# E13He Lab Report

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## 1 Switching Phenomena

Group #13

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**Overview of Tasks**

***Task 1***

Measure the voltage drop across a capacitor ($500\ pF < C < 10\ nF$), which is discharged through a resistor with an as small as possible rating.

***Task 2***

Develop a model for your observations.

***Task 3***

Perform further measurements to test the model. Analyze the data quantitatively.

***Task 4***

When do you observe a time constant $\tau = RC$? Make at least one measurement in this regime.

### 1.1 Task 1

#### 1.1.1 Measure the voltage drop across a capacitor in a Mosfet Switch Circuit

***Task Definition***

The goal of task 1 is to set up a MOSFET- switch circuit and measure the voltage drop across the capacitor as it discharges through a resistor with the smallest possible rating. The collected data is then fitted to a model equation and used to determine the capacitance $C$.

***Theoretical Basis***

The following model equation is used to fit the data. It describes how voltage $U_c$ varies across a capacitor as it discharges over time $t$:

$$U_c = U_0 \exp\left(-\frac{t}{RC}\right)$$

$$\frac{U_c}{U_0} = \exp\left(-\frac{t}{RC}\right) \tag{1.1}$$

- $U_c$ : Voltage across capacitor at a given time $t$
- $U_0$ : Maximum voltage across capacitor
- $R$ : Resistance of discharge resistor
- $C$ : Capacitance

Eq 1.1 may be manipulated to yield the following linear relationship:

$$\ln(\frac{U_c}{U_0}) = kt$$

$$k = -\frac{1}{RC}$$

$$C = -\frac{1}{Rk} \tag{1.2}$$

Hence, the uncertainty in $C$ is given by:

$$\mu_C = \sqrt{\left(\frac{\partial C}{\partial k}\mu_k\right)^2 + \left(\frac{\partial C}{\partial R}\mu_R\right)^2}$$

- $\mu_k$ : Uncertainty of k from linear fit.

***Procedure***

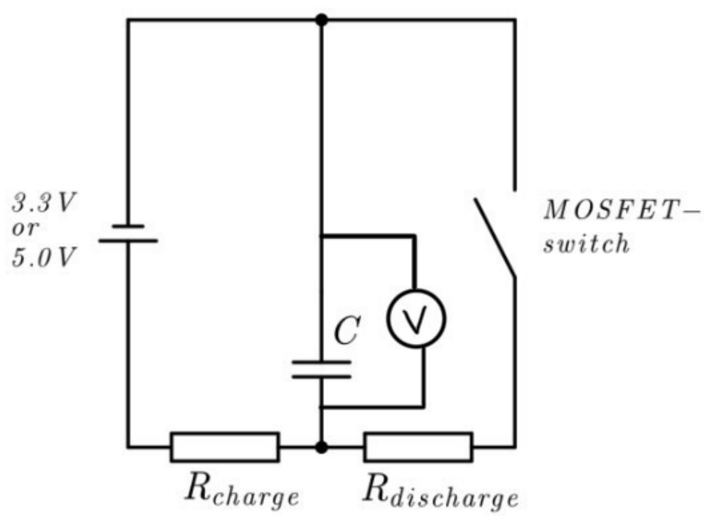


*Figure* 1.1: MOSFET Switch Circuit Diagram

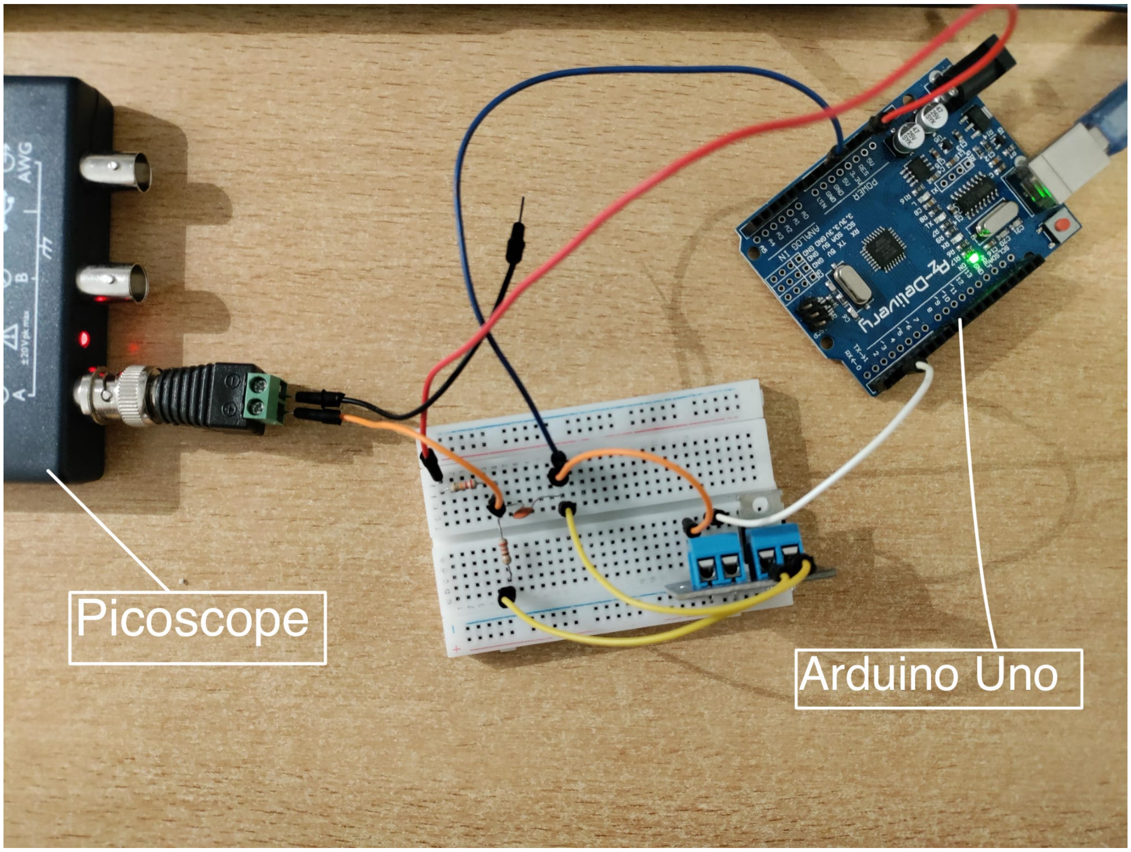


*Figure* 1.2: MOSFET Switch Circuit Set-Up

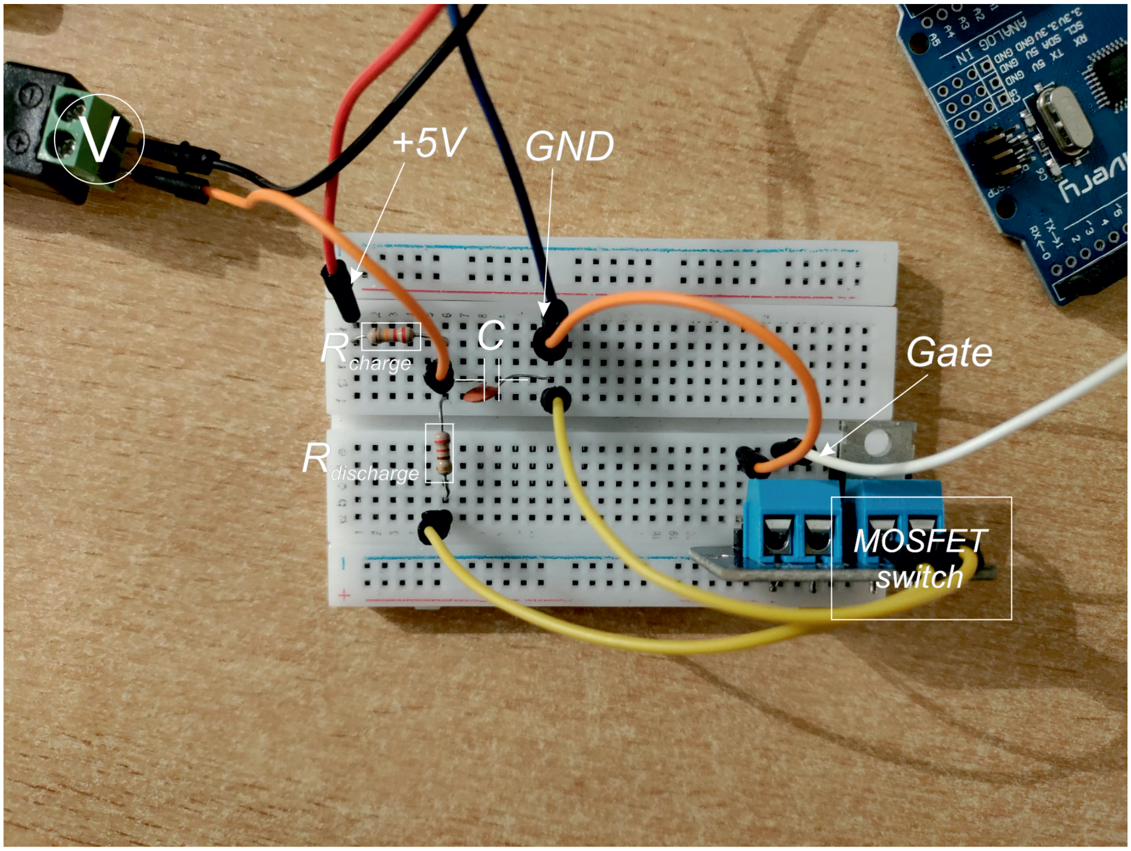


*Figure* 1.3: MOSFET Switch Circuit Closeup

1. The circuit was set up following the circuit diagram **Fig 1.1**.
2. The Arduino Uno provided the 5V DC source used to charge the capacitor, and to operate the MOSFET switch, while the PicoScope functioned as the voltmeter. **Fig 1.2**
3. A 22 $k\Omega$ and 10 $\Omega$ resistor was used as $R_{charge}$ and $R_{discharge}$ respectively **Fig 1.3**. The purpose of using a high resistance value for $R_{charge}$ was to impede the flow of current back into the power source during capacitor discharge.
4. Measurements of the voltage across the capacitor ($U_c$) as a function of time ($t$) were recorded using PicoScope 7 software installed on a host connected to the PicoScope 2000.
5. Vary $R_{discharge}$ and repeat the the steps above.

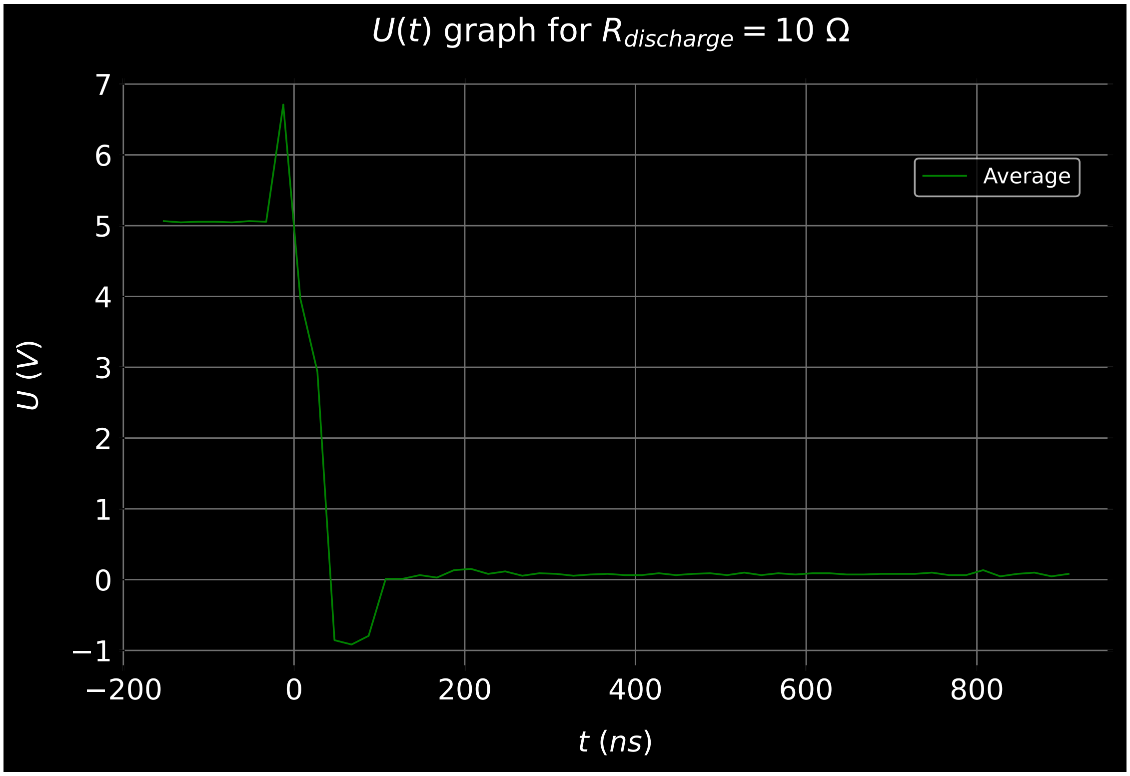


*Figure* 1.4: Average of 10 measurement samples. ($R = 10\ \Omega$)

## 1.2 Task 2

### 1.2.1 Model Observations

***Task Definition***

The goal of task 2 is to develop an accurate model for our observations.

***Theoretical Basis***

6 sets of measurements corresponding to different values of $R_{discharge}$ were taken in task 1.

Each set of measurements is fitted to Eq 1.1 to develop 6 distinct models.

The accuracy of a model is determined based on its root-mean square error (RMSE):

$$RMSE = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(y_i - \hat{y}_i)^2} \tag{2.1}$$

A higher RMSE value means that there is a greater deviation between the measurements and the fit line. This suggests that the built model does not accurately represent the observations.

Conversely, a lower RMSE value suggests that the built model is more accurate.

The value of $C$ may also be determined from the fit.

***Procedure***

1. Each set of measurements from task 1 is fitted to Eq1.1.
2. Subsequently, the value of $C$ for each model is determined from their respective fit along with their corresponding RMSE values.
3. The best model is selected with the lowest RMSE value, and its associated $C$ value is considered to be the actual capacitance denoted by $C_{best}$.

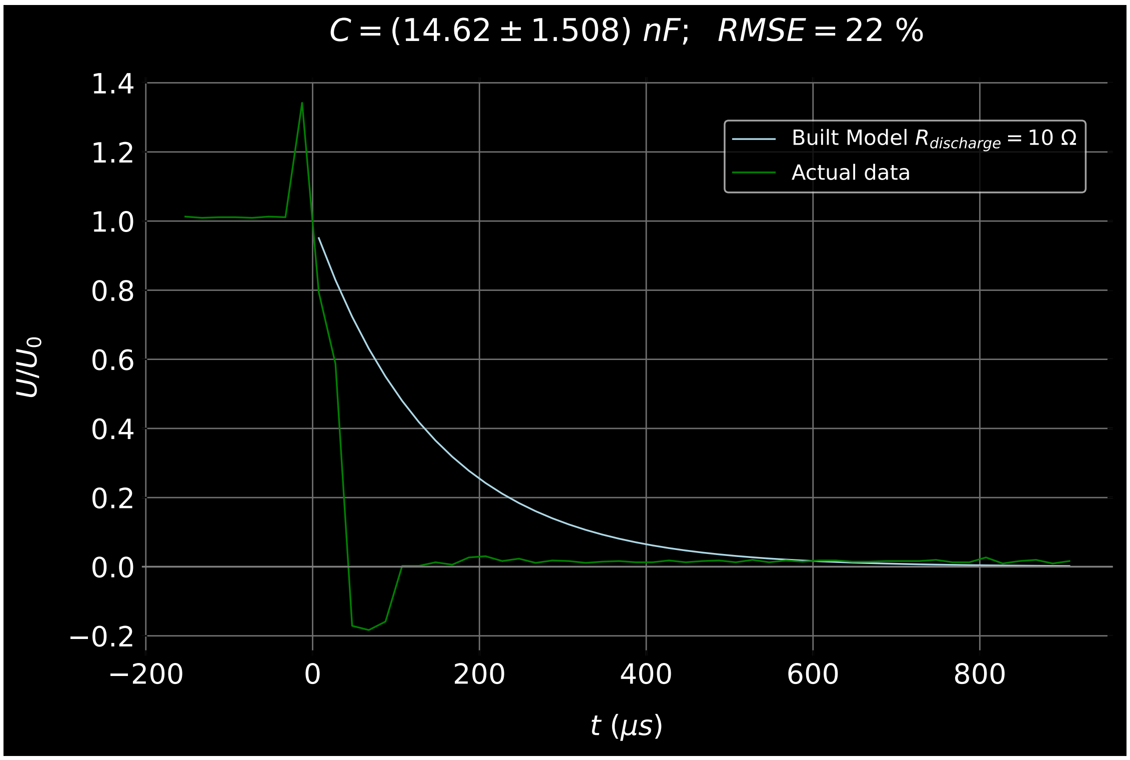


*Figure* 2.1: Fitted model of $R_{discharge} = 10\ \Omega$ data.

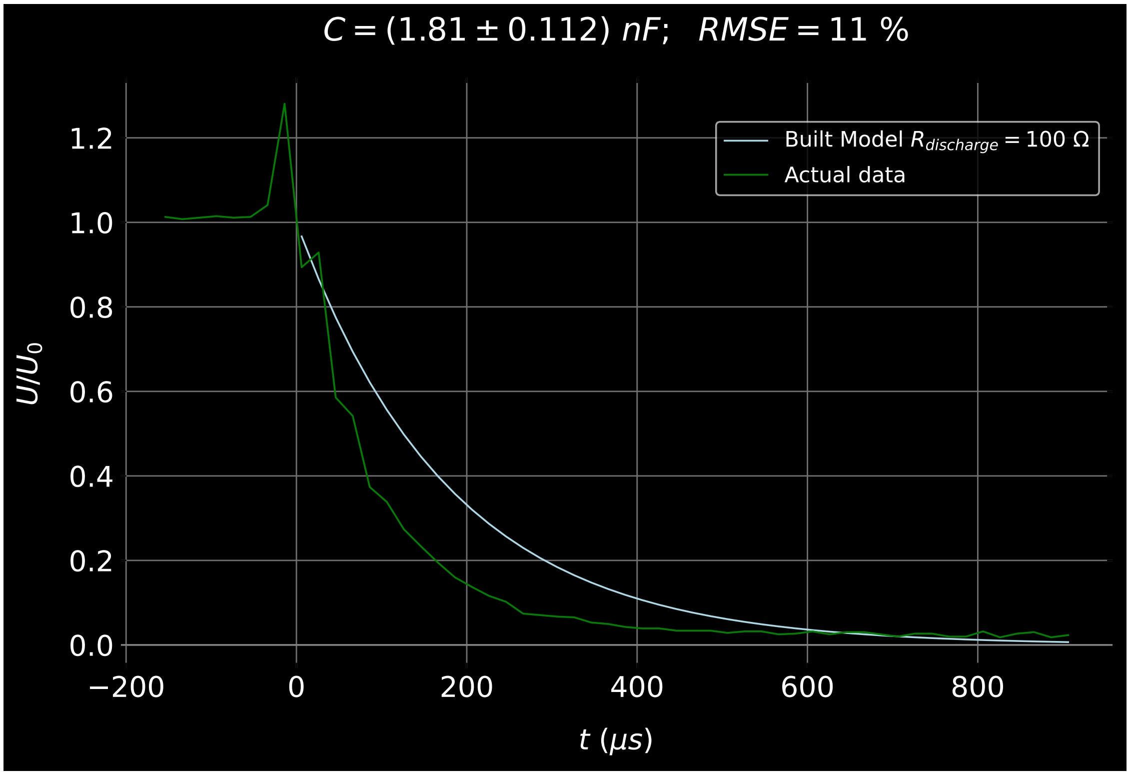


*Figure* 2.2: Fitted model of $R_{discharge} = 100\ \Omega$ data.

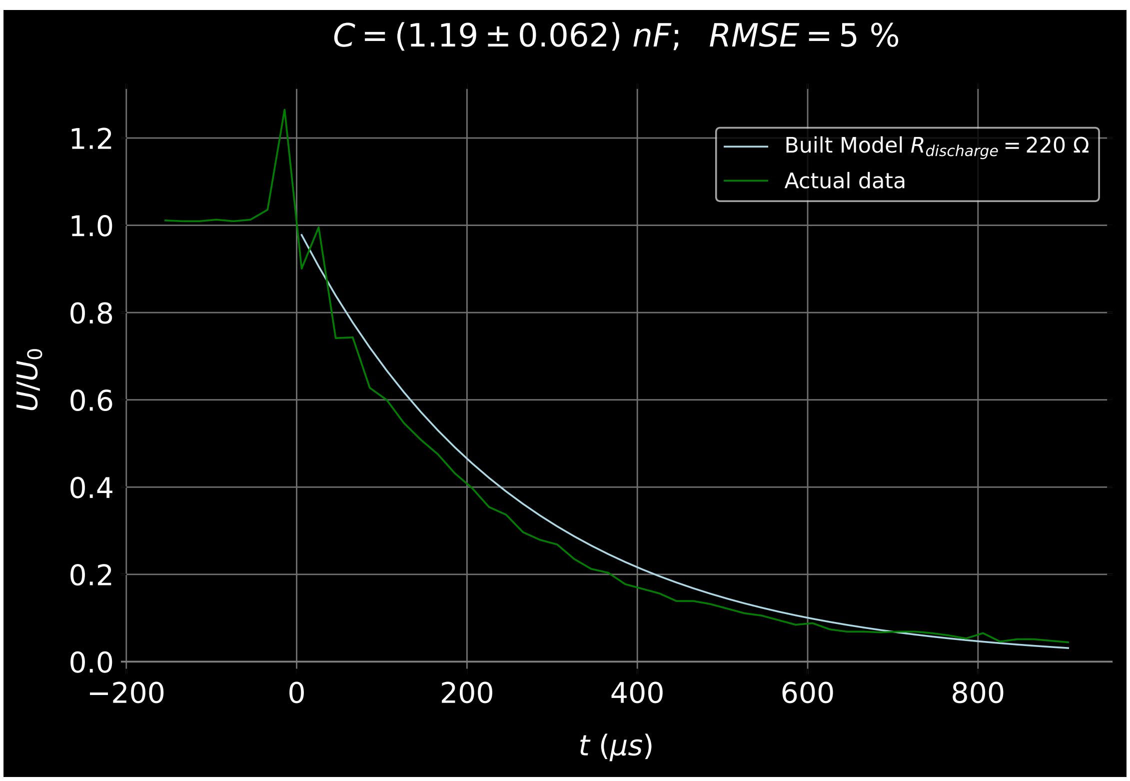

*Figure* 2.3: Fitted model of $R_{discharge} = 220\ \Omega$ data.

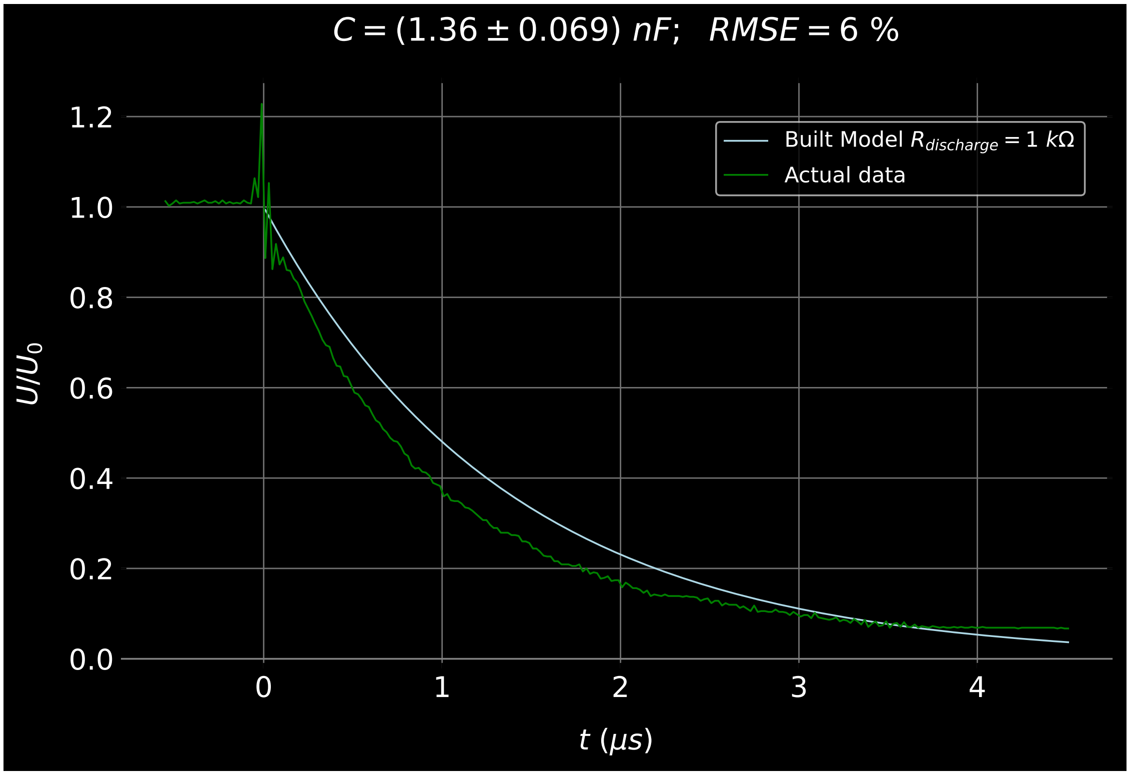


*Figure* 2.4: Fitted model of $R_{discharge} = 1\ k\Omega$ data.

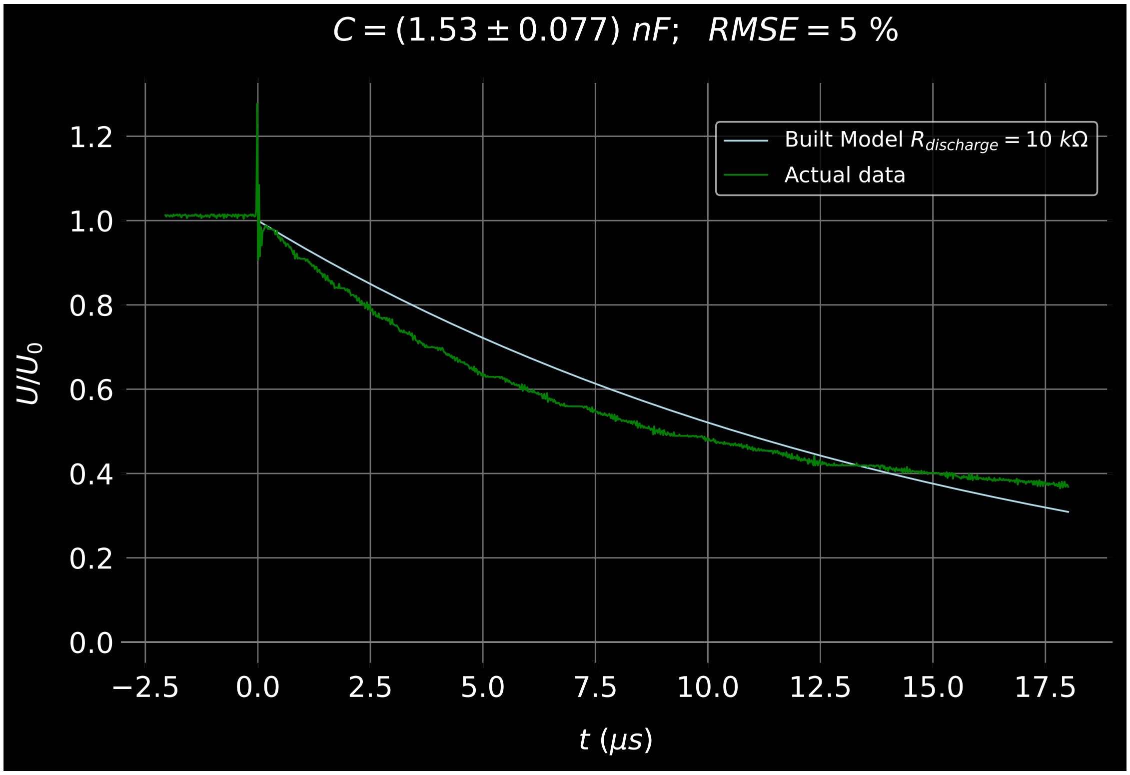


*Figure* 2.5: Fitted model of $R_{discharge} = 10\ k\Omega$ data.

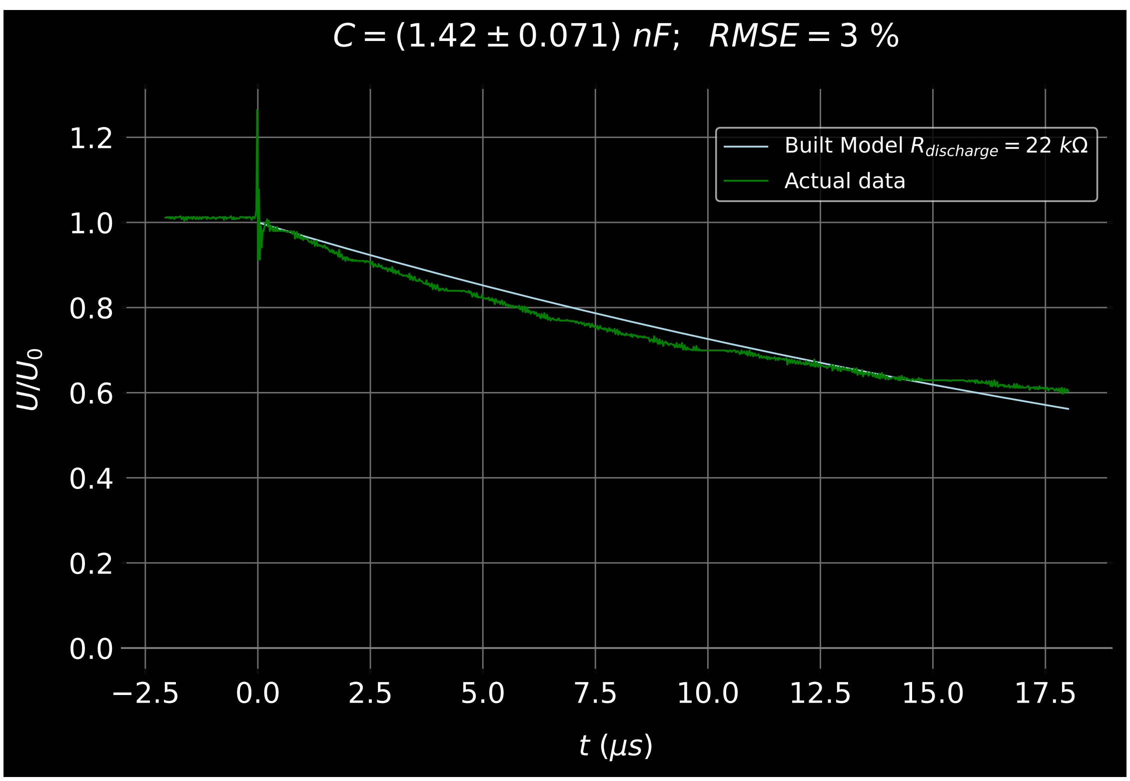

*Figure* 2.6: Fitted model of $R_{discharge} = 22\ k\Omega$ data.

Hence, one can observe from the figures above, that the model corresponding to $R = 22\ k\Omega \pm 5\ \%$ has the lowest error value with:

$$C_{Best} = (1.42 \pm 0.071)\ nF$$

## 1.3 Task3

### 1.3.1 Test Model

***Task Definition***

The goal of task 3 is to test the accuracy of the best model from task 2.

***Theoretical Basis***

To test the accuracy of the best model, $C_{Best} = 1.42nF$ was fixed and linear regression is applied for each set of measurements.

It is hypothesized that the best model has a higher accuracy compared to the built models in task 2.

Validation occurs if the the best model's fitting error is less than the built model's for each set of measurements.

Additionally, this ensures the value of $R$ obtained via the best model is in agreement with the actual resistances used in the experiment.

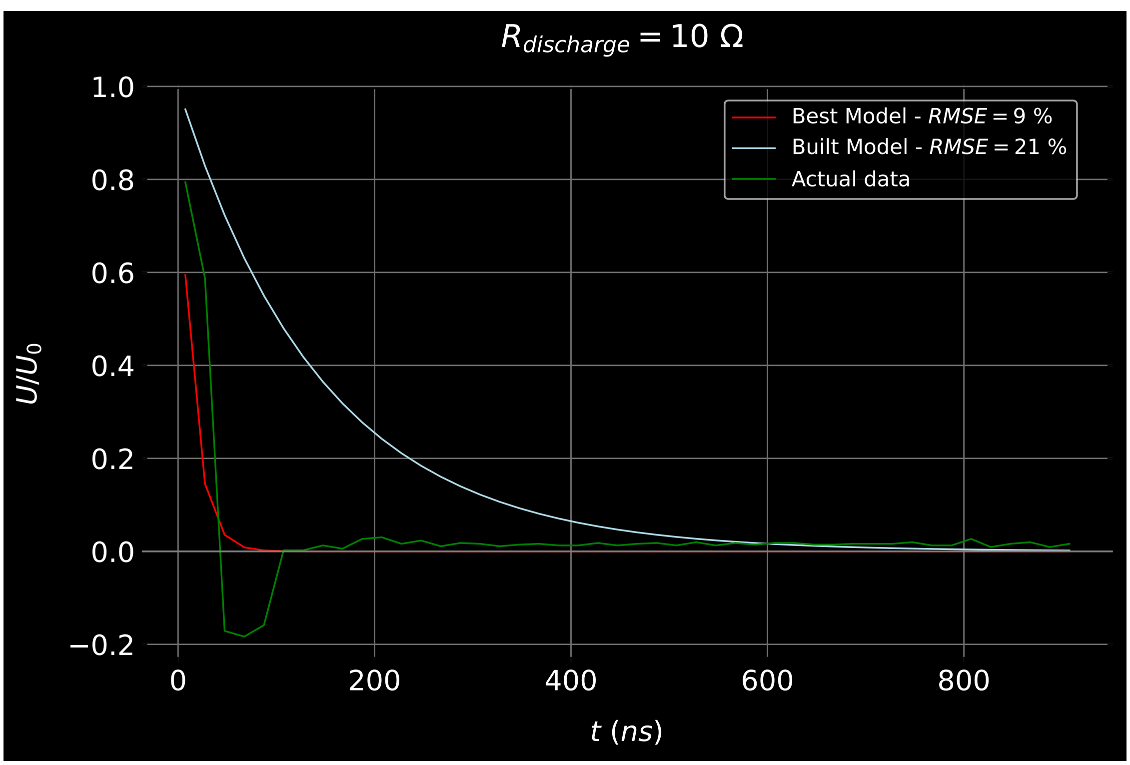

*Figure* 3.1: Comparison of Best model with Built model ($R_{discharge} = 10\ \Omega$).

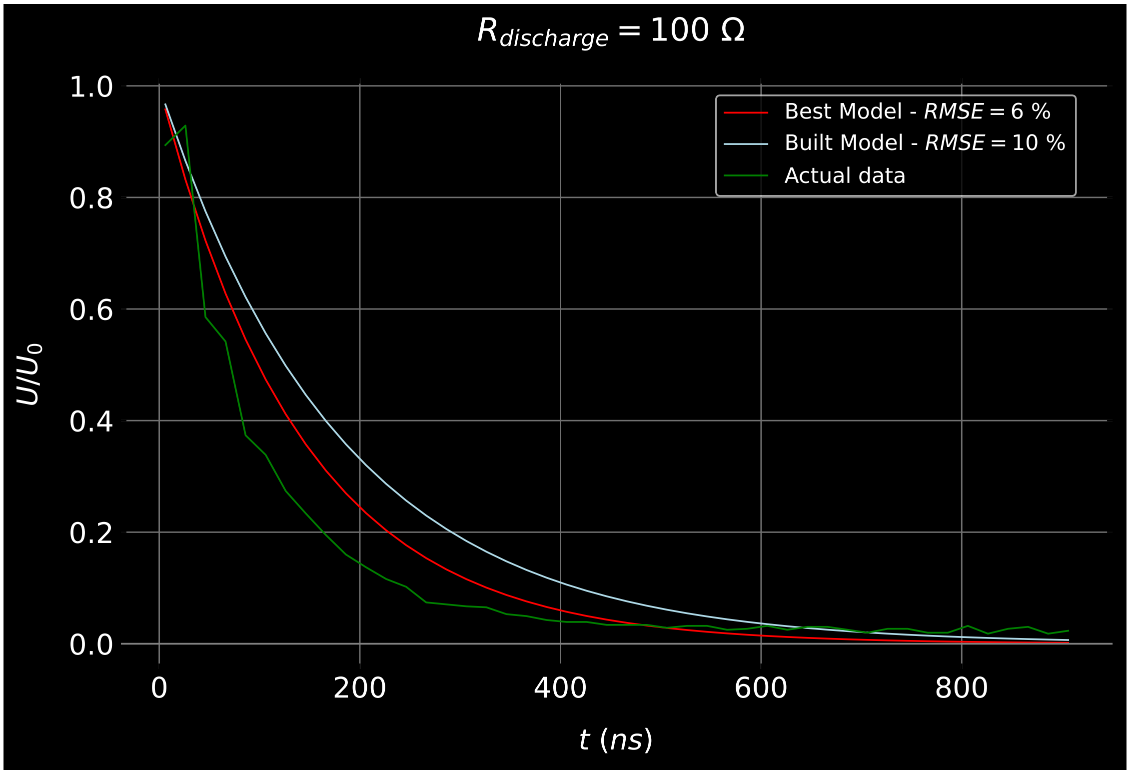


*Figure* 3.2: Comparison of Best model with Built model ($R_{discharge} = 100\ \Omega$).

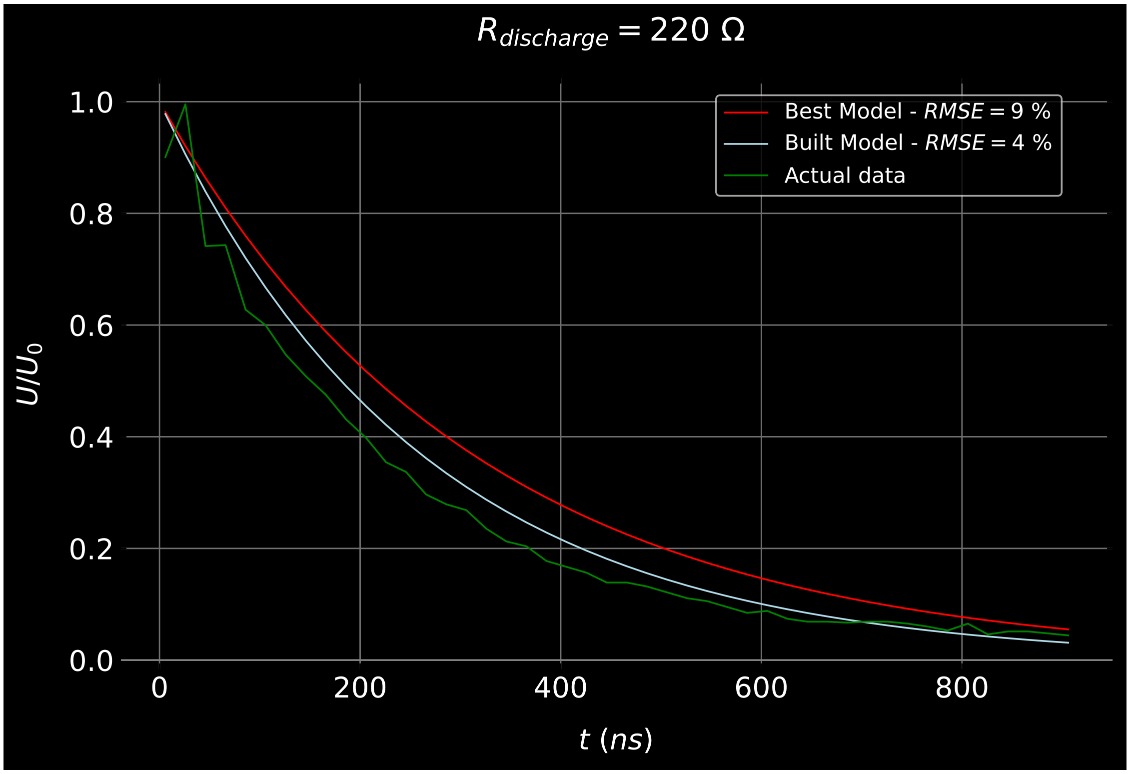


*Figure* 3.3: Comparison of Best model with Built model ($R_{discharge} = 220\ \Omega$).

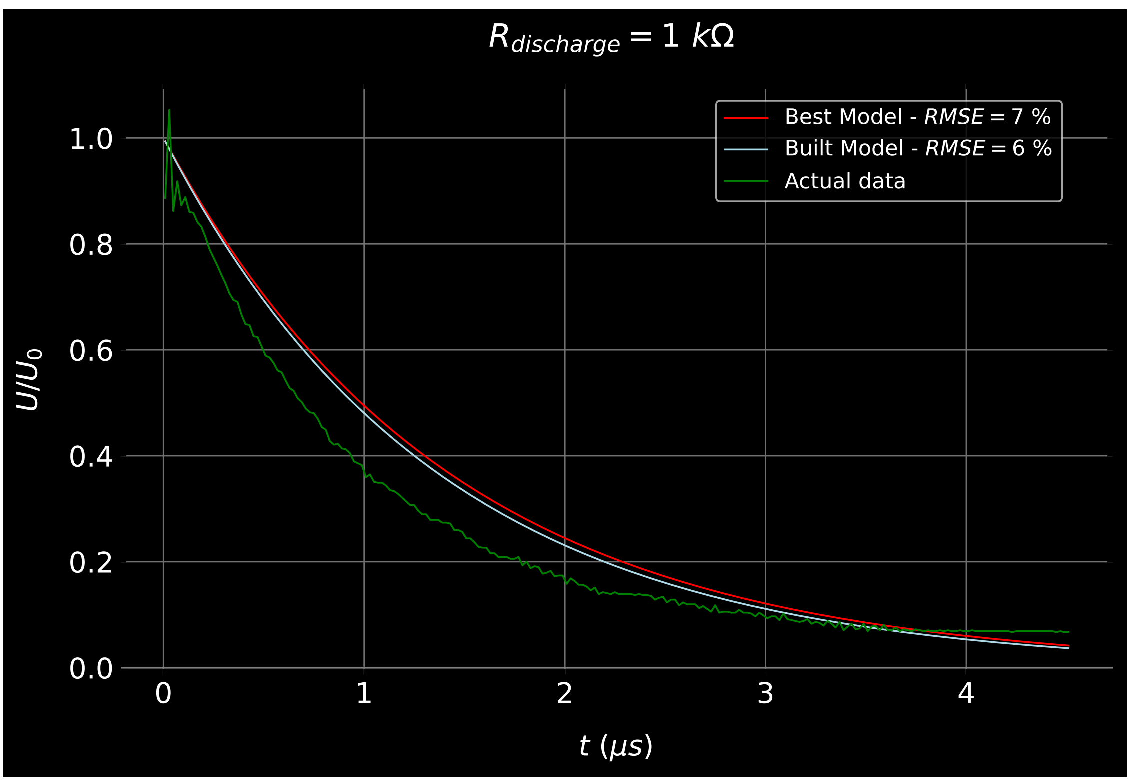

*Figure* 3.4: Comparison of Best model with Built model ($R_{discharge} = 1\ k\Omega$).

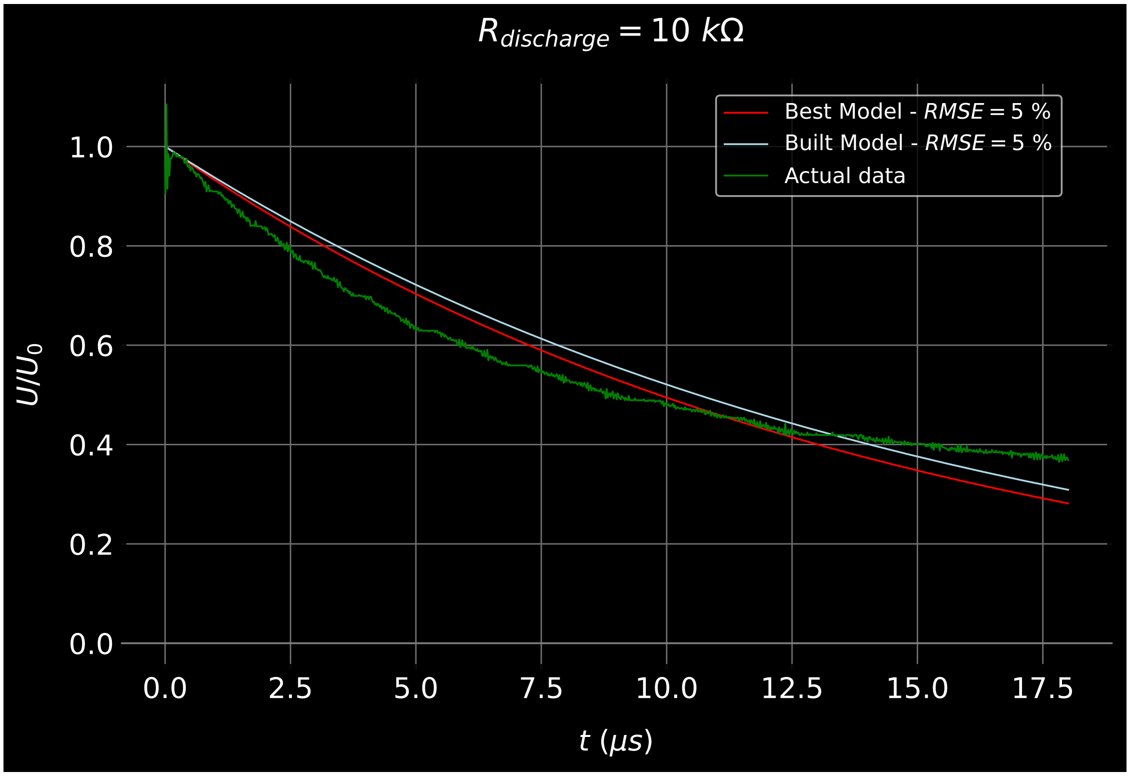


*Figure* 3.5: Comparison of Best model with Built model ($R_{discharge} = 10\ k\Omega$).

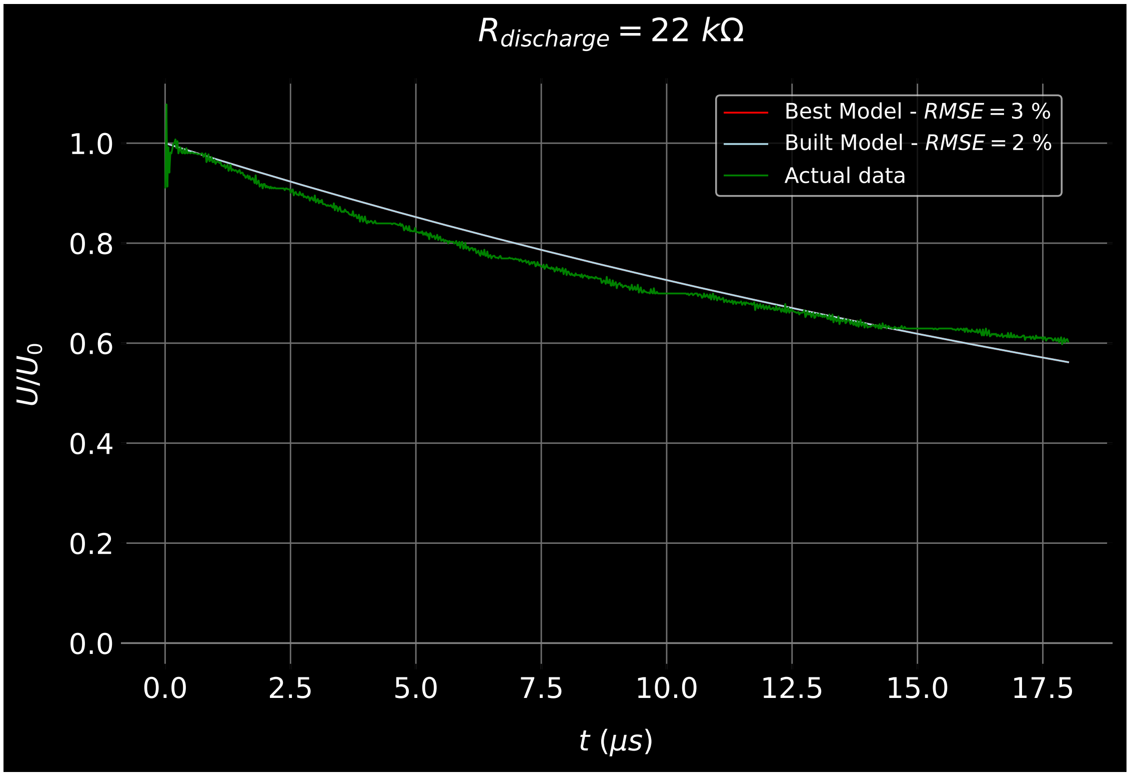


*Figure* 3.6: Comparison of Best model with Built model ($R_{discharge} = 22\ k\Omega$).

***Analysis***

The hypothesis proposed is validated for all values of $R_{discharge}$ except when it is equal to 220 Ω. This abnormal result is unexpected and requires further investigation.

## 1.4 Task 4

### 1.4.1 Compare Time Constant ($\tau$)

**Task Definition**

The goal of task 4 is to calculate and compare the time constant $\tau$ for the best model.

***Theoretical Basis***

The time constant $\tau$ is the amount of time needed for a discharging capacitor to reach 36.8 % of its maximum voltage.

$$\tau = R_{discharge}C \tag{4.1}$$

- $R_{discharge} = 22k\Omega$
- $C_{best} = 1.42nF$

The capacitor in our best model is unable to fully discharge because $R_{charge} = R_{discharge}$.

Instead, the capacitor is only able to reach a minimum value of $U_c = \frac{U_0}{2}$

This is mathematically captured by modifying Eq 1.1 to account for the offset yielding:

$$U_c = \frac{U_0}{2} \exp\left(-\frac{t}{RC}\right) + \frac{U_0}{2} \tag{4.2}$$

- $U_0 = 5V$

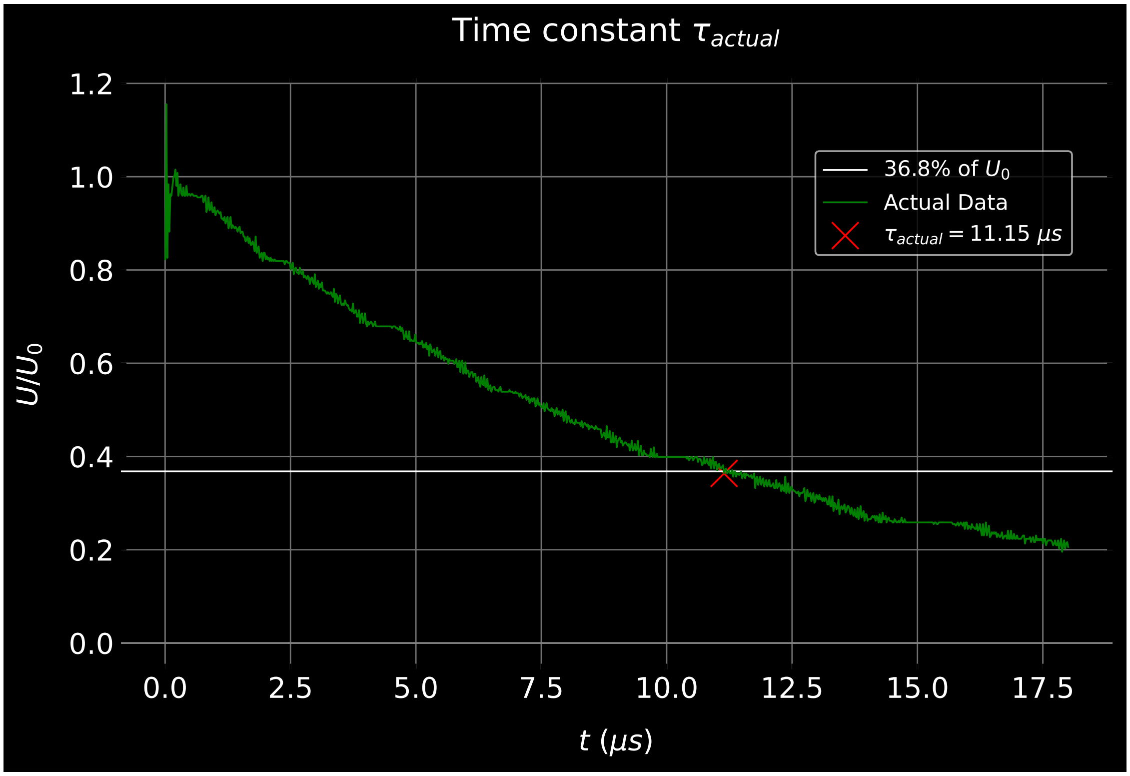


*Figure* 4.1: Determining $\tau_{actual}$. ($R_{discharge} = 22\ k\Omega$).

***Analysis***

1- Using $C_{best}$ and Eq 4.1, the theoretical value of $\tau$ was determined to be $\tau_{Theory} = 31.2\mu s$

2- Using the measurement data of $R_{discharge} = 22k\Omega$ and Eq 4.2, the actual value of $\tau$ was determined be $\tau_{actual} = 11.15\mu s$

3- This significant discrepency between the actual and theoretical values of $\tau$ is likely caused by using Eq 1.1 as the fitting equation instead of Eq 4.2. Hence, the mentioned offset was never considered when building the best model.