

(2014) identified non-convergence as an issue that may cause GANs to underfit. In general, simultaneous gradient descent on two players' costs is not guaranteed to reach an equilibrium. Consider for example the value function $v(a, b) = ab$, where one player controls a and incurs cost ab , while the other player controls b and receives a cost $-ab$. If we model each player as making infinitesimally small gradient steps, each player reducing their own cost at the expense of the other player, then a and b go into a stable, circular orbit, rather than arriving at the equilibrium point at the origin. Note that the equilibria for a minimax game are not local minima of v . Instead, they are points that are simultaneously minima for both players' costs. This means that they are saddle points of v that are local minima with respect to the first player's parameters and local maxima with respect to the second player's parameters. It is possible for the two players to take turns increasing then decreasing v forever, rather than landing exactly on the saddle point where neither player is capable of reducing its cost. It is not known to what extent this non-convergence problem affects GANs.

Goodfellow (2014) identified an alternative formulation of the payoffs, in which the game is no longer zero-sum, that has the same expected gradient as maximum likelihood learning whenever the discriminator is optimal. Because maximum likelihood training converges, this reformulation of the GAN game should also converge, given enough samples. Unfortunately, this alternative formulation does not seem to improve convergence in practice, possibly due to suboptimality of the discriminator, or possibly due to high variance around the expected gradient.

In realistic experiments, the best-performing formulation of the GAN game is a different formulation that is neither zero-sum nor equivalent to maximum likelihood, introduced by Goodfellow *et al.* (2014c) with a heuristic motivation. In this best-performing formulation, the generator aims to increase the log probability that the discriminator makes a mistake, rather than aiming to decrease the log probability that the discriminator makes the correct prediction. This reformulation is motivated solely by the observation that it causes the derivative of the generator's cost function with respect to the discriminator's logits to remain large even in the situation where the discriminator confidently rejects all generator samples.

Stabilization of GAN learning remains an open problem. Fortunately, GAN learning performs well when the model architecture and hyperparameters are carefully selected. Radford *et al.* (2015) crafted a deep convolutional GAN (DCGAN) that performs very well for image synthesis tasks, and showed that its latent representation space captures important factors of variation, as shown in figure 15.9. See figure 20.7 for examples of images generated by a DCGAN generator.

The GAN learning problem can also be simplified by breaking the generation