14.5.1 Estimating the Score

Score matching (Hyvärinen, 2005) is an alternative to maximum likelihood. It provides a consistent estimator of probability distributions based on encouraging the model to have the same **score** as the data distribution at every training point $\boldsymbol{x}$. In this context, the score is a particular gradient field:

$$\nabla_{\boldsymbol{x}} \log p(\boldsymbol{x}). \tag{14.15}$$

Score matching is discussed further in section 18.4. For the present discussion regarding autoencoders, it is sufficient to understand that learning the gradient field of $\log p_{\text{data}}$ is one way to learn the structure of p_{data} itself.

A very important property of DAEs is that their training criterion (with conditionally Gaussian $p(\boldsymbol{x} \mid \boldsymbol{h})$) makes the autoencoder learn a vector field $(g(f(\boldsymbol{x})) - \boldsymbol{x})$ that estimates the score of the data distribution. This is illustrated in figure 14.4.

Denoising training of a specific kind of autoencoder (sigmoidal hidden units, linear reconstruction units) using Gaussian noise and mean squared error as the reconstruction cost is equivalent (Vincent, 2011) to training a specific kind of undirected probabilistic model called an RBM with Gaussian visible units. This kind of model will be described in detail in section 20.5.1; for the present discussion it suffices to know that it is a model that provides an explicit $p_{\text{model}}(\boldsymbol{x}; \boldsymbol{\theta})$. When the RBM is trained using **denoising score matching** (Kingma and LeCun, 2010), its learning algorithm is equivalent to denoising training in the corresponding autoencoder. With a fixed noise level, regularized score matching is not a consistent estimator; it instead recovers a blurred version of the distribution. However, if the noise level is chosen to approach 0 when the number of examples approaches infinity, then consistency is recovered. Denoising score matching is discussed in more detail in section 18.5.

Other connections between autoencoders and RBMs exist. Score matching applied to RBMs yields a cost function that is identical to reconstruction error combined with a regularization term similar to the contractive penalty of the CAE (Swersky *et al.*, 2011). Bengio and Delalleau (2009) showed that an autoencoder gradient provides an approximation to contrastive divergence training of RBMs.

For continuous-valued $\boldsymbol{x}$, the denoising criterion with Gaussian corruption and reconstruction distribution yields an estimator of the score that is applicable to general encoder and decoder parametrizations (Alain and Bengio, 2013). This means a generic encoder-decoder architecture may be made to estimate the score