



Figure 14.6: An illustration of the concept of a tangent hyperplane. Here we create a one-dimensional manifold in 784-dimensional space. We take an MNIST image with 784 pixels and transform it by translating it vertically. The amount of vertical translation defines a coordinate along a one-dimensional manifold that traces out a curved path through image space. This plot shows a few points along this manifold. For visualization, we have projected the manifold into two dimensional space using PCA. An  $n$ -dimensional manifold has an  $n$ -dimensional tangent plane at every point. This tangent plane touches the manifold exactly at that point and is oriented parallel to the surface at that point. It defines the space of directions in which it is possible to move while remaining on the manifold. This one-dimensional manifold has a single tangent line. We indicate an example tangent line at one point, with an image showing how this tangent direction appears in image space. Gray pixels indicate pixels that do not change as we move along the tangent line, white pixels indicate pixels that brighten, and black pixels indicate pixels that darken.