Using Derivatives for Approximations

Approximations

- Sometimes functions are hard to evaluate and we want to use a simpiler function instead
- We usally only care about the function, ***f***, near some point ***a***
- So, as long as we find another function with similar properties as ***f***, then we are okay

0^{th} Order Approximations

- In this approximation, all we care about is that the new function ***g*** has the same value of ***f*** at the point ***a***
- This implies that $g(a) = f(a)$
- What is the simpliest function that satisfies this?
- A constant function

1^{st} Order Approximations

- In this case we want the values of f and it's approcimation, g, to be equal at a, and in addition we also want their deriviatives to be the same too
- $\boldsymbol{g}(\boldsymbol{a}) = \boldsymbol{f}(\boldsymbol{a})$ and $\boldsymbol{g'}(\boldsymbol{a}) = \boldsymbol{f'}(\boldsymbol{a})$
- Since we know that f' is the slope then the simpliest function would be a line that goes through ($\boldsymbol{a}$, $\boldsymbol{f}(\boldsymbol{a})$) with the slope $\boldsymbol{f'}(\boldsymbol{a})$

1st Order Approximations

- We can put this into point-slope formula and obtiain:
- $g(x) – f(a) = f'(a)\ (x – a)$
- $g(x) = f(a) + f'(a)\ (x – a)$
- This is also the Tangent Line at ***a***

n^{th} Order Approximations

- We can keep doing this to keep getting a better approximation
- If we do this infinitely we get a Taylor's Series which you will cover in Calculus II

Differentials

- These approximations have some error
- The Differential is the amount of error when you start moving away from ***a***
- We can calculate this by doing
  - error = $|g(x + h) - f(x+h)|$

Examples

$$f(x)=x^2 \, at \, (1,1)$$

$$f(x)=\frac{x^2+1}{x+1} \, at \, (0,1)$$

$$f(x)=\sqrt{x} \, at \, (1,1)$$

$$f(x)=\tan(x) \, at \, (\pi/4,1)$$

$$f(x)=e^x \, at \, (1,e)$$

$$f(x)=x^5 e^{x/2} \, at \, (2,32e)$$

Questions?