O11e Lab Report

May 21, 2024

1 Optical Polarisation

Group #13

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Overview of Tasks

Task 0

Calculate the light intensity I for the following forms of polarised light passing through a linear analyser as a function of the analyser's angle θ with the vertical direction.

Consider:

(a) Linear polarised light without analyser

(b) Circularly polarised light without analyser

(c) Linearly polarised light passing first through a $\frac{\lambda}{4}$ - waveplate before entering the linear analyser.

- Consider the special cases when $\alpha = \phi$, $\alpha = \phi \pm \frac{\pi}{2}$, $\alpha = \phi \pm \pi$ and $\alpha = \phi \pm \frac{\pi}{4}$.

(d) Circularly polarised light passing first through a $\frac{\lambda}{4}$ – waveplate before entering the analyser.

- Cosider the case of $\phi = \pm\frac{\pi}{4}$

Task 1

Experimentally measure the light intensity as a function of the analyser angle for linear polarised light that passed through a $\frac{\lambda}{4}$ – plate under angles of

- $\phi = 0^\circ$
- $\phi = 30^\circ$
- $\phi = 45^\circ$

Compare the measurements to the calculations from the first task by making nonlinear fits to the data or by plotting the theoretical prediction curves.

Task 2

Characterise the state of polarisation of three unknown black boxes in front of a light source using appropriate measurements.

1.1 Task 0: Calculate the light intensity I of polarised light

Task Definition

Calculate the theoretical light intensity I for the following forms of polarised light passing through a linear analyser as a function of the analyser's angle θ with the vertical direction.

(a) collimated LED light source, (b) polariser/black box, (c) $\lambda/4$ – plate, (d) analyser, (e) lens and (f) photodiode.

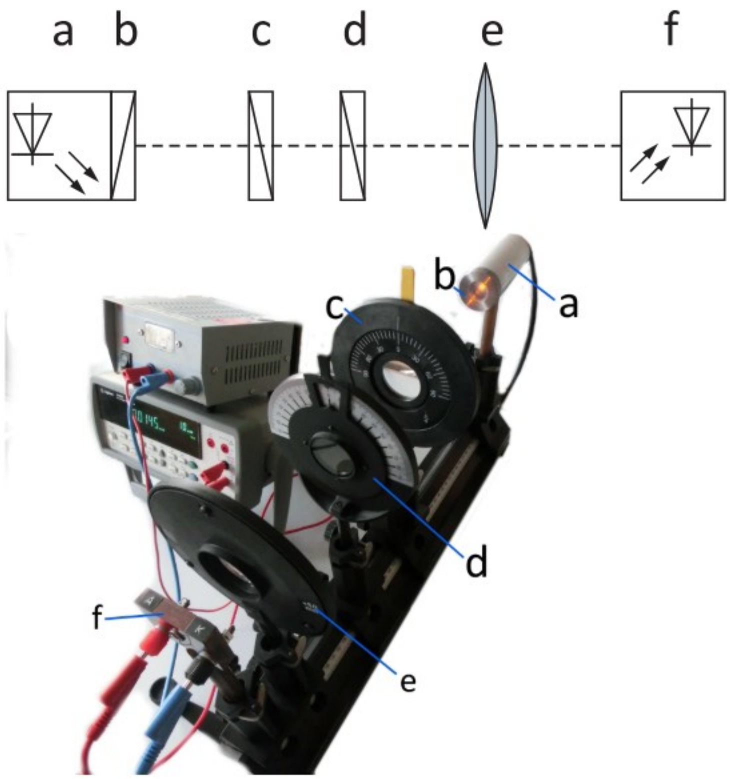

Figure 1.1: Optical Analyser Setup

Figure 1.2: Polarisers

Theoretical Basis

In this experiment, unpolarised light passes through 3 optical elements in the following sequence: (Polariser→Quarter Waveplate→Analyzer)

The theory of Jones Calculas states that electromagnetic waves are represented by Jones Vectors while the effects of optical elements may be characterized by Jones matrices.

Jones Vector

An electromagnetic wave's Jones Vector is given by:

$$J_\alpha = \begin{pmatrix} sin\alpha \\ cos\alpha \end{pmatrix}$$

- α : Angle between polariser's transmission axis and the vertical

The Jones vectors for vertical linearly, right circular and left circularly polarised light are respectively given by:

$$J_y = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \qquad J_+ = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -i \end{pmatrix} \qquad J_- = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ +i \end{pmatrix}$$

Jones Matrices

- Analyser:

$$M_\theta = \begin{pmatrix} sin^2(\theta) & sin(\theta)cos(\theta) \\ sin(\theta)cos(\theta) & cos^2(\theta) \end{pmatrix}$$

- $\frac{\lambda}{4}$- Waveplate:

$$M^\phi_{\lambda/4} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 + icos(2\phi) & -isin(2\phi) \\ -isin(2\phi) & 1 - icos(2\phi) \end{pmatrix}$$

ϕ : Angle between fast axis of waveplate and vertical

θ : Angle between analyzer's transmission axis and vertical

Therefore, the final Jones Vector J_T after passing through all 3 optical elements is given by:

$$J_T = M_\theta M^\phi_{\lambda/4} J_\alpha \tag{1.1}$$

It is also given that the transmitted light intensity:

$$I_T \propto |J_T|^2$$

$$I_T = I_0 |J_T|^2 \tag{1.2}$$

Tranmistted Light intensity I_T

We are asked to compute the general expressions for following cases of $I_T(\theta)$. This may be easily done by applying Eq 1.2.

(a) Linear polarised light with no quarter-waveplate:

$$I_T = I_0 |M_\theta J_\alpha|^2$$

(b) Circularly polarised light with no quarter waveplate:

$$I_T = I_0 |M_\theta J_+|^2$$

$$I_T = I_0 |M_\theta J_-|^2$$

(c) Linearly polarised light passing first through a /4 - plate before entering the linear analyser.

(d) $\alpha = \phi$:

$$I_T = I_0 |M_\theta M_{\frac{\lambda}{4}}^{\alpha} J_\alpha|^2$$

(ii) $\alpha = \phi \pm \pi$:

$$I_T = I_0 |M_\theta M_{\frac{\lambda}{4}}^{\alpha \pm \pi} J_\alpha|^2$$

(iii) $\alpha = \phi \pm \frac{\pi}{4}$:

$$I_T = I_0 |M_\theta M_{\frac{\lambda}{4}}^{\alpha \pm \frac{\pi}{4}} J_\alpha|^2$$

(iv) Circularly polarised light passing first through a /4 – plate before entering the analyser.

(v) $\phi = \pm\frac{\pi}{4}$:

$$I_T = I_0 |M_\theta M_{\frac{\lambda}{4}}^{\pm \frac{\pi}{4}} J_\alpha|^2$$

Procedure

1. The transmission axes of LPOL and the analyzer must first be calibrated without the quarter-waveplate. **Fig 1.2**
2. Unpolarized Light is passed through LPOL which makes it polarised with respect to the transmission axis of LPOL.
3. Subsequently, the polarized light is incident on the analyzer with ($\theta = 0^\circ$), and the light's intensity is measured using a connected voltmeter.
4. LPOL is slowly rotated until the voltmeter reaches its maximum value. This implies that both LPOL and the analyzer have the same transmission axes. ($\theta = \alpha = 0^\circ$)
5. Thereafter, the quarter-waveplate is placed back into the setup, and the angle ϕ is configured to a preset value of either ($0^\circ, 30^\circ$ or 45°) **Fig 1.1**
6. The analyzer's angle θ is varied from (-90° to 90°), and the light's intensity I_T is measured using the voltmeter. This is done for each value of ϕ, and the measurements are used to plot graphs of $I_T(\theta)$.

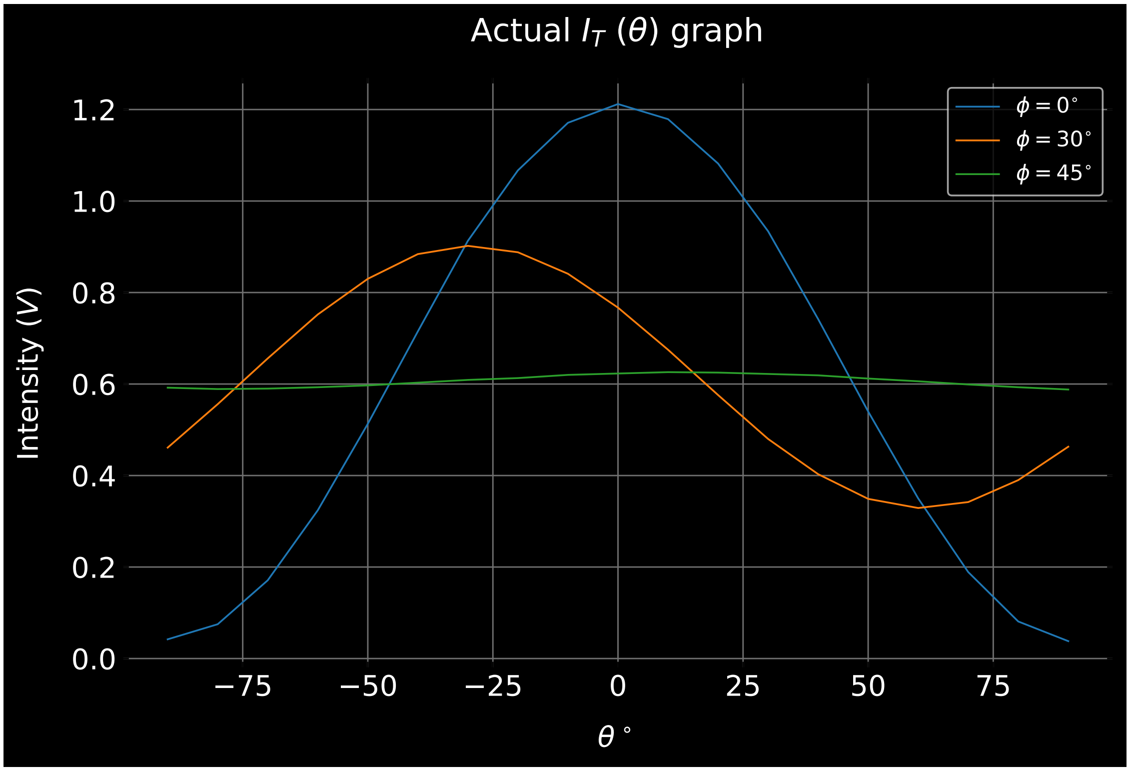

Figure 1.3: Measured I_T vs θ graph

1.2 Task 1

Task Definition

Experimentally measure the light intensity as a function of the analyser angle for linear polarised light that passed through a $\frac{\lambda}{4}$ − plate under angles of $\phi = 0^\circ, 30^\circ$ and 45°.

Theoretical Basis

During this experiment, the value of J_T may be calculated as follows:

$$J_y = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$M_{\lambda/4}^{\phi} = \begin{bmatrix} \frac{\sqrt{2}(i\cos(2\phi)+1)}{2} & -\frac{\sqrt{2}i\sin(2\phi)}{2} \\ -\frac{\sqrt{2}i\sin(2\phi)}{2} & \frac{\sqrt{2}(-i\cos(2\phi)+1)}{2} \end{bmatrix}$$

$$M_\theta = \begin{bmatrix} \sin^2(\theta) & \sin(\theta)\cos(\theta) \\ \sin(\theta)\cos(\theta) & \cos^2(\theta) \end{bmatrix}$$

$$J_T^{\phi} = M_\theta M_{\lambda/4}^{\phi} J_y = \begin{bmatrix} \frac{\sqrt{2}(-i\cos(2\phi)+1)\sin(\theta)\cos(\theta)}{2} - \frac{\sqrt{2}i\sin(2\phi)\sin^2(\theta)}{2} \\ \frac{\sqrt{2}(-i\cos(2\phi)+1)\cos^2(\theta)}{2} - \frac{\sqrt{2}i\sin(2\phi)\sin(\theta)\cos(\theta)}{2} \end{bmatrix}$$

Hence, the generalized fitting function is given by :

$$I_T = A|J_T(\phi)|^2 + B \tag{1.3}$$

- A : Scaling parameter
- B : Intercept parameter

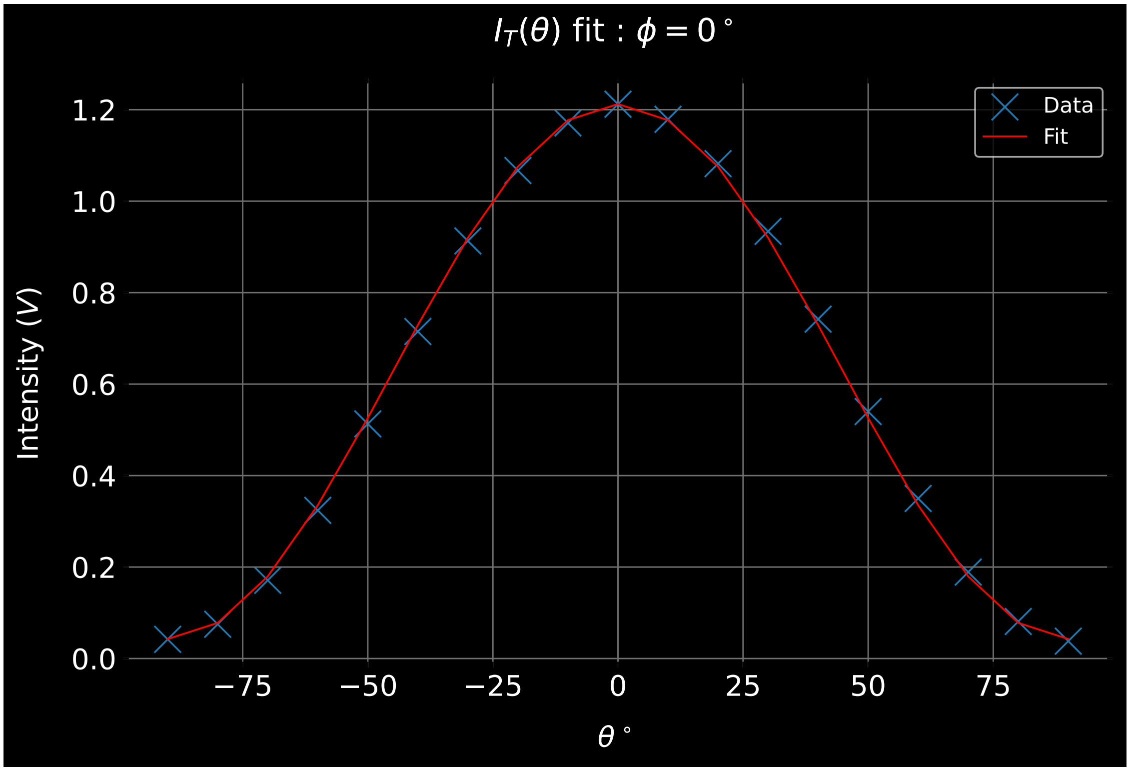

Figure 1.4: Light intensity vs analyser angle fitting for $\phi = 0°$.

$$J_T^0 = \begin{bmatrix} \frac{\sqrt{2}\cdot(1-i)\sin(\theta)\cos(\theta)}{2} \\ \frac{\sqrt{2}\cdot(1-i)\cos^2(\theta)}{2} \end{bmatrix}$$

$$I_T(\theta) = A|J_T^0(\theta)|^2 + B$$

where,

- $A = (1.17 \ \pm 0.006) \ V$

- $B = (0.04 \ \pm 0.004) \ V$

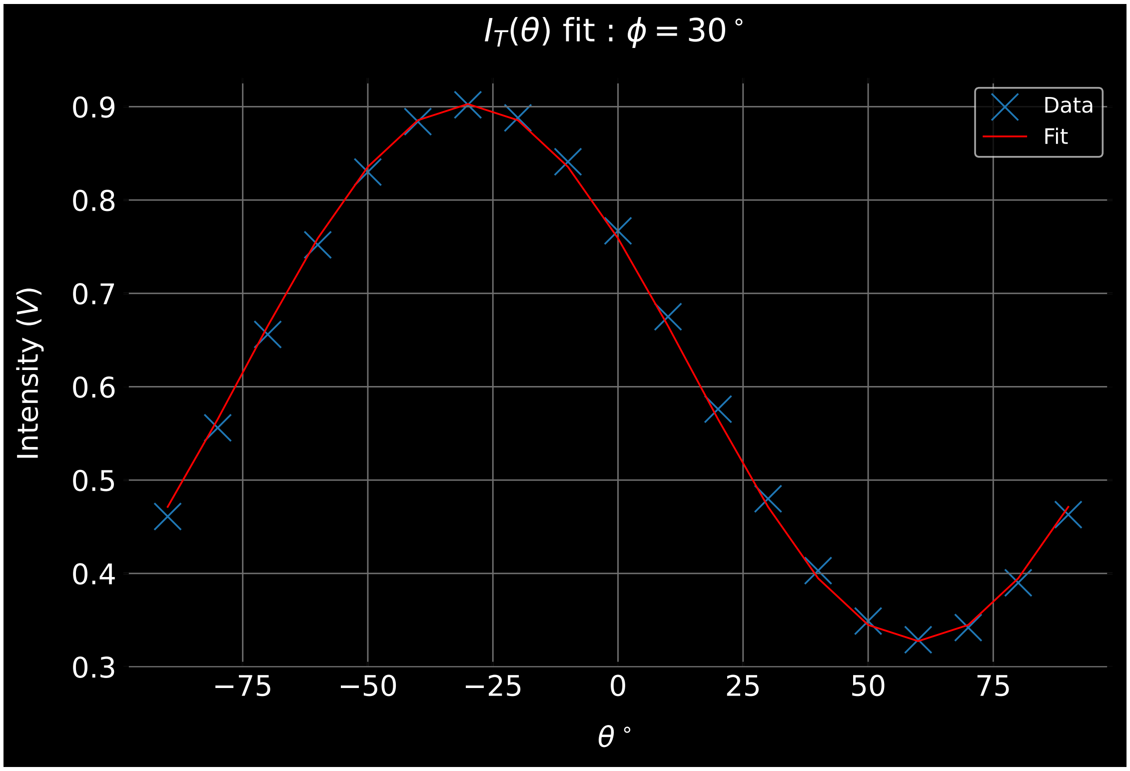

Figure 1.5: Light intensity vs analyser angle fitting for $\phi = 30°$.

$$J_T^{\pi/6} = \begin{bmatrix} 0.433012701892219\sqrt{2}i\sin^2(\theta) + \frac{\sqrt{2}\cdot(1-0.5i)\sin(\theta)\cos(\theta)}{2} \\ 0.433012701892219\sqrt{2}i\sin(\theta)\cos(\theta) + \frac{\sqrt{2}\cdot(1-0.5i)\cos^2(\theta)}{2} \end{bmatrix}$$

$$I_T(\theta) = A|J_T^{\pi/6}(\theta)|^2 + B$$

where,

- $A = (1.15\ \pm 0.010)\ V$

- $B = (0.04\ \pm 0.005)\ V$

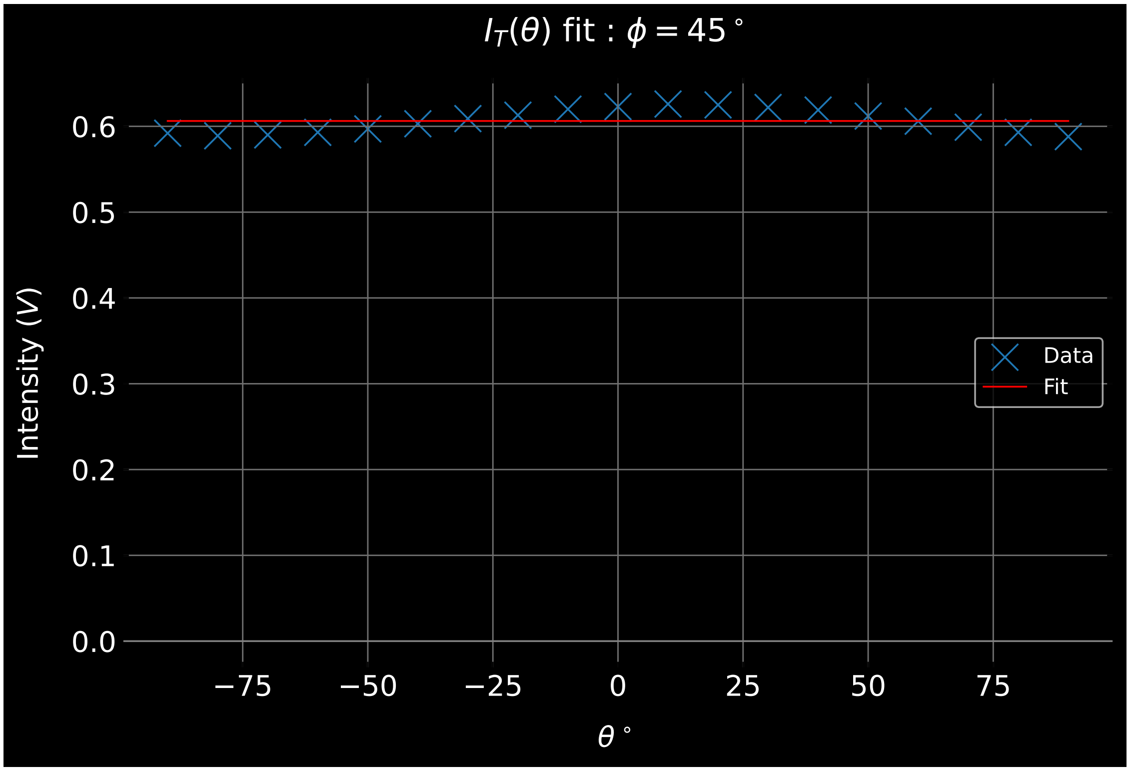

Figure 1.6: Light intensity vs analyser angle fitting for $\phi = 45°$.

$$J_T^{\pi/4} = \begin{bmatrix} -0.5\sqrt{2}i\sin^2(\theta) + \frac{\sqrt{2}\cdot(1-6.12323399573677\cdot 10^{-17}i)\sin(\theta)\cos(\theta)}{2} \\ -0.5\sqrt{2}i\sin(\theta)\cos(\theta) + \frac{\sqrt{2}\cdot(1-6.12323399573677\cdot 10^{-17}i)\cos^2(\theta)}{2} \end{bmatrix}$$

$$I_T(\theta) = A|J_T^{\pi/4}(\theta)|^2 + B$$

where,

- $A = (1.00 \ \pm 59209273773809.164)\ V$

- $B = (0.11 \ \pm 29604636103271.688)\ V$

Analysis

- It has been observed that the actual measurements are accurately represented by the theoretical fitting functions derived from Jones vectors and matrices.
- A significant uncertainty is observed in parameters A and B corresponding to $\phi = \frac{\pi}{4}$. This is unusual and caused by python's inability to make a zero approximation for the imaginary component in $J_T^{\frac{\pi}{4}}(\theta)$. This leads to a failure to accurately determine the uncertainty of the fitting parameters.

Sources of Errors

Multiple sources of error were observed during the experiment which contributed to uncertainty in the recorded measurements of I_T.

- The room's brightness introduced a systemic error which was measured to be $\approx +0.004V$
- The voltmeter's resolution was $0.001V$ which introduced an error of $\pm 0.0005V$
- The optical analyser's resolution was 1° which introduced an error of $\pm 0.5^\circ$
- The quarter waveplate's resolution was 5° which introduced an error of $\pm 2.5^\circ$

Theerefore, the room's brightness is the largest contributor to the relative error in I_T.

1.3 Task 2

Task Definition

Characterise the state of polarisation of three unknown black boxes in front of a light source using appropriate measurements.

Theoretical Basis

1. The procedure in Task 0 is repeated using unknown polarisers (BBC 1-3).
2. Measurements are carried out similar to Task 1.
3. The data is analysed using **Fig2.1** to determine the state of polarisation of the unknown polarisers.

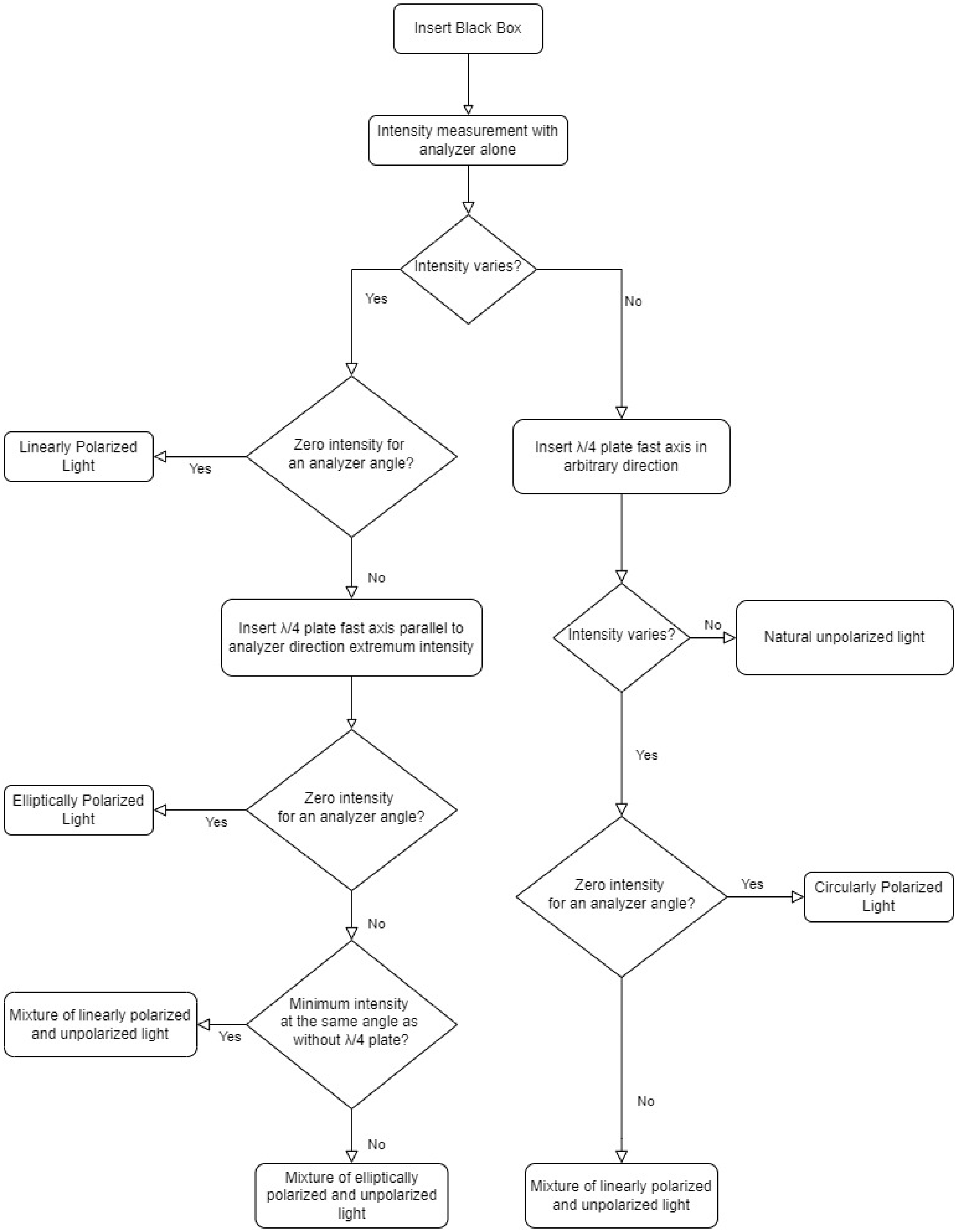

Figure 2.1: Black Box polarization determination flowchart.

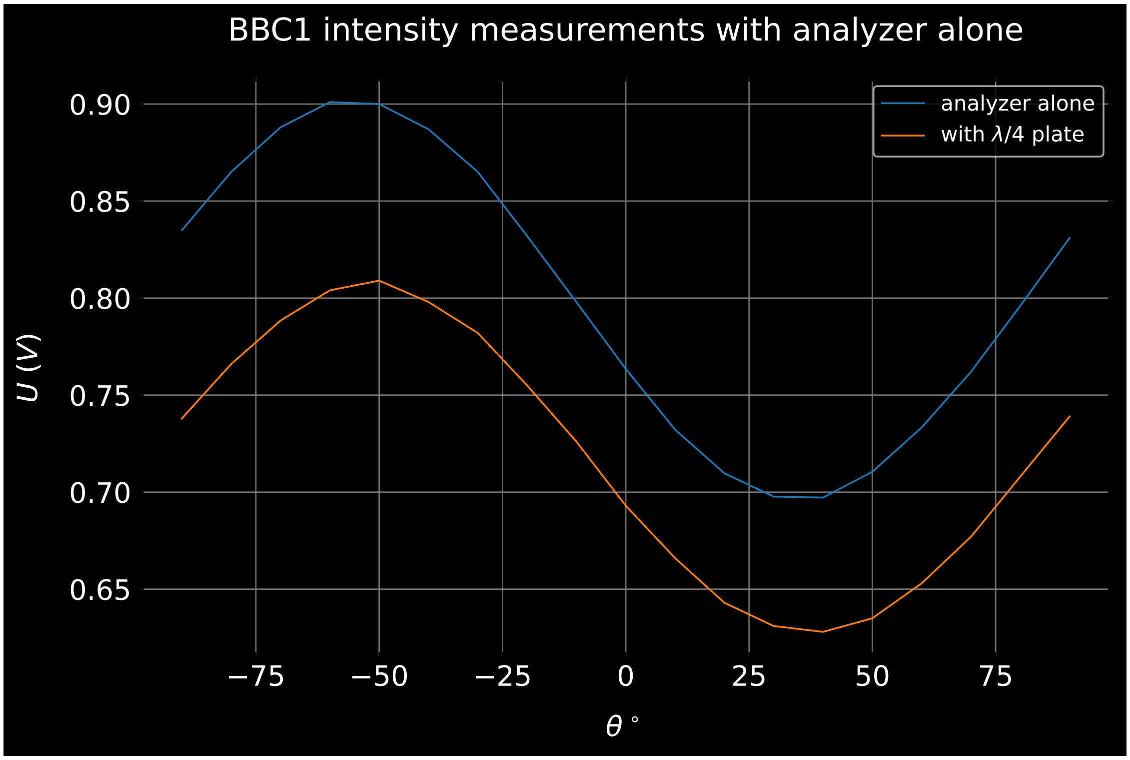

Figure 2.2: BBC1 Light intensity vs analyser angle data plot.

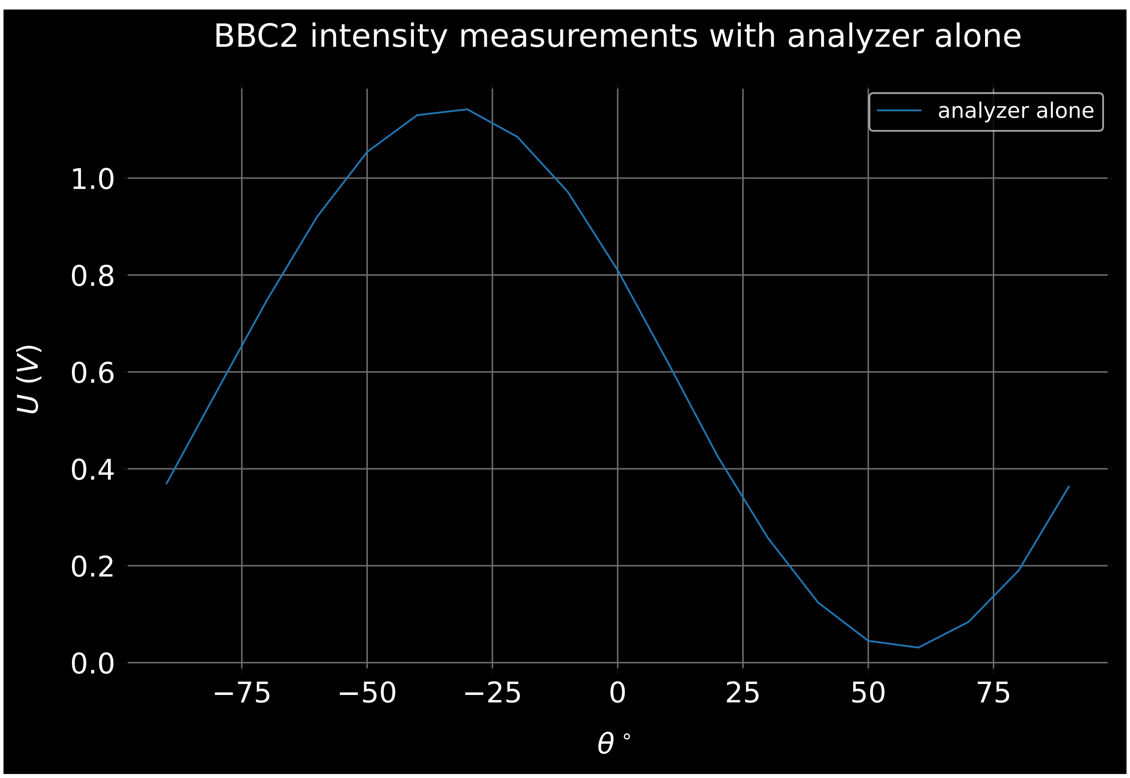

Figure 2.3: BBC2 Light intensity vs analyser angle data plot.

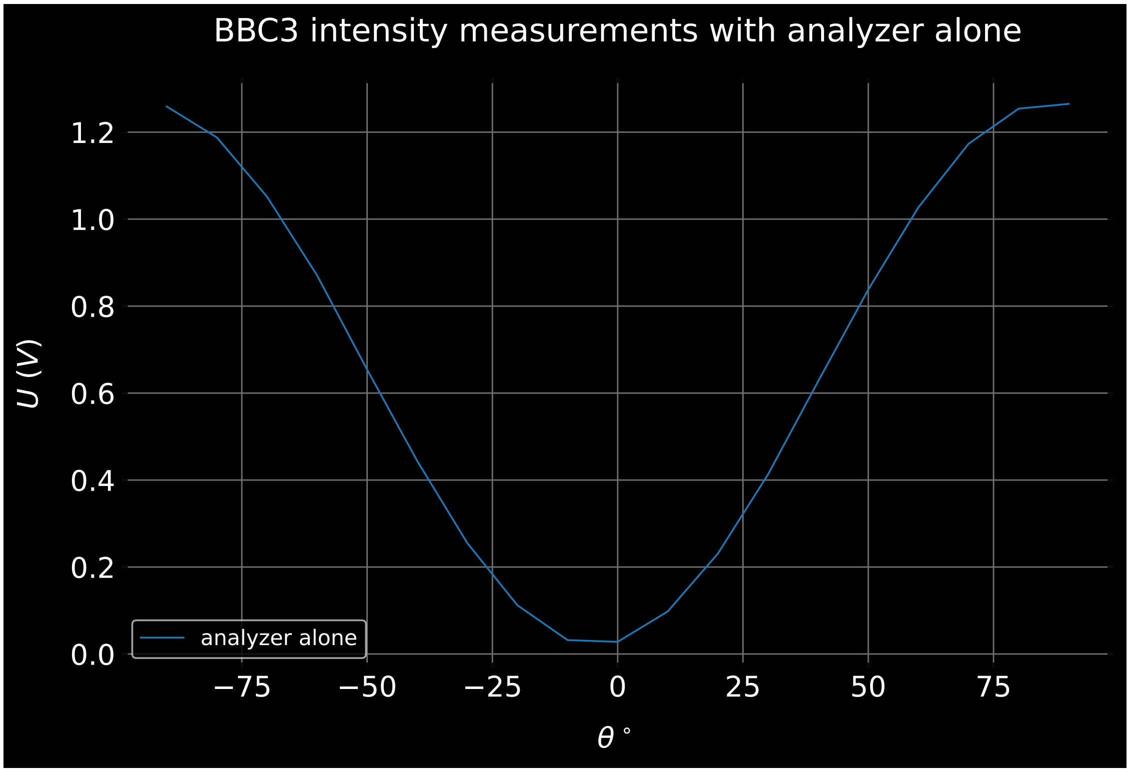

Figure 2.4: BBC3 Light intensity vs analyser angle data plot.

Analysis

BBC1:

Measurements: - Intensity varies with analyzer alone, and non-zero minimum observed at $\theta = 37^{\circ}$ - Minimum intensity observed at the same angle with $\lambda/4$ waveplate

Conclusion: - BBC1 Polariser produces mixture of linearly polarized and unpolarized light.

BBC2 and BBC3:

Measurements:

- Using analyser alone, the measured Intensity ($I_T = 0$) at $\theta = 60^{\circ}$ and $\theta = -5^{\circ}$ for BBC2 and BBC3 polarisers respectively.

Conclusion: - BBC2&3 Polariser produces linearly polarised light.

References

1) O11e Lab instruction