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Financial Econometrics Assignment
Group A-Team 12

Abstract

This assignment sets out to analyse 2,768 observations of adjusted daily returns for Starbucks. The dataset is a time series; therefore, we are observing a single individual at multiple time periods: from the last quarter of 2008 to the start of the first quarter of 2019.

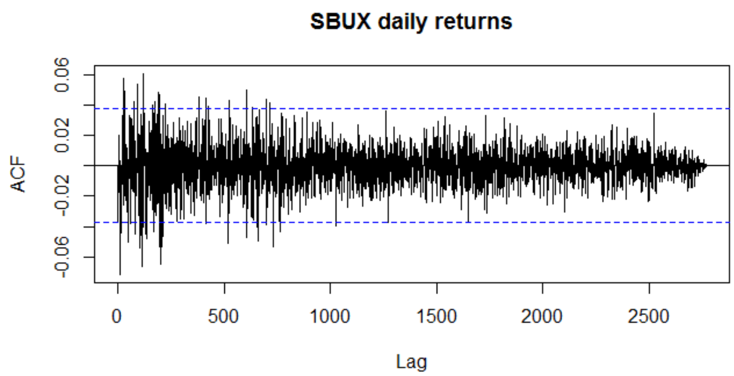
In order to firstly deduce whether this dataset presents the features in line with the seven stylised facts, all the data used is in the functional form of daily log-returns. Therefore, throughout this assignment, all graphs and tables have been computed for the log-returns, even if not explicitly stated.

We will start by firstly, observing stationarity of returns, followed by asymmetry and heavy tails, high frequency non-Gaussianity, autocorrelation, volatility clustering and ARCH effect, and finally the leverage effect. In addition, questions two will use the Fama and French factors in order to estimate the Capital Asset Pricing Model, and so as to discuss the characteristics of the coefficients. Followed by the third question, which sets out to estimate at two different ARCH and GARCH specifications.

Finally, we have applied a similar analysis to two sub-samples from the same dataset set used through Questions 1 to 3. This is in order to compare any differences or similarities with the observed characteristics of the daily returns for the whole period.

Question 1
Descriptive Statistics

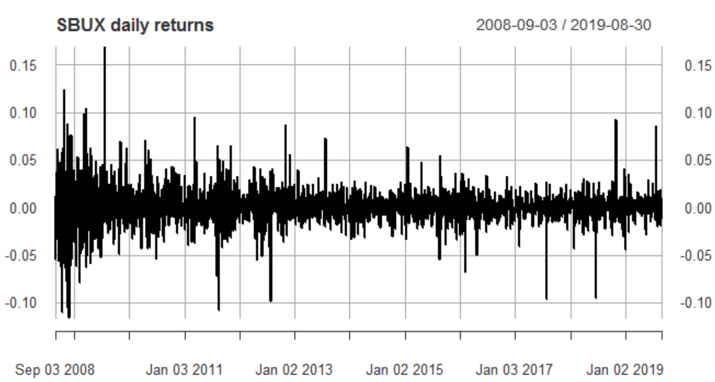
Estimates	daily
Mean	0.096136014
St.Deviation	1.879582736
Diameter.C.I.Mean	0.070020708
Skewness	0.288506286
Kurtosis	11.47728452
Excess.Kurtosis	8.477284525
Min	-11.62437552
Quant.5%	-2.681801845
Quant.25%	-0.734602736
Median.50%	0.087080861
Quant.75%	0.909529251
Quant.95%	2.777962463
Max	16.87280968
Jarque.Bera.stat.JB	8326.754854
Jarque.Bera.pvalue.X100	2.20E-16
Lillie.test.stat.D	0.094643921
Lillie.test.pvalue.X100	5.32604E-65
N.obs	2768

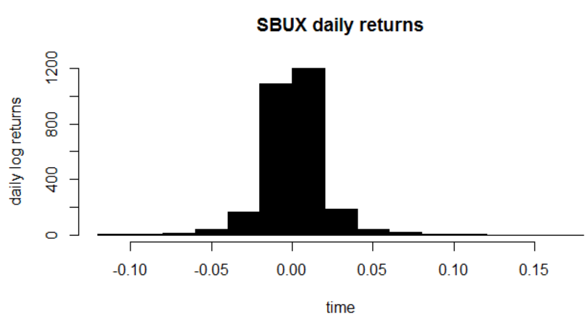


Before analysing the data more closely, it is worth observing the summary statistics first in order to gauge the nature of this dataset, as well as to compare it to the graphical results, which will follow. The distribution of daily returns has a very small and positive mean 0.096% compared to its standard deviation, which is around 1.88%. Nonetheless, as the magnitude of daily returns is small with respect to its standard deviation (0.096 vs 1.88), we can assume the log-return to be 0.

The log-returns show weak evidence of positive skewness (caused by very positive and unexpected news which generated large positive returns) and there is evidence for excess kurtosis. **As the first two moments (mean and standard deviation) of the returns are finite, and the ACF of returns declines fast towards zero (in contrast to a random walk process).**

Stylized Fact 2 - Returns are Stationary

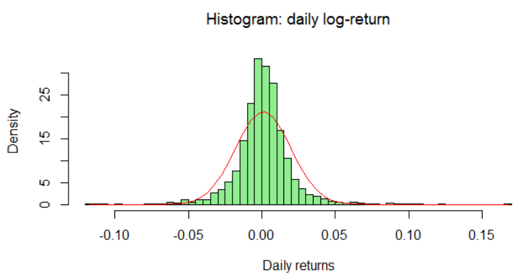


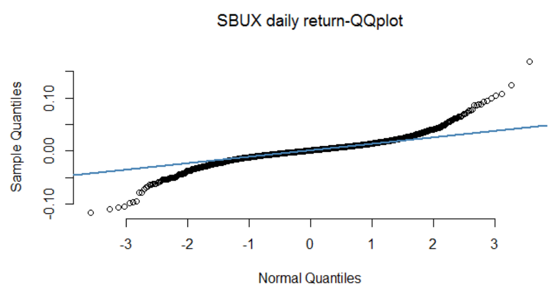


Stationarity is a probabilistic measure of regularity and can be exploited to estimate consistently unknown parameters and characterize the dependence between observations across time.

The Starbucks daily log-returns graph seems to oscillate around zero in a constant level in the analyzed period, reflecting a possible stationary process of its returns, and consequently, a constant mean over time. The histogram of Starbucks daily returns also proves that, even though with more positive values, that most are aggregated around a certain value. Therefore, with the analysis of both the estimates and graphs, we conclude that the returns are stationary.

Stylized Facts 3 and 4 - Asymmetry and Heavy Tails

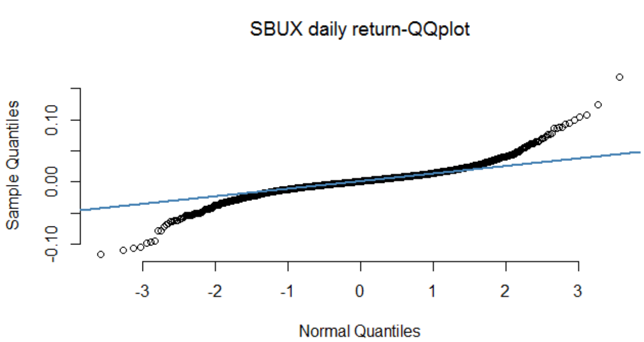


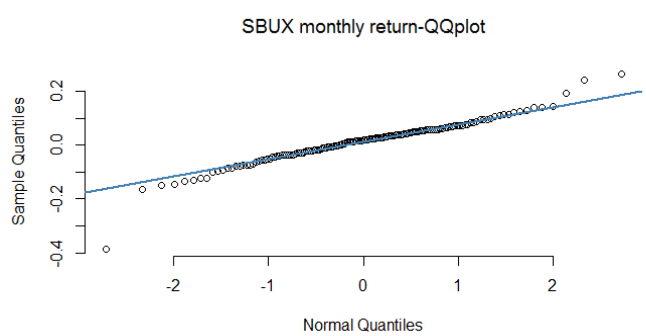


When observing the histogram with a plotted normal distribution, the returns are more concentrated around the mean, with a little more positive than negative returns, suggesting a slight positive skewness since the tail on the right side of the distribution is longer. This contradicts the stylized fact 3, which should be negatively skewed.

The blue line on the QQ-plot represents the value your data points would take, were it normally distributed. Upon observing Starbucks' QQ-plot, it presents fat tails as the plots corresponding to the return distribution are not aligned with the normal distribution line.

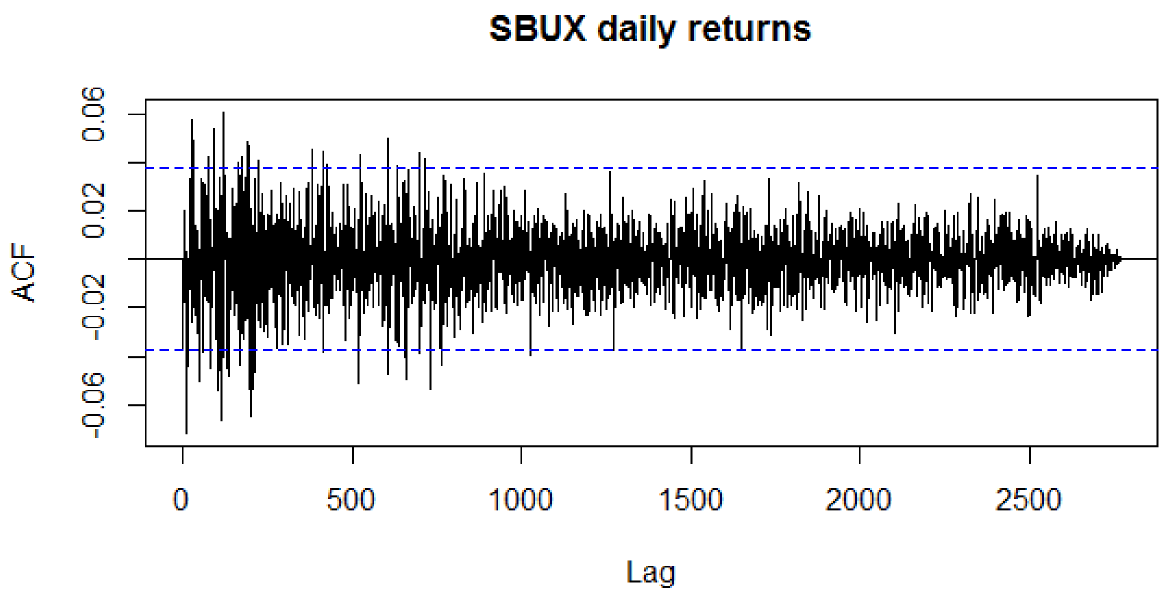
Stylized Fact 5 - High Frequency Non-Gaussianity





The daily returns indeed do not follow a gaussian distribution according to the daily QQ-plot. According to the values estimated table: **JarqueBera p-value (2.2E-16)** and the **Lillie test p-value (5.33E-65)** computed are both below 1%, therefore, the returns do not follow a normal distribution. When aggregating the daily into monthly, it is immediately observed that the transformation aligned more with the normal distribution line, which may prove that low time frequency is gaussian. Nevertheless, **JarqueBera p-value** and the **Lillie test p-value** for monthly returns are still below 1%, suggesting also that monthly returns are not normally distributed.

Stylized Fact 6 - Returns are Not Autocorrelated

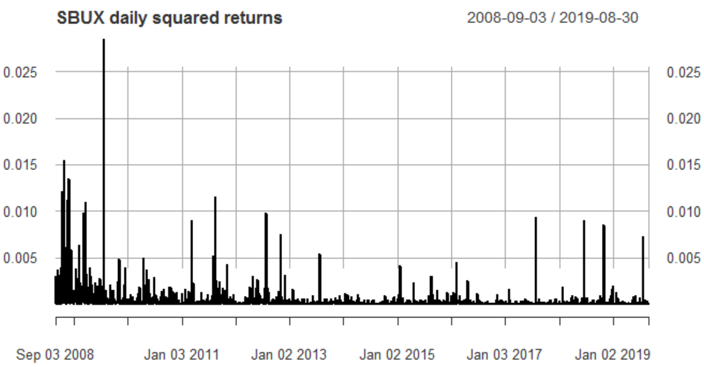


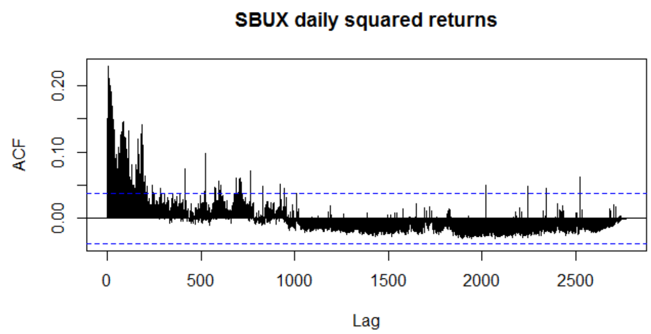
Observing the graph, which plots the correlation between returns at different lags, we can conclude that at lags below 1000, there is some significant autocorrelation between the log-returns, and after that, it slowly decays to 0 as the lags increase. This means that ACF are significantly different from 0, however it appears that they are very small in absolute values even if they are significant, which means that they are extremely close to the critical point, therefore we can conclude that it is insignificant.

X-squared = 5.9281, df = 5, p-value = 0.3133

Performing the Box-Peirce also shows that we do not reject the null, which means that the autocorrelation up to lag 5 is not significant.

Stylized Fact 7 - Volatility Clustering and ARCH Effect



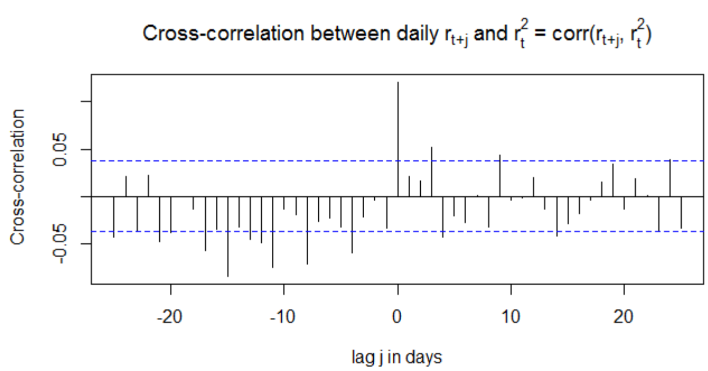


We can notice that some volatility clustering occurs after the financial crisis around september 2008, leading to significant autocorrelation which can be observed in the graph. The rest of our time period is also what can be expected and periods with higher volatility leads to autocorrelation in the squared returns.

Usually the autocorrelations are very persistent and decay very slowly to 0 as k increases, indicating possible long term-memory. The ACF in our sample decays to 0 at around k=200 which is a slow process, however the decay is not consistent.

We also tested the ACF with the box-pierce with lag 1 and 5.
X-squared = 62.397, df = 1, p-value = 2.776e-15
X-squared = 445.97, df = 5, p-value < 2.2e-16.
P-values below all the normal significance levels, implies that we have significant autocorrelation for lag 1 and 5.

Stylized Fact 8 - Leverage Effect



In order to look at the leverage effect we needed to plot the cross-correlation between daily rRt+j and r2= corr(rt+j,rt2) with lag j, -25 and j + 25.Looking at the graph we have significant cross-correlation for almost all the lags, and almost all of the different lags i negatively autocorrelated. This is proof for the leverage effect, indicating that Starbuck's returns are negatively correlated with the changes in their respective volatilities.

Question 2

```
> summary(lm.CAPM.d)
lm(formula = ExRt ~ Mkt.RF, data = Starbucks.FF.d)
Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  0.06534    0.02533    2.58  0.00991 **
Mkt.RF       1.06101    0.02266   46.83  < 2e-16 ***
Residual standard error: 2.101 on 6887 degrees of freedom
Multiple R-squared:  0.2415,  Adjusted R-squared:  0.2414
F-statistic:  2193 on 1 and 6887 DF,  p-value: < 2.2e-16
```

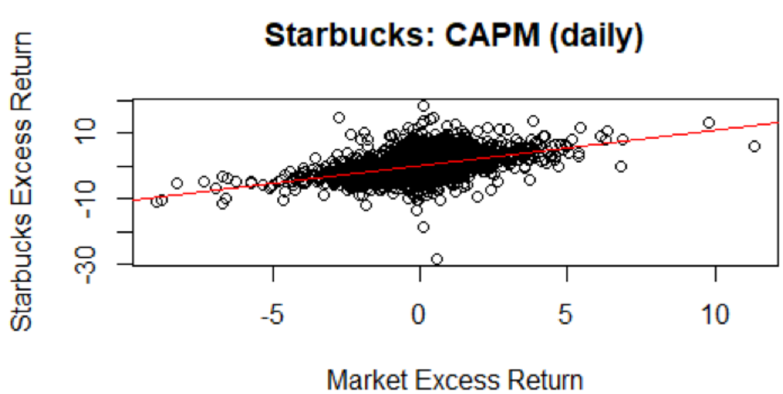
Above is the regression analysis for Starbucks based on daily data since their IPO. The regressor is the Mkt.RF which has an estimate of 1.06101. The regressors p-value is highly significant with a value of 2.2e-16, and the conclusion can be drawn that "Mkt.RF" is significantly different from zero and thus explains the model.

Hypothesis test for α:
H_0: $\alpha_i = 0$
H_1: $\alpha_i \neq 0$

The α coefficients p-value is below all traditional significance levels but very close to 1%, thus meaning that the α coefficient is different from zero, which is not compatible with the CAPM.

CAPM holds the assumption that $E[\varepsilon_{i,t}] = 0$ and $Cov(\varepsilon_{i,t}, z_{m,t}) = 0$ is compatible or equivalent to the CAPM if $\alpha = 0$. As we rejected the null hypothesis with $\alpha = 0$, this shows that the CAPM is not valid.

Below is the plotted "abline"/the fitted regression line from R and the real values of the excess return of starbucks.



H_0: $\alpha_i = 0$
H_1: $\alpha_i \neq 0$

```
> summary(lm.FF3FM.d)

Call:
lm(formula = ExRt ~ Mkt.RF + SMB + HML, data = Starbucks.FF.d)

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  0.06622      0.02531   2.616  0.00891
Mkt.RF       1.05618 0.02267  46.586  < 2e-16
SMB          0.08381 0.04366   1.919  0.05497
HML     -0.126220.04190  -3.012  0.00260

(Intercept) **
Mkt.RF      ***
SMB     .
HML     **
---
 Residual standard error: 2.099 on 6885 degrees of freedom
Multiple R-squared:  0.2432,  Adjusted R-squared:  0.2428
F-statistic: 737.3 on 3 and 6885 DF,  p-value: < 2.2e-16
```

Regarding the coefficients of the FAMA-FRENCH model, different results are presented.
Based on the same assumptions as shown previously, we reject the null hypothesis that the regressors are equal to zero if the p-value is below our significance level.
We conclude that α, Mkt.RF and HML has p-values below 1% significance. However, SMB has a p-value of 0,0597 which means that the null is not rejected for 5% significance, while at 10% significance the null is rejected. At 5% significance the SMB factor in Fama-French model does not help in the prediction of the fitted values in the regression for Starbucks.

Concerning the goodness of fit for the model below is the output from the regressions.

CAPM: Multiple R-squared: 0.2415, Adjusted R-squared: 0.2414
Fama-French: Multiple R-squared: 0.2432, Adjusted R-squared: 0.2428

Adjusted R-squared is very similar to R-squared because of daily data → large number of observations

In this case, the adjusted R-square is used to compare the goodness of fit for these two models. As seen, the adjusted R-square is almost the same as R-square. Both the CAPM and Fama-French models have goodness-of-fit leves that hoovers around typical levels for CAPM and Fama-French regressions, although it still means that there are a lot of error/noise in the regression for both models. Thus, both the CAPM and the Fama-French has a low degree of explanation in predicting the returns of Starbucks although the Fama-French is marginally higher degree of explanation.

Question 3 - GARCH Models

The analysis of conditional heteroskedasticity of the error terms was performed by GARCH (1,1) and GARCH (2,1). Below are the results estimates for the sample data of Starbucks in the analyzed period.

```
*---------------------------------*
*          GARCH Model Fit        *
*---------------------------------*

Conditional Variance Dynamics
-----------------------------------
GARCH Model     : sGARCH(1,1)
Mean Model      : ARFIMA(0,0,0)
Distribution    : norm

Optimal Parameters
------------------------------------
        Estimate  Std. Error  t value Pr(>|t|)
mu      0.021746    0.006375   3.4110 0.000647
omega   0.001063    0.000382   2.7843 0.005364
alpha1  0.067656    0.009694   6.9789 0.000000
beta1   0.931344    0.010751  86.6279 0.000000
```

```
*---------------------------------*
*          GARCH Model Fit        *
*---------------------------------*

Conditional Variance Dynamics
-----------------------------------
GARCH Model     : sGARCH(2,1)
Mean Model      : ARFIMA(0,0,0)
Distribution    : norm

Optimal Parameters
------------------------------------
        Estimate  Std. Error   t value Pr(>|t|)
mu      0.021771    0.006378  3.413514 0.000641
omega   0.001064    0.000432  2.465358 0.013688
alpha1  0.067681    0.012435  5.442863 0.000000
alpha2  0.000000    0.015855  0.000004 0.999997
beta1   0.931319    0.013527 68.846628 0.000000
```

In model GARCH (1,1), all of the optimal parameters are relevant at the 1% significance level. On the other hand, GARCH (2,1) seems to differ in two parameters: omega and alpha2. The former is only statistically significant at the 5% significance level and the latter is not at all significant (p-value = 0.999). According to the results in both models, the beta coefficient seems to be higher than the alpha optimal parameters, with beta larger than 0.9 and alpha below 0.1 for daily returns and its respective sum to be generally close to 1, as expected in theory.

```
Information Criteria
------------------------------------

Akaike       1.2234
Bayes        1.2320
Shibata      1.2234
Hannan-Quinn 1.2265
```

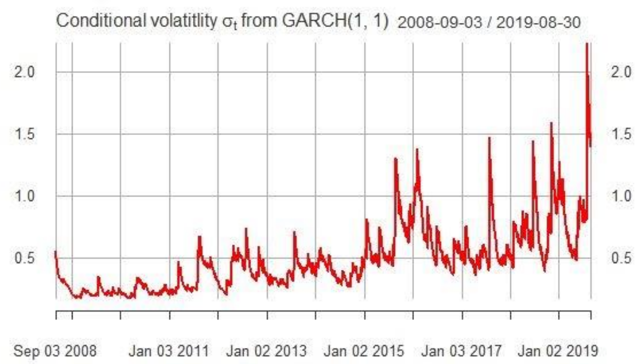
```
Information Criteria
------------------------------------

Akaike       1.2245
Bayes        1.2352
Shibata      1.2245
Hannan-Quinn 1.2284
```

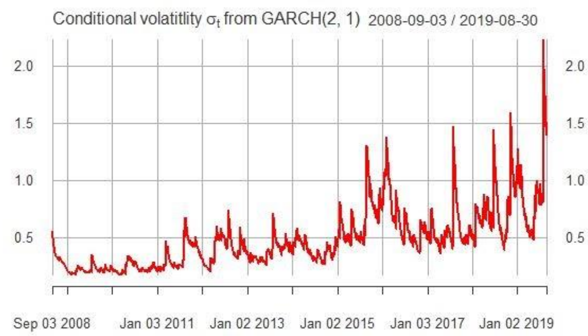
Information Criteria: According to the results, all the coefficients in GARCH (1,1) are lower than GARCH (2,1), suggesting the former to be a better fit of the model.

```
Weighted Ljung-Box Test on Standardized Residuals
------------------------------------
                        statistic p-value
Lag[1]                      1.464  0.2263
Lag[2*(p+q)+(p+q)-1][2]     1.744  0.3090
Lag[4*(p+q)+(p+q)-1][5]     4.222  0.2277
d.o.f=0
H0 : No serial correlation
```

```
Weighted Ljung-Box Test on Standardized Residuals
------------------------------------
                        statistic p-value
Lag[1]                      1.467  0.2258
Lag[2*(p+q)+(p+q)-1][2]     1.749  0.3081
Lag[4*(p+q)+(p+q)-1][5]     4.236  0.2260
d.o.f=0
H0 : No serial correlation
```

According to the weighted Ljung-Box Test on Standardized Residuals, all of the respective p-values suggests that there is no serial correlation. As a consequence, the respective t-statistics of the residuals are below the critical point. The result is in accordance with theory as it is expected to not have autocorrelation over time.

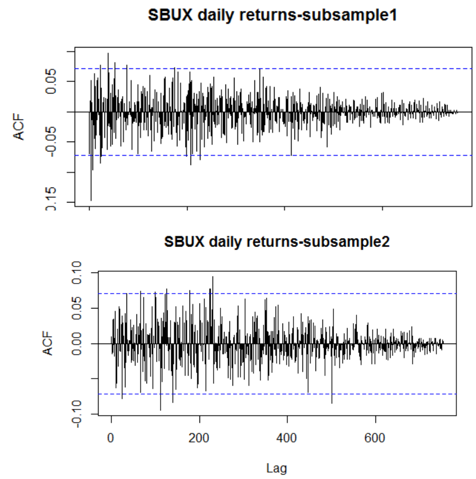




The distribution of conditional volatilities for both models seem to be similar. Also, a peculiar observation is that as time passes, the volatility increases. The graphs clearly show a separation between a time period in which volatility is following a consistent pattern and a period (after 2015) with huge volatility fluctuations, suggesting specific strategies and events that have influenced the risk and return associated with the company. The general increase in volatility starting from 2015 relates to the fact that Starbucks has been able to penetrate other important markets (EMEA region), leading to higher profits and, consequently, the demand for the stock and its price has seen huge increases along these four years. A reason that might explain the increase in volatility is: the price of the stock in year 2014 was around 33-38$ and currently is trading around 80-90$, which represents a huge spread. More specifically, the huge increase in risk (which is a result of the fluctuation in price) associated with the stock from the beginning of 2019 can be explained by the fact that the firm has consistently beaten the expectations of the analysts and the market

Question 4 - Comparing Two Sub-samples: Descriptive Statistics

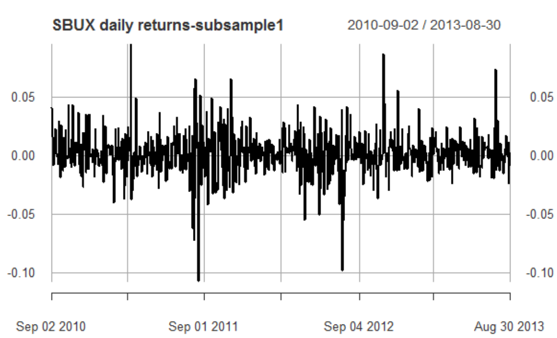
Estimates	Subsample 1	Subsample 2
Mean	0.150542494	0.073714704
St.Deviation	1.786580461	1.252151858
Diameter.C.I.Mean	0.127521966	0.089375693
Skewness	-0.074524263	-0.388165448
Kurtosis	8.47099963	18.52362239
Excess.Kurtosis	5.47099963	15.52362239
Min	-10.76599927	-9.699221684
Quant.5%	-2.545873476	-1.638243562
Quant.25%	-0.790057497	-0.500279654
Median.50%	0.127647278	0.079679902
Quant.75%	1.000845966	0.715073879
Quant.95%	2.883205407	1.776838788
Max	9.46786821	9.2624162
Jarque.Bera.stat.JB	941.0564799	7589.812438
Jarque.Bera.pvalue.X100	2.20E-16	2.20E-16
Lillie.test.stat.D	0.071521175	0.086769405
Lillie.test.pvalue.X100	8.71894E-08	1.02E-12
N.obs	754	754

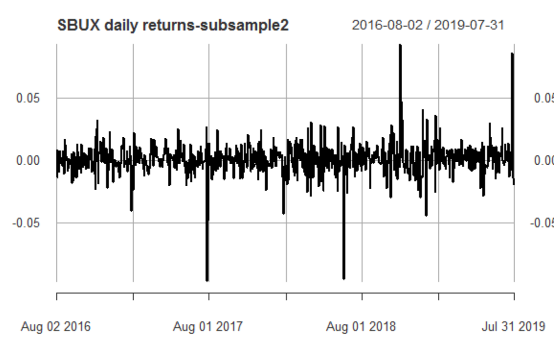


As we did earlier for the entire dataset, let us first take a look at the summary statistics. Sample 1 has a higher mean log-return than sample 2. When compared to their respective estimates, we get the same results as mentioned in the sample data. However, the skewness of both samples is weak and negative, and is therefore in line with the theory. Moreover, there is also strong evidence of excess kurtosis. The Lillie and Jarque Bera tests both conclude that the subsamples do not follow a normal distribution as their p-value is below the 5% significance level for the same reason in the sample data.

As the first two moments of the returns are finite, and the ACF of returns declines fast towards zero, we conclude that both processes are stationary and is in accordance with the original sample.

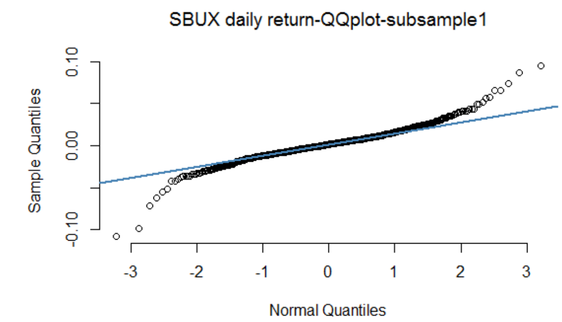
Stylized Facts 2,3 and 4

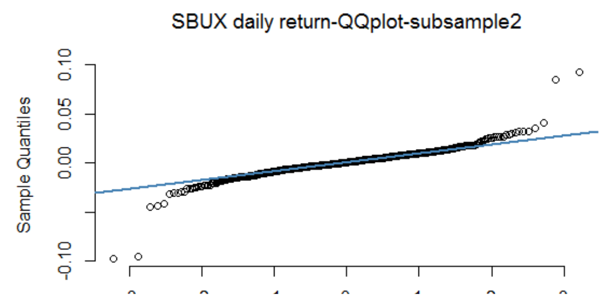


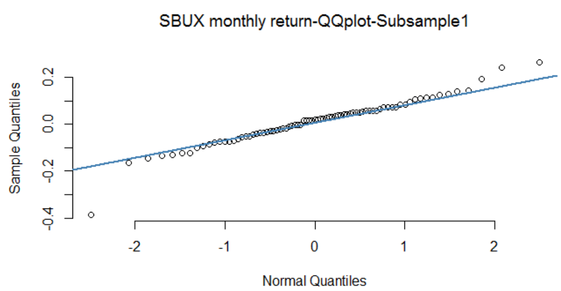


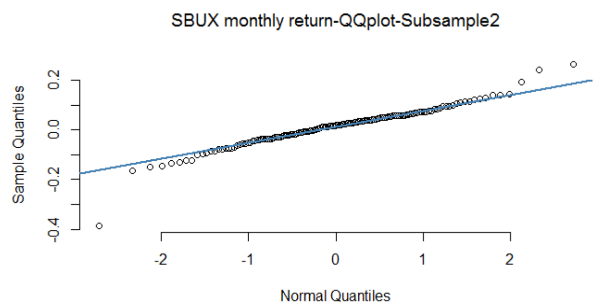
Even though the distribution of the returns fluctuates more with the first sub-sample, it is around zero in a constant level in both the analyzed periods, and consequently a constant mean over time and it is similar to the distribution in the whole sample.

Stylized Fact 5 - Aggregational Gaussianity



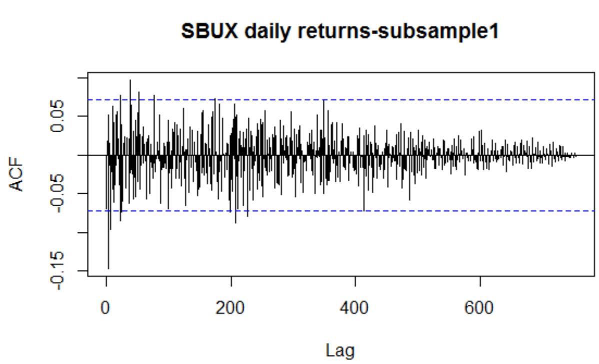


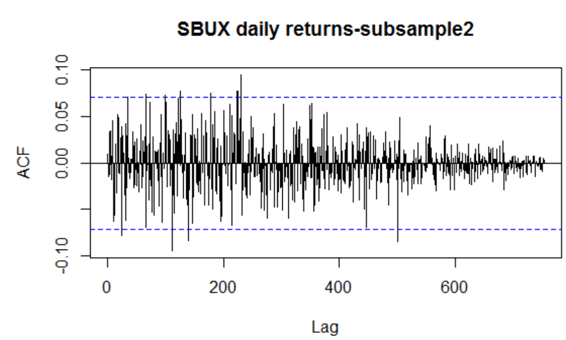




By observation of the QQ-plots, the daily return of the subsamples (particularly subsample 1) does not follow a gaussian distribution. According to the estimates table, **JarqueBera p-value** and the **Lillie test p-value** computed are both below 1%, therefore, the returns do not follow a normal distribution. When compounded into monthly, the distribution assimilated more with the normal distribution line, which shows some gaussian characteristics. Nevertheless, **JarqueBera p-value** and the **Lillie test p-value** for monthly returns are still below 1%, suggesting also that monthly returns are not normally distributed.

Stylized Fact 6 - Returns are Not Autocorrelated





If we look at the graphs for the sub-samples, we see similar results as for the whole sample. There is some observable autocorrelation at the lower lags and then they slowly decay to zero.

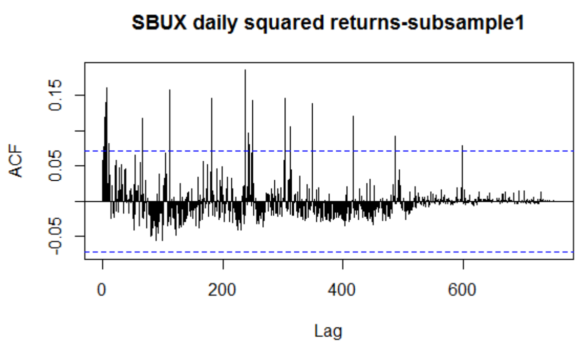
In order to test this we carried out another box-pierce test on the two sub-samples, and obtained the following results:

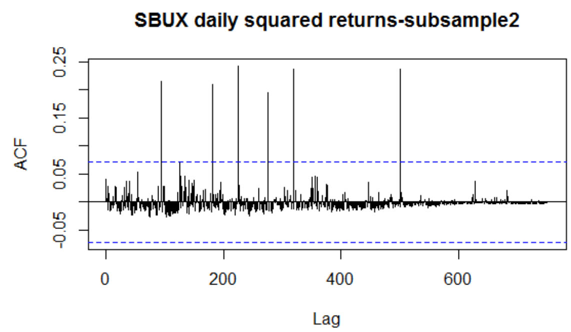
X-squared = 0.068163, df = 1, p-value = 0.794

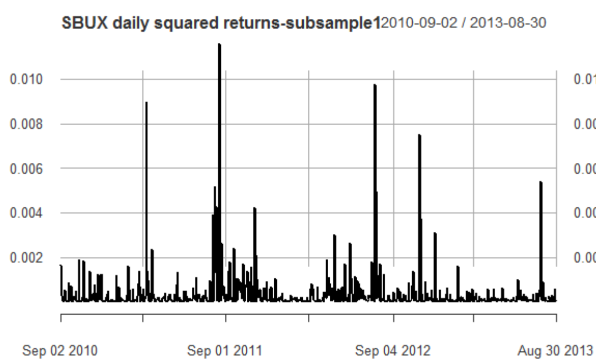
X-squared = 3.6404, df = 1, p-value = 0.05639

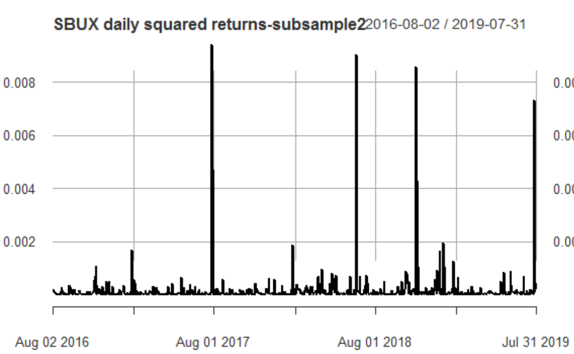
Hence, we conclude that both samples have p-values above a 5% significance level. This leads us to not rejecting the null, so they do not have significant autocorrelation in the returns as expected.

Stylized Facts 7 - Volatility (Sub-samples)





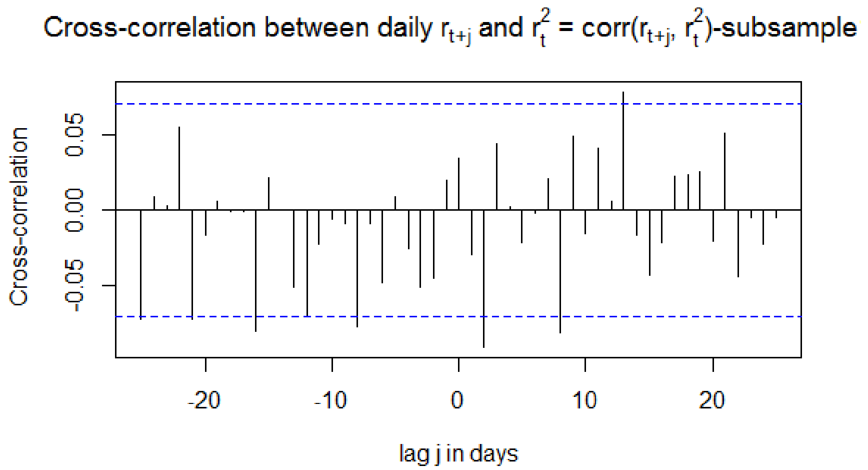


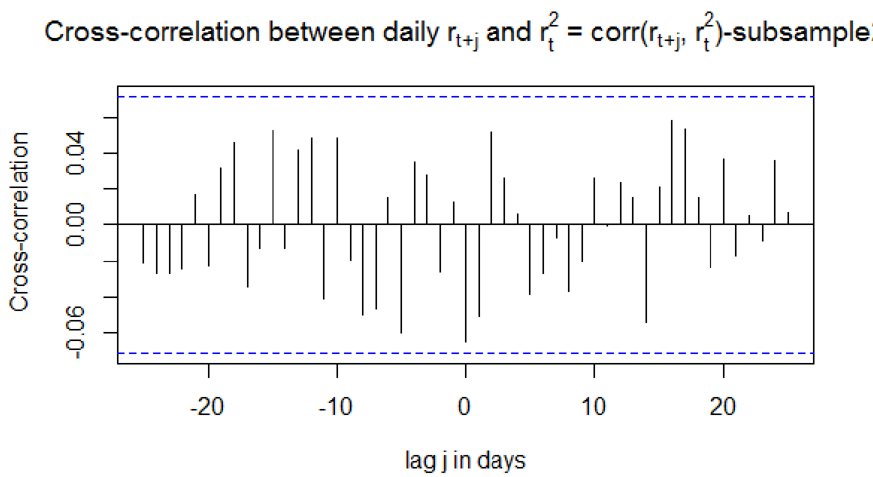


Subsample 1: X-squared = 32.412, df = 5, p-value = 4.923e-06
Subsample 2: X-squared = 1.9922, df = 5, p-value = 0.8502

The results obtained show us that subsample 1 is in line with the entire sample, however, subsample 2 shows us something completely different, with a p-value of 0.85. This can probably be explained by the low volatility we notice in the squared returns, which is the meaning of this stylized fact. High volatility leads to autocorrelation of the squared returns and low volatility leads to no autocorrelation.

Stylized Facts 8 - Leverage effect (Sub-samples)





In the cross correlation we notice that first subsample looks similar to the entire sample, however, the second subsample has a lot more positive cross-correlation, this also has to do with the low volatility that we also noticed in the subsample in question 2. It is worth noting, that differences were primarily found in the second sub-sample, with an observably more positive cross-correlation than the original whole dataset, as well as clear differences in the volatility.

Conclusion

When comparing to the theoretical stylized facts, we conclude that most of them are applied to our empirical study of Starbucks returns. Slight deviations arise from asymmetry in distribution, which revealed to be positive in contrast to the theoretical negative skewness. Moreover, taking random subsamples (3 years) of the data, the skewness was adjusted and turned negative, therefore, specific moments of the performance of the stock brought negative returns. Additionally, when analyzing the subsamples, all were according to the theory.

When regressing the returns to the CAPM and Fama French models, both had a normal level of goodness of fit. TheR2 also very similar between the models with a slight advantage to the Fama French. The divergence is the fact that the intercept in the CAPM model was significantly different from zero, which it is not supposed to be in theory, but in reality, it should be observable.

Finally, volatility related to the stock is quite high as a result of management and strategies implemented by the company, which boosted and improved their overall performance during the last 4 years. Key indicators reflecting business and operation have been consistently outperforming analyst expectations and the fact that it is focusing on penetrating the other markets(EMEA) around the world seem to confirm this hypothesis.