# Trabajo Práctico Nº 2

## Cátedra: Robótica - Plan 95A

## Grupo Nº

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**Fecha Aprobación:**

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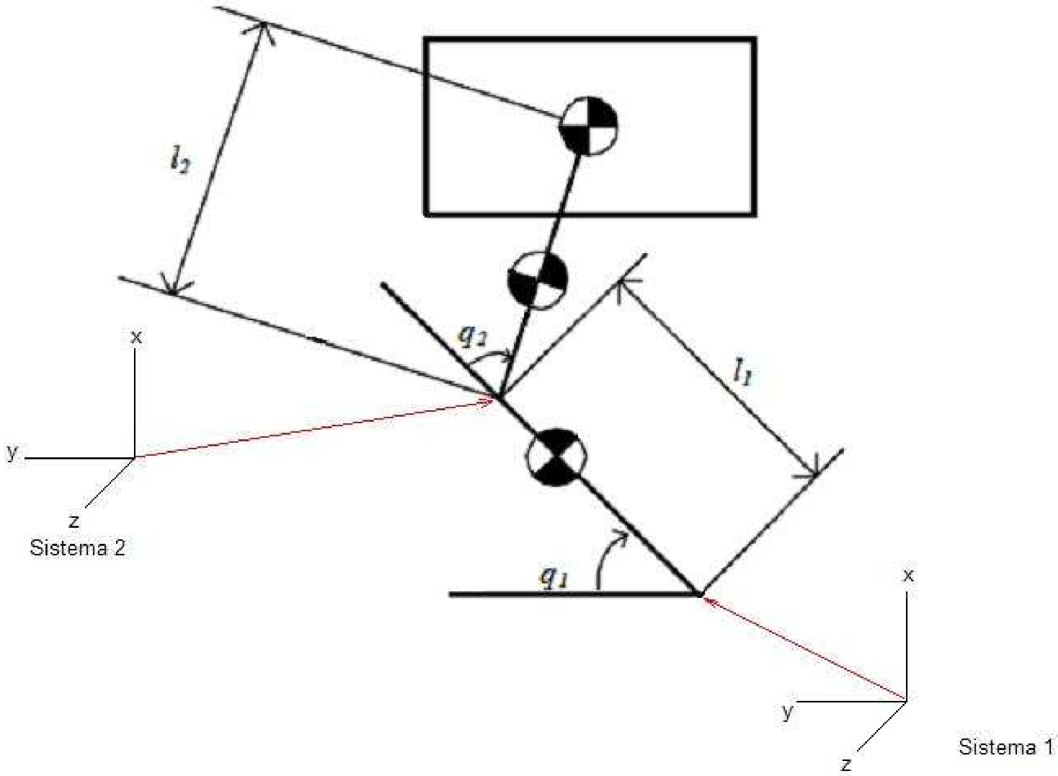
# Cinemática directa del robot




| Sistema | $\theta_i$ | $d_i$ | $a_i$ | $\alpha_i$ |
|---|---|---|---|---|
| 1 | Q1 | 0 | 0 | 0° |
| 2 | 0 | 0 | L1 | 0 |
| 3 | Q2 | 0 | 0 | 0° |

$$\begin{bmatrix} C\theta_i & -C\alpha_i S\theta_i & S\alpha_i S\theta_i & a_i C\theta_i \\ S\theta_i & C\alpha_i C\theta_i & -S\alpha_i C\theta_i & a_i S\theta_i \\ 0 & S\alpha_i & C\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^0A_1 = \begin{bmatrix} C_1 & -S_1 & 0 & 0 \\ S_1 & C_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad ^1A_2 = \begin{bmatrix} 1 & 0 & 0 & L_1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$
{}^{2}A_{3} = \begin{bmatrix} C_2 & -S_2 & 0 & 0 \\ S_2 & C_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^{1}A_{3} = \begin{bmatrix} 1 & 0 & 0 & L_1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C_2 & -S_2 & 0 & 0 \\ S_2 & C_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} C_2 & -S_2 & 0 & L_1 \\ S_2 & C_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^{0}A_{3} = \begin{bmatrix} C_1 & -S_1 & 0 & 0 \\ S_1 & C_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C_2 & -S_2 & 0 & L_1 \\ S_2 & C_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} C_{1+2} & -S_{1+2} & 0 & L_1C_1 \\ S_{1+2} & C_{1+2} & 0 & L_1S_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

Como se observa, el sistema condice con los ejes de referencia que se han propuesto.

## Dinámica del robot

Se utilizará el método de Lagrange-Euler para resolver el sistema dinámico:

$$
{}^{0}A_{1} = \begin{bmatrix} C_1 & -S_1 & 0 & 0 \\ S_1 & C_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad {}^{0}A_{2} = \begin{bmatrix} C_{1+2} & -S_{1+2} & 0 & L_1C_1 \\ S_{1+2} & C_{1+2} & 0 & L_1S_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

### *Matrices $U_{ij}$*

$$
U_{11} = \frac{\partial\left({}^{0}A_1\right)}{\partial\sigma_1} = \begin{bmatrix} -S_1 & -C_1 & 0 & 0 \\ C_1 & -S_1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad U_{12} = \frac{\partial\left({}^{0}A_1\right)}{\partial\sigma_2} = [0]
$$

$$U_{21}=\frac{\partial\left({}^{0}A_{2}\right)}{\partial\sigma_{1}}=\begin{bmatrix}-S_{1+2} & -C_{1+2} & 0 & -L_{1}S_{1}\\ C_{1+2} & -S_{1+2} & 0 & L_{1}C_{1}\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\end{bmatrix}$$

$$U_{22}=\frac{\partial\left({}^{0}A_{2}\right)}{\partial\sigma_{2}}=\begin{bmatrix}-S_{1+2} & -C_{1+2} & 0 & 0\\ C_{1+2} & -S_{1+2} & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\end{bmatrix}$$

## *Matrices $U_{ijk}$*

$$U_{111}=\frac{\partial\left(U_{11}\right)}{\partial\sigma_{1}}=\begin{bmatrix}-C_{1} & S_{1} & 0 & 0\\ -S_{1} & -C_{1} & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\end{bmatrix} \qquad U_{112}=\frac{\partial\left(U_{12}\right)}{\partial\sigma_{2}}=[0]$$

$$U_{121}=\frac{\partial\left(U_{12}\right)}{\partial\sigma_{1}}=[0] \qquad U_{122}=\frac{\partial\left(U_{12}\right)}{\partial\sigma_{2}}=[0]$$

$$U_{211}=\frac{\partial\left(U_{21}\right)}{\partial\sigma_{1}}=\begin{bmatrix}-C_{1+2} & S_{1+2} & 0 & -L_{1}C_{1}\\ -S_{1+2} & -C_{1+2} & 0 & -L_{1}S_{1}\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\end{bmatrix} \qquad U_{212}=\frac{\partial\left(U_{21}\right)}{\partial\sigma_{2}}=\begin{bmatrix}-C_{1+2} & S_{1+2} & 0 & 0\\ -S_{1+2} & -C_{1+2} & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\end{bmatrix}$$

$$U_{221}=\frac{\partial\left(U_{22}\right)}{\partial\sigma_{1}}=\begin{bmatrix}-C_{1+2} & S_{1+2} & 0 & 0\\ -S_{1+2} & -C_{1+2} & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\end{bmatrix} \qquad U_{222}=\frac{\partial\left(U_{22}\right)}{\partial\sigma_{2}}=\begin{bmatrix}-C_{1+2} & S_{1+2} & 0 & 0\\ -S_{1+2} & -C_{1+2} & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\end{bmatrix}$$

## *Matrices de pseudoinercias $J_i$*

$$\mathbf{J}_i = \begin{bmatrix} \int x_i^2 \mathrm{dm} & \int x_i y_i \mathrm{dm} & \int x_i z_i \mathrm{dm} & \int x_i \mathrm{dm} \\ \int y_i x_i \mathrm{dm} & \int y_i^2 \mathrm{dm} & \int y_i z_i \mathrm{dm} & \int y_i \mathrm{dm} \\ \int z_i x_i \mathrm{dm} & \int z_i y_i \mathrm{dm} & \int z_i^2 \mathrm{dm} & \int z_i \mathrm{dm} \\ \int x_i \mathrm{dm} & \int y_i \mathrm{dm} & \int z_i \mathrm{dm} & \int \mathrm{dm} \end{bmatrix}$$

$$J_1 = \begin{bmatrix} Lg_1^2 m_1 & 0 & 0 & Lg_1 m_1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ Lg_1 m_1 & 0 & 0 & m_1 \end{bmatrix}$$

$$J_2 = \begin{bmatrix} Lg_2^2 m_2 & 0 & 0 & Lg_2 m_2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ Lg_2 m_2 & 0 & 0 & m_2 \end{bmatrix}$$

## *Matrices de inercia $D[d_{ij}]$*

### $\mathbf{d_{11}}$

$$\mathrm{d}_{11} = \sum_{k=\max(1,1)}^{2} \mathrm{Traza}\left(\mathbf{U}_{k1}\mathbf{J}_k\mathbf{U}_{k1}^T\right) = \mathrm{Tr}\left(\mathbf{U}_{11}\mathbf{J}_1\mathbf{U}_{11}^T\right) + \mathrm{Tr}\left(\mathbf{U}_{21}\mathbf{J}_2\mathbf{U}_{21}^T\right) =$$

$$d_{11} = Tr\left(U_{11} * J_1 * U_{11}^T\right) + Tr\left(U_{21} * J_2 * U_{21}^T\right)$$

$$U_{11} * J_1 * U_{11}^T = \begin{bmatrix} -S_1 & -C_1 & 0 & 0 \\ C_1 & -S_1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} Lg_1^2 m_1 & 0 & 0 & Lg_1 m_1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ Lg_1 m_1 & 0 & 0 & m_1 \end{bmatrix} \begin{bmatrix} -S_1 & C_1 & 0 & 0 \\ -C_1 & -S_1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} -S_1 Lg_1^2 m_1 & 0 & 0 & -S_1 Lg_1 m_1 \\ C_1 Lg_1^2 m_1 & 0 & 0 & C_1 Lg_1 m_1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} -S_1 & C_1 & 0 & 0 \\ -C_1 & -S_1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} S_1^2 Lg_1^2 m_1 & x & x & x \\ x & C_1^2 Lg_1^2 m_1 & x & x \\ x & x & 0 & x \\ x & x & x & 0 \end{bmatrix}$$

$$\boxed{Tr\left(U_{11} * J_1 * U_{11}^{T}\right) = Lg_1^{\,2} m_1}$$

$$Tr\left(U_{21} * J_2 * U_{21}^{T}\right) = \begin{bmatrix} -S_{1+2} & -C_{1+2} & 0 & -L_1S_1 \\ C_{1+2} & -S_{1+2} & 0 & L_1C_1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} Lg_2^{\,2} m_2 & 0 & 0 & Lg_2 m_2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ Lg_2 m_2 & 0 & 0 & m_2 \end{bmatrix} \begin{bmatrix} -S_{1+2} & C_{1+2} & 0 & 0 \\ -C_{1+2} & -S_{1+2} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -L_1S_1 & L_1C_1 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} -\left(S_{1+2} Lg_2^{\,2} m_2 + L_1S_1 Lg_2 m_2\right) & 0 & 0 & -\left(S_{1+2} Lg_2 m_2 + L_1 S_1 m_2\right) \\ C_{1+2} Lg_2^{\,2} m_2 + L_1 C_1 Lg_2 m_2 & 0 & 0 & C_{1+2} Lg_2 m_2 + L_1 C_1 m_2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} -S_{1+2} & C_{1+2} & 0 & 0 \\ -C_{1+2} & -S_{1+2} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -L_1S_1 & L_1C_1 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} A & x & x & x \\ x & B & x & x \\ x & x & 0 & x \\ x & x & x & 0 \end{bmatrix}$$

$$A = S_{1+2}\left(S_{1+2} Lg_2^{\,2} m_2 + L_1 S_1 Lg_2 m_2\right) + L_1 S_1 \left(S_{1+2} Lg_2 m_2 + L_1 S_1 m_2\right)$$
$$B = C_{1+2}\left(C_{1+2} Lg_2^{\,2} m_2 + L_1 C_1 Lg_2 m_2\right) + L_1 C_1 \left(C_{1+2} Lg_2 m_2 + L_1 C_1 m_2\right)$$

$$Tr\left(U_{21} * J_2 * U_{21}^{T}\right) = Lg_2^{\,2} m_2 + 2L_1 Lg_2 m_2 \left(C_{1+2} C_1 + S_{1+2} S_1\right) + L_1^{\,2} m_2$$

$$\boxed{Tr\left(U_{21} * J_2 * U_{21}^{T}\right) = Lg_2^{\,2} m_2 + 2L_1 Lg_2 m_2 C_2 + L_1^{\,2} m_2}$$
$$\boxed{Tr\left(U_{11} * J_1 * U_{11}^{T}\right) = Lg_1^{\,2} m_1}$$
$$\boxed{Tr\left(U_{21} * J_2 * U_{21}^{T}\right) = Lg_2^{\,2} m_2 + 2L_1 Lg_2 m_2 C_2 + L_1^{\,2} m_2}$$

$$\boxed{d_{11} = Lg_1^{\,2} m_1 + m_2\left(L_1^{\,2} + Lg_2^{\,2} + 2L_1 Lg_2 C_2\right)}$$

**$d_{12}$**

$$d_{12} = \sum_{k=\max(1,2)}^{2} \mathrm{Traza}\left(U_{k2} J_k U_{k1}^{T}\right) =$$

$$Tr\left(U_{12} J_1 U_{11}^{T}\right) + Tr\left(U_{22} J_2 U_{21}^{T}\right) = Tr\left(U_{22} J_2 U_{21}^{T}\right)$$

$$Tr\left(U_{22}J_2U_{21}^T\right)=\begin{bmatrix}-S_{1+2} & -C_{1+2} & 0 & 0\\ C_{1+2} & -S_{1+2} & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\end{bmatrix}\begin{bmatrix}Lg_2{}^2m_2 & 0 & 0 & Lg_2m_2\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\\ Lg_2m_2 & 0 & 0 & m_2\end{bmatrix}\begin{bmatrix}-S_{1+2} & C_{1+2} & 0 & 0\\ -C_{1+2} & -S_{1+2} & 0 & 0\\ 0 & 0 & 0 & 0\\ -L_1S_1 & L_1C_1 & 0 & 0\end{bmatrix}=$$

$$\begin{bmatrix}-S_{1+2}Lg_2{}^2m_2 & 0 & 0 & -S_{1+2}Lg_2m_2\\ C_{1+2}Lg_2{}^2m_2 & 0 & 0 & C_{1+2}Lg_2m_2\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\end{bmatrix}\begin{bmatrix}-S_{1+2} & C_{1+2} & 0 & 0\\ -C_{1+2} & -S_{1+2} & 0 & 0\\ 0 & 0 & 0 & 0\\ -L_1S_1 & L_1C_1 & 0 & 0\end{bmatrix}=$$

$$\begin{bmatrix}S_{1+2}{}^2Lg_2{}^2m_2+S_{1+2}Lg_2m_2L_1S_1 & x & x & x\\ x & C_{1+2}{}^2Lg_2{}^2m_2+C_{1+2}Lg_2m_2L_1C_1 & x & x\\ x & x & 0 & x\\ x & x & x & 0\end{bmatrix}$$

$$Tr\left(U_{22}J_2U_{21}^T\right)=S_{1+2}{}^2Lg_2{}^2m_2+S_1S_{1+2}L_1Lg_2m_2+C_{1+2}{}^2Lg_2{}^2m_2+C_1C_{1+2}L_1Lg_2m_2=$$

$$Tr\left(U_{22}J_2U_{21}^T\right)=m_2\left(Lg_2{}^2+L_1Lg_2C_2\right)$$

$$\boxed{d_{12}=m_2\left(Lg_2{}^2+L_1Lg_2C_2\right)}$$

## $d_{21}$

$$d_{21}=\sum_{k=\max(2,1)}^{2}\mathrm{Traza}\left(\mathbf{U}_{k1}\mathbf{J}_k\mathbf{U}_{k2}{}^T\right)=$$

$$Tr\left(U_{11}J_1U_{12}{}^T\right)+Tr\left(U_{21}J_2U_{22}{}^T\right)=Tr\left(U_{21}J_2U_{22}{}^T\right)$$

$$Tr\left(U_{21}J_2U_{22}^T\right)=\begin{bmatrix}-S_{1+2} & -C_{1+2} & 0 & -L_1S_1\\ C_{1+2} & -S_{1+2} & 0 & L_1C_1\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\end{bmatrix}\begin{bmatrix}Lg_2{}^2m_2 & 0 & 0 & Lg_2m_2\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\\ Lg_2m_2 & 0 & 0 & m_2\end{bmatrix}\begin{bmatrix}-S_{1+2} & C_{1+2} & 0 & 0\\ -C_{1+2} & -S_{1+2} & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\end{bmatrix}=$$

$$\begin{bmatrix}-\left(S_{1+2}Lg_2{}^2m_2+L_1S_1Lg_2m_2\right) & 0 & 0 & -\left(S_{1+2}Lg_2m_2+L_1S_1m_2\right)\\ C_{1+2}Lg_2{}^2m_2+L_1C_1Lg_2m_2 & 0 & 0 & C_{1+2}Lg_2m_2+L_1C_1m_2\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\end{bmatrix}\begin{bmatrix}-S_{1+2} & C_{1+2} & 0 & 0\\ -C_{1+2} & -S_{1+2} & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\end{bmatrix}=$$

$$\begin{bmatrix} A & x & x & x \\ x & B & x & x \\ x & x & 0 & x \\ x & x & x & 0 \end{bmatrix}$$

$$A = S_{1+2}\left(S_{1+2}Lg_2{}^2 m_2 + L_1 S_1 Lg_2 m_2\right) = S_{1+2}{}^2 Lg_2{}^2 m_2 + S_{1+2} S_1 L_1 Lg_2 m_2$$
$$B = C_{1+2}\left(C_{1+2}Lg_2{}^2 m_2 + L_1 C_1 Lg_2 m_2\right) = C_{1+2}{}^2 Lg_2{}^2 m_2 + C_{1+2} C_1 L_1 Lg_2 m_2$$

$$Tr\left(U_{21} J_2 U_{22}{}^T\right) = Lg_2{}^2 m_2 + L_1 Lg_2 m_2 C_2$$

$$\boxed{d_{21} = m_2\left(Lg_2{}^2 + L_1 Lg_2 C_2\right)}$$

**$d_{22}$**

$$\mathrm{d}_{22} = \sum_{k=\max(2,2)}^{2} \mathrm{Traza}\left(\mathbf{U}_{k2}\mathbf{J}_k\mathbf{U}_{k2}{}^{\mathrm{T}}\right) =$$

$$d_{22} = Tr\left(U_{12} J_1 U_{12}{}^T\right) + Tr\left(U_{22} J_2 U_{22}{}^T\right) = Tr\left(U_{22} J_2 U_{22}{}^T\right)$$

$$Tr\left(U_{22} J_2 U_{22}{}^T\right) = \begin{bmatrix} -S_{1+2} & -C_{1+2} & 0 & 0 \\ C_{1+2} & -S_{1+2} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} Lg_2{}^2 m_2 & 0 & 0 & Lg_2 m_2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ Lg_2 m_2 & 0 & 0 & m_2 \end{bmatrix} \begin{bmatrix} -S_{1+2} & C_{1+2} & 0 & 0 \\ -C_{1+2} & -S_{1+2} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} =$$

$$\begin{bmatrix} -S_{1+2} Lg_2{}^2 m_2 & 0 & 0 & -S_{1+2} Lg_2 m_2 \\ C_{1+2} Lg_2{}^2 m_2 & 0 & 0 & -S_{1+2} Lg_2 m_2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} -S_{1+2} & C_{1+2} & 0 & 0 \\ -C_{1+2} & -S_{1+2} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} S_{1+2}{}^2 Lg_2{}^2 m_2 & x & x & x \\ x & C_{1+2}{}^2 Lg_2{}^2 m_2 & x & x \\ x & x & 0 & x \\ x & x & x & 0 \end{bmatrix}$$

$$Tr\left(U_{22} J_2 U_{22}{}^T\right) = Lg_2{}^2 m_2$$

$$\boxed{d_{22} = Lg_2{}^2 m_2}$$

## *Matriz de inercia*

$$D = \begin{bmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{bmatrix}$$

Con:

$$d_{11} = Lg_1{}^2 m_1 + m_2\left(L_1{}^2 + Lg_2{}^2 + 2L_1 Lg_2 C_2\right)$$
$$d_{12} = m_2\left(Lg_2{}^2 + L_1 Lg_2 C_2\right)$$
$$d_{21} = m_2\left(Lg_2{}^2 + L_1 Lg_2 C_2\right)$$
$$d_{22} = Lg_2{}^2 m_2$$

## Matrices de Coriolis

**$h_{111}$**

$$h_{111} = \sum_{j=\max(1,1,1)}^{2} \mathrm{Traza}\left(\mathbf{U}_{j11}\mathbf{J}_j\mathbf{U}_{j1}^{\mathrm{T}}\right) = \mathrm{Tr}\left(\mathbf{U}_{111}\mathbf{J}_1\mathbf{U}_{11}^{\mathrm{T}}\right) + \mathrm{Tr}\left(\mathbf{U}_{211}\mathbf{J}_2\mathbf{U}_{21}^{\mathrm{T}}\right) =$$

$$Tr\left(U_{111}J_1U_{11}^{\;T}\right) = \begin{bmatrix} -C_1 & S_1 & 0 & 0 \\ -S_1 & -C_1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} Lg_1^{\;2}m_1 & 0 & 0 & Lg_1m_1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ Lg_1m_1 & 0 & 0 & m_1 \end{bmatrix} \begin{bmatrix} -S_1 & C_1 & 0 & 0 \\ -C_1 & -S_1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} -C_1Lg_1^{\;2}m_1 & 0 & 0 & -C_1Lg_1m_1 \\ -S_1Lg_1^{\;2}m_1 & 0 & 0 & -S_1Lg_1m_1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} -S_1 & C_1 & 0 & 0 \\ -C_1 & -S_1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} C_1S_1Lg_1^{\;2}m_1 & x & x & x \\ x & -C_1S_1Lg_1^{\;2}m_1 & x & x \\ x & x & 0 & x \\ x & x & x & 0 \end{bmatrix} = 0$$

$$Tr\left(U_{211}J_2U_{21}^{\;T}\right) = \begin{bmatrix} -C_{1+2} & S_{1+2} & 0 & -L_1C_1 \\ -S_{1+2} & -C_{1+2} & 0 & -L_1S_1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} Lg_2^{\;2}m_2 & 0 & 0 & Lg_2m_2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ Lg_2m_2 & 0 & 0 & m_2 \end{bmatrix} \begin{bmatrix} -S_{1+2} & C_{1+2} & 0 & 0 \\ -C_{1+2} & -S_{1+2} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -L_1S_1 & L_1C_1 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} -\left(C_{1+2}Lg_2^{\;2}m_2 + L_1C_1Lg_2m_2\right) & 0 & 0 & -\left(C_{1+2}Lg_2m_2 + L_1C_1m_2\right) \\ -\left(S_{1+2}Lg_2^{\;2}m_2 + L_1S_1Lg_2m_2\right) & 0 & 0 & -\left(S_{1+2}Lg_2m_2 + L_1S_1m_2\right) \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} -S_{1+2} & C_{1+2} & 0 & 0 \\ -C_{1+2} & -S_{1+2} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -L_1S_1 & L_1C_1 & 0 & 0 \end{bmatrix} =$$

$$\begin{bmatrix} A & x & x & x \\ x & B & x & x \\ x & x & 0 & x \\ x & x & x & 0 \end{bmatrix} =$$

$$A = S_{1+2}\left(C_{1+2}Lg_2^{\;2}m_2 + L_1C_1Lg_2m_2\right) + L_1S_1\left(C_{1+2}Lg_2m_2 + L_1C_1m_2\right)$$

$$B = -C_{1+2}\left(S_{1+2}Lg_2^{\;2}m_2 + L_1S_1Lg_2m_2\right) - L_1C_1\left(S_{1+2}Lg_2m_2 + L_1S_1m_2\right)$$

$$Tr\left(U_{211}J_2U_{21}^{\;T}\right) = 0$$

$$\boxed{h_{111} = 0}$$

**h$_{112}$**

$$h_{112} = \sum_{j=\max(1,1,2)}^{2} \mathrm{Traza}\left(\mathbf{U}_{j12}\mathbf{J}_j\mathbf{U}_{j1}^{T}\right) =$$

$$h_{112} = Tr\left(U_{112}J_1U_{11}^{T}\right) + Tr\left(U_{212}J_2U_{21}^{T}\right) = Tr\left(U_{212}J_2U_{21}^{T}\right)$$

$$Tr\left(U_{212}J_2U_{21}^{T}\right) = \begin{bmatrix} -C_{1+2} & S_{1+2} & 0 & 0 \\ -S_{1+2} & -C_{1+2} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} Lg_2{}^2m_2 & 0 & 0 & Lg_2m_2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ Lg_2m_2 & 0 & 0 & m_2 \end{bmatrix} \begin{bmatrix} -S_{1+2} & C_{1+2} & 0 & 0 \\ -C_{1+2} & -S_{1+2} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -L_1S_1 & L_1C_1 & 0 & 0 \end{bmatrix} =$$

$$\begin{bmatrix} -C_{1+2}Lg_2{}^2m_2 & 0 & 0 & -C_{1+2}Lg_2m_2 \\ -S_{1+2}Lg_2{}^2m_2 & 0 & 0 & -S_{1+2}Lg_2m_2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} -S_{1+2} & C_{1+2} & 0 & 0 \\ -C_{1+2} & -S_{1+2} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -L_1S_1 & L_1C_1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} A & x & x & x \\ x & B & x & x \\ x & x & 0 & x \\ x & x & x & 0 \end{bmatrix} =$$

$$A = S_{1+2}C_{1+2}Lg_2{}^2m_2 + C_{1+2}S_1Lg_2L_1m_2$$

$$B = -C_{1+2}S_{1+2}Lg_2{}^2m_2 - S_{1+2}C_1Lg_2L_1m_2$$

$$Tr\left(U_{212}J_2U_{21}^{T}\right) = Lg_2L_1m_2\left(S_1C_{1+2} - C_1S_{1+2}\right)$$

$$\boxed{h_{112} = -m_2Lg_2L_1S_2}$$

**h$_{121}$**

$$h_{121} = \sum_{j=\max(1,2,1)}^{2} \mathrm{Traza}\left(\mathbf{U}_{j21}\mathbf{J}_j\mathbf{U}_{j1}^{T}\right) =$$

$$h_{121} = Tr\left(U_{121}J_1U_{11}^{T}\right) + Tr\left(U_{221}J_2U_{21}^{T}\right) = Tr\left(U_{221}J_2U_{21}^{T}\right)$$

$$\boxed{h_{121} = -m_2Lg_2L_1S_2}$$

**$h_{122}$**

$$h_{122}=\sum_{j=\max(1,2,2)}^{2}\mathrm{Traza}\left(\mathbf{U}_{j22}\mathbf{J}_{j}\mathbf{U}_{j1}^{\mathrm{T}}\right)=$$

$$h_{122}=Tr\left(U_{122}J_1U_{11}{}^{T}\right)+Tr\left(U_{222}J_2U_{21}{}^{T}\right)=Tr\left(U_{222}J_2U_{21}{}^{T}\right)$$

$$Tr\left(U_{222}J_2U_{21}{}^{T}\right)=Tr\left(U_{221}J_2U_{21}{}^{T}\right)$$

$$\boxed{h_{122}=-m_2Lg_2L_1S_2}$$

**$h_{211}$**

$$h_{211}=\sum_{j=\max(2,1,1)}^{2}\mathrm{Traza}\left(\mathbf{U}_{j11}\mathbf{J}_{j}\mathbf{U}_{j2}^{\mathrm{T}}\right)=$$

$$h_{211}=Tr\left(U_{111}J_1U_{12}{}^{T}\right)+Tr\left(U_{211}J_2U_{22}{}^{T}\right)=Tr\left(U_{211}J_2U_{22}{}^{T}\right)$$

$$Tr\left(U_{211}J_2U_{22}{}^{T}\right)=\begin{bmatrix}-C_{1+2} & S_{1+2} & 0 & -L_1C_1\\ -S_{1+2} & -C_{1+2} & 0 & -L_1S_1\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\end{bmatrix}\begin{bmatrix}Lg_2{}^2m_2 & 0 & 0 & Lg_2m_2\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\\ Lg_2m_2 & 0 & 0 & m_2\end{bmatrix}\begin{bmatrix}-S_{1+2} & C_{1+2} & 0 & 0\\ -C_{1+2} & -S_{1+2} & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\end{bmatrix}=$$

$$\begin{bmatrix}-\left(C_{1+2}Lg_2{}^2m_2+L_1C_1Lg_2m_2\right) & 0 & 0 & -\left(C_{1+2}Lg_2m_2+L_1C_1m_2\right)\\ -\left(S_{1+2}Lg_2{}^2m_2+L_1S_1Lg_2m_2\right) & 0 & 0 & -\left(S_{1+2}Lg_2m_2+L_1S_1m_2\right)\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\end{bmatrix}\begin{bmatrix}-S_{1+2} & C_{1+2} & 0 & 0\\ -C_{1+2} & -S_{1+2} & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\end{bmatrix}=$$

$$Tr\left(U_{211}J_2U_{22}{}^{T}\right)=S_{1+2}\left(C_{1+2}Lg_2{}^2m_2+L_1C_1Lg_2m_2\right)-C_{1+2}\left(S_{1+2}Lg_2{}^2m_2+L_1S_1Lg_2m_2\right)=$$
$$L_1Lg_2m_2\left(S_{1+2}C_1-C_{1+2}S_1\right)$$

$$\boxed{h_{211}=L_1Lg_2m_2S_2}$$

**$h_{212}$**

$$h_{212}=\sum_{j=\max(2,1,2)}^{2}\mathrm{Traza}\left(\mathbf{U}_{j12}\mathbf{J}_{j}\mathbf{U}_{j2}^{\mathrm{T}}\right)=$$

$$h_{212} = Tr\left(U_{112}J_1U_{12}{}^T\right) + Tr\left(U_{212}J_2U_{22}{}^T\right) = Tr\left(U_{212}J_2U_{22}{}^T\right)$$

$$\begin{bmatrix} -C_{1+2} & S_{1+2} & 0 & 0 \\ -S_{1+2} & -C_{1+2} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} Lg_2{}^2m_2 & 0 & 0 & Lg_2m_2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ Lg_2m_2 & 0 & 0 & m_2 \end{bmatrix}\begin{bmatrix} -S_{1+2} & C_{1+2} & 0 & 0 \\ -C_{1+2} & -S_{1+2} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} -C_{1+2}Lg_2{}^2m_2 & 0 & 0 & -C_{1+2}Lg_2m_2 \\ -S_{1+2}Lg_2{}^2m_2 & 0 & 0 & -S_{1+2}Lg_2m_2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} -S_{1+2} & C_{1+2} & 0 & 0 \\ -C_{1+2} & -S_{1+2} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} =$$

$$\begin{bmatrix} S_{1+2}C_{1+2}Lg_2{}^2m_2 & x & x & x \\ x & -S_{1+2}C_{1+2}Lg_2{}^2m_2 & x & x \\ x & x & 0 & x \\ x & x & x & 0 \end{bmatrix}$$

$$\boxed{h_{212} = 0}$$

**$h_{221}$**

$$h_{221} = \sum_{j=\max(2,2,1)}^{2} \text{Traza}\left(\mathbf{U}_{j21}\mathbf{J}_j\mathbf{U}_{j2}{}^{\mathbf{T}}\right) =$$

$$h_{221} = Tr\left(U_{121}J_1U_{12}{}^T\right) + Tr\left(U_{221}J_2U_{22}{}^T\right) = Tr\left(U_{221}J_2U_{22}{}^T\right)$$

$$\boxed{h_{221} = 0}$$

**$h_{222}$**

$$h_{222} = \sum_{j=\max(2,2,2)}^{2} \text{Traza}\left(\mathbf{U}_{j22}\mathbf{J}_j\mathbf{U}_{j2}{}^{\mathbf{T}}\right) =$$

$$h_{222} = Tr\left(U_{122}J_1U_{12}{}^T\right) + Tr\left(U_{222}J_2U_{22}{}^T\right) = Tr\left(U_{222}J_2U_{22}{}^T\right) =$$

$$\boxed{h_{222} = 0}$$

***$h_1$***

$$h_1 = \sum_{k=1}^{2}\sum_{m=1}^{2} h_{1km}\dot{q}_k\dot{q}_m =$$

$$h_1 = h_{111}\overset{*}{\sigma}_1{}^2 + h_{112}\overset{*}{\sigma}_1\overset{*}{\sigma}_2 + h_{121}\overset{*}{\sigma}_2\overset{*}{\sigma}_1 + h_{122}\overset{*}{\sigma}_2{}^2$$

$$h_{111} = 0$$
$$h_{112} = -m_2 Lg_2 L_1 S_2$$
$$h_{121} = -m_2 Lg_2 L_1 S_2$$
$$h_{122} = -m_2 Lg_2 L_1 S_2$$

$$\boxed{h_1 = -2m_2 Lg_2 L_1 S_2 \overset{*}{\sigma}_1\overset{*}{\sigma}_2 - m_2 Lg_2 L_1 S_2 \overset{*}{\sigma}_2{}^2}$$

***$h_2$***

$$h_2 = \sum_{k=1}^{2}\sum_{m=1}^{2} h_{2km}\dot{q}_k\dot{q}_m =$$

$$h_2 = h_{211}\overset{*}{\sigma}_1{}^2 + h_{212}\overset{*}{\sigma}_1\overset{*}{\sigma}_2 + h_{221}\overset{*}{\sigma}_2\overset{*}{\sigma}_1 + h_{222}\overset{*}{\sigma}_2{}^2$$

$$h_{211} = L_1 Lg_2 m_2 S_2$$
$$h_{212} = 0$$
$$h_{221} = 0$$
$$h_{222} = 0$$

$$\boxed{h_2 = L_1 Lg_2 m_2 S_2 \overset{*}{\sigma}_1{}^2}$$

## Matriz Gravedad

$$G = [-g \quad 0 \quad 0 \quad 0]$$

$$c_i = \sum_{j=1}^{N} \left( - m_j \mathbf{g} \mathbf{U}_{ji} \, {}^j\mathbf{r}_j \right)$$

***c₁***

$$c_1 = \sum_{j=1}^{2} \left( - m_j \mathbf{g} \mathbf{U}_{j1} \, {}^j\mathbf{r}_j \right) =$$

$$c_1 = -m_1 G U_{11} \, {}^1r_1 - m_2 G U_{21} \, {}^2r_2$$

En donde:

$${}^1r_1 = \begin{bmatrix} Lg_1 \\ 0 \\ 0 \\ 1 \end{bmatrix} \qquad {}^2r_2 = \begin{bmatrix} Lg_2 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$m_1 G U_{11} \, {}^1r_1 = m_1 \begin{bmatrix} -g & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} -S_1 & -C_1 & 0 & 0 \\ C_1 & -S_1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} Lg_1 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$m_1 G U_{11} \, {}^1r_1 = m_1 \begin{bmatrix} -g & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} -S_1 Lg_1 \\ C_1 Lg_1 \\ 0 \\ 0 \end{bmatrix} = m_1 g Lg_1 S_1$$

$$m_2 G U_{21} \, {}^2r_2 = m_2 \begin{bmatrix} -g & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} -S_{1+2} & -C_{1+2} & 0 & -L_1 S_1 \\ C_{1+2} & -S_{1+2} & 0 & L_1 C_1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} Lg_2 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$m_2 G U_{21} \, {}^2r_2 = m_2 \begin{bmatrix} -g & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} -S_{1+2} Lg_2 - L_1 S_1 \\ C_{1+2} Lg_2 + L_1 C_1 \\ 0 \\ 0 \end{bmatrix} = m_2 g \left( S_{1+2} Lg_2 + L_1 S_1 \right)$$

$$\boxed{c_1 = m_1 g L g_1 S_1 + m_2 g\left(S_{1+2} L g_2 + L_1 S_1\right)}$$

***c2***

$$\mathrm{c}_2 = \sum_{j=1}^{2}\left(-\mathrm{m}_j \mathbf{g} \mathbf{U}_{j2}\, {}^{j}\mathbf{r}_j\right) =$$

$$c_2 = -m_1 G U_{12}\, {}^{1}r_1 - m_2 G U_{22}\, {}^{2}r_2$$

$$m_1 G U_{11}\, {}^{1}r_1 = 0$$

$$m_2 G U_{22}\, {}^{2}r_2 = m_2 \begin{bmatrix} -g & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} -S_{1+2} & -C_{1+2} & 0 & 0 \\ C_{1+2} & -S_{1+2} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} Lg_2 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$m_2 G U_{22}\, {}^{2}r_2 = m_2 \begin{bmatrix} -g & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} -S_{1+2} L g_2 \\ C_{1+2} L g_2 \\ 0 \\ 0 \end{bmatrix} = m_2 g L g_2 S_{1+2}$$

$$\boxed{c_2 = m_2 g L g_2 S_{1+2}}$$

# Resultado Final

$$\boldsymbol{\tau} = \mathbf{D}\ddot{\mathbf{q}} + \mathbf{H} + \mathbf{C}$$

$$\begin{bmatrix} T_1 \\ T_2 \end{bmatrix} = \begin{bmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{bmatrix} \begin{bmatrix} \overset{**}{\sigma}_1 \\ \overset{**}{\sigma}_2 \end{bmatrix} + \begin{bmatrix} h_1 \\ h_2 \end{bmatrix} + \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}$$

Con:

$$d_{11} = L g_1^{\,2} m_1 + m_2\left(L_1^{\,2} + L g_2^{\,2} + 2 L_1 L g_2 C_2\right)$$
$$d_{12} = m_2\left(L g_2^{\,2} + L_1 L g_2 C_2\right)$$
$$d_{21} = m_2\left(L g_2^{\,2} + L_1 L g_2 C_2\right)$$
$$d_{22} = L g_2^{\,2} m_2$$

$$h_1 = -2m_2 Lg_2 L_1 S_2 \overset{*}{\sigma}_1 \overset{*}{\sigma}_2 - m_2 Lg_2 L_1 S_2 \overset{*}{\sigma}_2{}^2$$
$$h_2 = L_1 Lg_2 m_2 S_2 \overset{*}{\sigma}_1{}^2$$

$$c_1 = m_1 g Lg_1 S_1 + m_2 g\left(S_{1+2} Lg_2 + L_1 S_1\right)$$
$$c_2 = m_2 g Lg_2 S_{1+2}$$

## *Especificaciones constructivas*

Table 1: One-legged robot specifications

| | |
|---|---|
| Mass(kg) | 1.336(kg) |
| Width | 0.18(m) |
| Depth | 0.04(m) |
| Height | 0.36(m) |
| $l_0$ (Body link) | 0.18(m) |
| $l_1$ (2nd thigh length) | 0.18(m) |
| $l_2$ (thigh length) | 0.13(m) |
| $m_0$ (body mass) | 0.235(kg) |
| $m_1$ (2nd thigh mass ) | 0.465(kg) |
| $m_2$ (thigh mass) | 0.450(kg) |
| $M_p$ (frame mass) | 0.185(kg) |
| $I_1$ | 2nd thigh inertia |
| $I_2$ | thigh inertia |
| $lg_1$ | gravity point of 2nd thigh |
| $lg_2$ | gravity point of thigh |

Suponiendo un $Lg_x=L_x/2$, obtenemos que:

$$d_{11} = 3766.5*10^{-6} + 16481.25*10^{-6} + 10530*10^{-6}*C_2$$
$$d_{12} = 1901.25*10^{-6} + 5265*10^{-6}*C_2$$
$$d_{21} = 1901.25*10^{-6} + 5265*10^{-6}*C_2$$
$$d_{22} = 1901.25*10^{-6}$$

$$h_1 = -10530*10^{-6} S_2 \overset{*}{\sigma}_1 \overset{*}{\sigma}_2 - 5265*10^{-6} S_2 \overset{*}{\sigma}_2{}^2$$
$$h_2 = 5265*10^{-6} S_2 \overset{*}{\sigma}_1{}^2$$

$$c_1 = 410130*10^{-6} S_1 + 286650*10^{-6} S_{1+2} + 793800*10^{-6} S_1$$
$$c_2 = 286650*10^{-6} S_{1+2}$$

$$\begin{bmatrix} T_1 \\ T_2 \end{bmatrix} = \begin{bmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{bmatrix} \begin{bmatrix} \overset{**}{\sigma}_1 \\ \overset{**}{\sigma}_2 \end{bmatrix} + \begin{bmatrix} h_1 \\ h_2 \end{bmatrix} + \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}$$

$$T_1 = d_{11} \overset{**}{\sigma}_1 + d_{12} \overset{**}{\sigma}_2 + h_1 + c_1$$
$$T_1 = \left(3766.5*10^{-6} + 16481.25*10^{-6} + 10530*10^{-6} * C_2\right) * \overset{**}{\sigma}_1 + \left(1901.25*10^{-6} + 5265*10^{-6} * C_2\right) * \overset{**}{\sigma}_2$$
$$-10530*10^{-6} S_2 \overset{*}{\sigma}_1 \overset{*}{\sigma}_2 - 5265*10^{-6} S_2 \overset{*}{\sigma}_2{}^2 + 4010130*10^{-6} S_1 + 286650*10^{-6} S_{1+2} + 793800*10^{-6} S_1$$

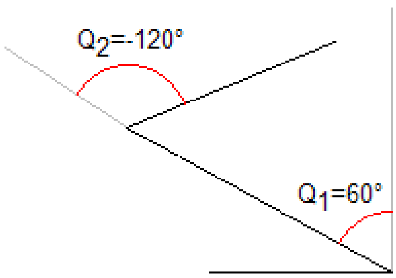
$$T_2 = d_{21} \overset{**}{\sigma}_1 + d_{22} \overset{**}{\sigma}_2 + h_2 + c_2$$
$$T_2 = \left(1901.25*10^{-6} + 5265*10^{-6} * C_2\right) \overset{**}{\sigma}_1 + 1901.25*10^{-6} \overset{**}{\sigma}_2 + 5265*10^{-6} S_2 \overset{*}{\sigma}_1{}^2 + 286650*10^{-6} S_{1+2}$$

## Calculo de los torques.

Se partirá de una posición inicial para el talón y la rodilla, para luego llevarla a una posición final de salto. Para lo mismo, se utilizará ángulos iniciales de $q_1$=60° (a 30° del plano según nuestros ejes de refere ncia) y $q_2$=-120°. Como posiciones finales, se buscará una pierna completamente extendida, esto quiere decir, $q_1$=$q_2$=0°

Se buscará que las aceleraciones que se apliquen logren despegar del suelo al robot de una pierna, por lo que se podrán probar varias aceleraciones con este fin.

Se ha optado por generar una trayectoria que tarde 0.5seg,



## *Aceleraciones, velocidades y posiciones angulares*

### Motor N°1 (talón)

$Q_{1i} = 60°$

$Q_{1f} = 0°$

$\Delta t = 0.5s$

$a = A_0 sen(2\pi f.t) = A_0 sen(4\pi.t)$

$v = \int a.dt = C - \frac{A_0}{4\pi}\cos(\pi.t)$

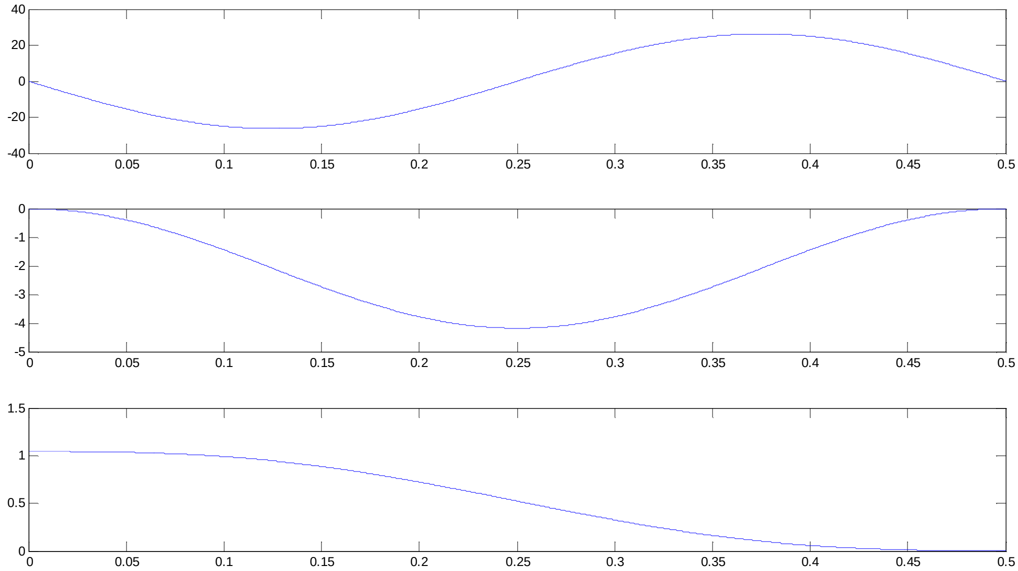
$p = \int v.dt = B + C.t - \frac{A_0}{16\pi^2}\sin(\pi.t)$

$p_i(t=0) = 60° \Rightarrow B = 1.047$

$p_f(t=0.5) = -60° = C.0.5seg \Rightarrow C = -2,09$

$\mathrm{v}_f(t=0.5) = 0\frac{rad}{seg} = C - \frac{A_0}{4\pi} \Rightarrow A_0 = 4\pi.C = -26.26$

$$\boxed{\begin{aligned} a_1 &= -26.26 sen(4\pi.t) \\ v_1 &= -2,09\frac{\text{rad}}{\text{seg}} + 2,09\cos(4\pi.t) \\ p_1 &= 1.047 - 2,09\frac{\text{rad}}{\text{seg}}t + 0,16\sin(4\pi.t) \end{aligned}}$$

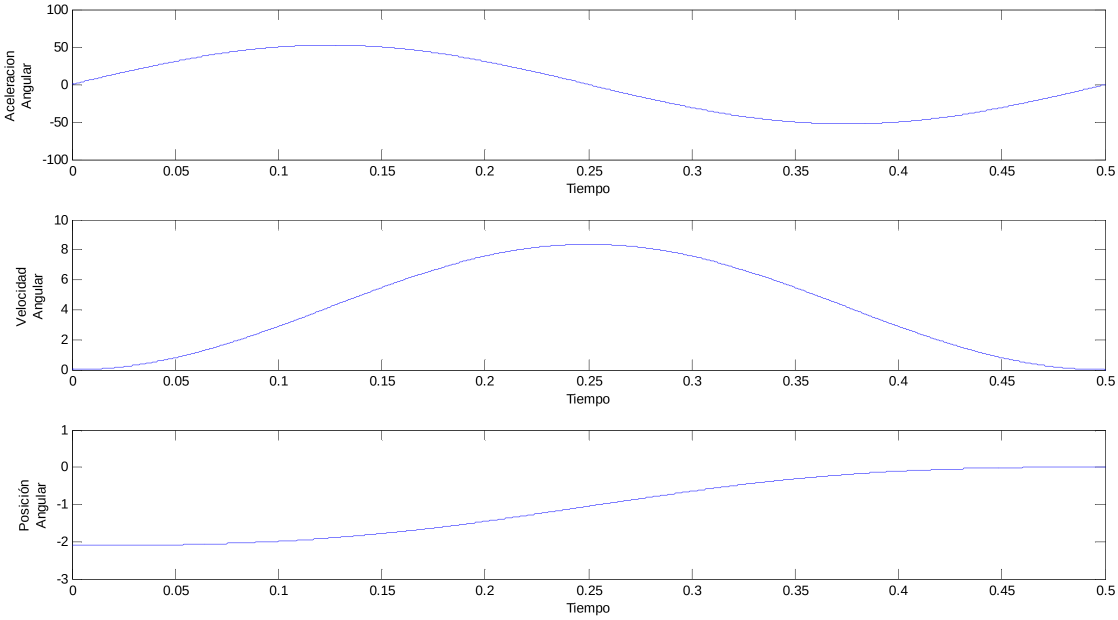


## Motor N°2 (rodilla)

$$Q_{2i} = -120°$$
$$Q_{2f} = 0°$$
$$\Delta t = 0.5s$$

$$a = A_0 sen(4\pi.t)$$
$$v = \int a.dt = C - \frac{A_0}{4\pi}\cos(4\pi.t)$$
$$p = \int v.dt = D + C.t - \frac{A_0}{(4\pi)^2}\sin(4\pi.t)$$
$$p_i(t=0) = -120° = D \Rightarrow D = -2{,}094\frac{\text{rad}}{\text{seg}} \Rightarrow p_f(t=0.5) = D + C * 0.5 = 0 \Rightarrow C = 4{,}188\frac{\text{rad}}{\text{seg}}$$
$$\text{v}_f(t=0.5) = 0\frac{rad}{seg} = C - \frac{A_0}{4\pi} \Rightarrow A_0 = 4\pi.C = 52{,}63\frac{\text{rad}}{\text{seg}^2}$$

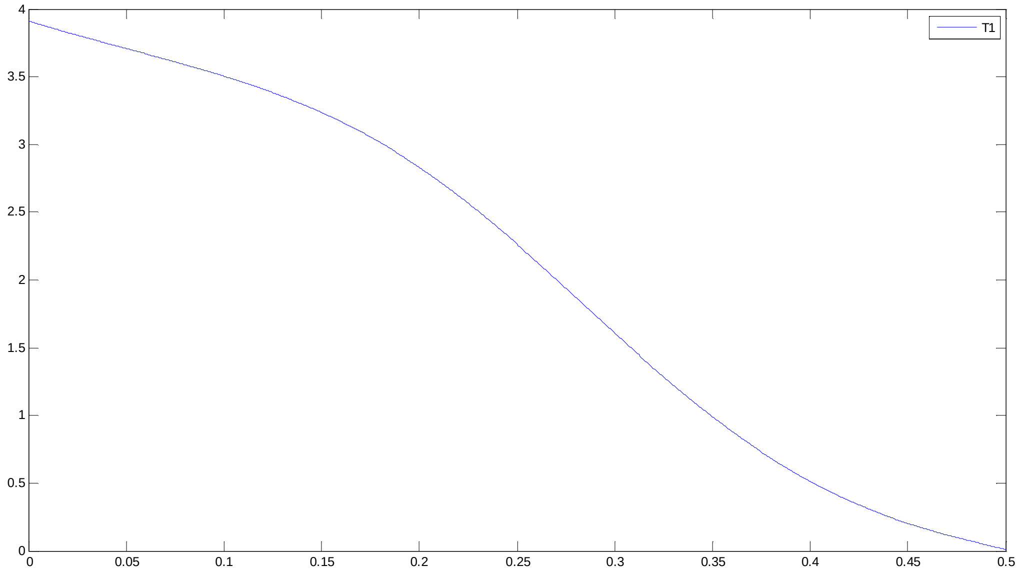
$$\boxed{\begin{aligned} a &= 52{,}63 sen(4\pi.t)\frac{\text{rad}}{\text{seg}^2} \\ v &= 4{,}188\frac{\text{rad}}{\text{seg}} - 4{,}188\cos(4\pi.t) \\ p &= -2{,}094\frac{\text{rad}}{\text{seg}} + 4{,}188\frac{\text{rad}}{\text{seg}}t - 0{,}333\sin(4\pi.t) \end{aligned}}$$



## *Torques*

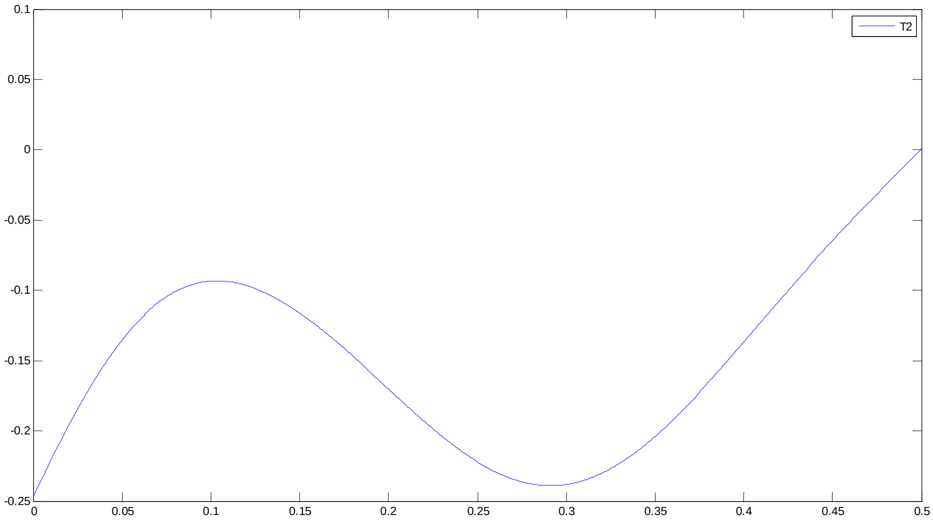
### Motor N°1 (talón)

Aplicando la formula obtenida en el punto anterior, y en base a las aceleraciones, velocidades y posiciones angulares que hemos supuesto para el salto, obtenemos:




En donde el torque máximo requerido para el motor 1 es de 3.9122Nm.

### Motor N°2 (rodilla)




Dándonos un torque máximo para el motor 2 de 0.2476Nm.

## Conclusiones

Observamos que el motor del talón (motor N°1) es el que absorbe prácticamente todo el esfuerzo a la hora del salto. Esto en si es una ventaja, ya que disminuye drásticamente el tamaño del otro motor, reduciendo el peso y los costos de desarrollo. Aunque en el caso de querer realizar un robot entero, los motores se incrementarán sensiblemente por los pesos.

Vemos como los torques varían con el tiempo, y esto es debido a los efectos de la gravedad y de las fuerzas centrifugas y de coriolisis. El efecto de que los torques sean negativos, es meramente por los sistemas utilizados, siendo en realidad lo que se toma para dimensionar al motor el valor máximo del extremo.

Se ha logrado alcanzar un resultado bastante similar a la de la tesis propuesta, aunque algunos términos difieren levemente. Por último, el motor dio un poco menor que el de la tesis, debido a que se tomaron tiempos más largos para alcanzar la normal. Al incrementar la velocidad de reacción que se quiere, aumentarían los torques necesarios de los motores.

# Simulación de Control PWM:

A partir del módulo PWM provisto en la Application Note de ATMEL (pwm_fpga.vhd), se expandió el mismo para manejar un esquema de seis transistores, con aplicación práctica como drive de motor CC.

Las señales de entrada se mantuvieron acordes al módulo original. Además de la señal de reset y de clock, se cuenta con un variable de 8 bits correspondiente al duty cycle.

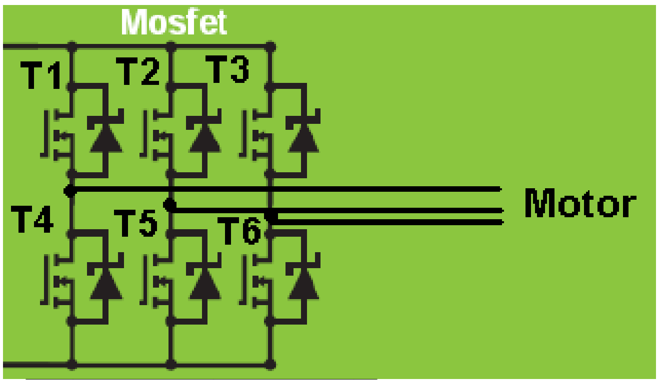
**Table 1.** Data Values for Different Duty Cycles

| Data Value | Duty Cycle (%) |
|---|---|
| 11100110 | 90 |
| 11000000 | 75 |
| 10000000 | 50 |
| 01000000 | 25 |
| 00011001 | 10 |

## *Esquemático:*

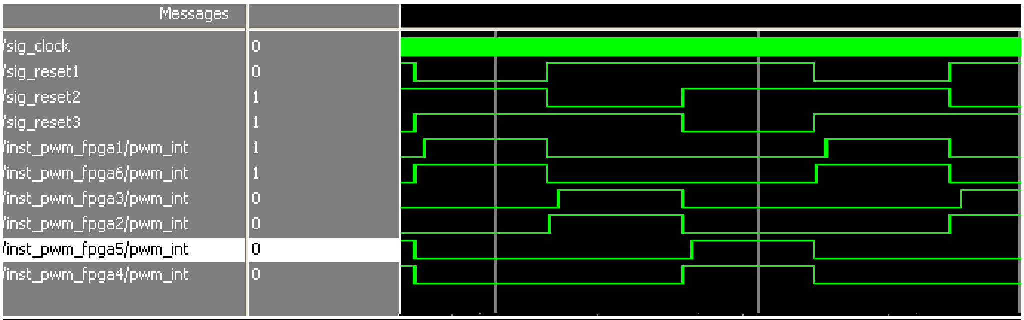
La señal modulada será ingresada en los gates de los transistores T1, T2 y T3, mientras que los transistores T4, T5 y T6 conmutan como llaves on / off durante la totalidad del ciclo.

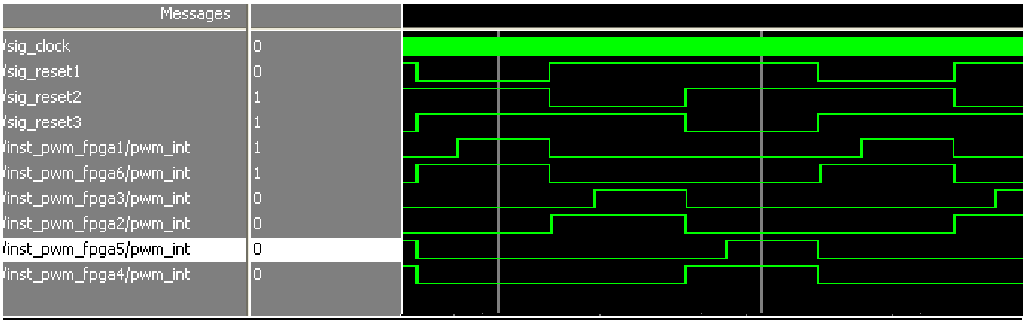
La corriente circula al cerrarse las siguientes combinaciones, alternadas en forma consecutiva: T1-T6, T2-T4 y por último T3-T4.

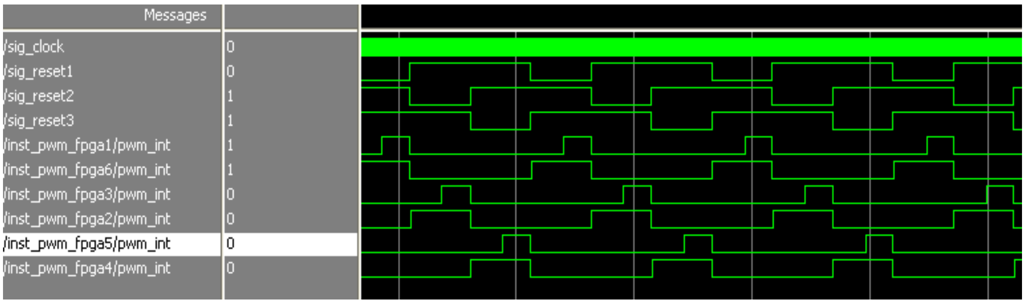


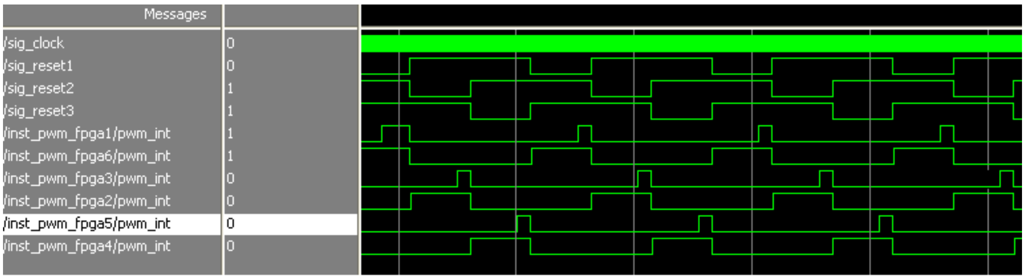
## *Resultados de la Simulación*

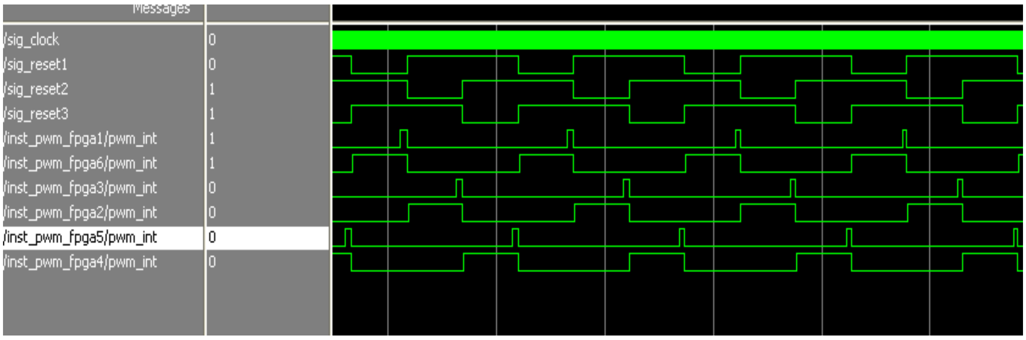
A partir del loop desarrollado en el test bench se barrieron todos los valores posibles de duty cycle. Se muestran capturas de distintos valores, correspondientes a valores de duty altos, medios y bajos.






## *Archivo VHDL*

**“pwm_test_bench_final.vhd”**

```
LIBRARY ieee;
use ieee.std_logic_1164.all;
ENTITY pwm_fpga_test_bench IS
END pwm_fpga_test_bench;
ARCHITECTURE arch_test_bench OF pwm_fpga_test_bench IS
COMPONENT pwm_fpga
PORT (
      clock: in std_logic;
      reset: in std_logic;
      data_value: in std_logic_vector(7 downto 0);
      pwm: out std_logic
);
END COMPONENT;

-- Declaracion de señales para uso interno.

SIGNAL sig_clock    : std_logic;
SIGNAL sig_reset1   : std_logic;
SIGNAL sig_reset2   : std_logic;
SIGNAL sig_reset3   : std_logic;
SIGNAL sig_data_value: std_logic_vector(7 downto 0);
SIGNAL sig_100 : std_logic_vector(7 downto 0);
SIGNAL sig_pwm1,sig_pwm2,sig_pwm3,sig_pwm4,sig_pwm5,sig_pwm6
      : std_logic;
shared variable ENDSIM: boolean:=false;
  constant clk_period:TIME:=100 ns;

BEGIN
clk_gen: process

BEGIN
```

```
		If ENDSIM = FALSE THEN
			sig_clock <= '1';
			wait for clk_period/2;
			sig_clock <= '0';
			wait for clk_period/2;
		else
			wait;
		end if;
	end process;
```

**-- A partir del modulo pwm_fpga basado en al app. note**
**-- de ATMEL, se instancian 6 modulos idénticos, uno por transistor.**

```
inst_pwm_fpga1 : pwm_fpga
PORT MAP(
		clock		=> sig_clock,
		reset		=> sig_reset1,
		data_value	=> sig_data_value,
		pwm	=> sig_pwm1
	);

inst_pwm_fpga2 : pwm_fpga
PORT MAP(
		clock		=> sig_clock,
		reset		=> sig_reset2,
		data_value	=> sig_100,
		pwm	=> sig_pwm2
	);
inst_pwm_fpga3 : pwm_fpga
PORT MAP(
		clock		=> sig_clock,
		reset		=> sig_reset2,
		data_value	=> sig_data_value,
		pwm	=> sig_pwm3
	);
inst_pwm_fpga4 : pwm_fpga
PORT MAP(
		clock		=> sig_clock,
		reset		=> sig_reset3,
		data_value	=> sig_100,
		pwm	=> sig_pwm4
	);

inst_pwm_fpga5 : pwm_fpga
PORT MAP(
		clock		=> sig_clock,
```

```
        reset     => sig_reset3,
        data_value     => sig_data_value,
        pwm  => sig_pwm5
    );

inst_pwm_fpga6 : pwm_fpga
PORT MAP(
        clock     => sig_clock,
        reset     => sig_reset1,
        data_value     => sig_100,
        pwm  => sig_pwm6
    );

stimulus_process: PROCESS
```

-- Se inicializan las variables auxiliares necesarias.
-- La varrable sig_data_value corresponde al duty_cycle actual.
-- La variable sig_100 se inicializa para el menor duty cycle disponible.
-- Se resetean las tres ramas, y por último se esperan 100ns antes de comenzar el lazo.

```
  VARIABLE b1,b2,b3,b4,b5,b6,b7,EnableRama   : integer;
  BEGIN
  EnableRama:=0;
  b1:=0; b2:=0; b3:=0; b4:=0; b5:=0; b6:=0; b7:=0;
  sig_data_value <= "00000000";
  sig_100 <= "00000001";
     sig_reset1 <= '1';
     sig_reset2 <= '1';
     sig_reset3 <= '1';
     wait for 100 ns;
```

-- Se incrementan los vectores binarios
-- hasta alcanzar el valor máximo.

```
     for I in 0 to 255 loop
     b1:= b1+1; b2:= b2+1; b3:= b3+1; b4:= b4+1; b5:= b5+1;
b6:= b6+1;
     b7:= b7+1;

     sig_data_value(0) <=  not sig_data_value(0);
```

-- Se controla si se ha cumplido el tiempo del ciclo
-- correspondiente a la parte inactiva. En caso
-- afirmativo, se invierte el estado.

```
     if(b1 = 2) then
       sig_data_value(1) <=  not sig_data_value(1);
```

```
      b1:=0;
      end if;

    if(b2 = 4) then
      sig_data_value(2) <=  not sig_data_value(2);
      b2:=0;
      end if;

    if(b3 = 8) then
      sig_data_value(3) <=  not sig_data_value(3);
      b3:=0;
      end if;

    if(b4 = 16) then
      sig_data_value(4) <=  not sig_data_value(4);
      b4:=0;
      end if;

    if(b5 = 32) then
      sig_data_value(5) <=  not sig_data_value(5);
      b5:=0;
      end if;

    if(b6 = 64) then
      sig_data_value(6) <=  not sig_data_value(6);
      b6:=0;
      end if;

    if(b7 = 128) then
      sig_data_value(7) <=  not sig_data_value(7);
      b7:=0;
    end if;

-- Por cada loop, habilito una a una las ramas del puente.
-- Solo puede haber una habilitada por vez.
-- Al completar una vuelta, reinicio la secuencia.

    if(EnableRama = 0) then
    sig_reset1 <= '0';
    sig_reset2 <= '1';
    sig_reset3 <= '1';
    end if;

    if(EnableRama = 1) then
    sig_reset1 <= '1';
    sig_reset2 <= '0';
    sig_reset3 <= '1';
```

```
      end if;

      if(EnableRama = 2) then
      sig_reset1 <= '1';
      sig_reset2 <= '1';
      sig_reset3 <= '0';
      end if;

      EnableRama := EnableRama+1;
      if(EnableRama = 3) then
        EnableRama:=0;
      end if;

-- Espero a que termine el periodo actual. (256 pulsos de clock)
-- Sincroniza con el próximo período.

      wait for 25600 ns;
      end loop;


-- Se resetean las tres ramas del puente.

      sig_reset1 <= '1';
      sig_reset2 <= '1';
      sig_reset3 <= '1';
wait;
 END PROCESS stimulus_process;
 END arch_test_bench;
```