



Figure 16.10: Examples of complete graphs, which can describe any probability distribution. Here we show examples with four random variables. (*Left*)The complete undirected graph. In the undirected case, the complete graph is unique. (*Right*)A complete directed graph. In the directed case, there is not a unique complete graph. We choose an ordering of the variables and draw an arc from each variable to every variable that comes after it in the ordering. There are thus a factorial number of complete graphs for every set of random variables. In this example we order the variables from left to right, top to bottom.

using directed models, while other distributions can be represented more efficiently using undirected models. In other words, directed models can encode some independences that undirected models cannot encode, and vice versa.

Directed models are able to use one specific kind of substructure that undirected models cannot represent perfectly. This substructure is called an **immorality**. The structure occurs when two random variables a and b are both parents of a third random variable c , and there is no edge directly connecting a and b in either direction. (The name “immorality” may seem strange; it was coined in the graphical models literature as a joke about unmarried parents.) To convert a directed model with graph \mathcal{D} into an undirected model, we need to create a new graph \mathcal{U} . For every pair of variables x and y , we add an undirected edge connecting x and y to \mathcal{U} if there is a directed edge (in either direction) connecting x and y in \mathcal{D} or if x and y are both parents in \mathcal{D} of a third variable z . The resulting \mathcal{U} is known as a **moralized graph**. See figure 16.11 for examples of converting directed models to undirected models via moralization.

Likewise, undirected models can include substructures that no directed model can represent perfectly. Specifically, a directed graph \mathcal{D} cannot capture all of the conditional independences implied by an undirected graph \mathcal{U} if \mathcal{U} contains a **loop** of length greater than three, unless that loop also contains a **chord**. A loop is a sequence of variables connected by undirected edges, with the last variable in the sequence connected back to the first variable in the sequence. A chord is a connection between any two non-consecutive variables in the sequence defining a loop. If \mathcal{U} has loops of length four or greater and does not have chords for these loops, we must add the chords before we can convert it to a directed model. Adding