Project 7
MFE 405: Computational Finance
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This is a summary of the project for data visualisation, for detail implementation and result, please refer to the print out of the program

Qn1, all implementation in C++
**Explicit Finite-Difference method:**
$\sigma\sqrt{dt}$

| Price | Pay Off | Black Scholes | Error In Percentage |
|---|---|---|---|
| **4** | 5.8017 | 5.80199 | -4.99829E-05 |
| **5** | 4.80183 | 4.80199 | -3.33195E-05 |
| **6** | 3.80195 | 3.80206 | -2.89317E-05 |
| **7** | 2.80462 | 2.80536 | -0.000263781 |
| **8** | 1.84357 | 1.84427 | -0.000379554 |
| **9** | 1.02501 | 1.02443 | 0.000566169 |
| **10** | 0.466006 | 0.464696 | 0.002819047 |
| **11** | 0.172229 | 0.171537 | 0.004034115 |
| **12** | 0.052905 | 0.0524596 | 0.008490343 |
| **13** | 0.0137246 | 0.0136507 | 0.005413642 |
| **14** | 0.00314309 | 0.00310847 | 0.011137312 |
| **15** | 0.000731843 | 0.000634385 | 0.153625953 |
| **16** | 0.000518862 | 0.000117774 | 3.405573386 |

$\sigma\sqrt{(3dt)}$

| Price, | Pay Off | Black Scholes | Error In Percentage |
|---|---|---|---|
| **4** | 5.80121 | 5.80199 | -0.000134437 |
| **5** | 4.80121 | 4.80199 | -0.000162433 |
| **6** | 3.80129 | 3.80206 | -0.000202522 |
| **7** | 2.80478 | 2.80536 | -0.000206747 |
| **8** | 1.84431 | 1.84427 | 2.16888E-05 |
| **9** | 1.02461 | 1.02443 | 0.000175707 |
| **10** | 0.464327 | 0.464696 | -0.000794068 |
| **11** | 0.171789 | 0.171537 | 0.001469071 |
| **12** | 0.0529175 | 0.0524596 | 0.008728622 |
| **13** | 0.0139156 | 0.0136507 | 0.019405598 |
| **14** | 0.00327371 | 0.00310847 | 0.053157984 |
| **15** | 0.00069426 | 0.000634385 | 0.094376443 |
| **16** | 0.00013879 | 0.000117774 | 0.178460441 |

$\sigma\sqrt{(4dt)}$

| Price, | Pay Off | Black Scholes | Error In Percentage |
|---|---|---|---|
| 4 | 5.8012 | 5.80199 | -0.00013616 |
| 5 | 4.8012 | 4.80199 | -0.000164515 |
| 6 | 3.80128 | 3.80206 | -0.000205152 |
| 7 | 2.80472 | 2.80536 | -0.000228135 |
| 8 | 1.84424 | 1.84427 | -1.62666E-05 |
| 9 | 1.02442 | 1.02443 | -9.76153E-06 |
| 10 | 0.464332 | 0.464696 | -0.000783308 |
| 11 | 0.172169 | 0.171537 | 0.003684336 |
| 12 | 0.0528479 | 0.0524596 | 0.007401886 |
| 13 | 0.0139581 | 0.0136507 | 0.022518992 |
| 14 | 0.00323025 | 0.00310847 | 0.03917683 |
| 15 | 0.00068398 | 0.000634385 | 0.078173349 |
| 16 | 0.00013313 | 0.000117774 | 0.130376823 |

**Implicit Finite-Difference method:**

$\sigma\sqrt{dt}$

| Price, | Pay Off | Black Scholes | Error In Percentage |
|---|---|---|---|
| 4 | 5.80121 | 5.80199 | -0.000134437 |
| 5 | 4.80121 | 4.80199 | -0.000162433 |
| 6 | 3.80129 | 3.80206 | -0.000202522 |
| 7 | 2.80475 | 2.80536 | -0.000217441 |
| 8 | 1.84408 | 1.84427 | -0.000103022 |
| 9 | 1.02466 | 1.02443 | 0.000224515 |
| 10 | 0.464875 | 0.464696 | 0.000385198 |
| 11 | 0.172039 | 0.171537 | 0.002926482 |
| 12 | 0.0529019 | 0.0524596 | 0.00843125 |
| 13 | 0.0139218 | 0.0136507 | 0.019859787 |
| 14 | 0.00323378 | 0.00310847 | 0.040312437 |
| 15 | 0.000680477 | 0.000634385 | 0.072656195 |
| 16 | 0.000132919 | 0.000117774 | 0.128593747 |

$\sigma\sqrt{(3dt)}$

| Price, | Pay Off | Black Scholes | Error In Percentage |
|---|---|---|---|
| 4 | 5.80121 | 5.80199 | -0.000134437 |
| 5 | 4.80121 | 4.80199 | -0.000162433 |
| 6 | 3.80129 | 3.80206 | -0.000202522 |
| 7 | 2.80478 | 2.80536 | -0.000206747 |
| 8 | 1.84431 | 1.84427 | 2.16888E-05 |
| 9 | 1.02461 | 1.02443 | 0.000175707 |

| | | | |
|---|---|---|---|
| **10** | 0.464327 | 0.464696 | -0.000794068 |
| **11** | 0.171789 | 0.171537 | 0.001469071 |
| **12** | 0.0529175 | 0.0524596 | 0.008728622 |
| **13** | 0.0139157 | 0.0136507 | 0.019412924 |
| **14** | 0.00327374 | 0.00310847 | 0.053167636 |
| **15** | 0.00069429 | 0.000634385 | 0.09442531 |
| **16** | 0.00013883 | 0.000117774 | 0.178740639 |

$\sigma\sqrt{(4dt)}$

| Price, | Pay Off | Black Scholes | Error In Percentage |
|---|---|---|---|
| **4** | 5.80121 | 5.80199 | -0.000134437 |
| **5** | 4.80121 | 4.80199 | -0.000162433 |
| **6** | 3.8013 | 3.80206 | -0.000199892 |
| **7** | 2.80481 | 2.80536 | -0.000196053 |
| **8** | 1.84441 | 1.84427 | 7.59108E-05 |
| **9** | 1.02437 | 1.02443 | -5.85692E-05 |
| **10** | 0.464052 | 0.464696 | -0.001385852 |
| **11** | 0.172012 | 0.171537 | 0.002769082 |
| **12** | 0.0528786 | 0.0524596 | 0.007987099 |
| **13** | 0.0140392 | 0.0136507 | 0.028460079 |
| **14** | 0.00328428 | 0.00310847 | 0.056558371 |
| **15** | 0.0007077 | 0.000634385 | 0.115574927 |
| **16** | 0.00014131 | 0.000117774 | 0.1998149 |

**Crank-Nicolson Finite Difference method:**

$\sigma\sqrt{(dt)}$

| Price, | Pay Off | Black Scholes | Error In Percentage |
|---|---|---|---|
| **4** | 5.8012 | 5.80199 | -0.00013616 |
| **5** | 4.8012 | 4.80199 | -0.000164515 |
| **6** | 3.80128 | 3.80206 | -0.000205152 |
| **7** | 2.80466 | 2.80536 | -0.000249522 |
| **8** | 1.84391 | 1.84427 | -0.000195199 |
| **9** | 1.02471 | 1.02443 | 0.000273323 |
| **10** | 0.465154 | 0.464696 | 0.000985591 |
| **11** | 0.172196 | 0.171537 | 0.003841737 |
| **12** | 0.0528718 | 0.0524596 | 0.007857475 |
| **13** | 0.0138403 | 0.0136507 | 0.013889398 |
| **14** | 0.00317942 | 0.00310847 | 0.022824734 |
| **15** | 0.000656768 | 0.000634385 | 0.035282991 |
| **16** | 0.000124879 | 0.000117774 | 0.060327407 |

$\sigma\sqrt{(3}dt)$

| Price, | Pay Off | Black Scholes | Error In Percentage |
|---|---|---|---|
| 4 | 5.8012 | 5.80199 | -0.00013616 |
| 5 | 4.8012 | 4.80199 | -0.000164515 |
| 6 | 3.80128 | 3.80206 | -0.000205152 |
| 7 | 2.80469 | 2.80536 | -0.000238829 |
| 8 | 1.84414 | 1.84427 | -7.04886E-05 |
| 9 | 1.02466 | 1.02443 | 0.000224515 |
| 10 | 0.464607 | 0.464696 | -0.000191523 |
| 11 | 0.171946 | 0.171537 | 0.002384325 |
| 12 | 0.0528871 | 0.0524596 | 0.008149128 |
| 13 | 0.0138342 | 0.0136507 | 0.013442534 |
| 14 | 0.00321958 | 0.00310847 | 0.035744273 |
| 15 | 0.00067058 | 0.000634385 | 0.057059987 |
| 16 | 0.00013067 | 0.000117774 | 0.109463888 |

$\sigma\sqrt{(4}dt)$

| Price, | Pay Off | Black Scholes | Error In Percentage |
|---|---|---|---|
| 4 | 5.8012 | 5.80199 | -0.00013616 |
| 5 | 4.8012 | 4.80199 | -0.000164515 |
| 6 | 3.80128 | 3.80206 | -0.000205152 |
| 7 | 2.80472 | 2.80536 | -0.000228135 |
| 8 | 1.84424 | 1.84427 | -1.62666E-05 |
| 9 | 1.02442 | 1.02443 | -9.76153E-06 |
| 10 | 0.464332 | 0.464696 | -0.000783308 |
| 11 | 0.172169 | 0.171537 | 0.003684336 |
| 12 | 0.0528479 | 0.0524596 | 0.007401886 |
| 13 | 0.0139581 | 0.0136507 | 0.022518992 |
| 14 | 0.00323025 | 0.00310847 | 0.03917683 |
| 15 | 0.00068398 | 0.000634385 | 0.078173349 |
| 16 | 0.00013313 | 0.000117774 | 0.130376823 |

Comment:
In general when the put option is in the money and at the money, the error is very small using all of the three method. However, as the stock price goes further up, the option is way out of the money, the error starts to increase for all three method. Also because the option price is way too small (almost zero.)

Based on the data, the error also reduces for Implicit and Crank Nicolson methods in general. But that is not the case for Explicit Finite-Difference method. This might have to do with unconditional convergence for Implicit and Crank Nicolson methods.

## Qn 2.

Explicit Finite-Difference method: $\sigma\sqrt{(dx)}$

```
ds_05 <- read.csv("~/Documents/ucla/Dropbox/Quarter2/Computational Finance/HW/ComputationalFinanceProje
ds_10 <- read.csv("~/Documents/ucla/Dropbox/Quarter2/Computational Finance/HW/ComputationalFinanceProje
ds_125 <- read.csv("~/Documents/ucla/Dropbox/Quarter2/Computational Finance/HW/ComputationalFinanceProj

priceRange <- seq(4,16,1)
colnames(ds_05) <- c("S","Method","type","OptionP")
colnames(ds_10) <- c("S","Method","type","OptionP")
colnames(ds_125) <- c("S","Method","type","OptionP")
color1 <- c("red","blue","yellow")
```

At S = \$10

At ds = 0.25

Call:

EFD = \$0.66231800 IFD = \$0.661739000 CNFD = \$0.662029000

Put:

EFD = \$0.481483000 IFD = \$0.480550000 CNFD = \$0.481011000

At ds = 1

Call:

EFD = \$0.624669000 IFD = \$0.623914000 CNFD = \$0.624292000

Put:

EFD = \$0.440889000 IFD = \$0.439993000 CNFD = \$0.440441000

At ds = 1.25

Call:

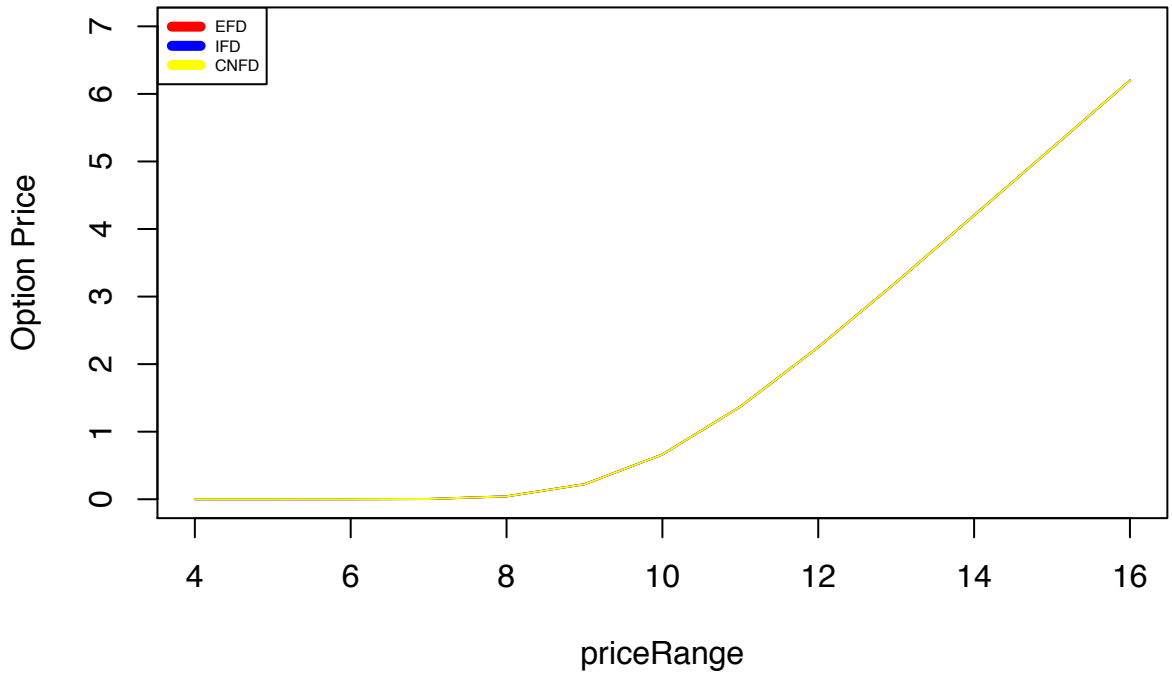
EFD = \$0.597990000 IFD = \$0.597213000 CNFD = \$0.59760200

Put:

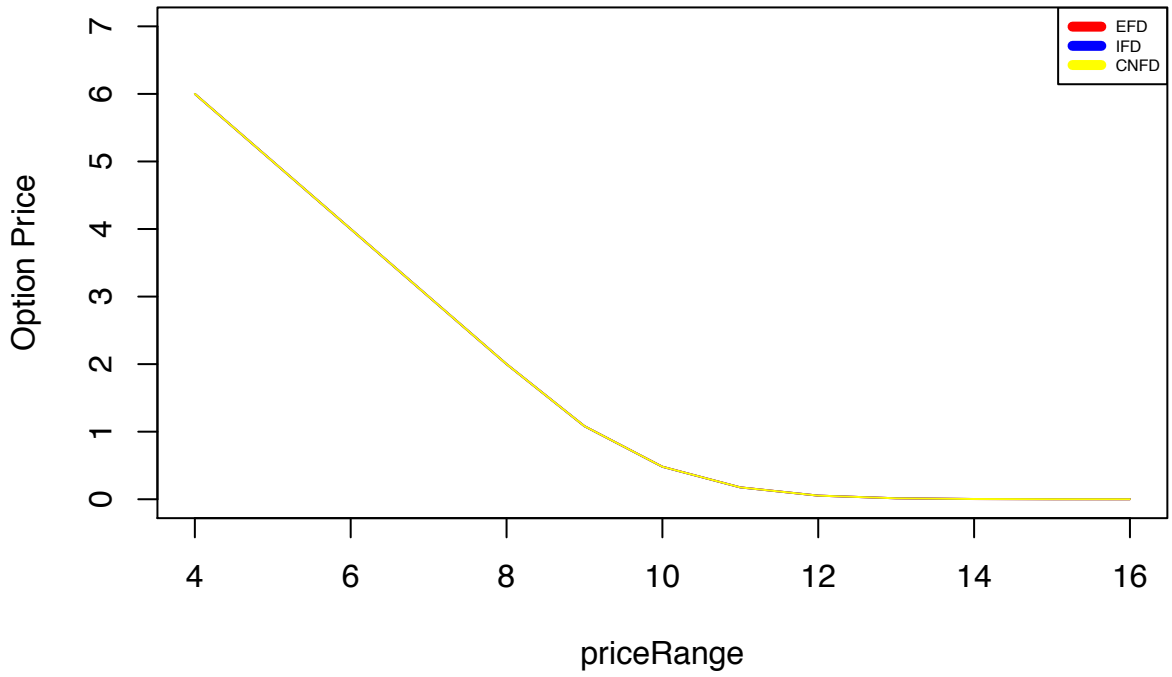
EFD = \$0.416292000 IFD = \$0.415392000 CNFD = \$0.41584200

European Vanilla Call is at 0.66 and Put at 0.46, hence we can observe that the apporximation becomes less accuraet when ds increases.

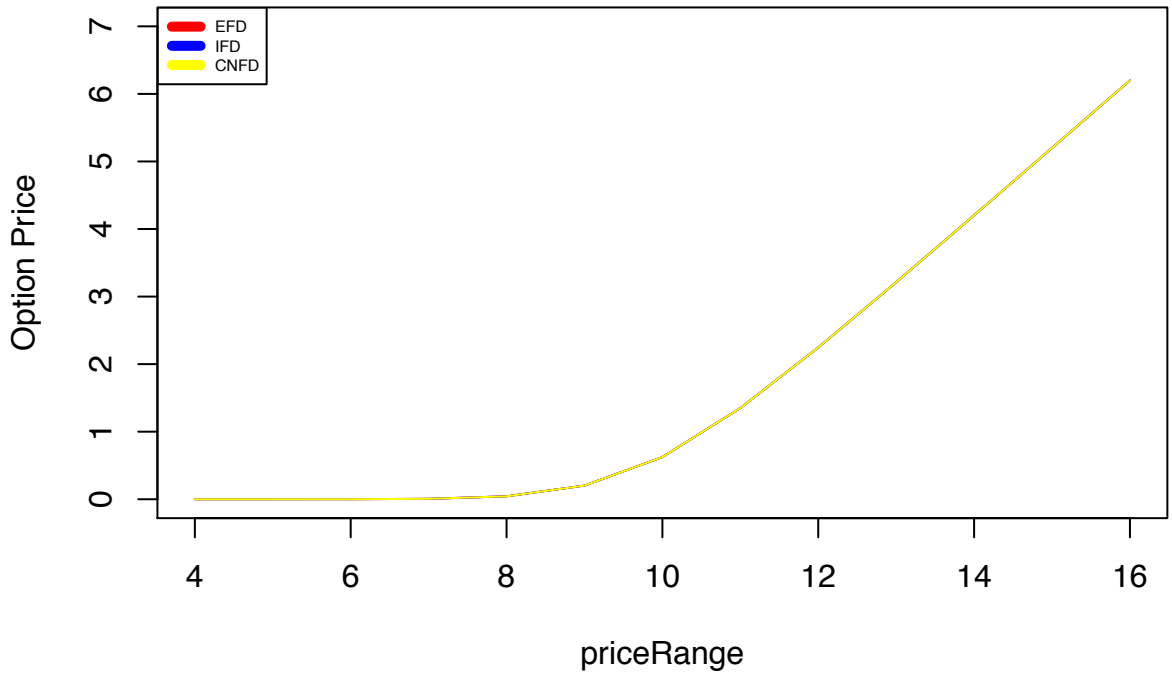
```
# Call ds =0.25
EFD <- ds_05[ds_05$Method=="EFD",]
IFD <- ds_05[ds_05$Method=="IFD",]
CNFD <- ds_05[ds_05$Method=="CNFD",]

EFDCall <- EFD[EFD$type=="Call",]
IFDCall <- IFD[IFD$type=="Call",]
CNFDCall <- CNFD[CNFD$type=="Call",]

plot(x= priceRange, y= EFDCall$OptionP,type = "l",ylim = c(0,7), ylab = "Option Price", main = "dS = 0.5
lines(x=priceRange, y= IFDCall$OptionP, type="l",col="blue")
lines(x=priceRange, y= CNFDCall$OptionP, type="l",col="yellow")
legend("topleft", c("EFD","IFD","CNFD"), col=color1, lwd=5,cex=0.5)
```



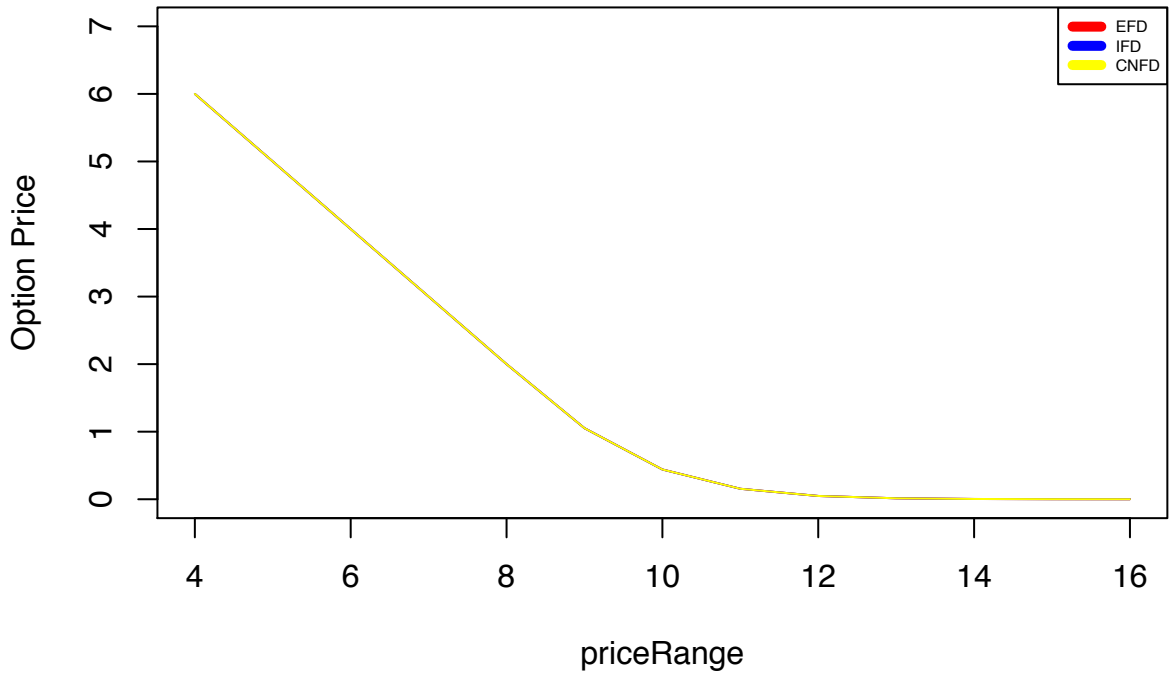

```
# Put ds =0.5
EFDPut <- EFD[EFD$type=="Put",]
IFDPut <- IFD[IFD$type=="Put",]
CNFDPut <- CNFD[CNFD$type=="Put",]

plot(x= priceRange, y= EFDPut$OptionP,type = "l",ylim = c(0,7), ylab = "Option Price", main = "dS = 0.5
lines(x=priceRange, y= IFDPut$OptionP, type="l",col="blue")
lines(x=priceRange, y= CNFDPut$OptionP, type="l",col="yellow")
legend("topright", c("EFD","IFD","CNFD"), col=color1, lwd=5,cex=0.5)
```



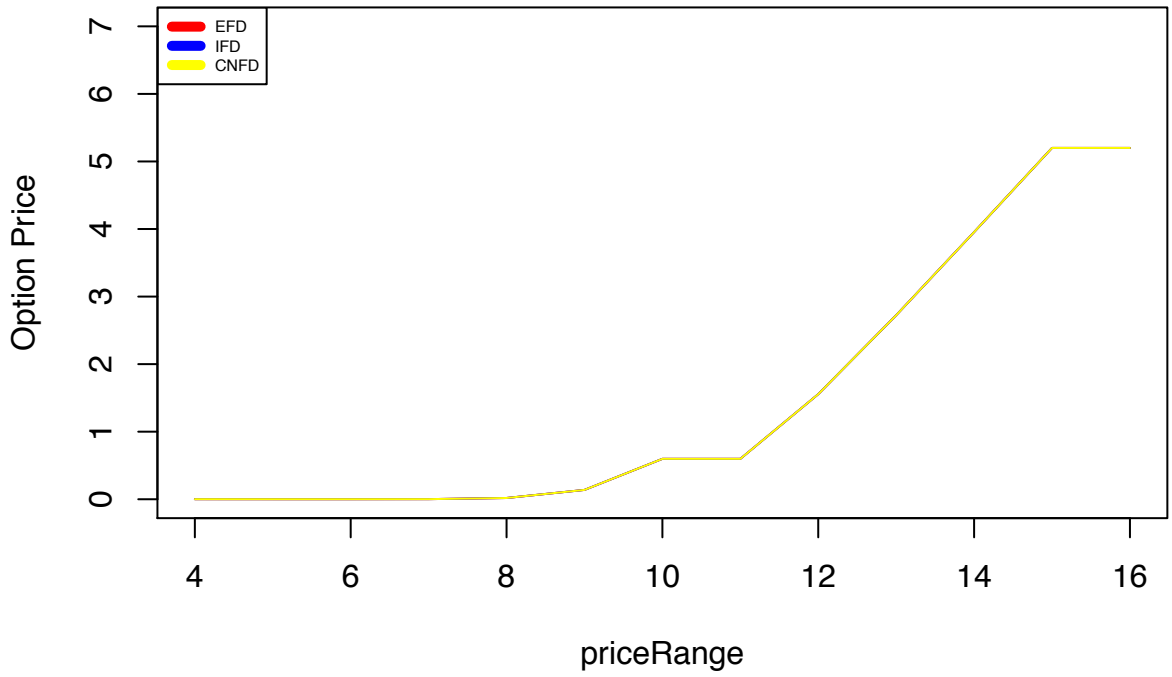

```
# Call ds =1
EFD_10 <- ds_10[ds_10$Method=="EFD",]
IFD_10 <- ds_10[ds_10$Method=="IFD",]
CNFD_10 <- ds_10[ds_10$Method=="CNFD",]

EFDCall_10 <- EFD_10[EFD_10$type=="Call",]
IFDCall_10 <- IFD_10[IFD_10$type=="Call",]
CNFDCall_10 <- CNFD_10[CNFD_10$type=="Call",]

plot(x= priceRange, y= EFDCall_10$OptionP,type = "l",ylim = c(0,7), ylab = "Option Price", main = "dS =
lines(x=priceRange, y= IFDCall_10$OptionP, type="l",col="blue")
lines(x=priceRange, y= CNFDCall_10$OptionP, type="l",col="yellow")
legend("topleft", c("EFD","IFD","CNFD"), col=color1, lwd=5,cex=0.5)
```



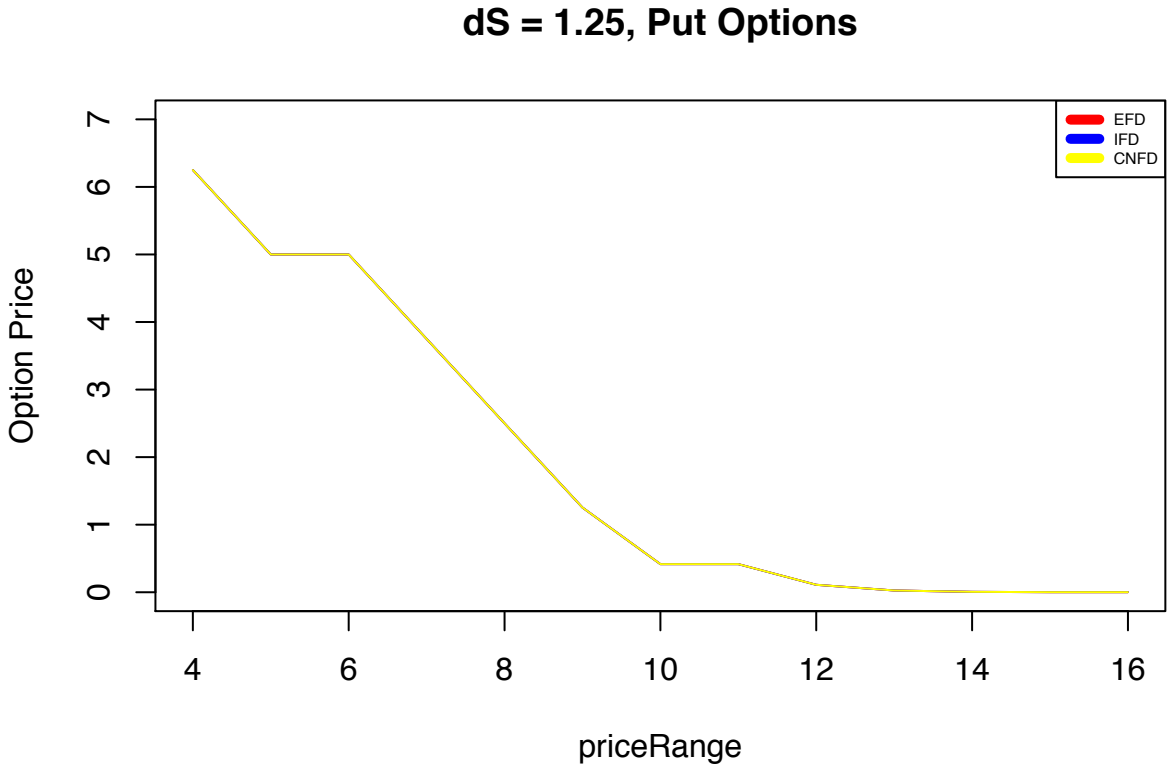

```
# put ds =1
EFDPut_10 <- EFD_10[EFD_10$type=="Put",]
IFDPut_10 <- IFD_10[IFD_10$type=="Put",]
CNFDPut_10 <- CNFD_10[CNFD_10$type=="Put",]

plot(x= priceRange, y= EFDPut_10$OptionP,type = "l",ylim = c(0,7), ylab = "Option Price", main = "dS =
lines(x=priceRange, y= IFDPut_10$OptionP, type="l",col="blue")
lines(x=priceRange, y= CNFDPut_10$OptionP, type="l",col="yellow")
legend("topright", c("EFD","IFD","CNFD"), col=color1, lwd=5,cex=0.5)
```




```
# Call ds =1.25
EFD_125 <- ds_125[ds_125$Method=="EFD",]
IFD_125 <- ds_125[ds_125$Method=="IFD",]
CNFD_125 <- ds_125[ds_125$Method=="CNFD",]

EFDCall_125 <- EFD_125[EFD_125$type=="Call",]
IFDCall_125 <- IFD_125[IFD_125$type=="Call",]
CNFDCall_125 <- CNFD_125[CNFD_125$type=="Call",]

plot(x= priceRange, y= EFDCall_10$OptionP,type = "l",ylim = c(0,7), ylab = "Option Price", main = "dS =
lines(x=priceRange, y= IFDCall_10$OptionP, type="l",col="blue")
lines(x=priceRange, y= CNFDCall_10$OptionP, type="l",col="yellow")
legend("topleft", c("EFD","IFD","CNFD"), col=color1, lwd=5,cex=0.5)
```




```
EFDPut_125 <- EFD_125[EFD_125$type=="Put",]
IFDPut_125 <- IFD_125[IFD_125$type=="Put",]
CNFDPut_125 <- CNFD_125[CNFD_125$type=="Put",]

plot(x= priceRange, y= EFDPut_125$OptionP,type = "l",ylim = c(0,7), ylab = "Option Price", main = "dS =
lines(x=priceRange, y= IFDPut_125$OptionP, type="l",col="blue")
lines(x=priceRange, y= CNFDPut_125$OptionP, type="l",col="yellow")
legend("topright", c("EFD","IFD","CNFD"), col=color1, lwd=5,cex=0.5)
```




Based on the data, the American Call and put price are very similar for all the three methods. And Values become less accurate as the ds increases. On the last graph when ds =1.25 the curve is not as smooth as when ds=0.25 and 1