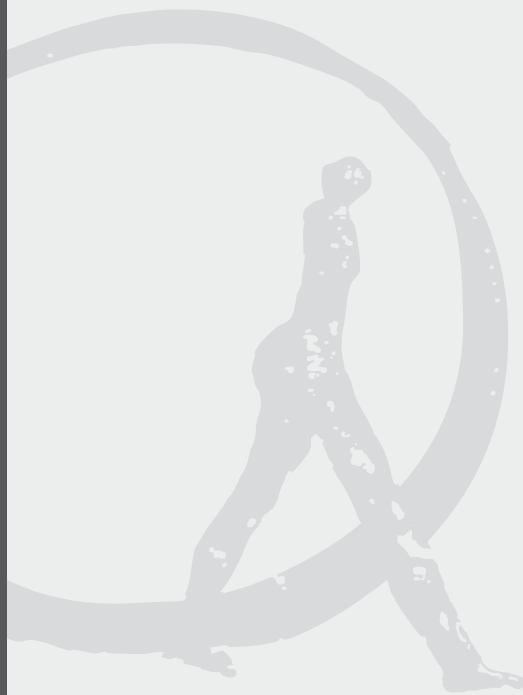


Extending Black-Litterman Analysis Beyond the Mean-Variance Framework

An Application to Hedge Fund Style Active Allocation Decisions



Lionel Martellini

Professor of Finance and Scientific Director of
the EDHEC Risk and Asset Management Research Centre

Volker Ziemann

Research Engineer at the EDHEC Risk and Asset Management Research Centre

Abstract

In this paper, we introduce a suitable extension of the Black-Litterman Bayesian approach to portfolio construction in the presence of non-trivial preferences about higher moments of asset return distributions. We also present an application to active style allocation decisions in the hedge fund universe. Overall the results in this paper suggest that significant value can be added in a hedge fund portfolio through the systematic implementation of active style allocation decisions provided that a sound investment process is implemented that accounts for both non-normality and parameter uncertainty in hedge fund return distributions.

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EDHEC pursues an active research policy in the field of finance. Its "Risk and Asset Management Research Centre" carries out numerous research programs in the areas of asset allocation and risk management in both the traditional and alternative investment universes.

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One of the by-products of the bull market of the 90's has been the consolidation of hedge funds as an important segment of financial markets. It was recently announced that the value of the hedge fund industry worldwide had passed the \$1 trillion mark for the first time, with approximately 7,000 hedge funds in the world.¹ There seem to be two main reasons behind the success of hedge funds in institutional money management. On the one hand, it is argued that hedge funds may provide abnormal risk-adjusted returns, due to the superior skills of hedge fund managers and flexibility in trading strategies. On the other hand, hedge funds seem to provide diversification benefits with respect to other existing investment possibilities. In a nutshell, the diversification argument states that investors can take advantage of hedge funds' linear and non-linear exposure to a great variety of risk factors, including volatility, credit and liquidity risk, etc., to reduce the risk of their overall portfolios.

In an attempt to fully capitalize on there diversification benefits in a top-down approach, investors or (funds of hedge funds) managers must be able to rely on robust techniques for the optimization of portfolios including hedge funds. Standard mean-variance portfolio selection techniques are known to suffer from a number of shortcomings, and the problems are exacerbated in the presence of hedge funds. It has been argued that the two aspects that require specific care are the presence of non-normally distributed assets and of parameter uncertainty.

First, because hedge fund returns are not normally distributed, a mean-variance optimization would be severely ill-adapted, except in the case of an investor endowed with quadratic preferences. That mixing hedge funds with traditional assets leads to a reduction in the volatility of the traditional portfolio with a constant return level has been shown by many (Terhaar et al. 2002), and originates from the fact that hedge funds (at least for some strategies), present low volatility together with low correlation with traditional asset classes.,

However, it can be shown, through a statistical model integrating fatter tails than those of the normal distribution, that minimising the second order moment (the volatility) can be accompanied by a significant increase in extreme risks (Sornette et al. 2000). This is confirmed in Amin and Kat (2003), where the authors find empirical evidence that low volatility is generally obtained at the cost of lower skewness and higher kurtosis. Consequently, as stressed in Cremers et al. (2005), in the presence of asymmetric and/or fat-tailed return distribution functions, the use of mean-variance analysis can potentially lead to a significant loss of utility for investors. As a result of the shortcomings of mean-variance optimization, many attempts have been made to better account for the specific risk features of hedge funds and to extend portfolio optimization techniques to account for the presence of fat-tailed distributions, mostly by introducing some risk objective (*Value-at-Risk* as in Favre and Galeano 2002, *Conditional Value-at-Risk* as in De Souza and Gokcan 2004 and Agarwal and Naik 2004, or the *omega ratio* as in Favre-Bulle and Pache 2003), more general than volatility, and incorporating the presence of non-trivial higher moments in asset returns.

Secondly, the problem of parameter uncertainty needs to be carefully addressed, as the lack of a long history and the non-availability of high frequency data imply that parameter estimation is a real challenge in the case of hedge fund returns. There are many reasons to believe that the main challenge for optimal allocation models is in fact in estimating expected returns, as opposed to higher moments, of asset class returns. First, there is a general consensus that expected returns are difficult to obtain with a reasonable estimation error, and this problem can only be more acute in the hedge fund universe where data is scarce and where a number of performance biases are present (Fung and Hsieh 2000, 2002). What makes the problem worse is that optimization techniques are very sensitive to differences in expected returns, so that portfolio optimizers typically allocate the largest fraction of capital to the asset class for which estimation error in the expected returns is the largest (Britten-Jones 1999 or Michaud 1998). Significant progress on the question of portfolio optimization in the presence of parameter uncertainty has been achieved in the influential work by Black and Litterman (1990, 1992), who introduce, in a static mean-variance setting, a methodology that allows investors to account for uncertainty in their priors on expected returns, expressed in terms of deviation from neutral equilibrium CAPM-based estimates.² On the one hand, the Black-Litterman approach has been found

1 - See the 2004 Alternative Fund Service Review Survey.

2 - For more details on dynamic allocation decisions with parameter uncertainty we refer the reader to Detemple (1985), Dothan and Feldman (1985), and Gennotte (1985), Brennan (1998), Brennan and Xia (2001), Cvitanic et al. (2005), and Xia (2001), in a continuous-time setting, or Kandel and Stambaugh (1996), Pastor and Stambaugh (1999, 2000), Barberis (2000), and Pastor (2000) in a discrete-time setting.

very appealing because of its technical tractability and conceptual simplicity, and has become a widely used tool in the context of active asset allocation decisions. On the other hand, it is not well suited for hedge fund portfolio optimization because of its focus on normally distributed assets.

In other words, while both problems (non-trivial preferences about higher moments of the asset return distribution and the presence of parameter uncertainty) have been studied independently, what is still missing for active style allocation in the hedge fund universe is a model that would take both of these two aspects into account. Our contribution to this literature is precisely to introduce an optimal allocation model that incorporates an answer to both challenges within a unified framework. To this end, we first introduce a suitable extension of the Black-Litterman Bayesian approach to portfolio construction that allows the incorporation of active views of hedge fund strategy performance in the presence of non-trivial preferences about higher moments of hedge fund return distributions. Our work is closely related to a recent paper by Giacometti et al. (2005), who extend the Black-Litterman model to a more general class of return distributions, the stable distributions, and solve the allocation problem for various risk measures including Value-at-Risk and Conditional-Value-at-Risk. We differ from this paper by using a non-parametric approach that can be used for general return distributions. Also related is a paper by Meucci (2006), who describes a procedure for extending the Black-Litterman framework to generic market distributions and shows an application to portfolio management in a thick-tailed environment. This extension of the Black-Litterman model, however, comes at the cost of analytical results, and numerical methods must be used to solve the problem. Because we focus on an approximation of the joint distribution of asset returns in terms of third- and fourth-order moments and co-moments (co-skewness and co-kurtosis), as opposed to the distribution itself, we are able to maintain analytical results.

We also present a numerical application illustrating how investors can use a multi-factor approach to generate such active views and dynamically adjust their allocation to various hedge fund strategies while staying coherent with a long-term strategic allocation benchmark. It is only by taking into account the exact nature and composition of an investor's existing portfolio, as opposed to regarding hedge fund investing from a stand-alone approach, that institutional investors can truly customize and maximize the benefits they can expect from investing in these modern forms of alternative investment strategies. Overall the results in this paper strongly suggest that significant value can be added in a hedge fund portfolio through the systematic implementation of active style allocation decisions, both at the strategic and tactical levels.

1. A BAYESIAN MODEL OF ACTIVE HEDGE FUND STYLE ALLOCATION DECISIONS

We first present a formal model of active style allocation that can be used in the hedge fund universe. We first review the Black-Litterman model, and then extend it to a setup where higher moments of return distribution are taken into account.

Review of the Black-Litterman approach

Since the seminal work of Markowitz (1952) there has been a strong consensus in portfolio management on the trade-off between expected return and risk. In the Markowitz world, the risk is represented by the standard deviation. Given the investor's specific risk aversion, optimal portfolios and the so-called efficient frontier can be derived. Based on this mean-variance approach, Sharpe (1964) and Lintner (1965) designed an equilibrium model, the Capital Asset Pricing Model (CAPM), the aim of which is to describe asset returns. Assuming homogenous beliefs, every investor holds the market portfolio derived from this equilibrium model.

Subsequently, Black and Litterman (1990, 1992) proposed a formal model based on the desire to combine neutral views consistent with market equilibrium and individual active views. They introduce confidence levels on the prior distribution and on individual beliefs and obtain the joint distribution. Based on a Bayesian approach, the expected return incorporates market views and individual expectations. The Black-Litterman approach in its original form can be summarized as the following multi-step process (Idzorek 2004).

Based on the risk aversion coefficient (λ), the historical variance covariance matrix (Σ) and the vector of market capitalization weights (w_M), the vector of implied equilibrium returns in excess of the risk-free rate can be obtained as: $\Pi = \lambda \Sigma w_M$.³ This, of course, is equivalent to using a standard Capital Asset Pricing Model (CAPM). The parameter λ can thus be rewritten as:

$$\lambda = \left(\frac{E(R_M) - R_F}{\sigma_M^2} \right)$$

where the indices F and M denote the risk-free rate and the market portfolio respectively. Individual active views, based on forecasting procedures, for example are then introduced. These k views can be either relative or absolute and are represented in the $k \times 1$ -vector Q . The $k \times n$ -projection matrix P will be used to define these views: $Q = P \cdot R_A$. A confidence level will be associated with each of the views expressed in Q . Thus, the individual beliefs can be described by a normal view distribution: $P \cdot R_A \sim N(Q, \Omega)$. In the same way, the confidence in the equilibrium model and the derived implied returns can be defined. Consequently, we obtain the prior equilibrium distribution: $R_N \sim N(\Pi, \tau \Sigma)$. Following the Bayesian rule the two distributions are combined (Satchell and Scowcroft 2000) to yield the following distribution:

$$R_{BL} \sim N \left(E(R_{BL}), \left[(\tau \Sigma)^{-1} + P' \Omega^{-1} P \right]^{-1} \right)$$

where

$$E(R_{BL}) = \left[(\tau \Sigma)^{-1} + P' \Omega^{-1} P \right]^{-1} \left[(\tau \Sigma)^{-1} \Pi + P' \Omega^{-1} Q \right]$$

(1)

This distribution incorporates both the neutral (equilibrium) and the active views. Taking this expected return as an input, the optimal Black-Litterman portfolio weights (w_{BL}) are then given by: $w_{BL} = (\lambda \Sigma)^{-1} E(R_{BL})$.

An Extension to Higher Moments

While the Black-Litterman model is well-suited to portfolio construction in the context of active asset allocation decisions, it suffers from an important limitation, namely that it is based on the Markowitz model, where volatility is used as the definition of risk. In order to apply this approach to hedge funds portfolio construction, we propose a simple extension of the original Black-Litterman model that relies on the use of the four-moment-Capital Asset Pricing Model, as opposed to the standard CAPM, in the estimation of the market neutral implied views.

The four-moment-Capital Asset Pricing Model is a straightforward extension of the standard CAPM for investors who have non-trivial preferences about higher-order moments of asset returns, which gives the following expression for expected returns μ in excess of the risk-free rate R_0 (Hwang and Satchell 1999):

$$\mu - R_0 = \alpha_1 \beta^{(2)} + \alpha_2 \beta^{(3)} + \alpha_3 \beta^{(4)}$$

with $\beta^{(i)}$ the vectors of portfolio betas defined as:

$$\beta^{(2)} = \frac{\Sigma w}{\mu^{(2)}(R_p)} \quad \beta^{(3)} = \frac{\Omega_w}{\mu^{(3)}(R_p)} \quad \beta^{(4)} = \frac{\Psi_w}{\mu^{(4)}(R_p)} \quad (2)$$

3 - More generally, this reverse-engineering mechanism can be applied to infer the expected return estimate that can support any given asset allocation, and not necessarily an asset allocation corresponding to market cap weightings.

where $\Omega_w (\Psi_w)$ is the vector of co-skewness parameters (co-kurtosis parameters) for the weight vector w (see the appendix for more details). The variables α_i can be understood as the risk premia associated with covariance, coskewness, and cokurtosis respectively.

One possibility to obtain the $\hat{\alpha}_i$ estimates is a simple GLS-regression. However, since our model will be applied to data including hedge funds, for which data is not available at a frequency greater than monthly, we are constrained to small samples. In this case, one is naturally led to mitigate sample risk through the introduction of a specific model, which of course comes at the cost of specification risk. As in Jondeau and Rockinger (2004), we assume a specific representative utility function. Due to its appealing parameter interpretation, we chose the constant absolute risk aversion function $U(W) = -e^{(-\lambda W)}$. The risk premia are then given by (Hwang and Satchell 1999 or Jondeau and Rockinger 2004):

$$\alpha_1 = \frac{\lambda \mu^{(2)}(R_p)}{A} \quad \alpha_2 = -\frac{\lambda^2 \mu^{(3)}(R_p)}{2A} \quad \alpha_3 = \frac{\lambda^3 \mu^{(4)}(R_p)}{6A}$$

with

$$A = 1 + \frac{\lambda^2}{2} \mu^{(2)}(R_p) - \frac{\lambda^3}{6} \mu^{(3)}(R_p) + \frac{\lambda^4}{24} \mu^{(4)}(R_p)$$

The risk aversion parameter λ can be calibrated with respect to historical data. Here we use longterm estimates based on stock return estimates over the 1900–2000 period for 16 countries by Dimson, Marsh, and Staunton (2002) to obtain $\lambda=2.14$, based on a 6.20% risk premium and 17% volatility (see 34–1, page 311 in Dimson, Marsh and Staunton 2002). The various co-moments are estimated over the whole sample period January 1997 to December 2004.

It is important to note that the vectors of portfolio betas as well as the risk premia (α_i) are functions in the weight vector w . Thus, equation (2) gives us a deterministic relationship between the expected returns (left-hand side) and the weighting vector (right-hand side). This equation can be used in two different ways: i) to obtain implied expected return estimates based on exogenous portfolio weights, or ii) to obtain the weight vector based on exogenous expected returns.

In what follows, we will create a strategic style allocation benchmark that will define the “neutral portfolio weights” based on a minimization of the portfolio Value-at-Risk. Using equation (2), these views will then be taken in order to obtain “neutral” expected returns (approach i)). We then introduce a prediction model that will be used to generate active views (expressed in terms of returns). The latter will be incorporated with the neutral views applying the known Bayesian approach (equation (1)). Finally, based on those Black-Litterman returns, equation (2) allows us to obtain the resulting Black-Litterman portfolio weights (approach ii)).

2. DESIGNING A STRATEGIC STYLE ALLOCATION BENCHMARK

The key to the implementation of a structured top-down approach to investing in hedge funds is to properly define a target asset allocation across various hedge fund styles. A variety of investment constraints and objectives need to be taken into account in the design of the target allocation. In particular, the target allocation should be designed so as to allow for an optimal mix with the client's existing stock and bond portfolio.

In the real world, investors are not only interested in maximizing expected return and minimizing volatility, but also in limiting the loss with a given probability $(1-\alpha)$. This is the reason our objective shall focus on a measure like Value-at-Risk at $(1-\alpha)\%$, which is defined as the negative value of the α -quantile of the underlying return distribution. Assuming a normal distribution this measure is simply given by: $Var(1-\alpha) = -(\mu + z_\alpha \sigma)$, with z_α the α -quantile of the standard normal distribution.

We now present a method for optimal allocation decisions to hedge funds that can be used in the absence of

specific views on hedge fund strategy returns when higher moments of hedge fund return distribution need to be taken into account. We focus on the most widely-used set of hedge fund strategies, namely Long/Short Equity, Event Driven, Relative Value, and CTA.⁴ We used the S&P500 index to proxy equity market returns, and a portfolio made up of 20% US Government bonds, 45% mortgage bonds, and 35% corporate bonds to proxy bond market returns (based on corresponding Lehman bond indices), while we use the corresponding the EDHEC Alternative Indexes as proxies for the return on the selected hedge fund strategies.⁵ As exhibit 1 shows, strategies such as Relative Value or Event Driven present low levels of volatility but significantly negative skewness and significantly positive excess kurtosis.

Exhibit 1: Analysis of the return distribution of monthly asset returns (from January 1997 through December 2004)

	Return	Volatility	Skewness Exc.	Kurtosis
Long/Short Equity	0.96%	2.14%	0.07	0.99
CTA Global	0.74%	2.70%	0.10	-0.16
Relative Value	0.81%	0.99%	1.25	3.55
Event Driven	0.93%	1.71%	-2.08	10.49
Proxy Stock Markets	0.63%	4.86%	0.50	0.04
Proxy Bond Markets	0.27%	1.03%	-0.79	1.41

This is the reason we should incorporate higher moments in the Value-at-Risk measure. This can be done through a method called a Cornish-Fisher expansion (Jaschke 2002 for a detailed description), which represents a convenient approximation of distribution percentiles in the presence of non-Gaussian higher moments.

The Cornish-Fisher expansion is derived from the general Gram-Charlier expansion, using the standard normal distribution as the reference function. For a four-moment approximation of α -percentiles the following formula is given by:

$$\tilde{z}_\alpha = z_\alpha + \frac{1}{6}(z_\alpha^2 - 1)S + \frac{1}{24}(z_\alpha^3 - 3z_\alpha)K - \frac{1}{36}(2z_\alpha^3 - 5z_\alpha)S^2$$

where S denotes the portfolio skewness, K the portfolio excess kurtosis and z_α the α -percentile of the standard normal distribution. \tilde{z}_α denotes the modified α -percentile.⁶ We then obtain the modified Value-at-Risk measure with confidence $(1-\alpha)$: $\text{VaR}_{\text{mod}}(1-\alpha) = -(\mu + \tilde{z}_\alpha \sigma)$, where σ (respectively, μ) denotes the portfolio standard deviation (respectively, expected return).

We also impose the following portfolio constraints: not less than 5% and not more than 40% is allocated to a single hedge fund strategy. It has actually been argued (Jagannathan and Ma 2003) that the presence of portfolio constraints, in addition to avoiding corner solutions in optimization techniques, allows one to achieve a better trade-off between specification error and sampling error similar to what can be achieved by statistical shrinkage techniques (see Jorion 1986 and Ledoit 1999).

We rebalance the optimally designed portfolio every 3 months, using a 36-month rolling window analysis in the optimization procedure. We further assume that the investor is not willing to allocate more than 15% to hedge funds. Since the history of the EDHEC Alternative Indexes starts in January 1997, the first calibration period was January 1997 – December 1999. We therefore obtain out-of-sample returns for the optimally designed portfolios from January 2000 onward.

In what follows this portfolio will be used as a strategic benchmark, from which neutral expected return estimates will be obtained.

4 - According to CSFB Tremont, these four broad categories made up 91 percent of assets under management in the hedge fund industry at the end of 2003. Likewise, in the CISDM hedge fund database, funds in these strategies constitute 85 percent of total assets under management by single hedge funds.

5 - For more information, see www.edhec-risk.com, where monthly data on EDHEC Alternative Indexes can be downloaded, as well as Amenc and Martellini (2003). In the case of Global Macro and CTA, we construct a portfolio that is equal weighted in the indexes for these two strategies.

6 - In what follows, we will use a confidence level of 95%.

3. GENERATING ACTIVE VIEWS ON HEDGE FUND STRATEGIES AND APPLICATION TO ACTIVE STYLE ALLOCATION DECISIONS

Investors and fund of hedge funds managers typically change their allocation to various hedge fund strategies in response to changes in their expectations about expected returns on these strategies. We now explain how the extended Black-Litterman model can be used to add the benefits of active style rotation decisions to the benefits of strategic allocation decisions.

There is now a consensus in empirical finance that expected asset returns, as well as variances and covariances, are, to some extent, predictable. Pioneering work on the predictability of asset class returns in the US market was carried out by Keim and Stambaugh (1986), Campbell (1987), Campbell and Shiller (1988), Fama and French (1989), and Ferson and Harvey (1991). Subsequently, some authors investigate this phenomenon on an international basis by studying the predictability of asset class returns in various national markets (Bekaert and Hodrick 1992, Ferson and Harvey 1993, 1995, Harvey 1995, and Harasty and Roulet 2000). While there has been a significant amount of research on the predictability of traditional asset classes, and the implications in terms of tactical asset allocation strategies, until recently very little was known about the predictability of returns emanating from alternative vehicles such as hedge funds. In a recent paper, Amenc, El Bied, and Martellini (2003) examine (lagged) multifactor models for the return on nine hedge fund indexes. The factors are chosen to measure the many dimensions of financial risks: market risks (proxied by stock prices, interest rates and commodity prices), volatility risks (proxied by implicit volatilities from option prices), default risks (proxied by default spreads) and liquidity risks (proxied by trading volume). They show that a parsimonious set of models captures a very significant amount of predictability for most hedge fund styles. They also find that the benefits in terms of tactical style allocation portfolios are potentially very large. Even more spectacular results are obtained both for an equity-oriented portfolio mixing traditional and alternative investment vehicles, and for a fixed-income-oriented portfolio mixing traditional and alternative investment vehicles.

The conclusion from this research is that there is at least as much evidence of predictability in hedge fund returns as there is in stock and bond returns. In what follows, we present the results of a simple experiment showing how the presence of predictability in hedge fund returns can be exploited in the context of active asset allocation. In an attempt to alleviate the risk of model specification, we perform a simple in-sample analysis using the historical relationship between a set of factors and hedge fund returns. The spirit of the experiment we now conduct is not in fact emphasize to on a specific econometric process that could be used to generate signals about hedge fund style returns but to point out that sophisticated and reliable active asset allocation models are available that can be used by investors or managers who have active views on the performance of hedge fund strategies.

Forming Active Views on Hedge Fund Returns

We now propose an approach in order to detect individual active views on various hedge fund styles. For each hedge fund strategy, we obtain a bullish, bearish or neutral view on the expected return based on a lagged factor analysis. The predictive factors we use are:

- Implied Volatility (VIX) - CBOE SPX Volatility VIX
- First differences of the implied volatility
- Commodity Index - Goldman Sachs
- Term Spread - Lehman US Treasury 5-7 years minus Lehman US Treasury 1-3 years
- Credit Spread - Lehman US Universal: High Yield Corp. Red Yield minus Lehman US Treasury 1-3 years
- Value vs. Growth - S&P 500 Barra/Value minus S&P 500 Barra/Growth
- Small Cap vs. Large Cap - S&P 600 Small Cap minus S&P 500 Composite
- S&P 500 Composite return
- T-Bill - Merrill Lynch T-Bill 3 month
- US Dollar - US MAJOR CURRENCY MAR73=100 (FED) EXCHANGE INDEX
- Bond return volatility - calculated over one month Lehman US Aggregate returns

The approach we take is based on a simple univariate conditional factor analysis. The idea is to consider each factor separately and to estimate the correlation between the hedge fund returns and the three-month lagged factor values. Over the sample period from October 1996 to September 2004, the values for each factor are classified into three different states of nature based on the corresponding distribution: low (0%-33%), medium (33%-66%) and high (66%-100%). Next, we consider the hedge fund returns for each strategy, conditional on the state of nature of the three-month lagged factor values. One of the letters H (high), M (medium) or L (low) is associated with each combination, as a function of the difference between the conditional and the unconditional mean:⁷

- H indicates that the conditional mean return on the strategy is significantly greater than the unconditional one, suggesting a bullish view according to the factor.⁸
- L indicates that the conditional mean is significantly smaller than the unconditional one, suggesting a bearish view according to the factor.
- M indicates that there is no significant difference between conditional and unconditional mean.

We report the result of this analysis in Exhibit 2.

	Implied Volatility			Diff. Implied Volatility			Commodity Index			Term Spread			Credit Spread			Value minus Growth			Small minus Large			S&P 500			T-Bill			US Dollar			Lehman Bond return		
	low	medium	high	low	medium	high	low	medium	high	low	medium	high	low	medium	high	low	medium	high	low	medium	high	low	medium	high	low	medium	high	low	medium	high	low	medium	high
Long/Short Equity	M	M	M	L	M	M	M	M	M	M	H	L	M	H	L	H	M	L	H	M	M	L	H	M	M	M	M	M	M	M	M	M	M
CTA Global	H	M	L	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	H	L	M	M	M	M	M	M	M	M	H	M	M	M	L
Relative Value	L	M	M	L	M	M	M	M	M	M	H	L	M	H	L	H	M	M	M	M	M	L	M	M	L	M	H	M	M	M	M	M	M
Event Driven	L	M	M	M	M	H	M	M	M	M	H	M	M	H	M	H	M	M	M	M	M	L	H	M	M	M	M	M	M	M	M	M	M

This suggests, for example, that when value and size spreads are tight, when markets are posting average returns, or when term and credit spreads are narrow, long/short equity funds tend to achieve high returns 3 months later. This is consistent with the fact that these funds typically hold—for liquidity issues—long positions in small companies (growth stocks) and short positions in large companies (value stocks). Similarly, when stock markets are moving strongly in one direction (i.e., bull or bear), the correlation between individual stocks tends to increase, producing profits, since returns generated on the long positions are offset by those generated on short positions. Finally, since small companies and growth stocks tend to show greater sensitivity to credit and term spreads, long/short equity funds are negatively exposed to the widening of these spreads.

Conditionally on the state of nature of the different factors, active views on hedge fund returns are determined by the combination of individual factor influences. More specifically, we assign the value +1 to H (high), the value -1 to L (low) and the value 0 to M and calculate the sum S of these numbers over all factors. The view on the strategy is bullish if this number is positive (indicating more H occurrences than L occurrences for the strategy at a given date) and bearish if it is negative (indicating more L occurrences than H occurrences for the strategy at a given date). The corresponding value Q_i in the view-vector will be defined as: $Q_i = (1 + x \cdot \text{sign}(S))$, where Π_i is the implied expected return on strategy i . This scheme implies that a bullish active bet is obtained (with respect to a neutral estimate) when S is positive, and a bearish view is obtained (with respect to a neutral estimate) when S is negative.

The confidence in our view will be determined by the corresponding i^{th} diagonal value Ω_{ii} in the variance-covariance matrix of the view-distribution, which we define by:

7 - Each combination is characterized by a strategy, a factor, and a state of nature. With four strategies, eleven factors, and three states of nature, we obtain 132 combinations.
8 - We use a 20% significance level so as to generate a reasonably large number of H and L values.

$$\Omega_{ii} = \left(1 - \frac{|S|}{K}\right) \Sigma_{ii}$$

where K is the total number of factors (11 in this case). Based on this formulation, it appears that if all factors agree on a bullish or bearish view for a given strategy, then $|S| = K$ and the uncertainty on the view goes to zero, as is normal since confidence is greatest in that case. Note also that we use the variance of the returns on the hedge fund strategy Σ_{ii} as a multiplicative scaling factor. The intuition is that an investor perceives more uncertainty on a view that relates to a riskier asset, as it is more likely to experience large moves in a given interval of time. Finally, the matrix P is defined so that each line corresponds to an active view and consists of zeros in all but the i th place (view on the i th asset).

The parameters α and τ determine the relative weight between neutral portfolio and view-adjusted weights, and these degrees of freedom can be adjusted to generate an active portfolio that will deviate more or less with respect to the benchmark portfolio. More specifically, since the standard deviation of the individual view is perfectly determined by the above relationship, we use the parameter τ in order to calibrate the model in terms of the trade-off between neutral and active views. The higher τ , the lower the relative confidence in the neutral view and the more weight is put on individual beliefs. Since the parameter α has a function similar to the parameter τ , we choose to fix the former arbitrarily at 40% whereas the value for τ will be discussed in the application below.

Implications for Active Style Allocation Decisions

Our opportunity set is composed of the same four hedge fund indices as before. The sample period is from January 1997 to December 2004. We use the optimal minimum VaR portfolio obtained previously as the reference benchmark portfolio from which neutral implied expected return estimates are extracted. As an attempt to take into account higher moments in hedge fund return distributions, as well as preferences as to these higher moments, we apply the fourmoment portfolio selection model presented above, and use the active views generated by applying the afore-mentioned prediction procedure.

We first use equation (2) to obtain the implied returns (Π) derived from the "neutral" weights of the minimum Value-at-Risk portfolio (w_N): $\Pi - R_o = \alpha_1 \beta^{(2)} + \alpha_2 \beta^{(3)} + \alpha_3 \beta^{(4)}$ (4), where alphas and betas are functions in w_N (see the appendix for more details). Having defined P , Q and Ω we apply equation (1) in order to obtain the Black-Litterman return vector $E(RBL)$. Finally, equation (2) will be used in order to obtain the resulting Black-Litterman portfolio: $E(R_{BL}) - R_o = \alpha_1 \beta^{(2)} + \alpha_2 \beta^{(3)} + \alpha_3 \beta^{(4)} + \varepsilon$, where alpha and beta parameters are functions of the Black-Litterman weight vector obtained by minimizing the sum of squared residuals ($SSR = \varepsilon' \varepsilon$).

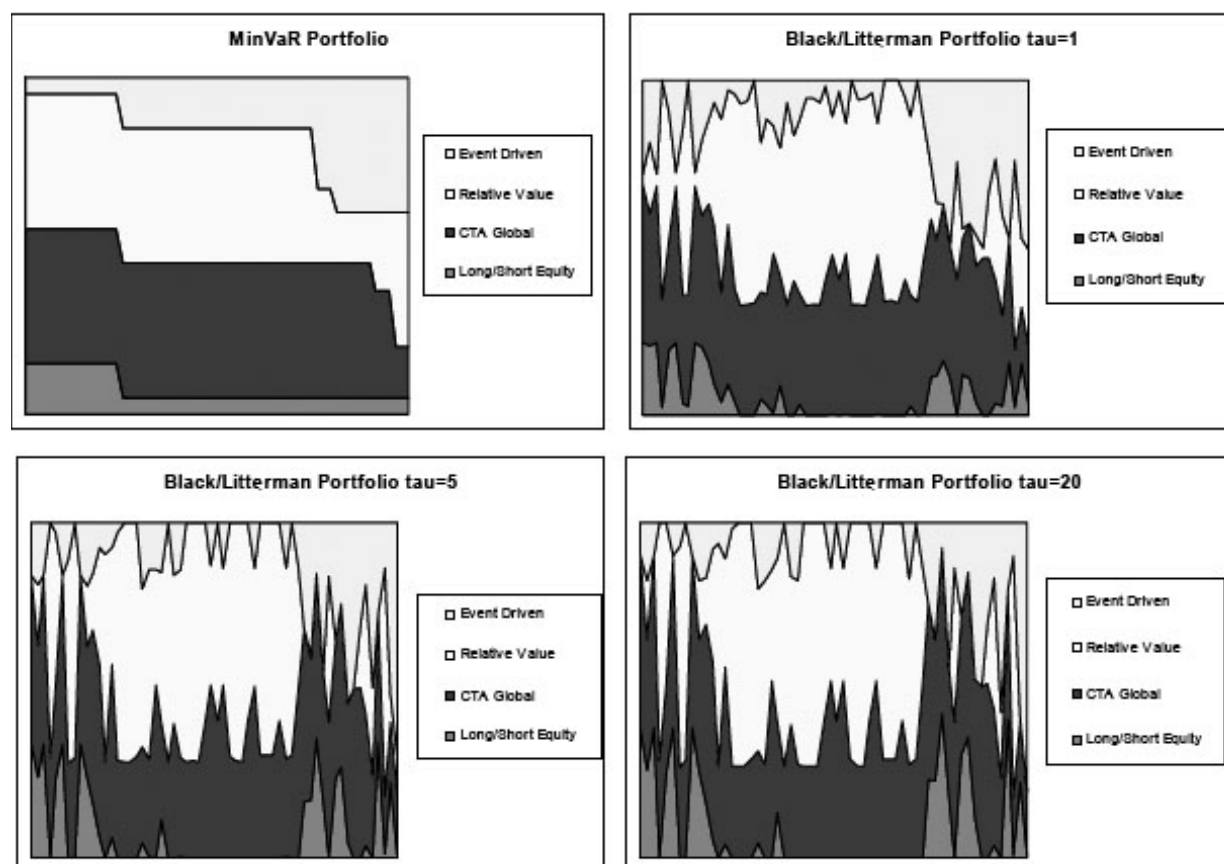
All elements except the parameter τ have already been determined. Our approach to this parameter consists of controlling the tracking error of the portfolio with respect to the benchmark strategic style allocation portfolio. We have considered three different Black-Litterman active portfolios associated with different levels of tracking error. The corresponding parameter values τ are 1, 5, and 20.

Exhibit 3. Performance of Active Allocation Strategies

	minVaR Portfolio	B&L Portfolio $\tau = 1$	B&L Portfolio $\tau = 5$	B&L Portfolio $\tau = 20$
Mean annual return	8.79%	9.79%	10.48%	10.71%
Volatility	4.29%	4.32%	4.39%	4.46%
VaR (95%)	1.33%	1.21%	1.17%	1.19%
Sharpe ratio ($r=3\%$)	1.58	1.80	1.93	1.95
Tracking error		0.86%	1.24%	1.37%
Information ratio		1.17	1.37	1.41

Exhibit 3 presents summary statistics for the corresponding portfolios. The active style selection process, combined with the Black-Litterman portfolio selection method, allows significant out-performance without a large increase in tracking error, as can be seen from the information ratio values. The excess performance and the tracking error, increase in τ , as expected.

Exhibit 4: Portfolio allocations



It should be emphasized at this point that the results presented here abstract away from a number of issues. First, because we have performed an in-sample factor analysis of hedge fund returns, we do not claim that the process used in this paper to generate active style allocation views is efficient and should be applied in practice. We believe instead that the implementation of dynamic asset allocation strategies requires the use of a more sophisticated econometric process, which should put great care on avoiding the pitfalls of data mining. Besides, our analysis does not take into account the presence of transaction costs. Because of significant shifts in asset allocation (as evidenced in exhibit 4), we expect such frictions to be a major issue when implementing a style rotation strategy in the hedge fund universe. It is clearly the case that investable media used in the process should display a high level of liquidity.

Overall, the purpose of the exercise was not to promote a specific style rotation strategy, but rather to illustrate the fact that, given a set of active views on hedge fund strategy returns, which may come from either a qualitative or a quantitative analysis, optimal active asset allocation decisions can be implemented on the basis of a sound and sophisticated investment process.

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APPENDIX

In this appendix are presented more details about the derivation of the 4-moment CAPM. Going back to the basics of utility-maximization, we note that the mean-variance approach only represents a second-order approximation of a general utility function in case of non-normal asset returns. In order to take higher moments into account, we consider any arbitrary utility function.

The investor is assumed to be maximizing the utility emanating from wealth invested in a portfolio with return denoted by R .

The fourth-order Taylor expansion is written as:

$$U(R) = \sum_{k=0}^4 \left[\frac{U^{(k)}(E(R))}{k!} (R - E(R))^k \right] + o[(R - E(R))^4]$$

where $U^{(k)}$ denotes the k^{th} derivative of the function U . Taking the expectation on both sides yields:

$$E[U(R)] \approx U(E(R)) + \frac{U^{(2)}(E(R))}{2} \mu^{(2)}(R) + \frac{U^{(3)}(E(R))}{6} \mu^{(3)}(R) + \frac{U^{(4)}(E(R))}{24} \mu^{(4)}(R)$$

with the centralized moments:

$$\begin{aligned} \mu^{(2)}(R) &= E[(R - E(R))^2] \\ \mu^{(3)}(R) &= E[(R - E(R))^3] \\ \mu^{(4)}(R) &= E[(R - E(R))^4] \end{aligned}$$

Note that the definitions of higher order central portfolio moments can be used to express in turn the variance $Var(R)$, skewness $S(R)$ and (excess) kurtosis $K(R)$ in the following way:

$$\begin{aligned} Var(R) &= \mu^{(2)}(R) \\ S(R) &= \frac{\mu^{(3)}(R)}{[\mu^{(2)}(R)]^{3/2}} \\ K(R) &= \frac{\mu^{(4)}(R)}{[\mu^{(2)}(R)]^2} - 3 \end{aligned}$$

Thus, we can approximate any utility function of a portfolio return as a function of expected portfolio return and standard deviation, but also as a function of third and fourth moments of the portfolio return distribution.

The new maximization problem yields: $\max_{\omega} \Phi(\mu(R_p), \mu^{(2)}(R_p), \mu^{(3)}(R_p), \mu^{(4)}(R_p))$,

such that $\sum_{i=1}^n w_i = 1 - w_0$, with:

$$\begin{aligned} \mu(R_p) &= E(w'R) + w_0 R_0 = w'E(R) + w_0 R_0 \\ \mu^{(2)}(R_p) &= w'E[(R - E(R))(R - E(R))]'w = w'\Sigma w \\ \mu^{(3)}(R_p) &= w'E[(R - E(R))(R - E(R)) \otimes (R - E(R))]'(w \otimes w) = w'\Omega_w \\ \mu^{(4)}(R_p) &= w'E[(R - E(R))(R - E(R)) \otimes (R - E(R)) \otimes (R - E(R))]'(w \otimes w \otimes w) = w'\Psi_w. \end{aligned}$$

and where \otimes denotes the Kronecker-product, $R=(R_1, \dots, R_n)'$ the vector of asset returns, $w=(w_1, \dots, w_n)'$ the vector of portfolio proportions invested in these assets, w_0 the position held in the risk-free asset (with return R_0) and

$\mu=(\mu_1,...,\mu_n)'$ the mean return vector. Ω_w is the vector of co-skewnesses for the weighting vector w and Ψ_w the vector of co-kurtosises respectively. We take this matrix representation from Jondeau and Rockinger (2004) who used it in a different context.

Solving the problem as in Hwang and Satchell (1999) yields equation (2).