in section 20.10.10 below, we can introduce a form of parameter sharing that brings both a statistical advantage (fewer unique parameters) and a computational advantage (less computation). This is one more instance of the recurring deep learning motif of *reuse of features*.

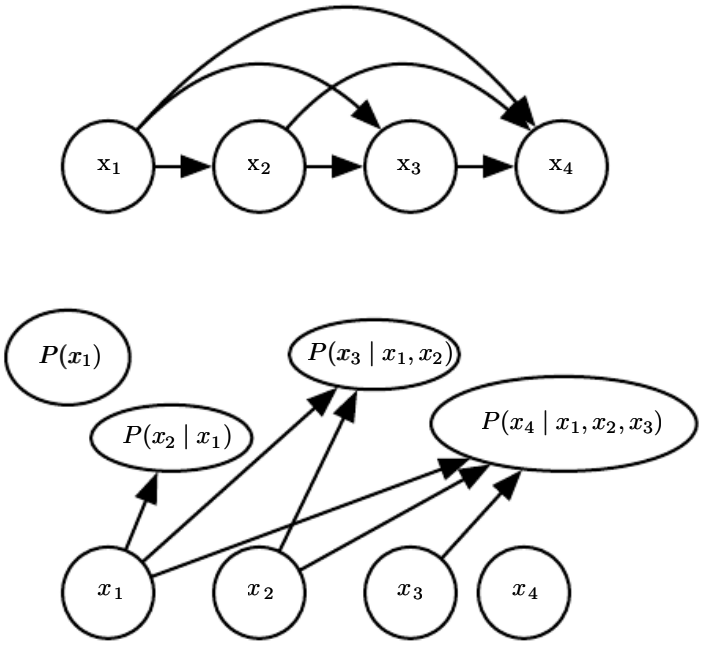

Figure 20.8: A fully visible belief network predicts the i-th variable from the $i-1$ previous ones. *(Top)*The directed graphical model for an FVBN. *(Bottom)*Corresponding computational graph, in the case of the logistic FVBN, where each prediction is made by a linear predictor.

20.10.8 Linear Auto-Regressive Networks

The simplest form of auto-regressive network has no hidden units and no sharing of parameters or features. Each $P(x_i \mid x_{i-1}, \ldots, x_1)$ is parametrized as a linear model (linear regression for real-valued data, logistic regression for binary data, softmax regression for discrete data). This model was introduced by Frey (1998) and has $O(d^2)$ parameters when there are d variables to model. It is illustrated in figure 20.8.

If the variables are continuous, a linear auto-regressive model is merely another way to formulate a multivariate Gaussian distribution, capturing linear pairwise interactions between the observed variables.

Linear auto-regressive networks are essentially the generalization of linear classification methods to generative modeling. They therefore have the same