Infinite Limits

Concept of Infinity

- How many numbers are there?
  - We can't really answer this question...
- So we create another new concept: Infinity
- There are a few differnt kinds of infinities
- But in this class we will deal with this one: ∞
- It means that x gets really big

Limits that approach ∞

- Examples: $\lim_{x \to 0} (\frac{1}{x^2}) = \infty$
- As x approaches 0 from either direction, f(x) keeps getting bigger

$$\lim_{x \to 0} (\frac{1}{x}) = DNE$$

- Since as you approach from the left you approach -∞, but as you approach from the right you apprach ∞
- These points are called Vertical Asymptotes

Infinte Limit Algebra

- Example

$$\underset{x\to 4}{limL}\frac{(x-2)(x+4)}{(x-4)(x+2)}\approx\underset{x\to 4}{limL}\frac{16}{(-0)(6)}=-\infty$$

$$\underset{x\to 4}{limR}\frac{(x-2)(x+4)}{(x-4)(x+2)}\approx\underset{x\to 4}{limR}\frac{16}{(+0)(6)}=\infty$$

$$\lim_{x\to 4}\frac{(x-2)(x+4)}{(x-4)(x+2)}\approx\lim_{x\to 4}\frac{16}{(0)(6)}=DNE$$

Questions?

Limits at Infinity

- If as x gets bigger and bigger f(x) approaches L then:

$$\lim_{x \to \infty} f\left(x\right) = L$$

- This is a Horizontal Asymptote

Polynomial Limits at Infinity

- Rules for polynomials

$$\lim_{x\to\infty} x^n = \infty \qquad \lim_{x\to-\infty} x^{2n+0} = \infty$$

$$\lim_{x\to\pm\infty} x^{-n} = 0 \qquad \lim_{x\to-\infty} x^{2n+1} = -\infty$$

$$\lim_{x\to\pm\infty} a_n x^n + a_{n-1} x^{n-1} + \ldots = \lim_{x\to\pm\infty} a_n x^n$$

- Look at the dominating term

Slant Asymptotes

- As x approaches ∞, f(x) approaches a function g(x)
- Formally this means: $\lim_{x \to \infty} \left(f(x) - g(x) \right) = 0$
- Example: $\lim_{x \to \infty} \frac{2\mathrm{x}^2 + 6\mathrm{x} - 2}{x + 1}$

Transcendental Limits

- Expontial functions keep growing as x grows as long as the exponent is positive
- Expontial functions approach 0 when x appoaches negative infinty
- sin(x) and cos(x) don't approach anything
- ln(x) approaches infinity as x grows, but very very slowly

Questions?