

Once the gradients on the internal nodes of the computational graph are obtained, we can obtain the gradients on the parameter nodes. Because the parameters are shared across many time steps, we must take some care when denoting calculus operations involving these variables. The equations we wish to implement use the `bprop` method of section 6.5.6, that computes the contribution of a single edge in the computational graph to the gradient. However, the $\nabla_{\mathbf{W}} f$ operator used in calculus takes into account the contribution of \mathbf{W} to the value of f due to *all* edges in the computational graph. To resolve this ambiguity, we introduce dummy variables $\mathbf{W}^{(t)}$ that are defined to be copies of \mathbf{W} but with each $\mathbf{W}^{(t)}$ used only at time step t . We may then use $\nabla_{\mathbf{W}^{(t)}}$ to denote the contribution of the weights at time step t to the gradient.

Using this notation, the gradient on the remaining parameters is given by:

$$\nabla_{\mathbf{c}} L = \sum_t \left(\frac{\partial \mathbf{o}^{(t)}}{\partial \mathbf{c}} \right)^\top \nabla_{\mathbf{o}^{(t)}} L = \sum_t \nabla_{\mathbf{o}^{(t)}} L \quad (10.22)$$

$$\nabla_{\mathbf{b}} L = \sum_t \left(\frac{\partial \mathbf{h}^{(t)}}{\partial \mathbf{b}^{(t)}} \right)^\top \nabla_{\mathbf{h}^{(t)}} L = \sum_t \text{diag} \left(1 - \left(\mathbf{h}^{(t)} \right)^2 \right) \nabla_{\mathbf{h}^{(t)}} L \quad (10.23)$$

$$\nabla_{\mathbf{v}} L = \sum_t \sum_i \left(\frac{\partial L}{\partial o_i^{(t)}} \right) \nabla_{\mathbf{v} o_i^{(t)}} = \sum_t (\nabla_{\mathbf{o}^{(t)}} L) \mathbf{h}^{(t)\top} \quad (10.24)$$

$$\nabla_{\mathbf{W}} L = \sum_t \sum_i \left(\frac{\partial L}{\partial h_i^{(t)}} \right) \nabla_{\mathbf{W}^{(t)} h_i^{(t)}} \quad (10.25)$$

$$= \sum_t \text{diag} \left(1 - \left(\mathbf{h}^{(t)} \right)^2 \right) (\nabla_{\mathbf{h}^{(t)}} L) \mathbf{h}^{(t-1)\top} \quad (10.26)$$

$$\nabla_{\mathbf{U}} L = \sum_t \sum_i \left(\frac{\partial L}{\partial h_i^{(t)}} \right) \nabla_{\mathbf{U}^{(t)} h_i^{(t)}} \quad (10.27)$$

$$= \sum_t \text{diag} \left(1 - \left(\mathbf{h}^{(t)} \right)^2 \right) (\nabla_{\mathbf{h}^{(t)}} L) \mathbf{x}^{(t)\top} \quad (10.28)$$

We do not need to compute the gradient with respect to $\mathbf{x}^{(t)}$ for training because it does not have any parameters as ancestors in the computational graph defining the loss.