E17e Lab Report

May 27, 2024

1 E17e Fourier Analysis of Coupled Electric Oscillations

Group #13

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Overview of Tasks

1. Measure the time traces and frequency spectra of the free beat oscillations of the low-point circuit for ten different coupling capacitance values. Plot the ten frequency spectra into one graph. Determine the frequencies of the in-phase and out-of-phase oscillation modes. Calculate the coupling factors from the measured frequencies. Fit the theoretical expressions to the data to determine the value of the capacitance C.

2. Measure the time traces and frequency spectra of the free beat oscillations of the high-point circuit for ten different coupling capacitance values. Plot the ten frequency spectra into one graph. Determine the frequencies of the in-phase and out-of-phase oscillation modes. Calculate the coupling factors from the measured frequencies. Fit the theoretical expressions to the data to determine the value of the capacitance C.

3. Measure the beat period of the high-point circuit for one selected coupling factor. Compare the values to those obtained from the frequencies of the in-phase and out-of-phase oscillation modes. Make plots of both the time trace and the spectrum.

4. Measure the coupling factor for two inductively coupled resonant circuits as a function of the distance between the inductance coils. Plot the frequency spectra in one graph. Plot the coupling factor as a function of distance and analyze the distance dependence.

1.1 Task 1

Task Definition

- Measure the time traces and frequency spectra of the free beat oscillations of the low-point circuit for ten different coupling capacitance C_k values.
- Plot the ten frequency spectra into one graph.
- Determine the frequencies of the in-phase f_1 and out-of-phase f_2 oscillation modes.
- Calculate the coupling factors $k_{C,T}$ from the measured frequencies.
- Fit the theoretical expressions to the data to determine the value of the capacitance C.

Theoretical Basis

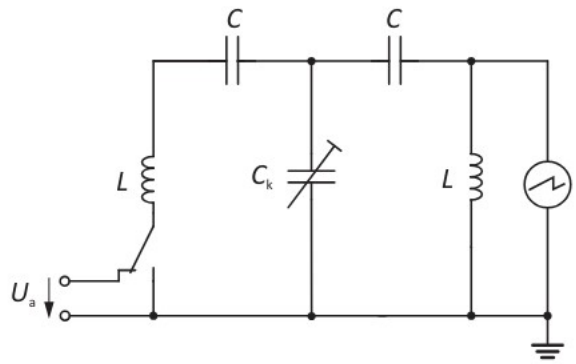

Figure 1.1: Low-Point Circuit [1]

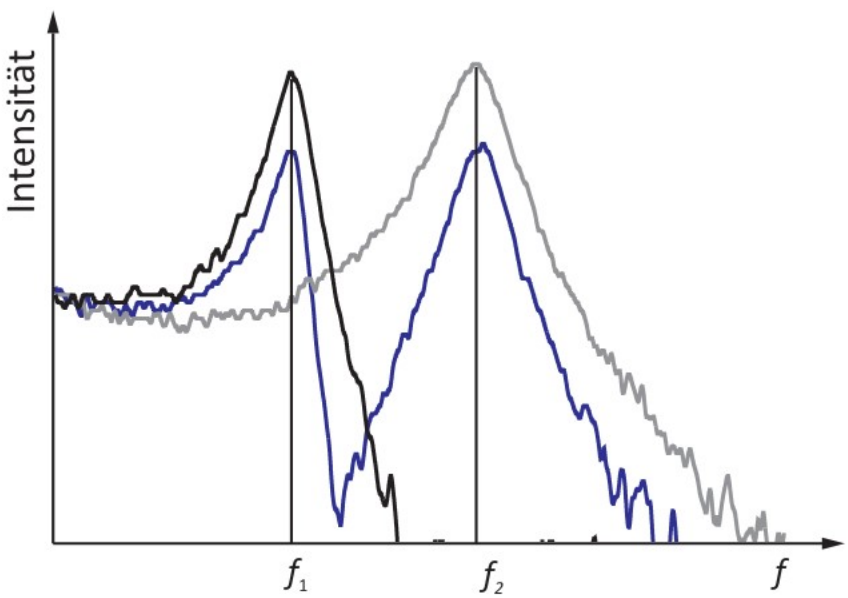

Figure 1.2: Frequency Spectra: Beat Oscillation (Blue Curve) [1]

In task 1, a low-point resonant circuit is set up which produces beats. **Fig 1.1**.

A beat has an in-phase-oscillation f_1 and out-of-phase oscillation f_2 frequency, which depends on the total capacitance C and C_k **Fig 1.2**.

The values of f_1 and f_2 are related by the coupling constant $k_{C,T}$ given by:

$$k_{C,T} = \left| \frac{\omega_1^2 - \omega_2^2}{\omega_1^2 - \omega_2^2} \right| \tag{1.1}$$

$$\omega_i = 2\pi f_i$$

- ω_1 : In-phase angular frequency
- ω_2 : Out-of-phase angular frequency

Additionally, for a low-point circuit, the relationship between $k_{C,T}$, C and C_k is given by:

$$k_{C,T} = \frac{C}{C + C_k} \tag{1.2}$$

- C : Capacitance of Fixed Capacitor
- C_k : Capacitance of Coupling Capacitor
- $k_{C,T}$: Coupling factor of low-point circuit

Procedure

1. The circuit was set up following the circuit diagram **Fig 1.1**, and the value of C_k is varied using a capacitor decade.
2. The corresponding time trace and frequency spectrum was recorded using PicoScope 7 software installed on a host connected to the PicoScope 2000.
3. Based on frequency spectra **Fig 1.2**, the frequencies f_1 and f_2 were determined using `find_peaks` method from `scipy.signal` python module library.
4. The value of $k_{C,T}$ was calculated using Eq 1.1.
5. The process was repeated for 10 values of C_k
6. Linear regression was performed using the values of C_k, $k_{C,T}$ and fitting function 1.2.
7. Finally, the value of C is determined from the fit.

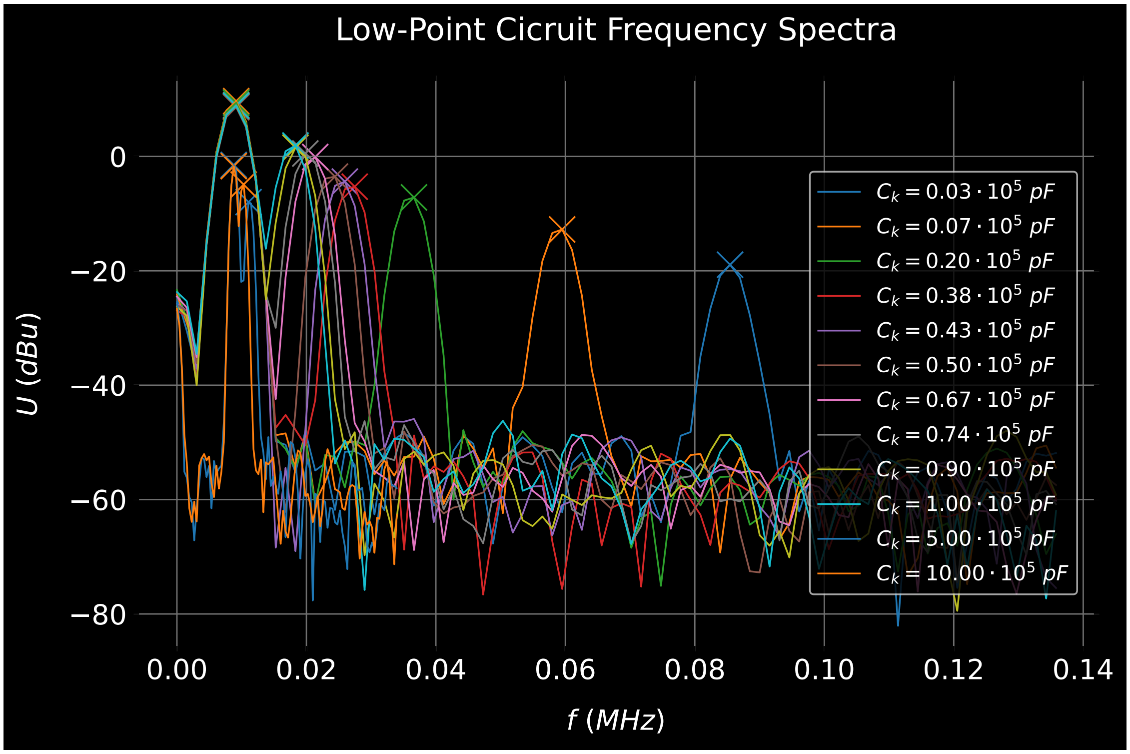

Figure 1.2: Frequency spectra plot of 12 measurements of low-point circuit.

Capacitance from fitting:

$$C = (0.15 \pm 0.003)\ \mu F$$

1.2 Task 2

Task Definition

- Measure the time traces and frequency spectra of the free beat oscillations of the low-point circuit for ten different coupling capacitance C_k values.
- Plot the ten frequency spectra into one graph.
- Determine the frequencies of the in-phase f_+ and out-of-phase f_- oscillation modes.
- Calculate the coupling factors $k_{C,H}$ from the measured frequencies.
- Fit the theoretical expressions to the data to determine the value of the capacitance C.

Theoretical Basis

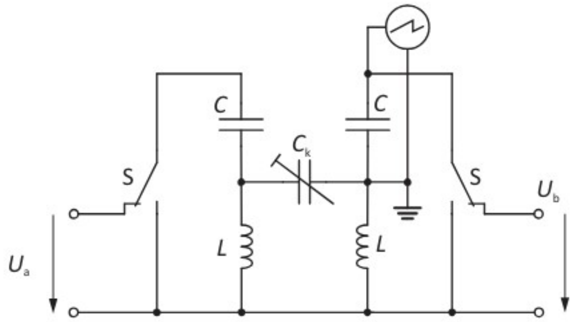

Figure 2.1: High-Point Circuit [1]

The coupling factor $k_{C,H}$ of high-point circuit:

$$k_{C,H} = \frac{\omega_2^2 - \omega_1^2}{\omega_2^2 + \omega_1^2} = \frac{C_K}{C + C_K} \tag{2.1}$$

Procedure

1. The procedure has been carried out similar to Task1, and uses Eq 2.1 as the fitting function instead.

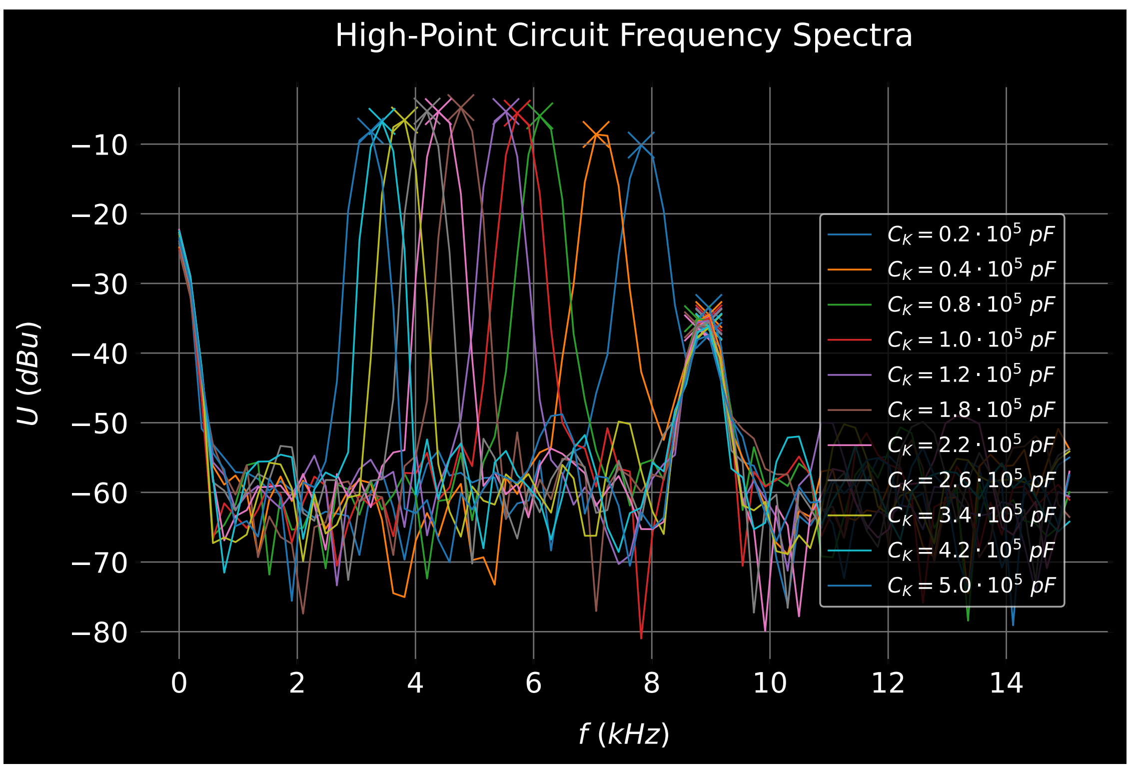

Figure 2.2: Frequency spectra plot of ten measurements of high-point circuit.

Capacitance from fitting:

$$C = (0.15 \pm 0.002)\ \mu F$$

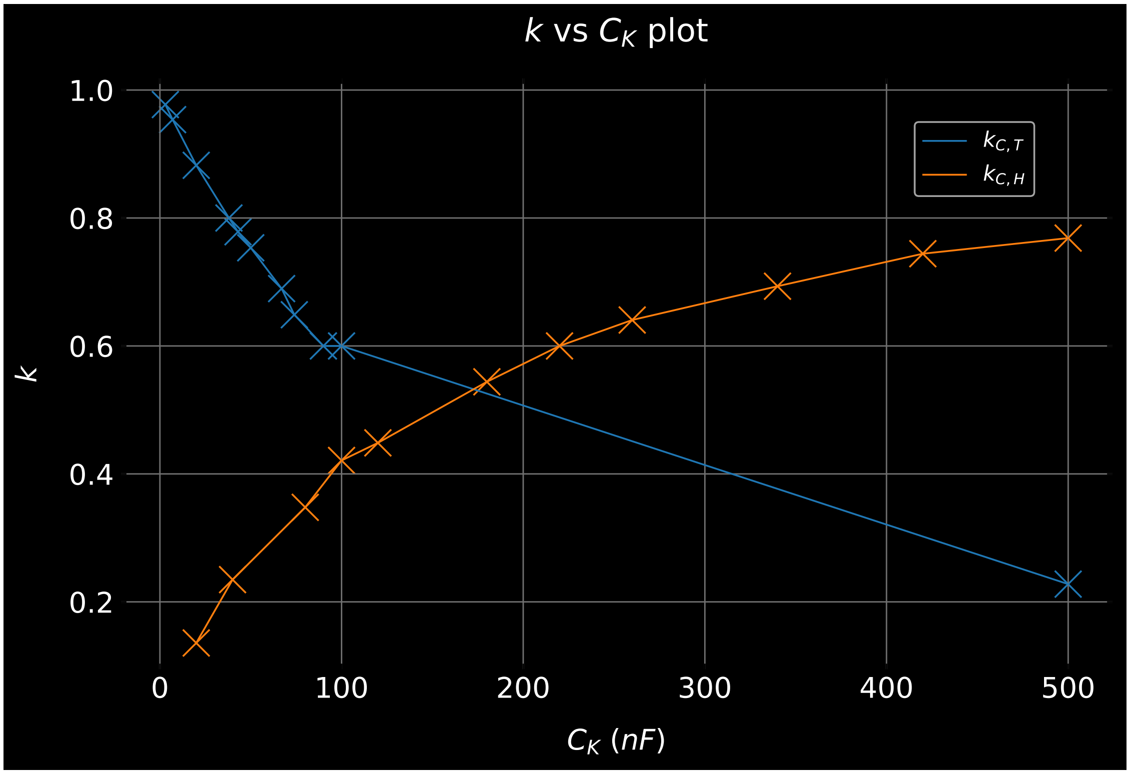

Figure 2.3: Coupling factor as a function of coupling capacitance.

1.3 Task3

Task Definition

Measure the beat period of the high-point circuit for one selected coupling factor. Compare the values to those obtained from the frequencies of the in-phase and out-of-phase oscillation modes. Make plots of both the time trace and the spectrum.

Theoretical Basis

Beat mode is produced by short circuiting the right switch of the high-point circuit (**Fig.2.2**).

Angular frequency of the beat is:

$$\omega_S = \omega_2 - \omega_1 = \frac{2\pi}{T_S}$$

- T_S : Beat period

Procedure

1. The measurements in beat mode were carried out for the coupling capacitance $C_K = 28 \cdot 10^3 pF$.
2. Angular frequecies have been determined using `find_peaks` on the frequency spectra.
3. The coupling factor $K_{C,H}$ and capacitance C value were evaluated using Eq 2.1.
4. Using the measured angular frequencies the oscillation period T and beat period T_S were computed.

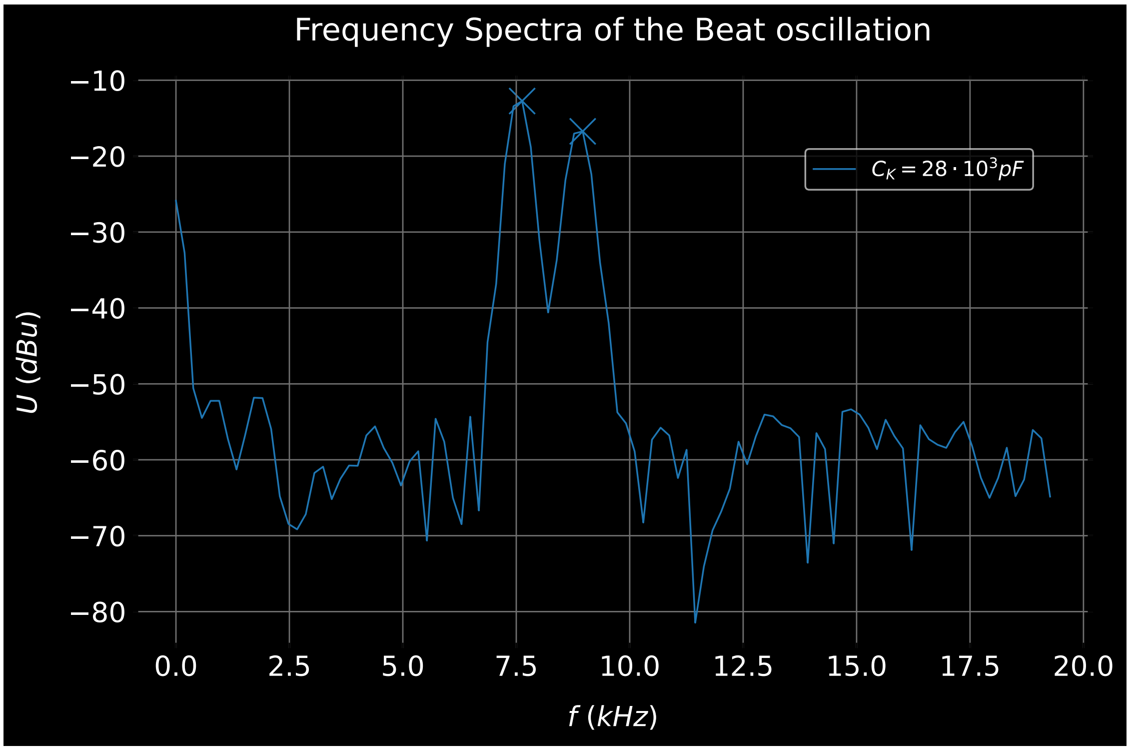

Figure 3.1: Frequency spectra plot of the high-point circuit.

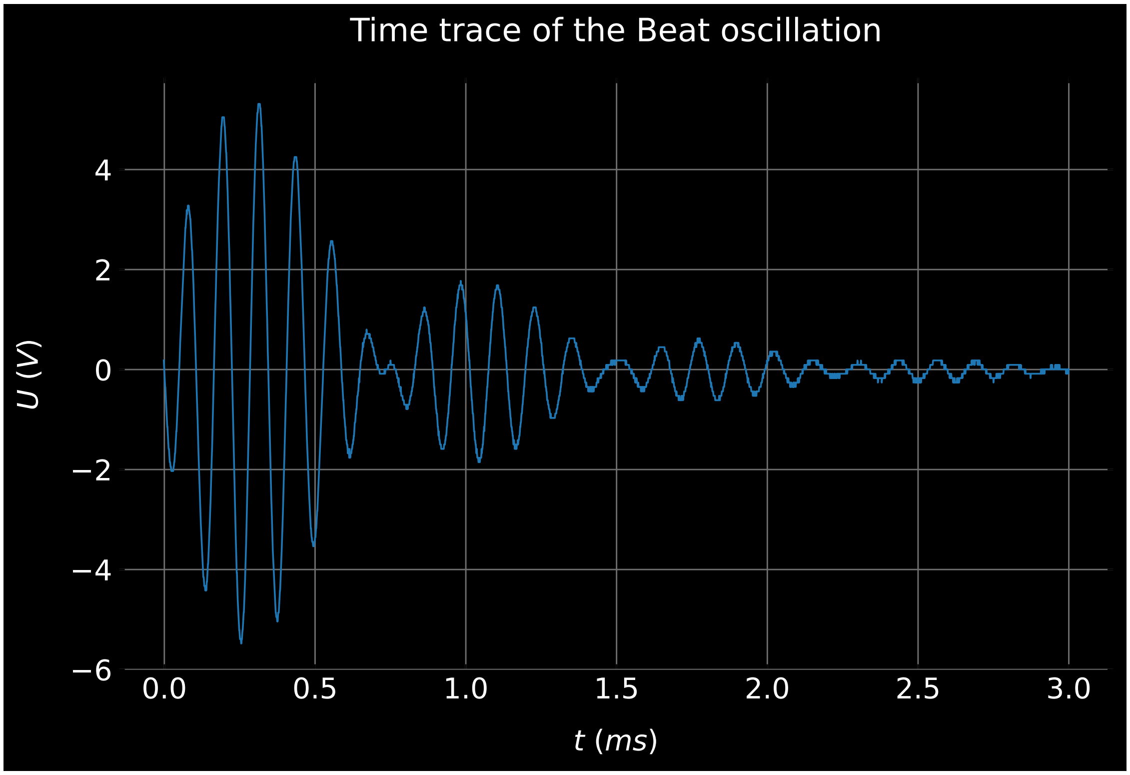

Figure 3.2: Time trace plot of the high-point circuit.

Analysis

Angular frequency of out-of-phase mode is $w_2 = 56.33\ rad \cdot ms^{-}$

Angular frequency of in-phase mode is $w_1 = 47.94\ rad \cdot ms^{-}$

The beat period:

$$T_S = \frac{2\pi}{\omega_2 - \omega_1} = 0.75ms$$

Whereas, the oscillation period:

$$T = \frac{2\pi}{\omega_2} = 0.11ms$$

Hence, in a beat there are $\frac{T_S}{T} \approx 6$ oscillations.

The coupling factor:

$$k_{C,H} = \frac{\omega_2^2 - \omega_1^2}{\omega_2^2 + \omega_1^2} = 0.1599$$

The capacitance:

$$C = C_k \left(\frac{1}{k_{C,H}} - 1 \right) = 0.15\ \mu F$$

It is observed, that the obtained result for the capacitance value is in agreement with the fitted result obtained in Task1 and Task2.

1.4 Task 4

Task Definition

Measure the coupling factor k_L for two inductively coupled resonant circuits as a function of the distance d between the inductance coils. Plot the frequency spectra in one graph. Plot the coupling factor as a function of distance and analyze the distance dependence.

Theoretical Basis

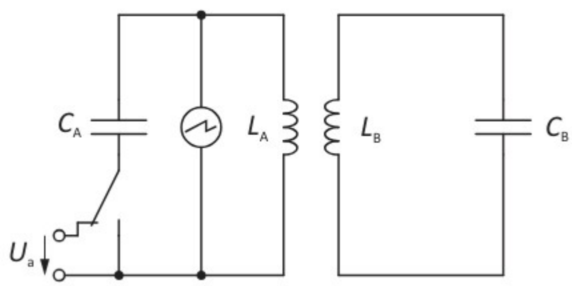

Figure 4.1: Coupled Resonant Circuit [1]

In Task4, a coupled resonant circuit is set up which produces beats similar to those in previous tasks. **Fig 4.1**.

The following conditions were imposed during the experiment:

- $C_A = C_B = C$
- $L_A = L_B$

Under such conditions, the coupling factor k_L and mutual inductance M are given by:

$$\omega_1 = \frac{\omega_0}{\sqrt{1+k_L}}$$

$$k_L = \frac{\omega_0^2}{\omega_1^2} - 1 \tag{4.1}$$

$$M = k_L \cdot L \tag{4.2}$$

- $\omega_0 = 1/\sqrt{LC}$: Uncoupled angular frequncy measured during experiment
- ω_1 : In-phase angular frequency
- k_L : Coupling factor of Resonant Circuit
- M : Mutual Inductance

Procedure

1. The circuit was set up following the circuit diagram **Fig 4.1**, and the distance d between inductance coils L_1 and L_2 was recorded.
2. The corresponding frequency spectrum was recorded using PicoScope 7 software installed on a host connected to the PicoScope 2000.
3. Subsequently, the frequency f_1 is determined by analyzing the peaks in the frequency spectra **Fig 1.2**.
4. The values of k_L and M are calculated using Eq 4.1 and Eq 4.2 respectively.
5. The process was repeated for 8 values of d, and a graph of $M(d)$ is plotted.
6. This graph is fitted to a power law.

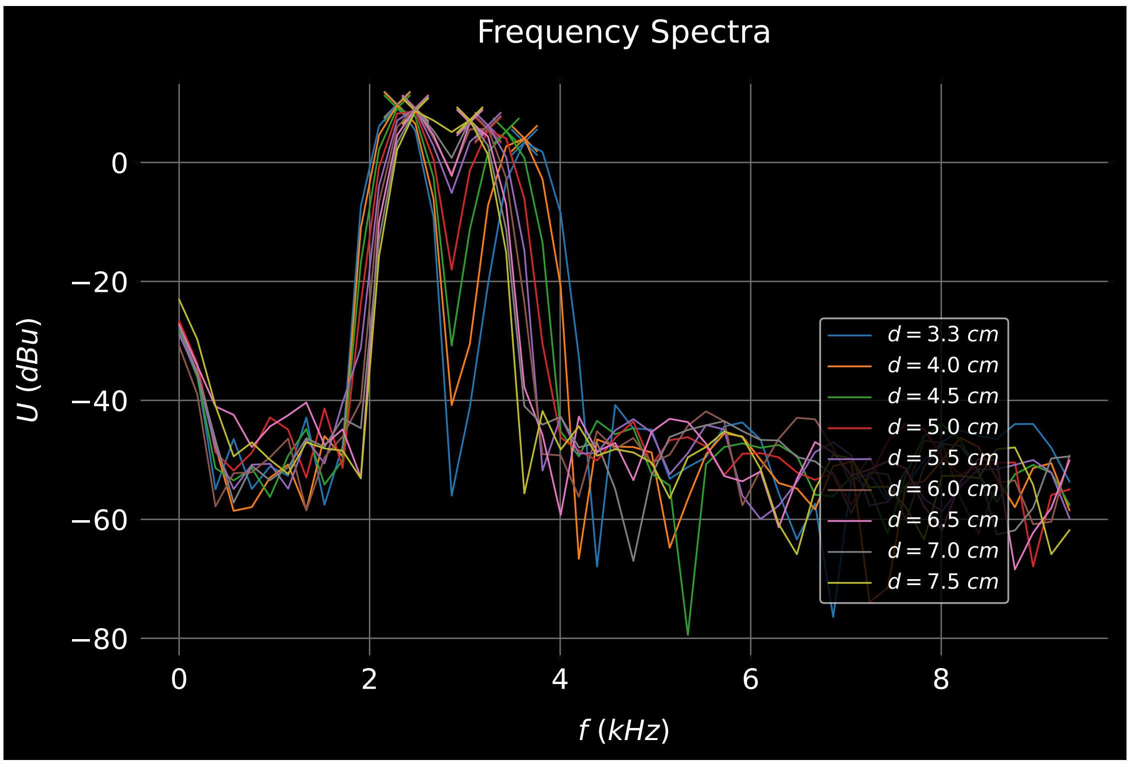

Figure 4.1: Frequency spectra plot of two inductively coupled resonant circuits.

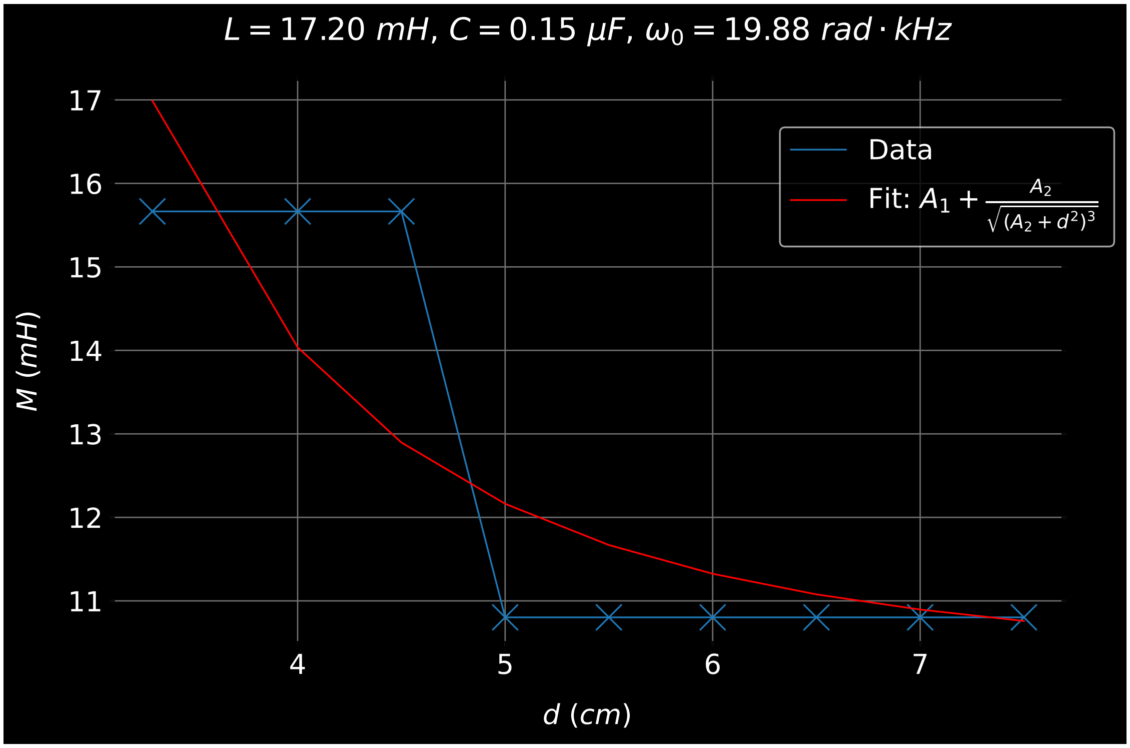

Figure 4.2: Mutual inductance as a function of distance between inductance coils.

$$A_1 = 10.16 \pm 0.766$$

$$A_2 = 253.98 \pm 67.785$$

1.5 References

1) E17e Lab instruction