consistent estimators can differ in their **statistic efficiency**, meaning that one consistent estimator may obtain lower generalization error for a fixed number of samples m, or equivalently, may require fewer examples to obtain a fixed level of generalization error.

Statistical efficiency is typically studied in the **parametric case** (like in linear regression) where our goal is to estimate the value of a parameter (and assuming it is possible to identify the true parameter), not the value of a function. A way to measure how close we are to the true parameter is by the expected mean squared error, computing the squared difference between the estimated and true parameter values, where the expectation is over m training samples from the data generating distribution. That parametric mean squared error decreases as m increases, and for m large, the Cramér-Rao lower bound (Rao, 1945; Cramér, 1946) shows that no consistent estimator has a lower mean squared error than the maximum likelihood estimator.

For these reasons (consistency and efficiency), maximum likelihood is often considered the preferred estimator to use for machine learning. When the number of examples is small enough to yield overfitting behavior, regularization strategies such as weight decay may be used to obtain a biased version of maximum likelihood that has less variance when training data is limited.

5.6 Bayesian Statistics

So far we have discussed **frequentist statistics** and approaches based on estimating a single value of $\boldsymbol{\theta}$, then making all predictions thereafter based on that one estimate. Another approach is to consider all possible values of $\boldsymbol{\theta}$ when making a prediction. The latter is the domain of **Bayesian statistics**.

As discussed in section 5.4.1, the frequentist perspective is that the true parameter value $\boldsymbol{\theta}$ is fixed but unknown, while the point estimate $\hat{\boldsymbol{\theta}}$ is a random variable on account of it being a function of the dataset (which is seen as random).

The Bayesian perspective on statistics is quite different. The Bayesian uses probability to reflect degrees of certainty of states of knowledge. The dataset is directly observed and so is not random. On the other hand, the true parameter $\boldsymbol{\theta}$ is unknown or uncertain and thus is represented as a random variable.

Before observing the data, we represent our knowledge of $\boldsymbol{\theta}$ using the **prior probability distribution**, $p(\boldsymbol{\theta})$ (sometimes referred to as simply "the prior"). Generally, the machine learning practitioner selects a prior distribution that is quite broad (i.e. with high entropy) to reflect a high degree of uncertainty in the