Recall that the Gaussian probability density function is given by

$$p(x^{(i)}; \mu, \sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{1}{2}\frac{(x^{(i)} - \mu)^2}{\sigma^2}\right). \tag{5.29}$$

A common estimator of the Gaussian mean parameter is known as the **sample mean**:

$$\hat{\mu}_m = \frac{1}{m}\sum_{i=1}^{m} x^{(i)} \tag{5.30}$$

To determine the bias of the sample mean, we are again interested in calculating its expectation:

$$\text{bias}(\hat{\mu}_m) = \mathbb{E}[\hat{\mu}_m] - \mu \tag{5.31}$$

$$= \mathbb{E}\left[\frac{1}{m}\sum_{i=1}^{m} x^{(i)}\right] - \mu \tag{5.32}$$

$$= \left(\frac{1}{m}\sum_{i=1}^{m} \mathbb{E}\left[x^{(i)}\right]\right) - \mu \tag{5.33}$$

$$= \left(\frac{1}{m}\sum_{i=1}^{m} \mu\right) - \mu \tag{5.34}$$

$$= \mu - \mu = 0 \tag{5.35}$$

Thus we find that the sample mean is an unbiased estimator of Gaussian mean parameter.

Example: Estimators of the Variance of a Gaussian Distribution As an example, we compare two different estimators of the variance parameter σ^2 of a Gaussian distribution. We are interested in knowing if either estimator is biased.

The first estimator of σ^2 we consider is known as the **sample variance**:

$$\hat{\sigma}^2_m = \frac{1}{m}\sum_{i=1}^{m} \left(x^{(i)} - \hat{\mu}_m\right)^2, \tag{5.36}$$

where $\hat{\mu}_m$ is the sample mean, defined above. More formally, we are interested in computing

$$\text{bias}(\hat{\sigma}^2_m) = \mathbb{E}[\hat{\sigma}^2_m] - \sigma^2 \tag{5.37}$$