Implicit Derivatives

Explicit vs. Implicit Functions

- All the functions so far have been Explicit Functions
  - They came in the form y = f(x)
- We could also write functions another way
  - 0 = f(x, y)
- Examples:

$$0 = x^2 + y^2 - 1 \qquad 0 = x - y^2$$

$$0 = x^2 + y - \sin(x\,y) \qquad 0 = x^2 + x\,y - y^3 - 7$$

Differentiate Implicit Functions

- You differentiate implicit functions the same as explicit functions
  - Don't forget, y is a function of x so you need to apply the chain rule
  - You need to differentiate both sides

Example

$$\frac{d}{dx}0=\frac{d}{dx}\left(x^2+y^2-1\right)$$

$$0=\frac{d}{dx}\left(x^2\right)+\frac{d}{dx}\left(y^2\right)-\frac{d}{dx}\left(1\right)$$

$$0=2x+\frac{d}{dx}\left(y^2\right)-0$$

$$0=2x+2y\,y'-0$$

$$-2\mathrm{x}=2y\,y'$$

$$y'=\frac{-2\mathrm{x}}{2\mathrm{y}}$$

$$y'=\frac{-x}{y}$$

Example

$$\frac{d}{dx}0=\frac{d}{dx}x^2+\frac{d}{dx}y-\frac{d}{dx}\sin(xy)$$

$$0=2x+y'-\cos(xy)(1y+xy')$$

$$0=2x+y'-y\cos(xy)-xy'\cos(xy)$$

$$0=(2x-\cos(xy)y)+(1-\cos(xy)x)y'$$

$$-2x+\cos(xy)y=(1-\cos(xy)x)y'$$

$$y'=\frac{-2x+\cos(xy)y}{1-\cos(xy)x}$$

$$y'=\frac{\cos(xy)y-2x}{\cos(xy)x-1}$$

Problems

$$x = e^{y} \qquad x\,y = 5 \qquad (x+y)^{3} = 3$$

$$\sqrt{x^{2}+y^{2}} = 1 \qquad e^{y\cos(x)} = 15 \qquad \frac{y}{x} = 2$$

$$\ln(x+y) = \sin(x) \qquad x = \sqrt{\frac{y-1}{y+1}} \qquad \sin(x\,y)\cos(x) = \frac{1}{2}$$

Questions?