Related Rates

What are Related Rates

- This ususally occurs when two functions of time have a relation to each other
  - Like the radius of a circle and its area
  - Or the legs of a triangle and its hypotenuse
- Let's try some problems to see how to solve them

Related Rates Example 1

- The sides of a square increase at a rate of 1 cm/s
  - What rate is the area increasing when the sides are 10 cm long?
  - What rate are the diagonals growing at the same time?

Related Rates Example 1

- The sides of a square increase at a rate of 1 cm/s
  - What rate is the area increasing when the sides are 10 cm long?
    - A(t) = x(t)^2
    - dA(t)/dt = 2 x(t) dx(t)/dt
    - dA(t)/dt = 2 (10 cm) (1 cm/s) = 20 cm^2/s
  - What rate are the diagonals growing at the same time?
    - D(t) = 2^(1/2) x(t)
    - D'(t) = 2^(1/2) x'(t) = 2^(1/2) (1 cm/s) = 2^(1/2) cm/s

Related Rates Example 2

- If the surface area of a sphereical balloon increases more than 1 cm^2/s it will pop
  - What is the the fastest you can pump the balloon without it poping when it is 10cm in radius?? (in cm^3/s)

Related Rates Example 2

- If the surface area of a sphereical balloon increases more than 80 π cm^2/s it will pop
  - What is the the fastest you can pump the balloon without it poping when it is 10cm in radius? (in cm^3/s)
  - S(t) = 4 π r(t)^2 A(t) = 4/3 π r(t)^3
  - S' = 8 π r r' A' = 4 π r^2 r'
  - 80π cm^2/s = 8π(10cm)r' A' = 4 π 100 cm^2 r'
  - r' = (80 π cm^2/s) / (80 π cm)
  - r' = 1 cm/s A' = 4 π 100 cm^2 1 cm/s
  - A' = 400 π cm^3/s

Related Rates Example 3

- A 1.5m person walks 1 m/s towards a streetlight that is 4m above the ground. What is the rate of change in the length of the person's shadow when they are 6m away from the base of the light? How fast is the tip of their shadow moving?

Related Rates Example 3

- $4 / (x(t) + L(t)) = 1.5 / L(t)$
- $L = 3/5\ x$
- $L' = 3/5\ x'$
- $L' = 3/5\ (-1\ m/s)$
- $L' = -3/5\ m/s$

- $y = x + L$
- $y' = x' + L'$
- $y' = -1\ m/s - 3/5\ m/s = -8/5\ m/s$

Questions?