ML Exercise 6

6.2.a

Show that in the 2D case we have that $\frac{1}{\sqrt{(2\pi)^D|\mathbf{\Sigma}|}} = \frac{1}{2\pi\sigma_1\sigma_2\sqrt{1-\rho^2}}$

$$|\mathbf{\Sigma}| = |\begin{bmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 \\ \rho\sigma_1\sigma_2 & \sigma_2^2 \end{bmatrix}|$$

$$= \sigma_1^2\ \sigma_2^2 - \rho^2\ \sigma_1^2\ \sigma_2^2$$

$$= \sigma_1^2\ \sigma_2^2\ (1-\rho^2)$$

Adding the new term back to the eq.:

$$\frac{1}{\sqrt{(2\pi)^2\ \sigma_1^2\ \sigma_2^2\ (1-\rho^2)}} = \frac{1}{\sqrt{(2\pi)^2}\ \sqrt{\sigma_1^2}\ \sqrt{\sigma_2^2}\ \sqrt{1-\rho^2}} = \frac{1}{2\pi\ \sigma_1^2\ \sigma_2^2\ \sqrt{1-\rho^2}}$$

6.3.a

Intter term represents the "outputs" of the previous layer.

$$y(\mathbf{x}, \mathbf{w}) = \sum_{j=1}^{2} w_j^{(2)}\, h\left(\sum_{i=1}^{2} w_{ji}^{(1)}\, x_i + w_{j0}^{(1)}\right) + w_0^{(2)}$$

$$= w_1^{(2)}\ h\left(w_{11}^{(1)}\, x_1 + w_{12}^{(1)}\, x_2 + w_{10}^{(1)}\right) + w_2^{(2)}\ h\left(w_{21}^{(1)}\, x_1 + w_{22}^{(1)}\, x_2 + w_{20}^{(1)}\right) + w_0^{(2)}$$

$$= 1\ h\,(1x_1 + 1x_2 + 0) + (-2)\ h\,(1x_1 + 1x_2 - 1) + 0$$

$$= h\,(x_1 + x_2) - 2\ h\,(x_1 + x_2 - 1)$$

$$y((0,0), \mathbf{w}) = h\,(0+0) - 2\ h\,(0+0-1) = 0$$

$$y((1,0), \mathbf{w}) = h\,(1+0) - 2\ h\,(1+0-1) = 1 - 0 = 1$$

$$y((0,1), \mathbf{w}) = h\,(0+1) - 2\ h\,(0+1-1) = 1 - 0 = 1$$

$$y((1,1), \mathbf{w}) = h\,(1+1) - 2\ h\,(1+1-1) = 2 - 2 = 0$$

6.3.b