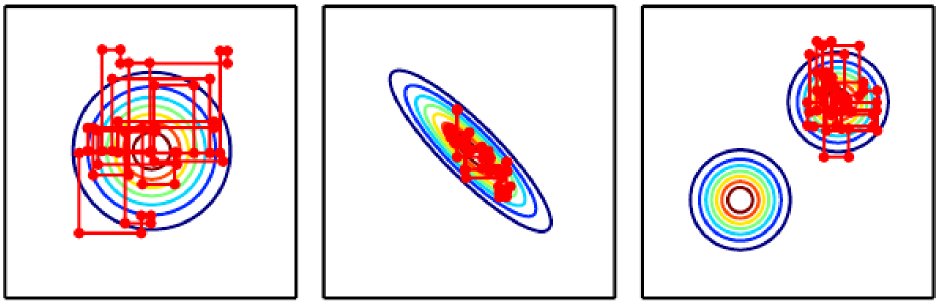

Figure 17.1: Paths followed by Gibbs sampling for three distributions, with the Markov chain initialized at the mode in both cases. *(Left)*A multivariate normal distribution with two independent variables. Gibbs sampling mixes well because the variables are independent. *(Center)*A multivariate normal distribution with highly correlated variables. The correlation between variables makes it difficult for the Markov chain to mix. Because the update for each variable must be conditioned on the other variable, the correlation reduces the rate at which the Markov chain can move away from the starting point. *(Right)*A mixture of Gaussians with widely separated modes that are not axis-aligned. Gibbs sampling mixes very slowly because it is difficult to change modes while altering only one variable at a time.

According to $P_{\text{model}}(\mathrm{a} \mid \mathrm{b})$, both variables should have the same sign. This means that Gibbs sampling will only very rarely flip the signs of these variables.

In more practical scenarios, the challenge is even greater because we care not only about making transitions between two modes but more generally between all the many modes that a real model might contain. If several such transitions are difficult because of the difficulty of mixing between modes, then it becomes very expensive to obtain a reliable set of samples covering most of the modes, and convergence of the chain to its stationary distribution is very slow.

Sometimes this problem can be resolved by finding groups of highly dependent units and updating all of them simultaneously in a block. Unfortunately, when the dependencies are complicated, it can be computationally intractable to draw a sample from the group. After all, the problem that the Markov chain was originally introduced to solve is this problem of sampling from a large group of variables.

In the context of models with latent variables, which define a joint distribution $p_{\text{model}}(\boldsymbol{x}, \boldsymbol{h})$, we often draw samples of $\boldsymbol{x}$ by alternating between sampling from $p_{\text{model}}(\boldsymbol{x} \mid \boldsymbol{h})$ and sampling from $p_{\text{model}}(\boldsymbol{h} \mid \boldsymbol{x})$. From the point of view of mixing