Matrix product operations have many useful properties that make mathematical analysis of matrices more convenient. For example, matrix multiplication is distributive:

$$\boldsymbol{A}(\boldsymbol{B}+\boldsymbol{C}) = \boldsymbol{A}\boldsymbol{B} + \boldsymbol{A}\boldsymbol{C}. \tag{2.6}$$

It is also associative:

$$\boldsymbol{A}(\boldsymbol{B}\boldsymbol{C}) = (\boldsymbol{A}\boldsymbol{B})\boldsymbol{C}. \tag{2.7}$$

Matrix multiplication is *not* commutative (the condition $\boldsymbol{A}\boldsymbol{B} = \boldsymbol{B}\boldsymbol{A}$ does not always hold), unlike scalar multiplication. However, the dot product between two vectors is commutative:

$$\boldsymbol{x}^\top \boldsymbol{y} = \boldsymbol{y}^\top \boldsymbol{x}. \tag{2.8}$$

The transpose of a matrix product has a simple form:

$$(\boldsymbol{A}\boldsymbol{B})^\top = \boldsymbol{B}^\top \boldsymbol{A}^\top. \tag{2.9}$$

This allows us to demonstrate equation 2.8, by exploiting the fact that the value of such a product is a scalar and therefore equal to its own transpose:

$$\boldsymbol{x}^\top \boldsymbol{y} = \left(\boldsymbol{x}^\top \boldsymbol{y}\right)^\top = \boldsymbol{y}^\top \boldsymbol{x}. \tag{2.10}$$

Since the focus of this textbook is not linear algebra, we do not attempt to develop a comprehensive list of useful properties of the matrix product here, but the reader should be aware that many more exist.

We now know enough linear algebra notation to write down a system of linear equations:

$$\boldsymbol{A}\boldsymbol{x} = \boldsymbol{b} \tag{2.11}$$

where $\boldsymbol{A} \in \mathbb{R}^{m\times n}$ is a known matrix, $\boldsymbol{b} \in \mathbb{R}^m$ is a known vector, and $\boldsymbol{x} \in \mathbb{R}^n$ is a vector of unknown variables we would like to solve for. Each element x_i of $\boldsymbol{x}$ is one of these unknown variables. Each row of $\boldsymbol{A}$ and each element of $\boldsymbol{b}$ provide another constraint. We can rewrite equation 2.11 as:

$$\boldsymbol{A}_{1,:}\boldsymbol{x} = b_1 \tag{2.12}$$

$$\boldsymbol{A}_{2,:}\boldsymbol{x} = b_2 \tag{2.13}$$

$$\dots \tag{2.14}$$

$$\boldsymbol{A}_{m,:}\boldsymbol{x} = b_m \tag{2.15}$$

or, even more explicitly, as:

$$\boldsymbol{A}_{1,1}x_1 + \boldsymbol{A}_{1,2}x_2 + \cdots + \boldsymbol{A}_{1,n}x_n = b_1 \tag{2.16}$$