

examples, with each example surrounded by other highly similar examples that may be reached by applying transformations to traverse the manifold. The second argument in favor of the manifold hypothesis is that we can also imagine such neighborhoods and transformations, at least informally. In the case of images, we can certainly think of many possible transformations that allow us to trace out a manifold in image space: we can gradually dim or brighten the lights, gradually move or rotate objects in the image, gradually alter the colors on the surfaces of objects, etc. It remains likely that there are multiple manifolds involved in most applications. For example, the manifold of images of human faces may not be connected to the manifold of images of cat faces.

These thought experiments supporting the manifold hypotheses convey some intuitive reasons supporting it. More rigorous experiments (Cayton, 2005; Narayanan and Mitter, 2010; Schölkopf *et al.*, 1998; Roweis and Saul, 2000; Tenenbaum *et al.*, 2000; Brand, 2003; Belkin and Niyogi, 2003; Donoho and Grimes, 2003; Weinberger and Saul, 2004) clearly support the hypothesis for a large class of datasets of interest in AI.

When the data lies on a low-dimensional manifold, it can be most natural for machine learning algorithms to represent the data in terms of coordinates on the manifold, rather than in terms of coordinates in  $\mathbb{R}^n$ . In everyday life, we can think of roads as 1-D manifolds embedded in 3-D space. We give directions to specific addresses in terms of address numbers along these 1-D roads, not in terms of coordinates in 3-D space. Extracting these manifold coordinates is challenging, but holds the promise to improve many machine learning algorithms. This general principle is applied in many contexts. Figure 5.13 shows the manifold structure of a dataset consisting of faces. By the end of this book, we will have developed the methods necessary to learn such a manifold structure. In figure 20.6, we will see how a machine learning algorithm can successfully accomplish this goal.

This concludes part I, which has provided the basic concepts in mathematics and machine learning which are employed throughout the remaining parts of the book. You are now prepared to embark upon your study of deep learning.