

- *Memory: the cost of storing the representation:* For all but very small values of  $n$  and  $k$ , representing the distribution as a table will require too many values to store.
- *Statistical efficiency:* As the number of parameters in a model increases, so does the amount of training data needed to choose the values of those parameters using a statistical estimator. Because the table-based model has an astronomical number of parameters, it will require an astronomically large training set to fit accurately. Any such model will overfit the training set very badly unless additional assumptions are made linking the different entries in the table (for example, like in back-off or smoothed  $n$ -gram models, section 12.4.1).
- *Runtime: the cost of inference:* Suppose we want to perform an inference task where we use our model of the joint distribution  $P(\mathbf{x})$  to compute some other distribution, such as the marginal distribution  $P(x_1)$  or the conditional distribution  $P(x_2 \mid x_1)$ . Computing these distributions will require summing across the entire table, so the runtime of these operations is as high as the intractable memory cost of storing the model.
- *Runtime: the cost of sampling:* Likewise, suppose we want to draw a sample from the model. The naive way to do this is to sample some value  $u \sim U(0, 1)$ , then iterate through the table, adding up the probability values until they exceed  $u$  and return the outcome corresponding to that position in the table. This requires reading through the whole table in the worst case, so it has the same exponential cost as the other operations.

The problem with the table-based approach is that we are explicitly modeling every possible kind of interaction between every possible subset of variables. The probability distributions we encounter in real tasks are much simpler than this. Usually, most variables influence each other only indirectly.

For example, consider modeling the finishing times of a team in a relay race. Suppose the team consists of three runners: Alice, Bob and Carol. At the start of the race, Alice carries a baton and begins running around a track. After completing her lap around the track, she hands the baton to Bob. Bob then runs his own lap and hands the baton to Carol, who runs the final lap. We can model each of their finishing times as a continuous random variable. Alice's finishing time does not depend on anyone else's, since she goes first. Bob's finishing time depends on Alice's, because Bob does not have the opportunity to start his lap until Alice has completed hers. If Alice finishes faster, Bob will finish faster, all else being