then increasing the value of that feature increases the value of our prediction $\hat{y}$. If a feature receives a negative weight, then increasing the value of that feature decreases the value of our prediction. If a feature's weight is large in magnitude, then it has a large effect on the prediction. If a feature's weight is zero, it has no effect on the prediction.

We thus have a definition of our task T: to predict y from $\boldsymbol{x}$ by outputting $\hat{y} = \boldsymbol{w}^\top \boldsymbol{x}$. Next we need a definition of our performance measure, P.

Suppose that we have a design matrix of m example inputs that we will not use for training, only for evaluating how well the model performs. We also have a vector of regression targets providing the correct value of y for each of these examples. Because this dataset will only be used for evaluation, we call it the **test set**. We refer to the design matrix of inputs as $\boldsymbol{X}^{(\text{test})}$ and the vector of regression targets as $\boldsymbol{y}^{(\text{test})}$.

One way of measuring the performance of the model is to compute the **mean squared error** of the model on the test set. If $\hat{\boldsymbol{y}}^{(\text{test})}$ gives the predictions of the model on the test set, then the mean squared error is given by

$$\text{MSE}_{\text{test}} = \frac{1}{m} \sum_i (\hat{\boldsymbol{y}}^{(\text{test})} - \boldsymbol{y}^{(\text{test})})_i^2. \tag{5.4}$$

Intuitively, one can see that this error measure decreases to 0 when $\hat{\boldsymbol{y}}^{(\text{test})} = \boldsymbol{y}^{(\text{test})}$. We can also see that

$$\text{MSE}_{\text{test}} = \frac{1}{m} ||\hat{\boldsymbol{y}}^{(\text{test})} - \boldsymbol{y}^{(\text{test})}||_2^2, \tag{5.5}$$

so the error increases whenever the Euclidean distance between the predictions and the targets increases.

To make a machine learning algorithm, we need to design an algorithm that will improve the weights $\boldsymbol{w}$ in a way that reduces MSE_{test} when the algorithm is allowed to gain experience by observing a training set $(\boldsymbol{X}^{(\text{train})}, \boldsymbol{y}^{(\text{train})})$. One intuitive way of doing this (which we will justify later, in section 5.5.1) is just to minimize the mean squared error on the training set, $\text{MSE}_{\text{train}}$.

To minimize $\text{MSE}_{\text{train}}$, we can simply solve for where its gradient is $\boldsymbol{0}$:

$$\nabla_{\boldsymbol{w}} \text{MSE}_{\text{train}} = 0 \tag{5.6}$$

$$\Rightarrow \nabla_{\boldsymbol{w}} \frac{1}{m} ||\hat{\boldsymbol{y}}^{(\text{train})} - \boldsymbol{y}^{(\text{train})}||_2^2 = 0 \tag{5.7}$$

$$\Rightarrow \frac{1}{m} \nabla_{\boldsymbol{w}} ||\boldsymbol{X}^{(\text{train})} \boldsymbol{w} - \boldsymbol{y}^{(\text{train})}||_2^2 = 0 \tag{5.8}$$