E12He

July 3, 2024

1 E12He RLC Resonant Circuits

Group #13

NG Kuok Fung | #3797903

Edgar Harutyunyan | #3775171

Overview of Tasks

1. Series resonant circuit: Measure the frequency-dependent current draw of an RLC series resonant circuit for two different values of the damping resistance R_d in a frequency range around the resonance frequency f_0 such that $0.2 \leq \frac{I(f)}{I(f_0)} \leq 1$

2. Evaluation of the series resonant circuit:

2a. Plot both resonance curves in one graph.

2b. Use the FWHM to determine the damping constants δ and the quality factors Q.

2c. Fit an appropriate model function to the measured resonance curves and determine f_0 and δ.

2d. Calculate the capacitance of the capacitor using the Thomson equation.

3. Parallel resonant circuit: Measure the frequency-dependent current draw of a circuit in which the capacitor and coil are connected in parallel and in series with the resistor R_d.

4. Evaluation of the parallel resonant circuit:

4a. Derive an expression for the impedance of this circuit.

4b. Fit this model curve to the measured data. Determine L, C, R_{sp} and R_d as well as the resonance frequency f_0.

4c. Compare the resonance frequency with the value obtained in task 2b

1.1 Task 1: Series Resonant Circuit Measurement

Task Definition

Series resonant circuit: Measure the frequency-dependent current draw of an RLC series resonant circuit for two different values of the damping resistance R_d in a frequency range around the resonance frequency f_0 such that $0.2 \leq \frac{I(f)}{I(f_0)} \leq 1$

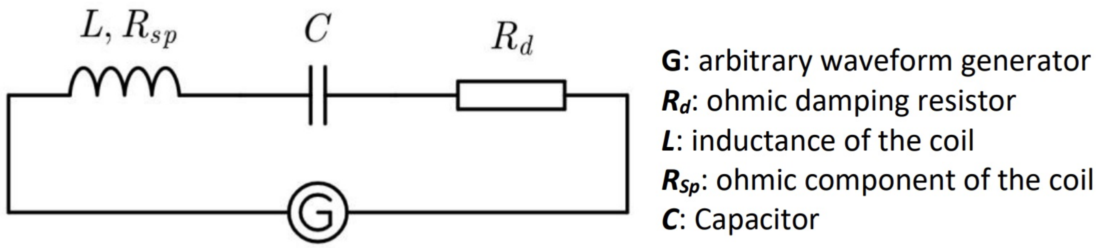


Figure 1.1: Series RLC Circuit Diagram

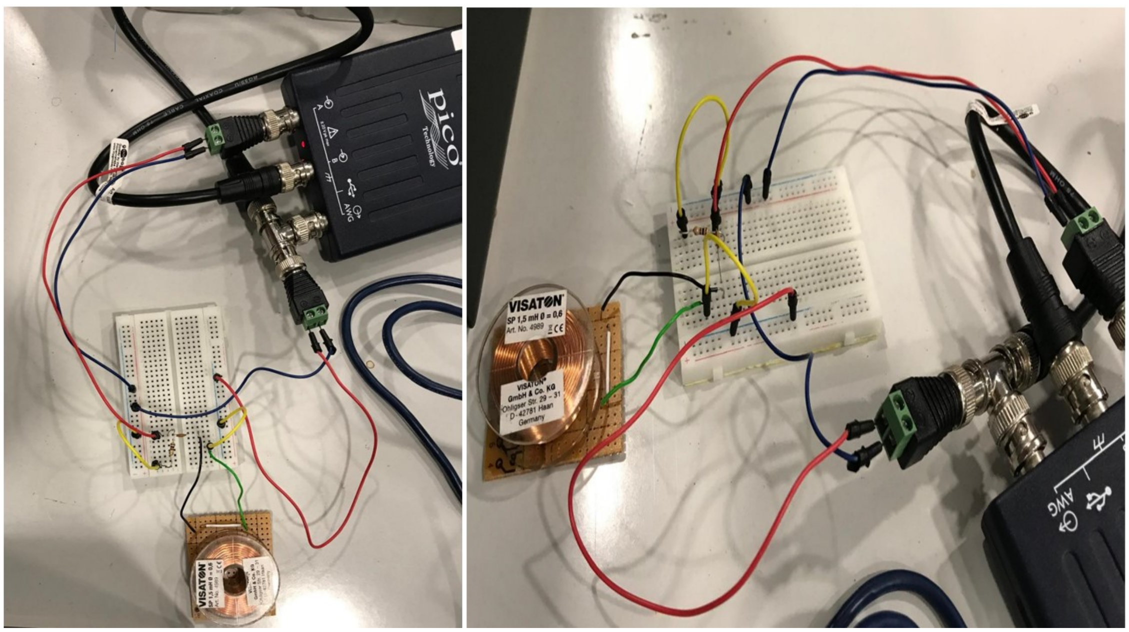


Figure 1.2: Series RLC Setup

Procedure

1. An RLC Series Circuit **Fig 1.2** is setup using the circuit diagram in **Fig 1.1**.
2. Sine waves of varying freqeuncies between 1 to 50kHz are generated by the picoscope, and supplied into the circuit. The peak to peak voltage of the output is then measured via Channel A of the picoscope.
3. The maximal value of the peak to peak voltage corresponding to $U(f_0)$ is found and used to plot a normalized graph of $\frac{U(f)}{U(f_0)}$ against f is plotted.
4. The above was repeated for three values of R_d, namely 100Ω, 220Ω and 1000Ω.

1.2 Task 2: Evaluation of Series Resonant Circuit

1.2.1 Task 2a: Resonance Curves

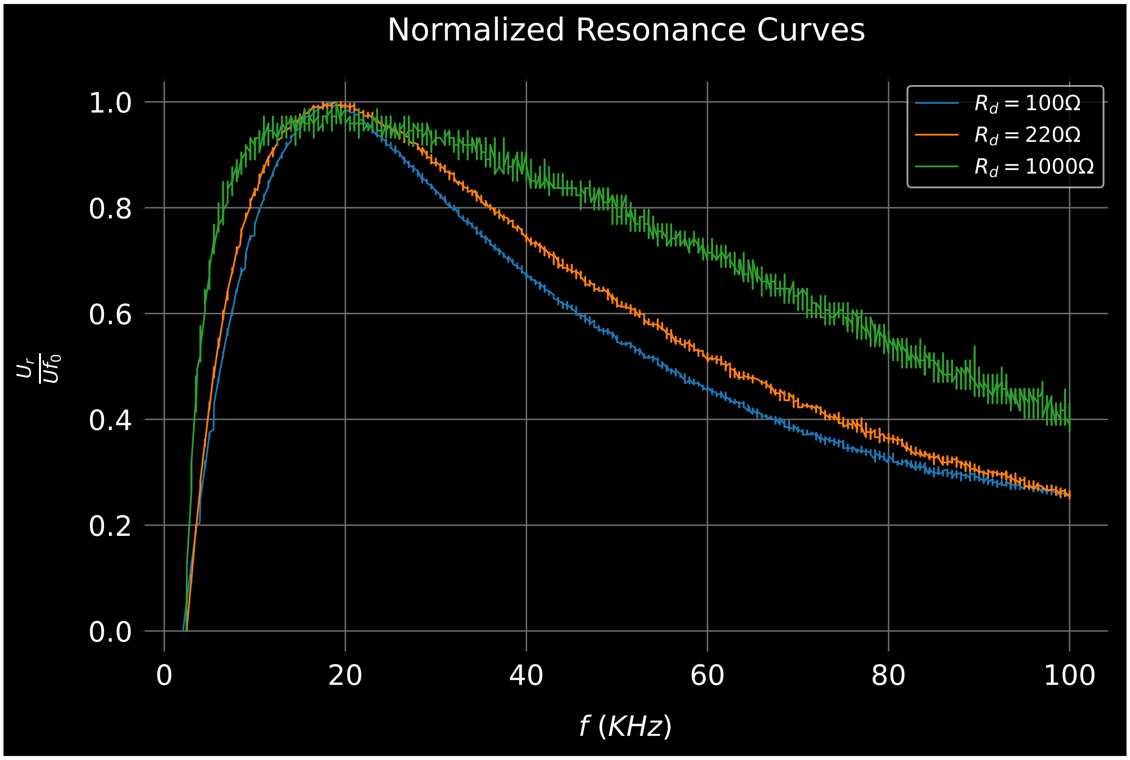


Figure 2.1: Plot of Resonance Curves for both damping resistance.

1.2.2 Task 2b: Quality Factor and Damping Constant

Task Definition

Determine the FWHM (Δf), then determine the Quality factor (Q) and damping constant (δ).

Theoretical Basis

The full width at half maximum (FWHM) (Δf) is defined to be the width of a resonance curve at half of the height of the dissipated power.

$$P = \frac{U^2}{R} \implies U = \sqrt{PR}$$

At $P = \frac{1}{2}P_{max}$:

$$U_{max} = \sqrt{2PR}$$

U_{max} is known to be the voltage across the resistor at resonance, and is denoted by U_{f_0}

Hence:

$$\frac{U}{U_{f_0}} = \frac{1}{\sqrt{2}}$$

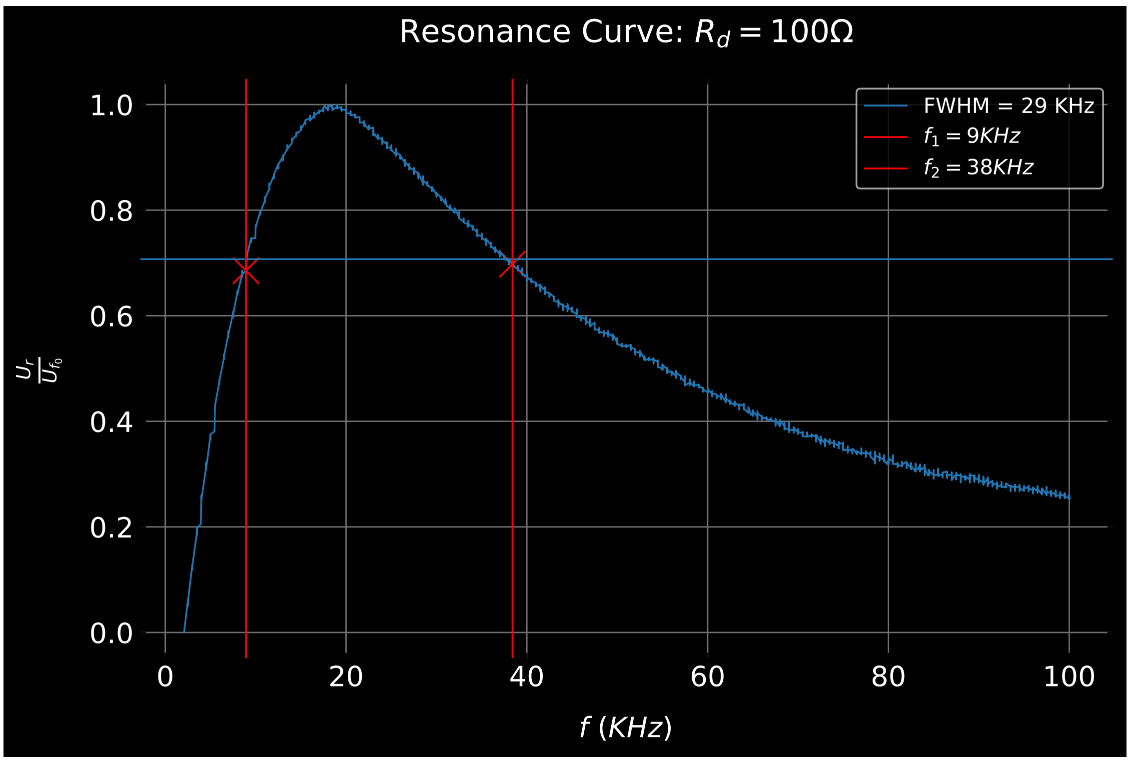
Frequencies at this value of $\frac{U}{U_{f_0}}$ are denoted by f_1 and f_2 and thier difference is Δf

$$\Delta f = f_2 - f_1 \tag{2.1}$$

Additionally, the Quality factor Q and damping constant δ are given by the following equations:

$$Q = \frac{f_0}{\Delta f} \tag{2.2}$$

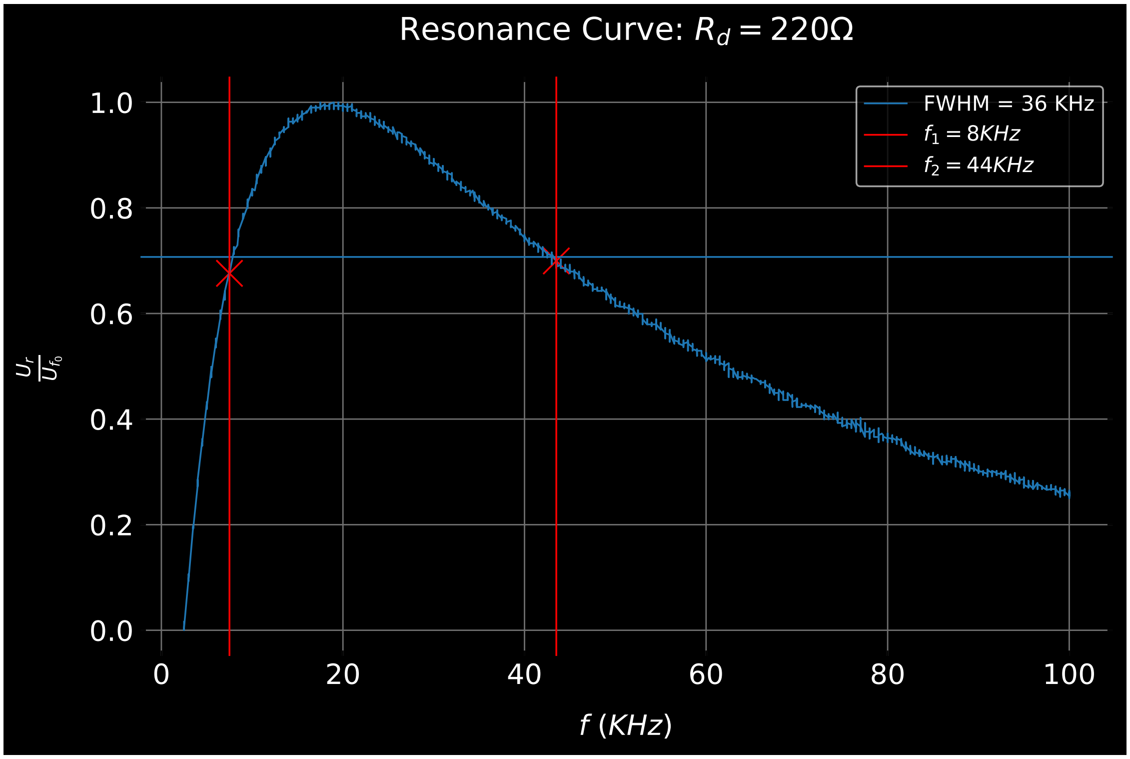
$$\delta = \pi \Delta f = \frac{\pi f_0}{Q} \tag{2.3}$$



$Figure$ 2.2: $R_d = 100\Omega$

$Q = 0.64$

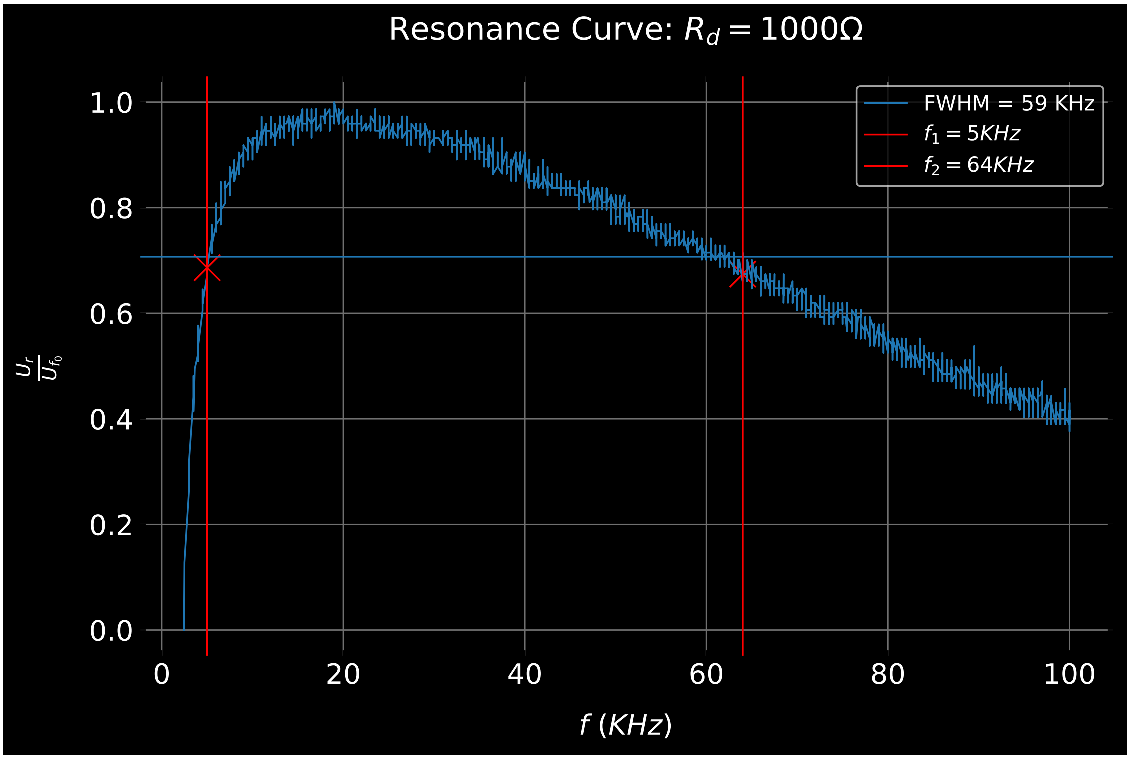
$\delta = 92.63\ kHz$



$Figure$ 2.3: $R_d = 220\Omega$

$Q = 0.49$

$\delta = 113.11\ kHz$



$Figure$ 2.4: $R_d = 1000\Omega$

$Q = 0.32$

$\delta = 185.37\ kHz$

1.2.3 Task 2c: Fitting

Task Definition

Fit an appropriate model function to the measured resonance curves and determine f_0 and δ.

Derivation of Model Function

The total impendence Z_T of the series circuit is defined as:

$$Z_T = R_T + iX_T$$

$$R_T = R_{sp} + R_d$$

and

$$X_T = X_L - X_C$$

- R_T: Total Resistance

- R_{sp}: Resistance of inductor

- R_d: Resistance of resistor
- X_T: Total Reactance
- X_L: Reactance of inductor
- X_C: Reactance of Capacitor

Across the resistor:

$$U_r = I_T R_d$$

For an AC current:

$$I_T = \frac{U_0}{|Z_T|}$$

- U_0: Frequency of Generator
- I_T: Total Current

By Substitution:

$$U_r = \frac{U_0}{|Z_T|} R_d$$

Expanding:

$$U_r = \frac{U_0}{\sqrt{R_T^2 + X_T^2}} R_d \tag{2.4}$$

At Resonance:

$$X_L = X_C \implies X_T = 0 \implies |Z_T| = R_T$$

Eq 2.4 reduces to:

$$U_{f_0} = \frac{U_0 R_d}{R_T} \tag{2.5}$$

Dividing Eq 2.4 by Eq 2.5 yields the modelling equation:

$$\frac{U_r}{U_{f_0}} = \frac{R_T}{\sqrt{R_T^2 + X_T^2}} \tag{2.6}$$

where:

$$X_L = 2\pi f L$$

$$X_C = \frac{1}{2\pi f C}$$

$$f_0 = \frac{1}{\sqrt{2\pi L C}} \tag{2.7}$$

$$\delta = \frac{\pi f_0}{Q} \tag{2.8}$$

- f: Frequency
- f_0: Resonant Frequency
- L: Inducatance
- C: Capacitance

Procedure

1. Modelling equation Eq 2.6 is a function of freqeuncy f via X_T. Hence, the measurements are fitted to this equation.
2. The parameters L, C may be found via the fit, and used to determine the value of f_0 for each R_d.
3. Subsequently, the value of Q from task 2b and f_0 for each R_d can be used to determine the corresponding value of δ.

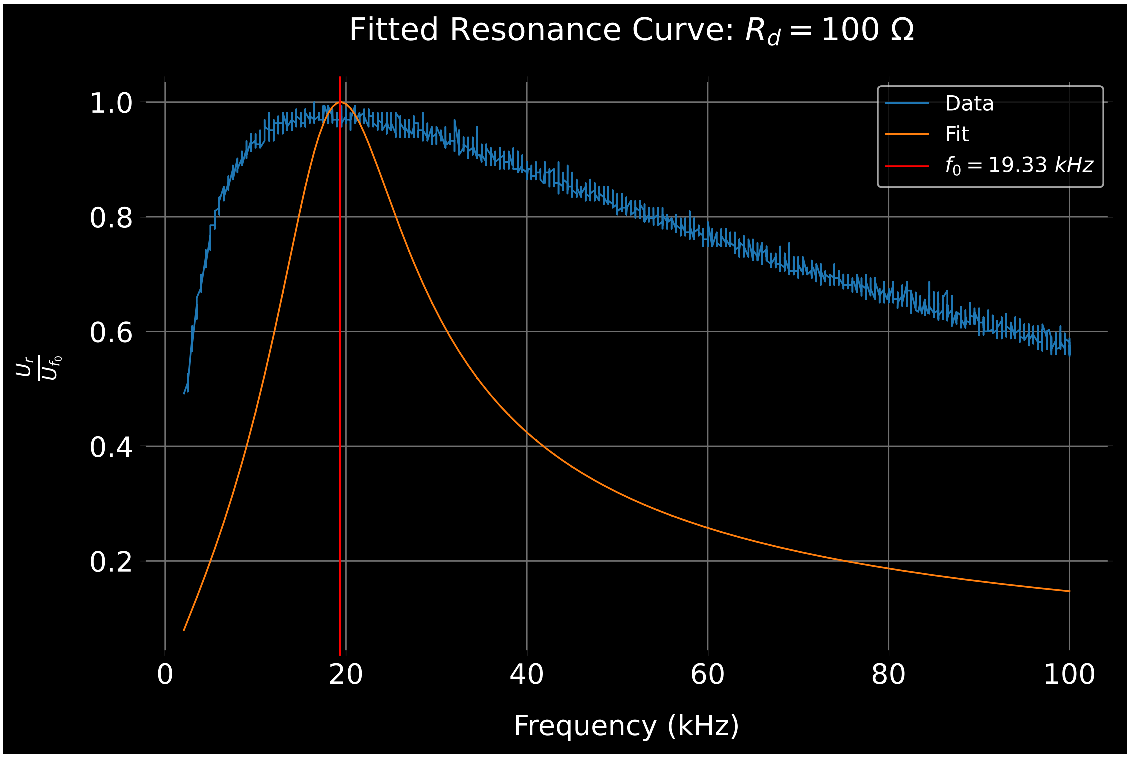


Figure 2.5: Fitted Resonance Curve: $R_d = 100\ \Omega$

$$R_d = (101 \pm 31.4)\ \Omega$$

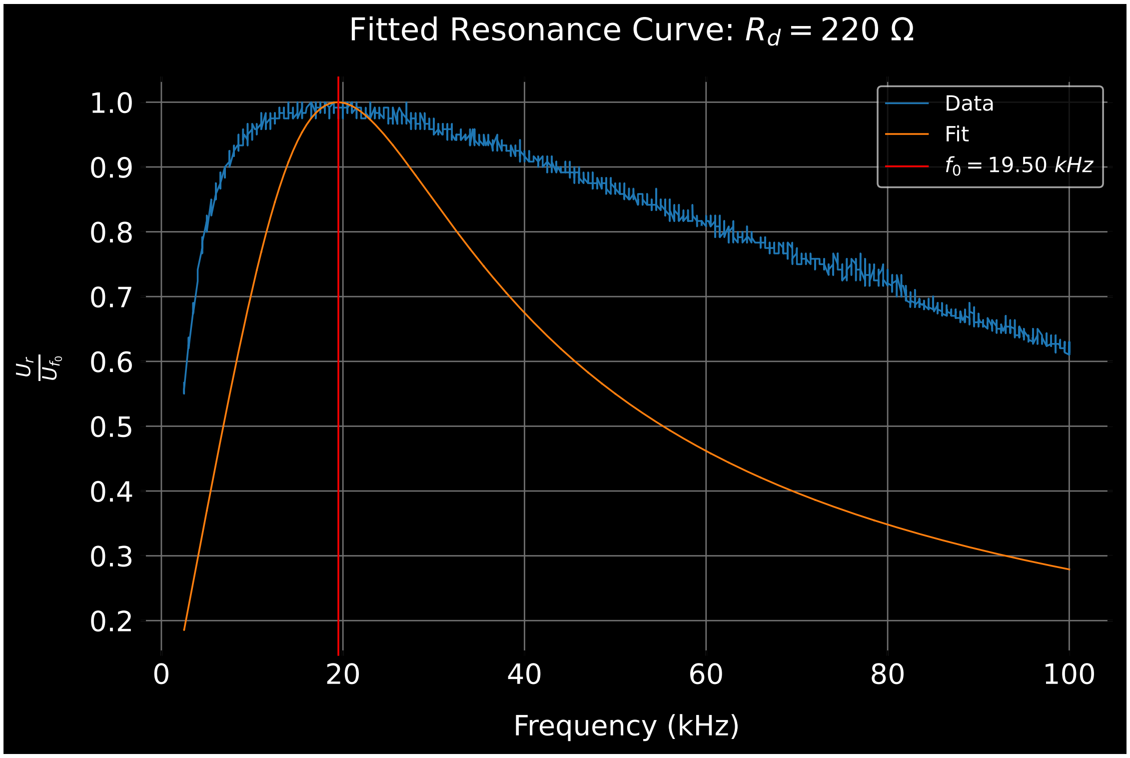
$$R_{sp} = (25 \pm 31.4)\ \Omega$$

$$L = (1.40 \pm 0.707)\ mH$$

$$C = (48.00 \pm 23.123)\ nF$$

$$\delta = 94.28\ kHz$$

$$f_0 = 19.33\ kHz$$



$Figure$ 2.6: Fitted Resonance Curve: $R_d = 220\ \Omega$

$$R_d = (221 \pm 95.9)\ \Omega$$

$$R_{sp} = (25 \pm 95.9)\ \Omega$$

$$L = (1.40 \pm 1.096)\ mH$$

$$C = (48.00 \pm 35.830)\ nF$$

$$\delta = 125.98\ kHz$$

$$f_0 = 19.50\ kHz$$

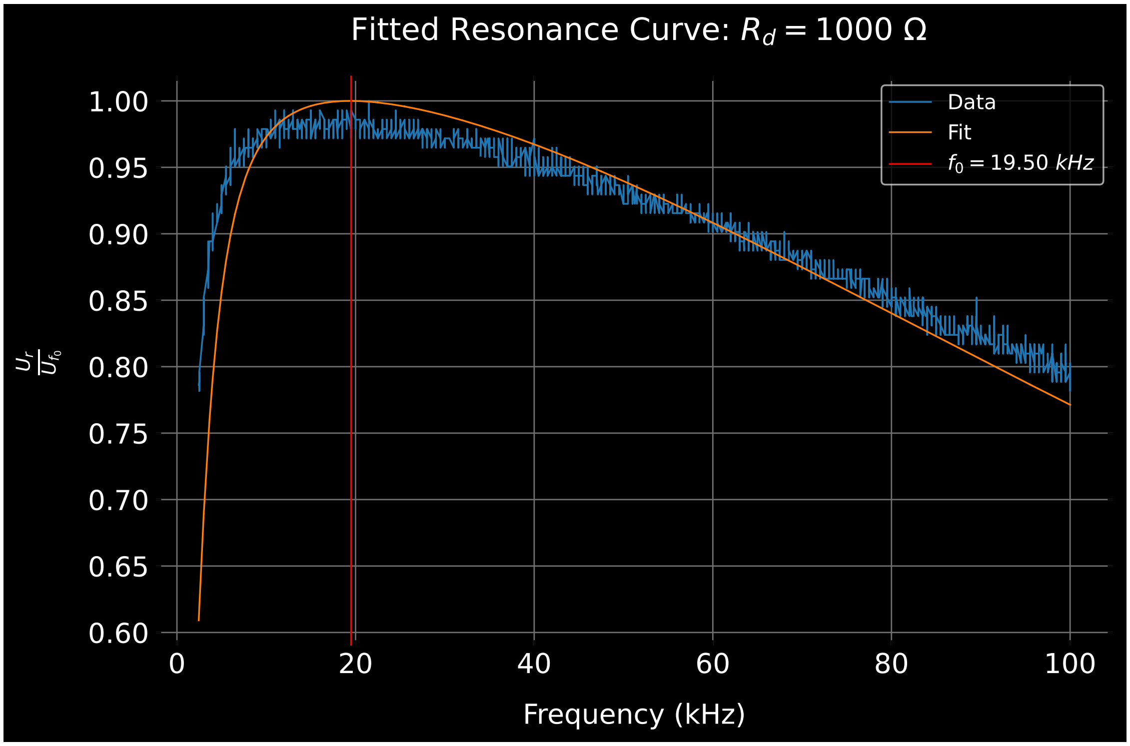


Figure 2.7: Fitted Resonance Curve: $R_d = 1000\ \Omega$

$$R_d = (1001 \pm 167.7)\ \Omega$$

$$R_{sp} = (25 \pm 167.7)\ \Omega$$

$$L = (1.40 \pm 0.458)\ mH$$

$$C = (48.00 \pm 14.782)\ nF$$

$$\delta = 190.24\ kHz$$

$$f_0 = 19.50\ kHz$$

Analysis

1. It is observed that the values of L and C found via fitting closely matches the actual values used in the experiment which were $L= 1.5mH $ and $C = 47nF$ respectively.
2. Similarly, the values of f_0 found via fitting is close to the value of f_0 computed via the Thomson Equation.

1.2.4 Task 2d: Thomson Equation

Task Definition

Calculate the capacitance C of the capacitor using the Thomson equation.

Theoretical Basis

1. The average resonant frequency f_{avg} is determined using the values of f_0 from the fittings in task 1c.
2. Additionally, the value of L from fit was also averaged, and used to determine C which closely matched the actual value of $C = 47nF$

Capacitance from Thomson's equation:

$$C = \frac{1}{(2\pi f_0)^2 L} = 49.34\ nF$$

1.3 Task 3: Parallel resonant circuit

Task Definition

Measure the frequency-dependent current draw of a circuit in which the capacitor and coil are connected in parallel and in series with the resistor R_d.

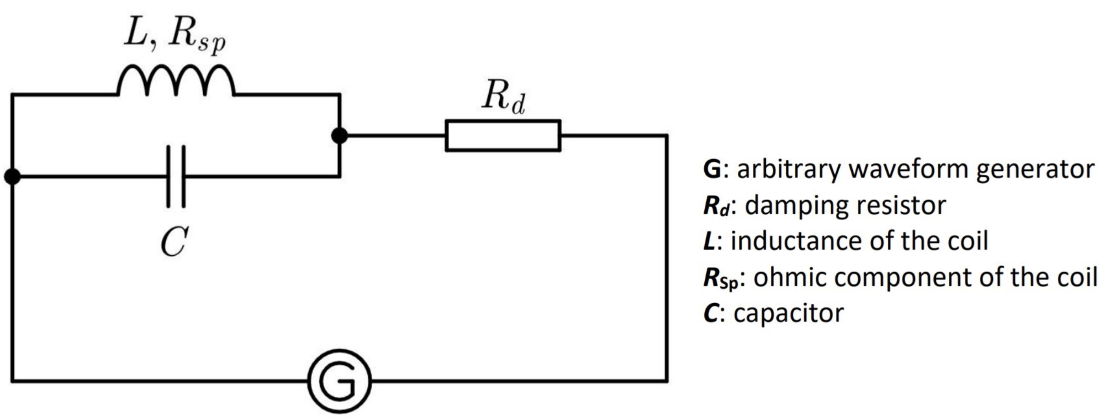


Figure 3.1: RLC Parallel Circuit Diagram

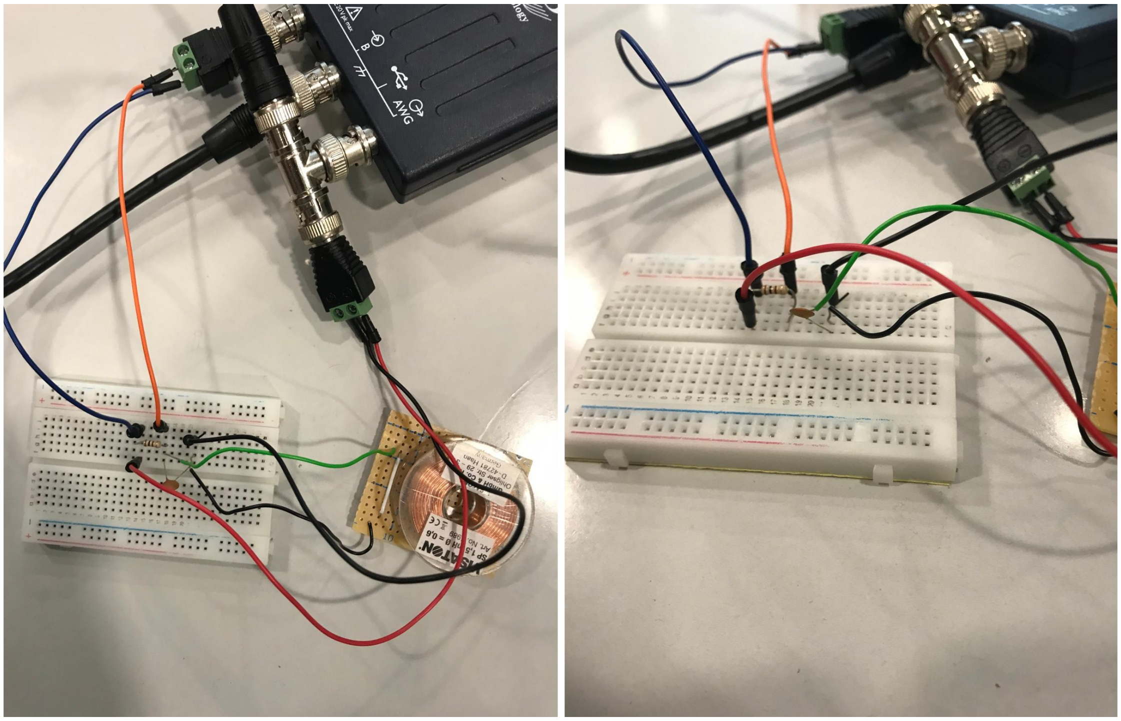


Figure 3.2: Parallel RLC Setup

Procedure

1. A RLC Parallel Circuit **Fig 3.2** is setup using the circuit diagram in **Fig 3.1**.
2. Sine waves of varying freqeuncies between 1 to 50kHz are generated by the picoscope, and supplied into the circuit.
3. The peak to peak voltage of the damping resistor is then measured via Channel A of the picoscope. Whereas, on Channel B the peak to peak voltage of the generator has been captured. The phase shift between Channel A and Channel B has been measured as well.
4. During data preprocessing, invalid values have been droped and the voltages have been normalized.

1.4 Task 4: Evaluation of the parallel resonant circuit

1.4.1 Task 4a

Task Definition

Derive an expression for the impedance of this circuit.

First consider the impedance of LC loop.

$$\frac{1}{Z_{||}} = \frac{1}{Z_C} + \frac{1}{Z_L}$$

where,

- Z_L : impedance of the coil.

$$Z_L = R_{sp} + iX_L = R_{sp} + i\omega L$$

- Z_C : impedance of the capacitor.

$$Z_C = iX_C = -i\frac{1}{\omega C}$$

Hence,

$$Z_{||} = \frac{1}{i\omega C + \frac{1}{R_{sp}+i\omega L}}$$

or in rectangular form:

$$Z_{||} = \frac{R_{sp}}{\alpha_\omega} + i\omega\frac{L - R_{sp}^2 C - \omega^2 L^2 C}{\alpha_\omega}$$

where $\alpha_\omega := (\omega C R_{sp})^2 + (\omega^2 LC - 1)^2$.

The total impedance of the circuit will be:

$$Z = R_d + Z_{||} = \left(R_d + \frac{R_{sp}}{\alpha_\omega}\right) + i\omega\frac{L - R_{sp}^2 C - \omega^2 L^2 C}{\alpha_\omega} \tag{4.1}$$

At resonance($\omega = \omega_0$), the impedance should be purely resistive, which means $Im(Z) = 0$.

$$L - R_{sp}^2 C - \omega_0^2 L^2 C = 0$$

Thus,

$$\omega_0 = \frac{1}{\sqrt{LC}}\sqrt{1 - \frac{R_{sp}^2 C}{L}} \tag{4.2}$$

Check that when one has an ideal inductance($R_{sp} = 0$), the resonant frequency recovers Thomson's equation $\omega_0 = \frac{1}{\sqrt{LC}}$

Note:

$$\alpha_{\omega_0} = \frac{C R_{sp}^2}{L}$$

To validate the results above, one can evaluate the Z, when $\omega \to 0$ (AC $\to$ DC). In this case, imaginary part of the Z becomes 0, and with $\alpha_{\omega\to 0} = 1$ one ends up

$$Z_{\omega\to 0} = R_d + R_{sp}$$

as expected for two series resistance.

The tangent of the phase shift ϕ between current and voltage is equal to the ratio of imaginary and real part of the impedance of a circuit.

$$\tan\phi = \frac{Im(Z)}{Re(Z)} = \frac{\omega(L - R_{sp}^2 C - \omega^2 L^2 C)}{\alpha_\omega R_d + R_{sp}} \tag{4.3}$$

1.4.2 Task 4b

Task Definition

Fit this model curve to the measured data. Determine L, C, R_{sp} and R_d as well as the resonance frequency f_0.

To determine the resonant frequency $\frac{U}{U_0}$ vs f graph is plotted. The resonant frequency f_0 corresponds to the minima of the curve.

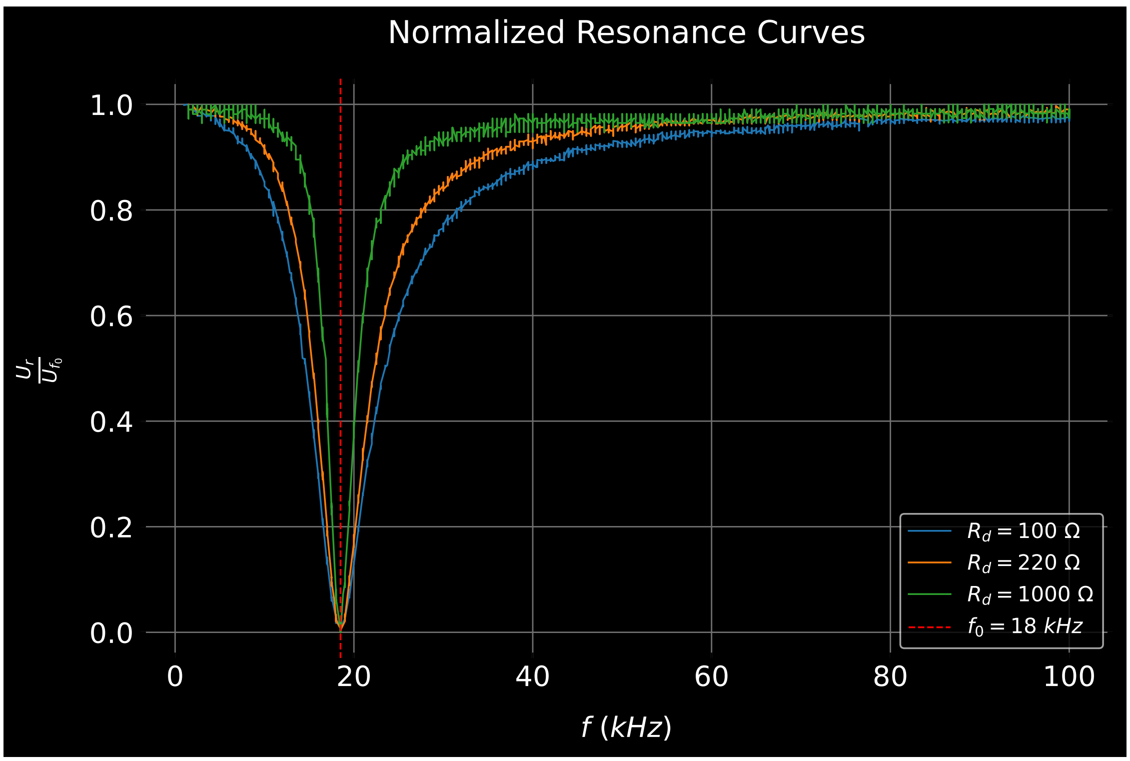
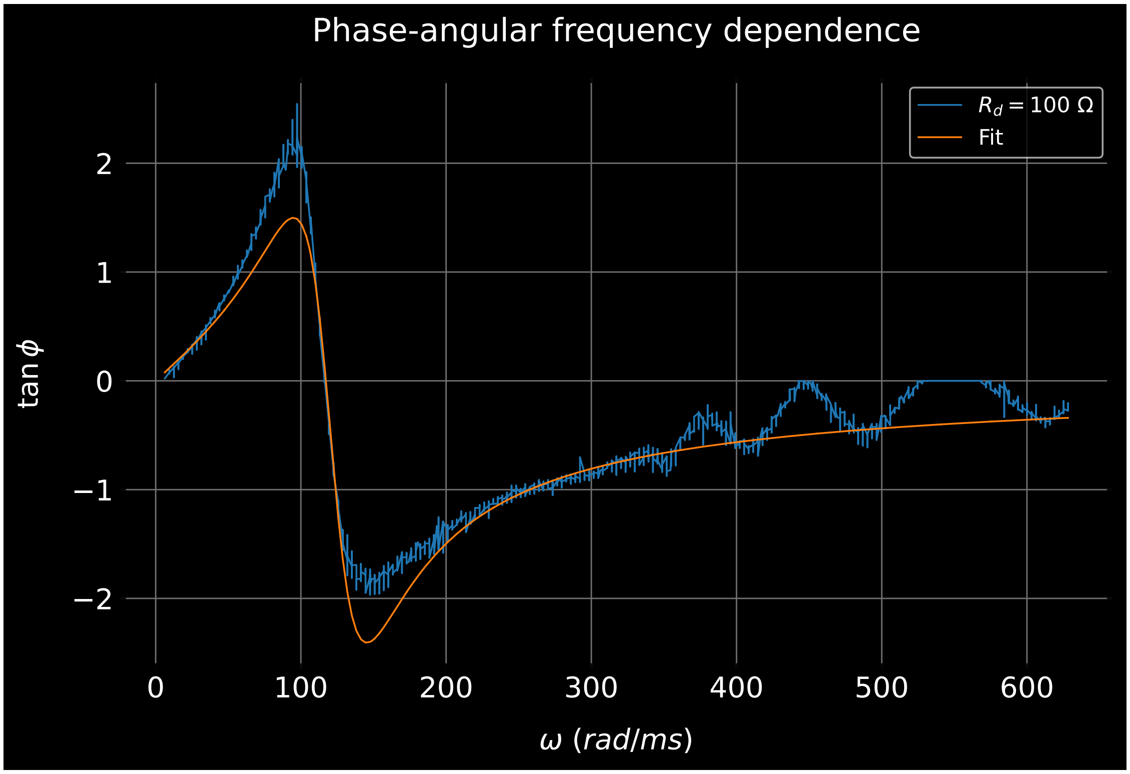


Figure 4.1: Plot of Resonance Curves for different damping resistance.

	R_d (Ω)	f_0 (kHz)
1	100	18.50
2	220	18.50
3	1000	18.50
Average		18.5

To determine L, C, R_{sp} and R_d Eq 4.3 is fitted using Trust Region Reflective algorithm [2].



$Figure\,4.2$: $\tan(\phi)$ vs ω fit for $R_d = 100\ \Omega$.

Fitted Parameters:

$$R_d = (101 \pm 3.9)\ \Omega$$

$$R_{sp} = (19 \pm 0.3)\ \Omega$$

$$L = (1.50 \pm 0.053)\ mH$$

$$C = (48.00 \pm 1.630)\ nF$$

Resonant frequency using Eq(4.2):

$$f_0 = (18.66 \pm 0.725)\ kHz$$

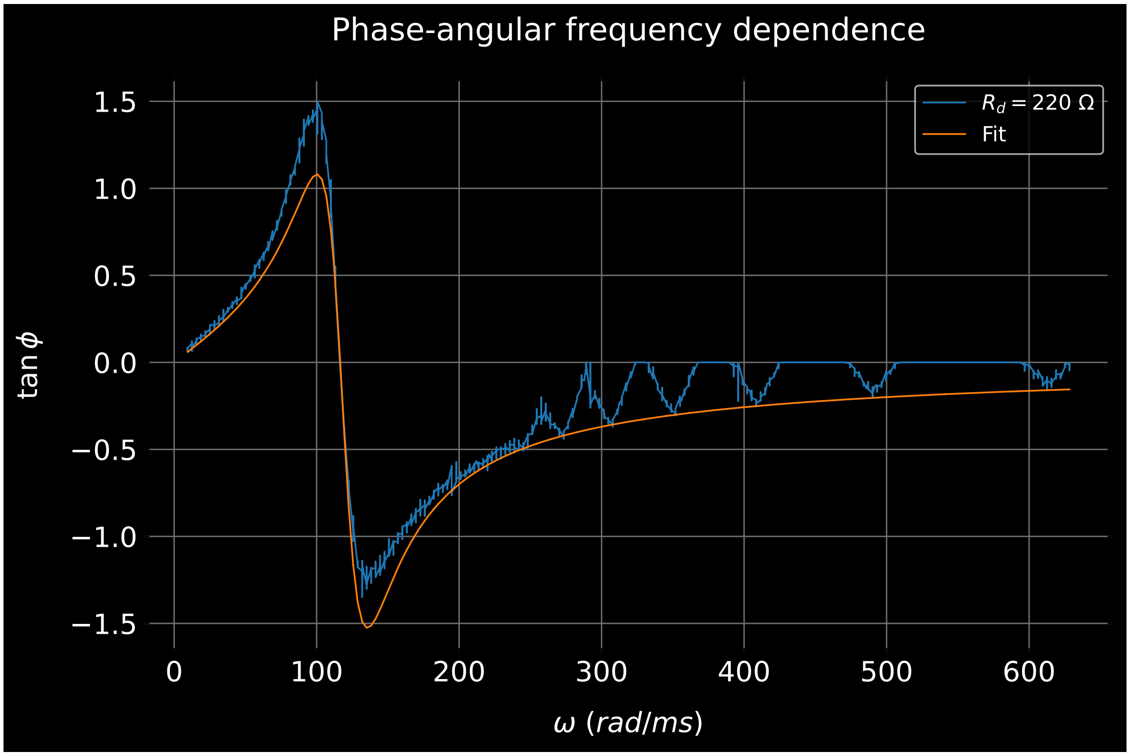


Figure 4.3: $\tan(\phi)$ vs ω fit for $R_d = 220\ \Omega$.

Fitted Parameters:

$$R_d = (221 \pm 6.3)\ \Omega$$

$$R_{sp} = (19 \pm 0.3)\ \Omega$$

$$L = (1.52 \pm 0.035)\ mH$$

$$C = (48.00 \pm 1.076)\ nF$$

Resonant frequency using Eq(4.2):

$$f_0 = (18.55 \pm 0.478)\ kHz$$

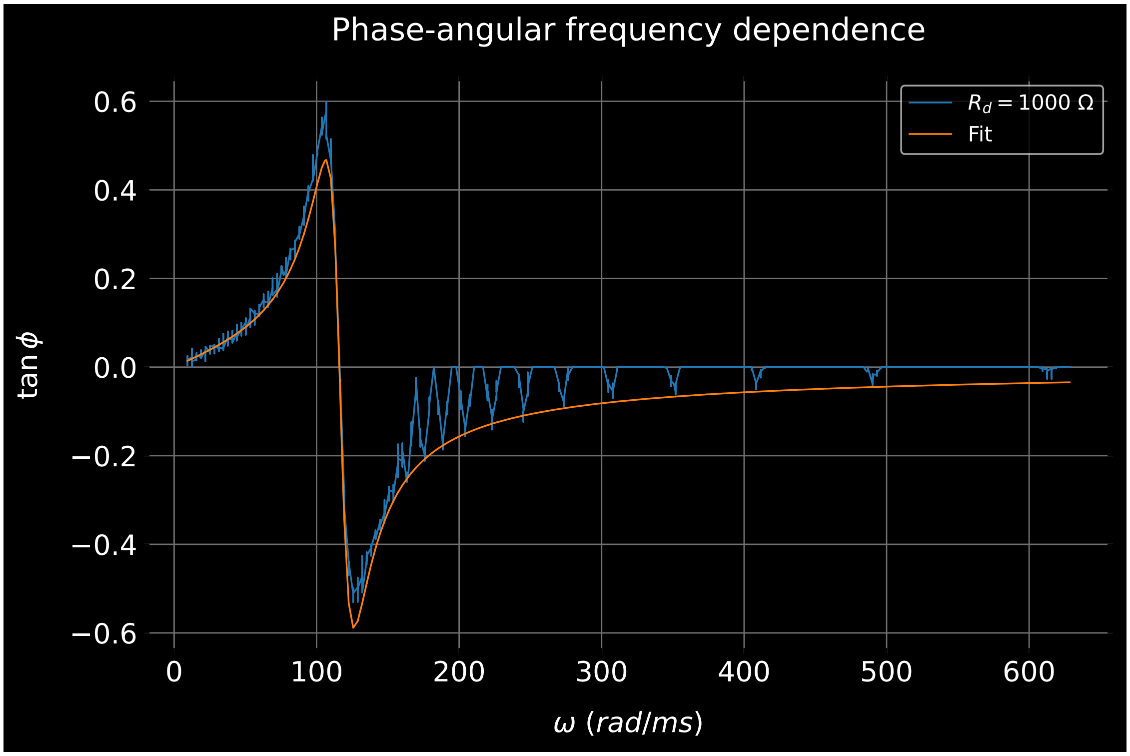


Figure 4.4: $\tan(\phi)$ vs ω fit for $R_d = 1000\ \Omega$.

Fitted Parameters:

$$R_d = (1001 \pm 15.7)\ \Omega$$

$$R_{sp} = (18 \pm 0.4)\ \Omega$$

$$L = (1.53 \pm 0.014)\ mH$$

$$C = (48.00 \pm 0.406)\ nF$$

Resonant frequency using Eq(4.2):

$$f_0 = (18.46 \pm 0.179)\ kHz$$

So, the overall result follows:

R_d (Ω)	R_{sp} (Ω)	L (mH)	C (nF)	f_0 (kHz)
101.00	19.02	1.50	48.00	18.66
221.00	18.54	1.52	48.00	18.55
1001.00	18.15	1.53	48.00	18.46
Average	18.6	1.5	48.0	18.6

References

1) E12He Lab instruction
2) Sorensen, D. C. Newton's Method with a Model Trust-Region Modification, report, September 1980; Argonne, Illinois. (https://digital.library.unt.edu/ark:/67531/metadc283479/: accessed June 30, 2024), University of North Texas Libraries, UNT Digital Library, https://digital.library.unt.edu; crediting UNT Libraries Government Documents Department.