internal state is thus updated as follows, but with a conditional self-loop weight $f_i^{(t)}$:

$$s_i^{(t)} = f_i^{(t)} s_i^{(t-1)} + g_i^{(t)} \sigma \left(b_i + \sum_j U_{i,j} x_j^{(t)} + \sum_j W_{i,j} h_j^{(t-1)} \right), \tag{10.41}$$

where b, U and W respectively denote the biases, input weights and recurrent weights into the LSTM cell. The **external input gate** unit $g_i^{(t)}$ is computed similarly to the forget gate (with a sigmoid unit to obtain a gating value between 0 and 1), but with its own parameters:

$$g_i^{(t)} = \sigma \left(b_i^g + \sum_j U_{i,j}^g x_j^{(t)} + \sum_j W_{i,j}^g h_j^{(t-1)} \right). \tag{10.42}$$

The output $h_i^{(t)}$ of the LSTM cell can also be shut off, via the **output gate** $q_i^{(t)}$, which also uses a sigmoid unit for gating:

$$h_i^{(t)} = \tanh\left(s_i^{(t)}\right) q_i^{(t)} \tag{10.43}$$

$$q_i^{(t)} = \sigma \left(b_i^o + \sum_j U_{i,j}^o x_j^{(t)} + \sum_j W_{i,j}^o h_j^{(t-1)} \right)$$
 (10.44)

which has parameters b^o , U^o , W^o for its biases, input weights and recurrent weights, respectively. Among the variants, one can choose to use the cell state $s_i^{(t)}$ as an extra input (with its weight) into the three gates of the *i*-th unit, as shown in figure 10.16. This would require three additional parameters.

LSTM networks have been shown to learn long-term dependencies more easily than the simple recurrent architectures, first on artificial data sets designed for testing the ability to learn long-term dependencies (Bengio *et al.*, 1994; Hochreiter and Schmidhuber, 1997; Hochreiter *et al.*, 2001), then on challenging sequence processing tasks where state-of-the-art performance was obtained (Graves, 2012; Graves *et al.*, 2013; Sutskever *et al.*, 2014). Variants and alternatives to the LSTM have been studied and used and are discussed next.

10.10.2 Other Gated RNNs

Which pieces of the LSTM architecture are actually necessary? What other successful architectures could be designed that allow the network to dynamically control the time scale and forgetting behavior of different units?