



Figure 7.9: Illustration of the main idea of the tangent prop algorithm (Simard *et al.*, 1992) and manifold tangent classifier (Rifai *et al.*, 2011c), which both regularize the classifier output function  $f(\mathbf{x})$ . Each curve represents the manifold for a different class, illustrated here as a one-dimensional manifold embedded in a two-dimensional space. On one curve, we have chosen a single point and drawn a vector that is tangent to the class manifold (parallel to and touching the manifold) and a vector that is normal to the class manifold (orthogonal to the manifold). In multiple dimensions there may be many tangent directions and many normal directions. We expect the classification function to change rapidly as it moves in the direction normal to the manifold, and not to change as it moves along the class manifold. Both tangent propagation and the manifold tangent classifier regularize  $f(\mathbf{x})$  to not change very much as  $\mathbf{x}$  moves along the manifold. Tangent propagation requires the user to manually specify functions that compute the tangent directions (such as specifying that small translations of images remain in the same class manifold) while the manifold tangent classifier estimates the manifold tangent directions by training an autoencoder to fit the training data. The use of autoencoders to estimate manifolds will be described in chapter 14.

estimate the manifold tangent vectors. The manifold tangent classifier makes use of this technique to avoid needing user-specified tangent vectors. As illustrated in figure 14.10, these estimated tangent vectors go beyond the classical invariants that arise out of the geometry of images (such as translation, rotation and scaling) and include factors that must be learned because they are object-specific (such as moving body parts). The algorithm proposed with the manifold tangent classifier is therefore simple: (1) use an autoencoder to learn the manifold structure by unsupervised learning, and (2) use these tangents to regularize a neural net classifier as in tangent prop (equation 7.67).

This chapter has described most of the general strategies used to regularize neural networks. Regularization is a central theme of machine learning and as such