Modelling and Comparing GARCH models on Hong Kong's Hang Seng Index Volatility

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Introduction

Stock exchange indexes have become useful resources to analyze the economic behavior of countries. Information on the returns and the volatility of these may be helpful in forecasting as well. The Hang Seng index (HSI) is the principal stock index of Hong Kong which compiles stock share prices of the most important firms of the autonomous region as well as mainland China. To follow the behavior of an economy its of paramount importance to explore stock indexes. This paper will provide a handful insight on the modelling of this index.

This paper focuses on making an econometric modelling of the Hang Seng index, Hong Kong's principal stock index, which selects the principal companies in Hong Kong over four industries: Commerce and Industry, Finance, Utilities, and Properties. This time series ranges from January 4, 2010 to February 28, 2018.

The term paper structure will be as following: First it will make a brief description of the data and a brief overview of the recent evolution of the HSI returns and volatility since 2010. Afterwards, the econometric modelling will take place to analyze the returns and volatility of the index diverse models. Finally, interpretations of the results of the models will be provided to decide which model better fits the HSI series. The estimations and tests of this paper were done with R/RStudio statistical software and with the use of specialized libraries for time series analysis (forecast, tseries, rugarch, moments, aTSA and FinTS)

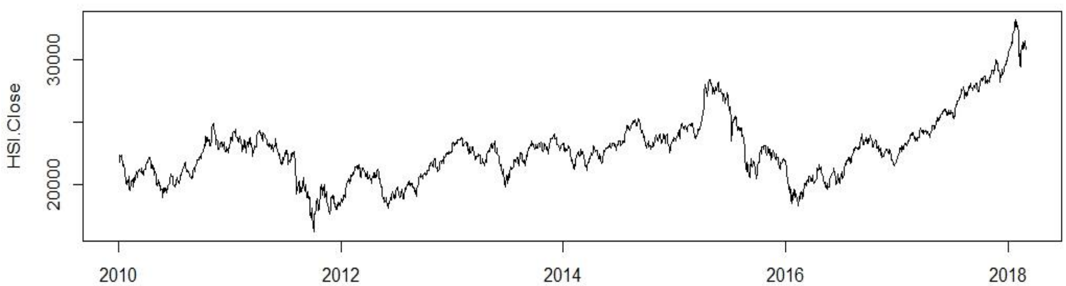
Descriptive Analysis

Before modelling the HSI it is important to understand the evolution of its returns. Therefore, plotting the time series may turn useful for an intuitive approach to the time series. According to graph 1, there are several trends in the price level. The first one on the second half of 2011, from May to September. Low economic growth in the region and after the US debt rating got downgraded to AA+ in August, the HSI, as well as other stock indexes, plummeted to its lowest price.[1] (16250.27). After a slow, but volatile growth in prices, a second downfall occurred in April 2015

[1] "Hong Kong's Hang Seng Posts Biggest 2-Day Fall Since Nov. 2009." Newsmax, August 8, 2011. https://www.newsmax.com/newsfront/2500-alltop-alu-asbnx/2011/08/08/id/406469/.

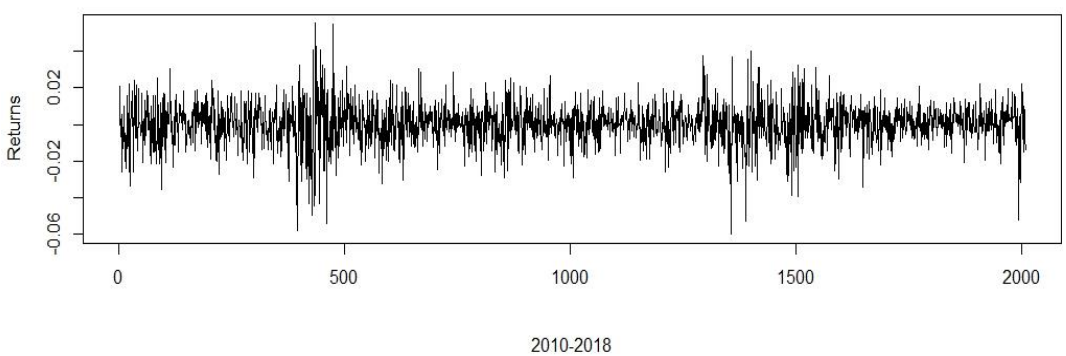
and a trend followed until 2016. Concerns about China's future because of its deaccelerating economy, a continuous drop in commodity prices such as oil and falling demand in the region generated uncertainty amongst investors.[2] After the shock, a price uprising follows until mid-January.

Graph 1: Returns of the Hang Seng Index (HSI)



It is also worth mentioning that volatility clusters exist in in two periods of the HSI. The first one between August 2011 and 2012 and the second one between 2015 and 2016. This clusters can be explained with the same series of events that led to the change in trends mentioned for Graph 1. Negative shocks tend to higher volatility than positive increases in the price.[3] It is useful to plot the logarithmic first differences of the time series in order to have a better understanding of the returns of the HSI series. (Graph 2)

Graph 2: Returns of the Hang Seng Index (^HSI)



Methodology

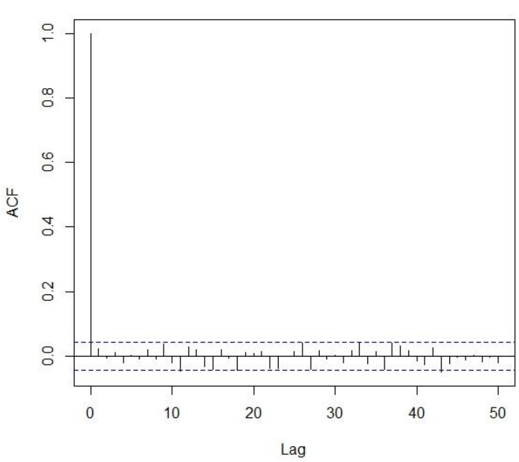
Before modelling the returns of the HSI, its relevant to assure if the time series to be modelled is stationary. After performing an Augmented Dickey-Fuller on the dataset it is confirmed that the series is nonstationary with a p-value = 0.835. Therefore, a logarithmic first-difference should make the index stationary. The ADF

[2] "Asian Markets Suffer Further Losses," January 21, 2016, sec. Business. https://www.bbc.com/news/business-35368484.

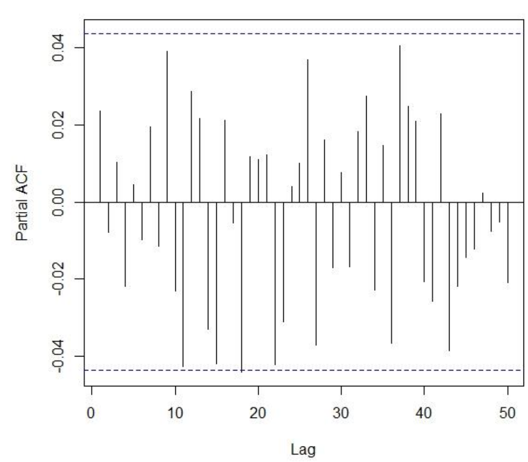
[3] Brooks, C, "Introductory Econometrics for Finance", *Cambridge University Press,* Third edition, Cambridge, 2014, pp 440

confirmed this with a p-value = 0.01. Intuitively, both the ACF and PACF of the time series can confirm that it's a stationary, short-term memory process.

Graph 3: ACF of the log-first-difference of the HSI



Graph 4: PACF of the log-first-difference of the HSI



Both correlograms show a low correlation between lags. In the case of the ACF, an exponential decay is present which means that the series is not time-dependent. As it was expected, the time series converges to a constant mean, variance is finite and correlations between lags disappear after the first lag, which means correlations are not dependent on previous lags.

Modelling the Hang Seng Index returns with ARIMA

Again, with the ACF and PACF functions we can intuitively determine which model should be used to model the HSI. As mentioned before, both plots show that correlations between lags drop significantly after the first lag. As there is not a visible pattern in the autocorrelation we can infer that an ARMA model of order (1,1,0) (i.e. an AR (1) with first-difference) should work best to model the returns of the HSI. Additionally, after looking for the best model according to the Akaike's Information Criteria (AIC) converged to the same result: an AR (1) model. Therefore, the fitted model is:

$$Y_t = \mu + Y_{t-1} + \varepsilon_{t-1}$$

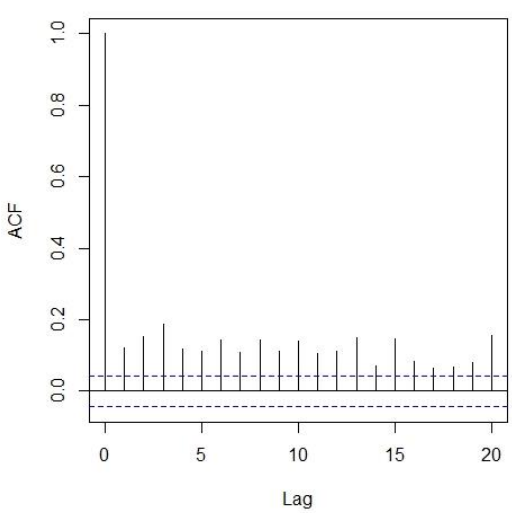
Y_t: Logarithmic first difference of HSI in time t

Even though this model is the best fit for the current data it fails to explain the returns of the Hang Seng Index. Nevertheless, this result was already expected, short term forecasting in a highly volatile index may not bring the best results, nor the best insight of the stock index. Therefore, the approach of the methodology should focus on the volatility instead. It is worth mentioning that the model assumes homoscedasticity for the error term. Nonetheless, in real terms this principle does not hold. Therefore, a process that considers heteroscedasticity among residuals may describe better the residuals volatility.

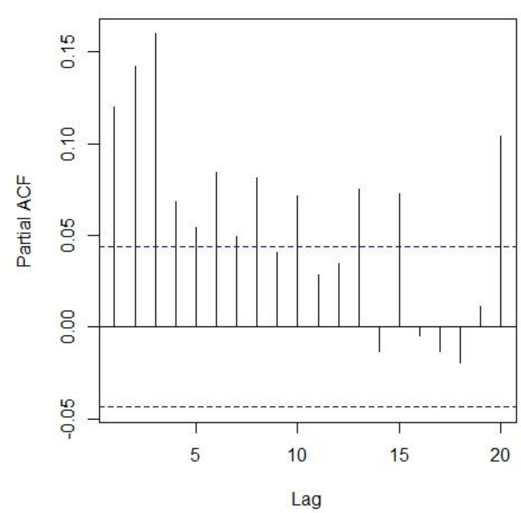
Modelling the Hang Seng Index volatility (GARCH)

After modelling the AR (1) process its necessary to analyze the volatility. First, it is necessary to test the residuals for autocorrelation among residuals. (e.g. white noise) The Box Ljung test fails to reject the null hypothesis at a p-value of 0.06. Once we proved that there is a no autocorrelation between the residuals we need to test if the squared residuals have arch effects. Therefore, we make an ARCH LM test. The test resulted in a p-value = 0, it is conclusive that there is a relation between the squared residuals in past lags and the actual period. (i.e. ARCH effects are present; the coefficients are significantly different from zero).

Graph 5: ACF of Squared Residuals



Graph 6: PACF of Squared Residuals



Both correlograms (ACF and PACF) of the squared residuals provide useful, but rather intuitive, insight about the order of the volatility of the Hang Seng index. The ACF decays exponentially after the first lag and the PACF goes down after the third lag. With this information it is plausible to infer that the best model for the stock index might be a GARCH (3,1). Nonetheless, taking more rigorous examinations (i.e. Akaike's Information Criteria) shows that the best model to work with is rather a GARCH (2,1).

ARIMA (1,1,0)-GARCH (2,1)

After comparing various models of several orders, the more adequate, according to the Akaike's Information Criteria (AIC) turns out to be a GARCH (2,1). Although is not as simple as a GARCH (1,1) it may be the best fitting model for this scenario.

$$Y_t = \mu + a_1 Y_{t-1} + \varepsilon_{t-1}$$

Y_t: Returns on HSI

$$\sigma_t^2 = \omega + \alpha_1 u_{t-1}^2 + \alpha_2 u_{t-2}^2 + \beta_1 \sigma_{t-1}^2$$

$$e_t = \sigma_t \epsilon_t$$

where ε is iid standard normal random variable.

ARIMA (1,1,0)-GARCH (3,1)

In order to explore the relevance of other variables, it may be interesting as well to analyze a model with higher orders to see if lags in longer periods may effectively explain the volatility as well.

$$Y_t = \mu + a_1 Y_{t-1} + \varepsilon_{t-1}$$

Y_t: Returns on HSI

$$\sigma_t^2 = \omega + \alpha_1 u_{t-1}^2 + \alpha_2 u_{t-2}^2 + \alpha_3 u_{t-3}^2 + \beta_1 \sigma_{t-1}^2$$

$$e_t = \sigma_t \epsilon_t$$

where ε is iid standard normal random variable.

TGARCH (2,1)

The previous models assume that changes in volatility are symmetrical. In other words, both a negative change and a positive change in the returns both have the same effect on volatility. In practice this assumption does not hold. Negative changes have a major effect over positive effects. In other words, a negative shock in a financial time series is likely to cause volatility to rise by more than a positive shock of the same magnitude.[4] This is usually associated with risk premiums which value more high-risk assets than low risk ones.[5] In this version of the model, a TGARCH takes into consideration the sign of the previous lags of the residuals (leverage effects) to capture volatility asymmetry, thus generating a more precise model.

$$Y_t = a_1 Y_{t-1} + \varepsilon_{t-1}$$

Y_t: Returns on HSI

$$\sigma_t^2 = \omega + \alpha_1 u_{t-1}^2 + \alpha_2 u_{t-2}^2 + \beta_1 \sigma_{t-1}^2 + \gamma_1 u_{t-1}^2 I_{t-1} + \gamma_2 u_{t-1}^2 I_{t-2}$$

$$e_t = \sigma_t \epsilon_t$$

[4] Brooks, C, "Introductory Econometrics for Finance", *Cambridge University Press,* Third edition, Cambridge, 2014, pp 439

[5] Ibid, 440

where ϵ_t is iid standard normal random variable. I_{t-1} and I_{t-2} are dummy variables that take the value of 1 if u_{t-1}, u_{t-2} are negative and 0 otherwise.

Results

TABLE 1: ARIMA-GARCH Models for the Volatility of the Hang Seng Index

Parameters	GARCH (2,1) ARIMA (1,1,0)	GARCH (3,1) ARIMA (1,1,0)	TARCH (2,1) ARIMA (1,1,0)
mu	0.0004315692*	0.0004357893*	
	(0.000231)	(0.000233)	
ar1	0.03838838*	0.03978684*	0.02351438
	(0.02172)	(0.021842)	(0.020951)
omega	0.000003	0.000003	0.000003
	(0.000002)	(0.000003)	(0.000003)
alpha1	0.000418884	0.0000008	0.00000007
	(0.012454)	(0.012914)	(0.022056)
alpha2	0.07409088***	0.06469503**	0.0172326
	(0.018289)	(0.025602)	(0.026500)
alpha3		0.01643061	
		(0.025325)	
beta1	0.9006055***	0.8916136***	0.9156052***
	(0.01629)	(0.020844)	(0.018128)
gamma1			0.01303929
			(0.034857)
gamma2			0.07474504
			(0.046054)
shape			7.237713***
AIC	-6.234366	-6.233373	-6.277504

Standard error in parentheses

*** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

The models presented were estimated for the 2010 observations of the logarithmic first differences of the Hang Seng Index. The table above exposes the final coefficients. In the first place, it is worth mentioning that the only parameter that remained significant among the three models was beta1, the coefficient linked to the conditional variance on the previous lag. Additionally, its significance was present at the 1% level.

The mean (mu) parameter remained significant at 10% level and remained positive in both models. Likewise, the ar1 coefficient, associated with the value of the index in the previous lag also was significative at the 10% level, there is a marginal change when applied to the TARCH but there is a loss in significance with a p-value = 0.26.

Regarding the parameters of the GARCH models, the omega coefficient remains low and consistent among the three models, although the HSI is presented in logarithmic first difference, the constant variance fails to explain its volatility.

In the same way, the previous lag squared residual of the models remained low and non-significant. The coefficient changes in the TARCH model but still fails to explain the volatility of the index.

Nonetheless, the second lag of the squared residuals turns out to be a strong coefficient that model the volatility. In the GARCH (2,1) remains significant at the 1% level and 5% in the GARCH (3,1). In the case of the TARCH model, this parameter loses significance. This effect may occur because of the additional weight that the asymmetries in the volatility carry. Once adjusted for these effects, it may be noticeable that this coefficient loses relevance.

Adding a third lag to the model didn't bring up new insight over the volatility of the Hang Seng Index. The alpha3 coefficient stays irrelevant. Therefore, a GARCH (2,1) may explain the changes in returns better than any higher order model. It is more effective and parsimonious.

The effects of the asymmetries in volatility might bring a more accurate insight over the stock index. The TARCH coefficients (gamma 1 and gamma 2) shows that once the model considers the asymmetries, the squared residual of the second lag becomes non-significant. In spite of that, this coefficient remains not significant as well. A more complex model may fail to explain the returns and its volatility even though it accounts for more adjustments. In that case we may end up preferring a simpler and more explanatory model (GARCH (2,1)) although the information criteria favors a TARCH process.

Conclusion

The aim of this term paper is to explore and compare various GARCH models and its variants to understand and describe effectively, the returns and the volatility of the Hang Seng Stock Index. In the first place it performed a brief description of the time series by describing trends and the main events that affected its behavior over time. Then it evaluated several GARCH models to find the better explanation for these fluctuations. Finally, it concludes that a GARCH (2,1) process better understands the evolution of the HSI.

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