Trig Derivatives

Rules of Derivatives so far

- Power Rule

$$\frac{d}{dx}x^n = n\,x^{n-1}$$

- Exponent Rule

$$\frac{d}{dx}b^x = \ln(b)\,b^x$$

- Contant Rule

$$\frac{d}{dx}(a\,f(x)) = a\,f'(x)$$

- Sum/Difference Rule

$$\frac{d}{dx}(f(x) \pm g(x)) = f'(x) \pm g'(x)$$

Product Rule

$$\frac{d}{dx}(f(x)g(x)) = f'(x)g(x) + f(x)g'(x)$$

Quotent Rule

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}$$

Sin and Cos

$$\frac{d}{dx}\sin(x)=\cos(x) \qquad \frac{d}{dx}\cos(x)=-\sin(x)$$

$$\frac{d}{dx}-\sin(x)=-\cos(x) \qquad \frac{d}{dx}-\cos(x)=\sin(x)$$

- Look at graphs:
  - Sin's slope starts at 1 going down
  - Cos's slope starts at 0 going down

Tangent

$$\frac{d}{dx}\tan(x)=\frac{d}{dx}\left(\frac{\sin(x)}{\cos(x)}\right)=\frac{\cos(x)\cos(x)+\sin(x)\sin(x)}{\cos^2(x)}$$

$$1+\frac{\sin^2(x)}{\cos^2(x)}=1+\tan^2(x)=sec^2(x)$$

- Just an application of the quotent rule

Other Trig Functions

$$\frac{d}{dx}\cot(x) = -csc^2(x) \qquad \frac{d}{dx}sec(x) = sec(x)\tan(x)$$

$$\frac{d}{dx}csc(x) = -csc(x)\cot(x)$$

$$\frac{d}{dx}\sin^{-1}(x) = \frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}\cos^{-1}(x) = \frac{-1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\tan^{-1}(x) = \frac{1}{x^2+1}$$

Examples

$$\frac{d}{dx}\sin^2(x)=\frac{d}{dx}\sin(x)\sin(x)=\cos(x)\sin(x)+\cos(x)\sin(x)=2\cos(x)\sin(x)$$

$$\frac{d}{dx}\cos^2(x)=\frac{d}{dx}\cos(x)\cos(x)=-\sin(x)\cos(x)-\sin(x)\cos(x)=-2\cos(x)\sin(x)$$

$$\frac{d}{dx}\tan^2(x)=\frac{d}{dx}\tan(x)\tan(x)=sec^2(x)\tan(x)+sec^2(x)\tan(x)=2\,sec^2(x)\tan(x)$$

Questions?

Chain Rule

$$\frac{d}{dx} f\left(g\left(x\right)\right) = f'\left(g\left(x\right)\right) g'\left(x\right)$$

$$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$$

- Deriviative of the outside leave the inside, times the deriviative of the inside

Chain Rule Example

$$\frac{d}{dx}e^{(x^2)}=\left(\frac{d}{du}e^{u}\right)u'=e^{u}2\mathrm{x}=2\,x\,e^{(x^2)}$$

$$\frac{d}{dx}\sin\left(x^5\right)=\left(\frac{d}{du}\sin u\right)u'=\cos\left(u\right)5\mathrm{x}^4=5\,x^4\cos\left(x^5\right)$$

$$\frac{d}{dx}\sin\left(\cos\left(\tan\left(x\right)\right)\right)=\left(\frac{d}{du}\sin\left(u\right)\right)\frac{d}{dx}\cos\left(\tan\left(x\right)\right)$$

$$=\left(\frac{d}{du}\sin\left(u\right)\right)\left(\frac{d}{dv}\cos\left(v\right)\right)v'=\cos\left(u\right)\left(-\sin\left(v\right)\right)sec^2\left(x\right)$$

$$=-\cos\left(\cos\left(\tan\left(x\right)\right)\right)\sin\left(\tan\left(x\right)\right)sec^2\left(x\right)$$

Questions?

More Chain Rule on Friday