E19e

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1 E19e Digital Oscilloscope

Group #13

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Overview of Tasks

1. Basics

1a. Measure a sine signal with a fixed range of $\pm 1\ V$, using a resolution of 8 bit, 12 bit and 14 bit. Determine the size of the discretization steps from the curves and compare with the nominal resolution.

1b. Measure the spectrum of a sine signal with a resolution of 14 bit using a Blackman window. Save the spectrum both with dbV units and on a linear scale. From the spectrum with linear scale, calculate the spectrum in dbV units and compare to the spectrum stored directly with a dbV scale.

2. Aliasing

2a. Choose oscilloscope parameters such that you have a spectrum with maximum frequency of 100 kHz, Blackman window and 16384 points. Measure the spectra of sine signals with frequencies of 10 kHz, 90 kHz, 110 kHz, 190 kHz, 210 kHz and 290 kHz.

What do you observe? From your observation, derive an equation for the apparent frequency as a function of the real generator frequency.

2b. Choose parameters such that you have a spectrum with maximum frequency of 100 kHz, Blackman window and 16384 points. Measure the spectra of square signals with frequencies of 5 kHz and 7 kHz.

Why do the spectra look so different? Measure the peak heights of the 10 peaks in the 5 kHz spectrum and of the leading 20 peaks in the 7 kHz spectrum. Plot the peak heights vs. frequency and compare with the theoretical expression. What do you observe? Explain.

3. Windowing

3a. Measure the spectra of a square signal using a rectangular, a Hann and a flat-top window. Use the following parameters: time constant of 1 ms, 20 kS, 16384 points in the spectrum and a frequency of 1 kHz.

Compare the spectra. What do you observe? Which windowing method has the worst amplitude resolution? Which windowing method has the worst frequency resolution?

3b. Measure the spectra of a sine signal using a rectangular, a triangular and a flat-top window. Use parameters such that you can see a distinct Fraunhofer pattern in case of the rectangular window. Compare the theoretical expressions for rectangular and triangular windowing quantitatively to the data.

4. Creating and analyzing a WAV file

Create a WAV file from an arbitrary function. Feed this sound into the oscilloscope and do a spectral analysis.

1.1 Task 1

1.1.1 Task 1a

Task Definition

Measure a sine signal with a fixed range of $\pm 1\ V$, using a resolution of 8 bit, 12 bit and 14 bit. Determine the size of the discretization steps from the curves and compare with the nominal resolution.

Theoretical basis

The discretization step is defined as:

$$\Delta V = \frac{V_{max} - V_{min}}{2^{N-1}}$$

where N is the number of bit resolution.

The interval is divided by the 2^{N-1} instead of 2^N to account the least significant bit(LSB) of digitalization [3].

Procedure

1. Each sine wave is analyzed by taking the difference between each consecutive point and plotting a histogram.
2. The bin corresponding to '0' value was discarded as noise.
3. The expected value of the discretization step is taken as the highest count bin.

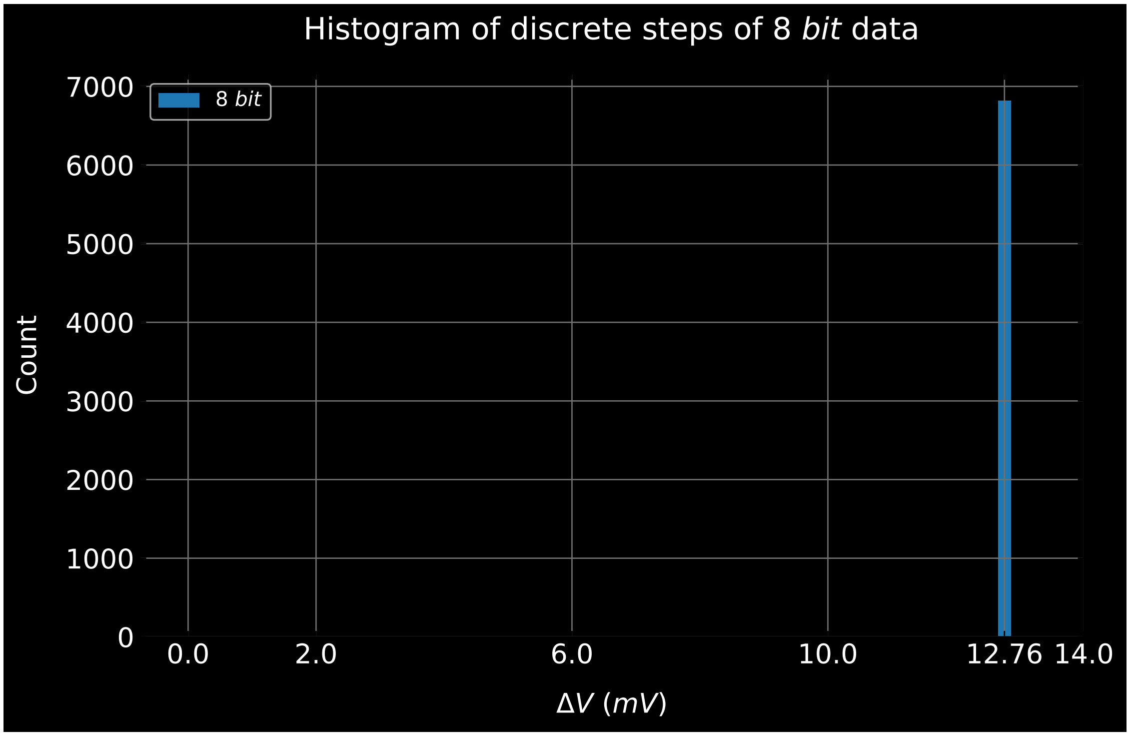


Figure 1.1: Histogram of differencies of 8 *bit* resolution sine wave.

Theoretical value of discretization:

$$\Delta V_{8\ bit} = 12.54\ mV$$

Computed value of discretization:

$$\Delta \tilde{V}_{8\ bit} = 12.76\ mV$$

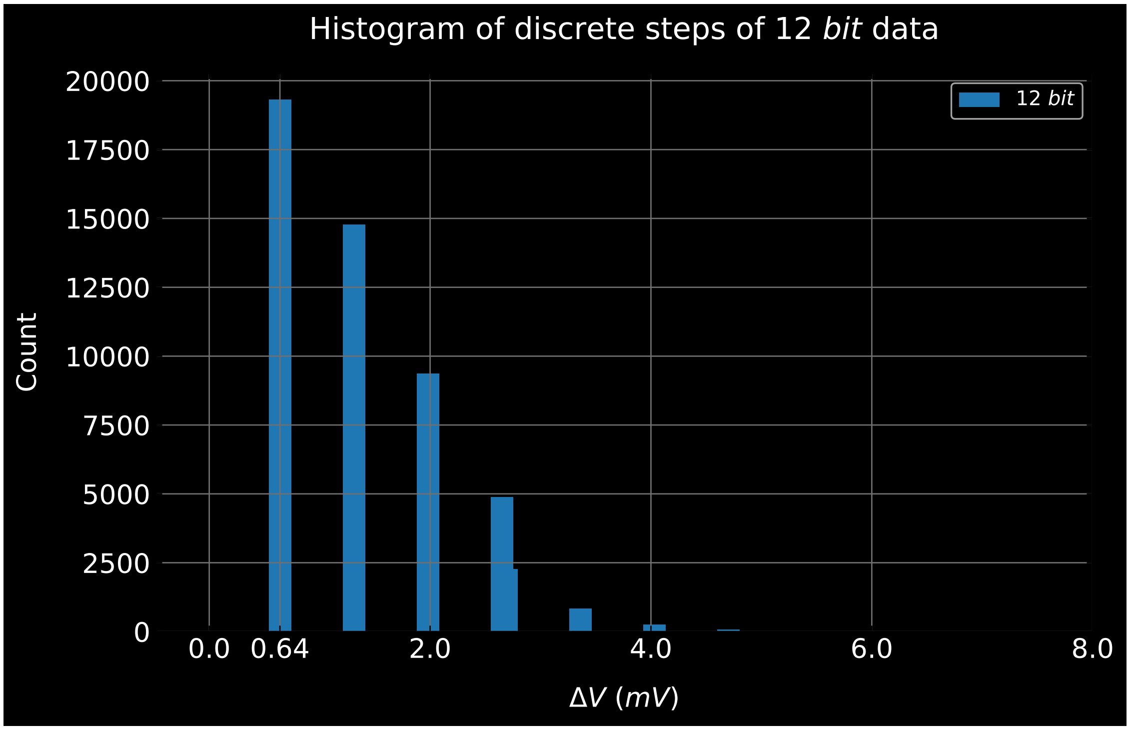


Figure 1.2: Histogram of differencies of 12 *bit* resolution sine wave.

Theoretical value of discretization:

$$\Delta V_{12\ bit} = 0.79\ mV$$

Computed value of discretization:

$$\Delta \tilde{V}_{12\ bit} = 0.64\ mV$$

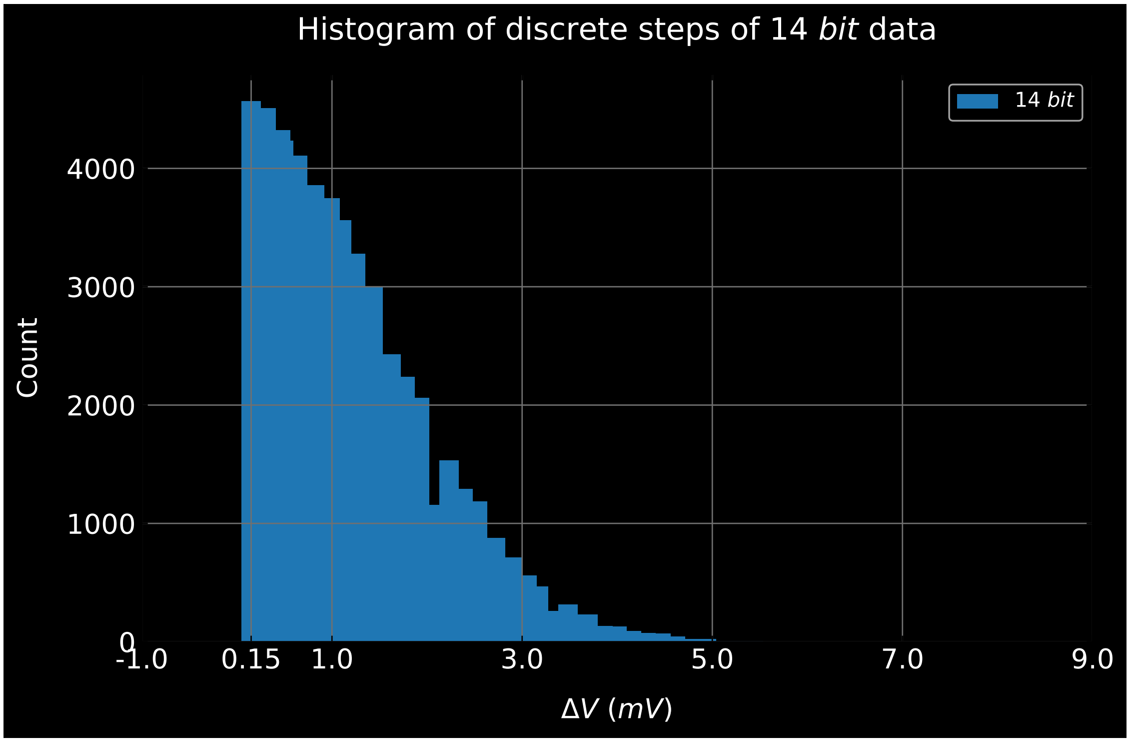


Figure 1.3: Histogram of differencies of 14 *bit* resolution sine wave.

Theoretical value of discretization:

$$\Delta V_{14\ bit} = 0.20\ mV$$

Computed value of discretization:

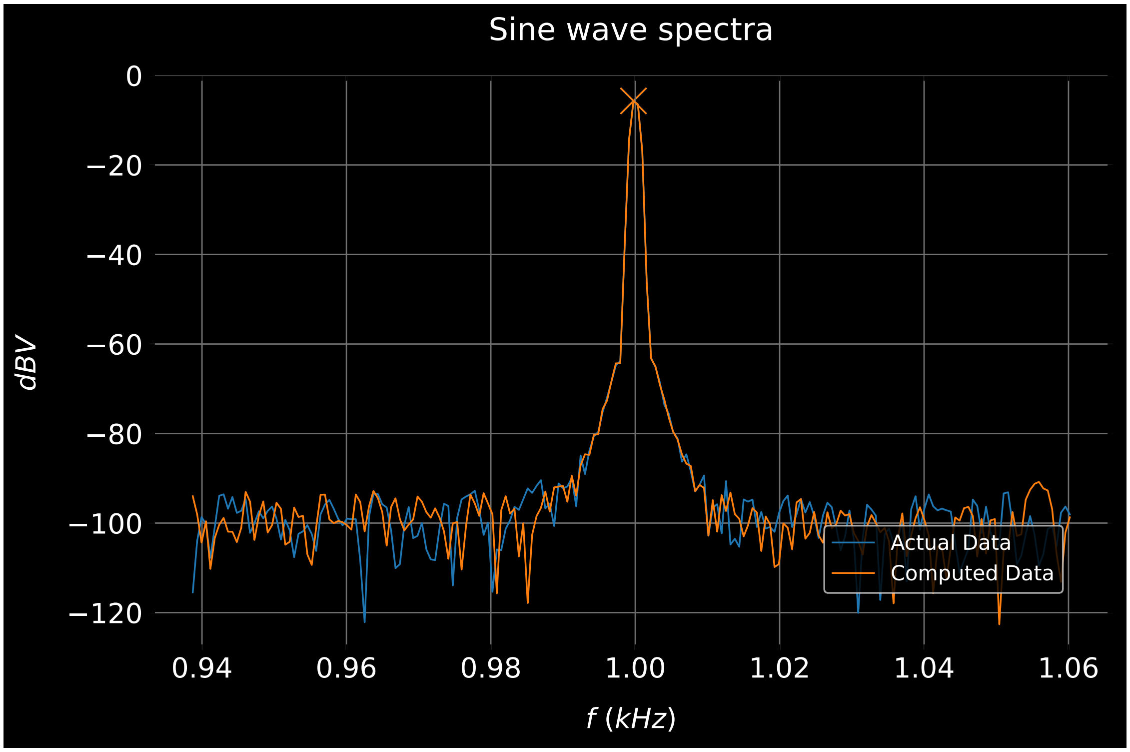
$$\Delta \tilde{V}_{14\ bit} = 0.15\ mV$$

1.1.2 Task 1b

Task Definition

Measure the spectrum of a sine signal with a resolution of 14 bit using a Blackman window. Save the spectrum both with dBV units and on a linear scale. From the spectrum with linear scale, calculate the spectrum in dBV units and compare to the spectrum stored directly with a dBV scale.

$$L = 20 \log_{10} \frac{V}{V_0}$$



6.1

Figure 1.4: Spectra of sine wave in logarithmic scale.

Frequency from computed data:

$$\tilde{f} = 1.00\ kHz$$

Frequency from actual data:

$$f = 1.00\ kHz$$

1.2 Task 2

1.2.1 Task 2a

Task Definition

- Choose oscilloscope parameters such that the frequency spectrum has a maximum frequency of 100 kHz, Blackman window and 16384 points. Measure the spectra of sine signals with frequencies of 10 kHz, 90 kHz, 110 kHz, 190 kHz, 210 kHz and 290 kHz.
- What do you observe? From your observation, derive an equation for the apparent frequency as a function of the real generator frequency.

Measurements

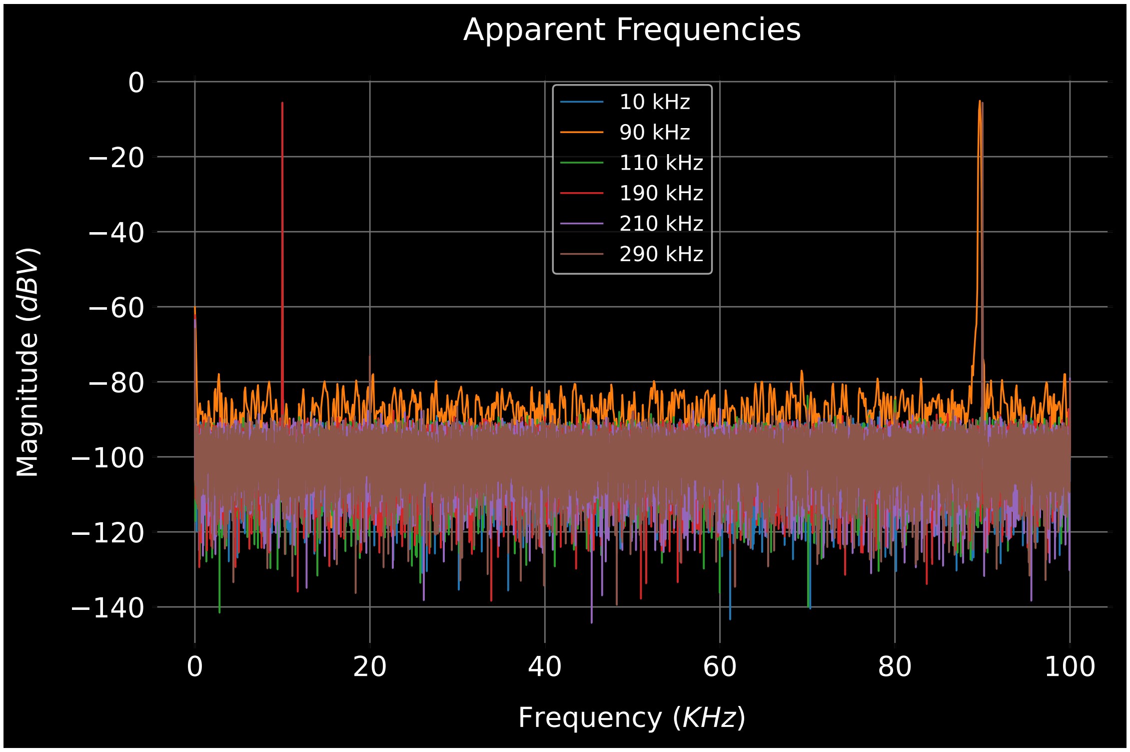


Figure 2.1: Spectra of sine wave with different frequencies.

#	Generator Freq f_g (kHz)	Apparent Freq f_p (kHz)
1	10	10
2	90	90
3	110	90
4	190	10
5	210	10
6	290	90

Analysis

1. During the experiment, a sampling freqeuncy of $f_s = 200\ kHz$ was used. Hence, the Nyquist Frequency is $f_c = 100\ kHz$
2. This means that all generated frequencies $f_g > f_c$ are aliased as per Nyquist theorem.
3. By observation of the measured values of f_p and Nyquist Theorem [2], the equation of the apparent frequency f_p as a function of the real generator frequency f_g is given by:

$$f_p = f_g - f_s \cdot NINT\left(\frac{f}{f_s}\right)$$

- f_p : Apparent frequency

- f_g : Real Generator frequency
- f_s : Sampling Freqeuency
- $NINT\left(\frac{f}{f_s}\right)$: Nearest Interger Function of $\frac{f}{f_s}$

1.2.2 Task 2b

Task Definition

- Choose parameters such that you have a spectrum with maximum frequency of 100 kHz, Blackman window and 16384 points. Measure the spectra of square signals with frequencies of 5 kHz and 7 kHz.

- Why do the spectra look so different? Measure the peak heights of the 10 peaks in the 5 kHz spectrum and of the leading 20 peaks in the 7 kHz spectrum. Plot the peak heights vs. frequency and compare with the theoretical expression. What do you observe? Explain.

Measurements (Square Signal Spectra)

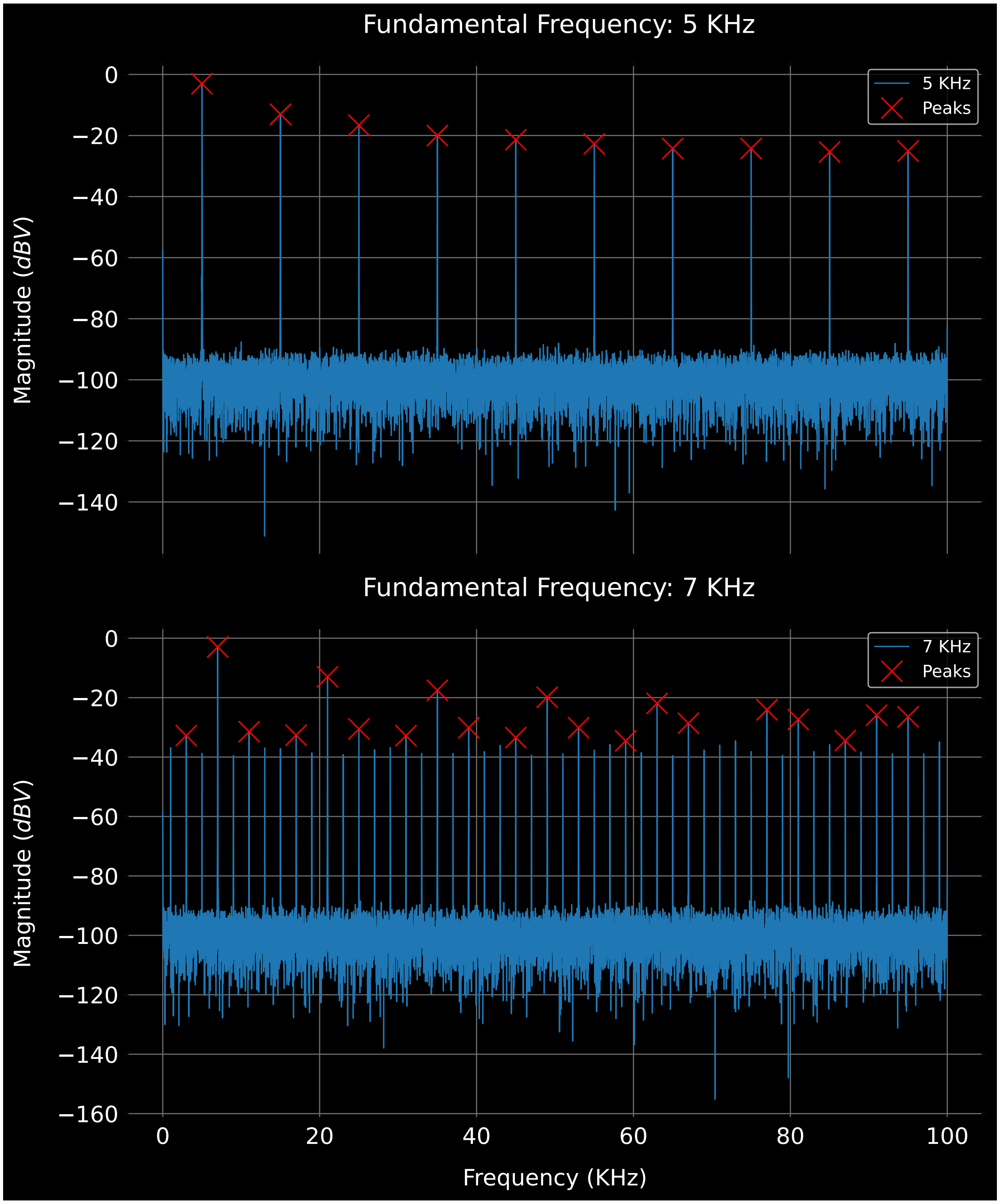


Figure 2.2: Spectra of the square signal with leading peaks.

Analysis of Frequency Spectra

1. During the experiment, two square signals with fundamental frequencies of 5 kHz and 7 kHz were analyzed using a Picoscope 5000.

2. Theoretically, a square signal is composed of a sum of sine waves at the fundamental frequency

and all its odd harmonics.

- An odd harmonic is a component frequency that is an odd integer multiple of the fundamental frequency.

3. Hence, it is expected that the frequency spectrum of a 5 kHz signal contains frequencies at (5, 15, 25, 35, 45, ...) kHz, while the spectrum of a 7 kHz signal contains frequencies at (7, 21, 35, ...) kHz.
4. A sampling rate $f_s = 200\ kHz$ was used, this means we have a Nyquist freqeuncy $f_c = 100\ kHz$. Frequencies above 100 kHz experience aliasing according to the Nyquist-Shannon sampling theorem.
5. For the 5 kHz signal, aliasing causes frequencies above 100 kHz to fold back, overlapping with the first 10 peaks.
6. For the 7 kHz signal, there is less overlap due to the higher fundamental frequency, resulting in more visible peaks in the frequency spectrum.

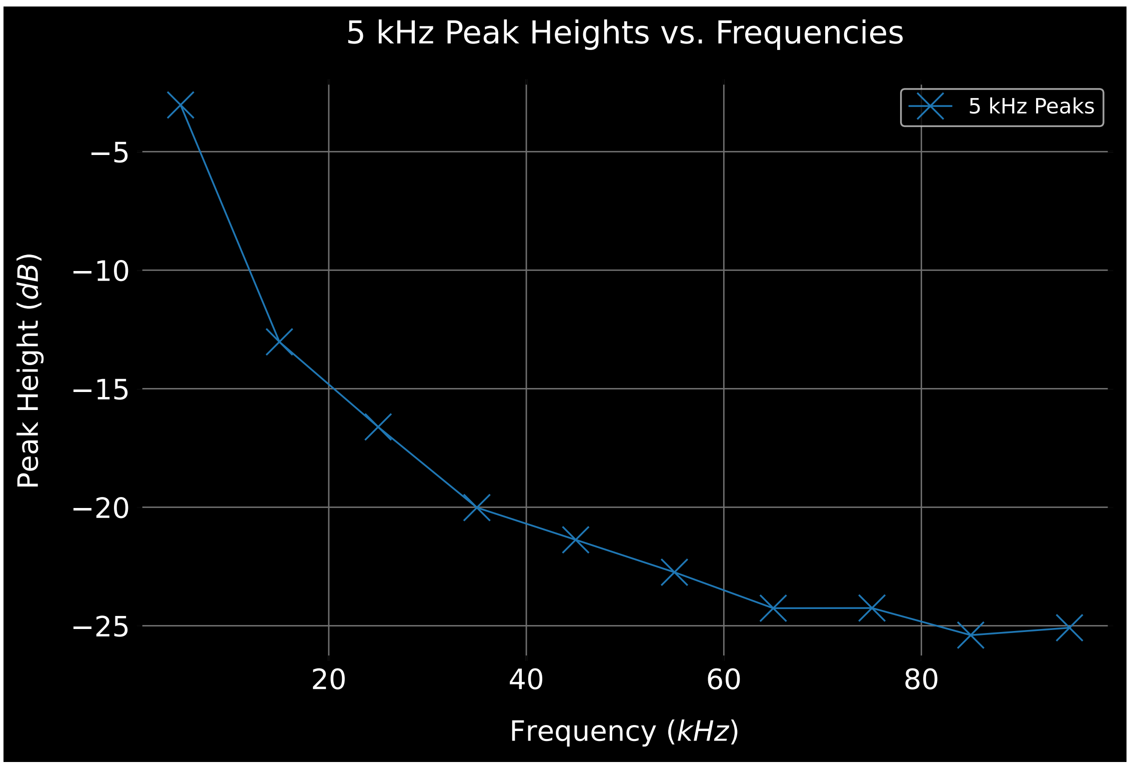


Figure 2.3: Peak Heights vs. Frequencies plot of the square wave with 5 kHz.

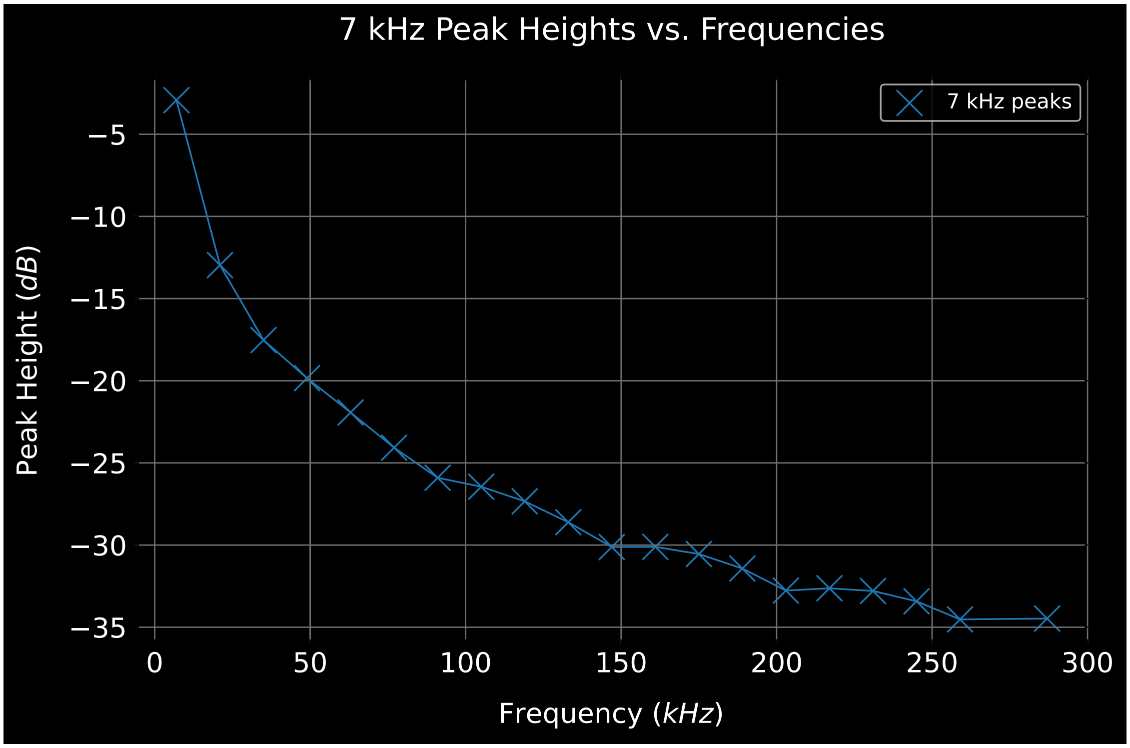


Figure 2.4: Peak Heights vs. Frequencies plot of the square wave with reconstructed original frequencies.

Analysis of Peak Height Graphs

1. Theoretically, the amplitude of the n-th harmonic is given by:

$$\frac{A_F}{n} \tag{2.1}$$

- n: Odd Interger
- A_f: Amplitude or Peak Height of Fundamental Freqeuncy

2. Therefore, it is expected that the peak heights will decay like $\frac{1}{f}$ as f increases.

3. From the peak height vs. frequency graphs, it is observed that both graphs exhibit the expected $\frac{1}{f}$ decay.

11.1

1.3 Task 3

1.3.1 Task 3a

Task Definition

Measure the spectra of a square signal using a rectangular, a Hann and a flat-top window along with the following parameters.

Use the following parameters: Time constant of 1 ms, 20 kS, 16384 points in the spectrum and a frequency of 1 kHz

Compare the spectra. What do you observe?

Explain which windowing method has the worst amplitude resolution, and which has the worst frequency resolution.

Measurements

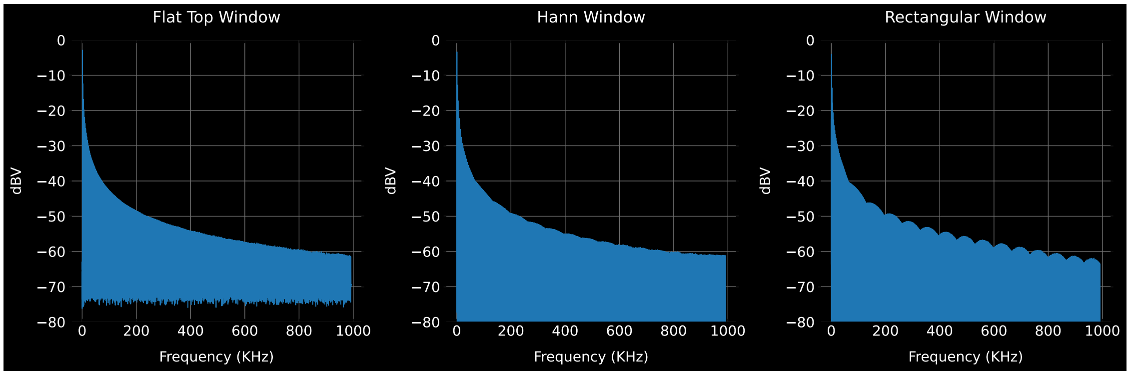


Figure 3.1: Comparison of the amplitude of the square wave with different windowing.

Amplitude Resolution Analysis

1. It is observed that the rectangular window has the worst amplitude resolution while the Flat Top window has the best amplitude resolution.
2. This is because the side lobes are most visible using the Rectangular window, and gets progressively less visible towards the left.
3. Theoretically, the presence of side lobes indicates leakage of signal energy from the main lobe to the adjacent frequency bins. The leakage causes the amplitude of the main lobe to decrease, and leads to ampltude distortion and inaccuracies in signal ampltude estimation.
4. Therefore, more visible side lobes indicates a lower ampltiude resolution because they make it difficult to accurately measure signal amplitudes.

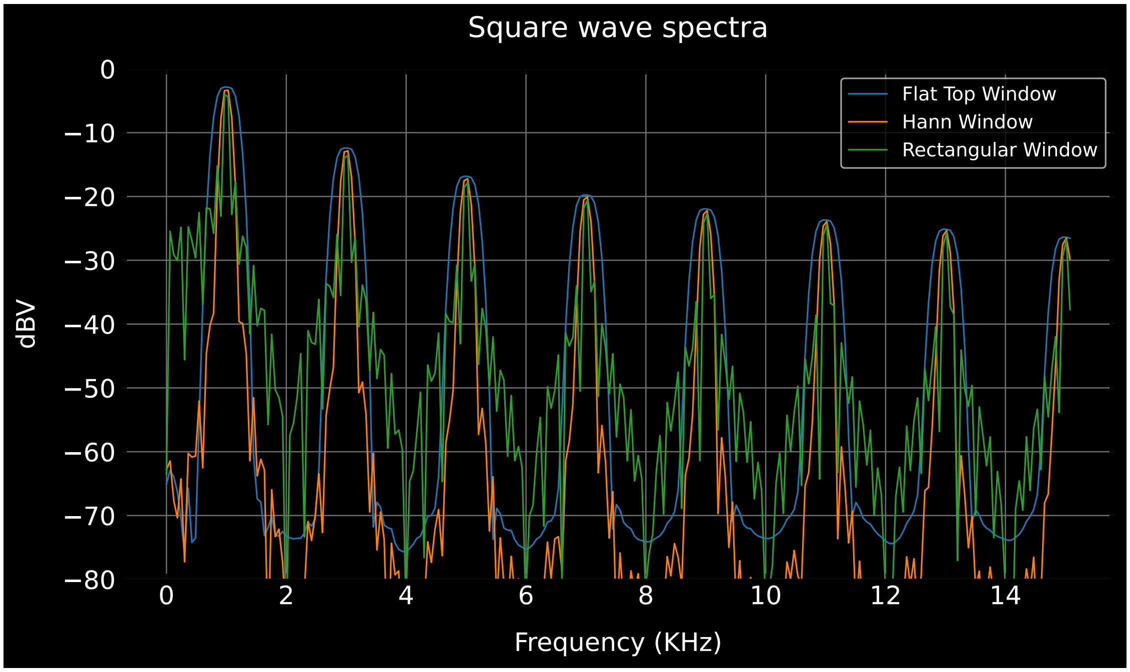


Figure 3.2: Comparison of the frequency resolution of the square wave with different windowing.

Frequency Resolution Analysis

1. It is observed that the Flat-Top window has the worst frequency resolution, whereas the rectangular window has the best frequency resolution.
2. This is because the Flat-Top window generates the most broad peaks whilst the Rectangular window generates the narrowest peaks.
3. In theory,an ideal Fourier Transform would yield sharp signal peaks resembling Dirac Delta Distributions. However, achieving this requires sampling the signal for an infinitely long duration of time, which is impractical.
4. Consequently, windowing functions which yield sharp peaks resembling Dirac Delta Distributions offer a greater frequency resolution, while those with broader peaks offer a poorer freqeuncy resolution.

13.1

1.3.2 Task3b

Task Definition

Measure the spectra of a sine signal using a rectangular, a triangular and a flat-top window. Use parameters such that you can see a distinct Fraunhofer pattern in case of the rectangular window. Compare the theoretical expressions for rectangular and triangular windowing quantitatively to the data.

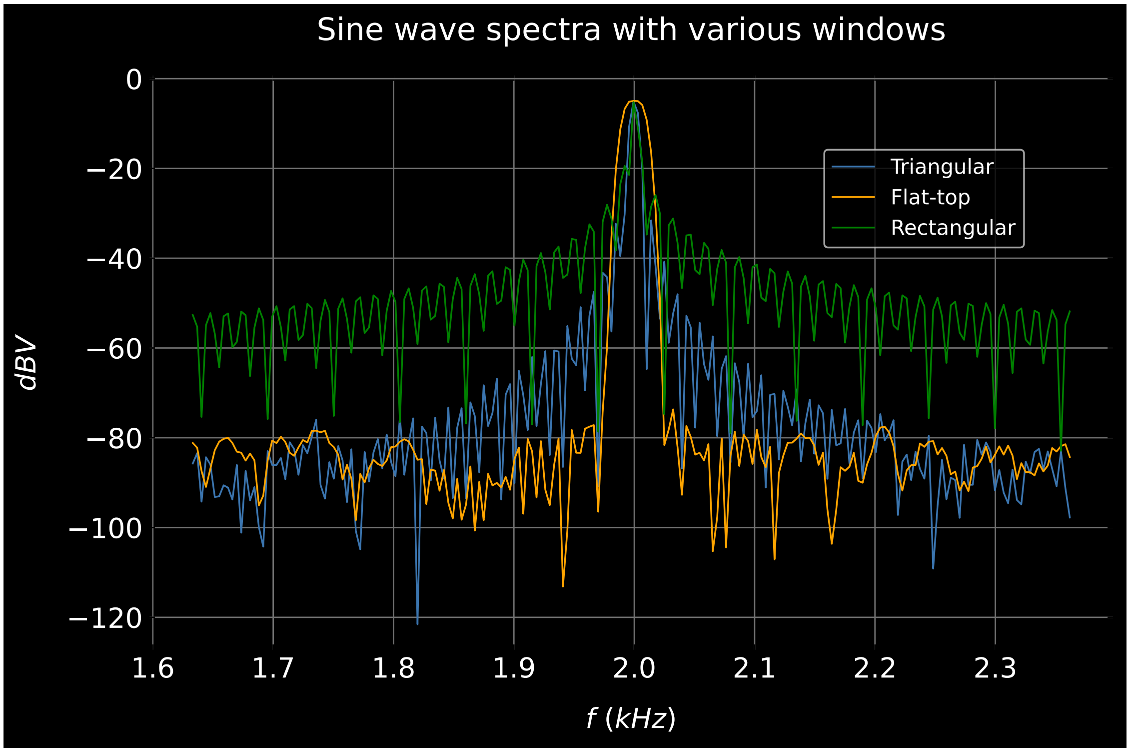


Figure 3.4: Spectra of sine wave with different windowing.

Analysis

For effective comparison of different window functions, several key properties are considered. One of them is the main lobe width, which describes the shape of the central response peak.

The main lobe width is defined as the extent (in FFT bins or frequency lines) where the window response falls to a specific level relative to the peak gain. The -6 dB main lobe width is analysed, which corresponds to a window response of half the peak value (0.5). [4]

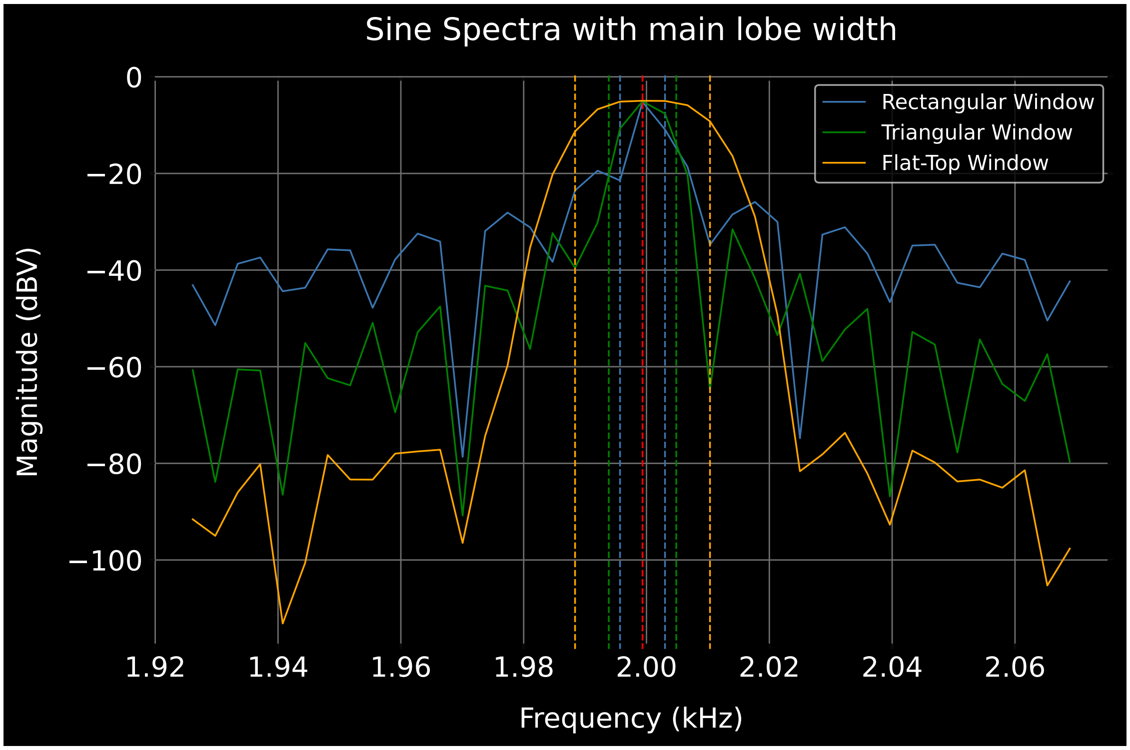


Figure 3.5: Frequency resolution comparison.

$$\Delta f_{Rectangular} = 7.32Hz$$

$$\Delta f_{Triangular} = 10.99Hz$$

$$\Delta f_{Flat-Top} = 21.97Hz$$

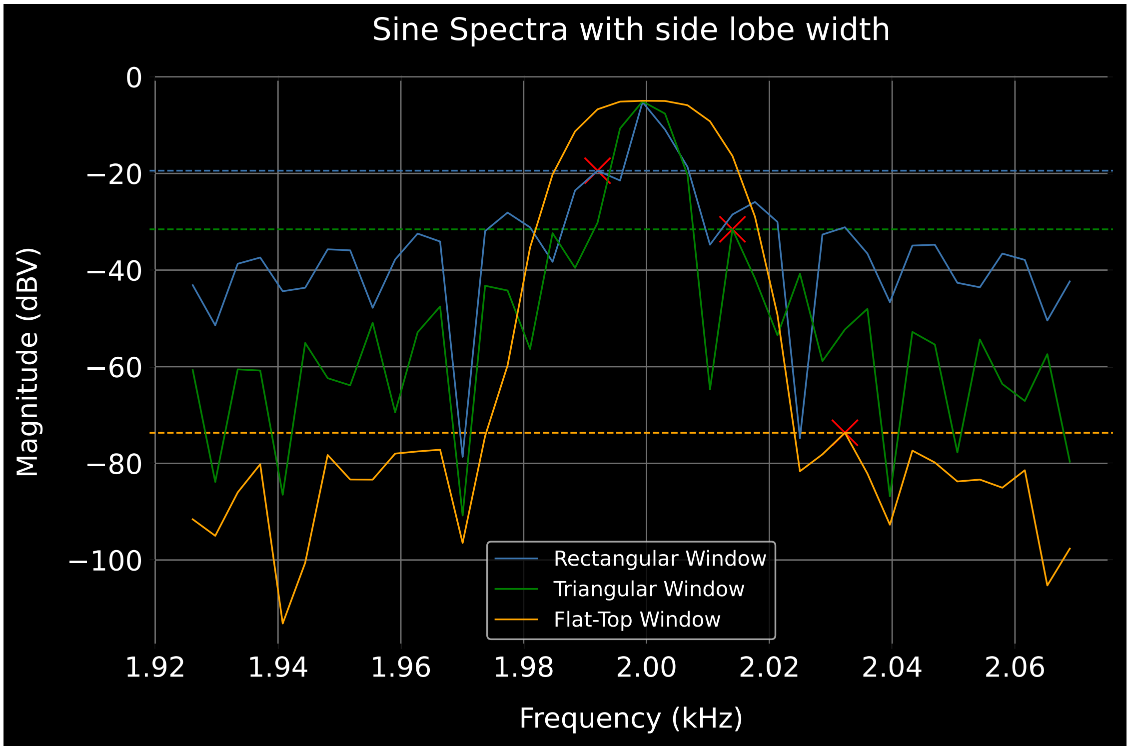


Figure 3.6: Amplitude resolution comparison.

16.1

$$L_{Rectangular} = -19.43\ dBV$$

$$L_{Triangular} = -31.56\ dBV$$

$$L_{Flat-Top} = -73.67\ dBV$$

In conclusion,

- Rectangular Window: The narrowest main lobe but higher side lobes, indicating good frequency resolution but poorer amplitude resolution.
- Triangular Window: Wider main lobe than the rectangular window, with lower side lobes, providing a balance between frequency and amplitude resolution.
- Flat-Top Window: Widest main lobe with the lowest side lobes(accurate amplitude) but poorer frequency resolution.

1.4 Task 4

Task Definition

Create a WAV file from an arbitrary function. Feed this sound into the oscilloscope and do a spectral analysis.

Procedure

1. During this task, a sound (WAV) file was generated the using the sinusodial function:

$$f(t) = \cos(2\pi \cdot 220 \cdot t) + \sin(2\pi \cdot 440 \cdot t)$$

- t : Time steps generated based on a sampling rate

2. The audio output of the sound file was analyzed using a Picoscope 5000.

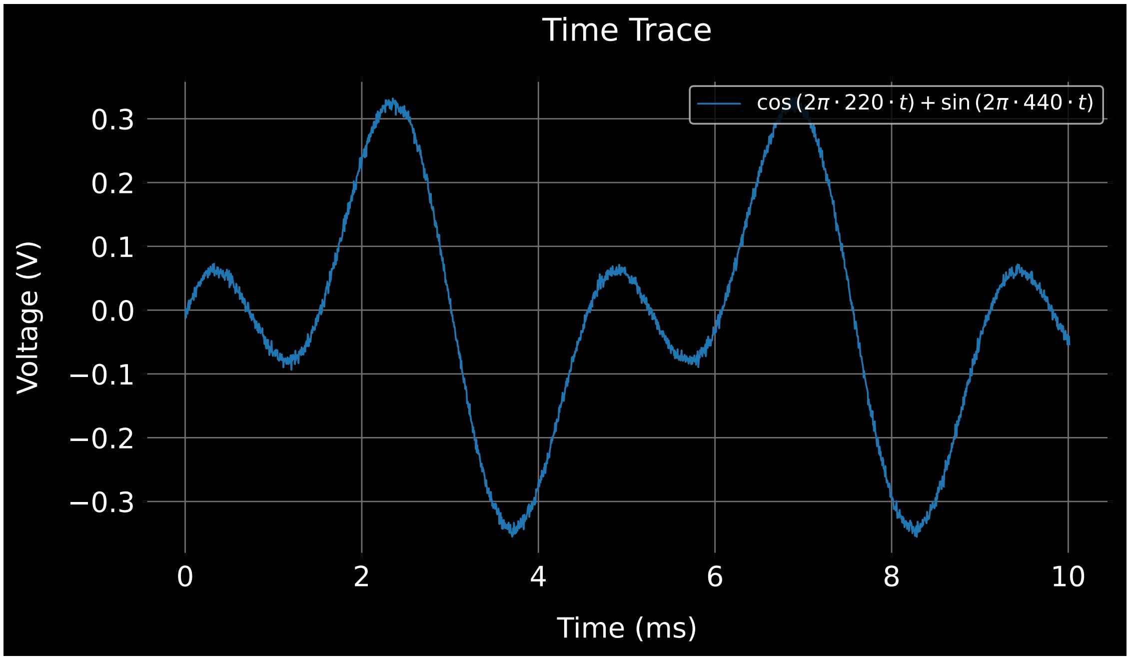


Figure 4.1: Time trace of the sound wave.

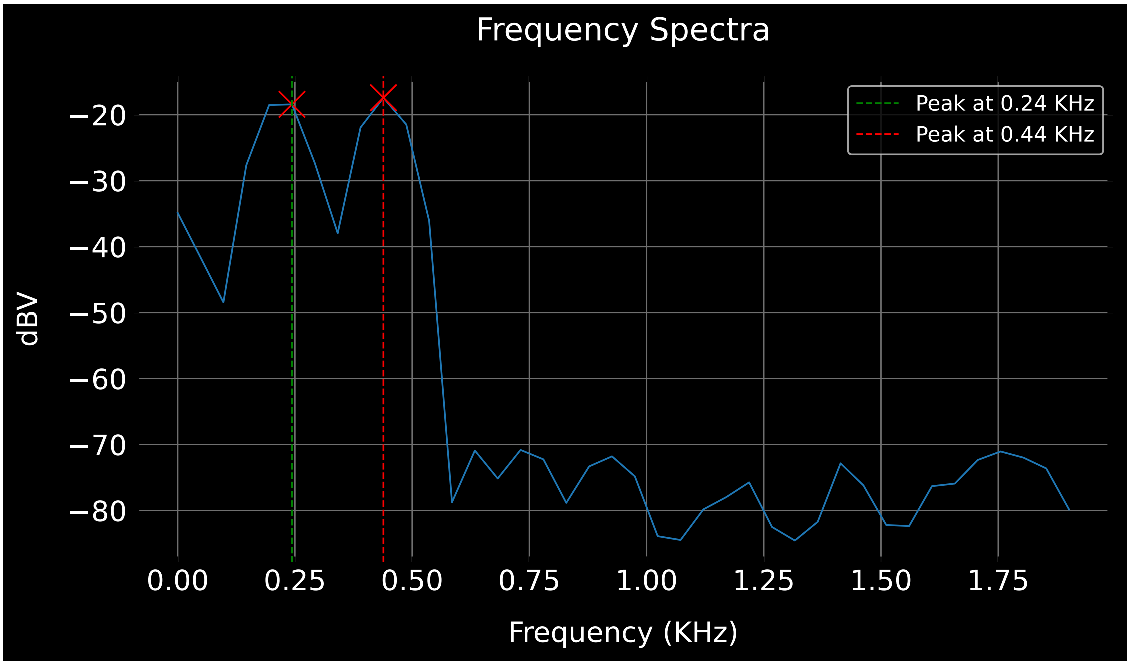


Figure 4.2: Frequency Spectra of the sound wave.

Analysis

1. Theoretically, the generated signal is a beat with frequencies 0.22 kHz and 0.44 kHz respectively.These frequencies correspond to the peaks in the signal's frequency spectra.
2. Upon inspection of the frequency spectra, it is observed that the peaks closely correspond to the theoretical freqeuncies as expected.

1.5 References

1) E19e Lab instruction
2) Nyquist Theorem
3) Picoscope Datasheet
4) Michael Cerna and Audrey F. Harvey, "The Fundamentals of FFT-Based Signal Analysis and Measurement", National Instrument, Application Note 041

Index der Kommentare

1.1 Test: 2.25 P

1.2 Formal things: 3 P

3.1 Ok.

4.1 Ok.

5.1 Fine. 1 P

5.2 Ok.

6.1 Ok. 1 P

7.1 Ok. 1 P

11.1 How do you see that from the graphs? It would be directly evident, when you made a log-log-plot.
1 P

13.1 Ok. 1 P

16.1 Here I wanted a direct comparison between the theory expressions and the measured data. See uploaded file.
0.5 P

18.1 Ok. 1 P

18.2 Nice report.