

Regardless of whether the state is continuous or discrete, all Markov chain methods consist of repeatedly applying stochastic updates until eventually the state begins to yield samples from the equilibrium distribution. Running the Markov chain until it reaches its equilibrium distribution is called “**burning in**” the Markov chain. After the chain has reached equilibrium, a sequence of infinitely many samples may be drawn from the equilibrium distribution. They are identically distributed but any two successive samples will be highly correlated with each other. A finite sequence of samples may thus not be very representative of the equilibrium distribution. One way to mitigate this problem is to return only every n successive samples, so that our estimate of the statistics of the equilibrium distribution is not as biased by the correlation between an MCMC sample and the next several samples. Markov chains are thus expensive to use because of the time required to burn in to the equilibrium distribution and the time required to transition from one sample to another reasonably decorrelated sample after reaching equilibrium. If one desires truly independent samples, one can run multiple Markov chains in parallel. This approach uses extra parallel computation to eliminate latency. The strategy of using only a single Markov chain to generate all samples and the strategy of using one Markov chain for each desired sample are two extremes; deep learning practitioners usually use a number of chains that is similar to the number of examples in a minibatch and then draw as many samples as are needed from this fixed set of Markov chains. A commonly used number of Markov chains is 100.

Another difficulty is that we do not know in advance how many steps the Markov chain must run before reaching its equilibrium distribution. This length of time is called the **mixing time**. It is also very difficult to test whether a Markov chain has reached equilibrium. We do not have a precise enough theory for guiding us in answering this question. Theory tells us that the chain will converge, but not much more. If we analyze the Markov chain from the point of view of a matrix \mathbf{A} acting on a vector of probabilities \mathbf{v} , then we know that the chain mixes when \mathbf{A}^t has effectively lost all of the eigenvalues from \mathbf{A} besides the unique eigenvalue of 1. This means that the magnitude of the second largest eigenvalue will determine the mixing time. However, in practice, we cannot actually represent our Markov chain in terms of a matrix. The number of states that our probabilistic model can visit is exponentially large in the number of variables, so it is infeasible to represent \mathbf{v} , \mathbf{A} , or the eigenvalues of \mathbf{A} . Due to these and other obstacles, we usually do not know whether a Markov chain has mixed. Instead, we simply run the Markov chain for an amount of time that we roughly estimate to be sufficient, and use heuristic methods to determine whether the chain has mixed. These heuristic methods include manually inspecting samples or measuring correlations between