

machines.

Lee *et al.* (2009) demonstrated that probabilistic max pooling could be used to build convolutional deep Boltzmann machines.³ This model is able to perform operations such as filling in missing portions of its input. While intellectually appealing, this model is challenging to make work in practice, and usually does not perform as well as a classifier as traditional convolutional networks trained with supervised learning.

Many convolutional models work equally well with inputs of many different spatial sizes. For Boltzmann machines, it is difficult to change the input size for a variety of reasons. The partition function changes as the size of the input changes. Moreover, many convolutional networks achieve size invariance by scaling up the size of their pooling regions proportional to the size of the input, but scaling Boltzmann machine pooling regions is awkward. Traditional convolutional neural networks can use a fixed number of pooling units and dynamically increase the size of their pooling regions in order to obtain a fixed-size representation of a variable-sized input. For Boltzmann machines, large pooling regions become too expensive for the naive approach. The approach of Lee *et al.* (2009) of making each of the detector units in the same pooling region mutually exclusive solves the computational problems, but still does not allow variable-size pooling regions. For example, suppose we learn a model with 2×2 probabilistic max pooling over detector units that learn edge detectors. This enforces the constraint that only one of these edges may appear in each 2×2 region. If we then increase the size of the input image by 50% in each direction, we would expect the number of edges to increase correspondingly. Instead, if we increase the size of the pooling regions by 50% in each direction to 3×3 , then the mutual exclusivity constraint now specifies that each of these edges may only appear once in a 3×3 region. As we grow a model's input image in this way, the model generates edges with less density. Of course, these issues only arise when the model must use variable amounts of pooling in order to emit a fixed-size output vector. Models that use probabilistic max pooling may still accept variable-sized input images so long as the output of the model is a feature map that can scale in size proportional to the input image.

Pixels at the boundary of the image also pose some difficulty, which is exacerbated by the fact that connections in a Boltzmann machine are symmetric. If we do not implicitly zero-pad the input, then there are fewer hidden units than visible units, and the visible units at the boundary of the image are not modeled

³The publication describes the model as a “deep belief network” but because it can be described as a purely undirected model with tractable layer-wise mean field fixed point updates, it best fits the definition of a deep Boltzmann machine.