in a Hessian matrix that is better conditioned. Melchior *et al.* (2013) experimentally confirmed that the conditioning of the Hessian matrix improves, and observed that the centering trick is equivalent to another Boltzmann machine learning technique, the **enhanced gradient** (Cho *et al.*, 2011). The improved conditioning of the Hessian matrix allows learning to succeed, even in difficult cases like training a deep Boltzmann machine with multiple layers.

The other approach to jointly training deep Boltzmann machines is the multi-prediction deep Boltzmann machine (MP-DBM) which works by viewing the mean field equations as defining a family of recurrent networks for approximately solving every possible inference problem (Goodfellow *et al.*, 2013b). Rather than training the model to maximize the likelihood, the model is trained to make each recurrent network obtain an accurate answer to the corresponding inference problem. The training process is illustrated in figure 20.5. It consists of randomly sampling a training example, randomly sampling a subset of inputs to the inference network, and then training the inference network to predict the values of the remaining units.

This general principle of back-propagating through the computational graph for approximate inference has been applied to other models (Stoyanov *et al.*, 2011; Brakel *et al.*, 2013). In these models and in the MP-DBM, the final loss is not the lower bound on the likelihood. Instead, the final loss is typically based on the approximate conditional distribution that the approximate inference network imposes over the missing values. This means that the training of these models is somewhat heuristically motivated. If we inspect the $p(\boldsymbol{v})$ represented by the Boltzmann machine learned by the MP-DBM, it tends to be somewhat defective, in the sense that Gibbs sampling yields poor samples.

Back-propagation through the inference graph has two main advantages. First, it trains the model as it is really used—with approximate inference. This means that approximate inference, for example, to fill in missing inputs, or to perform classification despite the presence of missing inputs, is more accurate in the MP-DBM than in the original DBM. The original DBM does not make an accurate classifier on its own; the best classification results with the original DBM were based on training a separate classifier to use features extracted by the DBM, rather than by using inference in the DBM to compute the distribution over the class labels. Mean field inference in the MP-DBM performs well as a classifier without special modifications. The other advantage of back-propagating through approximate inference is that back-propagation computes the exact gradient of the loss. This is better for optimization than the approximate gradients of SML training, which suffer from both bias and variance. This probably explains why MP-