

Chapter 4

Numerical Computation

Machine learning algorithms usually require a high amount of numerical computation. This typically refers to algorithms that solve mathematical problems by methods that update estimates of the solution via an iterative process, rather than analytically deriving a formula providing a symbolic expression for the correct solution. Common operations include optimization (finding the value of an argument that minimizes or maximizes a function) and solving systems of linear equations. Even just evaluating a mathematical function on a digital computer can be difficult when the function involves real numbers, which cannot be represented precisely using a finite amount of memory.

4.1 Overflow and Underflow

The fundamental difficulty in performing continuous math on a digital computer is that we need to represent infinitely many real numbers with a finite number of bit patterns. This means that for almost all real numbers, we incur some approximation error when we represent the number in the computer. In many cases, this is just rounding error. Rounding error is problematic, especially when it compounds across many operations, and can cause algorithms that work in theory to fail in practice if they are not designed to minimize the accumulation of rounding error.

One form of rounding error that is particularly devastating is **underflow**. Underflow occurs when numbers near zero are rounded to zero. Many functions behave qualitatively differently when their argument is zero rather than a small positive number. For example, we usually want to avoid division by zero (some