define a vector by writing out its elements in the text inline as a row matrix, then using the transpose operator to turn it into a standard column vector, e.g., $\boldsymbol{x} = [x_1, x_2, x_3]^\top$.

A scalar can be thought of as a matrix with only a single entry. From this, we can see that a scalar is its own transpose: $a = a^\top$.

We can add matrices to each other, as long as they have the same shape, just by adding their corresponding elements: $\boldsymbol{C} = \boldsymbol{A} + \boldsymbol{B}$ where $C_{i,j} = A_{i,j} + B_{i,j}$.

We can also add a scalar to a matrix or multiply a matrix by a scalar, just by performing that operation on each element of a matrix: $\boldsymbol{D} = a \cdot \boldsymbol{B} + c$ where $D_{i,j} = a \cdot B_{i,j} + c$.

In the context of deep learning, we also use some less conventional notation. We allow the addition of matrix and a vector, yielding another matrix: $\boldsymbol{C} = \boldsymbol{A} + \boldsymbol{b}$, where $C_{i,j} = A_{i,j} + b_j$. In other words, the vector $\boldsymbol{b}$ is added to each row of the matrix. This shorthand eliminates the need to define a matrix with $\boldsymbol{b}$ copied into each row before doing the addition. This implicit copying of $\boldsymbol{b}$ to many locations is called **broadcasting**.

2.2 Multiplying Matrices and Vectors

One of the most important operations involving matrices is multiplication of two matrices. The **matrix product** of matrices $\boldsymbol{A}$ and $\boldsymbol{B}$ is a third matrix $\boldsymbol{C}$. In order for this product to be defined, $\boldsymbol{A}$ must have the same number of columns as $\boldsymbol{B}$ has rows. If $\boldsymbol{A}$ is of shape $m \times n$ and $\boldsymbol{B}$ is of shape $n \times p$, then $\boldsymbol{C}$ is of shape $m \times p$. We can write the matrix product just by placing two or more matrices together, e.g.

$$\boldsymbol{C} = \boldsymbol{A}\boldsymbol{B}. \tag{2.4}$$

The product operation is defined by

$$C_{i,j} = \sum_k A_{i,k} B_{k,j}. \tag{2.5}$$

Note that the standard product of two matrices is *not* just a matrix containing the product of the individual elements. Such an operation exists and is called the **element-wise product** or **Hadamard product**, and is denoted as $\boldsymbol{A} \odot \boldsymbol{B}$.

The **dot product** between two vectors $\boldsymbol{x}$ and $\boldsymbol{y}$ of the same dimensionality is the matrix product $\boldsymbol{x}^\top \boldsymbol{y}$. We can think of the matrix product $\boldsymbol{C} = \boldsymbol{A}\boldsymbol{B}$ as computing $C_{i,j}$ as the dot product between row i of $\boldsymbol{A}$ and column j of $\boldsymbol{B}$.