O09e Lab Report

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1 Fringes of Equal Thickness

Group #13

NG Kuok Fung | #3797903

Edgar Harutyunyan | #3775171

Overview of Tasks

Task 1

Determine the radius (R) of a convex lens by measuring Newton´s rings using light of a given wavelength.

Task 2

Determine the thickness of a metal foil using the interference at wedge-shaped layers.

Task 3

Determine the thickness or the diameter of an object chosen by yourself, e.g. a strand of hair.

1.1 Task 1

1.1.1 Measurements of Newton rings

Task Definition

The goal of task 1 is to measure the radius (r_k) of the k-th Newton ring. Subsequently, the values of (r_k^2) are computed and plotted against (k) to determine the radius of curvature (R) of the convex lens.

Procedure

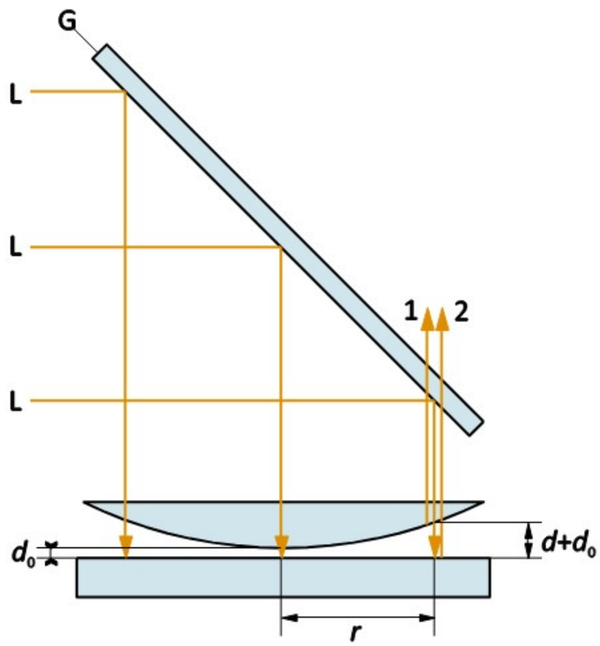

Figure 1.1: Lens [1]

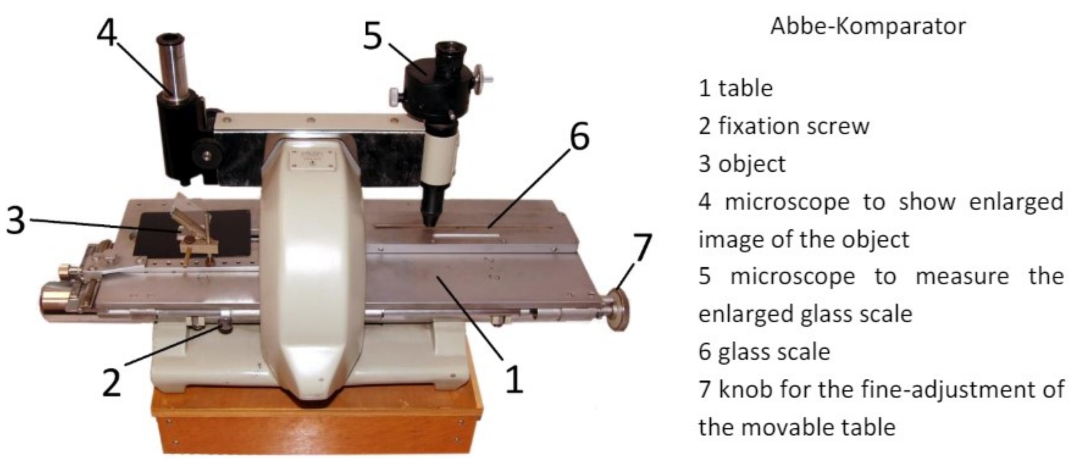

Figure 1.2: Abbe Comparator [1]

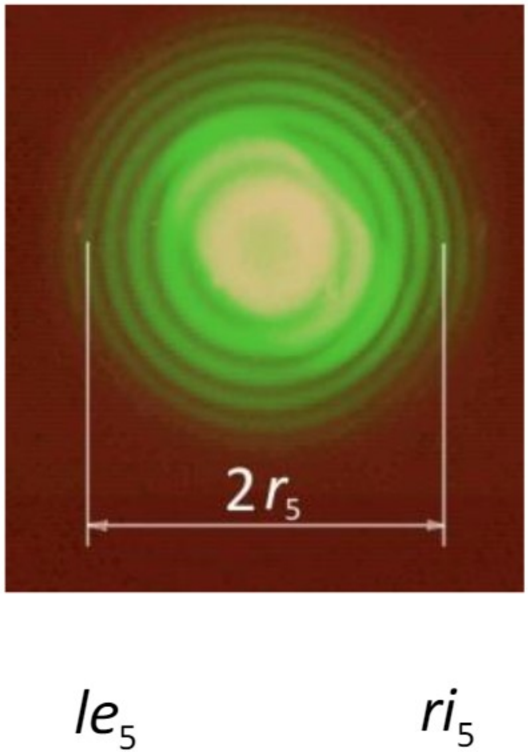

Figure 1.3: Ring [1]

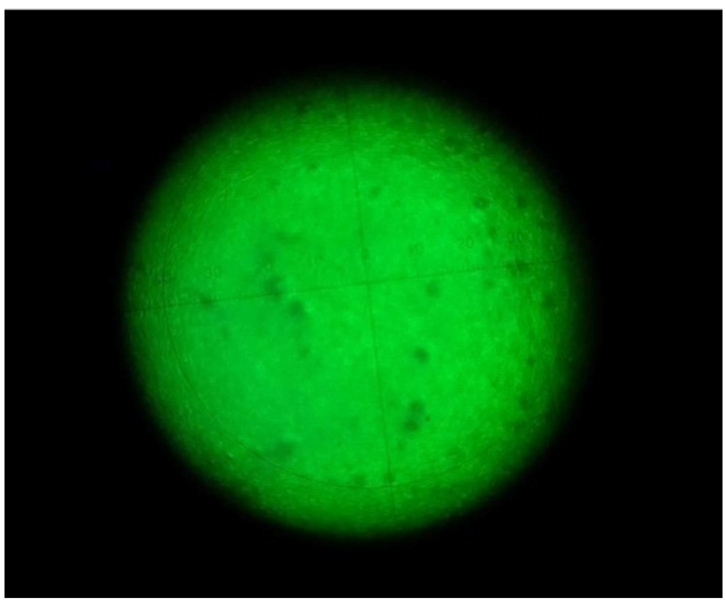

Figure 1.4: Pointer [1]

1. Place a clean convex lens onto a flat glass plate **Fig 1.1**. Insert it beneath the angled glass

plate (G) of the Abbe comparator, depicted in **Fig 1.2** - Label 3.

2. During the experiment, Newton rings are formed due to the path difference (Δx) between the interfering light rays **Fig 1.3**.
3. In the Abbe comparator's left microscope, there is a small cross **Fig 1.4**. Align this cross with the outer edge of a dark ring using the knob. Subsequently, measure the table position using the right microscope.
4. Adjust the cross to an adjacent dark ring and repeat the measurement.
5. If the measurement is to the right of the center of the Newton ring, it should be recorded as (ri_k). Conversely, if it is left of the center, it should be recorded as (le_k), where k represents the k-th dark ring.
6. Repeat this procedure for 10 dark rings, and measure both sides of each dark ring.
7. Calculate the radius (r_k) of each ring, and (r_k^2) using the formula: $r_k = |\mathrm{le}_k - \mathrm{ri}_k|$.

1.1.2 Determine radius of curvature (R)

Determine the thickness of a metal foil using the interference at wedge-shaped layers.

Theoretical Basis:

Two formulas were given for this experiment.

The first formula describes the optical path difference Δx between the light rays:

$$\Delta x = 2(d + d_o) + \frac{\lambda}{2} \tag{1.1}$$

The second relates the radius of the k-th newton ring (r_k) to the radius of curvature (R):

$$r_k^2 = d(2R - d) \tag{1.2}$$

- d : Ideal thickness due to the convex curvature
- d_0 : Contact distance between convex lens and glass plate
- λ : Wavelength of light used in the experiment

A relationship between r_k and R may be derived by imposing the condition for destructive interference: $\Delta x = (k + \frac{1}{2})\lambda$

Hence:

$$d = \frac{k\lambda}{4} - d_0 \tag{1.3}$$

It is also assumed that $R \ll d_0$

Substituing Eq.1.3 into Eq.1.2 with this assumption yields:

$$r_k^2 = 2R\frac{k\lambda}{4} - d_0 \tag{1.4}$$

Therefore:

$$r_k^2 = \frac{R\lambda}{2}k - 2d_0R \tag{1.5}$$

This implies that the gradient of a $r_k^2(k)$ graph ($\Delta := \frac{R\lambda}{2}$) can be used to compute R with uncertainty of μ_R given by:

$$\mu_R = \frac{\partial R}{\partial \Delta}\mu_\Delta = \frac{2}{\lambda}\mu_\Delta,$$

- μ_Δ : Uncertainty of the gradient from linear fit.

During this experiment, a green light from a mercury lamp was used.

$\lambda = 546\ nm$

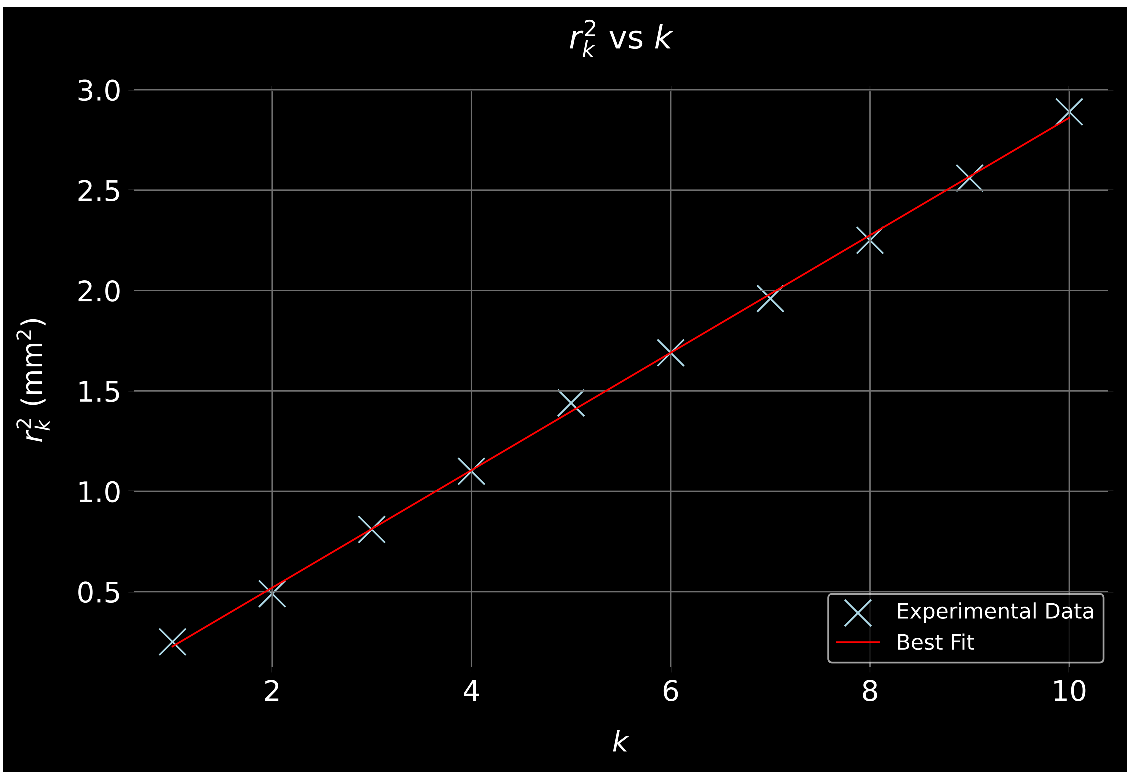

Figure 1.5: Squared radius of fringes w.r.t their number.

Gradient (Slope):

$$\Delta = (0.29 \pm 0.003)\ mm^2$$

$$R = (1.07 \pm 0.010)\ m$$

1.2 Task 2

1.2.1 Measuring positions (x_k)

Task Definition

Determine the thickness (D) of a metal foil by measuring the positions (x_k) of the k-th dark fringe formed by the interference pattern of light incident on the wedge-shaped layer.

Theoretical Basis

1. A wedge-shaped layer is an air gap between two parallel glass plates. Interference occurs in the wedge-shaped layer due to a gradual variation in the thickness between the glass layers.
2. This effect may be achieved by placing a thin object (i.e: metal foil or strand of hair) at one edge of the plates **Fig 2.1**.
3. Monochromatic light perperndicularly incident on the plates is reflected, this causes a phase difference between the light rays which leads to an interference pattern which may be obeserved using the Abbe comparator.
4. The frontal projection of the setup allows the for the derivation of a formula relating the the relation the thickness of an object D and the position of the k-th dark fringe x_k.

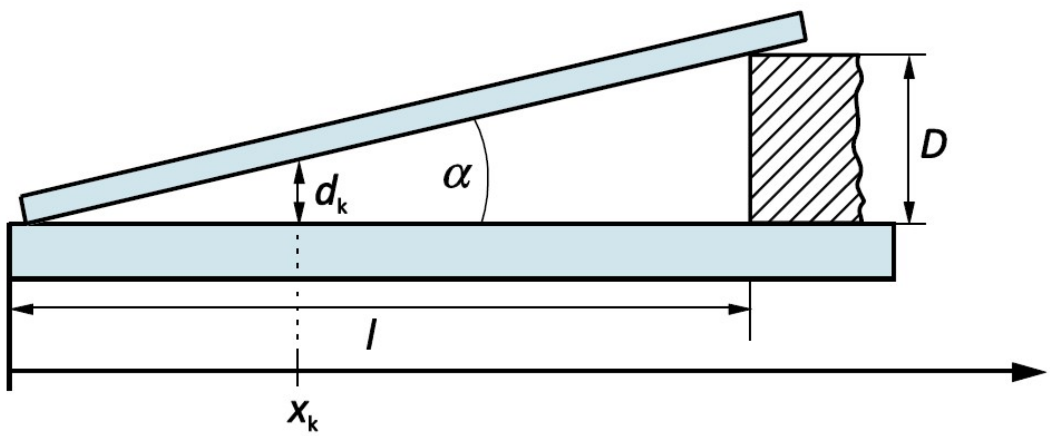

Figure 2.1: Frontal projection of the wedge shaped layer. [1]

The following formula may be derived by analyzing the right angled triangle in **Fig 2.1**:

$$\tan(\alpha) = \frac{D}{l} = \frac{d_k}{x_k}, \tag{2.1}$$

The phase difference (δ) for interference in wedge shaped layers was given as:

$$\delta = \frac{2\pi\Delta x}{\lambda} + \pi \tag{2.2}$$

- x_k = Position of k-th dark fringe
- d_k = Plate seperation at the k-th dark fringe
- D = Thickness of metal foil
- Δx = Path difference
- δ = Phase difference

Substituting the condition for destructive interference $\delta = (2k+1)\pi$ with $\Delta x = 2d_k$ into Eq.2.2

$$\frac{4\pi d_k}{\lambda} + \pi = \pi(2k+1)$$

$$2d_k = k\lambda$$

$$d_k = \frac{k\lambda}{2} \tag{2.3}$$

Substituting Eq.2.3 into Eq.2.1 provides the following relationship between D and x_k

$$x_k = \frac{l\lambda}{2D}k \tag{2.4}$$

where x_k is the position of the fringe d_k is the half path length of the light:

$\Delta x = 2d_k$ In the experiment the dark fringes have been measured. Thus from the destructive interference condition:

$$\frac{2\pi\Delta x}{\lambda} + \pi = \pi(2k+1)$$

$$\Delta x = k\lambda = 2d_k$$

$$d_k = \frac{k\lambda}{2}$$

Substituting the latter relation into the first angle ratio one ends up with:

$$x_k = \frac{l\lambda}{2D}k$$

Procedure

The Abbe comparator from task 1 is used to take measurements.

1) Place the wedge underneath the left microscope

2) Allign the cross under the left microscope until it lines up with an arbitrary dark fringe. This is the 0th dark fringe (k=0). Measure this initial position x_0 using the scale under the right micrscope of the Abbe comparator.

3) All future measurements of x_k are made with respect to the initial position x_0.

4) Move the cross to the adjacent dark fringe and repeat the measurement. Perform steps 1 to 4 at least 10 times.

1.2.2 Determine thickness (D) of metail foil

Theoretical Basis

From Eq.2.4, D is given by the gradient of an x_k against k graph:

$$D = \frac{l\lambda}{2\Delta}$$

- Δ : Slope of x_k against k graph
- l : Distance of metal foil from initial position x_0

During this experiment, a green light from a mercury lamp was used with wavelength ($\lambda = 546nm$).

The uncertainty of D is given by:

$$\mu_D = \sqrt{\left(\frac{\partial D}{\partial \Delta}\mu_\Delta\right)^2 + \left(\frac{\partial D}{\partial l}\mu_l\right)^2} = \sqrt{\left(\frac{l\lambda}{2\Delta^2}\mu_\Delta\right)^2 + \left(\frac{\lambda}{2\Delta}\mu_l\right)^2}$$

- $\mu_l = 0.5mm$: Uncertainity of Ruler
- μ_Δ : Uncertainty of Linear fit

$Figure\, 2$: Position of fringes with respect to their number.

$$l = (23.00 \pm 0.50)\ mm$$

The film thickness:

$$D = (27.97 \pm 0.769)\ \mu m$$

1.3 Task 3

Task Definition

The goal is to measure the thickness (D) of a strand of hair. The procedure and theoretical basis used in this task is identical to task 2.

For this experiment, the strand of hair was provided by NG Kuok Fung.

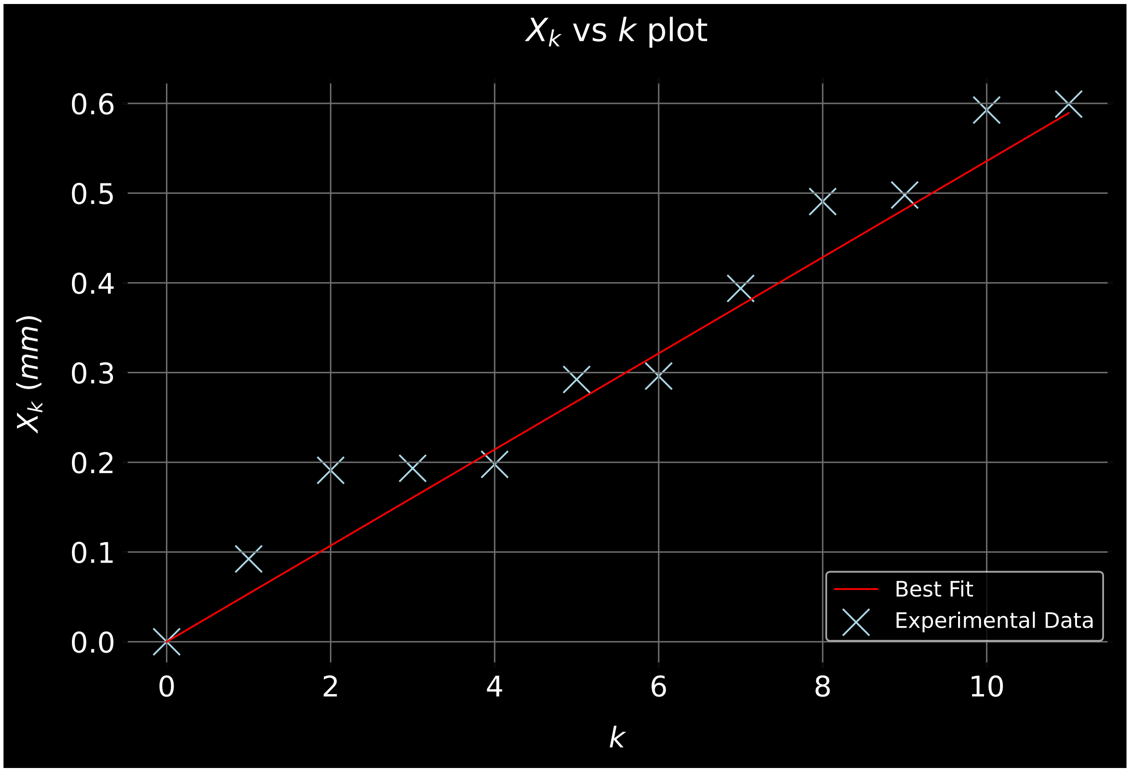

Figure 3: Position of fringes with respect to their number.

The hair thickness:

$$D = (117.22 \pm 6.671)\ \mu m$$

The uncertainity is relatively large due to difficulties with accurately allinging the microscope to the dark fringes. However, the value of D found is within an expected range of 80 to 120 micrmeters. [2]

1.4 References

1) O09e Lab instruction
2) **Research Progress in Composition, Classification and Influencing Factors of Hair** : Asian J Beauty Cosmetol. 2023;21 (3): 503-516. Publication Date (Web): 2023 September 26 (Review Article) doi:https://doi.org/10.20402/ajbc.2023.0001