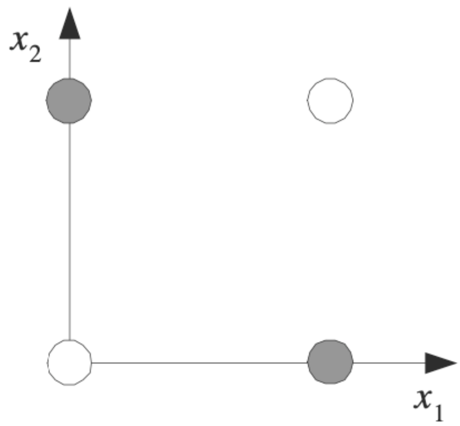
# ML Exercise 7

**7.2.a**

**Table 11.2** Input and output for the XOR function.

| $x_1$ | $x_2$ | $r$ |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 0 |




Grabbed from the previoues chapter

**7.2.b**

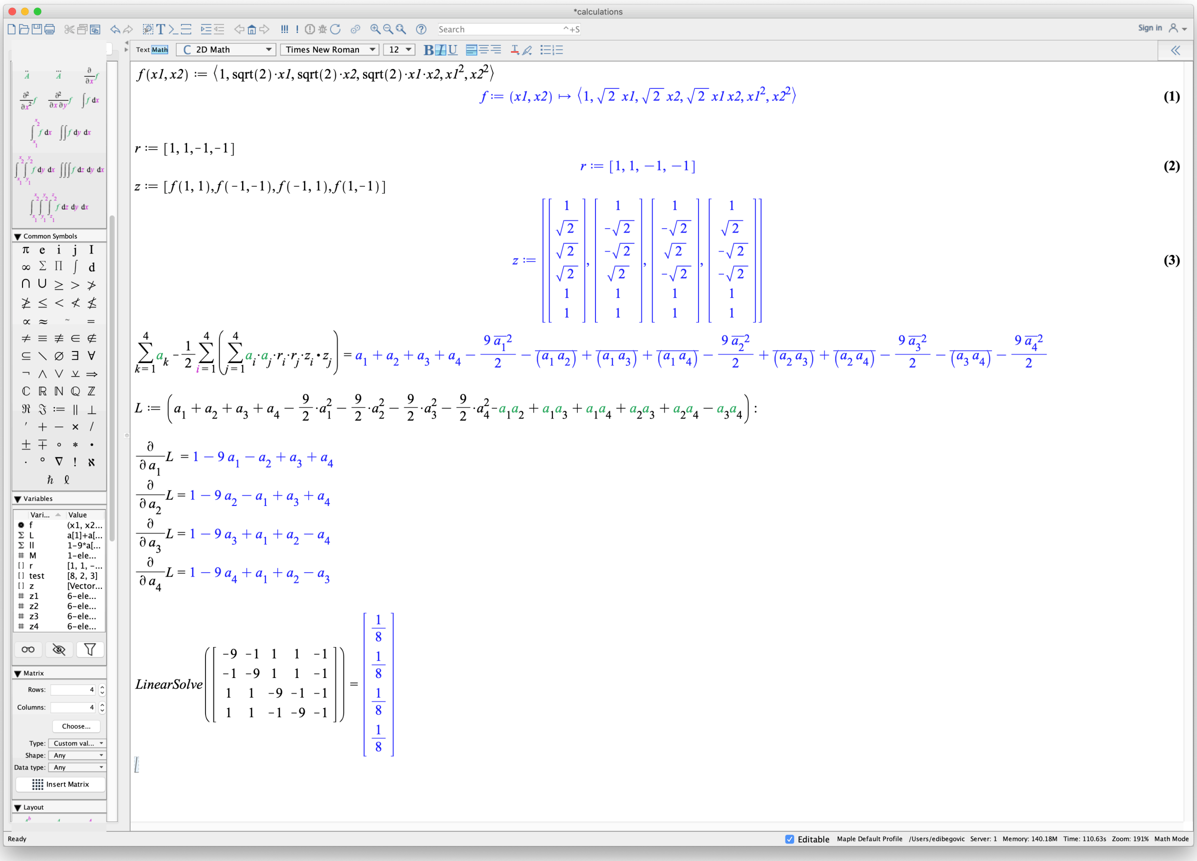
$\mathbf{z}_1 = [1, \sqrt{2}, \sqrt{2}, \sqrt{2}, 1, 1]^T$

$\mathbf{z}_2 = [1, -\sqrt{2}, -\sqrt{2}, \sqrt{2}, 1, 1]^T$

$\mathbf{z}_3 = [1, -\sqrt{2}, \sqrt{2}, -\sqrt{2}, 1, 1]^T$

$\mathbf{z}_4 = [1, \sqrt{2}, -\sqrt{2}, -\sqrt{2}, 1, 1]^T$

**7.2.c** + **7.2.d** + **7.2.e**




## 7.2.f

As all $\alpha$ are greater than 0, all four training points are thus support vectors.

## 7.2.g

$$\mathbf{w} = \sum_{i=1}^{4} \alpha_i r_i \phi(\mathbf{x}_i) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{\sqrt{2}}{2} \\ 0 \\ 0 \end{bmatrix}$$

## 7.2.h

Assuming we should use the polynomial kernel (of degree 2) instead of explicitly computing the feature vectors in $\mathbb{R}^6$ - that is, using the *kernel trick*:

$$g(\mathbf{x}) = \mathbf{w}^T\ \phi(\mathbf{x}) = \sum_{i=1}^{4} \alpha_i r_i \phi(\mathbf{x}_i)^T \phi(\mathbf{x}) = \sum_{i=1}^{4} \alpha_i r_i \mathbf{K}(\mathbf{x}_i, \mathbf{x}) = \sum_{i=1}^{4} \alpha_i r_i (\mathbf{x}_i^T \mathbf{x} + 1)^2$$

If it's simply implied that the discriminant function "takes" the origional attributes and we do the transformations as part of the function, we get:

$$g(\mathbf{x}) = \mathbf{w}^T\ \phi(\mathbf{x}) = \frac{\sqrt{2}}{2} \cdot \sqrt{2}x_1x_2 = x_1x_2$$

**7.2.i**

$g(\mathbf{x}_1) = 1 \cdot 1 = 1$

$g(\mathbf{x}_2) = -1 \cdot -1 = 1$

$g(\mathbf{x}_3) = -1 \cdot 1 = -1$

$g(\mathbf{x}_4) = 1 \cdot -1 = -1$