

training set? The field of **statistical learning theory** provides some answers. If the training and the test set are collected arbitrarily, there is indeed little we can do. If we are allowed to make some assumptions about how the training and test set are collected, then we can make some progress.

The train and test data are generated by a probability distribution over datasets called the **data generating process**. We typically make a set of assumptions known collectively as the **i.i.d. assumptions**. These assumptions are that the examples in each dataset are **independent** from each other, and that the train set and test set are **identically distributed**, drawn from the same probability distribution as each other. This assumption allows us to describe the data generating process with a probability distribution over a single example. The same distribution is then used to generate every train example and every test example. We call that shared underlying distribution the **data generating distribution**, denoted  $p_{\text{data}}$ . This probabilistic framework and the i.i.d. assumptions allow us to mathematically study the relationship between training error and test error.

One immediate connection we can observe between the training and test error is that the expected training error of a randomly selected model is equal to the expected test error of that model. Suppose we have a probability distribution  $p(\mathbf{x}, y)$  and we sample from it repeatedly to generate the train set and the test set. For some fixed value  $\mathbf{w}$ , the expected training set error is exactly the same as the expected test set error, because both expectations are formed using the same dataset sampling process. The only difference between the two conditions is the name we assign to the dataset we sample.

Of course, when we use a machine learning algorithm, we do not fix the parameters ahead of time, then sample both datasets. We sample the training set, then use it to choose the parameters to reduce training set error, then sample the test set. Under this process, the expected test error is greater than or equal to the expected value of training error. The factors determining how well a machine learning algorithm will perform are its ability to:

1. Make the training error small.
2. Make the gap between training and test error small.

These two factors correspond to the two central challenges in machine learning: **underfitting** and **overfitting**. Underfitting occurs when the model is not able to obtain a sufficiently low error value on the training set. Overfitting occurs when the gap between the training error and test error is too large.

We can control whether a model is more likely to overfit or underfit by altering its **capacity**. Informally, a model's capacity is its ability to fit a wide variety of