

# Mean Value Theorem

# Rolle's Theorem

If  $\mathbf{f}(\mathbf{x})$  is continuous and differentiable between  $[\mathbf{a}, \mathbf{b}]$  where  $\mathbf{f}(\mathbf{a}) = \mathbf{f}(\mathbf{b})$ ,  
then there is at least one point,  $c$ ,  
between  $\mathbf{a}$  and  $\mathbf{b}$  such that  $\mathbf{f}'(c) = 0$

# Mean Value Theorem

If  $\mathbf{f(x)}$  is continuous and differentiable between  $[a, b]$ , then there is at least one point,  $\mathbf{c}$ , between  $\mathbf{a}$  and  $\mathbf{b}$  such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

# So why do we care?

- Used to prove more useful theorems
- Examples
- If  $\mathbf{f}$  is differentiable then,  $\mathbf{f}'(\mathbf{x}) = 0$  iff  $\mathbf{f}(\mathbf{x}) = \mathbf{c}$
- If  $\mathbf{f}'(\mathbf{x}) = \mathbf{g}'(\mathbf{x})$  for all  $\mathbf{x}$  in  $[\mathbf{a}, \mathbf{b}]$  then  $\mathbf{f}(\mathbf{x}) = \mathbf{g}(\mathbf{x}) + \mathbf{c}$

# Example

- On certiant highways there are toll boths set up every 10 miles. If the speed limit is 60 mph on this stretch of road then is it logical that they give you a speeding ticket if you make it between the toll booths in less than 10 minutes?

Questions?