

example, we might say “Let $s \in \mathbb{R}$ be the slope of the line,” while defining a real-valued scalar, or “Let $n \in \mathbb{N}$ be the number of units,” while defining a natural number scalar.

- **Vectors:** A vector is an array of numbers. The numbers are arranged in order. We can identify each individual number by its index in that ordering. Typically we give vectors lower case names written in bold typeface, such as \mathbf{x} . The elements of the vector are identified by writing its name in italic typeface, with a subscript. The first element of \mathbf{x} is x_1 , the second element is x_2 and so on. We also need to say what kind of numbers are stored in the vector. If each element is in \mathbb{R} , and the vector has n elements, then the vector lies in the set formed by taking the Cartesian product of \mathbb{R} n times, denoted as \mathbb{R}^n . When we need to explicitly identify the elements of a vector, we write them as a column enclosed in square brackets:

$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}. \quad (2.1)$$

We can think of vectors as identifying points in space, with each element giving the coordinate along a different axis.

Sometimes we need to index a set of elements of a vector. In this case, we define a set containing the indices and write the set as a subscript. For example, to access x_1 , x_3 and x_6 , we define the set $S = \{1, 3, 6\}$ and write \mathbf{x}_S . We use the $-$ sign to index the complement of a set. For example \mathbf{x}_{-1} is the vector containing all elements of \mathbf{x} except for x_1 , and \mathbf{x}_{-S} is the vector containing all of the elements of \mathbf{x} except for x_1 , x_3 and x_6 .

- **Matrices:** A matrix is a 2-D array of numbers, so each element is identified by two indices instead of just one. We usually give matrices upper-case variable names with bold typeface, such as \mathbf{A} . If a real-valued matrix \mathbf{A} has a height of m and a width of n , then we say that $\mathbf{A} \in \mathbb{R}^{m \times n}$. We usually identify the elements of a matrix using its name in italic but not bold font, and the indices are listed with separating commas. For example, $A_{1,1}$ is the upper left entry of \mathbf{A} and $A_{m,n}$ is the bottom right entry. We can identify all of the numbers with vertical coordinate i by writing a “:” for the horizontal coordinate. For example, $\mathbf{A}_{i,:}$ denotes the horizontal cross section of \mathbf{A} with vertical coordinate i . This is known as the i -th **row** of \mathbf{A} . Likewise, $\mathbf{A}_{:,i}$ is