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Performance control and risk calibration in the Black-Litterman model

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Abstract

The authors show that risk aversion and prior estimation error input parameters of the Black-Litterman model that are arbitrarily fixed in existing practices should instead be carefully calibrated because they are related to the Sharpe performance ratio and Value at Risk or tail risk of the active portfolio. A related important insight is that these parameters are not entirely exogenous but are connected closely to the portfolio manager's inputs of subjective expected returns, as well as the degree of confidence over these subjective beliefs. The value of τ is closer to zero if the manager believes the initial estimates based on historical data are accurate compared to the subjective views and closer to one if the manager believes there is a fundamental shift in the market landscape such that past history should not be overly relied upon. The authors also show that in the event of an incorrect view, an unrealistically high Sharpe ratio and excessive risk taking can produce disastrous losses. Unifying parameter calibrations with performance and risk measures, the model is internally consistent and provides a powerful means for practical application.

The Black-Litterman model is an important and popular asset allocation model based on risk-return optimization using variance as a measure of risk (see Black and Litterman [1992]). Its key advantage over the Markowitz model is its capacity to enable the portfolio manager to adjust and fine-tune the inputs of subjective expected returns, as well as the degree of confidence over these subjective beliefs. These subjective beliefs can take flexible forms, such as inputs of particular assets' expected returns or inputs of expected spreads between two assets' returns.

However, for a number of subjective inputs, there is inadequate insight about how they are related in producing the final outputs of the model. Herold [2003] specifically addresses the difficulty of using Black-Litterman from the perspective of nonquantitative fund managers, highlighting the ambiguity in choosing and justifying the appropriate values for the parametric inputs. More recently, O'Toole [2013] formulated a derivation based on a risk-budgeting perspective to help clarify and demystify some of the calculations and resulting outputs of the model. Despite recent progress, there remain two input parameters within the Black-Litterman formulation that are determined in an ad hoc manner under existing practice.

As a short recap, in the Markowitz model, the mean return is given by $\hat{u} = r_F L + \lambda_M \Sigma W$, where r_F is the risk-free return rate, L is a vector of ones, λ_M is the market risk-aversion parameter, S is the covariance matrix of returns, and W is the market weights. Under Black-Litterman formulation, this return is a prior estimate and is related to the true return by $\hat{u} = \mu + \epsilon$, where μ is the true return and $\epsilon \sim N(0, \tau\Sigma)$. Here, τ represents the prior estimate error. Given the fund manager's subjective views, the true return can be estimated as μ^*, and the revised optimal portfolio weight can be obtained as $W_{BL} = \frac{1}{\lambda^*}\Sigma^{-1}(\mu^* - r_F L)$, where λ^* is the fund manager's idiosyncratic risk-aversion parameter.

As we have discussed, two ad hoc parameters are required in the Black-Litterman formulation. The first parameter is the multiplicative factor τ over the return covariance matrix; the resulting matrix is used to define the covariance matrix of the measurement errors of the true returns from the initial estimated mean returns. The second parameter is the portfolio fund manager's idiosyncratic risk-aversion parameter λ^*. For initial inputs, the returns covariance matrix could be derived from historical sampling estimates based on the recent past history or could be derived on conditional covariance modelling. The initial estimated mean return vector $\hat{u}$ could be derived similarly from a historical sample mean based on the recent past history or from utilizing the Sharpe [1964] capital asset pricing model (CAPM). The initial estimated mean returns $\hat{u}$ are combined with the portfolio manager's own subjective estimates of the true returns to produce a more accurate estimate of the expected true returns using the minimum least squares error criterion. Then, based on the risk-aversion parameter λ^* and the final estimate of true expected returns, optimal portfolio weights can be determined.

We contribute to the literature by illustrating how these two input parameters of λ^* and τ need not be, and indeed should not be, arbitrarily fixed. Our study shows that these two parameters are also related to the Sharpe performance ratio π and the value-at-risk (VaR) measure b of a portfolio. By choosing the appropriate λ^* and τ values, the portfolio manager can ensure that the resulting optimal portfolio attains a performance ratio that is consistent with expectations and that the portfolio also carries an acceptable tail risk. If λ^* and τ values are not suitably chosen, the output final return estimation and the resulting portfolio selection will possess ex ante either an unrealistically high Sharpe ratio or an excessively low Sharpe ratio. The portfolio may also have too high of a VaR or probability of a large loss that is not acceptable. We advocate suitable choices of these two parameter values for performance control and risk calibration in the Black-Litterman model.

EXISTING METHODS OF CALIBRATION

Black and Litterman [1990, 1992] extended the Markowitz mean-variance optimization framework to allow for incorporation of active investor views about future returns. They introduced confidence levels on the initial estimates of the mean returns of stocks in the active portfolio that could be implied returns derived from historical data based on the Sharpe CAPM. The active portfolio is not necessarily the universe of all securities in the market. It includes a risk-free asset, so that the portfolio manager can choose to increase or decrease risk exposures to the risky securities by increasing or decreasing total weights on the risky assets with the fund balance in the risk-free asset. The sum of weights in the risky assets plus the risk-free asset equals one.

The initial estimates û form the prior distribution of true return estimates $\mu \sim N(\hat{u}, \tau\Sigma)$, where Σ is the sampling estimate of the covariance matrix of the returns in the portfolio stocks, and τ is a multiplicative factor over the return covariance matrix such that $\tau\Sigma$ is the covariance matrix of the measurement errors of the true returns from û.

The portfolio manager's own subjective inputs about the true underlying returns μ act as posterior information, and the priors are updated by Bayesian analysis to obtain the final estimate of the true underlying returns. If the subjective inputs take the form $P\mu \sim N(Q, \Omega)$ where P is a matrix set of weights and Q represents active investment views, then the minimum least squares estimate of true mean is in the form of an expected mean of

$$\mu^* = \left[(\tau\Sigma)^{-1} + P'\Omega^{-1}P\right]^{-1}\left[(\tau\Sigma)^{-1}\hat{\mu} + P'\Omega^{-1}Q\right] \quad (1)$$

where sampling error of μ^* itself is negligible. This final output takes exactly the same form as if the model was solved using Theil's [1971] mixed estimation approach employing generalized least squares regression.

One can see in Equation 1 that in yielding the Bayesian update on the left-hand side, both the prior estimate of û and the posterior information of subjective input Q are utilized. Their weights in influencing the final true return estimates μ^* or $E(\mu)$ of individual assets are larger if the inputs are more accurate and smaller if the inputs are less accurate. Intuitively, a larger multiplier τ would indicate a less accurate and hence noisier estimate from û that is based on recent past historical data. A smaller τ would indicate a more accurate and less noisy estimate of û. Diagonal elements of Ω represent the variances of the portfolio manager's subjective belief in the returns $P\mu$. For most practical purposes, we treat subjective inputs for off-diagonal terms in Ω as zeros.

We then obtain the familiar Black-Litterman optimal portfolio weights as $W_{BL} = \frac{1}{\lambda^*}\Sigma^{-1}(\mu^* - r_F L)$, where r_F is the risk-free return rate over the portfolio investment horizon and L is a vector of ones. We see that ceteris paribus, higher μ^* leads to larger weights on the stocks (those with higher expected returns relative to others), and larger λ^* or higher portfolio manager risk aversion leads to lower weights on the risky stocks, shifting instead into more risk-free bonds. In most versions of the Black-Litterman model, Σ, being more stable and more accurately estimated using recent historical data or some form of conditional modelling, is not necessarily updated based on subjective views on the means.

In the equilibrium framework, the market risk-aversion parameter λ_M is the expected excess market return divided by the market return variance. In determining the final weights of W_{BL}, existing implementations of the Black-Litterman model typically use λ_M as an approximation or as a proxy for the idiosyncratic risk aversion of the portfolio manager, λ^*. See Martellini and Ziemann [2007] for an example of the many studies using such a proxy calibration of λ^*.

Some may use estimates of risk aversion from other economic studies in which the estimates may have little relevance to the actual idiosyncratic risk aversion of the portfolio manager making the investment decisions. The use of a market average proxy such as λ_M may not be appropriate for two reasons. First, the manager may be dealing with a smaller and more stylized portfolio that is different from the universe portfolio of all stocks and thus will have a different risk appetite with respect to the nature of the portfolio under management. Second, the styles and attributes of portfolio managers vary widely and are important factors affecting the success or failure of their portfolio selection. The market average of all portfolio managers' risk aversion will not be a suitable representation of a particular and unique portfolio manager's risk preference.

As for the choice of input parameter τ in the implementation of the Black-Litterman model, the existing literature has no consensus. Lee [2000] used values in the range (0.01, 0.05). Satchell and Scowcroft [2000] set τ equal to one. Blamont and Firoozy [2003] employed a τ value approximately equal to one divided by the number of observations in the time series sample. On the other hand, Da Silva, Lee, and Pornrojnangkool [2009] utilize a market-capitalization benchmark portfolio and its covariance matrix to set τ to attain a Sharpe ratio of 0.5, while purporting to maximize the information ratio. The latter is different from our approach to relate τ directly to the portfolio manager's active portfolio expected Sharpe ratio and to allow discretionary selection of τ.

CALIBRATING τ FOR PERFORMANCE CONTROL

The Black-Litterman portfolio weights W_{BL} depend on λ^* and $\mu^*(\tau)$. The expected or ex ante Sharpe ratio π is familiarly known to be $\sqrt{[\mu^* - r_F L]'\Sigma^{-1}[\mu^* - r_F L]}$ and is thus a function of τ. Lo [2002] reported annual Sharpe ratios of 0.50 to 1.12 for a number of mutual funds using a sampling period of several years up until 2000. A recent study by Bednarek, Patel, and Ramezani [2014] found that U.S. small-size, medium-size, large-size, and growth stock portfolios during the sampling period 1927 to 2013 showed ex post Sharpe ratios ranging from 0.37 to 0.84, with higher ratios in the 3- to 15-year horizon compared to short 1- to 2-year horizons or very long 25-year horizons.

Other public sources such as in Morningstar also indicate 0.25 to 1.0 as the usual performance range. Theoretically, the ratio could be negative if ex post return is below the risk-free rate of return, and it could also be extremely large. However, ex ante, portfolio managers form an appropriate view of the expected Sharpe ratio in the positive range that is consistent with their own beliefs of how well the stock market would perform and how confident they are of their beliefs. When portfolio managers specify a view, they often couple it with a target Sharpe ratio that they aim to achieve (see, for instance, Fabozzi, Focardi, and Kolm [2006]). The ex ante Sharpe ratio π can therefore be considered as a performance control because the portfolio manager would want to set a target for the ratio.

To examine the relationship between τ and π, we utilize three size-sorted portfolios in the Kenneth French data library[1]—namely stocks in the lower 30 percentiles, the middle 40 percentiles, and the upper 30 percentiles. Recent monthly samples from January 2014 to August 2015 were used to evaluate the covariance matrix of S. This showed

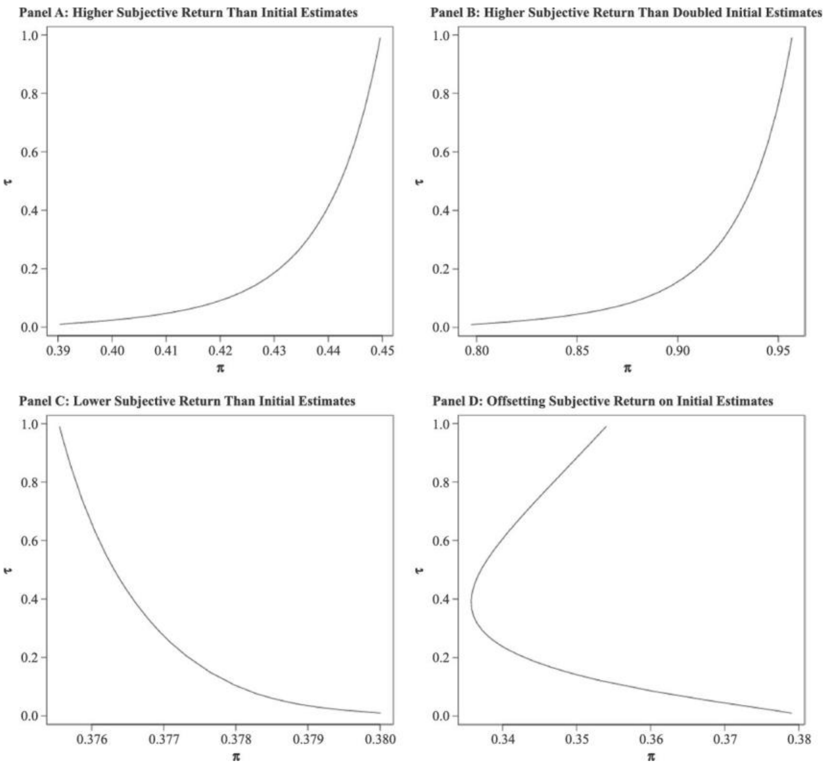
$$\Sigma = \begin{bmatrix} 0.0227 & 0.0174 & 0.0095 \\ 0.0174 & 0.0161 & 0.0109 \\ 0.0095 & 0.0109 & 0.0106 \end{bmatrix} \quad (2)$$

indicating fairly high correlations in the range of 0.6 to 0.9. Average annual 1-year Treasury bill rate for this period was 0.12%. Initial estimated mean returns of these three portfolios are 0.58%, 1.58%, and 3.11% on annual basis.[2] The mean returns for the period 2014-2015 are considerably smaller than for earlier periods. For illustration purposes, suppose a portfolio manager is trying to actively allocate portfolio weights among these three risky subportfolios as well as remaining funds in the risk-free asset, such as Treasury bills, for a horizon of up to a year. The subjective inputs take the form of strong views on the first two subportfolios with $Q' = [\mu_1, \mu_2]$ and uncertainty reflected in the diagonal elements of Ω as a small 0.001, thus indicating strong and confident views.

In Exhibit 1, we consider four different cases in Panels A-D. In Panel A, the subjective views are that the first two subportfolios should yield 2% higher returns than indicated in the initial estimates, namely $\mu_1 = 2.58\%$ and $\mu_2 = 3.58\%$. Panel A shows the graphical plot of τ versus π. With the given background data of $\hat{u}$ and Σ and the subjective inputs of *P*, *Q*, and Ω—which together determine the plausible range of the ex ante Sharpe ratio π—the exact π is then fixed precisely by selecting a specific value of τ as shown in the graph. A higher τ indicates less confidence with respect to the initial estimates $\hat{u}$; thus, more confident higher estimates of mean returns in *Q* would lead to higher updated estimates μ^* and hence higher π. This is shown in the upward sloping curve. If the portfolio manager believes that a Sharpe ratio of 0.44 is reasonable, then a value of 0.4 would be appropriate for τ. Too low of a value for τ may in this case produce unrealistically low π values.

Suppose the initial estimates of mean returns were double the values in Panel A, and the subjective μ_1 and μ_2 were also higher by 5%. Then as shown in Panel B of Exhibit 1, the π values would increase to the range of 0.80 to 1.0. With the same inputs used in Panel A, suppose the portfolio is less optimistic, with subjective fixes μ_1 and μ_2 at ¼% below the initial estimates of mean returns. The τ-versus-π relationship is shown in Panel C of Exhibit 1. In this case, the π values are lower in the range, at 0.375 to 0.380. In addition, the curve is now downward sloping because a higher τ indicates less confidence with respect to the initial estimates $\hat{u}$ —and so more confident lower estimates of mean returns in *Q* would lead to lower updated estimates of μ^*, and hence lower π.

Exhibit 1 Relationship between τ & π



Finally, in Exhibit 1, Panel D, we consider a case in which, as in Panel A, the portfolio manager subjectively believes that μ_1 should yield 2% higher than the initial estimate. However, in this case, the manager subjectively believes that μ_2 would underperform by 1% relative to the initial estimate. The resulting t-versus-p curve is shown in Panel D. In this case, for an expected Sharpe ratio to be higher than 0.36, the correct τ value input should be between 0.0 and 0.1.

The four cases illustrate that the value selected for input parameter τ should not be arbitrary. The value is very much linked to the other model inputs of $\hat{u}$, Σ, *P*, *Q*, and Ω and should be chosen to be consistent with a view of the expected Sharpe ratio target or expected portfolio performance.

CALIBRATING λ* FOR RISK MANAGEMENT

The input parameter of λ* or the portfolio manager's idiosyncratic risk aversion should be consistent with aversion to portfolio loss over the same horizon. The risk-aversion parameter λ* does not appear in the Sharpe ratio. This is because the portfolio performance is represented by the slope of the portfolio manager's efficient portfolios in the mean-variance return space, which is not connected to risk aversion. Portfolio managers' risk aversion is only relevant when they have to choose how much to allocate to risk-free assets versus risky assets along this efficient portfolio line.

The portfolio manager can control the risk of the portfolio by requiring the VaR of the portfolio not to exceed percentage amount β of the asset under management at the θ% confidence level. As Chow and Kritzman [2001] noted, VaR is a commonly used metric of risk exposure, and it is both natural and insightful for a portfolio manager to consider the risk budget as part of the process of portfolio optimization when determining the optimal allocations. VaR is the maximum-value loss that can occur with probability (1 − θ)%. VaR is typically expressed as a positive number. Without loss of generality, if we define the initial portfolio value as a unit dollar amount, then

$$\mathrm{VaR} = -(W'_{BL}\mu^* + [1 - W'_{BL}L]r_F + Z_\theta \sigma_P) \quad (3)$$

where $Z_\theta < 0$ is the standard normal value corresponding to a left tail area of (1 − θ)% and is the final portfolio return volatility. From these, we can derive

$$\mathrm{VaR} = -r_F + \frac{\pi}{\lambda^*}\left(|Z_\theta| - \pi\right) = \beta \quad (4)$$

For typical considerations such as θ = 0.975, $|Z_\theta| = 1.96 > \pi > 0$, so β > 0 with small risk-free rate r_F .

As seen in the previous section, once π is fixed with the choice of τ, then given θ, VaR or β decreases with an increase in risk aversion λ*, and vice versa. This is because a portfolio manager with a higher risk aversion λ* would adjust the portfolio so that VaR is smaller. Smaller VaR implies smaller loss at a given confidence level θ. We point out that in addition to λ*, β is also an implicit function of τ via π(τ). For a reasonably large π, given λ* and θ, VaR decreases with an increase in π. Intuitively, this is due to the dominating return distribution with higher portfolio performance, and thus a smaller loss at a given probability 1 − θ. However, if $\pi < \frac{|Z_\theta|}{2}$ is small initially, then marginal increases in π may lead to an increase in VaR because of increased tail spread as well as increased mean. The increased tail spread increases VaR for a given θ.

To illustrate the latter result, and also to check the robustness of our results in Exhibit 1, we create another example for an active portfolio investing in three assets whose return covariance matrix is as follows:

$$\Sigma = \begin{bmatrix} 0.0256 & 0.0159 & 0.0189 \\ 0.0159 & 0.0412 & 0.0334 \\ 0.0189 & 0.0334 & 0.0615 \end{bmatrix} \quad (5)$$

P = [0, -1, +1]’, Q = 0.05, Ω = 0.002, r_F = 0.02, initial estimated mean returns û = [0.0632, 0.0870, 0.1082]’, and θ = 0.975. For this case, the subjective view is that the true return of the third asset should be larger than that of the second asset by 5%, which is larger than the initial estimates indicate. The relationship between τ and π is shown in Exhibit 2, Panel A; and given τ (hence, π), the relationship of λ* and β is shown in Exhibit 2, Panel B.

Exhibit 2 λ* Calibration

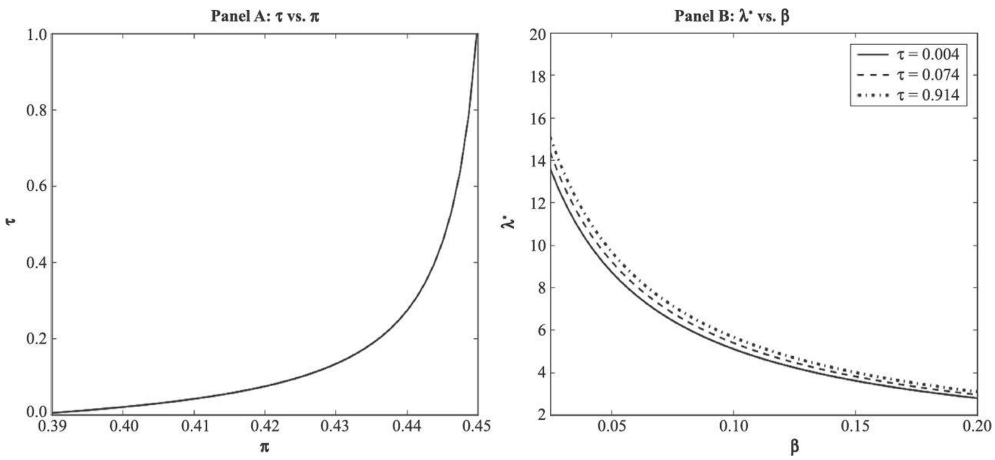


Exhibit 2, Panel A, shows similar results to those of Exhibit 1, Panel A, thus indicating the robustness of results. This is a case in which π is not high. From Panel B of Exhibit 2, we see that if the portfolio manager expects VaR or β to be a maximum of 5% of the portfolio value, the idiosyncratic risk aversion consistent with this is 9.0. If the portfolio manager expects VaR or β to be a maximum of 10% of the portfolio value, the idiosyncratic risk aversion consistent with this is 5.0. If the risk aversion is only 4.0, then VaR can increase to approximately 15% of the original portfolio value.

The curve in Panel B verifies that indeed β decreases with increase in risk aversion λ*. Moreover, because π is not high, for constant λ*, β increases with π, although the sensitivity is low. Thus, λ* is the key dominant determinant of β. If the portfolio manager decides to control risk specifically with a particular level of β, then the appropriate value of λ* should be chosen as input to the Black-Litterman model.

Suppose chosen value for τ is 0.074 such that π = 0.42, then the final true expected return estimate is μ* = [0.0646, 0.0834, 0.1210]'. Clearly, the subjective view of a larger spread between the expected returns of the second and third asset has resulted in a larger spread in this final estimated output μ*. If the chosen value for λ* is 2.5, with τ = 0.074 such that β = 0.20, then the resulting optimal portfolio weights[3] W_{BL} = [0.250, 0.087, 0.1210]'. If the chosen value for λ* is 5 such that β = 0.10, then the resulting optimal portfolio weights W_{BL} = [0.125, 0.043, 0.267]'. Higher risk aversion or higher input value of λ* leads to a lower sum total of weights in the risky assets and a higher remaining weight in the risk-free asset.

AN EXAMPLE OF IMPLEMENTATION

We present a more detailed example to demonstrate the Black-Litterman insights and implementation expounded in the previous sections. We also show how implementing the Black-Litterman model leads to improvement in the portfolio performance provided the subjective views are superior to the historical trend.

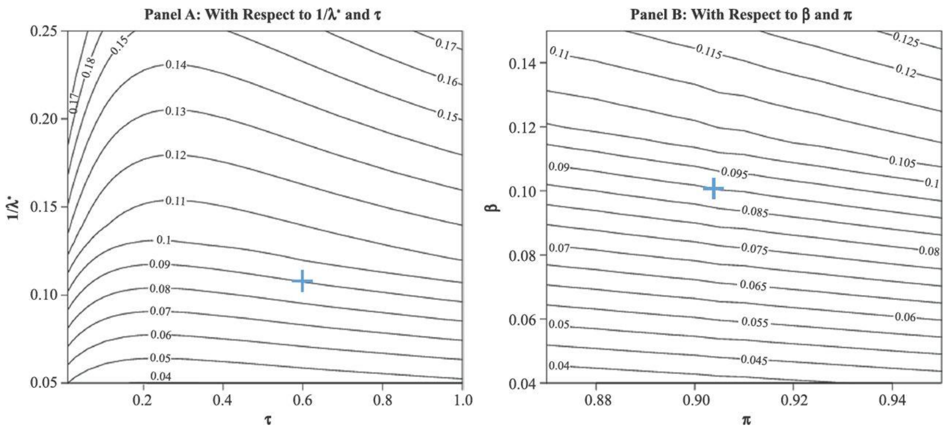
We use monthly return time series from 10 industries in the Kenneth French data library from January 2010 to December 2014 to construct the return covariance matrix Σ. Together with the S&P 500 market index returns and the U.S. Treasury bill rates proxying for risk-free returns, the betas of the 10 industry portfolio returns are computed. Their expected returns $\hat{u}$ based on the Sharpe CAPM are also derived. The market Sharpe ratio is 0.996 for this 5-year period, and the market risk-aversion parameter or excess market return per unit of variance is $\lambda_M = 7.65$. The Markowitz weights based on these initial $\hat{u}$ and λ_M were expected to produce an ex ante portfolio annualized return of 13.33%. If we use the subsequent realized returns of the 10 industrial sectors from January 2015 to July 2015 reported in the French database, the actual ex post return is an annualized 5.39%. The drop is mainly due to a very large fall in the returns across six of the sectors, which occurred at the beginning of what is currently known as the recessionary period prior to the growth slowdown of the People's Republic of China and the market drop worldwide post July 2015.

The annualized returns for 3 of the 10 industrial sectors—namely, durables, manufacturing, and health—are 20.5%, 15.3%, and 9.8%, respectively, based on the CAPM estimates. If the portfolio manager is correct in assessing a downturn in the stock market, he or she may reduce return expectations on durables and manufacturing sectors to 10% each but increase health sector subjective expected return to 15%.

SENSITIVITY OF EX ANTE PORTFOLIO RETURN TO τ, λ^*

By fixing Q = (0.10, 0.10, 0.15)' with a confidence in the subjective estimate variance being just 1% or about 20%-80% of variances in the stock returns, we can compute μ^*, followed by W_{BL} , and then the ex ante portfolio return $\mu_P^* = r_F + \frac{1}{\lambda^*}\pi^2$, which is a function of τ and $\frac{1}{\lambda^*}$. The relationship is shown in Panel A of Exhibit 3. $\mu_p{}^*$ increases in τ when $\tau > 0.25$ (a higher confidence of subjective inputs relative to initial return estimates) and also increases in $\frac{1}{\lambda^*}$ (less risk aversion or more risk-taking) at each τ. τ is typically confined to the range (0,1) because $\tau >$ 1 would imply a noisier estimate than the initial round of estimation, which would be counterintuitive.

Exhibit 3 Ex Ante Return Sensitivity



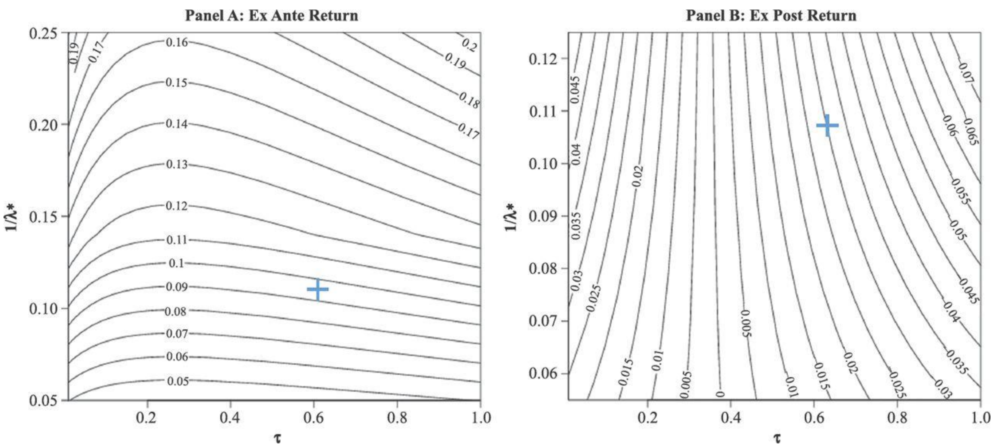
Panel B of Exhibit 3 shows that μ_p^* increases with VaR or β, and also with π. τ = 0.61 is mapped onto π = 0.905, and λ* = 9.1 is mapped onto β = 0.10. We assume that the parameter values τ = 0.61 and λ* = 9.1 are chosen because these imply π = 0.905, which is just a little smaller than 0.996 (for the past 5-year period) given the manager's outlook into the near future is not as optimistic. Choosing a higher τ, hence ex ante π, may not be realistic. The associated VaR or β of 10% also represents the maximum acceptable risk tolerance.

The pairs of parameter values are associated with an ex ante expected portfolio return of 9% per annum, seen at the "+" points on the graphs. Exhibit 3 also shows the important result that any pair of (τ, λ*), or equivalently (π, β), within reasonable ranges can result in a wide range of ex ante expected returns from 4% to 16% given P, Q, Ω, û, Σ, θ, and r_F .

EX POST PORTFOLIO RETURN BASED ON CHOICE OF τ, λ*

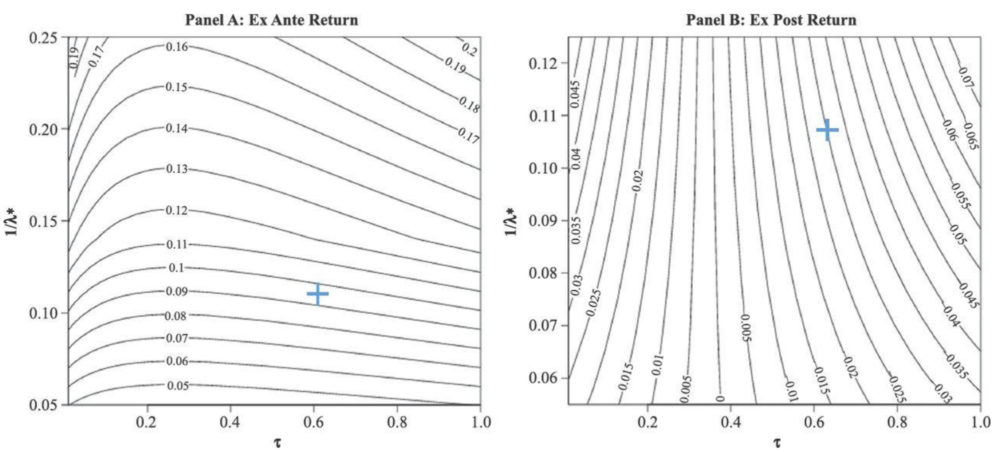
Using the average ex post return per industry group from January 2015 to July 2015, we compute the ex post optimal portfolio using the W_{BL} weights, including the risk-free asset. Because these are mainly asset management portfolios, we introduce a higher cost at 5% of borrowing relative to the risk-free investment rate of 0.02% during the first half of 2015 to prevent overly leveraged portfolios. As we can see in the ex post results shown in Panels A and B of Exhibit 4, the portfolio performs with a 12.5% return given these parameters. This is superior to the Markowitz portfolio return of 5.39%.

Exhibit 4 Ex Post Return Sensitivity



In Exhibit 5, we suppose that the manager's subjective views were not correct. *Q* is now (0.10, 0.20, 0.05)', which incorrectly predicts a stronger manufacturing sector and a weaker health sector. Using the same $\tau = 0.61$ and $\lambda^* = 9.1$ as in Exhibit 4, the ex ante expected return is 9.5%, as seen in Panel A. Because of the incorrect views, the ex post result shown in Panel B instead indicates a portfolio return loss of 3%. In Panel B, it is also seen that when the views expressed in *P* and *Q* and moderated by Ω are incorrect, a much higher τ or $1/\lambda^*$ would lead to a greater loss. Thus, choosing the correct parameter values of τ and λ^* requires consideration of the risks involved and is not a case of maximizing ex ante return based on the inputs.

Exhibit 5 Incorrect Subjective Views



CONCLUSION

The Black-Litterman model is an elegant, insightful extension to the traditional mean-variance optimal portfolio framework. However, one of the key reasons that practitioners have been relatively slow to adopt it is the need to specify a number of ad hoc input parameters. Most portfolio managers would refrain from arbitrarily choosing these parameters because of the lack of clarity and the corresponding ambiguity and uncertainty that arise. On the other hand, most, if not all, active portfolio managers have a more intuitive and better view on their target Sharpe ratio π and VaR measure β for performance and risk control. Therefore, this article proposes an intuitive framework to connect the ad hoc parameters to portfolio performance control and risk calibration. We show how these can be used to calibrate the values of the uncertainty factor τ and the idiosyncratic risk aversion λ^* in a manner that is internally consistent within the Black-Litterman formulation.

Our analyses reveal that the portfolio manager's choice of π uniquely determines the value of τ and that λ^* is predominantly determined by β. We show that realistic choices of π and β give rise to reasonable parameters of τ and λ^*, thereby giving portfolio managers a unique insight in their effort to reconcile their views of the market vis-à-vis the choices of Black-Litterman's parameters. The value of τ is closer to zero if the manager believes that the initial estimates based on historical data are accurate compared to the subjective views; and closer to one if the manager believes that there is a fundamental shift in the market landscape such that past history should not be overly relied upon. We also show that in the event of an incorrect view, an unrealistically high Sharpe ratio and taking too much risk can produce disastrous losses. By unifying parameter calibrations with performance and risk measures, the model is internally consistent and provides a powerful means for practical application.

ENDNOTES

[1]See http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html.

[2]The initial estimated mean returns are found via the security market line in Sharpe's CAPM, where mean or expected return of asset or subportfolio j equals $r_F + b_j E(r_M - r_F)$, where b_j is the CAPM beta and $E(r_M)$ is the expected market return in the investment horizon. An alternative is to use the historical sample means. When the entire universe of stocks in the market is considered in the mean-variance optimization, the initial estimate can also be written as $\hat{\mu} = r_F L + \lambda_M \Sigma W$, where λ_M is the slope of the security market line per unit of market volatility, and W is the historical set of weights.

[3] The three weights on the three risky assets sum up to 0.87, while the remaining 0.13 weight is on the risk-free asset. This portfolio-optimization formulation with total weight of 1 spread across all risky assets as well as the risk-free asset is typical as seen in Ingersoll [1987].

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