O04e

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1 O04e Focal Length and Principal Planes of a Lens System

Group #13

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Overview of Tasks

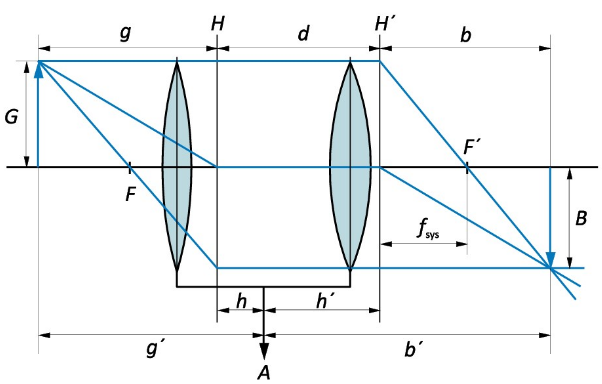
1. Determine the focal length and the position of the principal planes of a system consisting of two thin converging lenses for a given lens distance.
2. Determine the focal lengths of both lenses using Bessel's method. Measure the distance between the lenses and calculate the focal length of the lens system.
3. Draw a diagram to scale showing the positions of the principal planes and the focal points of the lens system; construct an image corresponding to one measured object distances from task 1. Determine the value of the linear magnification from the construction and compare to the experimental value.

1.1 Task 1

Task Definition

Determine the focal length of the system of 2 lenses f_{sys} and the position of the principal planes h and h' of a system consisting of two thin converging lenses for a given lens distance.

Theoretical Basis



Double lens system.

g' and *b*' denote the distances between the pointer *A* and the illuminated object *G* or the image *B*, respectively.

h and *h*' denote the distances between the pointer *A* and the principal planes *H* and *H*' of the lens system.

Figure 1.1: Double Lens System Diagram

Theoretical Basis

Linear Magnification γ

During the experiment, the following sign convention is used to measure object G and imae B sizes **Fig 1.1**.

- Object sizes (G) are positive because it lies above the principal axis.
- Image sizes (B) are negative because the image is inverted and lies beneath the principal axis.

The linear magnification is given by:

$$\gamma = \frac{B}{G} \tag{1.1}$$

Principal planes h and h'

Note: During the experiment, only the absolute value of the distances were measured.

The magnitude of h and h' provides the distance between the pointer A and the principal planes H and H' respectively.

The sign of h and h' indicates whether the principal planes lies to the right or left of pointer A.

With reference to Fig 1.1, note the following:

If $h' = b' - b$ is negative, then $b' < b \implies$ Principal plane H' lies to the left of pointer A.

If $h = g' - g$ is positive, then $g' > g \implies$ Principal plane H' lies to the left of pointer A.

- b' : Image distance
- g' : Object distance

The Fitting Functions are given as follows:

$$g' = g + h = f_{sys}\left(1 + \frac{1}{\gamma}\right) + h \tag{1.2}$$

$$b' = b + h' = f_{sys}(1 + \gamma) + h' \tag{1.3}$$

Procedure

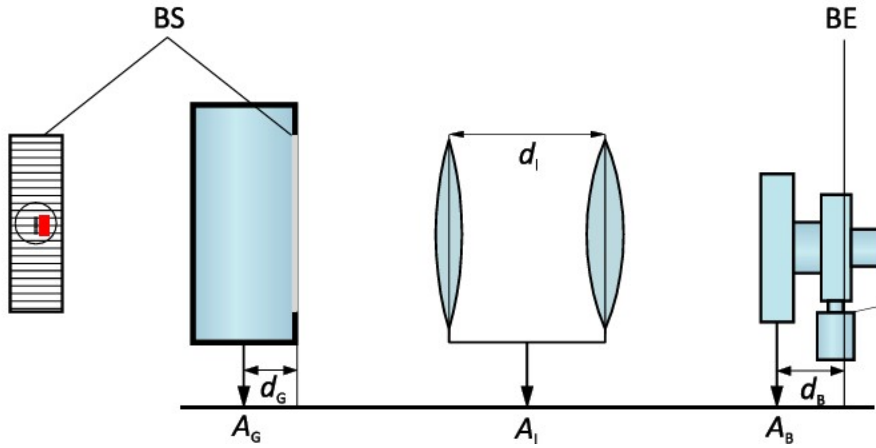
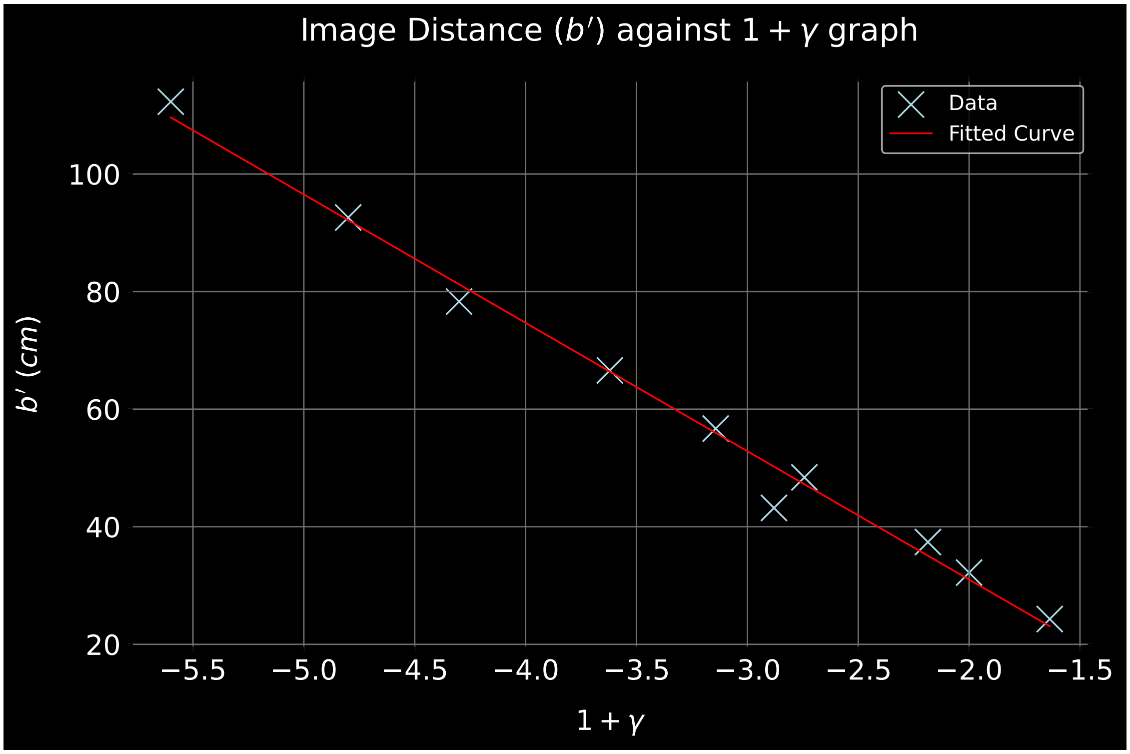


Figure 1.2: Double Lens System Experimental Setup

1. A double-lens system is picked with a fixed lens distance d_l, and correction values d_B and d_G **Fig 1.2**.
2. Adjust the lens system until a sharp image forms, then measure the object size (G) and the image size (B) using the micrometer labeled M in **Fig 1.2**.
3. Measure the distances d_1 and d_2 between pointers A_G and A_l and between A_B and A_l, respectively.
4. Calculate the object g' and image b' distances using the following formulas:

- $g' = |d_1| - d_G$
- $b' = |d_2| + d_B$

5. Calculate the linear magnification γ using Eq 1.1.
6. Repeat the procedure for 10 different values of g' and b'.
7. Perform linear regression using Eq 1.2 and Eq 1.3, and find the values of f_{sys}, h and h' from the fit.



$Figure$ 1.1: Image Distance (b') against $1+\gamma$ fit.

$$|f_{sys}| = (21.83 \pm 0.798)\ cm$$

$$h' = (-12.65 \pm 2.801)\ cm$$

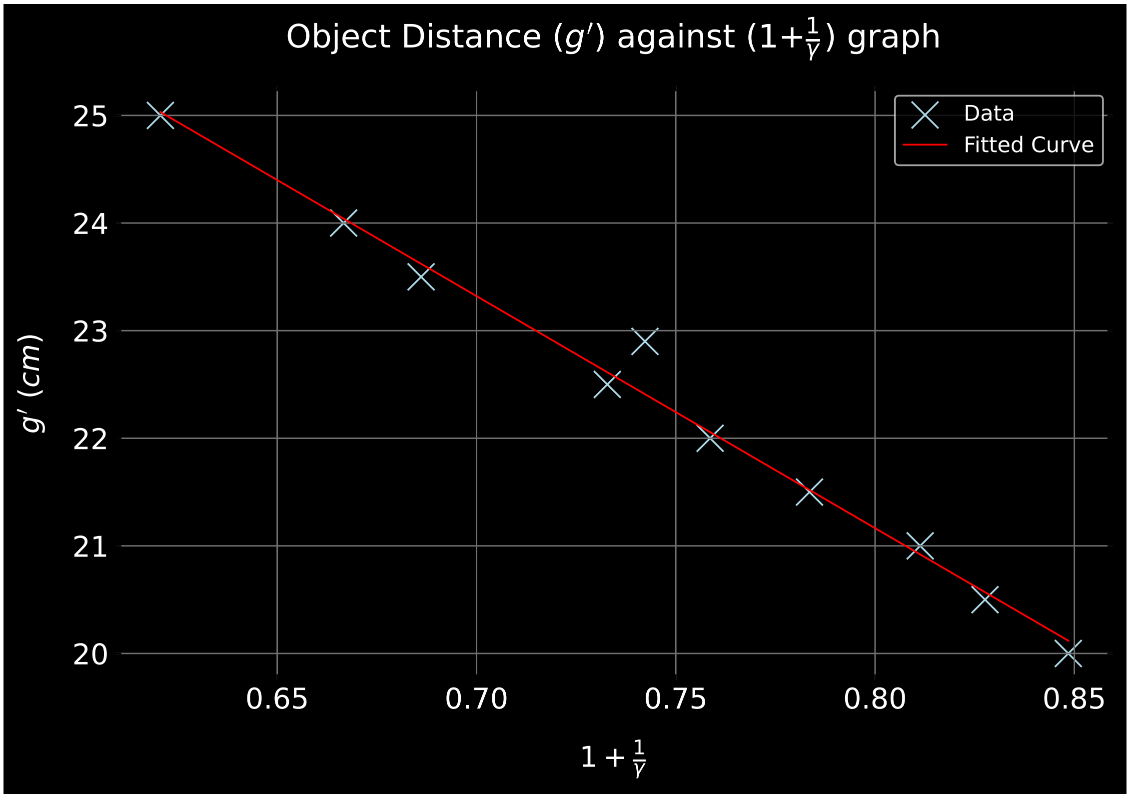


Figure 1.2: Object Distance (g') against ($1+\frac{1}{\gamma}$) graph fit.

$$|f_{sys}| = (21.57 \pm 0.876)\ cm$$

$$h = (38.42 \pm 0.658)\ cm$$

1.2 Task 2

Task Definition

Determine the focal lengths of both lenses f_1 and f_2 using Bessel's method. Measure the distance (a) between the lenses and calculate the focal length f_{sys} of the double-lens system from task 1.

Theoretical Basis

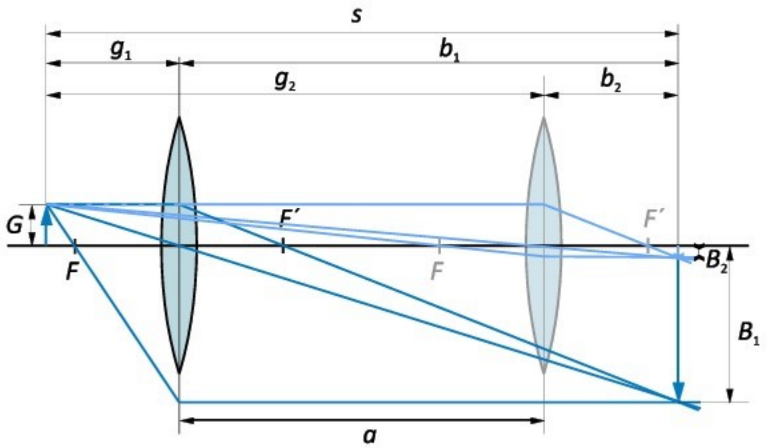


Figure 2.1: Bessel Method Setup

The Lens Equation:

$$\frac{1}{f} = \frac{1}{b} + \frac{1}{g}$$

- f : Focal Length of Lens
- b : Image Distance
- g : Object Distance

From Fig 2.1:

$$s = b + g$$

By Substitution:

$$\frac{1}{f} = \frac{s}{g(s-g)}$$

This yields:

$$0 = g^2 - sg + sf$$

Solving for g:

$$g_{1,2} = \frac{s}{2} \pm \sqrt{\frac{s^2}{4} - sf}$$

Hence:

$$g_1 - g_2 = \sqrt{s^2 - 4sf}$$

It is observed that:

$$a = g_1 - g_2 \tag{2.1}$$

Therefore:

$$f = \frac{s^2 - a^2}{4s} \tag{2.2}$$

Additionally, the focal length f_{sys} of a double-lens system is given by:

$$\frac{1}{f_{sys}} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d_l}{f_1 f_2} \tag{2.3}$$

- f_1 : Focal length of first lens
- f_2 : Focal length of second lens
- d_l : Distance between lenses

Procedure

1. The distance (S) between the object (G) and the screen (B) is measured and kept constant during the experiment. A lens from Task1 with focal length f_1 is then placed in-between the object and screen **Fig 2.1**.
2. The lens is adjusted until a sharp image forms on the screen, at which point the object distance (g_1) and the image distance (b_1) are recorded.
3. The lens is moved again until another sharp image forms, and the new distances (g_2) and (b_2) are recorded.
4. The distance (a) between the two different lens positions is found using Eq 2.1, and f_1 is determined using Eq 2.2.
5. The previous steps are repeated for 5 different values of (s), and the average value of (f_1) is determined.
6. This procedure is repeated using the second lens from Task1 with focal length (f_2).
7. The theoretical focal length f_{sys} of the double-lens system is determined using Eq 2.3.

Measurements

Lens 1

#	S (mm)	g_1 (mm)	b_1 (mm)	g_2 (mm)	b_2 (mm)	a (mm)	f_1 (mm)
1	650.0	98.0	552.0	545.0	105.0	447.0	85.65
2	750.0	96.0	654.0	647.0	103.0	551.0	86.30
3	850.0	94.0	756.0	749.0	101.0	655.0	86.32
4	550.0	102.0	448.0	441.0	109.0	339.0	85.26
5	450.0	110.0	340.0	330.0	120.0	220.0	85.61

Average value of $f_1 = 85.83$ mm

Lens 2

#	S (mm)	g_1 (mm)	b_1 (mm)	g_2 (mm)	b_2 (mm)	a (mm)	f_2 (mm)
1	1290.0	525.0	765.0	790.0	500.0	265.0	308.89
2	1390.0	446.0	944.0	940.0	450.0	494.0	303.61
3	1490.0	430.0	1060.0	1059.0	431.0	629.0	306.12
4	1590.0	427.0	1163.0	1176.0	414.0	749.0	309.29
5	1690.0	404.0	1286.0	1288.0	402.0	884.0	306.90

Average value of $f_2 = 306.96\ mm$

During the experiment:

$$d_l = 272\ mm$$

Therefore:

$$f_{sys} = 218.11\ mm$$
$$= 21.81 cm$$

Analysis

It is observed that Bessel's method yields approximately the same value for f_{sys} as found in task 1.

1.3 Task 3

Task Definition

Draw a diagram to scale showing the positions of the principal planes and the focal points of the lens system; construct an image corresponding to one measured object distances from task 1. Determine the value of the linear magnification from the construction and compare to the experimental value.

Procedure

1. A scale diagram has been drawn using values determined from tasks 1 and 2.
2. The following conditions were considred when constructing the scale diagram:

- Object distance: $g' = 250\ mm$
- Principal planes H and H' placed at positions h and h' respectively.
- Focal Lengths f_1 and f_2 of each lens taken from Task 2.

3. The scale diagram was used to find the image size B of an arbitrary object with object size G.
4. Subsequently, the theoretical linear magnification γ_{theory} was evaluated using Eq 1.1, and compared to the actual value of γ corresponding to $g' = 250\ mm$ from task 1.

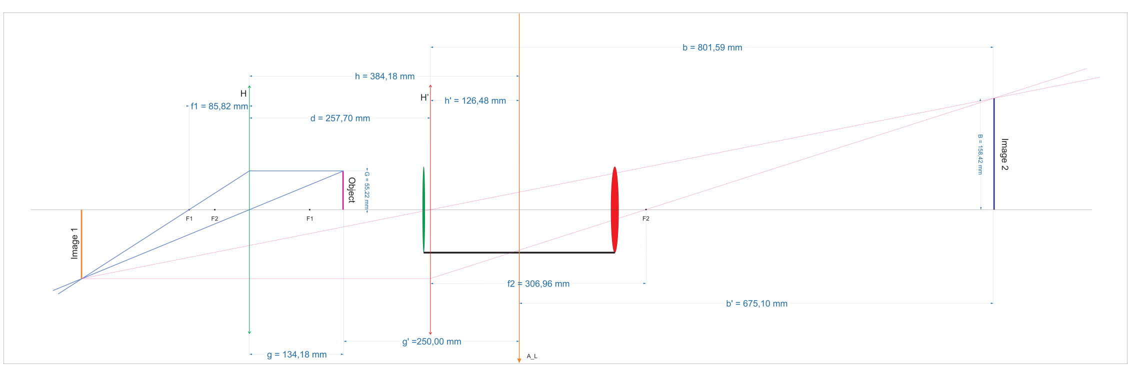


Figure 3.1: Diagram of the lens system for $g' = 250\ mm$ measurement value.

Analysis

From the **Fig.3.1**:

$$\gamma_{theory} = \frac{B}{G} = \frac{158.42\ mm}{55.22\ mm} = 2.87$$

From task 1:

$$\gamma_{actual} = 2.64$$

Hence, it is observed that the theoretical and actual value of γ are approximately equal as expected.

The slight difference between the values is likely due to not having considered the radius of curvature and the thickness of the lenses when evaluating γ_{theory}.