E11e

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1 Phase Shift in AC Circuits

Group #13

NG Kuok Fung | #3797903

Edgar Harutyunyan | #3775171

Overview of Tasks

1. Measure the phase shift ϕ between current and voltage as a function of frequency f for

1a. an RLC series circuit,

1b. an RC series circuit,

1c. an RL series circuit.

2. Analysis

2a. Plot the phase shifts of the three circuits in one graph.

2b. Determine the series resistance R from the phase shift of the RC-series circuit.

2c. Determine the inductance L of the coil

i. from the resonance frequency f_0 of the RLC series circuit

ii. by fitting of the phase shift of the RL series circuit.

2d. Fit the phase shift ϕ of the RLC series circuit treating the resistance R as a fitting parameter.

1.1 Task 1: Measurement of phase shift ϕ

Task Definition

Measure the phase shift ϕ between current and voltage as a function of frequency f for an RLC, RL and RC series circuit.

Theoretical Basis

$$\phi = \frac{\Delta t}{T}$$

- T: Period
- Δt: Time difference between 2 corresponding points on the input and output waveforms.

$$T = \frac{1}{f}$$

- f: Frequency of signal

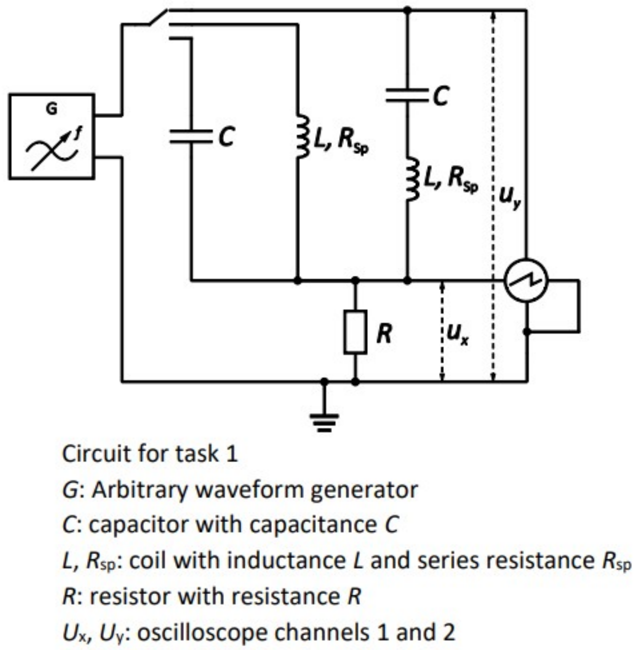

Figure 1.1: Series RLC,RL and RC Circuit Diagram

Procedure

1. The RLC, RL and RC Series Circuits were setup according to the circuit diagram in **Fig 1.1**.
2. A sine wave signal was generated, and fed into the circuit.
3. Subsequently, an oscilloscope was used to analyze the waveform of the generated signal via Channel B, and the output waveform from the circuit via Channel A.
4. The value of Δt was measured using the oscilloscope.
5. This was repeated for different values of f, and values of ϕ were determined using the given equations.

1.2 Task 2: Analysis

1.2.1 Task 2a: Plot the phase shifts of the three circuits in one graph.

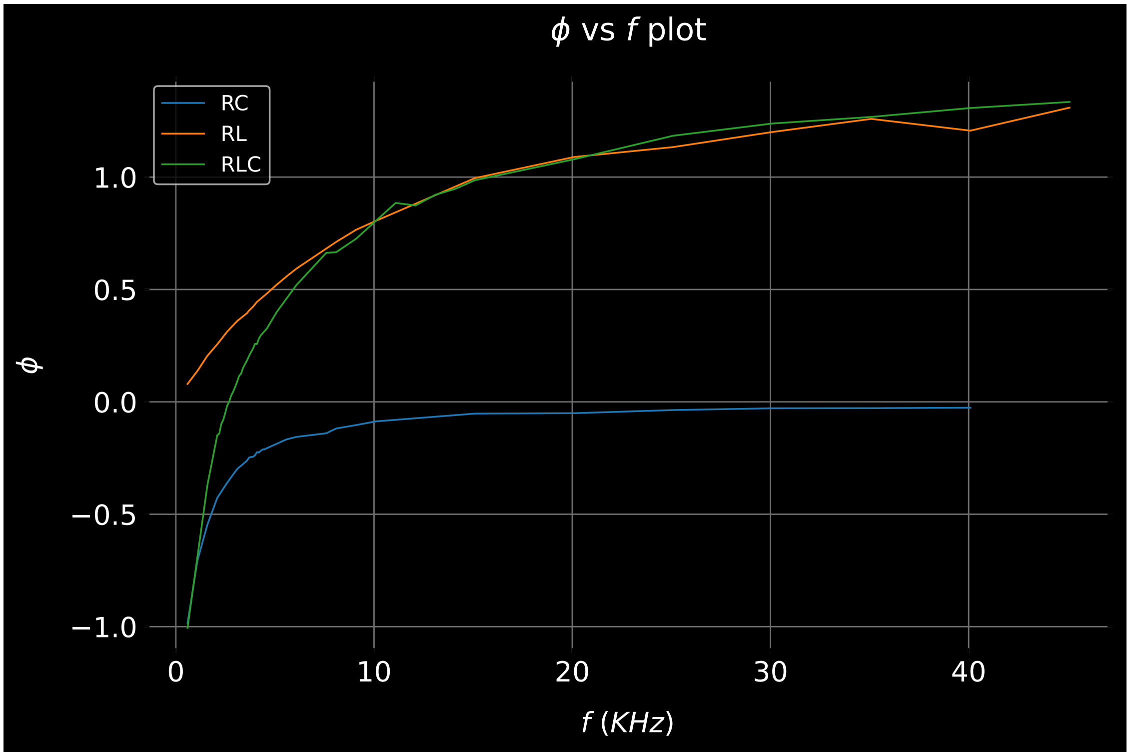

Figure 1.1: Phase Shift Curve

1.2.2 Task 2b: RC-series circuit analysis

Task Definition

Determine the series resistance R from the phase shift of the RC-series circuit.

Theoretical Basis

$$Z = R + iX_T$$

$$Im(Z) = X_T, \qquad Re(Z) = R$$

$$X_T = X_L - X_C$$

$$X_L = \omega L = 2\pi f L$$

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$$

$$\tan\phi = \frac{Im(Z)}{Re(Z)} = \frac{\omega L - \frac{1}{\omega C}}{R} \tag{2.1}$$

In RC circuit, the value of $L = 0\ H$.

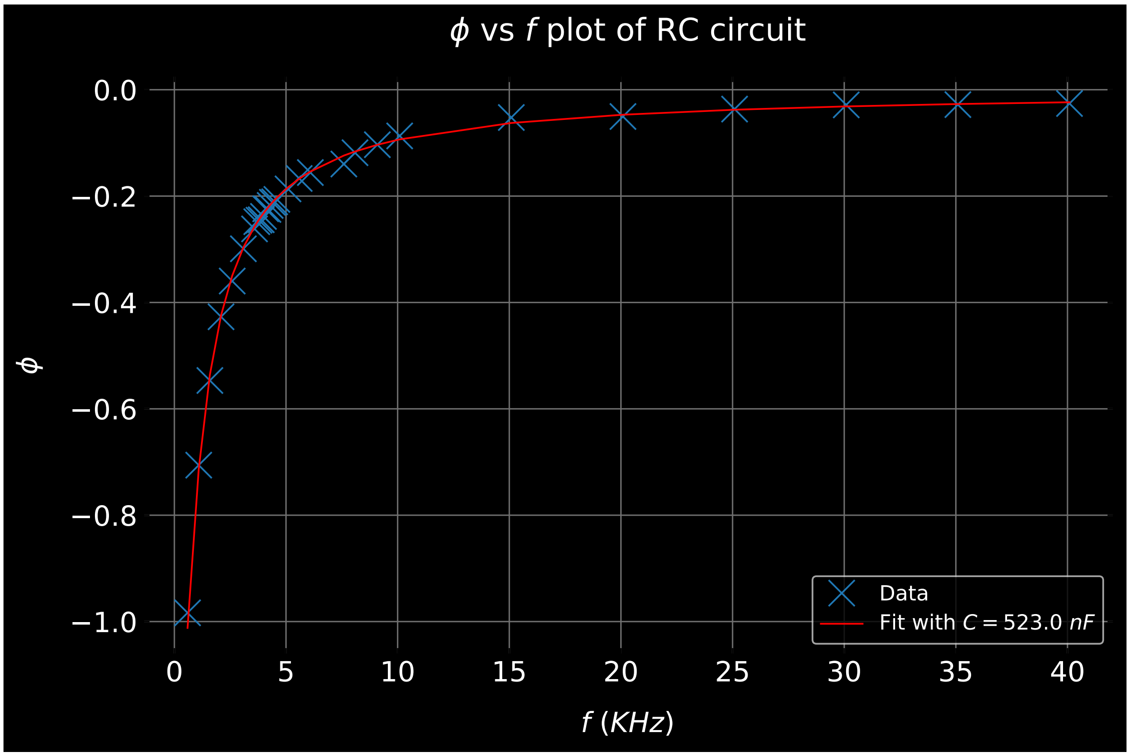

Figure 1.2: RC Fit

$$R = (322.19 \pm 1.924)\ \Omega$$

1.2.3 Task 2c:

Task Definition

Determine the inductance L of the coil

i. from the resonance frequency f_0 of the RLC series circuit

ii. by fitting of the phase shift of the RL series circuit.

Theoretical Basis

i.

At resonance, $f = f_0$ when $\phi = 0$. Hence, f_0 can be determined form the graph of ϕ against f for an RLC circuit.

The value of L may then be determined via the Thomson's Formula:

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

Hence:

$$L = \frac{1}{(2\pi f_0)^2 C}$$

$C = 0.523\ \mu F$

ii.

From Eq 2.1 for RL circuit, when $C = 0$:

$$\tan\phi = \frac{\omega L}{R + R_{sp}} \tag{2.2}$$

where R_{sp} is resistance of the coil.

R_{sp} can be estimated from RLC fit with R fixed value detemined from RC fit, L fixed value from Thomson's formula and C measured value. Then new L value has been determined by fitting Eq 2.2 for RL circuit:

$$\phi = arctan\frac{\omega L}{R + R_{sp}} \tag{2.3}$$

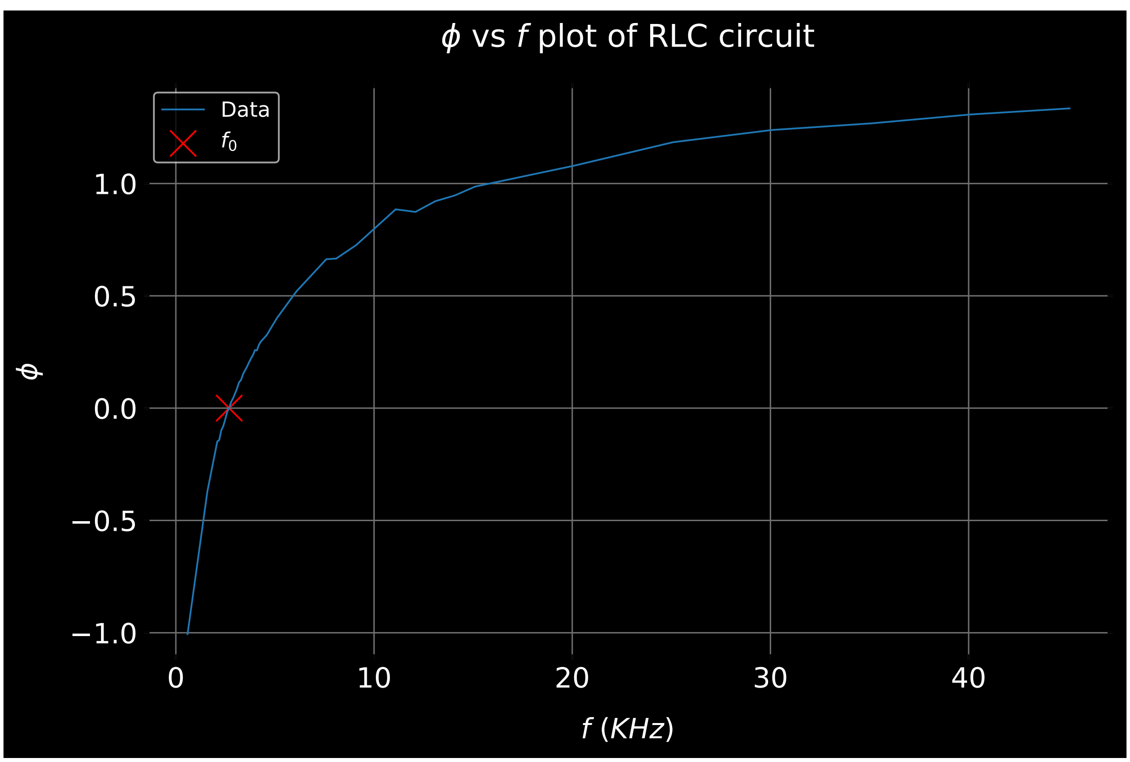

Figure 1.3: RLC data plot

Using Thomson's equation:

$$L = \frac{1}{(2\pi f_0)^2 C} = (6.69 \pm 0.249)\ mH$$

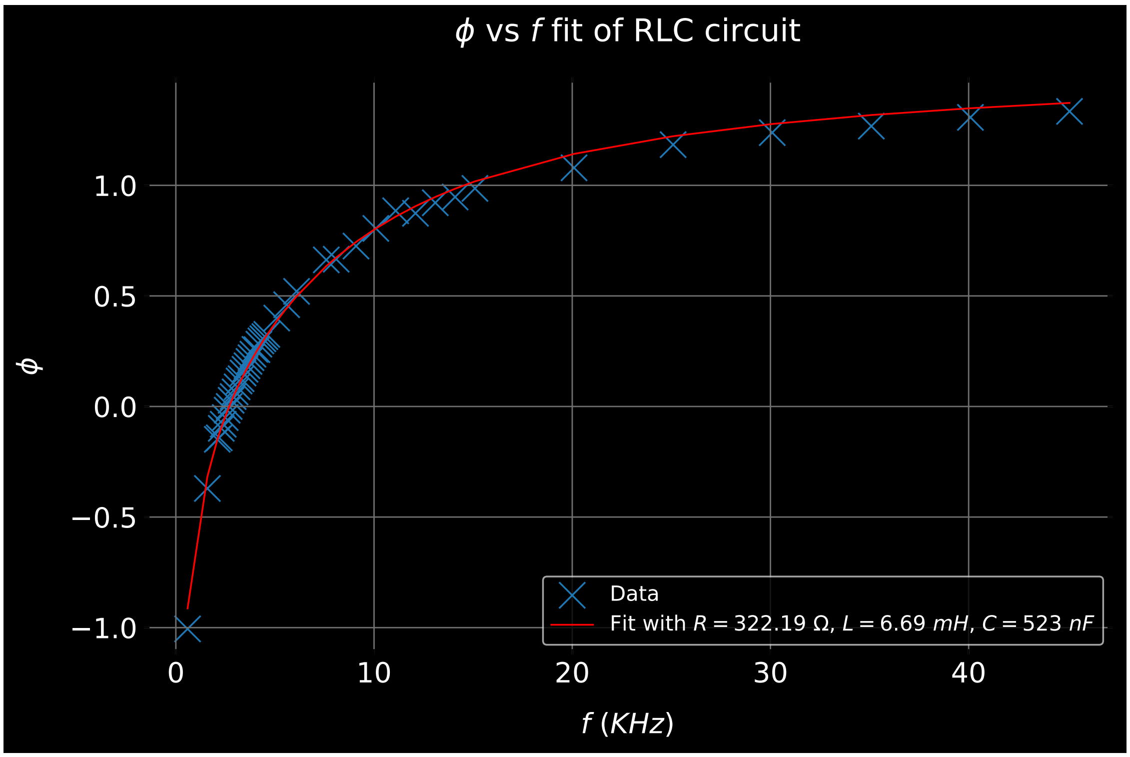

Figure 1.4: Fitting RLC circuit in order to estimate R_{sp} resistance.

$$R_{sp} = (56.69 \pm 5.155)\ \Omega$$

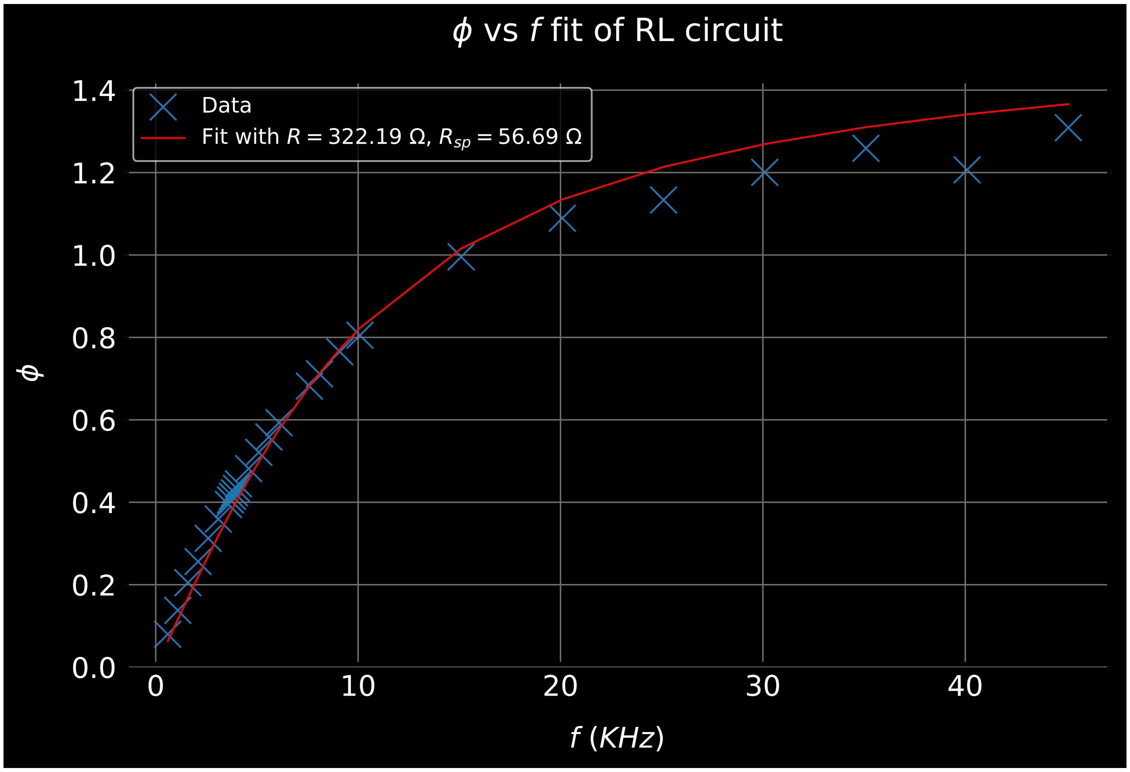

Figure 1.4: Fitting RL circuit.

$$L = (6.45 \pm 0.157)\ mH$$

1.2.4 Task 2d:

Task Definition

Fit the phase shift ϕ of the RLC series circuit treating the resistance R as a fitting parameter.

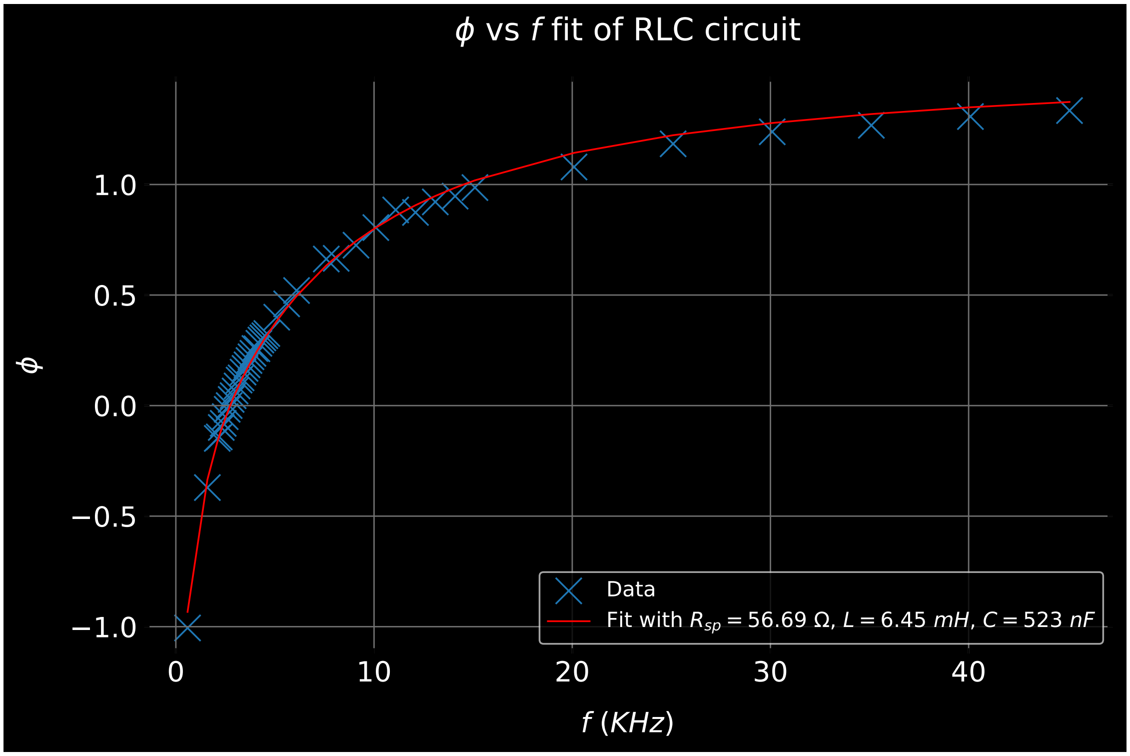

Figure 1.4: Fitting RLC circuit with fitting parameter R.

$$R = (307.34 \pm 5.002)\ \Omega$$

References

1) E11e Lab instruction