$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Figure 2.2: **Example identity matrix**: This is $\boldsymbol{I}_3$.

$$\boldsymbol{A}_{2,1}x_1 + \boldsymbol{A}_{2,2}x_2 + \cdots + \boldsymbol{A}_{2,n}x_n = b_2 \tag{2.17}$$

$$\ldots \tag{2.18}$$

$$\boldsymbol{A}_{m,1}x_1 + \boldsymbol{A}_{m,2}x_2 + \cdots + \boldsymbol{A}_{m,n}x_n = b_m. \tag{2.19}$$

Matrix-vector product notation provides a more compact representation for equations of this form.

## 2.3 Identity and Inverse Matrices

Linear algebra offers a powerful tool called **matrix inversion** that allows us to analytically solve equation 2.11 for many values of $\boldsymbol{A}$.

To describe matrix inversion, we first need to define the concept of an **identity matrix**. An identity matrix is a matrix that does not change any vector when we multiply that vector by that matrix. We denote the identity matrix that preserves $n$-dimensional vectors as $\boldsymbol{I}_n$. Formally, $\boldsymbol{I}_n \in \mathbb{R}^{n\times n}$, and

$$\forall \boldsymbol{x} \in \mathbb{R}^n, \boldsymbol{I}_n\boldsymbol{x} = \boldsymbol{x}. \tag{2.20}$$

The structure of the identity matrix is simple: all of the entries along the main diagonal are 1, while all of the other entries are zero. See figure 2.2 for an example.

The **matrix inverse** of $\boldsymbol{A}$ is denoted as $\boldsymbol{A}^{-1}$, and it is defined as the matrix such that

$$\boldsymbol{A}^{-1}\boldsymbol{A} = \boldsymbol{I}_n. \tag{2.21}$$

We can now solve equation 2.11 by the following steps:

$$\boldsymbol{A}\boldsymbol{x} = \boldsymbol{b} \tag{2.22}$$

$$\boldsymbol{A}^{-1}\boldsymbol{A}\boldsymbol{x} = \boldsymbol{A}^{-1}\boldsymbol{b} \tag{2.23}$$

$$\boldsymbol{I}_n\boldsymbol{x} = \boldsymbol{A}^{-1}\boldsymbol{b} \tag{2.24}$$