Convex Optimization

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The goal of this presentation is to convince you that mathematical optimization (also referred to as mathematical programming or just plain optimization) is interesting, broadly applicable, and very relevant to many of our members' interests and goals.

Optimization is a powerful approach to solving real-life, decision-based problems. Convex optimization is a subset of optimization and while the ideas underlying convex optimization are simple, the tool is rich enough to describe highly complex processes. Only recently, however, have algorithms and increased computational power allowed us to use this technique broadly in practice.

In optimization, one is concerned with minimizing an objective scalar value (or cost) over some subspace of possible solutions. In set notation, we minimize the function $f : X \in \mathbb{R}^n \to Y \in \mathbb{R}$ subject to a number of constraints. One of the interesting discoveries in the 20^{th} century was that it is not linearity that makes an optimization problem easy to solve, but convexity. This breakthrough in thought gave rise to modern convex optimization. Convex optimization problems are defined by convex objective functions f and constraints that describe convex sets. Celebrated convex optimization problems include linear programs and quadratic programs, the latter which contains the familiar least squares family of programs. Recent work has popularized more sophisticated but convex second-order cone programs and semi-definite programs. Convex optimization is preferred in particular because of powerful guarantees (provably global solutions, guaranteed constraint satisfaction) and efficient algorithms.

After describing the foundations of convex optimization, I will quickly move through specific examples to give an idea of how optimization is used in practice. Examples will be drawn from canonical teaching examples (support vector machines in machine learning, robust programming in finance, geometric programming in analog circuit design) as well as examples from my research/work (control for thermal ablation, image reconstruction, magnet design). Finally, only if time and interest permits, I have slides on more advanced topics in convex optimization such as duality and algorithms.

It is instructive to think of optimization as a generalization of linear algebra and, in some ways, signal processing – it solves the familiar problems in these fields, but allows us to go beyond. It is also important to be aware of related topics such as machine learning and graph theory. All of these areas influence and use each other, and a sound knowledge or at least acquaintance with all of these methods can make one very dangerous indeed.