# Appendix. The Interregional Computable General Equilibrium Model for Croatia: Theoretical Specification and Current Developments

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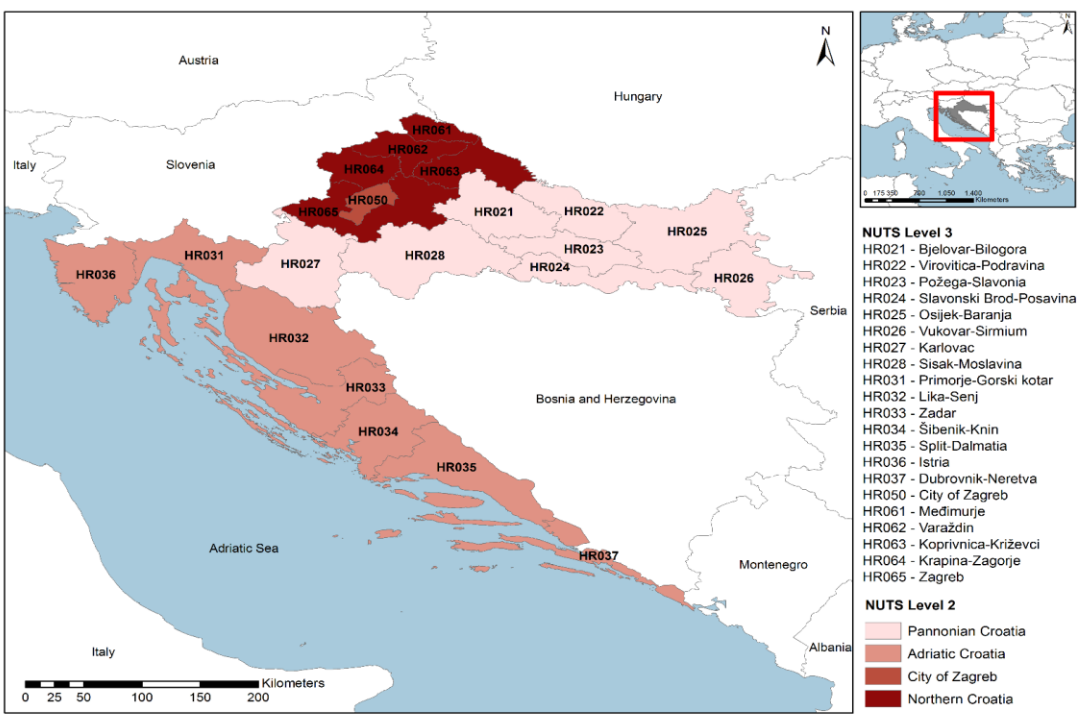
## 1. Introduction

In this appendix, we provide an overview of the specification of the linear form of the Interregional Computable General Equilibrium (ICGE) model for Croatia, based on the different groups of equations. The description is further presented around the TABLO file in Annex 1, which implements the model in GEMPACK. Attention is directed to the most relevant equations for the functioning mechanism of the model. The sets, variables and coefficients referring to each equation are explicitly declared. Variables are defined in percentage change form, unless specified otherwise.

Figure 1 shows the regional boundaries of Croatia (NUTS3)..

**Figure 1. Regional Setting of the ICGE Model**



## 2. The Johansen Approach[1]

The ICGE model for Croatia falls into a class of CGE models known as Johansen-type CGE models in the sense that the solutions are obtained by solving the system of linearized equations of the model.[2] A typical result shows the percentage change in the set of endogenous variables after a policy is carried out, compared to their values in the absence of such policy in a given environment. In Johansen-type CGE models, the system of equations of the model[3] can be written as:

$$\boldsymbol{F}(\boldsymbol{V}) = 0 \tag{1}$$

where $\boldsymbol{V}$ is an equilibrium vector of length *n* (number of variables), and $\boldsymbol{F}$ is a vector function of length *m* (number of equations), which is assumed to be differentiable. Regarding the dimensions, *n* and *m*, it is assumed that the total number of variables is greater than the total number of equations in the system (*n* > *m)*. Thus, (*n* - *m*) variables must be set exogenously. For calibration of the system, it is fundamental to assume that $\exists \boldsymbol{V} = \boldsymbol{V}^* \; s.t.\boldsymbol{F}(\boldsymbol{V}^*) = 0$ and the initial solution, $\boldsymbol{V}^*$, is known.

The Johansen approach consists of using a differential or log differential version of Eq. 1, which may be represented as:

$$\boldsymbol{A}(\boldsymbol{V})v = 0 \tag{2}$$

where $\boldsymbol{A}(\boldsymbol{V})$ is a (*m* x *n*) matrix containing partial derivatives or elasticities, and $v$ is appropriately calculated as changes, log-changes, or percentage changes in vector $\boldsymbol{V}$.

The procedure to obtain approximate estimates of (percentage) changes in endogenous variables is to evaluate $\boldsymbol{A}(.)$ on a known initial equilibrium vector $\boldsymbol{V^I}$and then solve Eq. 2. It is helpful to

[1] We have followed closely the notation and exposition of the Johansen approach presented in Dixon et al. (1992).
[2] More details can be found in Dixon *et al*. (1982, 1992), and Dixon & Parmenter (1996).
[3] A CGE model is simply the formalization of this general representation, along with the equilibrium constraints defined by the Walrasian paradigm. CGE models differ in how they define their analytical, functional and numerical structures.

partition matrix $\boldsymbol{A}$ and vector $\boldsymbol{v}$ into two parts each, separating the endogenous and exogenous variables. The endogenous and exogenous parts of the system are indexed $\alpha$ and $\beta$, respectively:

$$\boldsymbol{A}(\boldsymbol{V^I})v = \boldsymbol{A}_\alpha(\boldsymbol{V^I})v_\alpha + \boldsymbol{A}_\beta(\boldsymbol{V^I})v_\beta = 0 \tag{3}$$

$$\boldsymbol{v_\alpha} = -\boldsymbol{A}_\alpha(\boldsymbol{V^I})^{-1}\boldsymbol{A}_\beta(\boldsymbol{V^I})v_\beta \tag{4}$$

$$\boldsymbol{v_\alpha} = \boldsymbol{B}(\boldsymbol{V^I})v_\beta \tag{5}$$

where $\boldsymbol{A}_\alpha$ is ($m$ x $n$), $\boldsymbol{v}_\alpha$ is ($m$ x 1), $\boldsymbol{A_\beta}$ is ($m$ x $n$-$m$), $\boldsymbol{v_\beta}$ is ($n$-$m$ x $1$), and $\boldsymbol{B}(\boldsymbol{V^I})$ is defined as $\boldsymbol{A_\alpha}(\boldsymbol{V^I})^{-\mathbf{1}}\boldsymbol{A_\beta}(\boldsymbol{V^I})$**.**

## 3. Specification of the ICGE Model

We present the specification of the linearized form of the ICGE model, $A(V)v = 0$, based on different groups of equations. The notational convention uses uppercase letters to represent the levels of the variables and lowercase for their percentage-change representation. Superscripts *(u)*, *u = 0, 1j, 2j, 3, 4, 5,* 6 refer, respectively, to output (0) and the six different regional-specific users of the products identified in the model:[4] producers in sector *j* (1*j*), investors in sector *j* (2*j*), households (3), purchasers of exports (4), regional governments (5), and central government (6); the second superscript *(r)* identifies the domestic region where the user is located. Two subscripts identify inputs: the first *(i)* takes the values 1, ..., *g*, for commodities and *g + 1* for primary factors; the second subscript identifies the source of the input, being it from domestic region *b* (1*b*) or imported (2) or coming from labor (1) or capital (2), the two primary factors in the model. The symbol (•) is employed to indicate a sum over an index.

We define the following sets: $G = \{1, \dots, g\}$, where $g$ is the number of composite goods; $G^* = \{1, \dots, g, g+1\}$, where *g+1* is the number of composite goods and primary factors, with $G^* \supset G$;

[4] We have specified a seventh residual user, *(7)*, to deal with statistical discrepancies in the balancing of the model's absorption matrix based on the Croatian interregional input-output system (IIOS).

$H = \{1, \ldots, h\}$, where $h$ is the number of industries; $U = \{(3), (4b), (5), (6), (kj)\}$ for $k = (1), (2)$ and $j \in H$, is the set of all users in the model; $U^* = \{(3), (5), (6), (kj)\}$ for $k = (1), (2)$ and $j \in H$, with $U \supset U^*$, is the subset of domestic users; $S = \{1, \ldots, r, r + 1\}$, where $r+1$ is the number of all regions (including foreign); $S^* = \{1, \ldots, r\}$, with $S \supset S^*$, is the subset with the $r$ domestic regions; and $F = \{1, \ldots, f\}$ is the set of primary factors. In the ICGE model, $g = h = 54$, $r = 33$, and $f = 2$.

We model the sourcing of composite goods based on multilevel structures, enabling a significant number of substitution possibilities. We employ nested sourcing functions to create composite goods available for consumption in the regions of the model. We assume that domestic users such as firms, investors, households, and government, use combinations of composite goods specified within two-level Constant Elasticity of Substitution (CES) nests. At the bottom level, bundles of domestically produced goods are formed as combinations of goods from different regional sources. At the top level, substitution is possible between domestically produced and imported goods. Equations 6 and 7 describe the regional sourcing of domestic goods and the substitution between domestic and imported products.

$$x_{(i(1b))}^{(u)r} = x_{(i(1\bullet))}^{(u)r} - \sigma 1_{(i)}^{(u)r} \left( p_{(i(1b))}^{(u)r} - \sum_{l \in S^*} \left( \frac{V(i,1l,(u),r)}{V(i,1\bullet,(u),r)} \right) \left( p_{(i(1l))}^{(u)r} \right) \right)$$

$$i \in G;\ b \in S^*; (u) \in U^*;\ r \in S^* \tag{6}$$

where $x_{(i(1b))}^{(u)r}$ is the demand by the user *(u)* in region $r$ for good $i$ in the domestic region *(1b)*; $p_{(i(1b))}^{(u)r}$ is the price paid by user *(u)* in region $r$ for good $i$ in the domestic region *(1b)*; $\sigma 1_{(i)}^{(u)r}$ is a parameter measuring the user-specific elasticity of substitution between alternative domestic sources of commodity $i$, known as the regional trade Armington elasticity; and $V(i, 1l, (u), r)$ is an input-output flow coefficient that measures purchasers' value of good $i$ from domestic source $l$ used by the user *(u)* in region $r$.

$$x_{(is)}^{(u)r} = x_{(i\bullet)}^{(u)r} - \sigma 2_{(i)}^{(u)r} \left( p_{(is)}^{(u)r} - \sum_{l=1\bullet,2} \left( \frac{V(i,l,(u),r)}{V(i,\bullet,(u),r)} \right) \left( p_{(il)}^{(u)r} \right) \right)$$

$$i \in G;\ s = 1\bullet, 2; (u) \in U^*;\ r \in S^* \tag{7}$$

where $x_{(is)}^{(u)r}$ is the demand by the user *(u)* in region $r$ for either the domestic composite or the foreign good $i$; $p_{(is)}^{(u)r}$ is the price paid by user *(u)* in region $r$ for either the domestic composite or the foreign good $i$; $\sigma 2_{(i)}^{(u)r}$ is a parameter measuring the user-specific elasticity of substitution between the domestic bundle and imports of good $i$, known as the international trade Armington elasticity; and $V(i, l, (u), r)$ is an input-output flow coefficient that measures purchasers' value of good $i$ from either the aggregate domestic source or the foreign source $l$ used by the user *(u)* in region $r$.

In addition to goods used as intermediate inputs, firms in the model also demand primary factors of production. The equations that describe the industry $j$'s demands inputs are derived under the assumption of Leontief technology with Armington nests (imperfect substitution between inputs of the same type from different sources). In our specification of the nested production functions, we assume firms use combinations of composite intermediate inputs, formed according to Eq. 6 and Eq. 7, and primary factor composites. In the case of the primary factor bundle, substitution is possible among different types of primary factors. Eq. 8 specifies the model's substitution between labor and capital. It is derived under the assumption that industries choose their primary factor inputs to minimize costs subject to obtaining sufficient primary factor inputs to satisfy their technological requirements (nested Leontief/CES specification). We have included technical change variables to allow for factor-specific productivity shocks. We model the combination of intermediate inputs and the value-added (primary factors) aggregate in fixed proportions at the top of the nested production function, assuming no substitution between primary factors and other inputs. The Leontief specification is presented in Eq. 9. Due to data availability constraints, more flexible functional forms have rarely been introduced in multiregional models. In addition to a technical coefficient in the relation between the sectoral demand for the primary factor composite and the total output, we have also included a scale parameter. This modeling procedure is based on previous studies made by Haddad and Hewings (2005), which allows for the introduction of Marshallian agglomeration (external) economies by exploring the local properties of the CES function.

$$x_{(g+1,s)}^{(1j)r} - a_{(g+1,s)}^{(1j)r} = \alpha_{(g+1,s)}^{(1j)r} x_{(g+1,\bullet)}^{(1j)r} - \sigma 3_{(g+1)}^{(1j)r} \left( p_{(g+1,s)}^{(1j)r} + a_{(g+1,s)}^{(1j)r} - \sum_{l \in F} \left( \frac{V(g+1,l,(1j),r)}{V(g+1,\bullet,(1j),r)} \right) \left( p_{(g+1,l)}^{(1j)r} + a_{(g+1,l)}^{(1j)r} \right) \right)$$

$$j \in H;\ s \in F;\ r \in S^* \tag{8}$$

where $x_{(g+1,s)}^{(1j)r}$ is the demand by sector *j* in region *r* for each primary factor; $a_{(g+1,s)}^{(1j)r}$ is the exogenous sector-specific variable of (saving) technical change for primary factor *s* in region *r*; $p_{(g+1,s)}^{(1j)r}$ is the price paid by sector *j* in region *r* for primary factor *s*; $\sigma 3_{(g+1)}^{(1j)r}$ is a parameter measuring the sector-specific elasticity of substitution among different primary factors; and $V(g+1, l, (1j), r)$ is an input-output flow coefficient that measures purchasers' value of factor *l* used by sector *j* in region *r*.

$$x_{(i\bullet)}^{(1j)r} = \mu_{(g+1,\bullet)}^{(1j)r} z^{(1j)r} + a_{(i)}^{(1j)r}$$

$$j \in H;\ i \in G^*;\ r \in S^* \tag{9}$$

where $x_{(i\bullet)}^{(1j)r}$ is the demand by sector *j* in region *r* for the bundles of composite intermediate inputs and primary factors *i*; $z^{(1j)r}$ is the total output of sector *j* in region *r*; $a_{(i)}^{(1j)r}$ is the exogenous sector-specific variable of technical change for composite intermediate inputs and primary factors in region *r*; and $\mu_{(i\bullet)}^{(1j)r}$ is a scale parameter measuring the sector-specific returns to the composite of primary factors in each region.

Units of capital stock are created for industry *j* at minimum cost. Commodities are combined via a Leontief function, as specified in Eq. 10. In Eq. 6 and 7, regional and domestic-imported commodities are combined, respectively, via a CES specification (Armington assumption). No primary factors are used in capital creation. The use of these inputs is recognized through the capital goods-producing sectors in the model, mainly machinery and equipment industries, construction, and support services.

$$x_{(i\bullet)}^{(2j)r} = z^{(2j)r} + a_{(i)}^{(2j)r}$$

$j \in H;\ i \in G;\ r \in S^*$ (10)

where $x_{(i\bullet)}^{(2j)r}$ is the demand by sector *j* in region *r* for the bundles of composite capital goods *i*; $z^{(2j)r}$ is the total investment of sector *j* in region *r*; $a_{(i)}^{(2j)r}$ is the exogenous sector-specific variable of technical change for changing the composition of the sectoral unit of capital in region *r*.

In deriving the household demands for composite commodities, we assume that households in each region behave as a single, budget-constrained, utility-maximizing entity. The utility function is of the Stone-Geary or Klein-Rubin form. Equation 11 determines the optimal composition of household demand in each region. Total regional household consumption is determined as a function of real household income. The demands for the commodity bundles in the nesting structure of household demand follow the CES pattern established in Eq. 6 and Eq. 7, in which an activity variable and a price-substitution term play significant roles. In Eq. 11, consumption of each commodity *i* depends on two components: first, for the subsistence component, which is defined as the minimum expenditure requirement for each commodity, changes in demand are generated by changes in the number of households and tastes; second, for the luxury or supernumerary part of the expenditures in each good, demand moves with changes in the regional supernumerary expenditures, changes in tastes, and changes in the price of the composite commodity. The two components of household expenditures on the composite commodities are weighted by their respective shares in the total consumption of the composite commodity.

$$
\begin{aligned}
V(i,\bullet,(3),\mathrm{r})&\left(p_{(i\bullet)}^{(3)r}+x_{(i\bullet)}^{(3)r}-a_{(i\bullet)}^{(3)r}\right)\\
&=\gamma_{(i)}^{r}P_{(i\bullet)}^{(3)r}Q^{r}\left(p_{(i\bullet)}^{(3)r}+x_{(i\bullet)}^{(3)r}-a_{(i\bullet)}^{(3)r}\right)\\
&+\beta_{(i)}^{r}\left(C^{r}-\sum\nolimits_{j\in G}\gamma_{(j)}^{r}P_{(j\bullet)}^{(3)r}Q^{r}\left(p_{(j\bullet)}^{(3)r}+x_{(j\bullet)}^{(3)r}-a_{(i\bullet)}^{(3)r}\right)\right)
\end{aligned}
$$

$i \in G;\ r \in S^*$ (11)

where $p_{(i\bullet)}^{(3)r}$ is the price paid by household in region *r* for the composite good *i*; $x_{(i\bullet)}^{(3)r}$ is the household demand in region *r* for the composite good *i*; $a_{(i\bullet)}^{(3)r}$ is the commodity-specific variable of regional taste change; $Q^r$ is the number of households in region *r*; $C^r$ is the total expenditure by household in region *r*, which is proportional to regional labor income; $\gamma_{(i)}^r$ is the subsistence parameter in the linear expenditure system for commodity *i* in region *r*; $\beta_{(i)}^r$ is the parameter defined for commodity *i* in region *r* measuring the marginal budget shares in the linear expenditure system; and $V(i, \bullet, (3), \mathrm{r})$ is an input-output flow coefficient that measures purchasers' value of good *i* consumed by households in region *r*.

As noted by Peter *et al*. (1996), a feature of the Stone-Geary utility function is that only the above-subsistence, or luxury, component of real household consumption, $utility^{(r)}$, affects the per-household utility, as described in Eq. 12.

$$utility^{(r)} = \left(C^r - \sum_{j \in G} \gamma_{(j)}^r P_{(j\bullet)}^{(3)r} Q^r \left(p_{(j\bullet)}^{(3)r} + x_{(j\bullet)}^{(3)r} - a_{(i\bullet)}^{(3)r}\right)\right) - q^r - \sum_{i \in G} \beta_{(i)}^r \, p_{(i\bullet)}^{(3)r}$$
$$r \in S^*$$

(12)

where $q^r$ is the percentage change in the number of households in each region.

In Eq. 13, foreign demands (exports) for domestic good *i* depend on the percentage changes in a price and three-shift variables which allow for vertical and horizontal movements in the demand curves. The price variable that influences export demands is the purchaser's price in foreign countries, including the relevant taxes and margins. The parameter $\eta_{(is)}^r$ controls the sensitivity of export demand to price changes.

$$\left(x_{(is)}^{(4)r} - fq_{(is)}^{(4)r}\right) = \eta_{(is)}^r \left(p_{(is)}^{(4)r} - phi - fp_{(is)}^{(4)r}\right)$$
$$i \in G; \; r, s \in S^*$$

(13)

where $x_{(is)}^{(4)r}$ is foreign demand for domestic good *i* produced in region *s* and sold from region *r* (in the model, there are no re-exports, so that *r* = *s*); $p_{(is)}^{(4)r}$ is the purchasers' price in the domestic currency of exported good $i$ demand in region $r$; $phi$ is the nominal exchange rate; $fq_{(is)}^{(4)r}$ and $fp_{(is)}^{(4)r}$ are, respectively, quantity and price shift variables in foreign demand curves for regional exports.

Governments consume mainly public goods provided by the public administration sectors. Equations 14 and 15 show the movement of government consumption concerning movements in real tax revenue for regional governments and the central government, respectively.

$$x_{(is)}^{(5)r} = taxrev^r + f_{(is)}^{(5)r} + f^{(5)r} + f^{(5)}$$
$$i \in G;\ s = 1\text{b}, 2;\ r, b \in S^* \tag{14}$$

$$x_{(is)}^{(6)r} = nattaxrev + f_{(is)}^{(6)r} + f^{(6)r} + f^{(6)}$$
$$i \in G;\ s = 1\text{b}, 2;\ r, b \in S^* \tag{15}$$

where $x_{(is)}^{(5)r}$ and $x_{(is)}^{(6)r}$ are regional (5) and central (6) governments demand in region $r$ for good $i$ from region $s$; $f_{(is)}^{(5)r}$, $f^{(5)r}$ and $f^{(5)}$ are, respectively, commodity and source-specific shift terms for regional government expenditures in region $r$, shift term for regional governments expenditures in region $r$, and an overall shift term for regional governments expenditures. Similar shift terms ($f_{(is)}^{(6)r}$, $f^{(6)r}$ and $f^{(6)}$) appear in Eq. 15 related to central government expenditures. Finally, $taxrev^r$ is the percentage change in real revenue from indirect taxes in region $r$, and $nattaxrev$ refers to the percentage change in aggregate real revenue from indirect taxes, so that government demand moves with endogenous changes in regional and national tax bases.

The model's margins demand equations[5] show that the margins' demands are proportional to the commodity flows with which the margins are associated; moreover, a technology change component allows changes in the implicit transportation rate (Eq. 16).

$$x_{(m1)}^{(is)(u)r} = x_{(is)}^{(u)r} + a_{(m1)}^{(is)(u)r}$$

$m, i = 1, \ldots, g;$
$(u) = (3), (4b)$ for $b = 1, \ldots, r, (5)$ and $(kj)$ for $k = 1,2;$
$j = 1, \ldots, h;\ s = 1b, 2$ for $b = 1, r;$
$r = 1, \ldots, R$

(16)

where $x_{(m1)}^{(is)(u)r}$ is the demand for commodity *m1* to be used as a margin to facilitate the flow of *is* to *u* in region *r*; $x_{(is)}^{(u)r}$ is the demand by user *u* in region *r* for good *is*; and $a_{(m1)}^{(is)(u)r}$ is the technical change related to the demand for commodity *m1* to be used as a margin to facilitate the flow of *is* to *u* in region *r*.

Equation 17 specifies the sales tax rates for different users. They allow for variations in tax rates across commodities, their sources and destinations. Tax changes are expressed as percentage-point changes in the ad valorem tax rates.

$$t_{(is)}^{(u)r} = f_i + f_i^{(u)} + f_i^{(u)r}$$

$$i \in G;\ s = 1\text{b}, 2;\ b, r \in S^*;\ u \in U$$

(17)

where $t_{(is)}^{(u)r}$ is the power of the tax on sales of the commodity (*is*) to the user (*u*) in region *r*; and $f_i$, $f_i^{(u)}$, and $f_i^{(u)r}$ are different shift terms allowing percentage changes in the power of taxation.

[5] For completeness, we provide the usual specification of margins demand equation. However, in the current version of the model, there is no explicit margin commodity.

Equations 18 and 19 impose the equilibrium conditions in the market's domestic and imported commodities. Notice that there is no margin commodity in the model. Moreover, there is no secondary production in the model. In Eq. 18, demand equals supply for regional domestic commodities.

$$\sum_{j\in H} Y(l,j,r)x_{(l1)}^{(0j)r} = \sum_{(u)\in U} B(l,1b,(u),r)x_{(l1)}^{(u)r}$$

$$l \in G, b, r \in S^{*} \qquad (18)$$

where $x_{(l1)}^{(0j)r}$ is the output of domestic good *l* by industry *j* in region *r*; $x_{(l1)}^{(u)r}$ is the demand of the domestic good *l* by the user *(u)* in region *r*; $Y(l,j,r)$ is the input-output flow measuring the basic value of the output of domestic good *l* by industry *j* in region *r*; and $B(l,1,(u),r)$ is the input-output flow measuring the basic value of domestic good *l* used by *(u)* in region *r*.

Equation 19 imposes zero pure profits in importing. Where $p_{(i(2))}^{(0)}$ is the basic price in the domestic currency of good *i* from a foreign source; $p_{(i(2))}^{(w)}$ is the world Cost, Insurance and Freight (CIF) price of imported commodity *i*; $phi$ is the nominal exchange rate; and $t_{(i(2))}^{(0)}$ is the power of the tariff. i.e., one plus the tariff rate, on imports of *i*. Equation 18, thus, defines the basic price of a unit of imported commodity *i* – the revenue earned per unit by the importer – as the international CIF price converted to domestic currency, including import tariffs.

$$p_{(i(2))}^{(0)} = p_{(i(2))}^{(w)} - phi + t_{(i(2))}^{(0)}$$
$$i \in G \qquad (19)$$

Together with Eq. 18, Eqs. 20-21 constitute the model's pricing system. The price received for any activity equals the costs per unit of output. As can be noticed, the assumption of constant returns to scale adopted here precludes any activity variable from influencing basic prices, i.e., unit costs are independent of the scale at which activities are conducted. Thus, Eq. 19 defines the percentage change in the price received by producers in regional industry *j* per unit of output as equal to the percentage change in *j*'s costs, which are affected by changes in technology and input prices.

$$\sum_{l \in G} Y(l,j,r)\left(p_{(l1)}^{(0)r} + a_{(l1)}^{(0)r}\right) = \sum_{l \in G^*,F} \sum_{s \in S} V(l,s,(1j),r) p_{(ls)}^{(1j)r}$$
$$j \in H;\ r \in S^* \qquad (20)$$

where $p_{(l1)}^{(0)r}$ is the basic price of domestic good *i* in region *r*; $a_{(l1)}^{(0)r}$ refer to technological changes, measured as a weighted average of the different types of technical changes with influence on *j*'s unit costs; $p_{(ls)}^{(1j)r}$ is the unit cost of sector *j* in region *r*; $Y(l,j,r)$ is the input-output flow measuring the basic value of the output of domestic good *l* by industry *j* in region *r*; and $V(l,s,(1j),r)$ are input-output flows measuring purchasers' value of good or factor *l* from source *s* used by sector *j* in region *r*.

Equation 21 imposes zero pure profits in the distribution of commodities to different users. Prices paid for commodity *i* from region *s* in industry *j* in region *r* by each user equate to the sum of its basic value and the costs of the relevant taxes.

$$V(i,s,(u),r) p_{(is)}^{(u)r} = \big(B(i,s,(u),r) + T(i,s,(u),r)\big)\left(p_{(is)}^{(0)} + t_{(is)}^{(u)r}\right)$$
$$i \in G;\ s = 1\text{b},2;\ b,r \in S^*;\ u \in U \qquad (21)$$

where $p_{(is)}^{(u)r}$ is the price paid by user *(u)* in region *r* for good *(is)*; $p_{(is)}^{(0)}$is the basic price of domestic good *(is)*; $t_{(is)}^{(u)r}$ is the power of the tax on sales of the commodity *(is)* to the user *(u)* in region *r*;

$V(i,s,(u),r)$ are input-output flows measuring purchasers' value of good *i* from source *s* used by the user *(u)* in region *r*; $B(i,s,(u),r)$ is the input-output flow measuring the basic value of the good *(is)* used by *(u)* in region *r*; and $T(i,s,(u),r)$ is the input-output flow associated with tax revenue of the sales of *(is)* to *(u)* in region *r*.

The theory of investment allocation across industries is represented in Eqs. 22-25. The comparative-static nature of the model restricts its use to short-run and long-run policy analysis. There is no fixed relationship between capital and investment when running the model in the comparative-static mode. The user decides the required relationship based on the specific simulation requirements. Equation 21 defines the percentage change in the current rate of return on fixed capital in regional sectors. Under static expectations, rates of return are defined as the ratio between the rental values and the cost of a unit of capital in each industry – defined in Eq. 23 –, minus the depreciation rate.

$$r_{(j)}^{r} = \psi_{(j)}^{r}\left(p_{(g+1,2)}^{(1j)r} - p_{(k)}^{(1j)r}\right)$$
$$j \in H;\ r \in S^{*} \qquad (22)$$

where $r_{(j)}^{r}$ is the regional-industry-specific rate of return; $p_{(g+1,2)}^{(1j)r}$ is the rental value of capital in sector *j* in region *r*; $p_{(k)}^{(1j)r}$ is the cost of constructing units of capital for regional industries; and $\psi_{(j)}^{r}$ is a regional-industry-specific parameter referring to the ratio of the gross to the net rate of return.

Equation 22 defines $p_{(k)}^{(1j)r}$ as:

$$V(\bullet,\bullet,(2\mathrm{j}),\mathrm{r})\left(p_{(k)}^{(1j)r} - a_{(k)}^{(1j)r}\right) = \sum_{i\in G}\sum_{s\in S} V(\mathrm{i},\mathrm{s},(2\mathrm{j}),\mathrm{r})\left(p_{(is)}^{(2j)r} - a_{(is)}^{(2j)r}\right)$$
$$j \in H;\ r \in S^{*} \qquad (23)$$

where $p_{(is)}^{(2j)r}$ is the price paid by user *(2j)* in region *r* for good *(is)*; $a_{(k)}^{(1j)r}$ and $a_{(is)}^{(2j)r}$ are technical terms; and $V(i, s, (2j), r)$ represents input-output flows measuring purchasers' value of good *i* from source *s* used by the user (2*j*) in region *r*.

Equation 24 says that if the percentage change in the rate of return in a regional industry grows faster than the national average, capital stocks in that industry will increase at a higher rate than the average national stock. For industries with a lower-than-average increase in their rates of return to fixed capital, capital stocks increase at a lower-than-average rate, i.e., capital is attracted to higher return industries. The shift variable, $f_{(k)}^{(1j)r}$– exogenous in long-run simulation – allows shifts in the industry's rates of return.

$$r_{(j)}^{r} - \omega = \varepsilon_{(j)}^{r}\left(x_{(g+1,2)}^{(1j)r} - x_{(g+1,2)}^{(\bullet)r}\right) + f_{(k)}^{(1j)r}$$
$$j \in H;\ r \in S^{*} \qquad (24)$$

where $r_{(j)}^{r}$ is the regional-industry-specific rate of return; $\omega$ is the overall rate of return on capital; $x_{(g+1,2)}^{(1j)r}$ is the capital stock in industry *j* in region *r*; $f_{(k)}^{(1j)r}$ is the capital shift term in sector *j* in region *r*; and $\varepsilon_{(j)}^{r}$ measures the sensitivity of capital growth to rates of return of industry *j* in region $r$.

Equation 25 implies that the percentage change in an industry's capital stock, $x_{(g+1,2)}^{(1j)r}$, is equal to the percentage change in the industry's investments in the period, $z^{(2j)r}$.

$$z^{(2j)r} = x_{(g+1,2)}^{(1j)r} + f_{(k)}^{(2j)r}$$
$$j \in H;\ r \in S^{*} \qquad (25)$$

where $f_{(k)}^{(2j)r}$ allows for exogenous shifts in sectoral investments in region *r*.

Equation 26 defines the regional aggregation of labor prices (wages) across industries in the specification of the labor market. Equation 27 shows movements in regional wage differentials,

$wage\_diff^{(r)}$, defined as the difference between the movement in the aggregate regional real wage received by workers and the national real wage.

$$V(\mathrm{g}+1{,}1,\bullet,\mathrm{r})\left(p_{(g+1,1)}^{(\bullet)r}-a_{(g+1,1)}^{(\bullet)r}\right)=\sum\nolimits_{j\in H}V(\mathrm{g}+1{,}1,(1\mathrm{j}),\mathrm{r})\left(p_{(g+1,1)}^{(1j)r}-a_{(g+1,1)}^{(1j)r}\right)$$
$$r\in S^{*}\tag{26}$$

where $p_{(g+1,1)}^{(1j)r}$ is the wage in sector *j* in region *r*, $a_{(g+1,1)}^{(1j)r}$ is a technical term, and $V(g+1{,}1,(1j),r)$ represents input-output flows measuring sectoral labor payments in region *r*.

$$wage\_diff^{(r)}=p_{(g+1,1)}^{(\bullet)r}-cpi-natrealwage$$
$$r\in S^{*}\tag{27}$$

where $cpi$ is the national consumer price index, computed as the weighted average of $p_{(is)}^{(3)r}$ across regions *r* and consumption goods *(is)*; and $natrealwage$ is the national consumer real wage.

The regional population is defined by interacting demographic variables, including interregional migration. Links between regional population and regional labor supply are provided. Demographic variables are usually defined exogenously, and together with the specification of some of the labor market settings, labor supply can be determined together with interregional wage differentials or regional unemployment rates. In summary, labor supply and wage differentials determine unemployment rates, or labor supply and unemployment rates determine wage differentials.

Equation 28 defines the percentage-point change in regional unemployment rates in terms of percentage changes in labor supply and employed workers.

$$LABSUP(r)del\_unr^{(r)}=EMPLOY(r)\left(labsup^{(r)}-x_{(g+1,1)}^{(\bullet)r}\right)$$
$$r\in S^{*}\tag{28}$$

where $del\_unr^{(r)}$ measures percentage-point changes in the regional unemployment rate; $labsup^{(r)}$ is the variable for regional labor supply; and the coefficients $LABSUP(r)$ and $EMPLOY(r)$ are the benchmark values for regional labor supply and regional employment, respectively. The variable $labsup^{(r)}$ moves with regional workforce participation rate, proportional to the regional population and population of working age. Equation 28 defines regional population changes in the model as ordinary changes in the flows of net regional migration ($d\_rm^{(r)}$), net foreign migration ($d\_fm^{(r)}$), and natural population growth ($d\_g^{(r)}$).

$$POP(r)pop^{(r)} = d\_rm^{(r)} + d\_fm^{(r)} + d\_g^{(r)}$$
$$r \in S^* \qquad (29)$$

where $POP(r)$ is a coefficient measuring regional population in the benchmark year.

Equation 30 shows movements in per-household utility differentials, $util\_diff^{(r)}$, defined as the difference between the movement in regional utility and the overall national utility ($agg\_util$), including a shift variable, $futil^{(r)}$.

$$util\_diff^{(r)} = utility^{(r)} - agg\_util + futil^{(r)}$$
$$r \in S^* \qquad (30)$$

Finally, we can define changes in regional output as weighted averages of changes in regional aggregates, according to Eq. 31 below:

$$GRP^r grp^r = C^r x_{(\bullet\bullet)}^{(3)r} + INV^r z^{(2\bullet)r} + GOV^{(5)r} x_{(\bullet\bullet)}^{(5)r} + GOV^{(6)r} x_{(\bullet\bullet)}^{(6)r} + \left(FEXP^r x_{(\bullet\bullet)}^{(4)r} - FIMP^r x_{(\bullet 2)}^{(\bullet)r}\right) + \left(DEXP^r x_{(\bullet(1r))}^{(\bullet)s} - DIMP^r x_{(\bullet(1s))}^{(\bullet)r}\right)$$
$$r \in S^*; s \in S^* \text{ for } s \neq r \qquad (31)$$

where $grp^r$ is the percentage change in real Gross Regional Product (GRP) in region *r*; and the coefficients $GRP^r$, $INV^r$, $GOV^{(5)r}$, $GOV^{(6)r}$, $FEXP^r$, $FIMP^r$, $DEXP^r$, $DIMP^r$ represent, respectively, the following regional aggregates: investment, regional government spending, central government spending, foreign exports, foreign imports, domestic exports, and domestic imports. National output, $GDP$, is, thus, the sum of $GRP^r$ across all regions $r$. Notice that regional domestic trade balances cancel out.

To close the model, we set the following variables exogenously, which are usually exogenous both in short-run and long-run simulations: $a_{(g+1,s)}^{(1j)r}$, $a_{(i)}^{(1j)r}$, $a_{(i)}^{(2j)r}$, $a_{(i\bullet)}^{(3)r}$, $fq_{(is)}^{(4)r}$, $fp_{(is)}^{(4)r}$, $f_{(is)}^{(5)r}$, $f^{(5)r}$, $f^{(5)}$, $f_{(is)}^{(6)r}$, $f^{(6)r}$, $f^{(6)}$, $f_i$, $f_i^{(u)}$, $f_i^{(u)r}$, $p_{(i(2))}^{(w)}$, $t_{(i(2))}^{(0)}$, $a_{(l1)}^{(0)r}$, $a_{(k)}^{(1j)r}$, $a_{(is)}^{(2j)r}$, $a_{(g+1,1)}^{(\bullet)r}$, $\omega$, $f_{(k)}^{(2j)r}$, $d\_fm^{(r)}$, $d\_g^{(r)}$, and $futil^{(r)}$. To complete the short-run environment, we also set unchanged current stocks of capital ($x_{(g+1,2)}^{(1j)r}$), the national real wage ($natrealwage$), regional wage differentials, ($wage\_diff^{(r)}$), and regional population, by keeping regional migration unchanged ($d\_rm^{(r)}$).[6]

Other definitions of variables are computed using outcomes from simulations based on Eqs. 31. Of particular interest to our discussion is the definition of regional/national GDP and its components.

## 4. Calibration

The calibration of the ICGE model requires two subsets of data to define its numerical structure to implement the model empirically. First, we need information from an absorption matrix derived from interregional input-output sources (see Haddad et al., 2017 to calculate the coefficients of the model based on the following input-output flows (Table 1):

- $B(i, 1b, (u), r)$, with $i \in G^*, (u) \in U, b, r \in S^*$
- $M(i, s, (u), r)$, with $i \in G^*, s \in S, (u) \in U, r \in S^*$
- $T(i, s, (u), r)$, with $i \in G^*, s \in S, (u) \in U, r \in S^*$
- $V(i, s, (u), r)$, with $i \in G^*, s \in S, F, (u) \in U, r \in S^*$
- $Y(i, j, r)$, with $i \in G^*, j \in H, r \in S^*$

[6] In a long run closure, the assumptions on interregional mobility of capital and labor are relaxed by swapping variables $x_{(g+1,2)}^{(1j)r}$, $natrealwage$, $wage\_diff^{(r)}$ and $d\_rm^{(r)}$, for $f_{(k)}^{(1j)r}$, $del\_unr^{(r)}$ and $util\_diff^{(r)}$.

We complete this information with supplementary demographic data from the Croatian Bureau of Statistics (DZS) to calibrate the coefficients $LABSUP(r)$, $EMPLOY(r)$ and $POP(r)$, with $r \in S^*$. Because these estimates are based on snapshot observations for a single year revealing the economic structure of the economic system, this subset of data is denoted "structural coefficients" (Haddad and Araújo, 2024).

**Table 1. Aggregate Flows in the Absorption Matrix: Croatia, 2018**

| LABELS | User (1j)$^r$ | User (2j)$^r$ | User (3)$^r$ | User (4) | User (5)$^r$ | User (8)$^r$ | User (7) | TOTAL |
|---|---|---|---|---|---|---|---|---|
| **i∈G, s∈S*** | B(i,1b,(1j),r) | B(i,1b,(2j),r) | B(i,1b,(3),r) | B(i,1b,(4)) | B(i,1b,(5),r) | B(i,1b,(8),r) | B(i,1b,(7)) | B(i,1b,(•),•) |
| **i∈G, s∈S-S*** | B(i,2,(1j),r) | B(i,2,(2j),r) | B(i,2,(3),r) | - | B(i,2,(5),r) | B(i,2,(8),r) | - | B(i,2,(•),•) |
| **i∈G, s∈S** | M(i,s,(1j),r) | M(i,s,(2j),r) | M(i,s,(3),r) | M(i,s,(4)) | M(i,s,(5),r) | M(i,s,(8),r) | - | M(i,s,(•),•) |
| **i∈G, s∈S** | T(i,s,(1j),r) | T(i,s,(2j),r) | T(i,s,(3),r) | T(i,s,(4)) | T(i,s,(5),r) | T(i,s,(8),r) | - | T(i,s,(•),•) |
| **s∈F** | V(g+1,s,(1j),r) | - | - | - | - | - | - | V(g+1,s,(•),•) |
| **TOTAL** | Y(•,•,r) | V(•,•,(2j),r) | V(•,•,(3),r) | V(•,•,(4)) | V(•,•,(5),r) | V(•,•,(8),r) | V(•,•,(7)) | V(•,•,(•),•) |

| 2018 | User (1j)$^r$ | User (2j)$^r$ | User (3)$^r$ | User (4) | User (5)$^r$ | User (8)$^r$ | User (7) | TOTAL |
|---|---|---|---|---|---|---|---|---|
| **i∈G, s∈S*** | 226,280 | 53,316 | 193,596 | 83,959 | 68,750 | 4,300 | 24,112 | 654,313 |
| **i∈G, s∈S-S*** | 93,577 | 19,725 | 42,497 | - | 4,794 | 77 | - | 160,669 |
| **i∈G, s∈S** | - | - | - | - | - | - | - | 0 |
| **i∈G, s∈S** | 17,776 | 5,434 | 42,247 | 83,959 | 1,660 | 328 | - | 151,403 |
| **s∈F** | 316,681 | | | | | | - | 316,681 |
| **TOTAL** | 654,313 | 78,475 | 278,340 | 167,917 | 75,204 | 4,705 | 24,112 | 1,283,066 |

Values in current million units of Kuna

Note: **User(5)** consolidates users 5 and 6 from the specification of the model's equations. **User(8)** refers to NPISH demand, which follows household demand in the specification.

Source: Haddad, Eduardo A., Araújo, Inácio F., and Šimundić, Blanka (2023). Companion to the Interregional Input-Output System for Croatia, 2018, TD NEREUS 04-2023, The University of São Paulo Regional and Urban Economics Lab (NEREUS).

The second piece of information necessary to calibrate the model is represented by the subset of data defining various parameters, mainly elasticities. These are called "behavioral parameters". Empirical estimates for some of the parameters of the model are not available in the literature. We have thus relied on "best guesstimates" based on usual values employed in similar models (Dixon et al., 1982; Dixon et al., 2002). We set to 1.5 the values for both regional trade elasticities, $\sigma1_{(i)}^{(u)r}$ in Equation (6) and international trade elasticities, $\sigma2_{(i)}^{(u)r}$ in Equation (7). Substitution elasticity between primary factors, $\sigma3_{(g+1)}^{(1j)r}$ in Equation (8) was set to 0.5. The current version of the model runs under constant returns to scale, so that we set to 1.0 the values of $\mu_{(g+1,\bullet)}^{(1j)r}$ in Equation (9). The marginal budget shares in regional household consumption, $\beta_{(i)}^{r}$ in Equation (11), were calibrated from the input-output data, assuming the average budget share to be equal to the marginal budget share, and the subsistence parameter $\gamma_{(i)}^{r}$, also in Equation (11), was associated with a Frisch parameter equal to -2.0. The ratio of gross to net rate of return, $\psi_{(j)}^{r}$ in Equation (22), was set to 1.2. Finally, we set to 3.0 the parameter for sensitivity of capital growth to rates of return, $\varepsilon_{(j)}^{r}$ in Equation (24).

So far, we are in the process of estimating the export demand elasticities, $\eta_{(is)}^{r}$ in Equation (13).[7] Such price elasticities of export demand for Croatian products specify a relationship between the export price and the quantity exported of a specific commodity. For simplicity, the world is assumed to consist of one exporting country (Croatia) and one importer (the Rest of the World). The equation to be estimated takes the following form:

$$lnX_{it} = \alpha + \beta lnP_{it} + D_t + v_i + u_{it} \qquad (32)$$

where, $X_{it}$ is the export demand for a Croatian commodity *i* in time *t*; $P_{it}$ is the price of the commodity; $D_t$ is a time effect term; $v_i$ is a fixed effect that captures the influence of unobserved variables (such as exchange rates, transportation costs, policy, and other prices); $u_{it}$ is the error term; $\alpha$ and $\beta$ are coefficients to be estimated. The number of commodities defines the observations *i* in the classification system used in Croatia, over time *t*. The dependent variable ($X_{it}$) is the Free-on-Board Price (FOB) value of exports in USD. We can calculate the commodity price ($P_{it}$) as the value of exports divided by their weight.

[7] Current estimates are presented in the MDATA_2018.har file (https://github.com/edhaddad/BMCROG).

The price elasticity of export demand reflects the change in foreign demand caused by a foreign currency FOB price variation. The export-demand elasticity is negative, i.e., in Equation (13), exports are a negative function of their foreign currency prices on world markets. Dixon and Rimmer (2010) discuss the sensitivity of CGE models' results to the export-demand elasticities. The sensibility analysis or results from historical simulations are used to assess the parameters estimated in the econometric models (Dixon and Rimmer, 2013).

## 5. $CO_2$-emission module

We can now use the ICGE model to evaluate the impacts of carbon taxes imposed on the different sectors. We model carbon tax payments as an *ad valorem* production tax on the total cost of production in the polluting industries. Output price (adjusted for production taxes) of sector $j$, in region $r$, $PX_j^r$, is given by the weighted average of value-added prices and the cost of intermediate inputs:

$$PX_j^r\left(1 - carbontax_j^r\right) = X_j^{r\,-1}\left\{PV_j^r V_j^r + \textstyle\sum_i a_{ij}^{rs}\, PC_i^s X_j^r\right\} \tag{33}$$

where $carbontax_j^r$ is the rate of carbon tax on sector $j$ in region $r$.

In the ICGE model, governments consume mainly public goods provided by the public administration sectors. Equations 14 (34), which is the critical equation for the design of the simulations, shows the movement of central government consumption in relation to movements in movements in real tax revenues for the central government. Again:

$$x_{(is)}^{(5)r} = taxrev^r + f_{(is)}^{(5)r} + f^{(5)r} + f^{(5)}$$
$$i \in G;\ s = 1\mathrm{b}, 2;\ r, b \in S^* \tag{34}$$

where $x_{(is)}^{(5)r}$ is the regional governments demand in region $r$ for good $i$ from region $s$; $f_{(is)}^{(5)r}$ is a commodity and source-specific shift-term for regional government expenditure in region $r$, $f^{(5)r}$ is the shift-term for regional government expenditure in region $r$, and $f^{(5)}$is an overall shift term for regional government expenditure. Finally, $taxrev^r$ is the percentage change in real revenue from indirect taxes in region *r* so that government demand moves proportionally to endogenous changes in regional and national tax bases.

We define changes in regional output as weighted averages of changes in regional aggregates, according to Equation (35):

$$GRP^r grp^r = C^r x_{(\bullet\bullet)}^{(3)r} + INV^r z^{(2\bullet)r} + GOV^{(5)r} x_{(\bullet\bullet)}^{(5)r} + GOV^{(6)r} x_{(\bullet\bullet)}^{(6)r} + \left(FEXP^r x_{(\bullet\bullet)}^{(4)r} - FIMP^r x_{(\bullet 2)}^{(\bullet)r}\right) + \left(DEXP^r x_{(\bullet(1r))}^{(\bullet)s} - DIMP^r x_{(\bullet(1s))}^{(\bullet)r}\right)$$

$$r \in S^*; s \in S^* \text{ for } s \neq r \qquad (35)$$

where $grp^r$ is the percentage change in real Gross Regional Product in region *r*; and the coefficients $GRP^r$, $INV^r$, $GOV^{(5)r}$, $GOV^{(6)r}$, $FEXP^r$, $FIMP^r$, $DEXP^r$, and $DIMP^r$ represent, respectively, the following aggregates in region *r*: investment, local government spending, central government spending, foreign exports, foreign imports, domestic exports, and domestic imports. National output, $GDP$, is, thus, the sum of $GRP^r$ across all *r* regions. Note that regional domestic trade balances cancel out.

We can define scenarios for interregional transfers of carbon tax revenue in the context of our model specification. For instance, we assume that the central government transfers its annual carbon tax revenue to finance different components of regional government spending in region *r*. With this information, we can calculate the size of the "shock" by imposing region-specific changes in $f^{(5)r}$ (Eq. 34), the shift term for government expenditure in region *r* that are proportional to changes in $GOV^{(5)r}$ (Equation 35).

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