Functions and Sets

What is a Set?

- A Set is a collection of things
- A Set can't contain duplicates
- Ususally denoted in {}
- Examples:
  - {Apple, Orange, Bananna}
  - {Chairs in this room}
  - {1,2,3,4,5,6,7,8,9,...}
  - {} The empty set is a set
  - {1, 1, 1, 1, 1, 1} This is not a set

Sets we typically use in Math

- Natural Numbers ($\mathbb{N}$) = {1,2,3,4,5,...}
- Integers ($\mathbb{Z}$) = {...,-3,-2,-1,0,1,2,3,...}
- Rational Numbers ($\mathbb{Q}$) = {p/q | p,q $\in \mathbb{Z}$ and q ≠ 0}
- Real Numbers ($\mathbb{R}$) = Rationals + more stuff

What can we do with Sets?

- We can find the Union
  - $\{1,2,3\} \cup \{3,4,5\} = \{1,2,3,4,5\}$
- We can find the Intersection
  - $\{1,2,3\} \cup \{3,4,5\} = \{3\}$
- We can apply functions

Functions

- Take elements of one (or more) sets and map them into another set
- Examples:
  - Student's ID to Student's Name
  - Amount of water in water bottle to mass of water bottle
  - Integers to the "next" Integer
  - Two Reals to the sum of the two

Is it a function?

- A function must have a unique output for each input
- When you input a value you can only get one output
- Functions or not?
  - Student's grade to their name
  - Desk to the student sitting in them
  - Student to desk they are sitting in

Domain, Co-Domain and Range

- Domain is the Set of inputs to the function
- Co-Domain is the Set that the outputs are in
- Range is the Set of all possible outputs of the function
- Example:
  - Function from Capital letters to their lowercase counterparts
  - Domain = {A,B,C,D,E,F,G,H,I,J,K,L,M,N,O,P,Q,R,S,T,U,V,W,X,Y,Z}
  - Range = {a,b,c,d,e,f,g,h,i,j,k,l,m,n,o,p,q,r,s,t,u,v,w,x,y,z}
  - Co-Domain = {A,B,C,D,E,F,G,H,I,J,K,L,M,N,O,P,Q,R,S,T,U,V,W,X,Y,Z,a,b,c,d,e,f,g,h,i,j,k,l,m,n,o,p,q,r,s,t,u,v,w,x,y,z}

Composite Functions

- So what if we take the output of one function and feed it into another?
- Let f(x) be the function that takes a Student ID and returns the student, and g(x) takes a student and returns their favorite color
- What would happen if I first ran y = f(x) and then ran z = g(y)?
  - Ususlly denoted as z = g(f(x))
- What would happen if I did it in the other order z' = f(g(x))?

Functions in Math

- Most function we use in math take in and output numbers
- The simplist is the **Constant Function**, $f(x) = c$
- Next we can get **Linear Functions**,$f(x) = m\ x+b$
- Then **Quadratics**, $f(x) = a\ x^2 + bx + c$
- To generalize we can get **Polynomials**

$$f(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \ldots + a_n x^n$$

More Math Functions

- **Rational Functions** have the form $f(x) = p(x)/q(x)$
  - Look familair?
  - Undefined when $q(x) = 0$
- **Algebraic Functions**
  - Consrtucted using any combination of addition, subtraction, multiplication, division, expoentiation and roots
- We will cover additional functions later

Properties of Functions

- One-to-One Function
  - Each y in y = f(x) corrosponds to at most one x
- Onto Function
  - Each y in y = f(x) corrosponds to at least one x
- An Even Function is symmetric about the y-axis
  - Mathematiclly this means that f(x) = f(-x)
- An Odd Function is symmetric about the origin
  - Mathematiclly this means that f(-x) = -f(x)

Function Transformations

- Translation – Move the function around
  - f(x-k) + h
- Scaling – Stretch or compress the function
  - f(a * x) * b

Sketching Functions

Homework due Monday

- Homework 1
- Know your functions
  - Lines, Parabolas and Polynomials
  - Rational and Algebraic Functions
  - Be able to make a sketch