of obtaining a sample.

In an EBM, we can avoid this chicken and egg problem by sampling using a Markov chain. The core idea of a Markov chain is to have a state $\boldsymbol{x}$ that begins as an arbitrary value. Over time, we randomly update $\boldsymbol{x}$ repeatedly. Eventually $\boldsymbol{x}$ becomes (very nearly) a fair sample from $p(\boldsymbol{x})$. Formally, a Markov chain is defined by a random state $\boldsymbol{x}$ and a transition distribution $T(\boldsymbol{x}' \mid \boldsymbol{x})$ specifying the probability that a random update will go to state $\boldsymbol{x}'$ if it starts in state $\boldsymbol{x}$. Running the Markov chain means repeatedly updating the state $\boldsymbol{x}$ to a value $\boldsymbol{x}'$ sampled from $T(\mathbf{x}' \mid \boldsymbol{x})$.

To gain some theoretical understanding of how MCMC methods work, it is useful to reparametrize the problem. First, we restrict our attention to the case where the random variable $\mathbf{x}$ has countably many states. We can then represent the state as just a positive integer x. Different integer values of x map back to different states $\boldsymbol{x}$ in the original problem.

Consider what happens when we run infinitely many Markov chains in parallel. All of the states of the different Markov chains are drawn from some distribution $q^{(t)}(x)$, where t indicates the number of time steps that have elapsed. At the beginning, $q^{(0)}$ is some distribution that we used to arbitrarily initialize x for each Markov chain. Later, $q^{(t)}$ is influenced by all of the Markov chain steps that have run so far. Our goal is for $q^{(t)}(x)$ to converge to $p(x)$.

Because we have reparametrized the problem in terms of positive integer x, we can describe the probability distribution q using a vector $\boldsymbol{v}$, with

$$q(\mathrm{x} = i) = v_i. \tag{17.17}$$

Consider what happens when we update a single Markov chain's state x to a new state x'. The probability of a single state landing in state x' is given by

$$q^{(t+1)}(x') = \sum_x q^{(t)}(x) T(x' \mid x). \tag{17.18}$$

Using our integer parametrization, we can represent the effect of the transition operator T using a matrix $\boldsymbol{A}$. We define $\boldsymbol{A}$ so that

$$A_{i,j} = T(\mathbf{x}' = i \mid \mathbf{x} = j). \tag{17.19}$$

Using this definition, we can now rewrite equation 17.18. Rather than writing it in terms of q and T to understand how a single state is updated, we may now use $\boldsymbol{v}$ and $\boldsymbol{A}$ to describe how the entire distribution over all the different Markov chains (running in parallel) shifts as we apply an update:

$$\boldsymbol{v}^{(t)} = \boldsymbol{A}\boldsymbol{v}^{(t-1)}. \tag{17.20}$$