Risk

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Motivation

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- Today we explore how risk affects decision-making in strategic environments.
- We'll start by examining two examples where Nash Equilibrium exists—but might not be desirable.
- These examples highlight why players may deviate from equilibrium when outcomes are uncertain or risky.

Motivation

		COLUMN		
		A	B	C
ROW	A	(2, 2)	(3, 1)	(0, 2)
	B	(1, 3)	(2, 2)	(3, 2)
	C	(2, 0)	(2, 3)	(2, 2)

- Suppose you're the column player.
- Strategy C guarantees you a payoff of 2, regardless of what Row does.
- So—even if (A,A) is a Nash equilibrium—you might prefer to play C.
- Why? Because it's safer. Less payoff variance.

Motivation

		B	
		Left	Right
A	Up	(9, 10)	(8, 9.9)
	Down	(10, 10)	(-1000, 9.9)

- There is a Nash equilibrium in mixed strategies.
- But what if player A plays Down expecting B to play Right...
- ...and B accidentally chooses Left?
- Then A gets –1000. That's an extreme risk.
- Even if it's an equilibrium, is it reasonable to expect players to play it?

Motivation

- Not all Nash equilibria are equal from a risk perspective.
- Players may deviate from equilibrium if it exposes them to high downside risk.
- Strategic choices are shaped not just by expected payoffs—but by the variance and worst-case outcomes.
- Risk aversion plays a critical role in real-world strategy.

The Third-and-One Game: Setup and Payoffs

- Situation: 3rd down, 1 yard to go.
- **Offense** must choose between:

  Run – a safer option.
  Pass – a riskier option.
- **Defense** chooses whether to prepare for a Run or a Pass.
- Payoffs are based on the probability of getting a first down.

		Defense: Run	Defense: Pass
Offense	Run	(0.6V, -0.6V)	(0.7V, -0.7V)
	Pass	(0.8V, -0.8V)	(0.3V, -0.3V)

– No pure strategy Nash equilibrium (zero-sum game).

Solving with Mixed Strategies

- To find equilibrium, Offense randomizes: plays Run with probability p.
- Defense is indifferent when:

$$-0.6Vp - 0.8V(1-p) = -0.7Vp - 0.3V(1-p)$$

- Solving gives: $p = \frac{5}{6} \rightarrow$ Offense plays Run 5 out of 6 times, Pass 1 out of 6.

Surprising insight:

- ⋆ The result is **independent of** V (how big the stakes are).
- ⋆ Theory says the mix stays the same—even in the Super Bowl.
- ⋆ This contradicts common **risk-averse** intuition: higher stakes $\rightarrow$ safer play.
- ⋆ The model assumes no risk aversion (only expected payoffs).

Dealing with Risk

Strategies for Dealing with Risk

- A farmer faces uncertainty: high income (\$160,000) if weather is good, low income (\$40,000) if bad.
- Expected income is \$100,000, but there's significant risk (variance) around that average.
- To reduce risk, the farmer can try safer crops—but imagine it is too late for that.
- Remaining option: **transfer risk** to others (e.g., via insurance or contracts), typically in exchange for a fee or mutual risk sharing.

A. Sharing of Risk

- Sometimes, others face risks that are **negatively correlated** with yours.
- Example: You and your neighbor get good/bad weather in opposite years.
- By agreeing to share income (e.g., promising to pay each other in "lucky" years), you both eliminate risk: guaranteed \$100,000 each.
- Real-life analogy: **currency swaps**.
  - A U.S. firm and a European firm both face exchange rate risk in opposite directions.
  - By exchanging revenues, both can smooth out their profits.

A. Sharing Risk

- Even if risks are **independent**, sharing still reduces exposure to extreme outcomes.
- With a risk-sharing contract, each person earns:

		Neighbor: Lucky	Neighbor: Not
You	Lucky	(160k, 160k)	(100k, 100k)
	Not	(100k, 100k)	(40k, 40k)

- The middle outcome ($100k each) occurs in 2 out of 4 cases.
- Extreme outcomes ($160k or $40k) occur only 25% of the time (for each player).

A. Sharing Risk

- As long as your risks aren't perfectly **positively correlated**, sharing reduces exposure.
- Examples:
  - **Insurance**: pools independent risks across individuals.
  - **Portfolio diversification**: reduces exposure by mixing assets with different risk profiles.
- **But**: effective risk sharing depends on:
  - Observability of outcomes.
  - Enforcement of agreements.
- Incentives matter: people may renege or misreport unless trust or reputation supports the contract.

B. Paying to Reduce Risk

- Suppose your neighbor has a stable income (\$100k), but you face risky outcomes (\$160k or \$40k).
- You agree to pay or receive \$10k depending on who has bad luck:
  - Your income becomes: \$150k (good), \$50k (bad)
  - Your neighbor's income: \$90k (you lucky), \$110k (you unlucky)
- This reduces your income spread from \$120k to \$100k.
- You've **partially insured** your risk—and your neighbor gets compensated for taking on some of it.
- Do you think it makes sense for your neighbor to do this?

B. Paying to Reduce Risk

- In reality, you might pay a **premium** (e.g., \$250–\$3,000) to transfer risk.
- **Insurance markets** arise when many low-risk agents are willing to absorb pieces of others' risk.
- Competition and diversification make it possible to insure most risk at a low cost.
- Broader implication: **financial markets enable entrepreneurship** by letting the risk-averse offload exposure.
- These are not just forms of gambling—they're efficient **mechanisms for risk allocation**.

C. Manipulating Risk

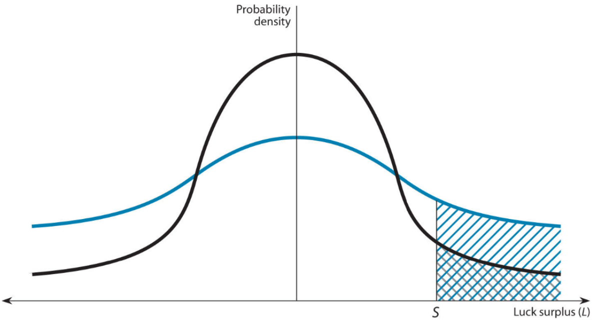
- In many contests, outcomes depend on both **skill** and **luck**.
- You win if:

$$\text{Your skill} + \text{Your luck} > \text{Rival's skill} + \text{Rival's luck}$$

- Rearranged: You win if your **luck surplus** exceeds your **skill deficit**.
- If you're the underdog (less skilled), your chance of winning depends on how much the game allows for extreme outcomes.
- This opens the door for strategic manipulation: you can **increase or decrease risk** to improve your chances.

C. Manipulating Risk

- When luck has low variance: outcomes cluster around the average.
- When luck has high variance: extreme values become more likely.
- **Underdogs** benefit from high variance—they need an upset.
- **Favorites** prefer low variance—it protects their skill edge.



C. Manipulating Risk

- **Underdogs** should take more risk—try uncertain or volatile strategies.
- **Favorites** should reduce risk—play it safe to protect their lead.
- You can also manipulate the **correlation** between your luck and your rival's:
  - Favorites want **high positive correlation** (shared outcomes).
  - Underdogs want **uncorrelated luck** to increase upset chances.
- Example: In sailboat races, the boat behind takes a different route to "de-correlate" luck from the leader.

Risk Aversion

Understanding Risk Aversion

- **Risk aversion** means a person prefers a sure amount of money over a risky gamble with the same expected value.
- Mathematically, it is represented by a **concave utility function**:

$$u''(x) < 0$$

- Don't worry about the math—just think of the shape:
  - The curve bends downward $\rightarrow$ each extra dollar gives you **less extra happiness**.
  - You care more about avoiding losses than making big gains.
- Examples:
  - A farmer prefers $100k for sure over a 50/50 chance of $160k or $40k.
  - A worker prefers a steady salary to a sales job with bonuses but big uncertainty.

Expected Utility and Risk Aversion

- A risk-averse person prefers a sure income over a risky one with the same expected value.
- Suppose utility is:

$$u(x) = \log(x)$$

- A farmer faces:
  - 50% chance of earning \$160,000 or \$40,000
  - Expected income:

$$\mathbb{E}[x] = 0.5 \times 160{,}000 + 0.5 \times 40{,}000 = 100{,}000$$

- The sure thing is \$100,000, or the lottery: 50% \$160k / 50% \$40k

Jensen's Inequality with Log Utility

- Compute utility of the sure thing:

$$u(100{,}000) = \log(100{,}000) \approx 11.51$$

- Compute expected utility of the risky income:

$$\mathbb{E}[u(x)] = 0.5\log(160{,}000) + 0.5\log(40{,}000)$$

$$\approx 0.5(11.98) + 0.5(10.60) = 11.29$$

- Conclusion:

$$\log(\mathbb{E}[x]) > \mathbb{E}[\log(x)] \Rightarrow 11.51 > 11.29$$

- The farmer would **prefer** the guaranteed \$100k over the risky option—even though the expected income is the same.

Why Take On Other People's Risk?

- If most people are risk-averse, they are willing to **pay** to avoid uncertainty.
- Some agents—like **insurance companies**, **banks**, or **well-diversified investors**—can absorb risk more easily.
- These agents are either:
  - Less risk-averse (or even risk-neutral), or
  - Able to **pool independent risks**, so their overall uncertainty is small.
- As a result, they can offer contracts that **reduce others' risk while making a profit**.
- Example: An insurer might collect \$300/year in premiums for covering a \$10,000 loss that almost never happens.