Spatial Analysis and Modeling (GIST 4302/5302)

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Outline of This Week

- Last week, we learned:
  - spatial point pattern analysis (PPA)
  - focus on location distribution of ‘events’
  - Measure the cluster (spatial autocorrelation)in point pattern
- This week, we will learn:
  - How to measure and detect clusters/spatial autocorrelation in areal data (regional data)

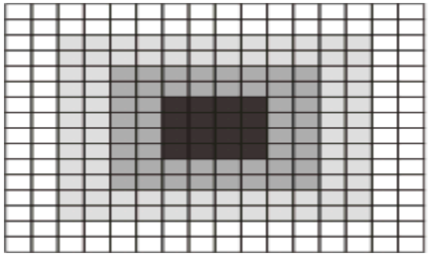
Spatial Autocorrelation

- Spatial autocorrelationship is everywhere
  - Spatial point pattern
    - K, F, G functions
    - Kernel functions
  - Areal/lattice (this topic)
  - Geostatistical data (next topic)

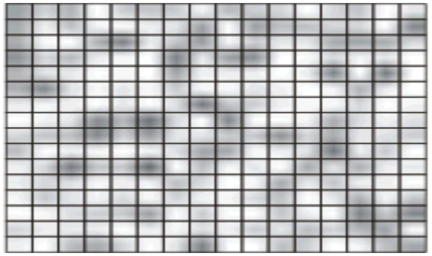
Spatial Autocorrelation of Areal Data

Spatial Autocorrelation

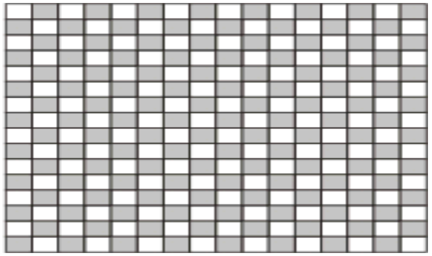
- Tobler's first law of geography
- Spatial auto/cross correlation



If like values tend to cluster together, then the field exhibits high **positive spatial autocorrelation**



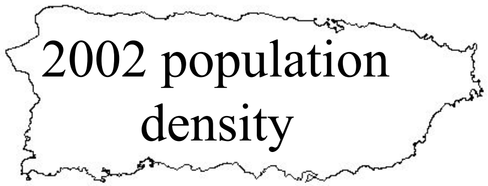
If there is no apparent relationship between attribute value and location then there is **zero spatial autocorrelation**

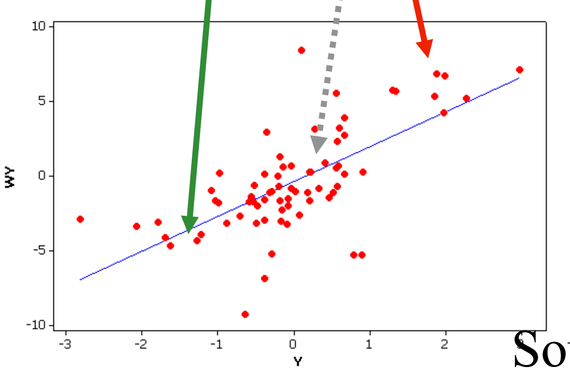


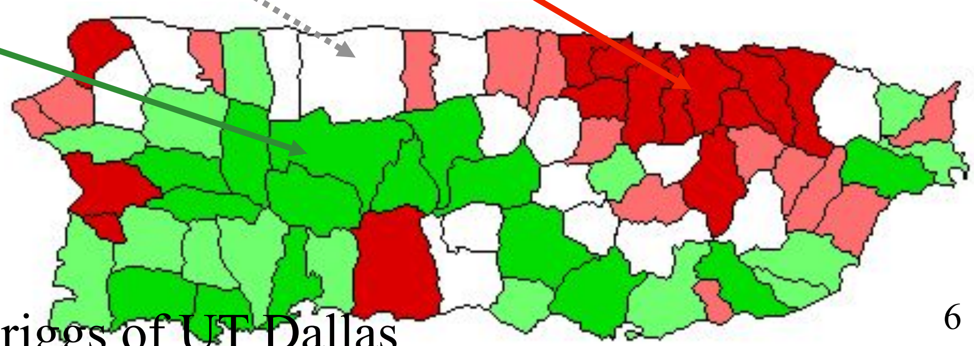
If like values tend to be located away from each other, then there is **negative spatial autocorrelation**

Positive spatial autocorrelation

- high values surrounded by nearby high values
- intermediate values surrounded by nearby intermediate values
- low values surrounded by nearby low values





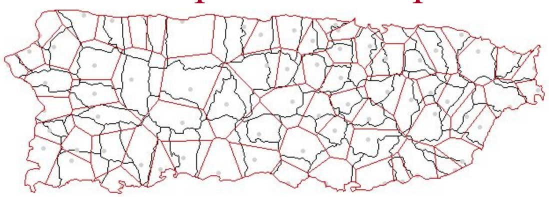


Source: Ron Briggs of UT Dallas

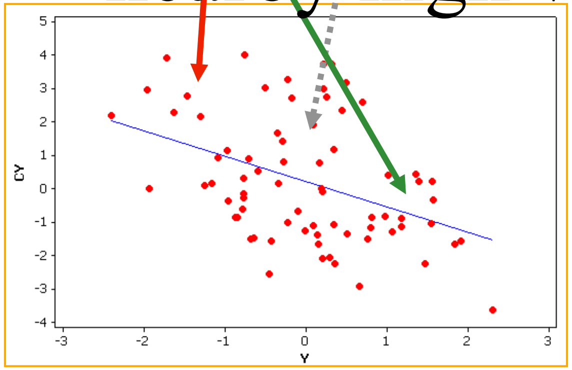
Negative spatial autocorrelation

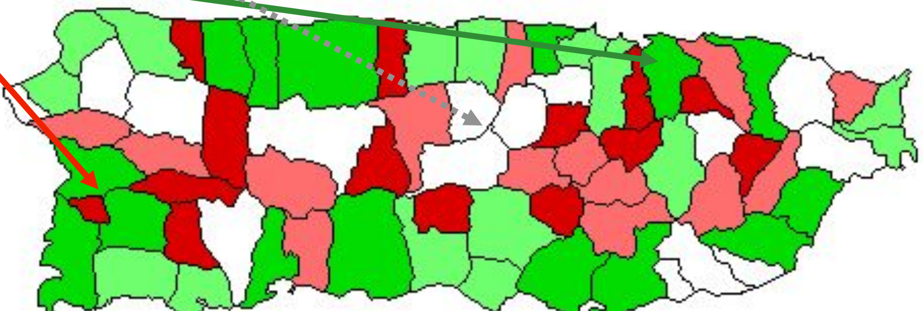
- high values surrounded by nearby low values
- intermediate values surrounded by nearby intermediate values
- low values surrounded by nearby high values

competition for space



Grocery store density





Source: Ron Briggs of UT Dallas

Measuring Spatial Autocorrelation:
the problem of measuring “nearness”

To measure spatial autocorrelation, we must know the “nearness” of our observations as we did for point pattern case

- Which points or polygons are “ near” or “next to” other points or polygons?
  - *Which states are near Texas?*
  - How to <u>measure</u> this?

Seems simple and obvious,

but it is not!

Spatial Weight Matrix

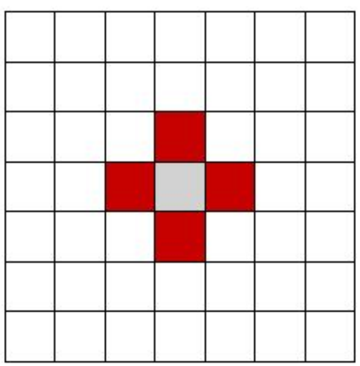
- **Core** concept in statistical analysis of areal data
- Two steps involved:
  - define which relationships between observations are to be given a nonzero weight, i.e., define spatial neighbors
  - assign weights to the neighbors

Spatial Neighbors

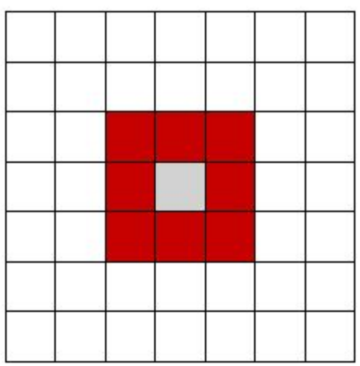
- **Contiguity-based neighbors**
  - Zone *i* and j are neighbors if zone i is contiguity or adjacent to zone *j*
  - But what constitutes contiguity?
- **Distance-based neighbors**
  - Zone i and j are neighbors if the distance between them are less than the threshold distance
  - But what distance do we use?

Contiguity-based Spatial Neighbors

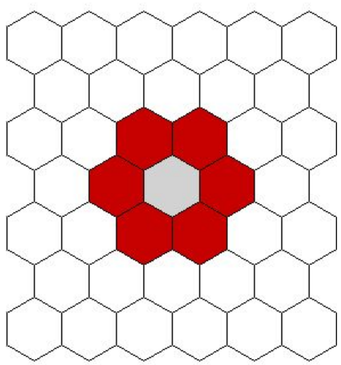
- Sharing a border or boundary
  - Rook: sharing a border
  - Queen: sharing a border <u>or</u> a point



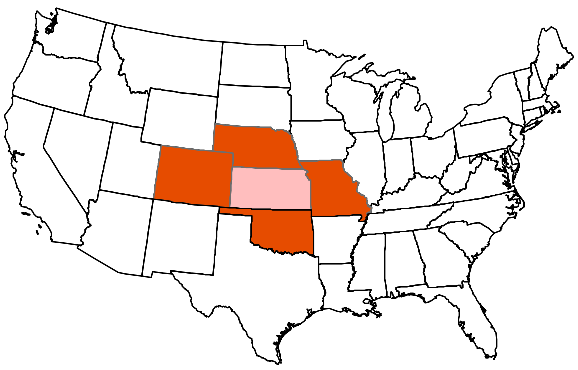
rook



queen



Hexagons

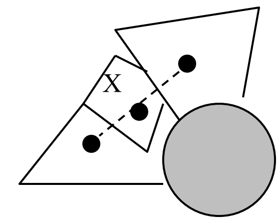


Irregular

Which use?

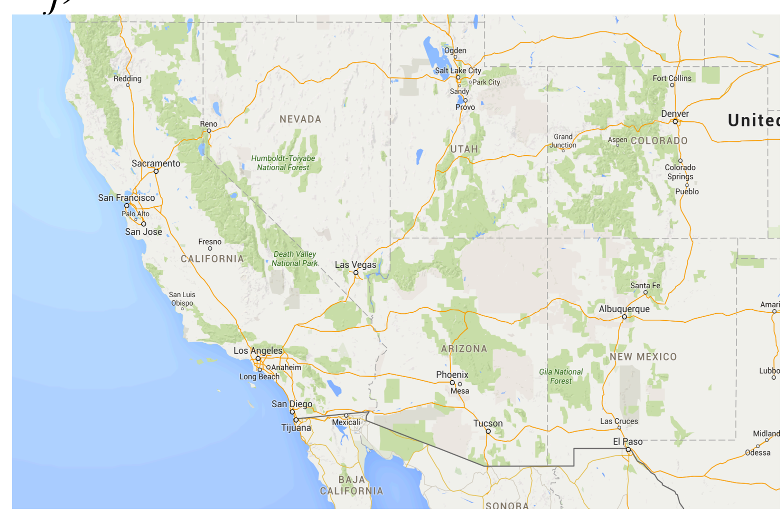
Problem Situations for Irregular Polygons

"Close" but no common border

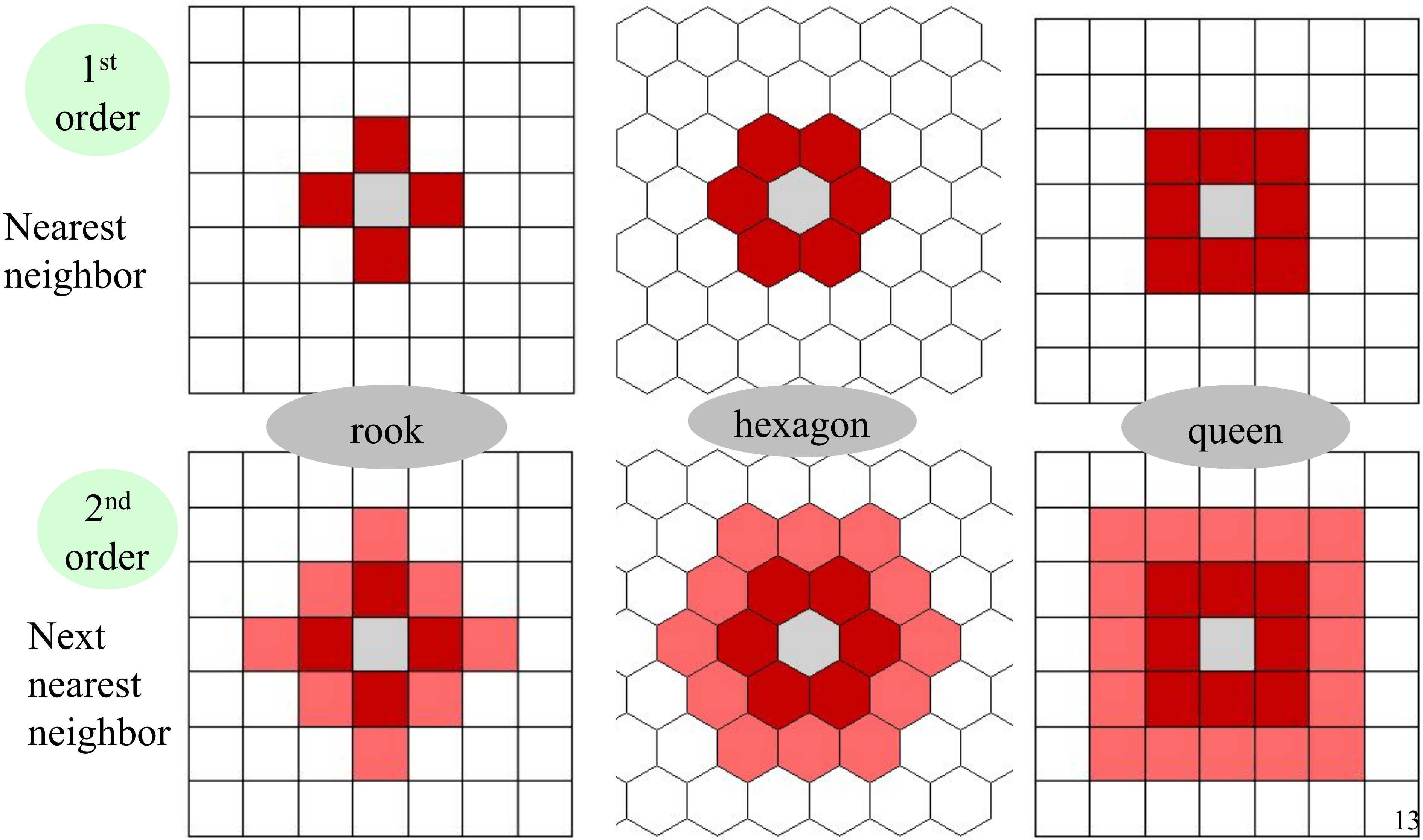


Length of border

- Is Arizona "as close to" California as to Utah?
- Base "closeness" on proportion of shared border, not just one (1) or zero (0)
- w_{ij} = *border length*$_{ij}$ */border length*$_j$)



Higher-Order Contiguity



Distance-based Neighbors

- How to measure distance between polygons?
- Distance metrics
  - 2D Cartesian distance (projected data)
  - 3D spherical distance/great-circle distance (lat/long data)
    - Haversine formula

Haversine formula:

$$a = \sin^2(\Delta\varphi/2) + \cos(\varphi_1).\cos(\varphi_2).\sin^2(\Delta\lambda/2)$$

$$c = 2.\mathrm{atan2}(\sqrt{a}, \sqrt{(1-a)})$$

$$d = R.c$$

where φ is latitude, λ is longitude, R is earth's radius (mean radius = 6,371km)

Distance-based Neighbors

- k-nearest neighbors

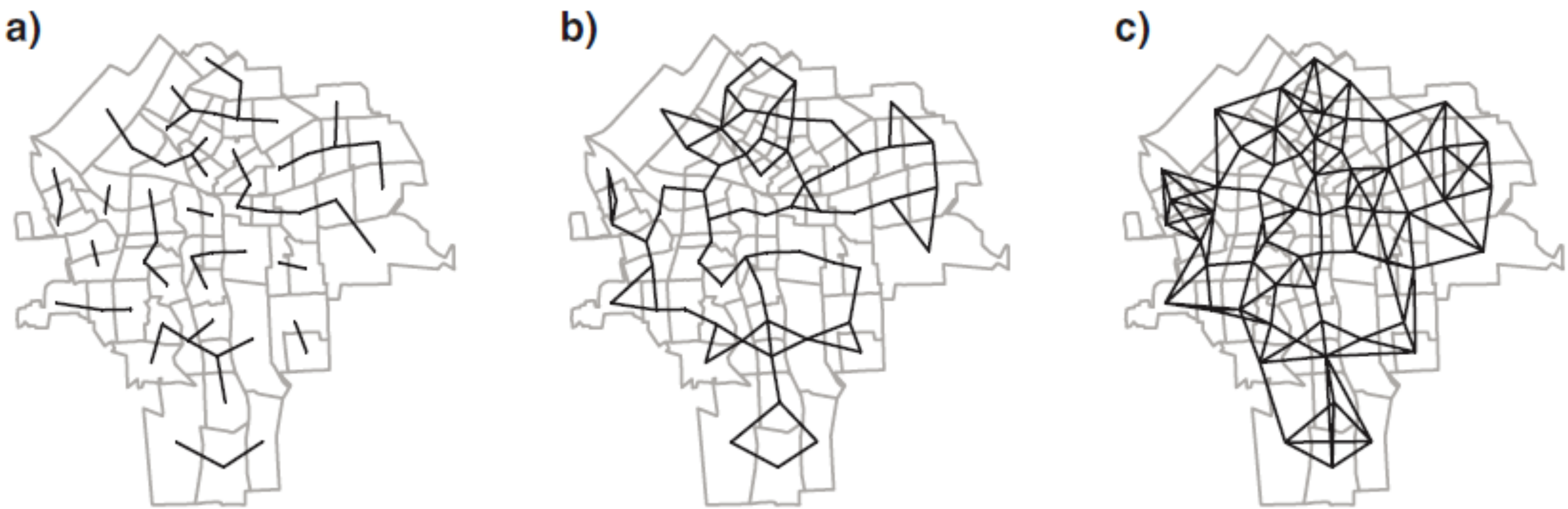


Fig. 9.5. (**a**) $k = 1$ neighbours; (**b**) $k = 2$ neighbours; (**c**) $k = 4$ neighbours

Source: Bivand and Pebesma and Gomez-Rubio

Distance-based Neighbors

- thresh-hold distance (buffer)

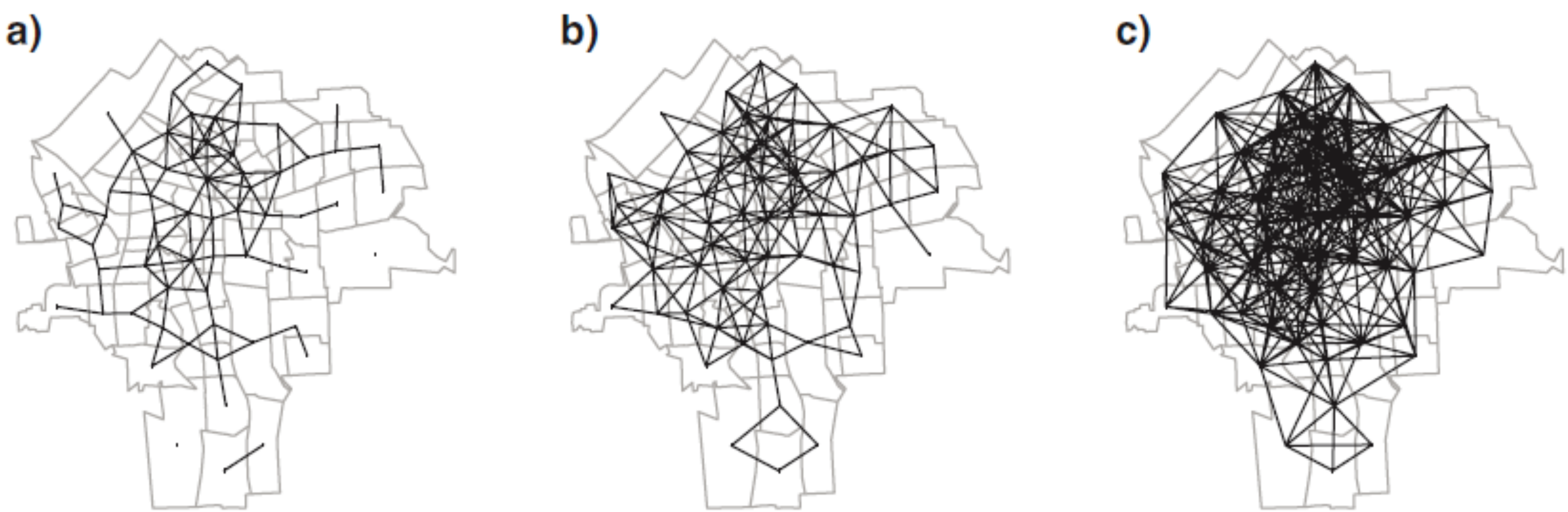
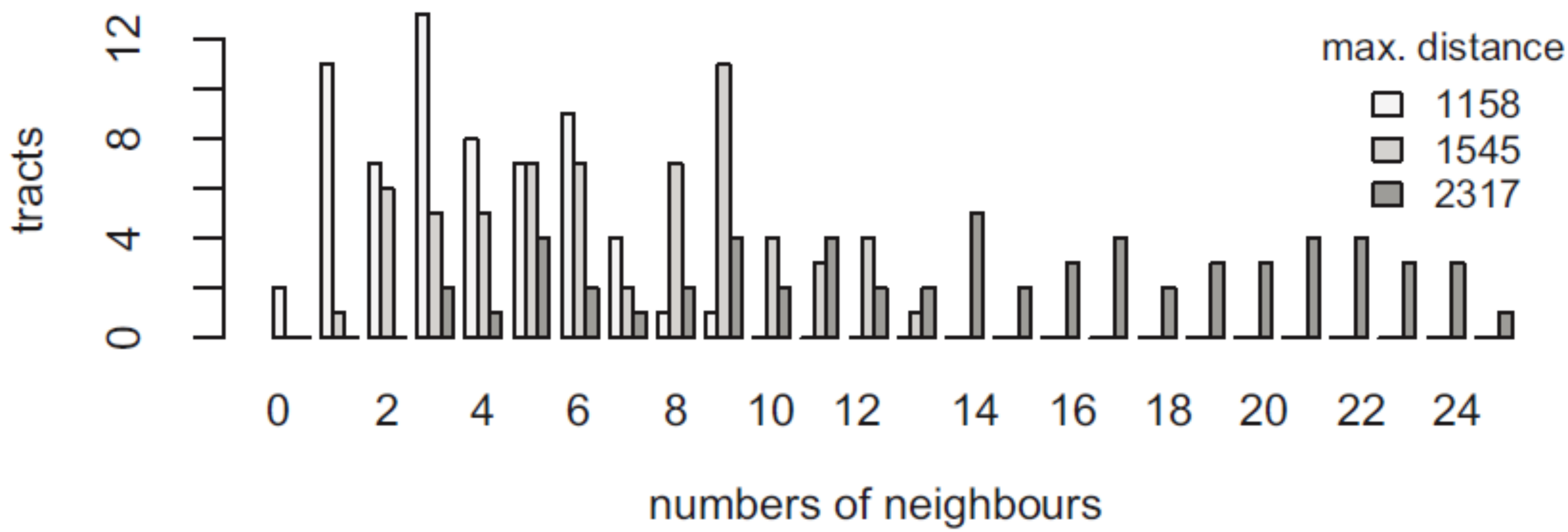


Fig. 9.6. (**a**) Neighbours within 1,158 m; (**b**) neighbours within 1,545 m; (**c**) neigh-
bours within 2,317 m

Source: Bivand and Pebesma and Gomez-Rubio

Neighbor/Connectivity Histogram



Source: Bivand and Pebesma and Gomez-Rubio

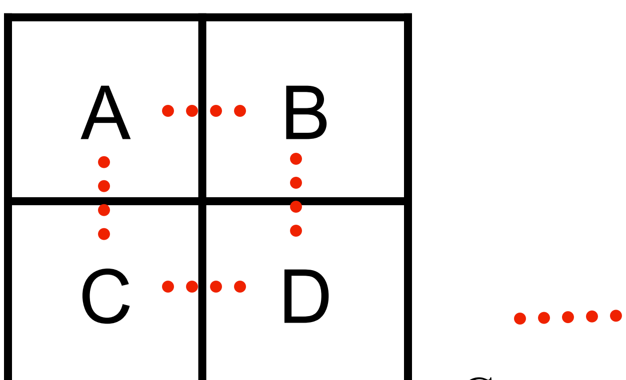
Spatial Weight Matrix

- Spatial weights can be seen as a list of weights indexed by a list of neighbors
- If zone j is not a neighbor of zone i, weights Wij will set to zero
  - The weight matrix can be illustrated as an image
  - Sparse matrix

A Simple Example for Rook case

- Matrix contains a:
  - 1 if share a border
  - 0 if do not share a border

4 areal units



Common border

4x4 matrix

$$
\mathbf{W} = \begin{array}{c|cccc} & A & B & C & D \\ \hline A & 0 & 1 & 1 & 0 \\ B & 1 & 0 & 0 & 1 \\ C & 1 & 0 & 0 & 1 \\ D & 0 & 1 & 1 & 0 \end{array}
$$

1 Washington		1				1																																											
2 Oregon	1		1		1	1																																											
3 California		1		1	1																																												
4 Arizona			1		1				1	1			1																																				
5 Nevada		1	1	1		1			1																																								
6 Idaho	1	1			1		1	1	1																																								
7 Montana						1		1								1	1																																
8 Wyoming						1	1		1				1		1	1																																	
9 Utah				1	1	1		1		1			1																																				
10 New Mexico				1					1		1	1	1																																				
11 Texas										1		1								1	1																											
12 Oklahoma										1	1		1	1						1	1																												
13 Colorado				1				1	1	1		1		1	1																																		
14 Kansas												1	1		1					1																													
15 Nebraska								1					1	1		1			1	1																													
16 South Dakota							1	1							1		1	1	1																														
17 North Dakota							1									1		1																															
18 Minnesota																1	1		1								1																						
19 Iowa															1	1		1		1						1	1																						
20 Missouri												1		1	1				1		1			1	1	1																							
21 Arkansas											1	1								1		1	1	1																									
22 Louisiana											1										1		1																										
23 Mississippi																					1	1		1									1																
24 Tennessee																				1	1		1		1								1	1		1	1												
25 Kentucky																				1				1		1			1	1	1						1												
26 Illinois																			1	1					1		1		1																				
27 Wisconsin																		1	1							1		1																					
28 Michigan																											1		1	1																			
29 Indiana																									1	1		1		1																			
30 Ohio																									1			1	1		1											1							
31 West Virginia																									1					1							1	1				1							
32 Florida																																	1	1															
33 Alabama																							1	1								1		1															
34 Georgia																								1								1	1		1	1													
35 South Carolina																																		1		1													
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37 Virginia																								1	1						1					1		1		1									
38 Maryland																															1						1		1	1		1							
39 Delaware																																						1			1	1							
40 District of Columbia																																					1	1											
41 New Jersey																																							1			1	1						
42 Pennsylvania																														1	1							1	1		1		1						
43 New York																																									1	1		1		1		1	
44 Connecticut																																											1		1	1			
45 Rhode Island																																												1		1			
46 Massachussets																																											1	1	1		1	1	
47 New Hampshire																																														1		1	1
48 Vermont																																											1			1	1		
49 Maine																																															1		

Sparse Contiguity Matrix for US States -- obtained from Anselin's web site (see powerpoint for link)										
Name	**Fips**	**Ncount**	**N1**	**N2**	**N3**	**N4**	**N5**	**N6**	**N7**	**N8**
Alabama	1	4	28	13	12	47				
Arizona	4	5	35	8	49	6	32			
Arkansas	5	6	22	28	48	47	40	29		
California	6	3	4	32	41					
Colorado	8	7	35	4	20	40	31	49	56	
Connecticut	9	3	44	36	25					
Delaware	10	3	24	42	34					
District of Columbia	11	2	51	24						
Florida	12	2	13	1						
Georgia	13	5	12	45	37	1	47			
Idaho	16	6	32	41	56	49	30	53		
Illinois	17	5	29	21	18	55	19			
Indiana	18	4	26	21	17	39				
Iowa	19	6	29	31	17	55	27	46		
Kansas	20	4	40	29	31	8				
Kentucky	21	7	47	29	18	39	54	51	17	
Louisiana	22	3	28	48	5					
Maine	23	1	33							
Maryland	24	5	51	10	54	42	11			
Massachusetts	25	5	44	9	36	50	33			
Michigan	26	3	18	39	55					
Minnesota	27	4	19	55	46	38				
Mississippi	28	4	22	5	1	47				
Missouri	29	8	5	40	17	21	47	20	19	31
Montana	30	4	16	56	38	46				
Nebraska	31	6	29	20	8	19	56	46		
Nevada	32	5	6	4	49	16	41			
New Hampshire	33	3	25	23	50					
New Jersey	34	3	10	36	42					
New Mexico	35	5	48	40	8	4	49			
New York	36	5	34	9	42	50	25			
North Carolina	37	4	45	13	47	51				
North Dakota	38	3	46	27	30					
Ohio	39	5	26	21	54	42	18			
Oklahoma	40	6	5	35	48	29	20	8		
Oregon	41	4	6	32	16	53				
Pennsylvania	42	6	24	54	10	39	36	34		
Rhode Island	44	2	25	9						
South Carolina	45	2	13	37						
South Dakota	46	6	56	27	19	31	38	30		
Tennessee	47	8	5	28	1	37	13	51	21	29
Texas	48	4	22	5	35	40				
Utah	49	6	4	8	35	56	32	16		
Vermont	50	3	36	25	33					
Virginia	51	6	47	37	24	54	11	21		
Washington	53	2	41	16						
West Virginia	54	5	51	21	24	39	42			
Wisconsin	55	4	26	17	19	27				
Wyoming	56	6	49	16	31	8	46	30		

Style of Spatial Weight Matrix

- Row
  - a weight of unity for each neighbor relationship
- Row standardization
  - Symmetry not guaranteed
  - can be interpreted as allowing the calculation of average values across neighbors
- General spatial weights based on distances

Row vs. Row standardization

A	B	C
D	E	F

Divide each number by the **row sum**

Total number of neighbors
--some have more than others

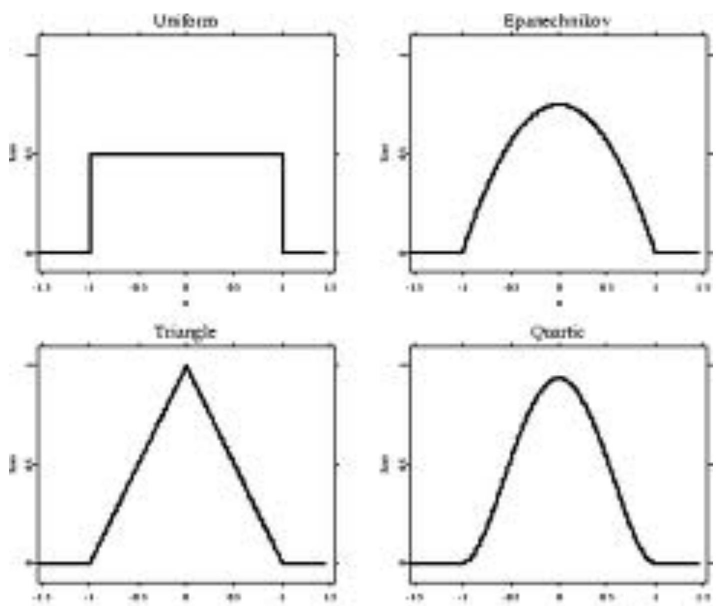
	A	B	C	D	E	F	Row Sum
A	0	1	0	1	0	0	2
B	1	0	1	0	1	0	***3***
C	0	1	0	0	0	1	2
D	1	0	0	0	1	0	2
E	0	1	0	1	0	1	***3***
F	0	0	1	0	1	0	2

Row standardized
--usually use this

	A	B	C	D	E	F	Row Sum
A	0.0	0.5	0.0	0.5	0.0	0.0	1
B	0.3	0.0	0.3	0.0	0.3	0.0	1
C	0.0	0.5	0.0	0.0	0.0	0.5	1
D	0.5	0.0	0.0	0.0	0.5	0.0	1
E	0.0	0.3	0.0	0.3	0.0	0.3	1
F	0.0	0.0	0.5	0.0	0.5	0.0	1

General Spatial Weights Based on Distance

- Decay functions of distance
  - Most common choice is the inverse (reciprocal) of the distance between locations i and j ($w_{ij} = 1/d_{ij}$)
  - Other functions also used
    - inverse of <u>squared</u> distance ($w_{ij} = 1/d_{ij}^2$), or
    - negative exponential ($w_{ij} = e^{-d}$ *or* $w_{ij} = e^{-d^2}$)



Measure of Spatial Autocorrelation

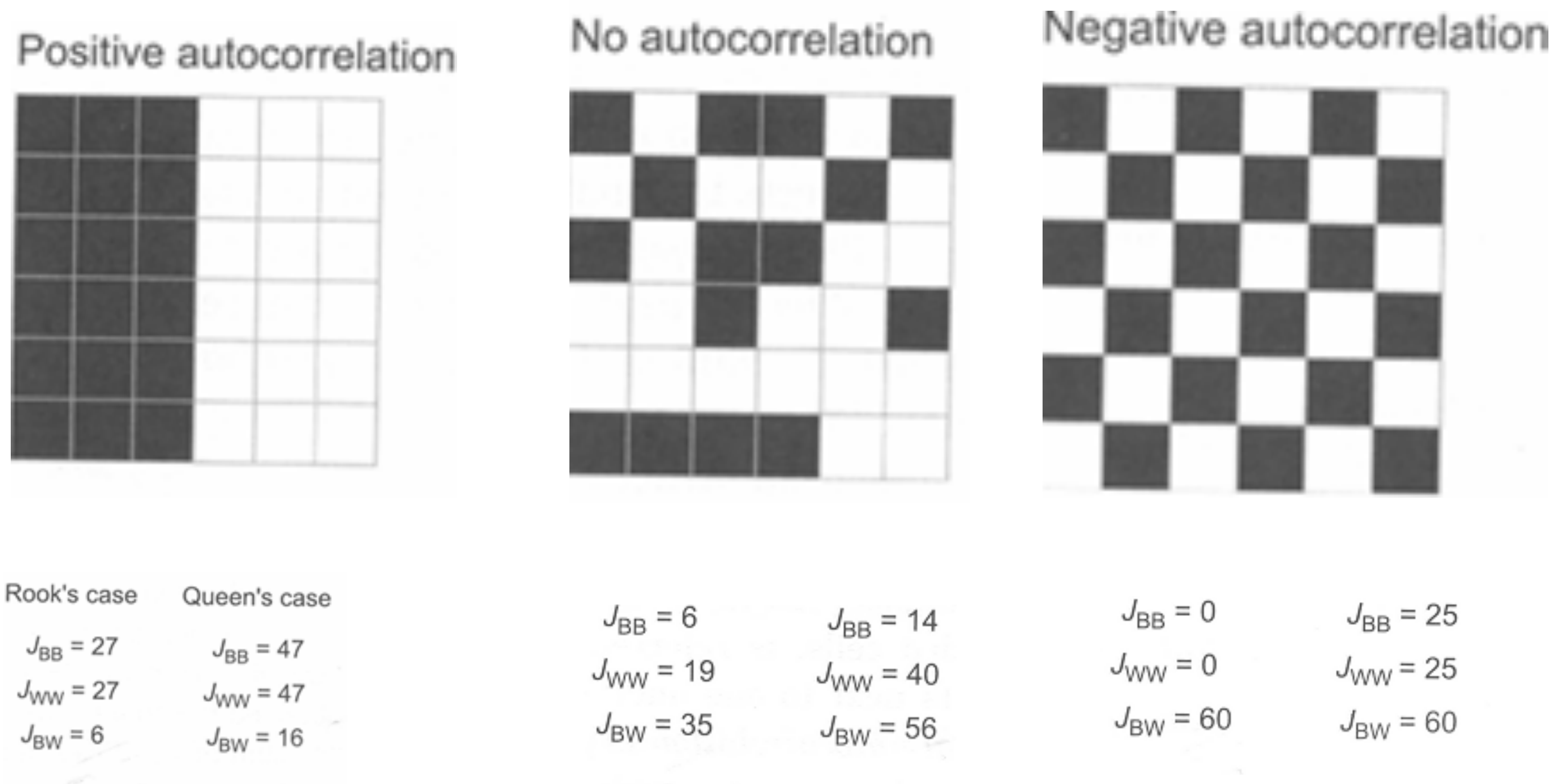
Global Measures and Local Measures

- Global Measures
  - A single value which applies to the entire data set
    - The same pattern or process occurs over the entire geographic area
    - An average for the entire area
- Local Measures
  - A value calculated for <u>each</u> observation unit
    - Different patterns or processes may occur in different parts of the region
    - A unique number for each location
- Global measures usually can be decomposed into a combination of local measures

Global Measures and Local Measures

- Global Measures
  - Join Count
  - Moran’s I (and Getis-Ord’s G)
- Local Measures
  - Local Moran’s I (and Getis-Ord’s G)

Join (or Joint or Joins) Count Statistic



- 60 for Rook Case
- 110 for Queen Case

Join Count: Test Statistic

Test Statistic given by: $Z = \frac{\text{Observed - Expected}}{\text{SD of Expected}}$

Expected = random pattern generated by tossing a coin in each cell.

Expected given by:

$$E(J_{BB}) = kp_B^2$$

$$E(J_{WW}) = kp_W^2$$

$$E(J_{BW}) = 2kp_Bp_W$$

Standard Deviation of Expected (standard error) given by:

$$E(s_{BB}) = \sqrt{kp_B^2 + 2mp_B^3 - (k+2m)p_B^4}$$

$$E(s_{WW}) = \sqrt{kp_W^2 + 2mp_W^3 - (k+2m)p_W^4}$$

$$E(s_{BW}) = \sqrt{2(k+m)p_Bp_W - 4(k+2m)p_B^2p_W^2}$$

Where: k is the total number of joins (neighbors)

p_B is the expected proportion Black, if random

p_W is the expected proportion White

m is calculated from k according to:

$$m = \frac{1}{2}\sum_{i=1}^{n} k_i(k_i - 1)$$

Gore/Bush Presidential Election 2000

	Actual
Jbb	60
Jgg	21
Jbg	28
Total	109

Join Count Statistic for Gore/Bush 2000 by State

candidates	probability
Bush	0.49885
Gore	0.50115

	Actual	Expected	Stan Dev	Z-score
Jbb	60	27.125	8.667	3.7930
Jgg	21	27.375	8.704	-0.7325
Jbg	28	54.500	5.220	-5.0763
Total	109	109.000		

- The expected number of joins is calculated based on the proportion of votes each received in the election (for Bush = 109*.499*.499=27.125)
- There are far more Bush/Bush joins (actual = 60) than would be expected (27)
  - Positive autocorrelation
- There are far fewer Bush/Gore joins (actual = 28) than would be expected (54)
  - Positive autocorrelation
- No strong clustering evidence for Gore (actual = 21 slightly less than 27.375)

Moran's I

- The most common measure of Spatial Autocorrelation
- Use for points <u>or</u> polygons
  - Join Count statistic only for polygons
- Use for a continuous variable (any value)
  - Join Count statistic only for binary variable (1,0)

Patrick Alfred Pierce Moran (1917-1988)

Formula for Moran's I

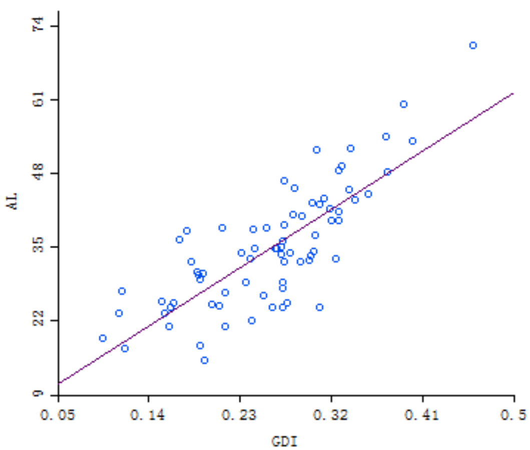
$$I = \frac{N \sum_{i=1}^{n} \sum_{j=1}^{n} w_{ij}(x_i - \bar{x})(x_j - \bar{x})}{\left(\sum_{i=1}^{n} \sum_{j=1}^{n} w_{ij}\right) \sum_{i=1}^{n} (x_i - \bar{x})^2}$$

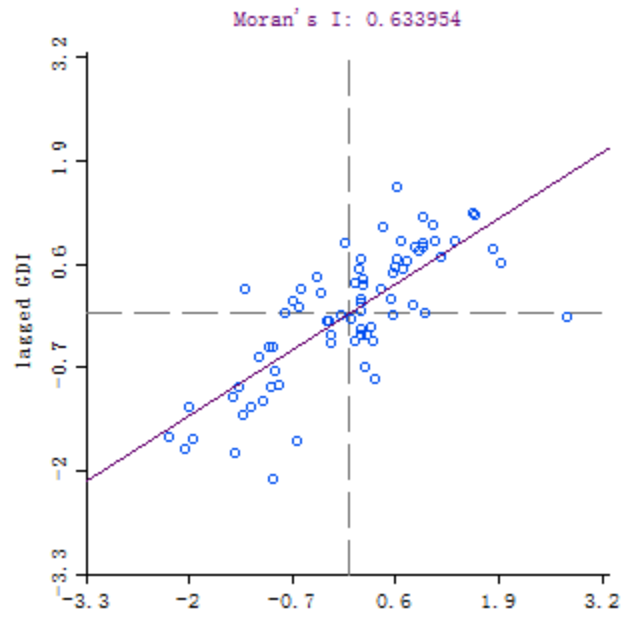
- Where:

N is the number of observations (points or polygons)
$\bar{x}$ is the mean of the variable
X_i is the variable value at a particular location
X_j is the variable value at another location
W_{ij} is a weight indexing location of *i* relative to *j*

Moran's *I* and Correlation Coefficient

- **Correlation Coefficient [-1, 1]**
  - Relationship between <u>two</u> different variables
- **Moran's I [-1, 1]**
- Spatial autocorrelation and often involves <u>one</u> (spatially indexed) variable only
- Correlation between observations of a spatial variable at location X and "spatial lag" of X formed by averaging all the observation at neighbors of X





$$\frac{\sum_{i=1}^{n} 1(y_i - \bar{y})(x_i - \bar{x})/n}{\sqrt{\frac{\sum_{i=1}^{n}(y_i - \bar{y})^2}{n}}\sqrt{\frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}}}$$

Correlation Coefficient

Note the similarity of the numerator (top) to the measures of spatial association discussed earlier if we view Yi as being the Xi for the neighboring polygon

(see next slide)

$$\frac{N\sum_{i=1}^{n}\sum_{j=1}^{n} w_{ij}(x_i - \bar{x})(x_j - \bar{x})}{(\sum_{i=1}^{n}\sum_{j=1}^{n} w_{ij})\sum_{i=1}^{n}(x_i - \bar{x})^2} = \frac{\sum_{i=1}^{n}\sum_{j=1}^{n} w_{ij}(x_i - \bar{x})(x_j - \bar{x}) / \sum_{i=1}^{n}\sum_{j=1}^{n} w_{ij}}{\sqrt{\frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}}\sqrt{\frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}}}$$

Spatial auto-correlation

Source: Ron Briggs of UT Dallas

Correlation Coefficient

$$\frac{\sum_{i=1}^{n} 1(y_i - \bar{y})(x_i - \bar{x})/n}{\sqrt{\frac{\sum_{i=1}^{n}(y_i - \bar{y})^2}{n}}\sqrt{\frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}}}$$

Spatial weights

Yi is the Xi for the neighboring polygon

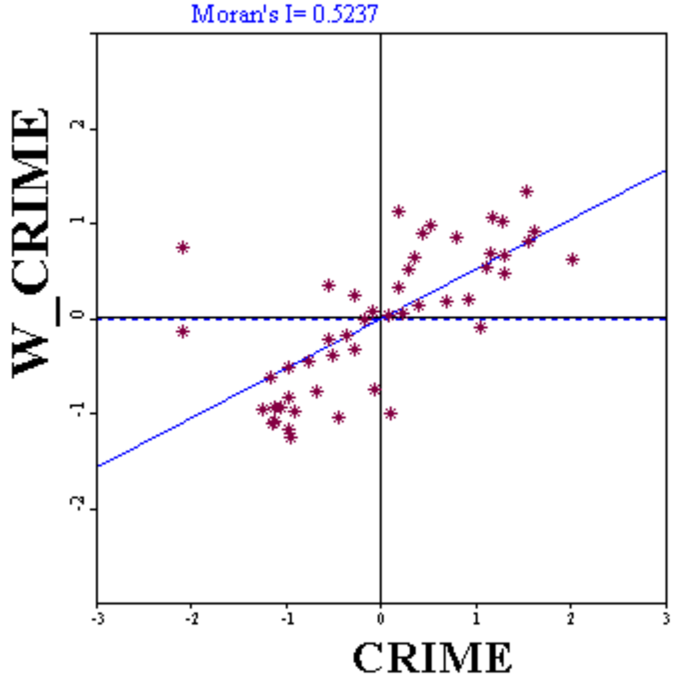
$$\frac{N\sum_{i=1}^{n}\sum_{j=1}^{n} w_{ij}(x_i - \bar{x})(x_j - \bar{x})}{(\sum_{i=1}^{n}\sum_{j=1}^{n} w_{ij})\sum_{i=1}^{n}(x_i - \bar{x})^2} = \frac{\sum_{i=1}^{n}\sum_{j=1}^{n} w_{ij}(x_i - \bar{x})(x_j - \bar{x})/\sum_{i=1}^{n}\sum_{j=1}^{n} w_{ij}}{\sqrt{\frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}}\sqrt{\frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}}}$$

Moran's I

Source: Ron Briggs of UT Dallas

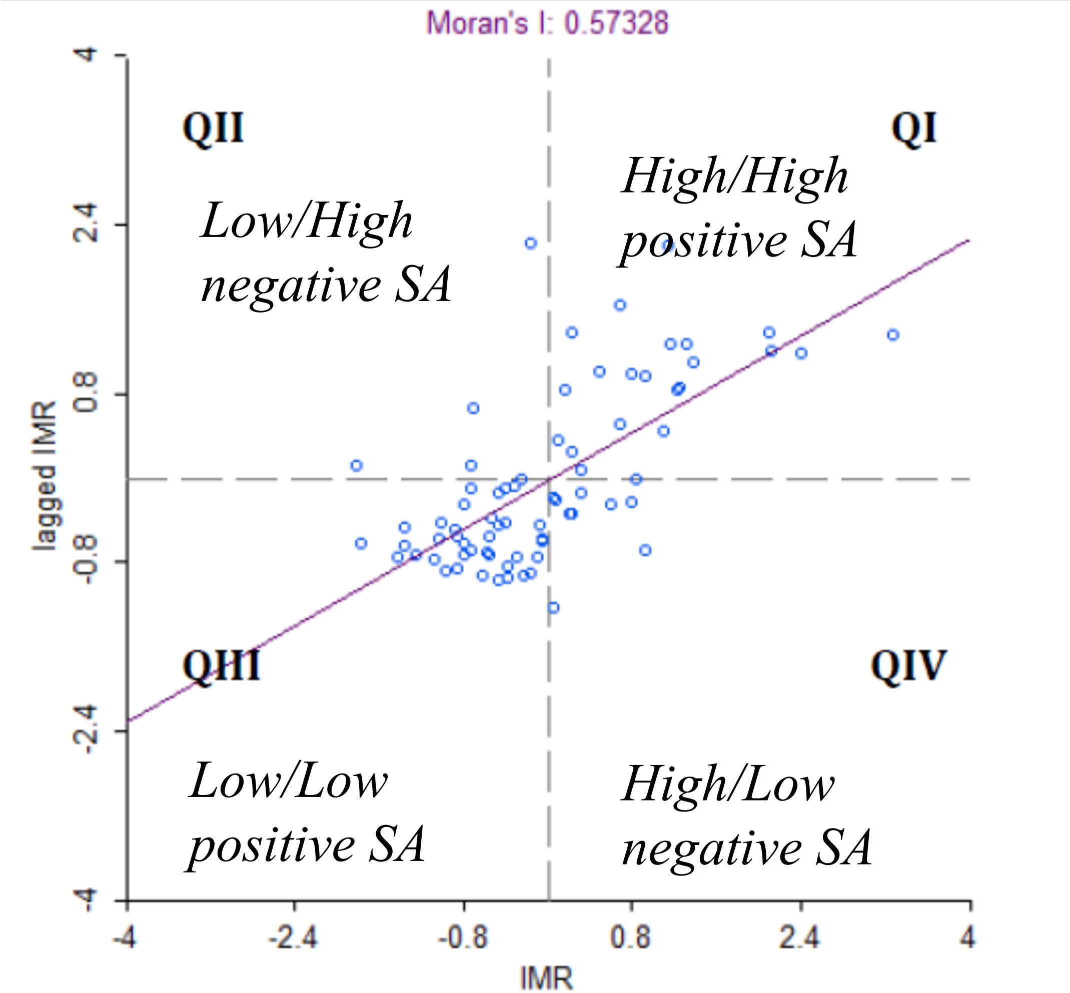
Moran Scatter Plots

We can draw a scatter diagram between these two variables (in standardized form): **X** and **lag-X** (or W_X)

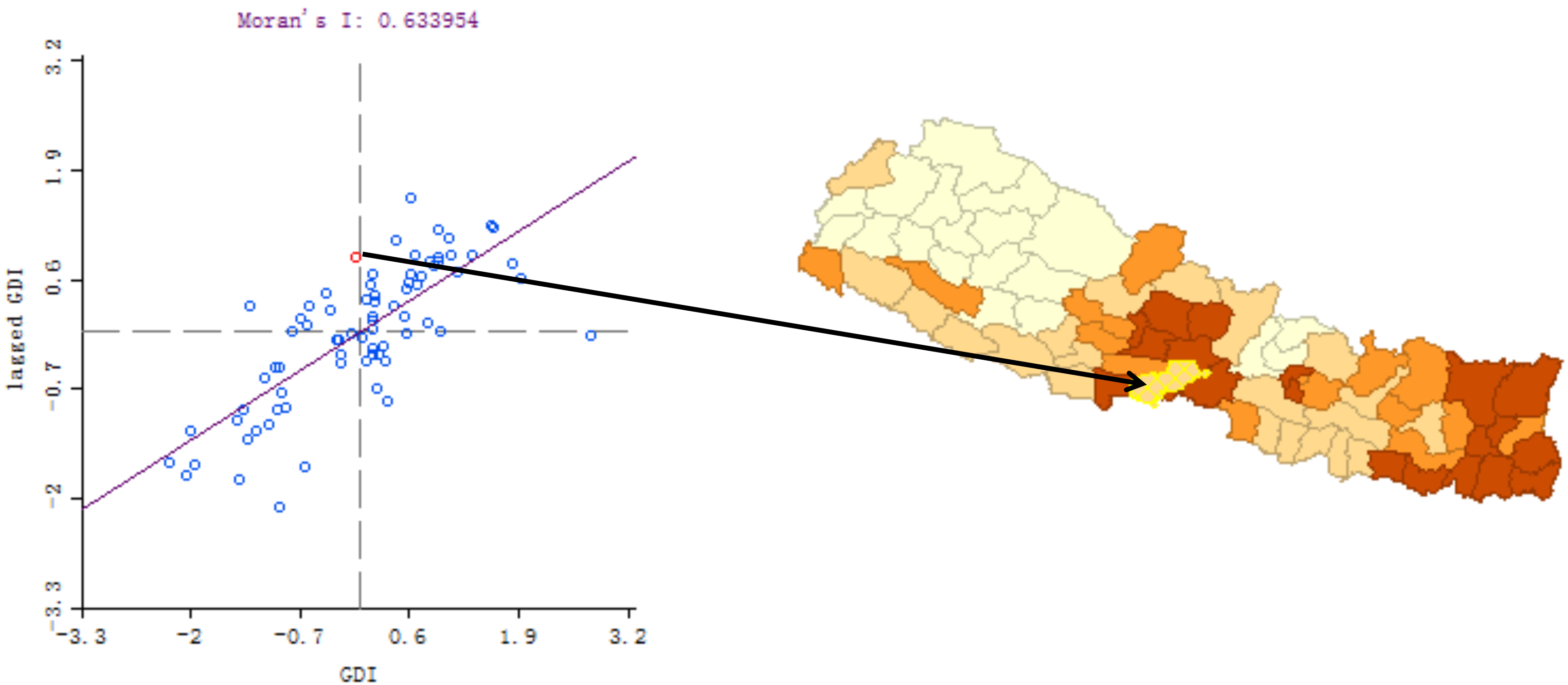


The <u>slope</u> of this *regression line* is Moran's I

Moran Scatter Plots



Moran Scatterplot: Example



Moran's I for rate-based data

- Moran's I is often calculated for rates, such as crime rates (e.g. number of crimes per 1,000 population) or infant mortality rates (e.g. number of deaths per 1,000 births)
- An adjustment should be made, especially if the denominator in the rate (population or number of births) varies greatly (as it usually does)
- Adjustment is know as the *EB adjust*ment:
  - see Assuncao-Reis *Empirical Bayes Standardization* Statistics in Medicine, 1999
- *GeoDA* software includes an option for this adjustment

Statistical Significance Tests for Moran's I

- Based on the normal frequency distribution with

$$Z = \frac{I - E(I)}{S_{error(I)}}$$

Where: I is the calculated value for Moran's I from the sample

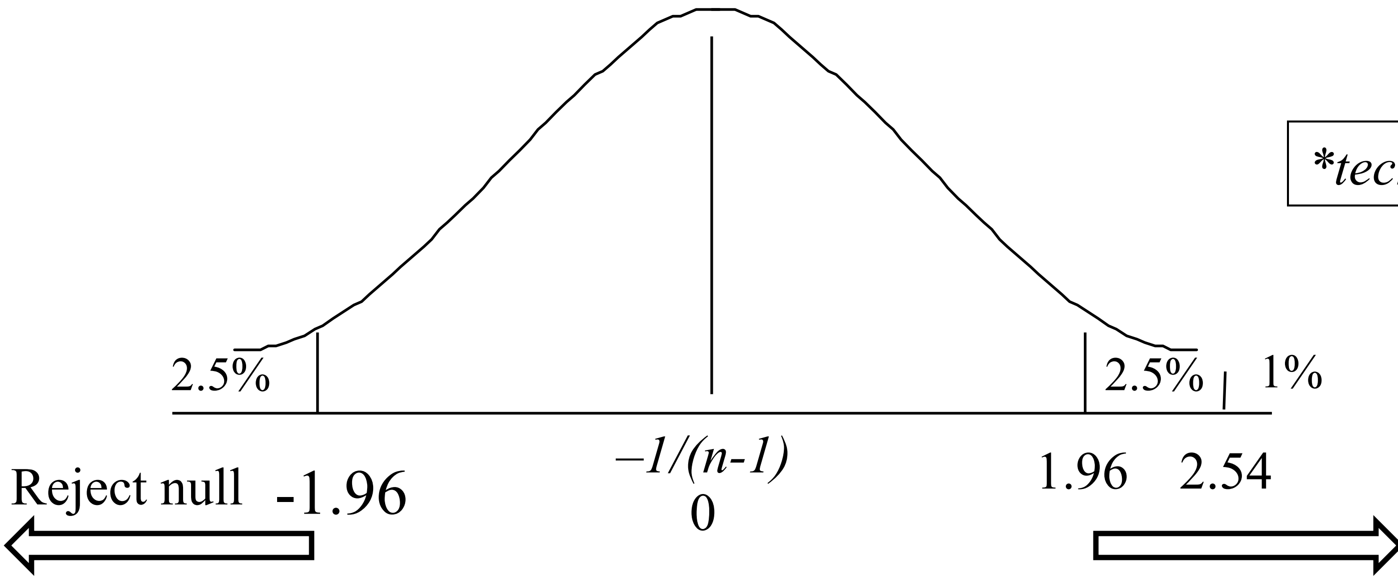
E(I) is the expected value if random

S is the standard error

- Statistical significance test
  - Monte Carlo test, as we did for spatial pattern analysis
  - Permutation test
    - Non-parametric
    - Data-driven, no assumption of the data
    - Implemented in GeoDa

Test Statistic for Normal Frequency Distribution

technically –1/(n-1)



Reject null at 1%

Null Hypothesis: no spatial autocorrelation

*Moran's I = 0

Alternative Hypothesis: spatial autocorrelation exists

*Moran's I > 0

Reject *Null Hypothesis* if Z test statistic > 1.96 (or < -1.96)

---less than a 5% chance that, in the population, there is no spatial autocorrelation

---95% confident that spatial auto correlation exits

Null Hypothesis: no spatial autocorrelation

*Moran's I = 0

Alternative Hypothesis: spatial autocorrelation exists

*Moran's I > 0

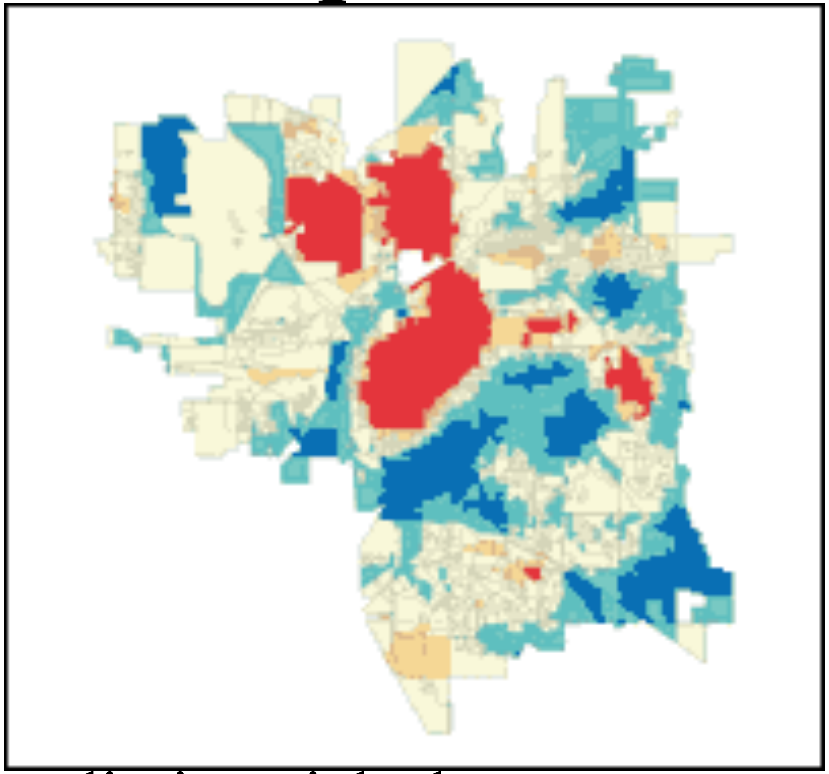
Reject *Null Hypothesis* if Z test statistic > 1.96 (or < -1.96)

---less than a 5% chance that, in the population, there is no spatial autocorrelation

---95% confident that spatial auto correlation exits

Hot Spots and Cold Spots

- What is a *hot spot*?
  - A place where high values cluster together
- What is a *cold spot*?
  - A place where low values cluster together



- Moran's I and Geary's C cannot distinguish them
  - They only indicate clustering
  - Cannot tell if these are hot spots, cold spots, or both

Getis-Ord General/Global G-Statistic

- The G statistic distinguishes between hot spots and cold spots. It identifies *spatial concentrations*.
  - G is relatively large if high values cluster together
  - G is relatively low if low values cluster together
- The General G statistic is interpreted relative to its *expected value*
  - The value for which there is no spatial association
  - G > (larger than) *expected value* ➔ potential "hot spots"
  - G < (smaller than) *expected value* ➔ potential "cold spots"
- Comments:
  - General G will not show negative spatial autocorrelation
  - Should only be calculated for ratio scale data
    - data with a "natural" zero such as crime rates, birth rates
  - Although it was defined using a contiguity (0,1) weights matrix, any type of spatial weights matrix can be used
    - ArcGIS gives multiple options

Local Measures of Spatial Autocorrelation

Local Indicators of Spatial Association (LISA)

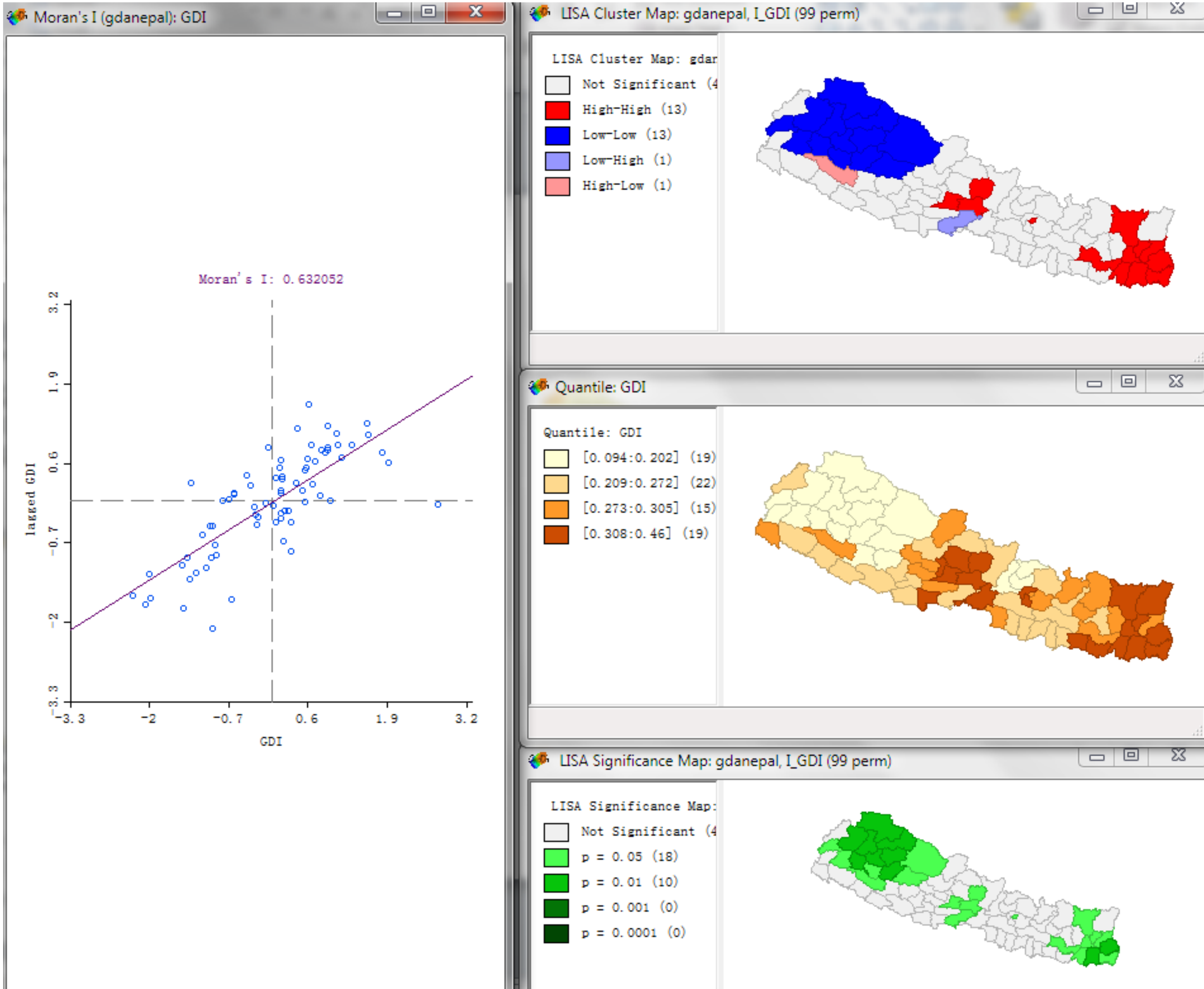
- Local versions of *Moran's I, and the Getis-Ord G statistic*
- Moran's I is most commonly used, and the local version is often called Anselin's LISA, or just LISA

See:
Luc Anselin 1995 *Local Indicators of Spatial Association-LISA* Geographical Analysis 27: 93-115

Local Indicators of Spatial Association (LISA)

- The statistic is calculated for **each** areal unit in the data
- For each polygon, the index is calculated based on neighboring polygons with which it shares a border
- A measure is available for each polygon, these can be mapped to indicate how spatial autocorrelation varies over the study region
- Each index has an associated test statistic, we can also map which of the polygons has a statistically significant relationship with its neighbors, and show type of relationship

Example:

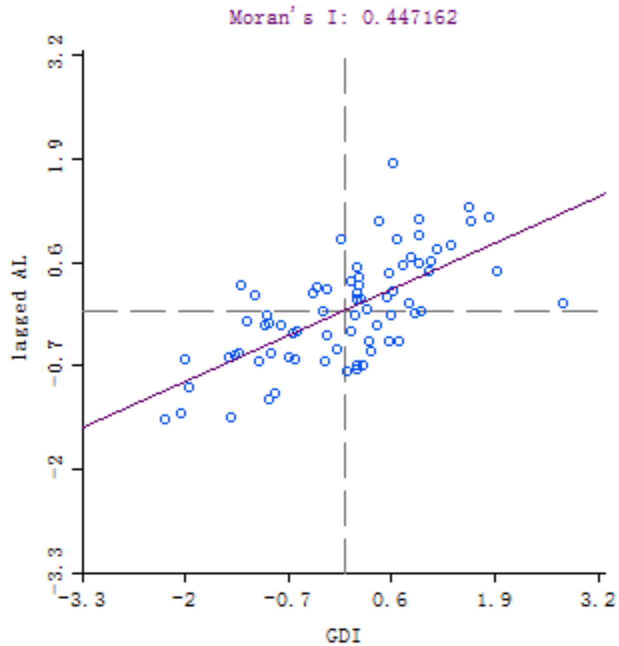


Moran's I (gdanepal): GDI
Moran's I: 0.632052
lagged GDI
GDI
-3.3
-2
-0.7
0.6
1.9
3.2
LISA Cluster Map: gdanepal, I_GDI (99 perm)
LISA Cluster Map: gdar
Not Significant (4
High-High (13)
Low-Low (13)
Low-High (1)
High-Low (1)
Quantile: GDI
Quantile: GDI
[0.094:0.202] (19)
[0.209:0.272] (22)
[0.273:0.305] (15)
[0.308:0.46] (19)
LISA Significance Map: gdanepal, I_GDI (99 perm)
LISA Significance Map:
Not Significant (4
p = 0.05 (18)
p = 0.01 (10)
p = 0.001 (0)
p = 0.0001 (0)

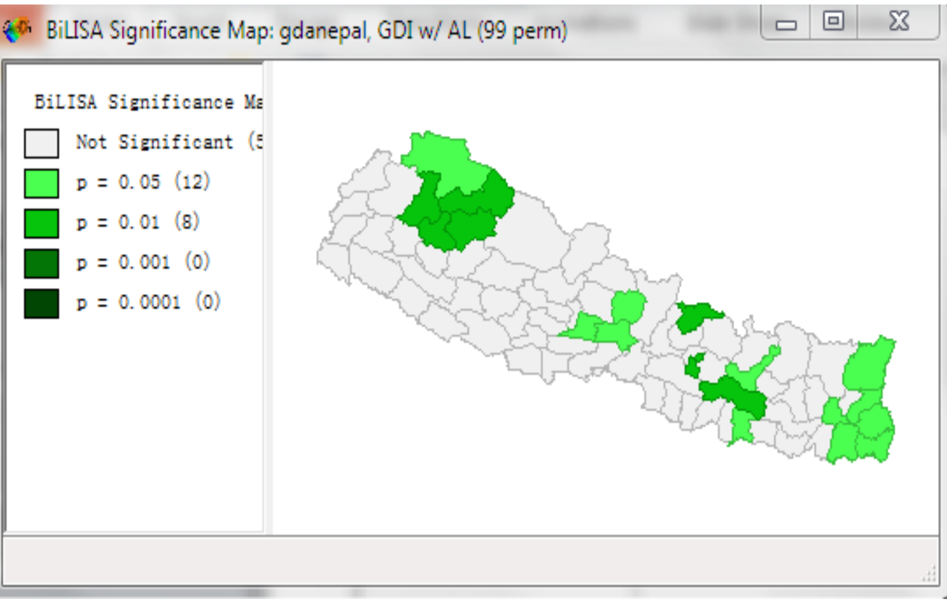
Bivariate LISA

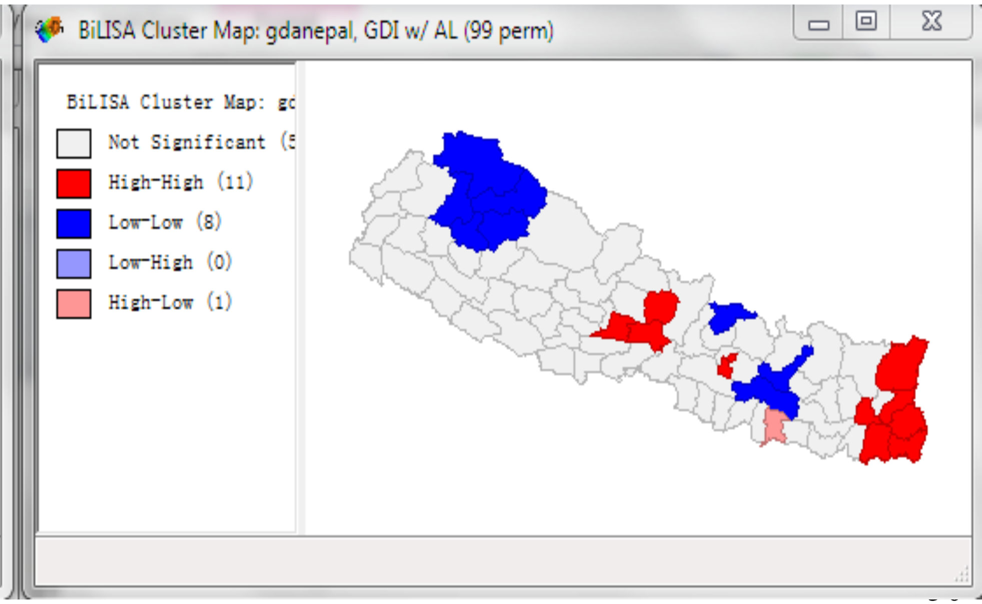
- Moran's I is the correlation between X and Lag-X--the same variable but in nearby areas
  - Univariate Moran's I
- Bivariate Moran's I is a correlation between X and a different variable in nearby areas.

Moran Scatter Plot for GDI vs AL



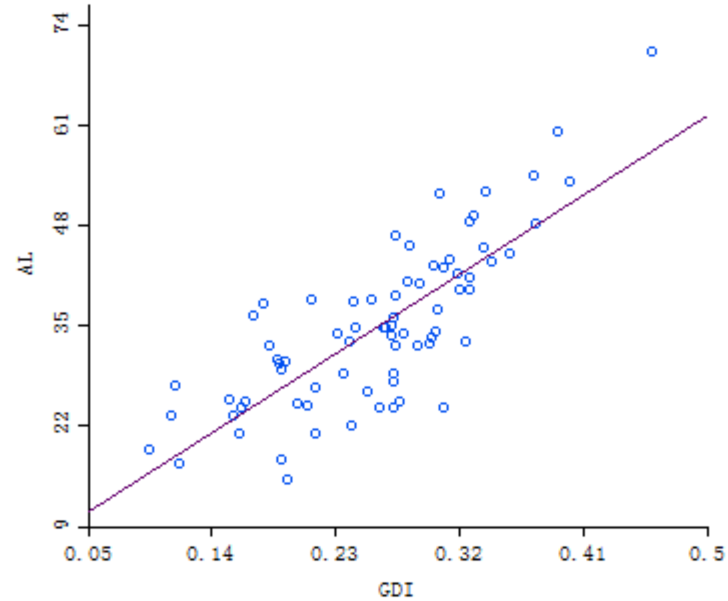
Moran Significance Map for GDI vs. AL

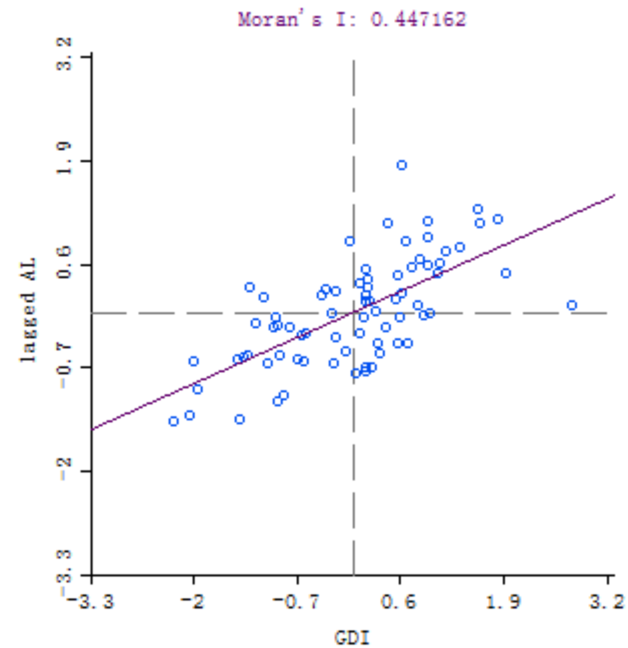




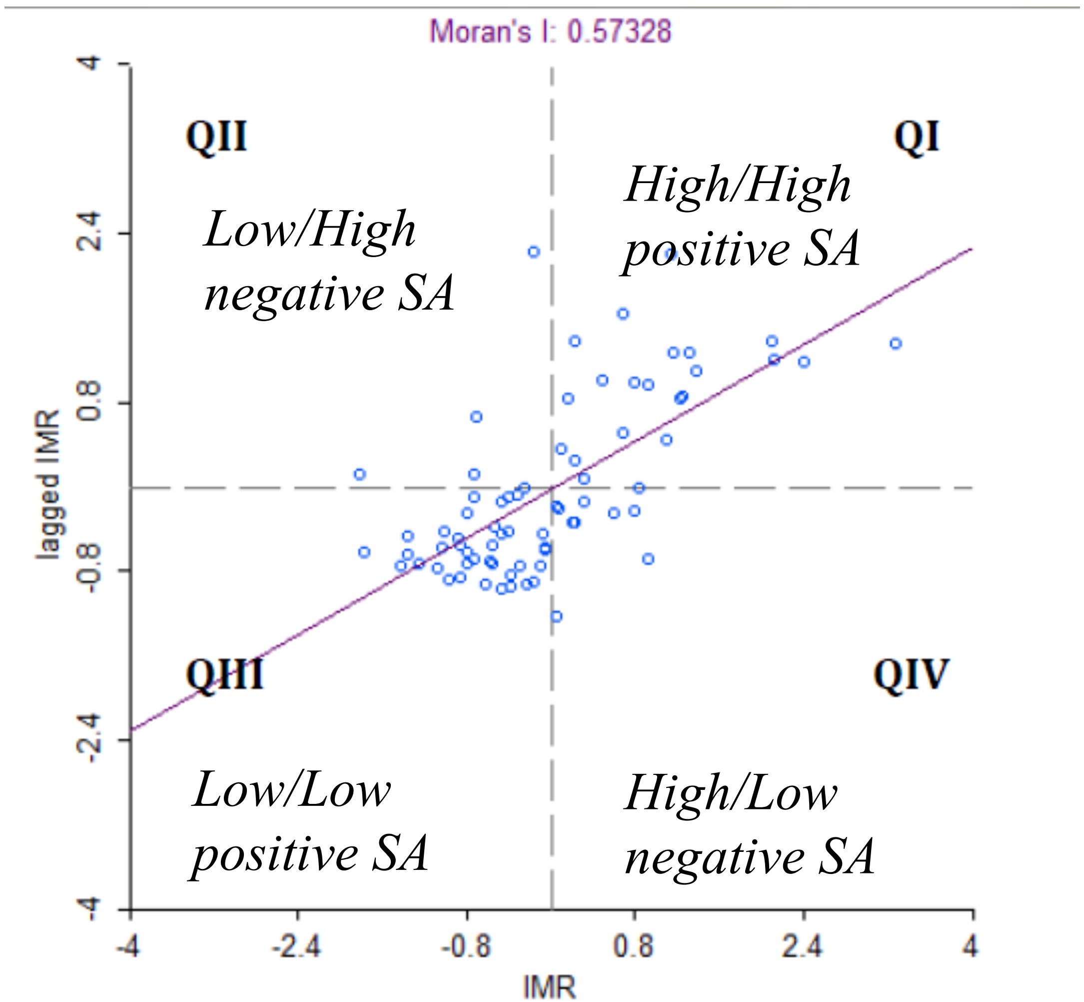
Bivariate LISA
and the Correlation Coefficie

- Correlation Coefficient is the relationship between two different variables in the same area
- Bivariate LISA is a correlation between two different variables in an area and in nearby areas.





Bivariate Moran Scatter Plot



Summary

- Spatial autocorrelation of areal data
- Spatial weight matrix
- Measures of spatial autocorrelation
- Global Measure
  - Moran's I/General G and G*
- Local
  - LISA: Moran's I/General G and G*
  - Bivariate LISA
  - Significance test

Spatial Regression

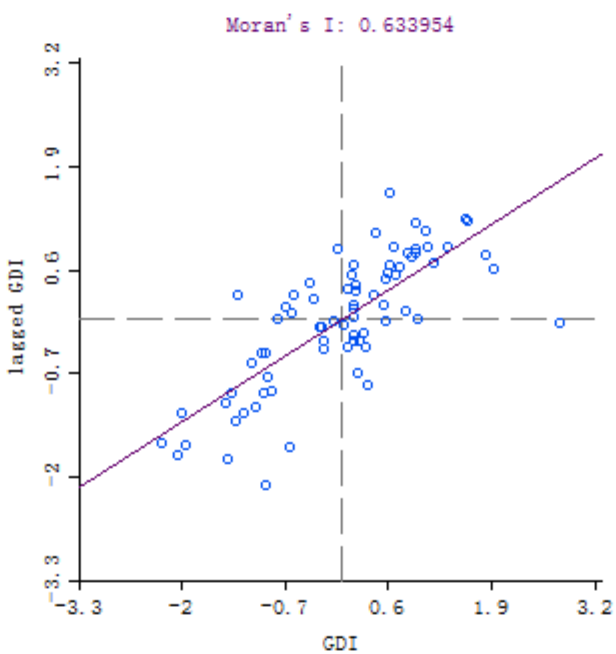
Spatial Autocorrelation vs Correlation

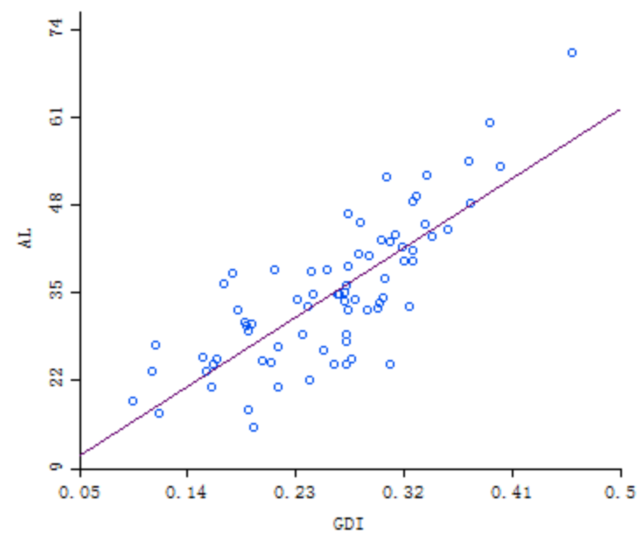
Spatial Autocorrelation:

shows the association or relationship between the <u>same</u> variable in “near-by” areas.

Standard Correlation

shows the association or relationship between two <u>different</u> variables





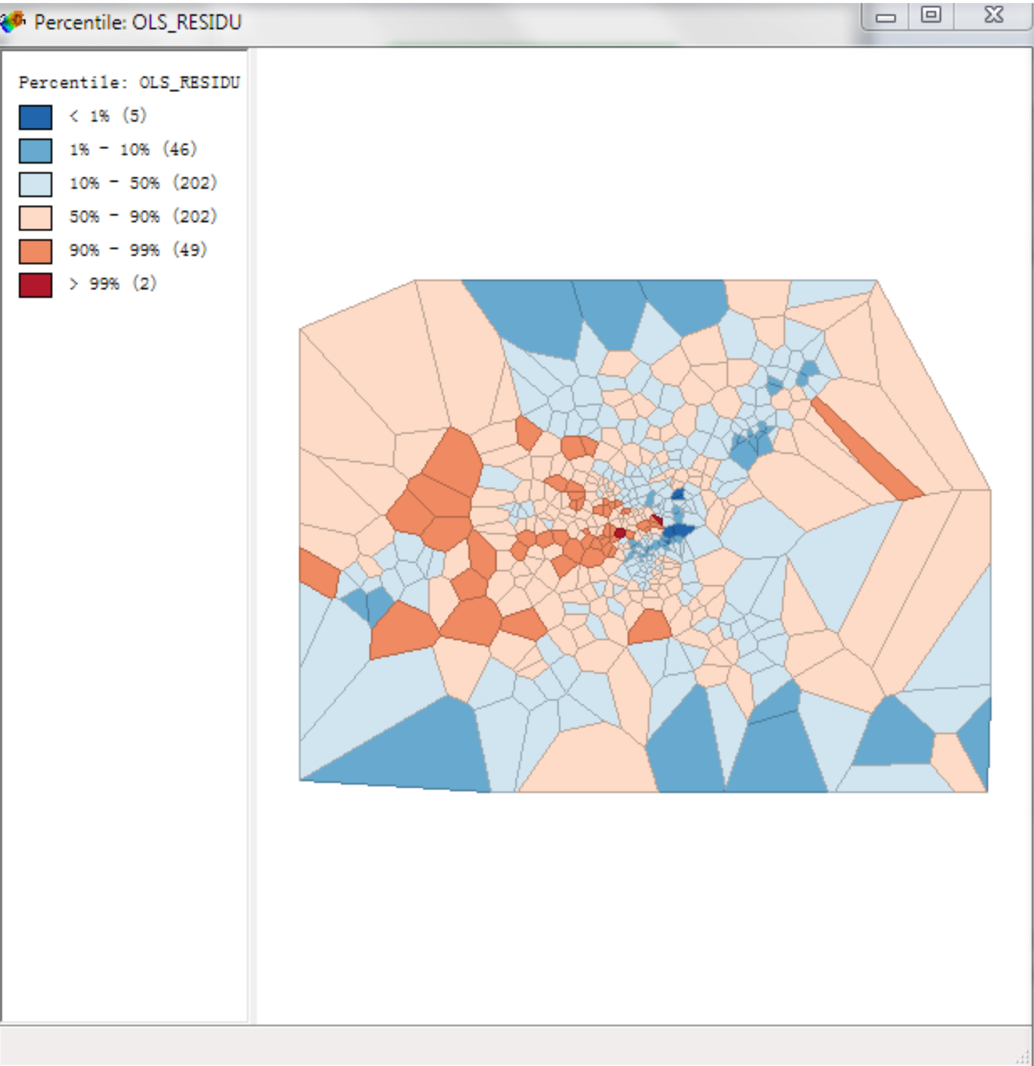
Consequences of Ignoring Spatial Autocorrelation

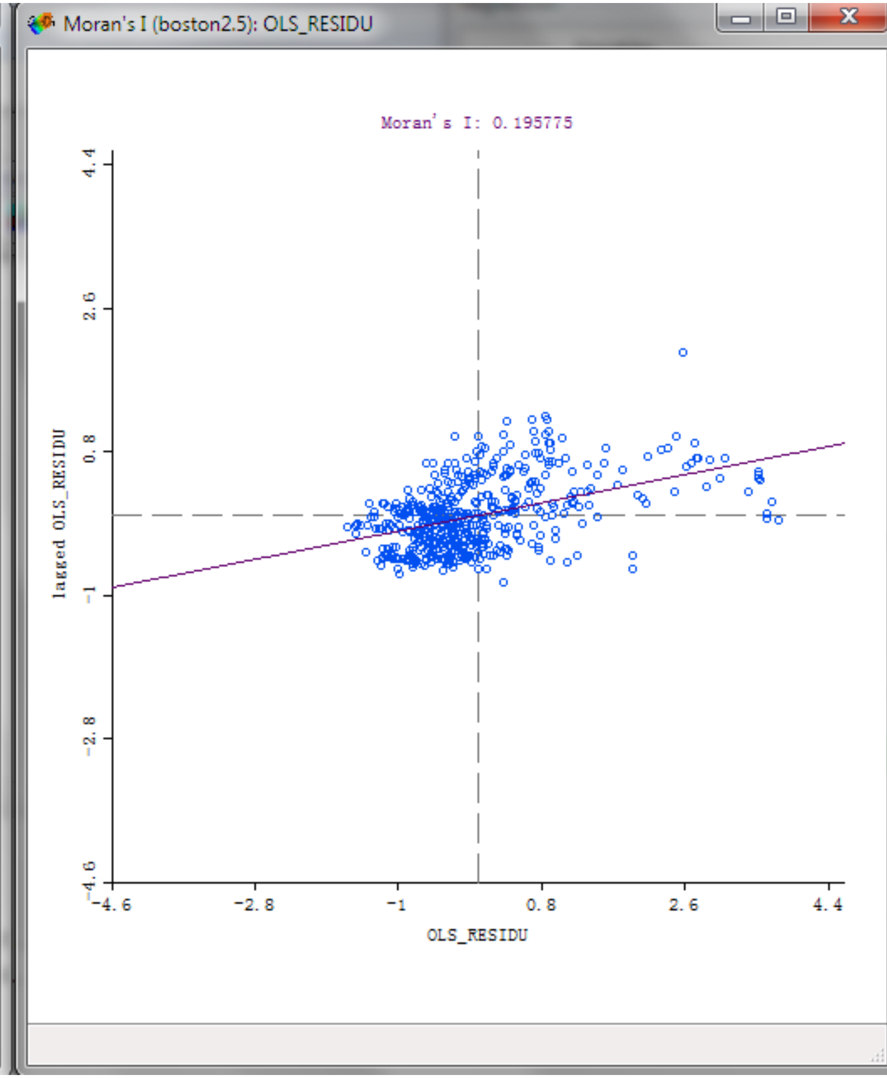
- correlation coefficients and coefficients of determination appear <u>bigger</u> than they really are
  - You think the relationship is stronger than it really is
  - the variables in nearby areas affect each other
- Standard errors appear <u>smaller</u> than they really are
  - *exaggerated precision*
  - You think your predictions are better than they really are since standard errors measure *predictive accuracy*
  - More likely to conclude relationship is *statistically significant*.

Diagnostic of Spatial Dependence

- **For correlation**
  - calculate Moran's I for each variable and test its statistical significance
  - If Moran's I is significant, you may have a problem!
- **For regression**
  - calculate the residuals

    map the residuals: do you see any spatial patterns?
  - Calculate Moran's I for the residuals: is it statistically significant?


Percentile: OLS_RESIDU
Percentile: OLS_RESIDU
< 1% (5)
1% - 10% (46)
10% - 50% (202)
50% - 90% (202)
90% - 99% (49)
> 99% (2)


Moran's I (boston2.5): OLS_RESIDU
Moran's I: 0.195775
lagged OLS_RESIDU
4.4
2.6
0.8
-1
-2.8
-4.6
-4.6
-2.8
-1
0.8
2.6
4.4
OLS_RESIDU

When (spatial) correlation happens

- Try to think of omitted variables and include them in a multiple regression.
  - Missing (omitted) variables may cause spatial autocorrelation
- Regression assumes all relevant variables influencing the dependent variable are included
  - If relevant variables are missing, model is *misspecified*

Spatial Regression Methods

- Spatial Econometrics Approaches
  - Lag model
  - Error model
- Spatial Statistics Approaches
  - Simultaneous Autoregressive Models (SAR)
    - A more general case of Spatial Econometrics
  - Conditional Autoregressive Models (CAR)
- Other methods:
  - Generalized linear model with mixed effects
  - Generalized additive model
  - Generalized Estimating Equations

Source: Briggs UT Dallas

Spatial Econometrics Approaches

- **Spatial lag model**

$$Y = \beta_0 + \lambda WY + X\beta + \varepsilon$$

values of the dependent variable in neighboring locations (WY) are included as an extra explanatory variable

  - these are the "spatial lag" of Y

- **Spatial error model**

$$Y = \beta_0 + X\beta + \rho W\varepsilon + \xi$$

ξ is "white noise"

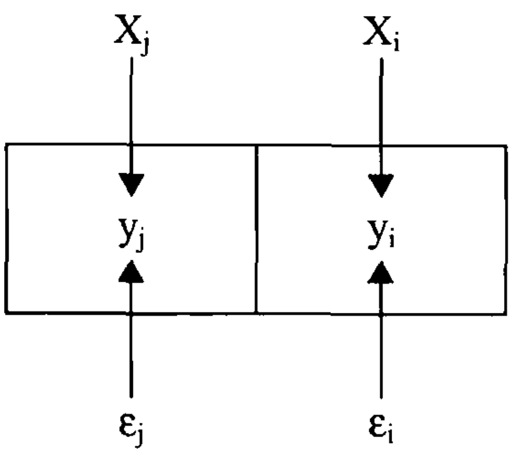
values of the residuals in neighboring locations ($W\varepsilon$) are included as an extra term in the equation;

  - these are "spatial error"

Spatial Lag and Spatial Error Models: *conceptual comparison*

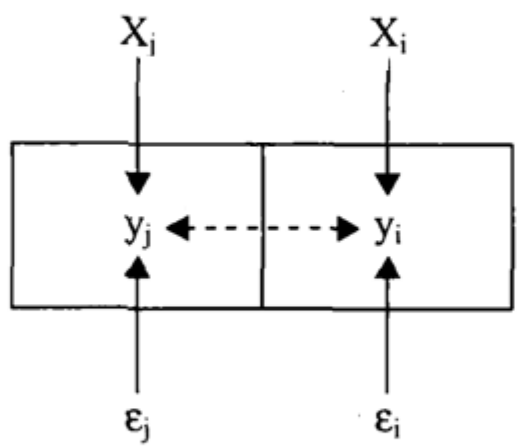
Ordinary Least Squares

OLS



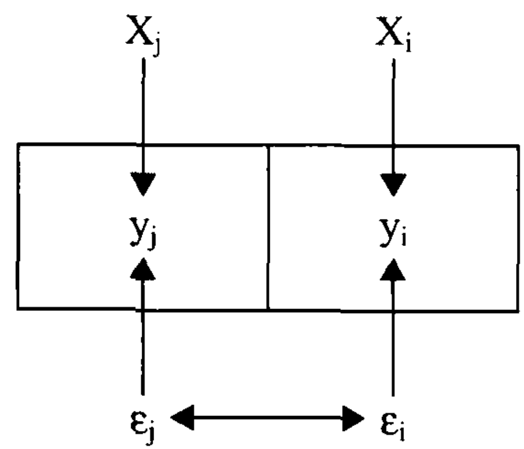
No influence from neighbors

SPATIAL LAG



Dependent variable influenced by neighbors

SPATIAL ERROR



Residuals influenced by neighbors

Baller, R., L. Anselin, S. Messner, G. Deane and D. Hawkins. 2001. *Structural covariates of US County homicide rates: incorporating spatial effects*,. Criminology , 39, 561-590

Source: Briggs UT Dallas

Spatial Lag Model

- Incorporates spatial effects by including a spatially lagged dependent variable as an additional predictor
- Outcome is dependent on the outcome for neighbors
- The ‘spatially lagged’ or ‘average neighbouring’ Wy is correlated with the unobserved error term, thus the model leads to biased and inefficient coefficients if using OLS

Spatial Error Model

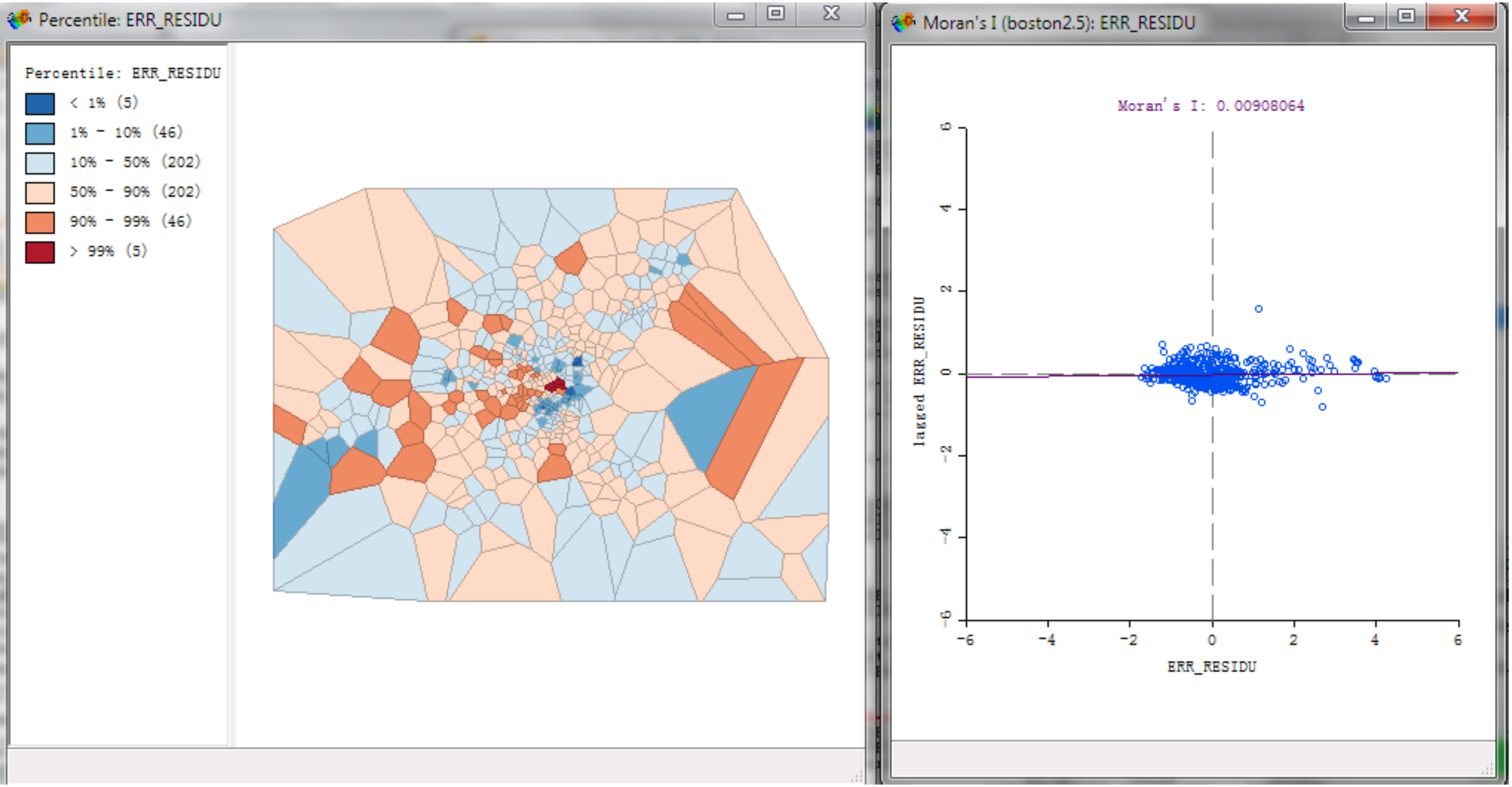
- Incorporates spatial effects through error term
- Unobserved factors in neighboring locations are correlated
- With spatial error violate the assumption that error terms are uncorrelated and coefficients are inefficient if using OLS

Lag or Error Model: *Which to use?*

- **Lag** model primarily controls spatial autocorrelation in the dependent variable
- **Error** model controls spatial autocorrelation in the residuals, thus it controls autocorrelation in both the dependent and the independent variables
- **Conclusion:** the error model is more robust and generally the better choice.
- **Statistical tests** called the *LM Robust* test can also be used to select
  - Will not discuss these

Regression Report

```
SUMMARY OF OUTPUT: ORDINARY LEAST SQUARES ESTIMATION
Data set              :  bostonpolygon
Dependent Variable    :       CMEDV  Number of Observations:  506
Mean dependent var    :     22.5289  Number of Variables   :    2
S.D. dependent var    :      9.1731  Degrees of Freedom    :  504

R-squared             :    0.184299  F-statistic           :     113.873
Adjusted R-squared    :    0.182680  Prob(F-statistic)     :4.16755e-024
Sum squared residual:       34730.7  Log likelihood        :    -1787.88
Sigma-square          :     68.9102  Akaike info criterion :     3579.76
S.E. of regression    :     8.30121  Schwarz criterion     :     3588.21
Sigma-square ML       :     68.6378
S.E of regression ML:       8.28479

-----------------------------------------------------------------------
    Variable   Coefficient      Std.Error    t-Statistic   Probability
-----------------------------------------------------------------------
    CONSTANT      41.39839       1.806375       22.91793     0.0000000
         NOX     -34.01786       3.187837      -10.67114     0.0000000
-----------------------------------------------------------------------


REGRESSION DIAGNOSTICS
MULTICOLLINEARITY CONDITION NUMBER   9.686514
TEST ON NORMALITY OF ERRORS
TEST                  DF          VALUE             PROB
Jarque-Bera            2           443.2973         0.0000000

DIAGNOSTICS FOR HETEROSKEDASTICITY
RANDOM COEFFICIENTS
TEST                  DF          VALUE             PROB
Breusch-Pagan test     1           1.131862         0.2873785
Koenker-Bassett test   1          0.4377741         0.5081988
SPECIFICATION ROBUST TEST
TEST                  DF          VALUE             PROB
White                  2           6.069546         0.0480856

DIAGNOSTICS FOR SPATIAL DEPENDENCE
FOR WEIGHT MATRIX : boston2.5.gwt
   (row-standardized weights)
TEST                          MI/DF       VALUE          PROB
Moran's I (error)           0.195775    15.2444755     0.0000000
Lagrange Multiplier (lag)       1      127.4022649     0.0000000
Robust LM (lag)                 1        1.7548967     0.1852623
Lagrange Multiplier (error)     1      207.8469315     0.0000000
Robust LM (error)               1       82.1995633     0.0000000
Lagrange Multiplier (SARMA)     2      209.6018282     0.0000000
```


Percentile: ERR_RESIDU
Percentile: ERR_RESIDU
< 1% (5)
1% - 10% (46)
10% - 50% (202)
50% - 90% (202)
90% - 99% (46)
> 99% (5)
Moran's I (boston2.5): ERR_RESIDU
Moran's I: 0.00908064
lagged ERR_RESIDU
ERR_RESIDU

- End of this topic