I Counting and basic laws of probability

I.1 5-card Poker Hands

a) The number of 5-card hands in a deck of 52 cards is given by $\binom{52}{5} = 2,598,960$ possible hands.

b) The probability of each event is $\frac{1}{\binom{52}{5}} \approx 0.00000038$.

c)
- A royal straigth flush can only be achieved in 4 different ways. The probability is then $\frac{4}{\binom{52}{5}} \approx 0.0000015$.
- The probability of drawing a 4 of a kind is $\frac{\binom{13}{1}\times\binom{4}{4}\times\binom{12}{1}\times\binom{4}{1}}{\binom{52}{5}} \approx 0.00024$.

I.2 Two cards in a deck

a) The probability that drawing two cards constitutes a pair is $\frac{3}{51} \approx 0.059$.

b) If they are of different suits, the probability is $\frac{3}{39} \approx 0.077$.

I.3 Conditional Probability

1. $P(A \mid B) > P(A) \Rightarrow \frac{P(B|A)P(A)}{P(B)} > P(A) \Rightarrow P(B \mid A)P(A) > P(A)P(B) \Rightarrow P(B \mid A) > P(B)$ So, yes. if the occurrence of B makes A more likely, then the occurrence of A makes B more likely.

2.
$$P(R=0) = \frac{P(R=0 \mid S=1) \times P(S=1)}{P(S=1 \mid R=0)} \tag{1}$$

First we find $P(R=0)$ to use in bayes rule in the next step since we were not given this value.

$$P(S=0 \mid R=0) = \frac{P(R=0 \mid S=0) \times P(S=0)}{P(R=0)} \tag{2}$$

Replacing $P(R=0)$ from Equation 1 gives us this:

$$P(S=0 \mid R=0) = \frac{P(R=0 \mid S=0) \times P(S=0) \times P(S=1 \mid R=0)}{P(R=0 \mid S=1) \times P(S=1)} \tag{3}$$

Which, when all known values are inserted looks like this:

$$\frac{P(S=0 \mid R=0)}{P(S=1 \mid R=0)} = 3 \tag{4}$$

Which is the same as this:

$$P(S=0 \mid R=0) = 3P(S=1 \mid R=0) \tag{5}$$

And since $P(S=0 \mid R=0) + P(S=1 \mid R=0) = 1$, we have:

$$P(S=0 \mid R=0) = 0.75 \tag{6}$$

II Bayesian Network Construction

Here is a list of some conditional independencies:

- Religion and history of illness.
- Fish-eating habits, fiber-eating habits and drinking habits.

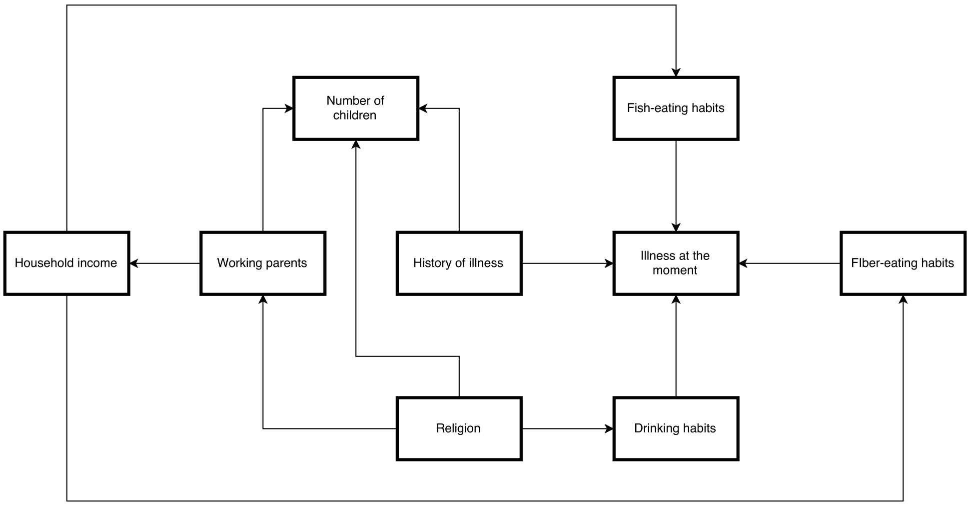

Figure 1: Bayesian network.

- History of illness, fish-eating habits, fiber-eating habits and drinking habits.
- Number of children and Illness at the moment.

I think the results are reasonable, as they model how I perceive the real world. One could argue that there should be drawn more dependencies. If a person eats a lot of fish, they are also probably eating a lot of fibers since they are both healthy. And all the habits can affect the history off illness, which could make one of the parents unable to work.

III Bayesian Network Application

I have choosen to solve this problem using GeNIe, although I have seen the problem before and know that you should always alter your choice.

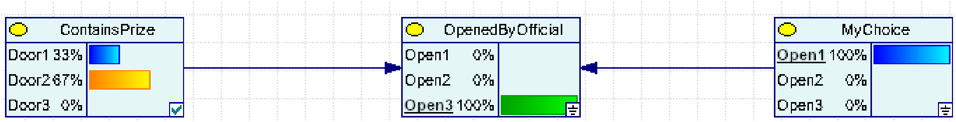

Figure 2: Example probabilites when I choose Door 1 and the official choose door 3.

As you can see, the probability for changing door when asked gives you a probability of $\frac{2}{3}$ instead of your original $\frac{1}{3}$.

ContainsPrize	Door 1			Door 2			Door 3		
MyChoice	1	2	3	1	2	3	1	2	3
Open 1	0	0	0	0	0.5	0	0	1	0.5
Open 2	0.5	0	1	0	0	0	1	0	0.5
Open 3	0.5	1	0	1	0.5	1	0	0	0

Table 1: The probabilities in OpenedByOfficial.