Part A

- The unobservable variable is whether or not it is raining.
- The observable varible is whether or not we see the director carrying an umbrella.
- Dynamic model:
$$P(X_t|X_{t-1}) = \begin{pmatrix} 0.7 & 0.3 \\ 0.3 & 0.7 \end{pmatrix} \tag{1}$$
Observation model:
$$P(E_t|X_t) = \begin{pmatrix} 0.9 & 0 \\ 0 & 0.2 \end{pmatrix} \tag{2}$$
- We assume that this is a first-order Markov process, meaning that the probability of rain today only depends on whether or not it rained yesterday. This would not do in the real world, but is sufficient for this example.

  We assume that the transition process is stationary. Since weather is controlled by the same physical laws every time this is an reasonable assumption.

  Since we base our sensor values on the current state variables, we also make a sensor Markov assumption.

Part B

```
$ ./forward_backward.py
1 [ 0.81818182  0.18181818]
2 [ 0.88335704  0.11664296]
3 [ 0.19066794  0.80933206]
4 [ 0.730794  0.269206]
5 [ 0.86733889  0.13266111]
```

From the table above we can se that $\boldsymbol{P}(\boldsymbol{X}_2|\boldsymbol{e}_{1:2}) \approx 0.883$ and that $\boldsymbol{P}(\boldsymbol{X}_5|\boldsymbol{e}_{1:5}) \approx 0.867$.

Part C

As we can see above, we got the result of $\boldsymbol{P}(\boldsymbol{X}_1|\boldsymbol{e}_{1:2}) = [0.883, 0.117]$

The backward messages looks like this:

```
$ ./forward_backward.py
(..)
Backward:
5 [1  1]
4 [ 0.69  0.41]
3 [ 0.4593  0.2437]
2 [ 0.090639  0.150251]
1 [ 0.06611763  0.04550767]
0 [ 0.04438457  0.02422283]
```

The Viterbi algorithm pseudocode:

Algorithm 1 Viterbi

function VITERBI(O, S, Π, Y, A, B)
  for each state $i \in \{1, 2, \ldots, K\}$ **do**
    $T_1[i, 1] \leftarrow \pi_i B_{iy1}$
    $T_2[i, 1] \leftarrow 0$
  end for
  for each observation $i \in \{2, 3, \ldots, T\}$ **do**
    for each state $j \in \{1, 2, \ldots, K\}$ **do**
      $T_1[j, i] \leftarrow B_{jyi} \cdot \max_k (T_1[k, i-1] \cdot A_{kj})$
      $T_2[j, i] \leftarrow arg \max_k (T_1[k, i-1] \cdot A_{kj})$
    end for
  end for
  $z_T \leftarrow \arg\ \max_k (T_1[k, T])$
  $x_T \leftarrow s_{z_t}$
  for $i \leftarrow T$, T-1,...,2 **do**
    $z_{i-1} \leftarrow T_2[z_1, i]$
    $x_{i-1}, \leftarrow s_{z_{i-1}}$
  end for
  return X
end function