



Figure 16.11: Examples of converting directed models (top row) to undirected models (bottom row) by constructing moralized graphs. *(Left)* This simple chain can be converted to a moralized graph merely by replacing its directed edges with undirected edges. The resulting undirected model implies exactly the same set of independences and conditional independences. *(Center)* This graph is the simplest directed model that cannot be converted to an undirected model without losing some independences. This graph consists entirely of a single immorality. Because a and b are parents of c , they are connected by an active path when c is observed. To capture this dependence, the undirected model must include a clique encompassing all three variables. This clique fails to encode the fact that $a \perp b$. *(Right)* In general, moralization may add many edges to the graph, thus losing many implied independences. For example, this sparse coding graph requires adding moralizing edges between every pair of hidden units, thus introducing a quadratic number of new direct dependences.