autoencoders thus provide yet another example of how useful properties can emerge as a byproduct of minimizing reconstruction error. They are also an example of how overcomplete, high-capacity models may be used as autoencoders so long as care is taken to prevent them from learning the identity function. Denoising autoencoders are presented in more detail in section 14.5.

14.2.3 Regularizing by Penalizing Derivatives

Another strategy for regularizing an autoencoder is to use a penalty Ω as in sparse autoencoders,

$$L(\boldsymbol{x}, g(f(\boldsymbol{x}))) + \Omega(\boldsymbol{h}, \boldsymbol{x}), \tag{14.10}$$

but with a different form of Ω:

$$\Omega(\boldsymbol{h}, \boldsymbol{x}) = \lambda \sum_i ||\nabla_{\boldsymbol{x}} h_i||^2. \tag{14.11}$$

This forces the model to learn a function that does not change much when $\boldsymbol{x}$ changes slightly. Because this penalty is applied only at training examples, it forces the autoencoder to learn features that capture information about the training distribution.

An autoencoder regularized in this way is called a **contractive autoencoder** or CAE. This approach has theoretical connections to denoising autoencoders, manifold learning and probabilistic modeling. The CAE is described in more detail in section 14.7.

14.3 Representational Power, Layer Size and Depth

Autoencoders are often trained with only a single layer encoder and a single layer decoder. However, this is not a requirement. In fact, using deep encoders and decoders offers many advantages.

Recall from section 6.4.1 that there are many advantages to depth in a feedforward network. Because autoencoders are feedforward networks, these advantages also apply to autoencoders. Moreover, the encoder is itself a feedforward network as is the decoder, so each of these components of the autoencoder can individually benefit from depth.

One major advantage of non-trivial depth is that the universal approximator theorem guarantees that a feedforward neural network with at least one hidden layer can represent an approximation of any function (within a broad class) to an