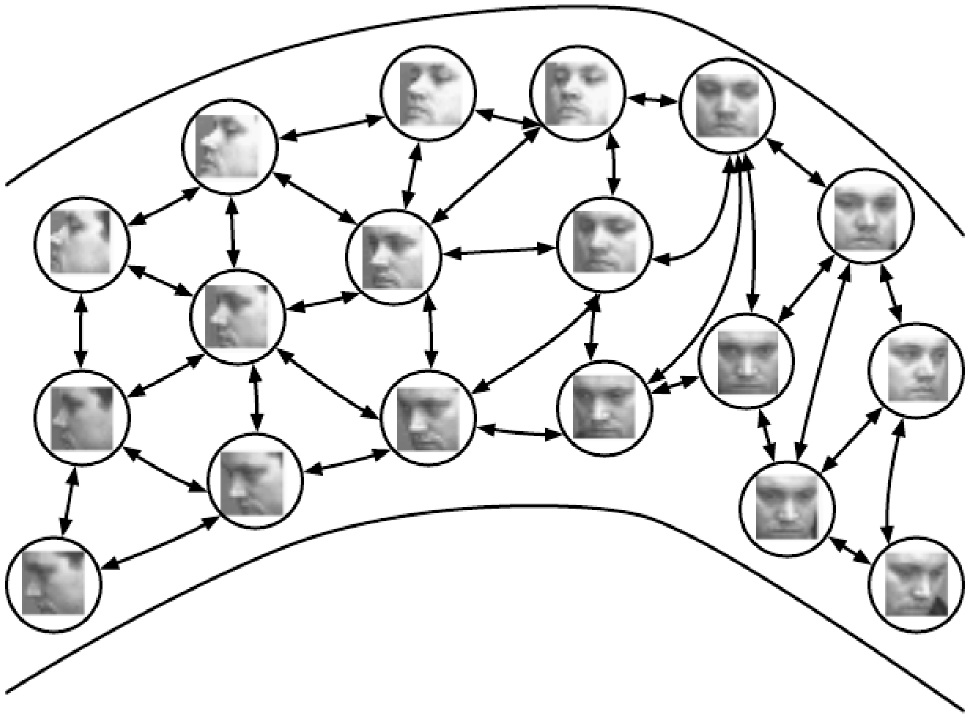

Figure 14.8: Non-parametric manifold learning procedures build a nearest neighbor graph in which nodes represent training examples a directed edges indicate nearest neighbor relationships. Various procedures can thus obtain the tangent plane associated with a neighborhood of the graph as well as a coordinate system that associates each training example with a real-valued vector position, or **embedding**. It is possible to generalize such a representation to new examples by a form of interpolation. So long as the number of examples is large enough to cover the curvature and twists of the manifold, these approaches work well. Images from the QMUL Multiview Face Dataset (Gong *et al.*, 2000).

and Niyogi, 2003; Donoho and Grimes, 2003; Weinberger and Saul, 2004; Hinton and Roweis, 2003; van der Maaten and Hinton, 2008) associate each of nodes with a tangent plane that spans the directions of variations associated with the difference vectors between the example and its neighbors, as illustrated in figure 14.8.

A global coordinate system can then be obtained through an optimization or solving a linear system. Figure 14.9 illustrates how a manifold can be tiled by a large number of locally linear Gaussian-like patches (or "pancakes," because the Gaussians are flat in the tangent directions).

However, there is a fundamental difficulty with such local non-parametric approaches to manifold learning, raised in Bengio and Monperrus (2005): if the manifolds are not very smooth (they have many peaks and troughs and twists), one may need a very large number of training examples to cover each one of