$$P(h_i^{(1)} = 1 \mid \boldsymbol{v}, \boldsymbol{h}^{(2)}) = \sigma\left(\boldsymbol{v}^\top \boldsymbol{W}_{:,i}^{(1)} + \boldsymbol{W}_{i,:}^{(2)} \boldsymbol{h}^{(2)}\right) \tag{20.27}$$

and

$$P(h_k^{(2)} = 1 \mid \boldsymbol{h}^{(1)}) = \sigma\left(\boldsymbol{h}^{(1)\top} \boldsymbol{W}_{:,k}^{(2)}\right). \tag{20.28}$$

The bipartite structure makes Gibbs sampling in a deep Boltzmann machine efficient. The naive approach to Gibbs sampling is to update only one variable at a time. RBMs allow all of the visible units to be updated in one block and all of the hidden units to be updated in a second block. One might naively assume that a DBM with l layers requires $l + 1$ updates, with each iteration updating a block consisting of one layer of units. Instead, it is possible to update all of the units in only two iterations. Gibbs sampling can be divided into two blocks of updates, one including all even layers (including the visible layer) and the other including all odd layers. Due to the bipartite DBM connection pattern, given the even layers, the distribution over the odd layers is factorial and thus can be sampled simultaneously and independently as a block. Likewise, given the odd layers, the even layers can be sampled simultaneously and independently as a block. Efficient sampling is especially important for training with the stochastic maximum likelihood algorithm.

20.4.1 Interesting Properties

Deep Boltzmann machines have many interesting properties.

DBMs were developed after DBNs. Compared to DBNs, the posterior distribution $P(\boldsymbol{h} \mid \boldsymbol{v})$ is simpler for DBMs. Somewhat counterintuitively, the simplicity of this posterior distribution allows richer approximations of the posterior. In the case of the DBN, we perform classification using a heuristically motivated approximate inference procedure, in which we guess that a reasonable value for the mean field expectation of the hidden units can be provided by an upward pass through the network in an MLP that uses sigmoid activation functions and the same weights as the original DBN. *Any* distribution $Q(\boldsymbol{h})$ may be used to obtain a variational lower bound on the log-likelihood. This heuristic procedure therefore allows us to obtain such a bound. However, the bound is not explicitly optimized in any way, so the bound may be far from tight. In particular, the heuristic estimate of Q ignores interactions between hidden units within the same layer as well as the top-down feedback influence of hidden units in deeper layers on hidden units that are closer to the input. Because the heuristic MLP-based inference procedure in the DBN is not able to account for these interactions, the resulting Q is presumably far