

Graphical Derivative

What a derivative looks like?

- Since derivative are the slope of a function this tells us wether a function is going up or down
- If the derivative is positive, then the function must be going up as x gets larger
- If the derivative is negative, then the function must be going down as x gets larger
- If the derivative is zero then the function must be horizontal as x gets larger

2nd Derivatives?

- The second derivative is the slope of the slope of the function
- This tells us how much the slope is changing
- A positive 2nd derivative tells us that the slope is getting more positive as x gets larger
- A negative 2nd derivative tells us that the slope is getting more negative as x gets larger
- A zero 2nd derivative tells us that the slope is not changing as x gets larger

Local Extrema

- Using these properties we can find where a function "maxes out" or "bottoms out" locally
- Extreme points only occur when the slope is zero
- We can find these points by solving $f'(x) = 0$
- Example:
 - Find extrema of $f(x) = Ax^3 + Bx^2 + Cx + D$
 - $f(x) = 1/(x^2 + 1)$

Local Extrema Meaning

- To figure out what kind of points we have we can use the 2nd Derivative test
- If the 2nd Derivative is positive, then we must have a minimum point
- If the 2nd Derivative is negative, then we must have a maximum point
- If the 2nd Derivative is zero, then we fail the 2nd Derivative test

•2nd Derivative test Failure

- We can use another test
- Take n derivatives until you get a constant, c
 - If n is odd then you have an Inflexion Point
 - If n is even and $c > 0$, then you have a Minimum
 - If n is even and $c < 0$, then you have a Maximum
- Examples:
 - $x^3 \rightarrow 3x^2 \rightarrow 6x \rightarrow 6 \rightarrow 0$, $n = 3$, Inflexion Point
 - $x^4 \rightarrow 4x^3 \rightarrow 12x^2 \rightarrow 24x \rightarrow 24$, $n=4$, $24 > 0$, Minimum

Global Extrema

- Now that we classified all of the points we can find the biggest maximum and the smallest minimum
- Don't forget to include the endpoints if you are looking on an interval
- The biggest maximum will give you the global maximum on the domain
- The smallest minimum will give you the global minimum on the domain

Questions?