Optimization

Optimization Joke

An engineer, a physicist, and a mathematician are shown a pasture with a herd of sheep, and told to put them inside the smallest possible amount of fence. The engineer is first. He herds the sheep into a circle and then puts the fence around them, declaring "a circle will use the least fence for a given area, so this is the best solution."

Optimization Joke

The physicist is next. She creates a circular fence of infinite radius around the sheep, and then draws the fence tight around the herd, declaring, "This will give the smallest circular fence around the herd."

Optimization Joke

The mathematician is last. After giving the problem a little thought, he puts a small fence around himself and then declares,"I define myself to be on the outside."

What is Optimization?

- An Optimization problem is a problem of the form:
- Minimize f(x) such that
  - $L_1 < g_1(x) < U_1$ and
  - ...
  - $L_n < g_n(x) < U_n$

What about Maximization?

- A Maximization problem is a minimization problem where you subsitute -f(x) for f(x)
- Example: Maximize the area of a rectangle with sides of length x and 1-x
  - f(x) = x(1-x) = - x^2 + x
  - f'(x) = -2x + 1 = 0 -> x = 1/2
- This is the same as Minimize the -(area of a rectangle) with sides of length x and 1-x
  - -f(x) = -x(1-x) = x^2 – x
  - -f'(x) = 2x - 1 = 0 -> x = 1/2

Optimization Example

- Two objects are 1cm apart. The first object has a charge of 2C while the second has a charge of -1C. If the force at a point near a charged object is of the form F = q/r^2, where q is the charge and r is the distance, where is the point that has the least amount of force? Most amount of force? What if it was 3C and 2C instead?

Optimization Example 2

- You are trying to take a piece of glass around a right angle turn in a hallway. If the first hallway is 2m wide, the adjoining hallway is 1.5m wide and the ceiling is 3m high, what is the biggest (most surface area) piece of glass that you can fit around the cornner without breaking it?

Optimization Example 4

- Given 400 identical coins of unit radius, what is the size of a rectangle with the smallest perimeter that can fit all of the coins?
  - Note coins can't overlap
- Parallelogram?
- Triangle?
- Circle?
- Does the number of coins make a difference?

Optimization Example 3

- Your average grade so far for a class is 79 and you only have a final that is worth 10% of your grade. You know that the class average is a 75 and you also know that the standard deviation for the class is 15. You know that your grade is normalized, i.e.
  - Grade = (score-average)/standard deviation
- If you get an 87 on the final, what is the worst normailized grade you can get?
  - What is the best grade you can get?

Questions?