ML Exercise 6

6.2.a

Show that in the 2D case we have that $rac{1}{\sqrt{(2\pi)^D|oldsymbol{\Sigma}|}}=rac{1}{2\pi\sigma_1\sigma_2\sqrt{1ho^2}}$

$$egin{align} |\mathbf{\Sigma}| &= |egin{bmatrix} \sigma_1^2 &
ho\sigma_1\sigma_2 \
ho\sigma_1\sigma_2 & \sigma_2^2 \end{bmatrix}| \ &= \sigma_1^2 \ \sigma_2^2 -
ho^2 \ \sigma_1^2 \ \sigma_2^2 \ &= \sigma_1^2 \ \sigma_2^2 \ (1-
ho^2) \end{aligned}$$

Adding the new term back to the eq.:

$$\frac{1}{\sqrt{(2\pi)^2 \; \sigma_1^2 \; \sigma_2^2 \; (1-\rho^2)}} = \frac{1}{\sqrt{(2\pi)^2} \; \sqrt{\sigma_1^2} \; \sqrt{\sigma_2^2} \; \sqrt{1-\rho^2}} = \frac{1}{2\pi \; \sigma_1^2 \; \sigma_2^2 \; \sqrt{1-\rho^2}}$$

6.3.a

Intter term represents the "outputs" of the previous layer.

$$egin{aligned} y(\mathbf{x},\mathbf{w}) &= \sum_{j=1}^2 w_j^{(2)} h\left(\sum_{i=1}^2 w_{ji}^{(1)} x_i + w_{j0}^{(1)}
ight) + w_0^{(2)} \ &= w_1^{(2)} \ h\left(w_{11}^{(1)} x_1 + w_{12}^{(1)} x_2 + w_{10}^{(1)}
ight) + w_2^{(2)} \ h\left(w_{21}^{(1)} x_1 + w_{22}^{(1)} x_2 + w_{20}^{(1)}
ight) + w_0^{(2)} \ &= 1 \ h\left(1x_1 + 1x_2 + 0
ight) + (-2) \ h\left(1x_1 + 1x_2 - 1
ight) + 0 \ &= h\left(x_1 + x_2
ight) - 2 \ h\left(x_1 + x_2 - 1
ight) \end{aligned}$$

$$y((0,0), \mathbf{w}) = h(0+0) - 2 h(0+0-1) = 0$$

 $y((1,0), \mathbf{w}) = h(1+0) - 2 h(1+0-1) = 1 - 0 = 1$
 $y((0,1), \mathbf{w}) = h(0+1) - 2 h(0+1-1) = 1 - 0 = 1$
 $y((1,1), \mathbf{w}) = h(1+1) - 2 h(1+1-1) = 2 - 2 = 0$