mass function and the reader must infer which probability mass function to use based on the identity of the random variable, rather than the name of the function; $P(\mathrm{x})$ is usually not the same as $P(\mathrm{y})$.

The probability mass function maps from a state of a random variable to the probability of that random variable taking on that state. The probability that $\mathrm{x} = x$ is denoted as $P(x)$, with a probability of 1 indicating that $\mathrm{x} = x$ is certain and a probability of 0 indicating that $\mathrm{x} = x$ is impossible. Sometimes to disambiguate which PMF to use, we write the name of the random variable explicitly: $P(\mathrm{x} = x)$. Sometimes we define a variable first, then use $\sim$ notation to specify which distribution it follows later: $\mathrm{x} \sim P(\mathrm{x})$.

Probability mass functions can act on many variables at the same time. Such a probability distribution over many variables is known as a **joint probability distribution**. $P(\mathrm{x} = x, \mathrm{y} = y)$ denotes the probability that $\mathrm{x} = x$ and $\mathrm{y} = y$ simultaneously. We may also write $P(x, y)$ for brevity.

To be a probability mass function on a random variable x, a function P must satisfy the following properties:

- The domain of P must be the set of all possible states of x.
- $\forall x \in \mathrm{x}, 0 \leq P(x) \leq 1$. An impossible event has probability 0 and no state can be less probable than that. Likewise, an event that is guaranteed to happen has probability 1, and no state can have a greater chance of occurring.
- $\sum_{x \in \mathrm{x}} P(x) = 1$. We refer to this property as being **normalized**. Without this property, we could obtain probabilities greater than one by computing the probability of one of many events occurring.

For example, consider a single discrete random variable x with k different states. We can place a **uniform distribution** on x—that is, make each of its states equally likely—by setting its probability mass function to

$$P(\mathrm{x} = x_i) = \frac{1}{k} \tag{3.1}$$

for all i. We can see that this fits the requirements for a probability mass function. The value $\frac{1}{k}$ is positive because k is a positive integer. We also see that

$$\sum_i P(\mathrm{x} = x_i) = \sum_i \frac{1}{k} = \frac{k}{k} = 1, \tag{3.2}$$

so the distribution is properly normalized.