section 4. Conclusions and future research are outlined in Section 5.

2. Background to electric buses and fast charging systems

There are currently three main types of electric buses on the market: hybrid electric buses, plug-in hybrid electric buses and battery electric buses: which are differentiated by design and operation (Stockes and Poger, 2013; Mahmoud et al., 2016). Battery electric buses, also called "all-electric" buses, are powered solely by an electric motor and have no onboard internal combustion engine. Compared with the other two types, battery electric buses have additional benefits such as zero tailpipe emissions and low-noise operations, and therefore have great potential to move the transportation sector away from a reliance on fossil fuels (Jerram and Gartner, 2012). Therefore, in this study, only battery electric buses are considered.

Battery electric buses are currently still in development or are being used as part of demonstration projects in most parts of the world. For instance, the Chinese government initiated the Ten Cities, Thousand Vehicles Program in 2009 to expedite the widespread adoption of all types of electric vehicles (PRTM, 2011). With this effort, China has become a leading developer of battery electric buses, with more than 10 battery electric bus manufacturers now operating (Stockes and Poger, 2013). At the same time, the United States, Canada, Europe and Japan are also producing and deploying battery electric buses, typically with the support of government funding (Jerram and Gartner, 2012).

However, the main problem with electric buses has been the limited driving range. Barnett (2015) reported that the 35-foot low-floor Proterra brand electric bus used in Seneca, South Carolina, USA, could only run for 35 miles (56 km) between charges. Similarly, the 22-foot Ebus brand electric bus used in Downey, California, USA, can only run for 45 miles (72 km) between charges (Stockes and Poger, 2013). Further, under safety considerations and to extend battery life, the deep discharging of batteries should be avoided. For example, electric buses with a maximum driving range of 200 km in Beijing are limited to driving 100 km on a full charge (PRTM, 2011).

This limited driving range means that it is difficult to continuously run an electric bus. To ensure that electric buses maintain high up-time operations, the energy consumed must be replenished quickly using either fast charging technology or by swapping the battery, whereby the existing battery is replaced with a fully-charged battery before it is fully depleted. Battery swapping has been employed in real-world services in Beijing (EV World, 2008) and Shanghai (United Nations Environment Programme, 2009). Range anxiety can also be remedied using fast charging technology. Different from the battery swapping mode, the energy recharged using the fast charging mode depends on the charging duration and the recharging efficiency; therefore, the battery may not be fully recharged after a fast charging service. With the development of fast charging technology, recharging has become increasingly faster. Stockes and Poger (2013) reported that a fast charging system was able to charge an electric bus with a range of 40 miles (64 km) in roughly 10 min. In Seneca, South Carolina, electric buses can be recharged using an automatic connection that links the buses to a high-capacity charger through an overhead system without the involvement of the driver, as shown in Fig. 1. The electric buses can reach at least a 92 percent state of charge (SOC) in as little as 6 min (Barnett, 2015).

In most public transit networks, there are transit centers at which several routes connect. For each bus, the layover time at the transit center is usually longer than at other stops to allow passengers to change routes. For instance, buses in the transit network in Davis, California usually stay for 10–15 min at the transit centers. This layover time in the transit centers allows time for multiple electric buses from different routes to be rapidly recharged as well as providing better charger station management. Transit center charger locations also facilitate easier charger maintenance. Therefore, transit centers are the candidate locations for the deployment of charging stations in this study.

3. Modeling

For the modeling, a situation in which the transit agency plans seeks to replace all conventional diesel buses with electric buses in an urban public transit network is considered, for which sensitivity analyses under various electrification rate scenarios are conducted. To ensure a smooth transition, the electric buses plan to operate on the already established timetable. As the electric bus driving ranges are shorter than the diesel buses, to ensure normal operations, several charging stations equipped with fast chargers are to be strategically placed at the transit centers. Given the fixed bus schedules, the transit agency needs to make the following decisions:

- How many charging stations are needed in the transit network?
- How many chargers are needed at each charging station?
- How can the electric buses be scheduled to ensure recharging without any delays or charging station congestion?

In this section, an optimization framework is developed. As the proposed framework is general, it can be applied to different types of transit networks.

3.1. Problem description

In this section, a single-depot transit network is considered (see Fig. 2)) to define the electric bus recharging scheduling problem. In this transit network, there is one depot where all electric buses are housed overnight. The electric buses leave the depot in the morning to operate a sequence of scheduled trips and then return to the same depot. A total of 5 routes are connected by 2 transit centers. Each route has one or two termini, which are the start or end point of a route. At least one terminus on each route is located at the transit centers in this network. A trip is defined as the short journey the bus operates along a route from one terminus to another (single-trip) or returning to the same terminus (round-trip), with one trip having a start time and an end time. For instance, Route 1: $A \rightarrow D \rightarrow A$, a trip starting at 7:00 am and ending at 7:50 am is different from the trip starting at 8:00 am and ending at 8:50 am. A trip-pair is defined as two consecutive trips operated by the same electric bus. For a trip-pair, the trip-interval is defined as the start time of the latter minus the end time of the former. It is assumed that one electric bus can only service one route, and there may be multiple electric buses on one route.

To power the electric buses, candidate charging stations equipped with several fast chargers are deployed at the transit centers. The electric buses can only be recharged during trip-intervals and cannot be recharged until they complete one or several whole trips before arriving at a charging station. The main tasks in this study are to schedule the electric buses to be recharged and to determine the number of the charging stations and needed chargers. To simplify the problem, several major assumptions are made:

(1) The electric buses operate according to the previously established timetable.
(2) All the electric buses are homogeneous and have the same driving range.
(3) The charge consumed is proportional to the driving distance.
(4) The energy recharged is proportional to the charging duration.

Fig. 1. An electric bus being recharged by a fast charger in Seneca.

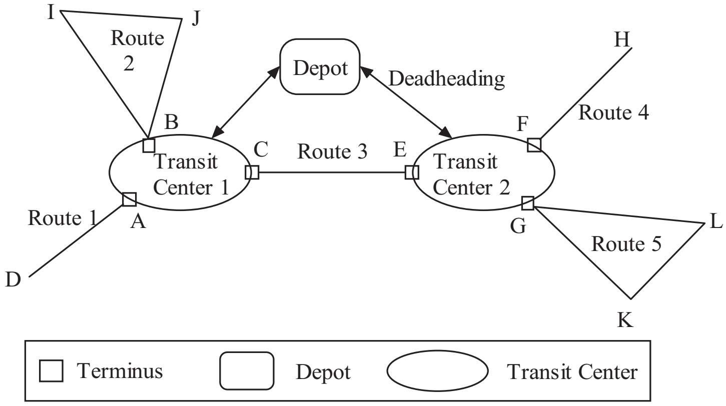


Fig. 2. Example of single-depot transit network architecture.

(5) All chargers are homogeneous fast chargers and each charger is equipped with one outlet.
(6) The recharging duration is fixed and the recharging process is continuous.

Note: For assumption (1), since all diesel buses are to be replaced by electric buses but the timetable remains unchanged, the number of electric buses needed is the same as the number of diesel buses, and the assignment of timetabled trips to each electric bus can also be predetermined. For assumptions (3) and (4), an electric scooter field-test was undertaken by Yu and Lu (2013) to clarify the relationship between energy consumption, bus driving distance and recharging duration. The experimental data showed that the relationship between the residual charge and bus driving distance was almost linear, meaning that the charge consumed was proportional to the bus driving distance, and that the extended distance per minute recharged was almost constant, findings which were in agreement with other similar research (Wang and Lin, 2013; Li and Huang, 2014; Huang and Zhou, 2015). In addition, to extend the battery life, most electric bus battery manufacturers recommend that deep discharging be avoided (Paul and Yamada, 2014); therefore, in this study, the lower limit for the remaining energy in the battery is set as 20% of capacity. New fast charging technology means that the recharging duration is now very short (e.g., usually several mins). Therefore, it is realistic to assume that the energy recharged is proportional to the recharging duration. For assumption (5), automatic connection fast chargers are considered, as shown in Fig. 1. Although there are now two or more outlet chargers available, most tend to be slower and the cost higher. Further, as the electric bus and the fast charger are relatively large, more room is needed to deploy a multiple outlet charger. However, this assumption can be relaxed in future studies as it does not change the structure of the proposed model. For assumption (6), as the recharging duration is very short, it is more convenient to operate the chargers if the recharging duration is fixed. The effects of recharging durations on the results are further examined in the sensitivity analyses.

3.2. Model formulation

For modeling convenience, the depots were divided into an origin depot and a destination depot, with the electric buses departing from the origin depot and returning to the destination depot. Let O be the set of origin depots, D be the set of destination depots and S be the set of scheduled trips. For each trip $i \in S$, let a_i and b_i be the start and end time of trip i. Let N indexed n be the set of candidate charging stations. To formulate the capacity at each charging station at any time, time was discretized into time nodes and one time step was predefined (e.g., one minute). Let T be the set of time nodes from the start time of the earliest trip to the end time of the latest trip. Let u_n be the charging duration for each recharging activity at charging station n. For each recharging activity, if the recharging start time is $t \in T$, the recharging end time is $t + u_n$. Travel from the end point of a trip to the charging stations is called deadheading. Let r_{in} be the deadheading travel time from the end point of trip $i \in S$ to charging station n. Let P be the set of trip-pairs, $(i,j) \in P$, which means that trip $j \in S \cup D$ is served immediately after trip $i \in S \cup O$ by the same electric bus. For a trip-pair $(i,j) \in P$, the trip-interval is defined as the start time of trip j minus the end time of trip i, i.e., $a_j - b_i$. An electric bus can be recharged during the trip-interval, but there may be multiple possible recharging start time nodes. For instance, for a 5-min trip-interval and 3-min recharging duration, the recharging activity could start at the first minute, the second minute or the third minute; therefore, there are three possible recharging start time nodes. To narrow the candidate recharging activity scope, A_n is introduced to denote the set of possible recharging activities at charging station n, where $(i,j,t) \in A_n$, which means that the electric bus is recharged at charging station n during the trip-interval between the trip-pair $(i,j) \in P$, from time $t \in T$ to time $t + u_n$, if $t \geqslant b_i + r_{in}$ and $t + u_n + r_{in} \leqslant a_j$. These two restrictions on t can guarantee no delays in a trip-pair. w_{ijnt} is also introduced to denote the recharging waiting time at charging station n. For an electric bus which serves trip i, the arrival time at charging station n should be $b_i + r_{in}$. If the bus starts recharging at time node $t \in T$, the recharging waiting time $w_{ijnt} = t - b_i - r_{in}, (i,j,t) \in A_n$.

A simple example is used in Fig. 3 to illustrate the recharging process between a trip-pair. In Fig. 3, (i,j) is a trip-pair served by an electric bus, and there is one charging station. Let the travel time between the charging station and the start/end point of a trip be one unit, and assume the recharging duration to be three units. The arrival time at the charging station is 6. As the start time of trip j is 11, the end time for the feasible recharging activity should be no later than 10. As a result, there are two possible recharging activities for the trip-pair (i,j); i.e., one starting at time 6 and ending at time 9 (node f), and the other starting at time 7 and ending at time 10 (node g). Note that the recharging waiting time for the former is zero, while the latter has a waiting time of one unit as the arrival time at the charging station is 6. Also note that there is no connection to node e and h. For node e, a recharging start time of 5 is invalid, as it is earlier than 6, which is when trip i arrives at the charging station. For node h, trip j arrives at the charging station at 12, which is later than 11, which is the start time for trip j. Therefore, in this example, $(i,j,5) \notin A_n, (i,j,6) \in A_n, (i,j,7) \in A_n, (i,j,8) \notin A_n$. Accordingly, $w_{ijn6} = 0, w_{ijn7} = 1$, where $n = 1$ as there is only one charging station.

Other parameters and variables are as follows:

Parameters:

β	maximum driving range for a fully-charged electric bus, in km;
β_0	the initial range for an electric bus at the depot, in km;
θ	recharging rate, i.e., the extended driving distance using energy charged per minute, in km/min;
d_i	length of trip $i \in S$, in km;
d_{in}	driving distance between the start/end point of trip $i \in S$ and charging station $n \in N$, in km;
c_w	cost of unit waiting time, in dollars/min;
$e_{\min}$	extended driving distance using the minimum energy in an electric bus, in km;
c_d	cost of unit driving distance, in dollars/km;
c_o	fixed cost per recharging activity; refers to charger startup and operation in dollars;
c_e	variable recharging costs in unit time; refers to the electricity costs in dollars/min;
c_{f1}	fixed costs per charger; includes purchase and installation costs in dollars;
c_{f2}	fixed costs per charging station; includes construction and land costs, in dollars;
c_{m1}	maintenance cost per charger in dollars;
c_{m2}	maintenance cost per charging station in dollars;
K_n	number of candidate chargers in the charging station $n \in N$;
$\overline{D}$	number of operating days per year;
$\overline{M}$	a sufficiently large positive number;
α	annualized factor.

Variables:

X_i^n	$= 1$ if the electric bus from trip $i \in S$ is recharged at charging station $n \in N$, 0 otherwise;
Y_{it}^{nk}	$= 1$ if the electric bus from trip $i \in S$ starts being recharged on the kth charger at charging station $n \in N$ at time $t \in T$, 0 otherwise, $k = 1, \ldots, K_n$;
E_i	extended bus driving distance using remaining onboard energy at the end of trip $i \in S$, in km;
Z_{nk}	$= 1$ if the kth charger at charging station $n \in N$ is used; 0 otherwise, $k = 1, \ldots, K_n$;
$Z\prime_n$	$= 1$ if charging station $n \in N$ is used; 0 otherwise.

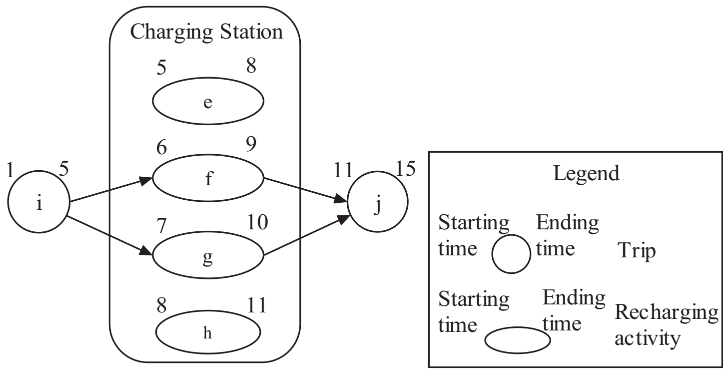


Fig. 3. An example of recharging activities between a trip-pair.

A mixed integer linear programming model was formulated. The complete model is provided in (1)–(15):

$$\min C = \sum_{i\in S}\sum_{n\in N}(c_d d_{in} + c_o + c_e u_n)\overline{D}X_i^n + \sum_{k=1}^{K_n}\sum_{n\in N}\sum_{(i,j,t)\in A_n} c_w w_{ijnt}\overline{D}Y_{it}^{nk} + \sum_{k=1}^{K_n}\sum_{n\in N}(\alpha c_{f1} + \overline{D}c_{m1})Z_{nk} + \sum_{n\in N}(\alpha c_{f2} + \overline{D}c_{m2})Z\prime_n \quad (1)$$

Subject to:

$$\sum_{n\in N} X_i^n \leqslant 1 \;\; \forall i \in S, \quad (2)$$

$$E_i + \sum_{n\in N}(\theta u_n - d_{in})X_i^n \leqslant \beta \;\; \forall i \in S, \quad (3)$$

$$E_j = E_i + \sum_{n\in N}(\theta u_n - d_{in})X_i^n - d_j \;\; \forall (i,j) \in P, \quad (4)$$

$$E_i \geqslant e_{\min}, \;\; \forall i \in S, \quad (5)$$

$$\sum_{k=1}^{K_n}\sum_{t:(i,j,t)\in A_n} Y_{it}^{nk} = X_i^n, \;\; \forall i \in S, n \in N, \quad (6)$$

$$\sum_{i:(i,j,t\prime)\in A_n}\sum_{t\prime=t-u_n+1}^{t} Y_{it\prime}^{nk} \leqslant 1, \;\; \forall k = 1,\ldots,K_n, n \in N t \in T \quad (7)$$

$$\sum_{(i,j,t)\in A_n} Y_{it}^{nk} \leqslant \overline{M}Z_{nk}, \;\; \forall k = 1,\ldots,K_n, n \in N, \quad (8)$$

$$\sum_{k}^{K_n} Z_{nk} \leqslant \overline{M}Z\prime_n, \;\; \forall n \in N, \quad (9)$$

$$Z_{nk} \leqslant Z_{n(k-1)}, \;\; \forall n \in N, k = 2,\ldots,K_n, \quad (10)$$

$$X_i^n = \{0,1\}, \;\; \forall i \in S, n \in N, \quad (11)$$

$$Y_{it}^{nk} = \{0,1\}, \;\; \forall i \in S, t \in T, k = 1,\ldots,K_n, n \in N, \quad (12)$$

$$Z_{nk} = \{0,1\}, \;\; \forall n \in N, k = 1,\ldots,K_n, \quad (13)$$

$$Z\prime_n = \{0,1\}, \;\; \forall n \in N, \quad (14)$$

$$E_i = \beta_0, \;\; \forall i \in O. \quad (15)$$

Objective (1) is to minimize the annual total electric bus recharging system operating costs, which are made up of: deadheading travel costs and recharging costs (expressed in the first term), recharging waiting costs (the second term), charger costs (the third term) and charging station costs (the last term). The recharging waiting costs are the costs incurred while waiting for a recharging activity. Note that an electric bus has to wait at the transit center for a fixed trip-interval regardless of whether it is being recharged. However, the recharging waiting penalty could result in more centralized recharging activities, thus allowing additional time to maintain the chargers. Therefore, it is meaningful to assign a penalty on the recharging waiting time.

Constraint (2) means that the electric bus cannot be recharged at more than one charging station at the same time. Constraint (3) means that the extended driving distance using the remaining energy plus the recharged energy cannot exceed the maximum driving range of the electric bus. Constraint (4) refers to energy conservation. Constraint (5) ensures that the remaining energy in an electric bus is no less than the minimum energy. Constraint (6) defines the relationship between variables Y and X. If an electric bus from trip i is recharged at charging station n (i.e., $X_i^n = 1$), there must be a t and a k, which enable $Y_{it}^{nk} = 1$; otherwise, all $Y_{it}^{nk} = 0$. Constraint (7) ensures that no more than one electric bus can be recharged on each individual charger at any time; in other words, the charging station capacity constraint has to be satisfied. Constraint (8) states the logical relationship between variables Y and Z. If a charger is not being used, there must be no recharging activity on this charger; if at least one recharging activity occurs on a charger, the charger must be used. To denote the logical relationship, a sufficiently large number $\overline{M}$ should be used as the cardinality for recharging activity set A_n (e.g., a total number of recharging activities) to ensure that there are sufficient recharging activities. Similarly, constraint (9) states the logical relationship between variables Z and Z; if a charging station is not used, all chargers at this charging station must be not used; if at least one charger is used in a charging station, the charging station must be used. Constraint (10) imposes an order constraint which states that the kth charger at charging station n cannot be used until the $(k-1)$th charger has been used. Constraints (11)–(14) are binary constraints. Constraint (15) sets an initial range for an electric bus at the depot.

3.3. Model generalization

The proposed framework in Section 3.2 can also be applied to a different general transit network that has a different network configuration, as shown in Fig. 4. The major difference between Figs. 4 and 2 is whether the termini are located at the transit center. In Fig. 4, electric buses depart from the depot in the morning to serve a sequence of scheduled trips and then return to the same depot. Here the transit center is not the start/end point of a trip. However, the layover time at the transit center is still longer than at other stops to enable passengers to change routes. To serve more electric buses, a charging station equipped with fast chargers is deployed at the transit center. The electric buses can only be recharged during the layover time at the transit center.

If the transit center is regarded as the midpoint, each trip can be divided into two half-trips. In a trip-pair, the remaining energy in the electric bus at the transit center plus the recharged energy should be sufficient to enable the bus to complete the second half-trip of the former trip and the first half-trip of the latter trip until returning to the charging station. If these two half-trips are combined, a "new" trip is identified, the distance for which is equal to the original trip. For example, assume that a trip-pair on Route 1 is: $A \rightarrow G \rightarrow A$. The first trip has a start time of 7:00 am and an end time of 7:50 am, and the latter trip has a start time of 8:00 am and an end time of 8:50 am. The arrival times at the transit center for the two trips are 7:20 am and 8:20 am and the layover time at the transit center is 10 min. Therefore, the electric bus can be recharged from 7:20–7:30 am or from 8:20–8:30 am. The "new" trip start time is 7:30 am and the end time is 8:20 am, the same duration as the original trip.

In this way, the transit network in Fig. 4 can be transformed into a similar network to Fig. 2 with adjustments. For example, the remaining energy when an electric bus starts its first "new" trip from the transit center should be equal to the energy at the depot minus the energy consumed traveling from the depot to the transit center. Similarly, the remaining energy when an electric bus completes its last "new" trip in the transit center should be sufficient for the electric bus to return to the depot from the transit center. δ is used to denote the energy consumed by traveling between the depot and the transit center. Eq. (15) is modified and a new constraint added (16). The new framework in 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14 and (15a), (16) can be applied to the transit network shown in Fig. 4.

$$E_i = \beta_0 - \delta, \ \forall i \in O, \tag{15a}$$

$$E_i \geqslant e_{\min} + \delta, \ \forall i \in D. \tag{16}$$

There are also two ways to relax the assumption that each charger has only one outlet. The first is to introduce a new index to label the outlets on each charger. To do this, the binary variable Y_{it}^{nk} is replaced with Y_{it}^{nkl}, where $Y_{it}^{nkl} = 1$ if the electric bus from trip $i \in S$ starts being recharged on the lth outlet of the kth charger of charging station $n \in N$ at time $t \in T$;

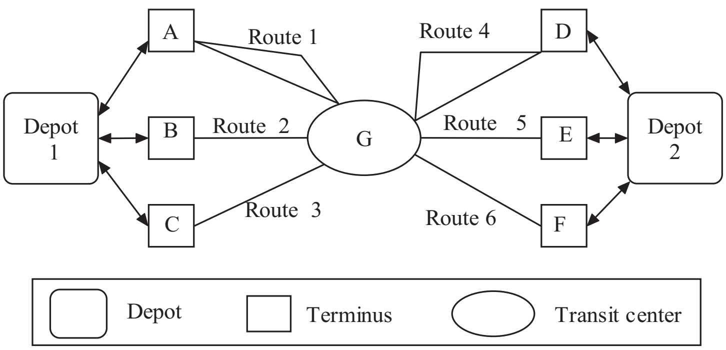


Fig. 4. A transit network at which the transit center is different from the terminus.

0 otherwise. The other is to only label the outlets, but not the chargers; in other words, an outlet is regarded as a new "charger". Compared with the first, the dimension of binary variable Y is reduced and the computation burden decreased. In this paper, the second approach is adopted to relax the assumption that there is only one outlet on a charger.

It is assumed that there are L outlets in a charging station labeled $1, \ldots, L$ in sequence and the binary variable Y_{it}^{nk} is redefined. $Y_{it}^{nk} = 1$ if the electric bus from trip $i \in S$ starts being recharged on the kth outlet at charging station $n \in N$ at time $t \in T$; 0 otherwise, $k = 1, \ldots, L$. The binary variable Z_{nk} is also redefined. $Z_{nk} = 1$ if the kth outlet at charging station $n \in N$ is used; 0 otherwise, $k = 1, \ldots, L$. Accordingly, the scope of k is modified as $k = 1, \ldots, L$ in constraints (7)–(10). Note that constraint (10) guarantees that the kth outlet at charging station n will not be used until the $(k-1)$th outlet has been used. Therefore, the proposed model can be generalized to transit networks where each charger may have multiple outlets.

In addition, the proposed model can also be generalized to a multi-depot transit network. In a multi-depot transit network, electric buses are housed at multiple depots overnight, which they leave in the morning to serve a sequence of scheduled trips, after which they return to the same depots. The difficulty for multi-depot transit networks is to ensure that each electric bus returns to the same depot it left. However, in this study, all electric buses operate according to the previously established timetable, meaning that the assigned trips for each electric bus are predetermined. The task is not to assign scheduled trips to electric buses, but to assign chargers to the electric buses. In a multi-depot transit network, the size of the problem becomes larger, but the structure of the proposed model does not change.

4. Case study

To test the applicability of the proposed optimization model and gain insights into real-world implementation, the optimization model was applied to the transit network in the city of Davis, California, as shown in Fig. 5.

4.1. Network description and data preparation

The transit network in Davis, California is a medium-sized network. There are two transit centers in Davis: the Memorial Union Terminal and the Silo Terminal. A total of 17 routes are connected by these two transit centers, of which B, E, F, G, K, M, P, Q and Z routes share the Memorial Union Terminal transit center, and A, C, D, J, L O, V and W routes share the Silo Terminal transit center. The terminus of each route is located in the corresponding transit center. For each route, a sequence of timetabled trips together with the start time and end time and the stop locations for each trip were obtained from *http://www.gtfs-data-exchange.com/agency/unitrans-davis*.

It was found that 574 timetabled trips were operated by 30 electric buses, of which 265 trips were connected by the Memorial Union Terminal and 309 trips were connected by the Silo Terminal. The trips serviced by each electric bus and the electric buses serviced on each route are listed in Table 1. Two consecutive trips serviced by the same bus were nominated a trip-pair; e.g. (4,18), (18,23) and (5,14). The set of trip-pairs was defined as P. Note that the pairs from the origin depot (denoted as o) to the first trip operated by each electric bus (e.g., (o,4) and (o,5)), and the pairs from the last trip operated by each electric bus to the destination depot (denoted as d) (e.g., (251,d) and (262,d)) were also included in set P. In addition, to calculate the distance of each route, the positions of two neighboring stops on Google Maps were first located using geographic latitude and longitude, after which the paths across each stop were mapped and the accumulated distance on each route along the mapped path measured. Each routeś distance is also shown in Table 1.

For these 574 timetabled trips, the earliest daily start time was 6:30 am, and the latest end time was 11:05 pm, so the transit service duration was 995 min. The earliest start time was set at 0, and the sampling time step was set at one minute, with the latest end time being 995. Table 2 lists the partial trips at both transit centers and the respective start times and end times.

Together with Tables 1 and 2, it can be seen that most trip-intervals in the Davis transit network ranged from 10 to 15 min. As mentioned in Section 2, electric buses in the Davis transit network can be recharged using fast chargers during these trip-intervals, for which one charging station at each transit center was deployed, with each being equipped with 3 homogeneous fast chargers. The 3 chargers at the Memorial Union Terminal charging station were labeled M_1, M_2, and M_3 and the 3 chargers at the Silo Terminal charging station were labeled S_1, S_2, and S_3. The distance and travel time between these two transit centers were 2 km and 10 min. As the termini were located in the corresponding transit centers, the distance between the termini and the corresponding charging station was ignored, so the values for r_{in} and d_{in} can also be determined.

The maximum driving range and the initial range for each electric bus were set at 150 km, and the minimum remaining range, $e_{\min}$, was set at 20 percent of the maximum driving range; i.e., 30 km. The fixed cost per recharging activity c_m, the electricity cost in unit time c_e, the unit waiting cost c_w, and the unit travel cost c_d were set at \$2, \$0.5 per minute, \$0.1 per minute, and 6.62 cents per km, respectively (Li, 2013). Based on an EV charging station infrastructure cost report (Agenbroad and Holland, 2014), the fixed costs per fast charger and per charging station were respectively estimated at \$15,000 and \$10,000. The daily maintenance costs per charger and per charging station were respectively estimated to be \$10. There were 360 operating days per year. According to Davis et al. (2015), as alternative fuel bus vehicles have an average life of 7.8 years, the fixed capital costs were annualized with an assumed 8-year useful life and a 10% interest rate, and the annualized cost factor α was calculated using the economic engineering formula (A/P, 10%, 8) = 0.1874. In addition,

Fig. 5. Transit network in Davis.

the charging duration for each recharging activity, u_n, was set at 5 min. Based on the fast charging technology specifications in Seneca transit network, the charging efficiency θ was set at 7 km/min.

4.2. Baseline results

The proposed model was programmed in AMPL (Fourer et al., 2011) and solved using the commercial solver CPLEX 12.4. All numerical experiments were run on a ThinkPad laptop with 4 GB of RAM and 2.40 GHz of CPU under a Windows 7 environment. There were 9427 constraints and 3,428,510 variables (including 12,453 binary variables) in this problem, which was solved in about 53 s. This section reports on the outcomes of the baseline case study.

The optimal objective was $\$176,230$, which was made up of the annualized charging costs at $\$149,040$, the annualized recharging waiting costs at $\$3,420$, the annualized charger costs at $\$12,822$ and the annualized charging station costs at $\$10,948$. Both transit centers needed to deploy the charging stations and only one charger was used at each charging station. At the Memorial Union Terminal charging station, charger M_1 was required to provide 52 recharging activities. At the Silo Terminal charging station, charger S_1 was required to provide 40 recharging activities. The recharging activities on these two chargers were plotted into Gantt charts, as shown in Fig. 6. From Fig. 6, it can be seen that all recharging activities on charger M_1 were provided to Memorial Union Terminal electric buses, and all recharging activities on charger S_1 were provided to Silo Terminal electric buses. The reason for this was that a traveling cost was incurred if the electric buses were recharged at the more distant charging station. In addition, some trip-intervals were insufficient to allow the electric buses to be recharged at the more distant charging station.

To study the recharging characteristics for the electric buses, all recharging activities for each electric bus were listed, as shown in Table 3. In Table 3, columns 1, 7 and 13 list the $\sharp$ of the electric buses and columns 2, 8 and 14 show the $\sharp$ of trips and the corresponding electric buses that need to be recharged after these trips. Columns 3, 9 and 15 show the remaining range for the corresponding electric buses before the recharging. Columns 4, 10 and 16 show the end times for these trips and columns 5, 11 and 17 show the recharging start times. The remaining columns show the recharging waiting time for each recharging activity.

From Table 3, it can be seen that the remaining range for the electric buses is unnecessarily close to the lowest remaining range limit (i.e., 30 km in this case) when the electric buses start recharging. Electric buses can be recharged even though the remaining range is sufficient for the next few trips as the proposed framework decides when to recharge the electric buses based on the total cost minimization. It can also be seen that most of the recharging waiting time is zero, indicating that all electric buses were recharged immediately after their corresponding trips. Because of the recharging waiting time penalty, the recharging activities should start as soon as the electric buses arrive at the charging stations; however, if two recharging activities are in conflict, the start time of one of these recharging activities needs to be adjusted. In this situation, one electric bus would have to wait until the other recharging activity is finished. For instance, the recharging activity after trip $\sharp$ 104 on charger M_1 is scheduled to start at time 355 for electric bus $\sharp$ 2; however, another recharging activity after trip $\sharp$ 107 on the

Table 1
Route information in Davis transit network.

	Route	♯ of Bus	♯ of trips	Length (km)
Memorial Union Terminal	B	1	4,18,23,38,43,60,63,78,84,100,103,118,123,138,150, 158,163,178,183,198,203,218,223,236,237,250,251	7
	E	2	5,14,24,34,44,54,64,74,85,94,104,114,124,134,144,154, 165,174,184,194,205,214,225,232,238,245,254,258,262	8
	F	3	6,15,25,35,45,55,65,75,86,95,105,115,125,135,145,155, 166,175,185,195,206,215,226,233,239,246,255	10
	G	4	3,21,33,51,68,81,93,111,128,141,153,171,188,201,213,231	9
		5	11,28,41,53,71,83,101,113,131,143,161,173,191, 204,221,242	
		6	13,31,48,61,73,91,108,121,133,151,164,181,193,211, 224,249,252,259,263	
	K	7	7,16,26,36,46,56,66,76,87,96,106,116,126,136,146,156,167, 176,186,196,207,216,227,234,240,247	10
	M	8	8,17,27,37,47,57,67,77,88,97,107,117,127,137,147, 157,168,177,187,197,208,217,228,235,241,248,253	8
	P	9	1,19,39,58,79,98,119,139,159,179,199,219	23
		10	9,29,49,69,89,109,129,148,169,189,209,229,243,256,260,264	
	Q	11	2,20,40,59,80,99,120,140,160,180,200,220	23
		12	10,30,50,70,90,110,130,149,170,190,210,230,244,257,261,265	
	Z	13	12,32,52,72,92,112,132,152,172,192,212	15
		14	22,42,62,82,102,122,142,162,182,202,222	
	A	15	266,284,306,327,352,373,398,421,444, 467,490,513,532,545	16
		16	273,295,317,341,364,385,409,430,455,477,501,524,536,551,561,571	
	C	17	270,282,290,304,312,329,337,350,361,375,381,396,404,419,433, 442,450,465,473,488,496,511,519,530,533,543,546,555,566	6
	D	18	272,281,292,303,314,326,339,349,363,372,383,395,406,418,427, 441,453,464,475,487,499,510,522,529,535,540,549,559,570	11
		19	275,286,297,308,320,331,343,354,366,377,389,400, 412,423,435,447,458,469,481,493,504,516	
		20	268,279,293,310,324,335,355,370,384,401,416,428,446, 463,480,492,508,521,538,550,560,569	
		21	276,287,301,315,332,348,360,379,393,407,424,439,452, 470,485,498,515,527,541,553,564,574	
		22	298,321,344,367,390,413,436,459,482,505	
	L	23	267,285,307,328,353,374,399,422,445,468,491,514	15
		24	274,296,318,342,365,386,410,431,456,478,502,525,537,552,562,572	
	O	25	319,336,359,387,411,432,457,479,503,526	21
	V	26	271,283,291,305,313,330,338,351,362,376,382,397,405,420,434, 443,451,466,474,489,497,512,520,531,534,544,547,556,557,565,567	3
		27	278,289,300,311,323,334,346,357,369,380,392,403,415,426,438, 449,461,472,484,495,507,518	
		28	269,280,294,309,325,340,356,371,388,402,417,429,448, 462,476,494,509,523,539,548,558,568	
		29	277,288,302,316,333,347,358,378,394,408,425,440,454, 471,486,500,517,528,542,554,563,573	
		30	299,322,345,368,391,414,437,460,483,506	

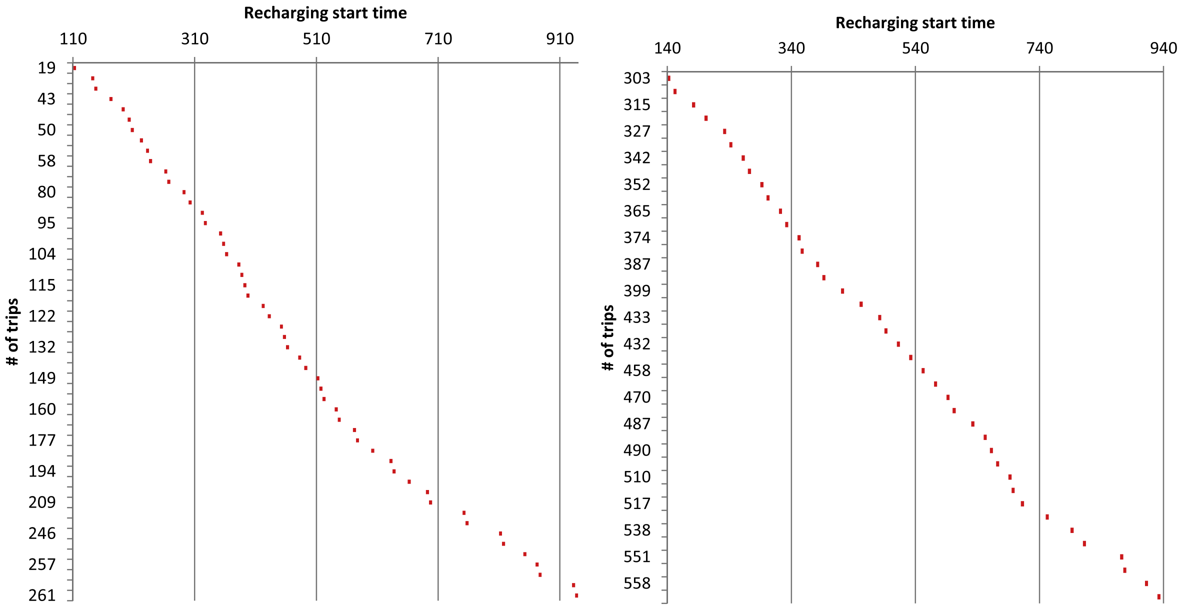
Table 2
Timetabled trips in the Davis transit network.

From Memorial Union Terminal									From Silo Terminal								
i	a_i	b_i	i	a_i	b_i	i	a_i	b_i	i	a_i	b_i	i	a_i	b_i	i	a_i	b_i
1	0	50	11	40	70	21	70	100	266	0	50	276	40	70	286	65	90
2	0	50	12	45	90	22	75	120	267	0	50	277	40	70	287	70	100
3	25	55	13	50	80	23	90	110 268	20	50	278	45	65	288	70	100	
4	30	50	14	55	80	24	90	115	269	20	50	279	50	80	289	75	95
5	30	55	15	55	80	⋮	⋮	⋮	270	30	50	280	50	80	⋮	⋮	⋮
6	30	55	16	55	80	261	880	930	271	30	50	281	55	80	570	940	965
7	30	55	17	55	80	262	905	930	272	30	55	282	60	80	571	940	990
8	30	55	18	60	80	263	940	965	273	30	80	283	60	80	572	940	990
9	30	80	19	60	110	264	940	990	274	30	80	284	60	110	573	965	995
10	30	80	20	60	110	265	940	990	275	40	65	285	60	110	574	965	995

Note: i is the ♯ of trips; a_i and b_i are the start time and end time for trip i.

same charger is also scheduled to start at time 355 for electric bus ♯ 8; therefore, it was necessary for electric bus ♯ 2 to wait 5 min before recharging.

It can also be seen that most electric buses were recharged several times over the day; however, electric buses ♯ 22, 26, 27 and 30 were not recharged as the maximum driving range for these electric buses covered the total driving distance, so recharging was unnecessary. To further study the relationship between the number of recharging activities and the total driving distance, the total driving distance for each electric bus was calculated by multiplying the length of the corresponding route by the number of operated trips, as illustrated in Fig. 7.



(a) The charger M_1 (b) The charger S_1

Fig. 6. Recharging activities on each charger in the Davis transit network.

Table 3
Recharging activities for each electric bus.

B	i	E_i	b_i	t	w	B	i	E_i	b_i	t	w	B	i	E_i	b_i	t	w
1	43	115	170	170	0		209	37	690	695	5		383	64	355	355	0
	103	108	350	350	0		229	49	750	755	5		395	88	390	390	0
2	104	62	355	360	5		243	61	810	815	5		487	35	630	630	0
	114	89	390	395	5		260	50	930	930	0		510	48	690	690	0
	194	60	630	635	5	11	59	58	230	230	0		540	39	810	810	0
	258	31	870	875	5		80	70	290	290	0	19	308	106	150	150	0
3	35	110	140	140	0		120	59	420	420	0		354	97	270	270	0
	95	85	320	325	5		160	48	540	540	0		377	110	330	330	0
	115	100	390	390	0		200	37	660	660	0		458	68	550	550	0
	215	35	690	690	0	12	50	81	200	205	5	20	428	30	490	490	0
	246	30	810	810	0		110	47	380	385	5		446	55	530	530	0
4	51	114	190	190	0		130	59	450	450	0		538	30	790	790	0
5	83	96	300	300	0		149	71	510	510	0	21	315	110	180	180	0
6	61	114	220	220	0		170	83	570	570	0		470	45	590	590	0
	252	32	850	850	0		230	49	750	750	0		498	60	670	670	0
7	36	110	140	145	5		257	38	870	870	0	23	374	60	350	350	0
	96	85	320	320	0		261	50	930	935	5		399	80	420	420	0
	156	60	510	515	5	13	132	45	460	460	0	24	342	90	260	260	0
	196	55	630	630	0		152	65	520	520	0		365	110	320	320	0
8	77	86	260	260	0	14	122	60	430	430	0		525	40	750	750	0
	107	97	355	355	0		142	80	490	490	0		562	30	930	930	0
	177	76	570	575	5	15	327	86	230	230	0	25	387	66	380	380	0
9	19	104	110	110	0		352	105	290	290	0		432	59	510	510	0
	58	93	230	235	5		490	44	660	660	0		457	73	570	570	0
	139	36	480	480	0	16	317	102	200	200	0	28	340	96	240	240	0
	159	48	540	545	5		409	73	450	450	0		494	41	650	650	0
	179	60	600	600	0		501	44	690	695	5		558	31	910	910	0
10	49	81	200	200	0		551	31	870	870	0	29	358	87	300	300	0
	69	93	260	265	5	17	433	60	480	480	0		517	32	710	710	0
	109	82	380	380	0		473	71	600	600	0		554	40	875	875	0
	129	94	450	455	5	18	303	106	140	140	0						

Note: B is the ♯ of electric buses and i is the ♯ of trips. The corresponding electric bus is recharged after trip i. E_i is the remaining range for the corresponding electric bus before recharging. b_i is the end time for trip i. t is the recharging start time. w is the recharging waiting time, where $w = t - b_i$.

From Fig. 7, it can be seen that the number of recharging activities is positively related to the total driving distance as the longer the total driving distance, the more recharging activities are required. This is because of the assumption that the energy consumed is proportional to the driving distance. For electric buses with the same maximum driving range, the longer the distance travelled, the more energy is consumed. As the energy to be recharged each time is fixed, more recharging activities are necessary to replenish the depleted energy. However, electric buses ♯ 22, 26, 27, and 30 do not require any recharging over the day because their total driving distances (i.e., 100, 93, 66, 90 km) are far less than the electric bus driving range (150 km.) From this, it can be concluded that electric buses may not need to be recharged over the day if battery capacities increase in the future.

4.3. Sensitivity analyses

Sensitivity analyses were conducted to understand the impact of the maximum driving range of the electric buses, the recharging duration, the initial range of the electric buses at the depot, the electricity pricing scheme and the electrification rate on the recharging scheduling strategies and annual costs, the results for which are presented in this section.

4.3.1. Maximum electric bus driving range

The sensitivity analysis on the maximum electric bus driving range was conducted under two scenarios. Under the first scenario, the proposed model was run for nine driving ranges (i.e., 100 km, 150 km, 200 km, 250 km, 300 km, 350 km, 400 km, 450 km, and 500 km) while the values for other parameters remained unchanged and were the same as those of the baseline problem. The resulting recharging activities and needed chargers were plotted, as shown in Fig. 8a. Under the second scenario, the recharging duration varied with different driving ranges. In general, the longer driving range yields the longer recharging duration if we want to achieve higher SOC. Considering that the maximum driving range and the recharging duration in the baseline problem were set as 150 km and 5 min, respectively, and most trip-intervals in the bus schedule were 10 min, the model was run for the aforementioned nine driving ranges, and the associated recharging durations were set as 4 min, 5 min, 6 min, 7 min, 8 min, 9 min, 10 min, 11 min, and 12 min, respectively. The values for other parameters were the same as those of the baseline problem. The resulting recharging activities and the needed chargers under the second scenario were shown in Fig. 8b.

It is obviously shown from Fig. 8 that the total recharging activities decreased quickly with an increase in the electric bus driving range under both scenarios, while the number of needed chargers decreased slightly from three chargers to none, which means the number of recharging activities was much more sensitive to the driving range than the number of needed chargers. Compared to Fig. 8a, the number of recharging activities in Fig. 8b decreased more sharply. When the maximum driving range increased from 100 km to 300 km, the total recharging activities reduced by 84.8% and 92%, respectively, under two scenarios. We noticed that when the driving range was less than 150 km, the number of recharging activities in Fig. 8a was less than the corresponding number in Fig. 8b. When the driving range was greater than 150 km, the number of recharging activities in Fig. 8a was greater than or equal to the corresponding number in Fig. 8b. The reason was that the longer recharging duration would lead to less recharging activities for the same driving range. The recharging duration was fixed at 5 min in the first scenario, while it varied from 4 to 12 min in the second scenario. Note that in Fig. 8b, there was no feasible solution when the driving range was 450 km and the corresponding recharging duration was 11 min, the reason behind which was that the recharging duration conflicted with the bus schedule, as most trip-intervals in the bus schedule were 10 min. However, when the driving range was 500 km and the corresponding recharging duration was 12 min, a feasible solution can be obtained, of which neither of recharging activity nor charger was needed. The reason was that the maximum driving range with 500 km was large enough to power an electric bus for the whole day without any recharging. With the developments in battery technology, the maximum driving range is expected to increasingly extend; however, electric buses with larger driving ranges are expected to be very expensive. Therefore, it is vital to strike an trade-off between the cost of the electric buses and the charging costs. In addition, for different maximum driving ranges, the number of needed chargers in Fig. 8a was almost the same as the corresponding number in Fig. 8b, which meant that for the same driving range, the number of needed chargers didn't present significant reduction with an increase in the recharging duration.

4.3.2. Recharging durations

The recharging duration was fixed in the proposed model; however, selecting the recharging duration value is crucial to the recharging strategy. In this section, the model was run using different recharging durations. When the recharging duration was less than 3 min or greater than 10 min, there was no feasible solution, so the results in this section examined different scenarios with recharging durations ranging from 3 to 10 min. Infeasible solutions were obtained when the recharging duration was too short because the energy recharged each time was insufficient to power the electric bus before the next recharging activity. Likewise, when the recharging duration was too long, the recharging activities could not be completed within the trip-interval of 10 min, meaning that they conflicted with the bus schedule. The number of recharging activities and needed chargers were plotted, as shown in Fig. 9.

From Fig. 9a, it is obvious that the total number of recharging activities decreased dramatically with an increase in the recharging duration. When the recharging duration increased from 3 to 10 min, the total recharging activities required reduced by about 65% because the energy recharged each time is proportional to the recharging duration; therefore, when the recharging duration was short, to maintain normal operations, an electric bus had to recharge more frequently.

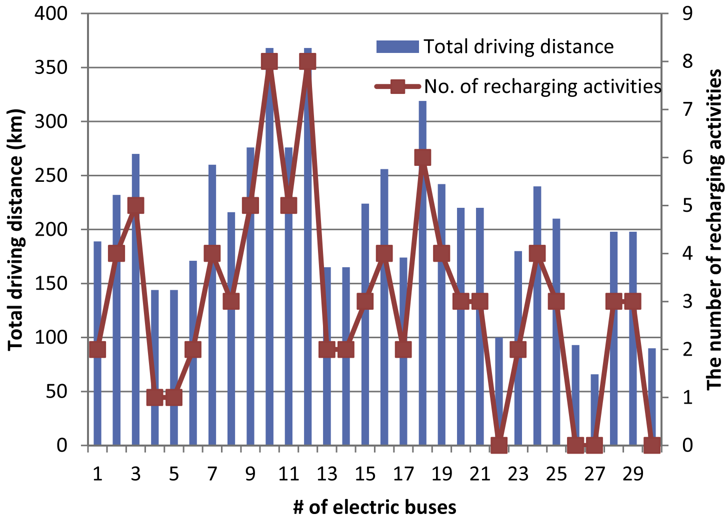
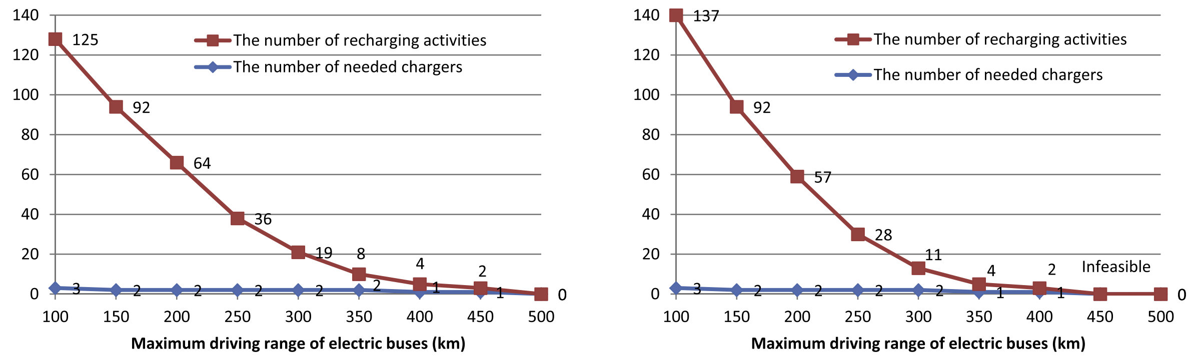


Fig. 7. Total driving distance and number of recharging activities for each electric bus.



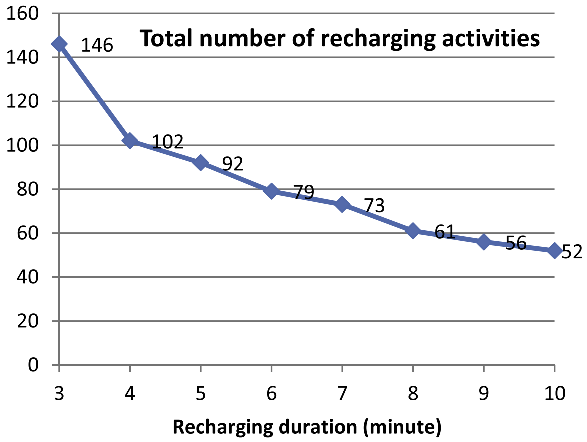
(a) Fixed recharging duration (b) Varied charging durations

Fig. 8. Effect of driving range on charging strategies.

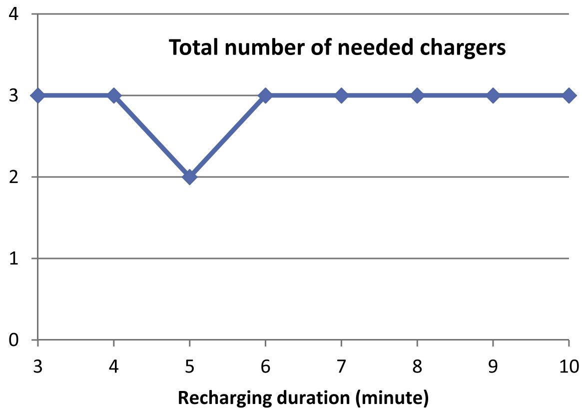
From Fig. 9b, it can be seen that the total number of needed chargers remained at three for all recharging durations except for those under 5 min, when only two chargers were needed. When the recharging duration is shorter, additional recharging activities are required, so additional chargers are needed; however, when the recharging duration is greater than 5 min, one additional charger is still needed even though less recharging activities take place. When the recharging duration is shorter, the buses can be recharged more flexibly as there are more candidate recharging start time nodes. As most trip intervals were 10 min in this case, when the recharging duration was 5 min, two recharging activities could be scheduled within a trip interval (one starting at the beginning of the trip interval and the other one starting at the sixth minute). When the recharging duration was greater than 5 min, there could be only one recharging activity within a trip interval; therefore, if longer recharging durations are required, additional chargers would be needed to satisfy the recharging demand.

The resulting costs for the different recharging durations were plotted, as shown in Fig. 10.

From Fig. 10, it can be seen that the annual total costs decreased by about 19% when the recharging duration increased from 3 to 5 min. When the recharging duration ranged from 5 to 10 min, the annual total costs varied slightly, indicating that the recharging duration had little effect on overall annual operating costs. As the annual recharging activity costs account for more than 80% of the annual total costs, managers need to focus cost reductions on this area. Recharging activity costs are made up of charger operating costs and electricity costs. As charger and clean energy technology develop, it is expected that these costs will fall in the near future. Compared with the other costs, the recharging waiting cost was found to be relatively insignificant, primarily because of the transit network characteristics. In this case, as the timetable was fixed, all electric buses had to operate according to the timetable, meaning that they arrived at the charging stations in certain time nodes. Accordingly, the candidate recharging start time nodes were limited. To maintain normal operations, as an electric bus was not permitted to wait too long time for a recharging activity, additional chargers were needed to satisfy the recharging demand. Further, as the electric buses had to remain in the transit centers during the trip intervals regardless of whether they needed to be recharged, the recharging waiting cost was not a "real" wait time cost. In this paper, as the recharging



(a) Recharging activities



(b) Needed chargers

Fig. 9. Effect of recharging duration on charging strategies in the Davis transit network.

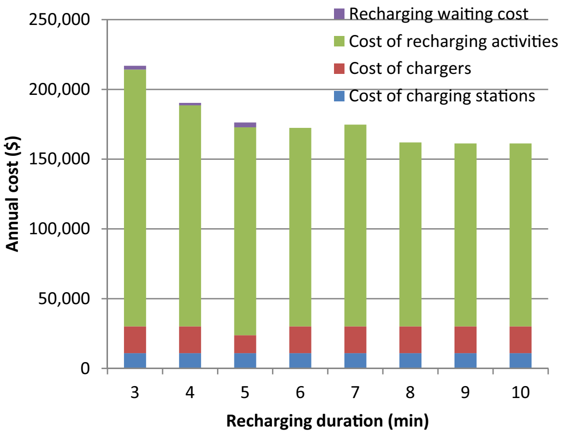


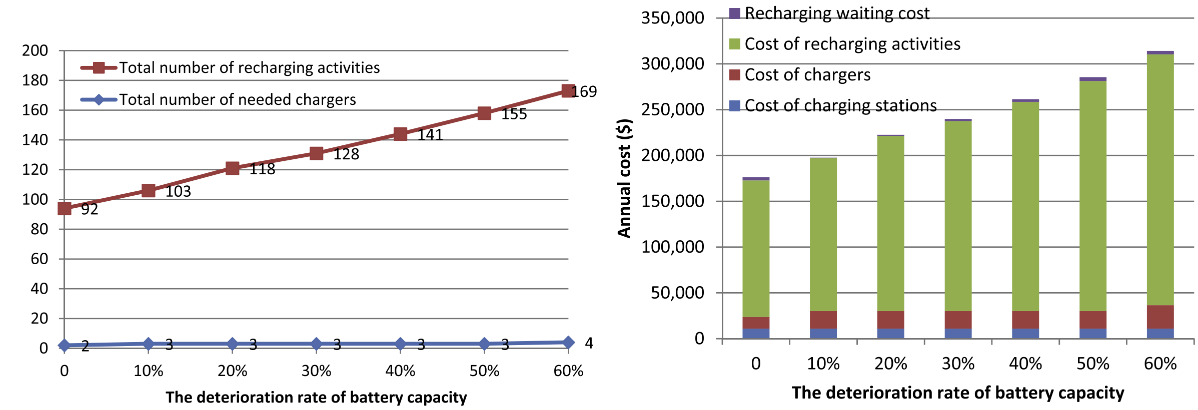
Fig. 10. Effect of recharging duration on operating costs.

waiting penalty implies that the recharging activities need to start as early as possible; the start time nodes are more centralized, allowing additional time for maintenance.

4.3.3. Initial range of the electric buses at the depot

In Section 4.1, the initial range of the electric buses at the depot was assumed to be identical to maximum driving range. Considering the deterioration in battery capacity over time, the model was run using different deterioration rates (i.e., 10%, 20%, ..., 100%). As the maximum driving range was 150 km, the corresponding initial ranges at the depot were 135 km, 120 km, 105 km, 90 km, 75 km, 60 km, 45 km, 30 km, 15 km and 0 km, respectively. As the charging stations were not located at the depot, when the deterioration rate was greater than 60%, the initial range was insufficient to power the bus before the next recharging activity. Therefore, scenarios in which the deterioration rates ranged from 10% to 60% were chosen, the results for which are shown in Fig. 11.

From Fig. 11a, it can be seen that the total number of recharging activities increased steadily with an increase in the deterioration rate. When the deterioration rate increased from 0 to 60%, the total recharging activities increased by nearly 84%, because the buses had to complete all scheduled trips regardless of the initial range at the depot. When the deterioration rate was high, the initial range at the depot was very low, meaning that additional recharging activities were required. Compared to the total recharging activities, the number of needed chargers also increased from two to four. From Fig. 11b, it can be seen that the annual total costs also increased steadily as the deterioration rate increased. However, only the recharging activity costs increased steadily while other costs varied slightly with the increase in the deterioration rate; as explained in Fig. 11a. From the analysis above, it is apparent that the total recharging activities are sensitive to the battery deterioration rate. In this case, as the cost per recharging activity was fixed, the annual cost of the recharging activities was shown to be sensitive to the battery deterioration rate.



(a) Recharging strategies (b) Annual costs

Fig. 11. Effect of initial range on recharging strategies and annual costs in the Davis transit network.

4.3.4. Electricity pricing scheme

The costs for one recharging activity in this paper is made up of both fixed and variable costs, with the fixed costs related to the charger operations and the variable costs related to the cost of electricity. Normally, electricity unit prices remain unchanged over a day; however, in some cases, utility companies adopt variable electricity pricing schemes to reduce electricity demand fluctuations.

In this section two different scenarios are discussed. The first is the baseline case presented in Section 4.1, in which the electricity cost in unit time, c_e, was set at \$0.5 per minute. In the second scenario, a varied electricity pricing scheme was adopted, with c_e fluctuating over time. It was assumed that c_e was \$1 per minute from 11:00 to 13:00, and 19:00 to 21:00, and \$0.2 per minute in other periods. Note that in these examples, 11:00, 13:00, 19:00, and 21:00 were respectively denoted by the time nodes 270, 390, 750, and 870. Let T_1 denote the period of a higher electricity price, $T_1 = [270, 390] \cup [750, 870]$. Let T_2 denote the period of a lower electricity price, $T_2 = [0, 270) \cup (390, 750) \cup (870, 995]$. The term $(\sum_{i\in S}\sum_{n\in N}(c_o + c_e u_n)\overline{D}X_i^n)$ was modified in the objective function to: $\sum_{k=1}^{3}\sum_{n=1}^{2}\sum_{(i,j,t)\in A_1|t\in T_1}(2 + 1 \times 5) \times 360 \times Y_{it}^{nk} + \sum_{k=1}^{3}\sum_{n=1}^{2}\sum_{(i,j,t)\in A_1|t\in T_2}(2 + 0.2 \times 5) \times 360 \times Y_{it}^{nk}$. Here, the first term was used to calculate the recharging activity costs during the high-cost period, and the second term was used to calculate the recharging activity costs during the low-cost period.

The revised optimization model was run and the results compared with the baseline results in Section 4.2. The second scenario also required two chargers and 92 recharging activities. However, the recharging activity arrangements were different for the two scenarios, as illustrated in Fig. 12. Compared with the results in Fig. 12, less than half the number of recharging activities was needed in the high-cost period under a varied electricity pricing scheme, even though the total number of recharging activities were the same as for the fixed price electricity scheme. In short, as a varied electricity pricing scheme can transfer recharging activities from the high-cost period to the low-cost period, a varied electricity pricing scheme could assist utility companies reduce the electricity load in peak periods.

4.3.5. Electrification rate

In Section 3, it was assumed that all conventional diesel buses were replaced with electric buses; however, the proposed model could also be applied to partly electrified transit networks. In this section, based on both the economic and social costs, various scenarios with different electrification rates were examined. Scenario 1 was the baseline problem in Section 4.1, in which all diesel busses were replaced with electric buses. In Scenario 2, the buses at the Memorial Union Terminal transit center were assumed to be electric buses and those in the Silo Terminal transit center were assumed to be diesel buses. Similarly, in Scenario 3, the buses at the Memorial Union Terminal transit center were assumed to be diesel buses and those at the Silo Terminal transit center were assumed to be electric buses. In Scenario 4, all buses with a driving distance of more than 250 km were assumed to be diesel buses, and the remaining buses were electric buses. Based on Fig. 7, buses ♯ 3, 7, 9, 10, 11, 12, 16 and 18 were diesel buses and the remaining buses were electric buses. In Scenario 5, all buses in the Davis transit network were diesel buses.

For the last four scenarios, it was assumed that there was a gasoline station deployed at the depot to refuel the diesel buses. The fixed capital costs and the daily maintenance costs for this gasoline station were set at \$10, 000 and \$10, respectively. All diesel buses were fully refueled at the depot and as their driving ranges were large enough to operate for the whole day, there was no need for refueling between trips. Based on Li (2013) and Stockes and Poger (2013), the diesel bus and electric bus were priced at \$300, 000 and \$450, 000 and the daily maintenance costs per diesel bus and electric bus were set at \$10. The fixed costs of the gasoline station and diesel buses were annualized with an assumed 12-year useful life at a 10% interest rate. The annualized factor was obtained using the engineering economic formula (A/P, 10%, 12) = 0.1468. Similarly,

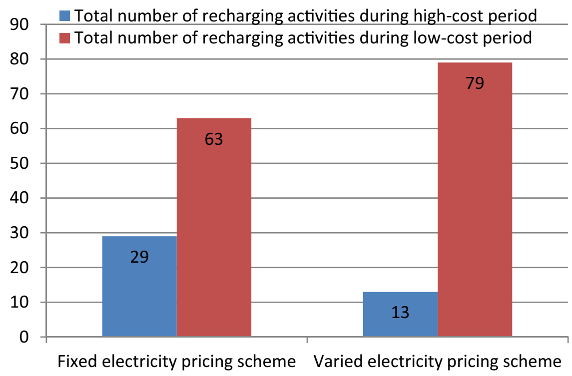


Fig. 12. Effect of the electricity pricing scheme on recharging activity arrangement.

the purchase costs of the electric buses were annualized using (A/P, 10%, 8) = 0.1874. Based on Li (2013), the unit travel cost was set at $0.68 per km for the diesel buses and 6.62 cents per km for the electric buses. The total driving distances for each bus are shown in Fig. 7.

Social costs were used to assess the impact of the tailpipe emissions on the environment, and were measured as a dollar figure for each tonne of each end-use pollutant type ($/tonne). Table 4 presents the rates for the major tailpipe emissions and the associated social costs for the diesel buses. The emissions rates were based on Li (2013) and the social costs were based on NERA Economic Consulting (Harrison et al., 2008). The social costs of the electric buses were considered to be zero in this study as battery-powered electric buses produce no zero tailpipe emissions.

The model was run for these five scenarios, and the resulting costs for each scenario are shown in Fig. 13. In Fig. 13, the station operating costs were all costs incurred at the charging/gasoline stations, and were made up of the charging/gasoline station construction and maintenance costs, charger purchases, installation and maintenance costs, recharging activity costs and recharging waiting time costs. Bus fleet costs consisted of the purchase and maintenance costs for all electric/diesel buses.

Table 4
Major tailpipe emissions rates and associated social costs for diesel buses.

Emission type	NO_x	PM	CO_2
Emission rate (g/km)[a]	17.8334	0.1988	1,739.839
Social cost ($/tonne)[b]	6,700	306,500	22

[a] From Li (2013).
[b] From Harrison et al. (2008).

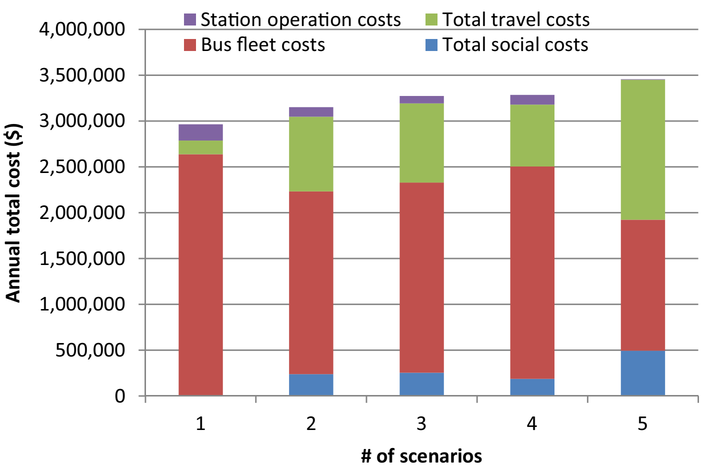


Fig. 13. Annual total costs under various scenarios at different electrification rates.

From Fig. 13, it can be seen that the annual total costs varied slightly for the different scenarios. Of these five scenarios, Scenario 1 had the lowest annual total costs and Scenario 5 had the highest, with Scenario 1 saving about $\$500,000$ annually compared to Scenario 5. Bus fleet costs accounted for about 89% of total costs in Scenario 1 and about 40% in Scenario 5, primarily because of the high cost of electric buses. The total travel costs accounted for about 44% in Scenario 5 and only 5% in Scenario 1, which was due to the much lower unit costs for electric buses compared to diesel buses. The total social costs in Scenario 5 were also the highest of all five scenarios because conventional diesel bus transit networks result in a high level of pollutants. Compared with Scenarios 2 and 3, the total cost of Scenario 4 was a little higher. In Scenario 4, the total driving distance for each diesel bus was greater than the total driving distance for each electric bus, the purchase costs for an electric bus were much higher than for a diesel bus, but the unit travel costs for the diesel buses was higher than for the electric buses; therefore the electric buses in Scenario 4 were not made full use of. The above analyses showed that the higher the electrification rate, the lower the annual total costs of the transit network.

The use of alternative fuel transit vehicles has been encouraged by governments in many countries. For instance, in the United States, electric bus purchases receive a Federal tax credit of up to $\$7,500$ (Davis et al., 2016). Further, as battery technology develops further, it is expected that electric bus prices will begin to fall. Therefore, it was found to be more economical and more environmentally friendly to use electric buses rather than diesel buses in the Davis transit network.

5. Conclusions

In this paper, an optimization charging scheduling framework for electric buses in an urban public transit network was proposed. A mixed integer programming optimization model which considered both the transit bus network configuration and the technical characteristics of fast charging systems was developed. The proposed optimization framework minimized the total costs of operating an electric bus recharging system and determined the planning decisions (i.e., location and capacity of the charging stations) and operational decisions (i.e., recharging schedules). The proposed optimization framework was demonstrated using a real-world transit network based in Davis, California. The results provide decision-makers with charging station locations, the number of needed chargers, the recharging schedule for the electric buses, as well as the corresponding costs. Sensitivity analyses indicated that the total number of recharging activities decreased quickly with an increase in the electric bus maximum driving range and the recharging duration. Compared to the recharging activities, the number of needed chargers varied slightly for different ranges and recharging durations. The battery deterioration rate at the depot was also found to affect the recharging strategies and annual total costs. More specifically, both the recharging activities and annual total costs increased steadily with an increase in the battery deterioration rate. Electricity pricing schemes were also found to affect electric bus recharging times; therefore, utility companies could reduce electricity loads in peak periods by adopting a varied electricity pricing scheme. The annual total costs under various diesel bus to electric bus transition scenarios were also examined, from which it was found that it was more economical and more environmentally friendly to utilize electric buses than diesel buses in the Davis transit network. The baseline results and sensitivity analyses provided the transit agency in Davis with comprehensive guidance for utilizing electric buses and developing a fast recharging system. Other interested transit agencies could also gain managerial insights from Davis, California.

There are several possible extensions to this study. First, the recharging duration was fixed in the proposed framework. However, recharging durations could be affected by many factors (e.g., remaining energy, bus capacity, length of the next trip). For the future study, the model will be revised so as to determine the optimal recharging duration. Second, the relationship between the energy recharged and the recharging duration was approximated with a linear function in this study. In the future, the non-linearity of the charging behavior will be considered. For instance, it is well believed that the first 80% SOC of recharging takes much shorter time than the last 20% SOC (Sweda et al., 2016). This is another policy question of the optimal recharging duration for the bus operation. Third, a charger can have multiple outlets so that more than one electric bus can be recharged simultaneously. This realistic consideration could result in more cost effective solutions; therefore, this model could be modified to reflect this change. Fourth, traffic conditions were not considered in the proposed model. In a real transit network, however, as traffic conditions are often very complex and can affect bus speeds and energy consumption, it is necessary to consider those factors in future research.

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