a small change in the output. Lipschitz continuity is also a fairly weak constraint, and many optimization problems in deep learning can be made Lipschitz continuous with relatively minor modifications.

Perhaps the most successful field of specialized optimization is **convex optimization**. Convex optimization algorithms are able to provide many more guarantees by making stronger restrictions. Convex optimization algorithms are applicable only to convex functions—functions for which the Hessian is positive semidefinite everywhere. Such functions are well-behaved because they lack saddle points and all of their local minima are necessarily global minima. However, most problems in deep learning are difficult to express in terms of convex optimization. Convex optimization is used only as a subroutine of some deep learning algorithms. Ideas from the analysis of convex optimization algorithms can be useful for proving the convergence of deep learning algorithms. However, in general, the importance of convex optimization is greatly diminished in the context of deep learning. For more information about convex optimization, see Boyd and Vandenberghe (2004) or Rockafellar (1997).

4.4 Constrained Optimization

Sometimes we wish not only to maximize or minimize a function $f(\boldsymbol{x})$ over all possible values of $\boldsymbol{x}$. Instead we may wish to find the maximal or minimal value of $f(\boldsymbol{x})$ for values of $\boldsymbol{x}$ in some set $\mathbb{S}$. This is known as **constrained optimization**. Points $\boldsymbol{x}$ that lie within the set $\mathbb{S}$ are called **feasible** points in constrained optimization terminology.

We often wish to find a solution that is small in some sense. A common approach in such situations is to impose a norm constraint, such as $||\boldsymbol{x}|| \leq 1$.

One simple approach to constrained optimization is simply to modify gradient descent taking the constraint into account. If we use a small constant step size ϵ, we can make gradient descent steps, then project the result back into $\mathbb{S}$. If we use a line search, we can search only over step sizes ϵ that yield new $\boldsymbol{x}$ points that are feasible, or we can project each point on the line back into the constraint region. When possible, this method can be made more efficient by projecting the gradient into the tangent space of the feasible region before taking the step or beginning the line search (Rosen, 1960).

A more sophisticated approach is to design a different, unconstrained optimization problem whose solution can be converted into a solution to the original, constrained optimization problem. For example, if we want to minimize $f(\boldsymbol{x})$ for