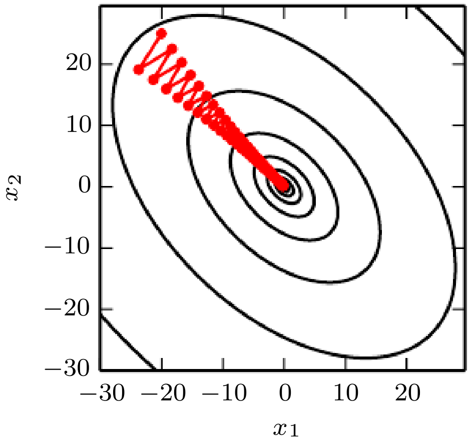

Figure 4.6: Gradient descent fails to exploit the curvature information contained in the Hessian matrix. Here we use gradient descent to minimize a quadratic function $f(\boldsymbol{x})$ whose Hessian matrix has condition number 5. This means that the direction of most curvature has five times more curvature than the direction of least curvature. In this case, the most curvature is in the direction $[1, 1]^\top$ and the least curvature is in the direction $[1, -1]^\top$. The red lines indicate the path followed by gradient descent. This very elongated quadratic function resembles a long canyon. Gradient descent wastes time repeatedly descending canyon walls, because they are the steepest feature. Because the step size is somewhat too large, it has a tendency to overshoot the bottom of the function and thus needs to descend the opposite canyon wall on the next iteration. The large positive eigenvalue of the Hessian corresponding to the eigenvector pointed in this direction indicates that this directional derivative is rapidly increasing, so an optimization algorithm based on the Hessian could predict that the steepest direction is not actually a promising search direction in this context.