Visualizing Frequency Distributions

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***Abstract*—Frequency distributions are visual displays that organize and present frequency counts so that the information can be interpreted more easily. Frequency distributions show the number of times a given quantity occurs in a set of data. The primary purpose of frequency distributions is to assist the researcher to make sense of the data collected.**

***Keywords*—Data visualization, Frequency distribution, Statistical chart.**

I. Introduction

In today's digital landscape, data has become complex and bulky as it continues to grow from independent sources. In fact, there is so much data available that the term Big Data has become mainstream across industries with data analytics as a driving force behind innovation. For companies hoping to leverage datasets, fully understanding them is key to effectively create strategic advantages in their respective industries. To accomplish this after the initial data collection phase organizing the data into a meaningful form so that a trend, if any, emerging out of the data is a critical step.

One of the most common methods used for organizing data are frequency distributions. A frequency distribution which is an overview of all the distinct values in some variable and the number of times they occur is a standard visualization technique. In practice frequency distributions are most commonly used to summarize categorical variables in datasets. If constructed well a frequency distribution is sometimes enough to make a detailed analysis of the structure of a population with respect to a given characteristic. Furthermore, one can easily spot whether observations are high or low and concentrated in one area or spread out across the entire scale.

II. Properties Of Frequency Distributions

There are four important characteristics of frequency distributions.

A. Measures of Central Location

Oftentimes when frequency distribution data is graphed it is common for a significant amount of data points to cluster around a central value. This clustering is known as the central location or central tendency of a frequency distribution. Once the value that a distribution centers around is known, it can be used to further characterize the rest of the data in the distribution. To calculate a central value several methods exist with each method producing somewhat of a different value. Collectively these methods can be refereed to as Measures of central location and the three most commonly used are:

1) Mean: the sum of all values divided by the total number of values.
2) Median: the middle number in an ordered data set.
3) Mode: the most frequent value.

These three measures are best used in combination with one another. This is because they have complementary strengths and limitations. The mode can be used for any level of measurement, but it is most meaningful for nominal and ordinal values. The median can only be used on data that exhibits some type of order and the mean can only be used on interval and ratio values of measurement because it requires equal spacing between adjacent values or scores in the scale. Most of the time depending on the dataset, only one or two of these measures are applicable at any given time.

B. Measures of Dispersion

A second property of frequency distributions is dispersion or variation, which is the spread of a distribution out from its central value. The dispersion of a frequency distribution is independent of its central location. Figure ?? illustrates this fact, by showing the graph of three theoretical frequency distributions that have the same central location but different amounts of dispersion. Some of the more common measures of dispersion that are used include the following:

1) Range: the difference between the largest and the smallest observation in the dataset.
2) Interquartile Range: the difference between the 25^{th} and 75^{th} percentile (also called the first and third quartile).
3) Standard Deviation: Measures the spread of data about the mean.

Much like measures of central location, measures of dispersion have their own strengths and weaknesses. The biggest advantage of the range is that it is easy to calculate but has many disadvantages to be aware of. For instance, it is very sensitive to outliers and does not use all the observations in a data set. Additionally, it is more informative to provide the actual minimum and the maximum values rather than providing the range a singular value.

The interquartile range has an important advantage given that it can be used as a measure of variability if there are extreme values in the dataset that are not recorded exactly. This leads to the other advantageous feature that the interquartile range is not affected by extreme values. However, the main disadvantage of the interquartile range is that it is not amenable to mathematical manipulation.

Standard Deviation (SD) is perhaps the most famous and widely used measure for dispersion calculation. The reason why is because if the observations are from a normal distribution, then, 68% of observations lie between mean ± 1 SD,

95% of observations lie between mean ± 2 SD and 99.7% of observations lie between mean ± 3 SD. The other advantage of SD is that along with the mean it can be used to detect skewness. However, its biggest disadvantage is its inability to be used as an appropriate measure of dispersion for skewed data.

C. Skewness

Skewness is a measure of symmetry, or more precisely, the lack of symmetry. A distribution is symmetric if it looks the same to the left and right of the center point. The skewness for a normal distribution is zero, and any symmetric data should typically have a skewness near zero. Negative values for the skewness indicate that a dataset has a majority of its data points skewed left while positive values indicate a majority of data points are skewed right. For context when we say "skewed left", we mean that the left tail is long relative to the right tail. Similarly, "skewed right" means that the right tail is long relative to the left tail. We can define the skewness with the formula:

$$Skewness = \sum \frac{(X_i - \bar{X})^3}{ns^3} \tag{1}$$

where n is the sample size, X_i is the i^{th} X value, X is the average and s is the sample standard deviation. However, most software tools such as Microsoft Excel take into account the sample size as well. Therefore, we can slightly modify the formula to the following:

$$\begin{aligned} Skewness &= \frac{n}{(n-1)(n-2)} \sum \frac{(X_i - \bar{X})^3}{s^3} \\ &= \frac{n}{s^3(n-1)(n-2)}(S_{above} - S_{below}) \end{aligned} \tag{2}$$

In practice, as the sample size increases the difference in the results that these two formulas produce is relatively small so either one can be used with confidence.

III. Displaying Frequency Distributions

Frequency distributions can be displayed in a table, or pictorial graphs to fully highlight a dataset.

A. Frequency tables

A frequency distribution is a table that shows "classes" or "intervals" of data entries with a count of the number of entries in each class. The frequency f of a class is the number of data entries in the class. Each class will have a "lower limit"' and an "upper limit" which can be interpreted as the lowest and highest numbers in each class. The class width is defined as the distance between the lower limits of consecutive classes. Before constructing a frequency table, some consideration should be given about the range of values in the dataset. In situations where there are to many class intervals, the likelihood of reducing the bulkiness of the data is highly unlikely. On the other hand, if the total number of classes is minimal, then the shape of the distribution itself cannot be successfully determined. Generally, for most datasets 614 intervals is considered an ideal benchmark. However, this should not be interpreted as the defacto standard as a lot depends on the dataset itself. With that being said, the following are a few general guidelines one can follow when constructing a frequency table.

1) The ideal number of classes can be determined or approximated by the formulas:

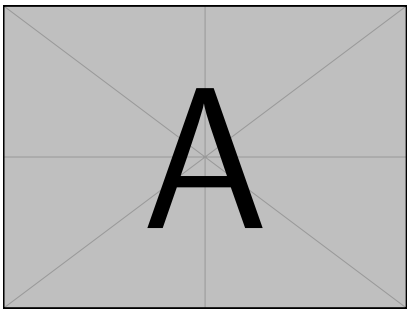
$$C = 1 + 3.3 \log n \tag{3}$$

$$C = \sqrt{n} \tag{4}$$

   where n is the total number of observations in the dataset.
2) Calculate the range of the data by finding the minimum and maximum data values.
3) Using the range, find the width of the classes which can be determined using the formula:

$$\text{Class Width} = \frac{\text{range}}{\text{number of classes}} \tag{5}$$

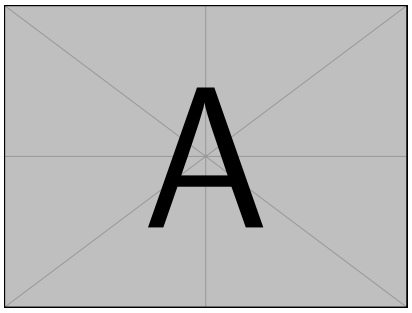
4) To find the class limits use the minimum data entry as the lower limit of the first class. Then to get the lower limit of the next class, add the class width. Continue until you reach the last class. Then find the upper limits of each class.



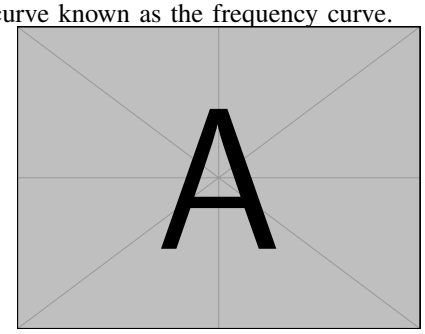
B. Frequency Distribution Graphs

A frequency distribution graph is a diagrammatic illustration of the information in the frequency table.

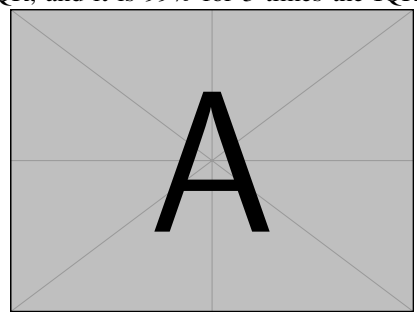
1) Histogram: A histogram is a graphical representation of the variable of interest in the X axis versus the number of observations (frequency) in the Y axis. Percentages can be used if the goal is to compare two histograms with a different number of subjects. Typically, a histogram is used to depict the frequency when data is measured against an interval or ratio scale. Coincidently, there is a striking resemblance between a bar diagram and a histogram. However, they are nothing alike with three important distinctions between them. First off, in a histogram, there is no gap between the bars as the variable is continuous. A bar diagram will oftentimes have a noticeable amount of space between the bars. Secondly, in histograms the width of the bars have meaning and do not need to be of equal length as this depends on the class interval. Whereas in a bar diagram all the bars widths are equal in length. Finally, the area of each bar corresponds to the frequency in a histogram whereas in a bar diagram, it is the height. Figure XX ...



2) Frequency Polygon: A frequency polygon is very similar to a histogram. In fact, they are almost identical except that frequency polygons can be used to compare sets of data or to display a cumulative frequency distribution. A cumulative distribution is a form of a frequency distribution that represents the sum of a class and all the classes below it. They are extremely useful when you need to determine the frequency up to a specific threshold or to easily compare two frequency distributions quickly. Visually, there is also a slight difference where histograms tend to have rectangles while a frequency polygon resembles a line graph. Constructing a frequency polygon is done by connecting all midpoints of the top of the bars in a histogram by a straight line without displaying the bars. Also, when the total frequency is large and the class intervals are narrow, the frequency polygon becomes a smooth curve known as the frequency curve.

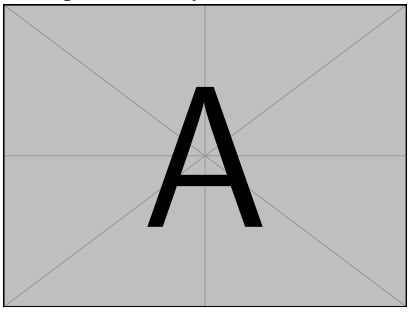


3) Box and whisker plot: This graph, first described by Tukey in 1977, can also be used to illustrate the distribution of data. There is a vertical or horizontal rectangle (box), the ends of which correspond to the upper and lower quartiles (75th and 25th percentile, respectively). Hence the middle 50% of observations are represented by the box. The length of the box indicates the variability of the data. The line inside the box denotes the median (sometimes marked as a plus sign). The position of the median indicates whether the data are skewed or not. If the median is closer to the upper quartile, then they are negatively skewed and if it is near the lower quartile, then positively skewed. The lines outside the box on either side are known as whiskers [Figure 3]. These whiskers are 1.5 times the length of the box, i.e., the interquartile range (IQR). The end of whiskers is called the inner fence and any value outside it is an outlier. If the distribution is symmetrical, then the whiskers are of equal length. If the data are sparse on one side, the corresponding side whisker will be short. The outer fence (usually not marked) is at a distance of three times the IQR on either side of the box. The reason behind having the inner and outer fence at 1.5 and 3 times the IQR, respectively, is the fact that 95% of observations fall within 1.5 times the IQR, and it is 99% for 3 times the IQR.[5]



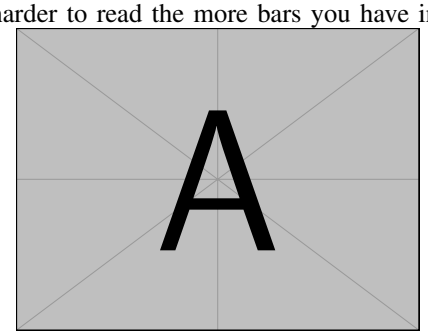
4) Bubble Chart: A Bubble Chart is a multi-variable graph that is a cross between a Scatterplot and a Proportional Area Chart. Like a Scatterplot, Bubble Charts use a Cartesian coordinate system to plot points along a grid where the X and Y axis are separate variables. However. unlike a Scatterplot, each point is assigned a label or category (either displayed alongside or on a legend). Each plotted point then represents a third variable by the area of its circle. Colours can also be used to distinguish between categories or used to represent an additional data variable. Time can be shown either by having it as a variable on one of the axis or by animating the data variables changing over time.

Bubble Charts are typically used to compare and show the relationships between categorised circles, by the use of positioning and proportions. The overall picture of Bubble Charts can be used to analyse for patterns/correlations. Too many bubbles can make the chart hard to read, so Bubble Charts have a limited data size capacity. This can be somewhat remedied by interactivity: clicking or hovering over bubbles to display hidden information, having an option to reorganise or filter out grouped categories. Like with Proportional Area Charts, the sizes of the circles need to be drawn based on the circles area, not its radius or diameter. Not only will the size of the circles change exponentially, but this will lead to misinterpretations by the human visual system.



5) Multi-set Bar Chart: Also known as a Grouped Bar Chart or Clustered Bar Chart. This variation of a Bar Chart is used when two or more data series are plotted side-by-side and grouped together under categories, all on the same axis. Like a Bar Chart, the length of each bar is used to show discrete, numerical comparisons amongst categories. Each data series is assigned an individual colour or a varying shade of the same colour, in order to distinguish them. Each group of bars are then spaced apart from each other. The use of

Multi-set Bar Charts is usually to compare grouped variables or categories to other groups with those same variables or category types. Multi-set Bar Charts can also be used to compare mini Histograms to each other, so each bar in the group would represent the significant intervals of a variable. The downside of Multi-set Bar Charts is that they become harder to read the more bars you have in one group.



IV. Conclusion

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