$$
\boldsymbol{A} = \begin{bmatrix} A_{1,1} & A_{1,2} \\ A_{2,1} & A_{2,2} \\ A_{3,1} & A_{3,2} \end{bmatrix} \Rightarrow \boldsymbol{A}^\top = \begin{bmatrix} A_{1,1} & A_{2,1} & A_{3,1} \\ A_{1,2} & A_{2,2} & A_{3,2} \end{bmatrix}
$$

Figure 2.1: The transpose of the matrix can be thought of as a mirror image across the main diagonal.

the i-th **column** of $\boldsymbol{A}$. When we need to explicitly identify the elements of a matrix, we write them as an array enclosed in square brackets:

$$
\begin{bmatrix} A_{1,1} & A_{1,2} \\ A_{2,1} & A_{2,2} \end{bmatrix}. \tag{2.2}
$$

Sometimes we may need to index matrix-valued expressions that are not just a single letter. In this case, we use subscripts after the expression, but do not convert anything to lower case. For example, $f(\boldsymbol{A})_{i,j}$ gives element (i,j) of the matrix computed by applying the function f to $\boldsymbol{A}$.

- **Tensors**: In some cases we will need an array with more than two axes. In the general case, an array of numbers arranged on a regular grid with a variable number of axes is known as a tensor. We denote a tensor named "A" with this typeface: $\mathsf{\mathbf{A}}$. We identify the element of $\mathsf{\mathbf{A}}$ at coordinates (i,j,k) by writing $\mathsf{A}_{i,j,k}$.

One important operation on matrices is the **transpose**. The transpose of a matrix is the mirror image of the matrix across a diagonal line, called the **main diagonal**, running down and to the right, starting from its upper left corner. See figure 2.1 for a graphical depiction of this operation. We denote the transpose of a matrix $\boldsymbol{A}$ as $\boldsymbol{A}^\top$, and it is defined such that

$$
(\boldsymbol{A}^\top)_{i,j} = A_{j,i}. \tag{2.3}
$$

Vectors can be thought of as matrices that contain only one column. The transpose of a vector is therefore a matrix with only one row. Sometimes we