Spatial Analysis and Modeling (GIST 4302/5302)

Guofeng Cao
Department of Geosciences
Texas Tech University

Outline of This Week

- Last week, we learned:
  - spatial point pattern analysis (PPA)
  - focus on location distribution of 'events'
  - Measure the cluster (spatial autocorrelation)in point pattern
- This week, we will learn:
  - How to measure and detect clusters/spatial autocorrelation in areal data (regional data)

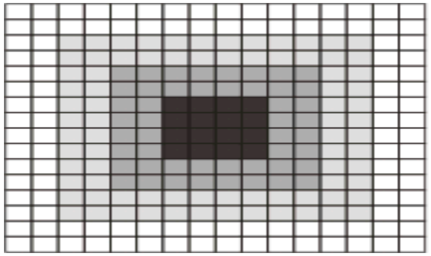
Spatial Autocorrelation

- Spatial autocorrelationship is everywhere
  - Spatial point pattern
    - K, F, G functions
    - Kernel functions
  - Areal/lattice (this topic)
  - Geostatistical data (next topic)

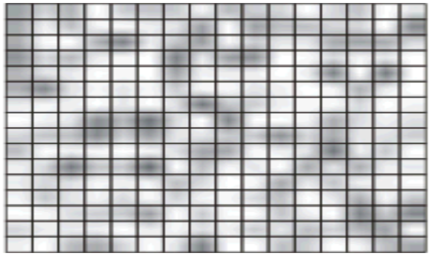
Spatial Autocorrelation of Areal Data

Spatial Autocorrelation

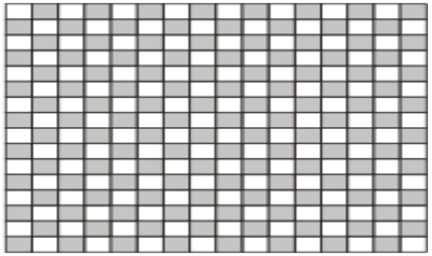
- Tobler’s first law of geography
- Spatial auto/cross correlation



If like values tend to cluster together, then the field exhibits high **positive spatial autocorrelation**



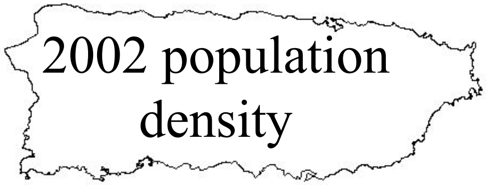
If there is no apparent relationship between attribute value and location then there is **zero spatial autocorrelation**



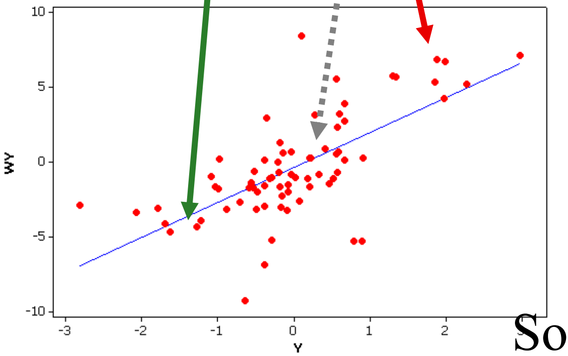
If like values tend to be located away from each other, then there is **negative spatial autocorrelation**

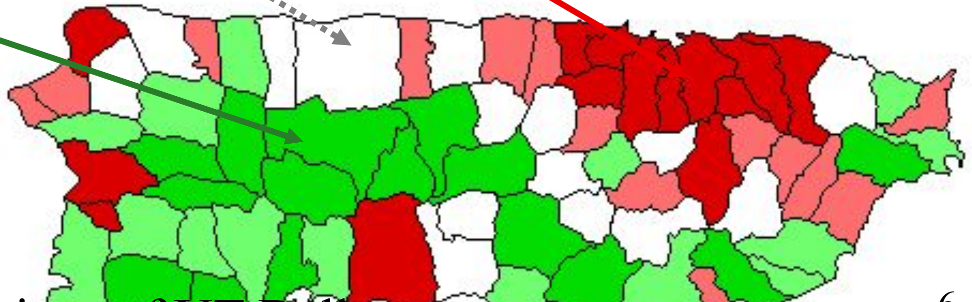
Positive spatial autocorrelation

- high values surrounded by nearby high values
- intermediate values surrounded by nearby intermediate values
- low values surrounded by nearby low values



2002 population density



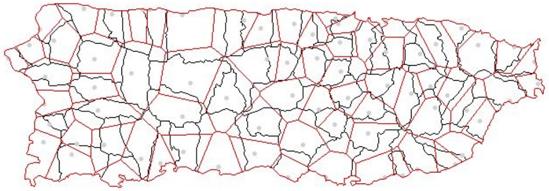


Source: Ron Briggs of UT Dallas

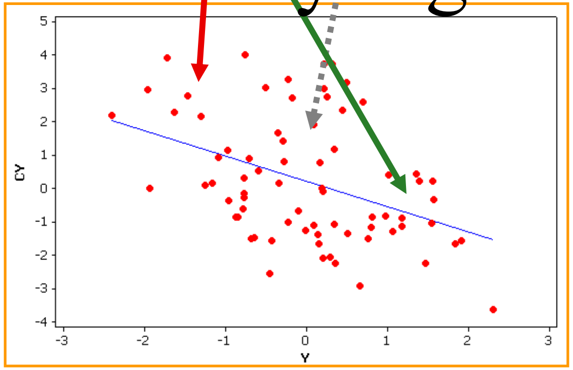
Negative spatial autocorrelation

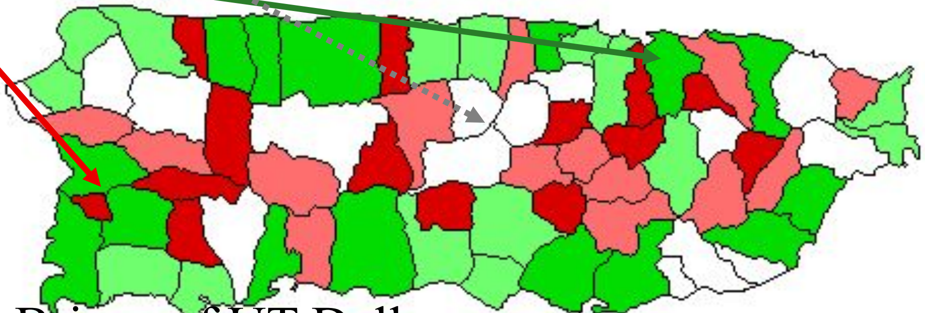
- high values surrounded by nearby low values
- intermediate values surrounded by nearby intermediate values
- low values surrounded by nearby high values

competition for space



Grocery store density





Source: Ron Briggs of UT Dallas

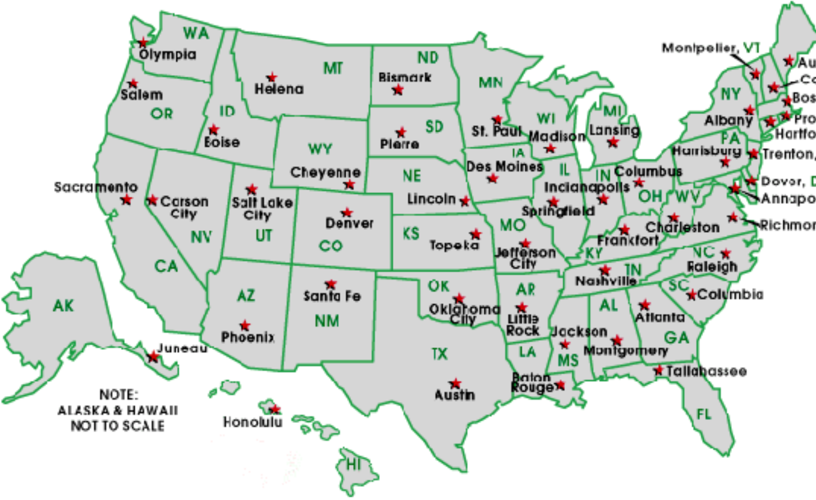
Measuring Spatial Autocorrelation:
the problem of measuring "nearness"

To measure spatial autocorrelation, we must know the "nearness" of our observations as we did for point pattern case

- Which points or polygons are " near" or "next to" other points or polygons?
  - *Which states are near Texas?*
  - How to <u>measure</u> this?

Seems simple and obvious,

but it is not!



Spatial Weight Matrix

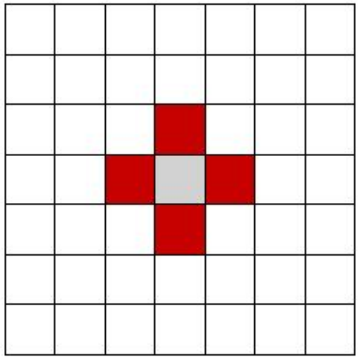
- **Core** concept in statistical analysis of areal data
- Two steps involved:
  - define which relationships between observations are to be given a nonzero weight, i.e., define spatial neighbors
  - assign weights to the neighbors

Spatial Neighbors

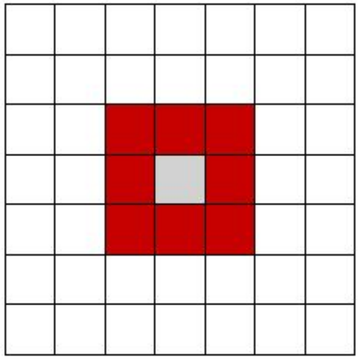
- **Contiguity-based neighbors**
  - Zone *i* and j are neighbors if zone i is contiguity or adjacent to zone *j*
  - But what constitutes contiguity?
- **Distance-based neighbors**
  - Zone i and j are neighbors if the distance between them are less than the threshold distance
  - But what distance do we use?

Contiguity-based Spatial Neighbors

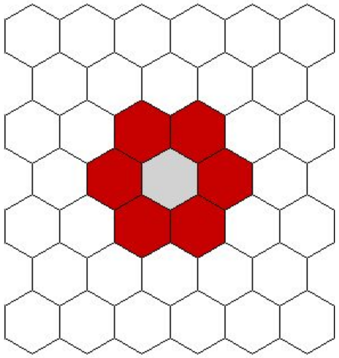
- Sharing a border or boundary
  - Rook: sharing a border
  - Queen: sharing a border or a point



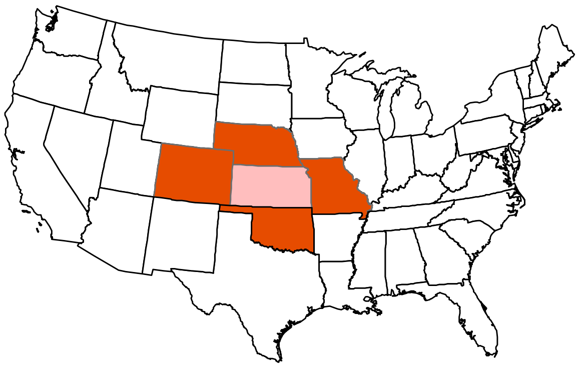
rook



queen



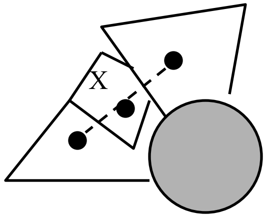
Hexagons



Irregular

Which use?

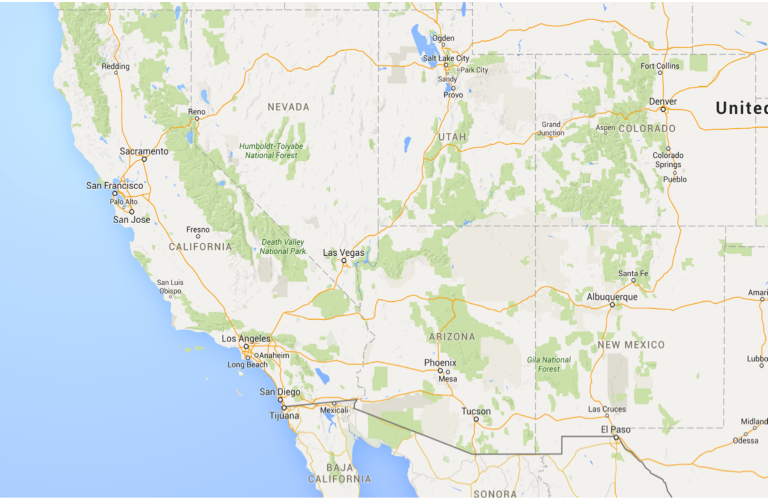
Problem Situations for Irregular Polygons



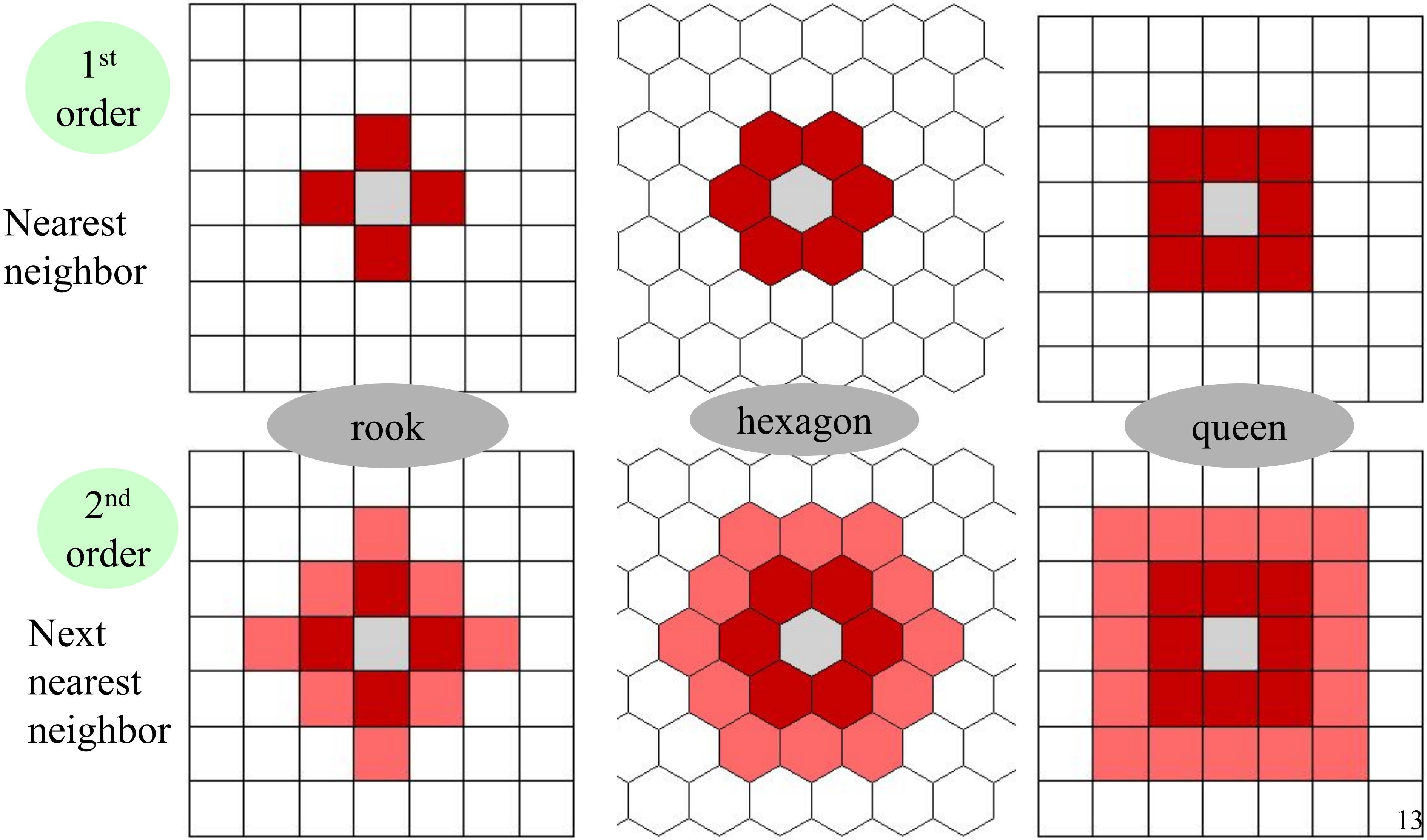
"Close" but no common border

Length of border

- Is Arizona"as close to" California as to Utah?
- Base "closeness" on proportion of shared border, not just one (1) or zero (0)
- $w_{ij} = \mathit{border\ length}_{ij} /\mathit{border\ length}_{j}$)



Higher-Order Contiguity



1st order

Nearest neighbor

rook

hexagon

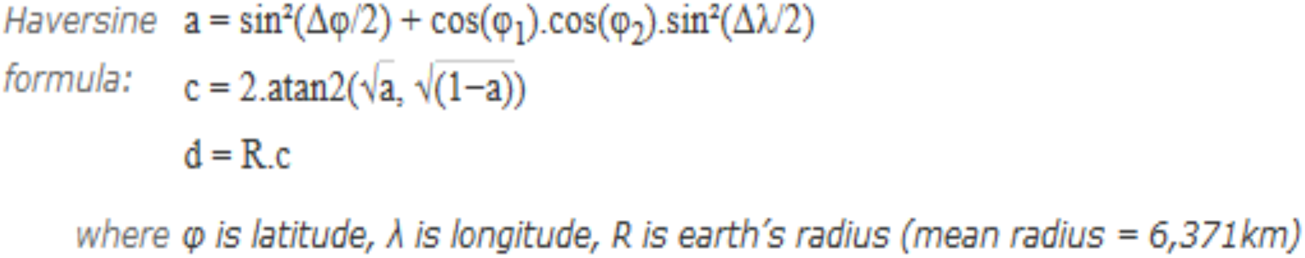
queen

2nd order

Next nearest neighbor

Distance-based Neighbors

- How to measure distance between polygons?
- Distance metrics
  - – 2D Cartesian distance (projected data)
  - – 3D spherical distance/great-circle distance (lat/long data)
    - Haversine formula



Haversine formula:

$a = \sin^2(\Delta\varphi/2) + \cos(\varphi_1).\cos(\varphi_2).\sin^2(\Delta\lambda/2)$

$c = 2.\text{atan2}(\sqrt{a}, \sqrt{(1-a)})$

$d = R.c$

where φ is latitude, λ is longitude, R is earth's radius (mean radius = 6,371km)

Distance-based Neighbors

- k-nearest neighbors

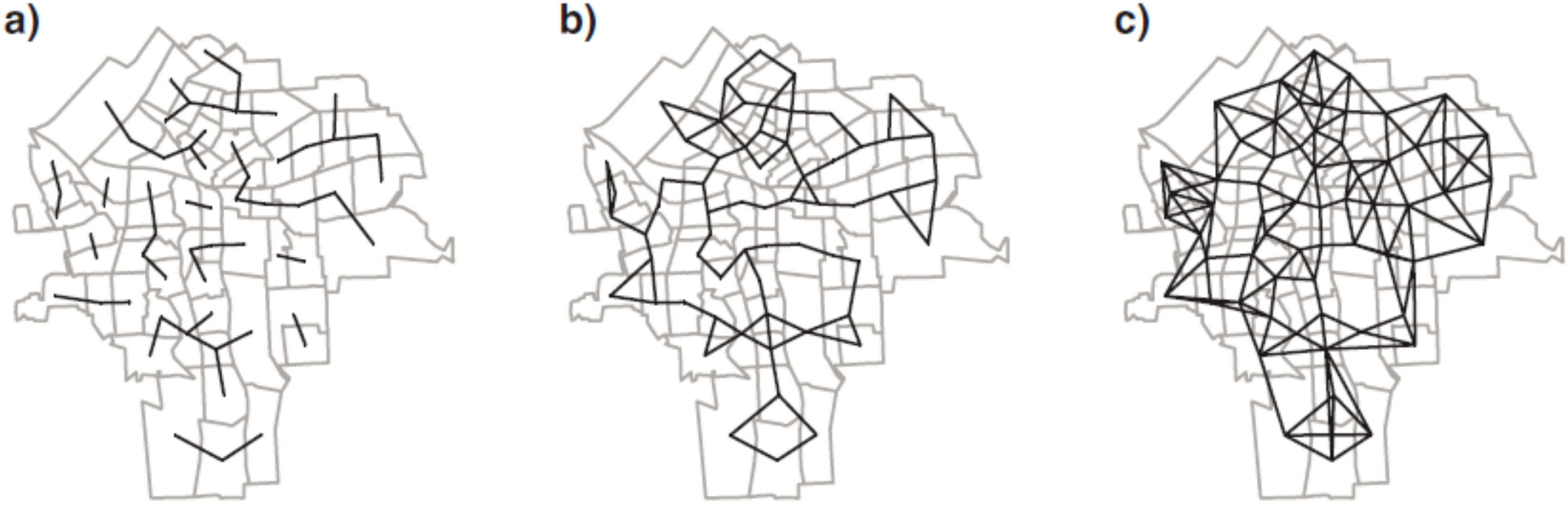


Fig. 9.5. (**a**) $k = 1$ neighbours; (**b**) $k = 2$ neighbours; (**c**) $k = 4$ neighbours

Source: Bivand and Pebesma and Gomez-Rubio

Distance-based Neighbors

- thresh-hold distance (buffer)

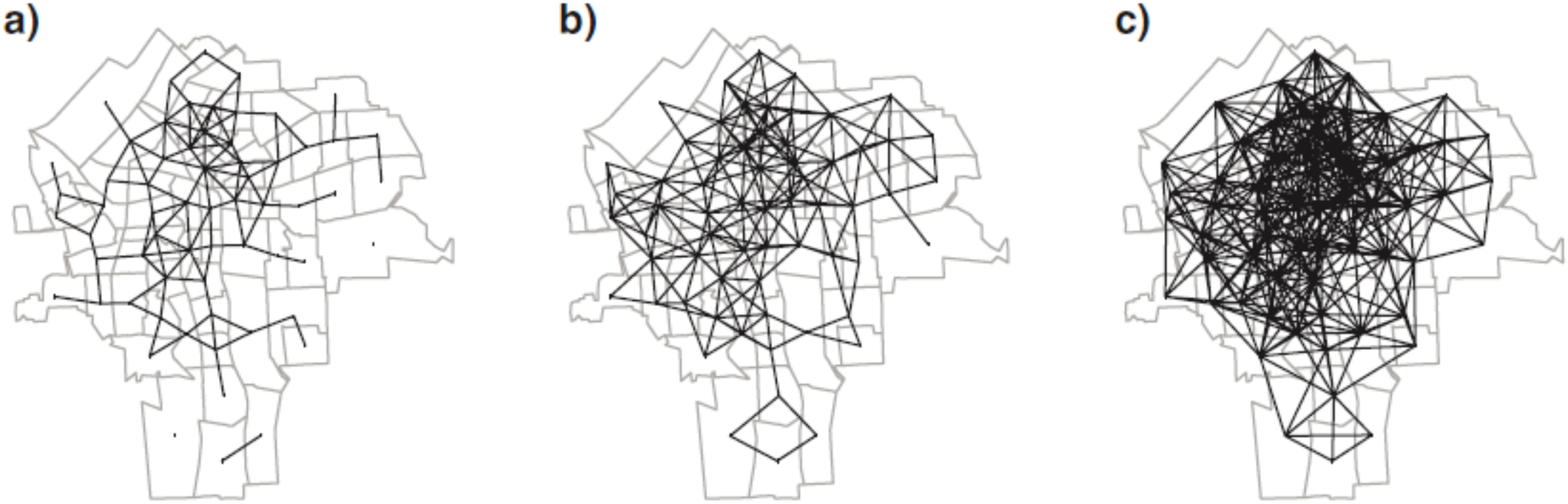
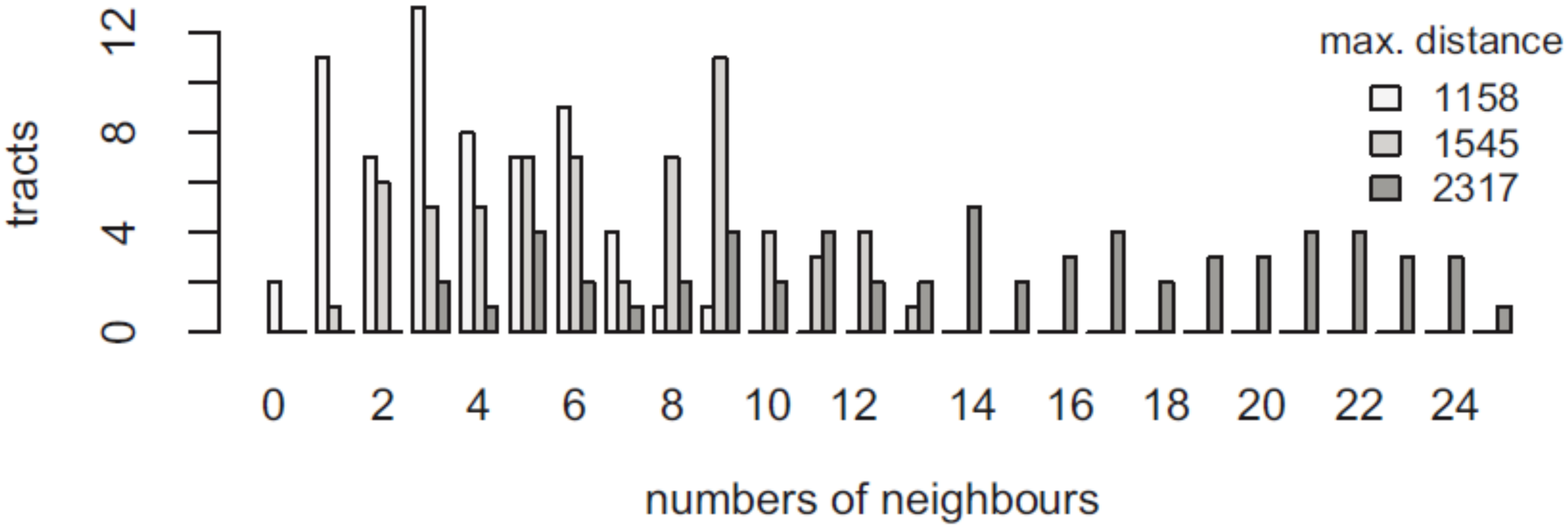


Fig. 9.6. (**a**) Neighbours within 1,158 m; (**b**) neighbours within 1,545 m; (**c**) neig
bours within 2,317 m

Source: Bivand and Pebesma and Gomez-Rubio

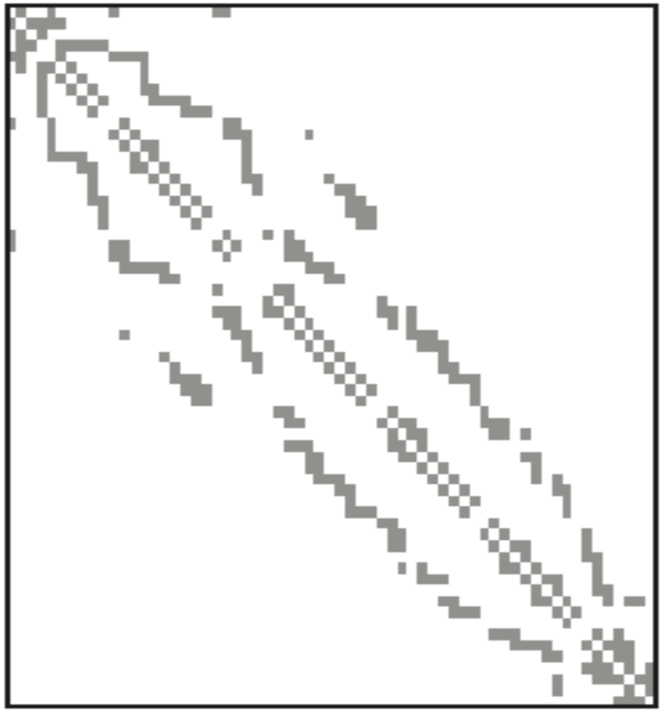
Neighbor/Connectivity Histogram



Source: Bivand and Pebesma and Gomez-Rubio

Spatial Weight Matrix

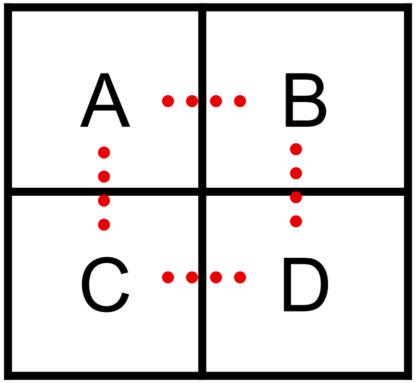
- Spatial weights can be seen as a list of weights indexed by a list of neighbors
- If zone j is not a neighbor of zone i, weights Wij will set to zero
  - – The weight matrix can be illustrated as an image
  - – Sparse matrix



A Simple Example for Rook case

- Matrix contains a:
  - 1 if share a border
  - 0 if do not share a border

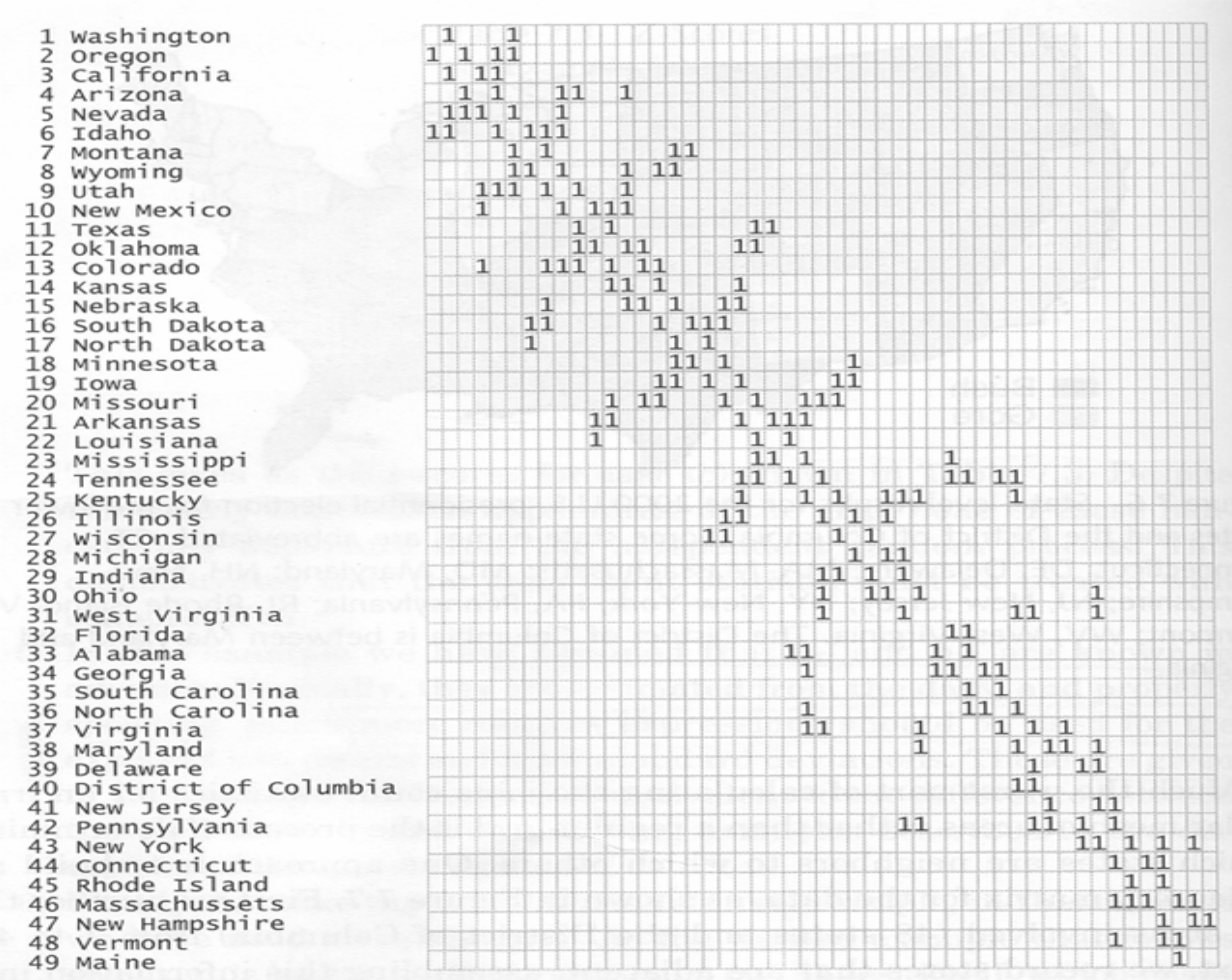
4 areal units



••••• Common border

4x4 matrix

$$\mathbf{W} = \begin{array}{ccccc} & A & B & C & D \\ A & 0 & 1 & 1 & 0 \\ B & 1 & 0 & 0 & 1 \\ C & 1 & 0 & 0 & 1 \\ D & 0 & 1 & 1 & 0 \end{array}$$



1 Washington		1				1																																												
2 Oregon	1		1		1	1																																												
3 California		1		1	1																																													
4 Arizona			1		1				1	1			1																																					
5 Nevada		1	1	1		1			1																																										
6 Idaho	1	1			1		1	1	1																																										
7 Montana						1		1								1	1																																		
8 Wyoming						1	1		1				1		1	1																																		
9 Utah				1	1	1		1		1				1																																					
10 New Mexico				1					1		1	1	1																																					
11 Texas										1		1									1	1																												
12 Oklahoma										1	1		1	1						1	1																												
13 Colorado				1				1	1	1		1		1	1																																		
14 Kansas												1	1		1					1																														
15 Nebraska								1					1	1		1			1	1																														
16 South Dakota							1	1							1		1	1	1																															
17 North Dakota							1								1		1																																
18 Minnesota																1	1		1								1																							
19 Iowa															1	1		1		1						1	1																							
20 Missouri												1		1	1				1		1			1	1	1																							
21 Arkansas												1	1								1		1	1	1																										
22 Louisiana												1										1		1																											
23 Mississippi																							1	1		1											1															
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27 Wisconsin																		1	1							1		1																					
28 Michigan																												1		1	1																				
29 Indiana																										1	1		1		1																				
30 Ohio																										1			1	1		1											1							
31 West Virginia																										1					1							1	1				1							
32 Florida																																		1	1																
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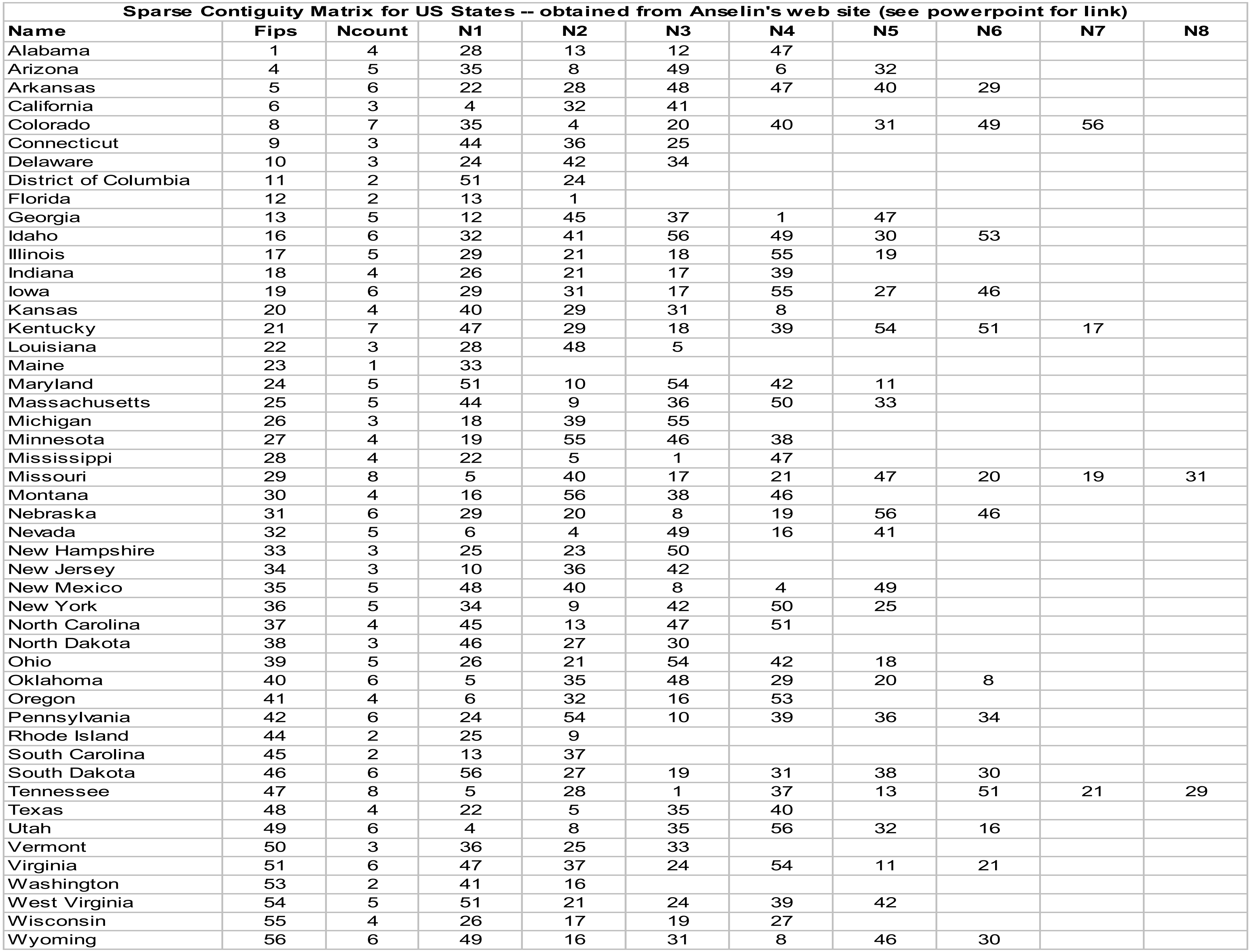
38 Maryland																																1						1		1	1		1							
39 Delaware																																							1				1	1								
40 District of Columbia																																						1	1												
41 New Jersey																																								1			1	1								
42 Pennsylvania																																1	1							1	1		1		1						
43 New York																																												1	1		1		1		1	
44 Connecticut																																														1		1	1			
45 Rhode Island																																															1		1			
46 Massachussets																																														1	1	1		1	1	
47 New Hampshire																																																	1		1	1
48 Vermont																																														1			1	1		
49 Maine																																																			1		



Sparse Contiguity Matrix for US States -- obtained from Anselin's web site (see powerpoint for link)

Name	Fips	Ncount	N1	N2	N3	N4	N5	N6	N7	N8
Alabama	1	4	28	13	12	47				
Arizona	4	5	35	8	49	6	32			
Arkansas	5	6	22	28	48	47	40	29		
California	6	3	4	32	41					
Colorado	8	7	35	4	20	40	31	49	56	
Connecticut	9	3	44	36	25					
Delaware	10	3	24	42	34					
District of Columbia	11	2	51	24						
Florida	12	2	13	1						
Georgia	13	5	12	45	37	1	47			
Idaho	16	6	32	41	56	49	30	53		
Illinois	17	5	29	21	18	55	19			
Indiana	18	4	26	21	17	39				
Iowa	19	6	29	31	17	55	27	46		
Kansas	20	4	40	29	31	8				
Kentucky	21	7	47	29	18	39	54	51	17	
Louisiana	22	3	28	48	5					
Maine	23	1	33							
Maryland	24	5	51	10	54	42	11			
Massachusetts	25	5	44	9	36	50	33			
Michigan	26	3	18	39	55					
Minnesota	27	4	19	55	46	38				
Mississippi	28	4	22	5	1	47				
Missouri	29	8	5	40	17	21	47	20	19	31
Montana	30	4	16	56	38	46				
Nebraska	31	6	29	20	8	19	56	46		
Nevada	32	5	6	4	49	16	41			
New Hampshire	33	3	25	23	50					
New Jersey	34	3	10	36	42					
New Mexico	35	5	48	40	8	4	49			
New York	36	5	34	9	42	50	25			
North Carolina	37	4	45	13	47	51				
North Dakota	38	3	46	27	30					
Ohio	39	5	26	21	54	42	18			
Oklahoma	40	6	5	35	48	29	20	8		
Oregon	41	4	6	32	16	53				
Pennsylvania	42	6	24	54	10	39	36	34		
Rhode Island	44	2	25	9						
South Carolina	45	2	13	37						
South Dakota	46	6	56	27	19	31	38	30		
Tennessee	47	8	5	28	1	37	13	51	21	29
Texas	48	4	22	5	35	40				
Utah	49	6	4	8	35	56	32	16		
Vermont	50	3	36	25	33					
Virginia	51	6	47	37	24	54	11	21		
Washington	53	2	41	16						
West Virginia	54	5	51	21	24	39	42			
Wisconsin	55	4	26	17	19	27				
Wyoming	56	6	49	16	31	8	46	30		

Style of Spatial Weight Matrix

- Row
  - a weight of unity for each neighbor relationship
- Row standardization
  - Symmetry not guaranteed
  - can be interpreted as allowing the calculation of average values across neighbors
- General spatial weights based on distances

Row vs. Row standardization

A	B	C
D	E	F

Divide each number by the **row sum**

Total number of neighbors
--some have more than others

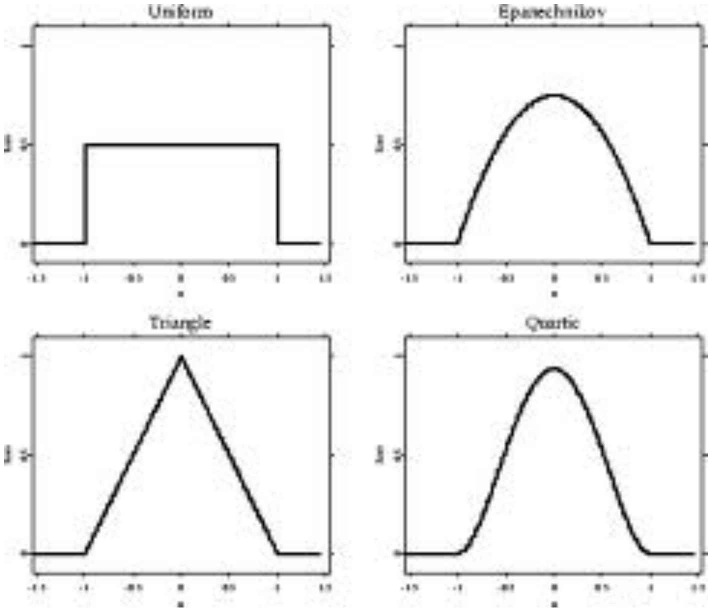
	A	**B**	**C**	**D**	**E**	**F**	**Row Sum**
A	0	1	0	1	0	0	2
B	1	0	1	0	1	0	***3***
C	0	1	0	0	0	1	2
D	1	0	0	0	1	0	2
E	0	1	0	1	0	1	***3***
F	0	0	1	0	1	0	2

Row standardized
--usually use this

	A	**B**	**C**	**D**	**E**	**F**	**Row Sum**
A	0.0	0.5	0.0	0.5	0.0	0.0	1
B	0.3	0.0	0.3	0.0	0.3	0.0	1
C	0.0	0.5	0.0	0.0	0.0	0.5	1
D	0.5	0.0	0.0	0.0	0.5	0.0	1
E	0.0	0.3	0.0	0.3	0.0	0.3	1
F	0.0	0.0	0.5	0.0	0.5	0.0	1

General Spatial Weights Based on Distance

- Decay functions of distance
  - Most common choice is the inverse (reciprocal) of the distance between locations i and j ($w_{ij} = 1/d_{ij}$)
  - Other functions also used
    - inverse of <u>squared</u> distance ($w_{ij} = 1/d_{ij}^2$), or
    - negative exponential ($w_{ij} = e^{-d}$ *or* $w_{ij} = e^{-d^2}$)



Measure of Spatial Autocorrelation

Global Measures and Local Measures

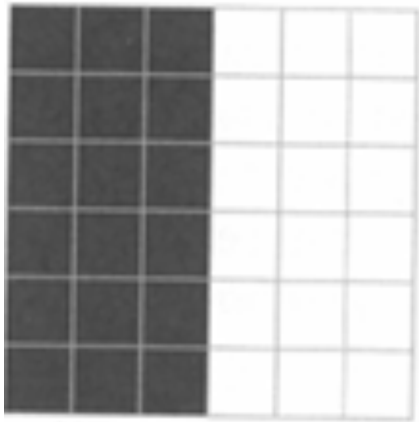
- Global Measures
  - A single value which applies to the entire data set
    - The same pattern or process occurs over the entire geographic area
    - An average for the entire area
- Local Measures
  - A value calculated for each observation unit
    - Different patterns or processes may occur in different parts of the region
    - A unique number for each location
- Global measures usually can be decomposed into a combination of local measures

Global Measures and Local Measures

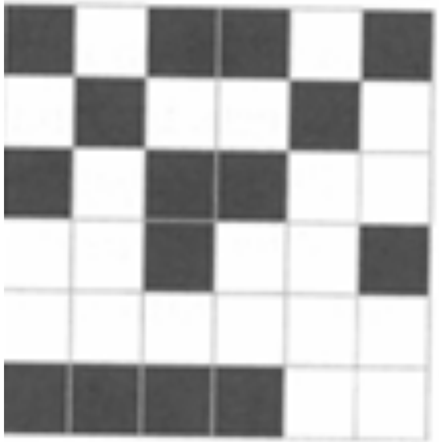
- Global Measures
  - Join Count
  - Moran’s I
- Local Measures
  - Local Moran’s I

Join (or Joint or Joins) Count Statistic

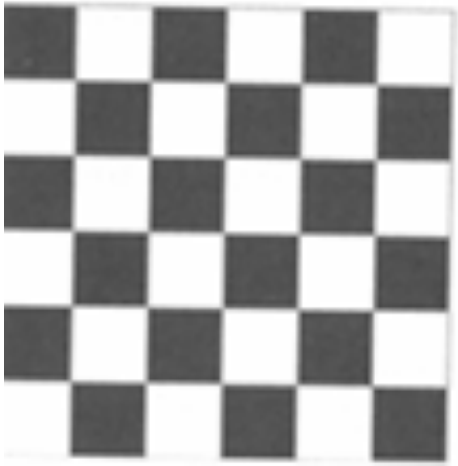
Positive autocorrelation



No autocorrelation



Negative autocorrelation



Rook's case	Queen's case
J_{BB} = 27	J_{BB} = 47
J_{WW} = 27	J_{WW} = 47
J_{BW} = 6	J_{BW} = 16

J_{BB} = 6	J_{BB} = 14
J_{WW} = 19	J_{WW} = 40
J_{BW} = 35	J_{BW} = 56

J_{BB} = 0	J_{BB} = 25
J_{WW} = 0	J_{WW} = 25
J_{BW} = 60	J_{BW} = 60

- 60 for Rook Case
- 110 for Queen Case

Join Count: Test Statistic

Test Statistic given by: $Z = \frac{\text{Observed - Expected}}{\text{SD of Expected}}$

Expected = random pattern generated by tossing a coin in each cell.

Expected given by:

$$E(J_{\mathrm{BB}}) = kp_B^2$$

$$E(J_{\mathrm{WW}}) = kp_{\mathrm{W}}^2$$

$$E(J_{\mathrm{BW}}) = 2kp_B p_W$$

Standard Deviation of Expected (standard error) given by:

$$E(s_{\mathrm{BB}}) = \sqrt{kp_{\mathrm{B}}^2 + 2mp_{\mathrm{B}}^3 - (k+2m)p_{\mathrm{B}}^4}$$

$$E(s_{\mathrm{WW}}) = \sqrt{kp_{\mathrm{W}}^2 + 2mp_{\mathrm{W}}^3 - (k+2m)p_{\mathrm{W}}^4}$$

$$E(s_{\mathrm{BW}}) = \sqrt{2(k+m)p_{\mathrm{B}}p_{\mathrm{W}} - 4(k+2m)p_{\mathrm{B}}^2 p_{\mathrm{W}}^2}$$

Where: k is the total number of joins (neighbors)

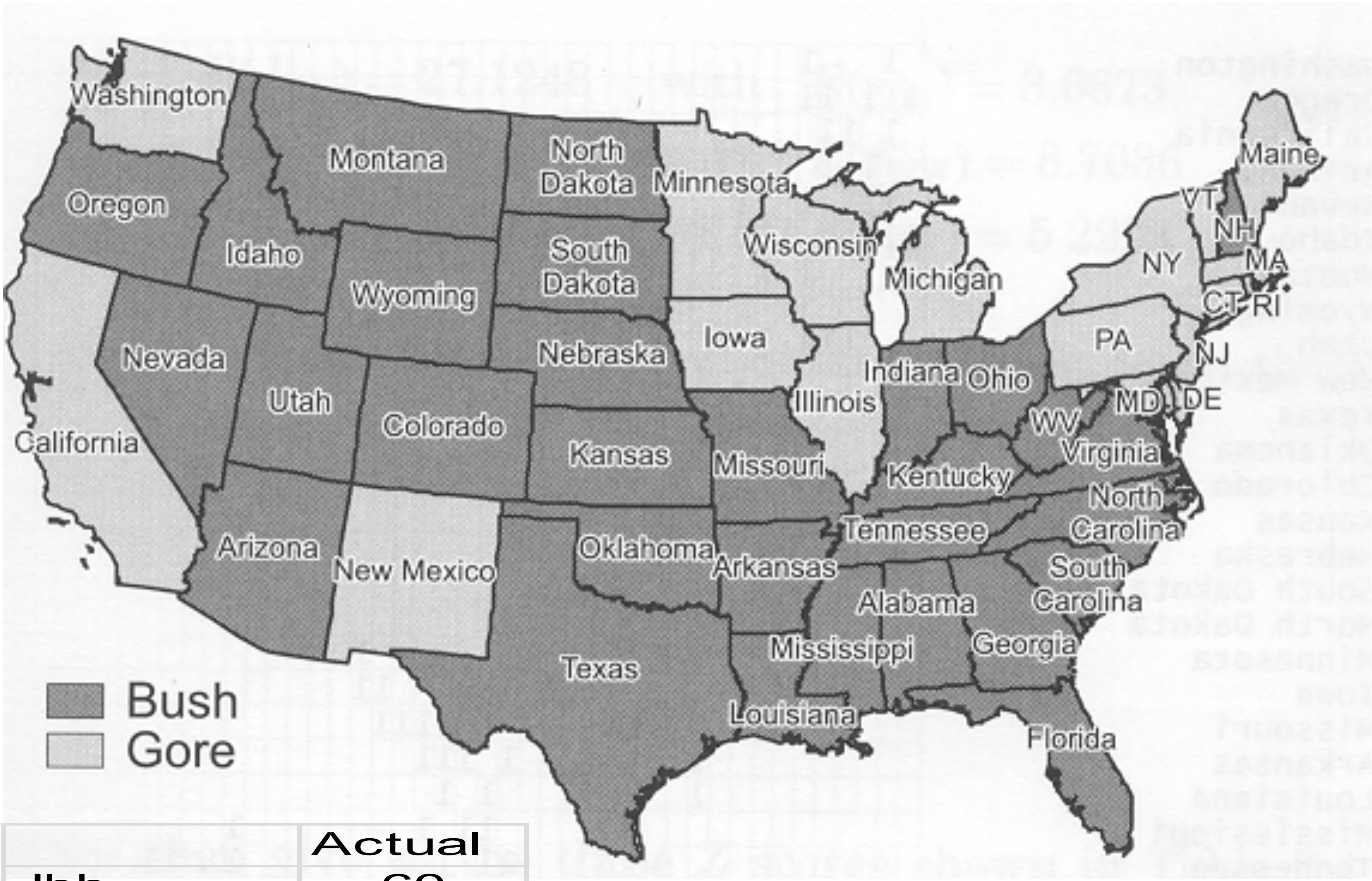
p_B is the expected proportion Black, if random

p_W is the expected proportion White

m is calculated from k according to:

$$m = \tfrac{1}{2}\sum_{i=1}^{n} k_i(k_i - 1)$$

Gore/Bush Presidential Election 2000



	Actual
Jbb	60
Jgg	21
Jbg	28
Total	109

Join Count Statistic for Gore/Bush 2000 by State

candidates	probability
Bush	0.49885
Gore	0.50115

	Actual	Expected	Stan Dev	Z-score
Jbb	60	27.125	8.667	3.7930
Jgg	21	27.375	8.704	-0.7325
Jbg	28	54.500	5.220	-5.0763
Total	109	109.000		

- The expected number of joins is calculated based on the proportion of votes each received in the election (for Bush = 109*.499*.499=27.125)
- There are far <u>more</u> Bush/Bush joins (actual = 60) than would be expected (27)
  - Positive autocorrelation
- There are far <u>fewer</u> Bush/Gore joins (actual = 28) than would be expected (54)
  - Positive autocorrelation
- No strong clustering evidence for Gore (actual = 21 slightly less than 27.375)

Moran’s I

- The most common measure of Spatial Autocorrelation
- Use for points or polygons
  - Join Count statistic only for polygons
- Use for a continuous variable (any value)
  - Join Count statistic only for binary variable (1,0)

Patrick Alfred Pierce Moran (1917-1988)

Formula for Moran’s I

$$I = \frac{N\sum_{i=1}^{n}\sum_{j=1}^{n} \mathrm{w}_{ij}(\mathrm{x}_i - \bar{\mathrm{x}})(\mathrm{x}_j - \bar{\mathrm{x}})}{(\sum_{i=1}^{n}\sum_{j=1}^{n} \mathrm{w}_{ij})\sum_{i=1}^{n}(\mathrm{x}_i - \bar{\mathrm{x}})^2}$$

- Where:

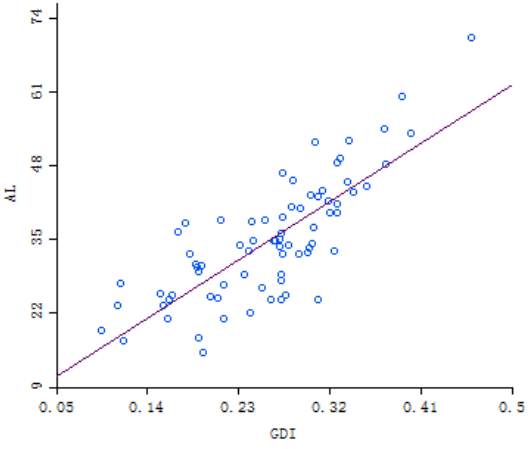
N is the number of observations (points or polygons)
$\bar{\mathrm{X}}$ is the mean of the variable
X_i is the variable value at a particular location
X_j is the variable value at another location
W_{ij} is a weight indexing location of *i* relative to *j*

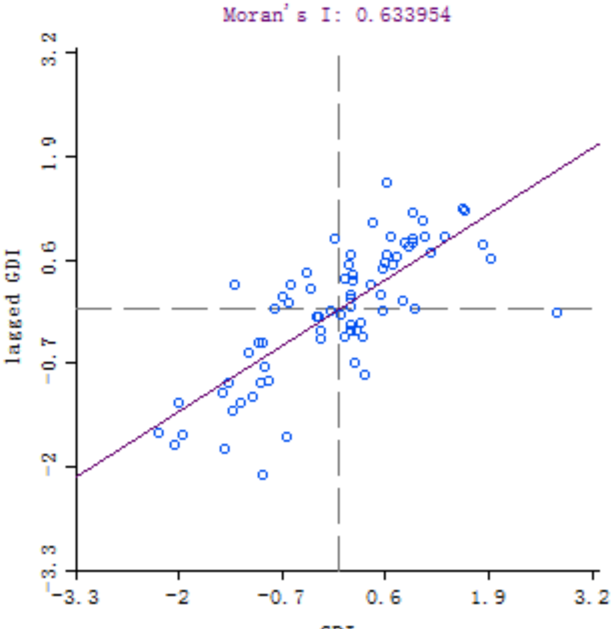
Moran’s *I* and Correlation Coefficient

- **Correlation Coefficient [-1, 1]**
  - Relationship between <u>two</u> different variables
- **Moran’s I [-1, 1]**

– Spatial autocorrelation and often involves <u>one</u> (spatially indexed) variable only

– Correlation between observations of a spatial variable at location X and “spatial lag” of X formed by averaging all the observation at neighbors of X





Correlation Coefficient

$$\frac{\sum_{i=1}^{n} 1(y_i - \bar{y})(x_i - \bar{x})/n}{\sqrt{\frac{\sum_{i=1}^{n}(y_i - \bar{y})^2}{n}}\sqrt{\frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}}}$$

Note the similarity of the numerator (top) to the measures of spatial association discussed earlier if we view Yi as being the Xi for the neighboring polygon

(see next slide)

$$\frac{N\sum_{i=1}^{n}\sum_{j=1}^{n} w_{ij}(x_i - \bar{x})(x_j - \bar{x})}{(\sum_{i=1}^{n}\sum_{j=1}^{n} w_{ij})\sum_{i=1}^{n}(x_i - \bar{x})^2} = \frac{\sum_{i=1}^{n}\sum_{j=1}^{n} w_{ij}(x_i - \bar{x})(x_j - \bar{x})/\sum_{i=1}^{n}\sum_{j=1}^{n} w_{ij}}{\sqrt{\frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}}\sqrt{\frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}}}$$

Spatial auto-correlation

Source: Ron Briggs of UT Dallas

Correlation Coefficient

$$\frac{\sum_{i=1}^{n} 1(y_i-\bar{y})(x_i-\bar{x})/n}{\sqrt{\frac{\sum_{i=1}^{n}(y_i-\bar{y})^2}{n}}\sqrt{\frac{\sum_{i=1}^{n}(x_i-\bar{x})^2}{n}}}$$

Spatial weights

Yi is the Xi for the neighboring polygon

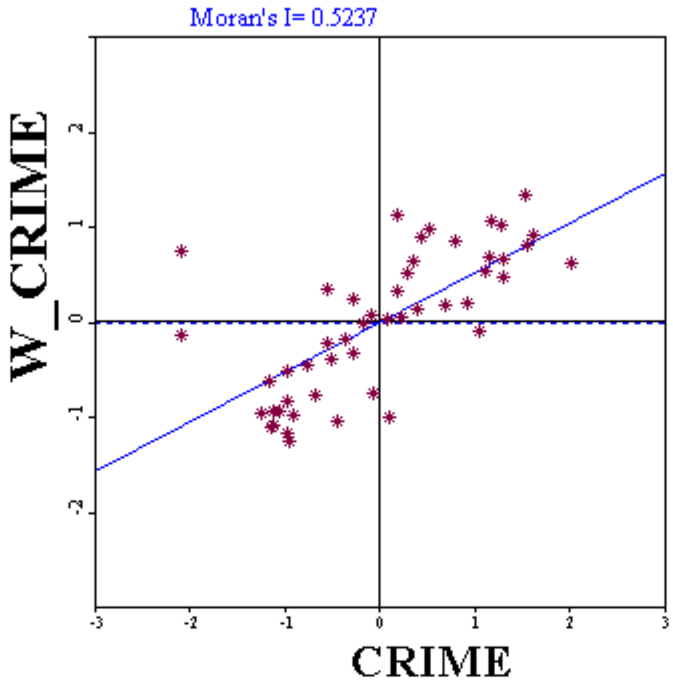
$$\frac{N\sum_{i=1}^{n}\sum_{j=1}^{n} w_{ij}(x_i-\bar{x})(x_j-\bar{x})}{(\sum_{i=1}^{n}\sum_{j=1}^{n} w_{ij})\sum_{i=1}^{n}(x_i-\bar{x})^2} = \frac{\sum_{i=1}^{n}\sum_{j=1}^{n} w_{ij}(x_i-\bar{x})(x_j-\bar{x})/\sum_{i=1}^{n}\sum_{j=1}^{n} w_{ij}}{\sqrt{\frac{\sum_{i=1}^{n}(x_i-\bar{x})^2}{n}}\sqrt{\frac{\sum_{i=1}^{n}(x_i-\bar{x})^2}{n}}}$$

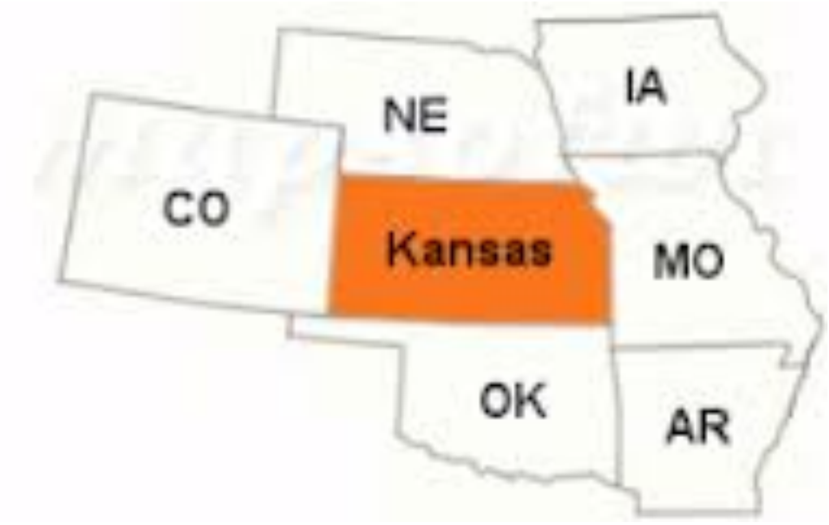
Moran’s I

Source: Ron Briggs of UT Dallas

Moran Scatter Plots

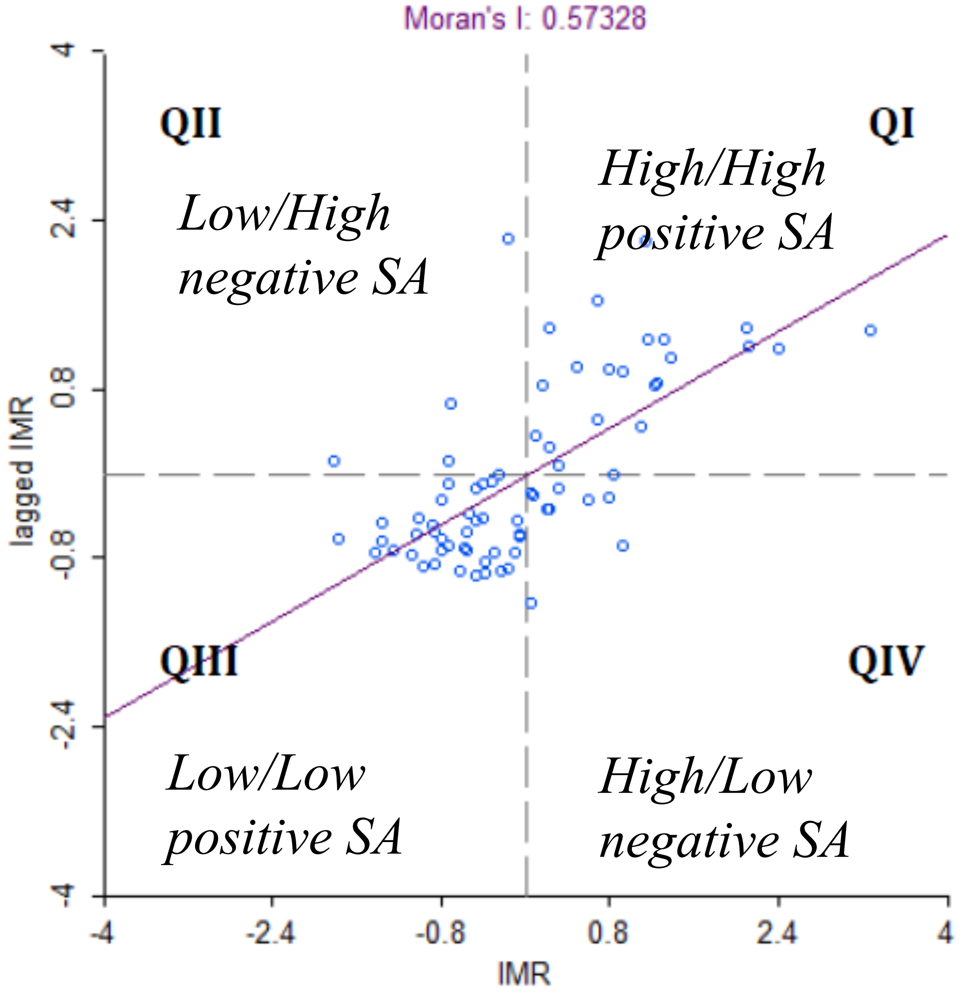
We can draw a scatter diagram between these two variables (in standardized form): **X** and **lag-X** (or W_X)





The slope of this *regression line* is Moran's I

Moran Scatter Plots



QII

Low/High negative SA

QI

High/High positive SA

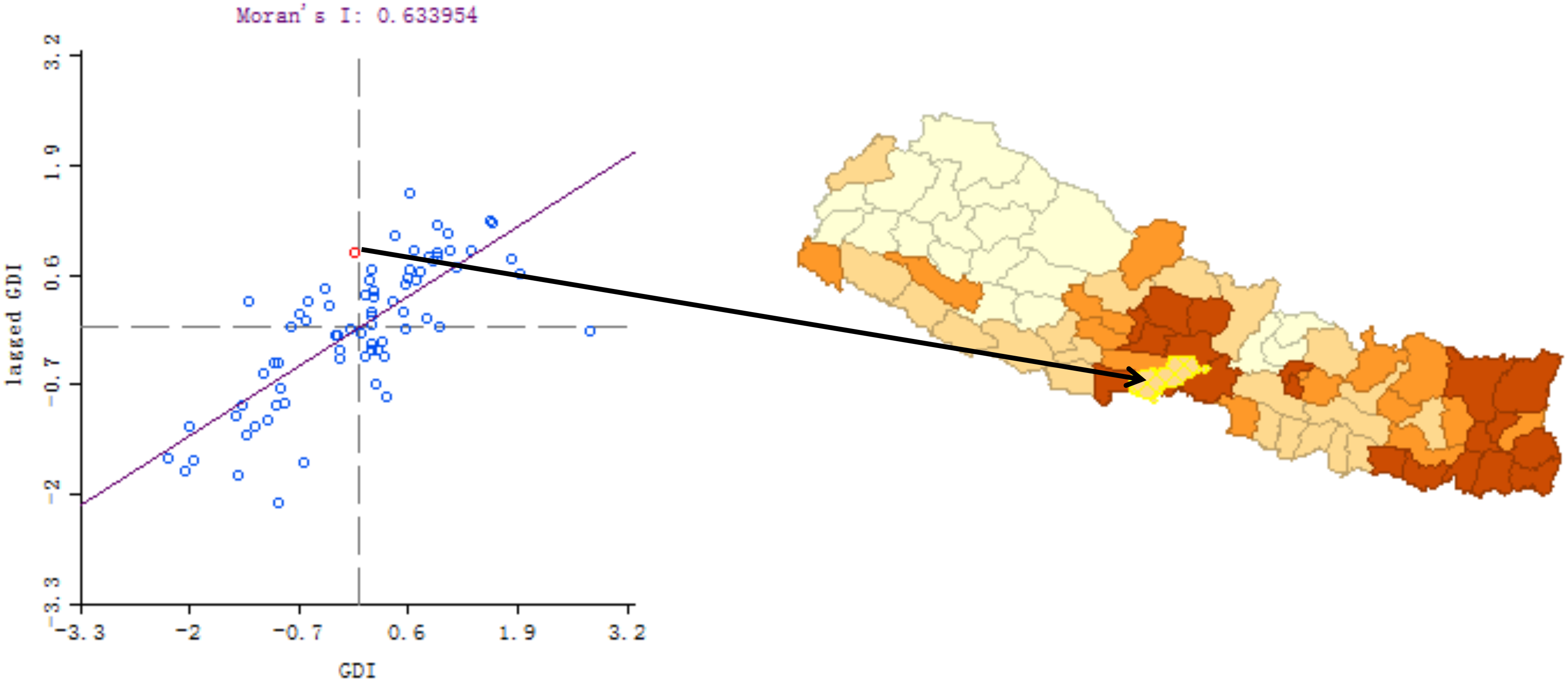
QIII

Low/Low positive SA

QIV

High/Low negative SA

Moran Scatterplot: Example



Statistical Significance Tests for Moran's I

- Based on the normal frequency distribution with

$$Z = \frac{I - E(I)}{S_{error(I)}}$$

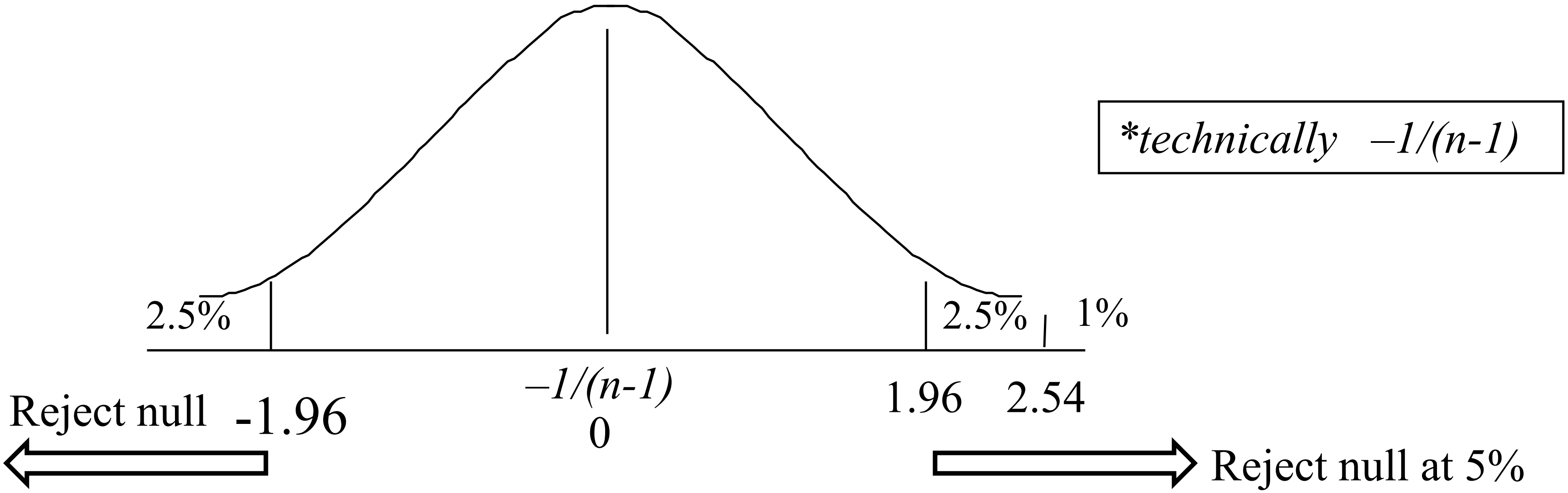
Where: I is the calculated value for Moran's I from the sample

E(I) is the expected value if random

S is the standard error

- Statistical significance test
  - Monte Carlo test, as we did for spatial pattern analysis
  - Permutation test
    - Non-parametric
    - Data-driven, no assumption of the data
    - Implemented in GeoDa

Test Statistic for Normal Frequency Distribution



2.5%

2.5%

1%

*technically –1/(n-1)

Reject null

-1.96

–1/(n-1)
0

1.96 2.54

Reject null at 5%

Reject null at 1%

Null Hypothesis: no spatial autocorrelation

*Moran' s I = 0

Alternative Hypothesis: spatial autocorrelation exists

*Moran' s I > 0

Reject *Null Hypothesis* if Z test statistic > 1.96 (or < -1.96)

---less than a 5% chance that, in the population, there is no spatial autocorrelation

---95% confident that spatial auto correlation exits

Null Hypothesis: no spatial autocorrelation

*Moran' s I = 0

Alternative Hypothesis: spatial autocorrelation exists

*Moran' s I > 0

Reject *Null Hypothesis* if Z test statistic > 1.96 (or < -1.96)

---less than a 5% chance that, in the population, there is no spatial autocorrelation

---95% confident that spatial auto correlation exits

Local Measures of Spatial Autocorrelation

Local Indicators of Spatial Association (LISA)

- Local versions of *Moran's I, and the Getis-Ord G statistic*
- Moran's I is most commonly used, and the local version is often called Anselin's LISA, or just LISA

See:
Luc Anselin 1995 *Local Indicators of Spatial Association-LISA* Geographical Analysis 27: 93-115

Local Indicators of Spatial Association (LISA)

- The statistic is calculated for **each** areal unit in the data
- For each polygon, the index is calculated based on neighboring polygons with which it shares a border
- A measure is available for each polygon, these can be mapped to indicate how spatial autocorrelation varies over the study region
- Each index has an associated test statistic, we can also map which of the polygons has a statistically significant relationship with its neighbors, and show type of relationship

Summary

- Spatial autocorrelation of areal data
- Spatial weight matrix
- Measures of spatial autocorrelation
- Global Measure
  - Moran's I
- Local
  - LISA: Moran's I
- Significance test

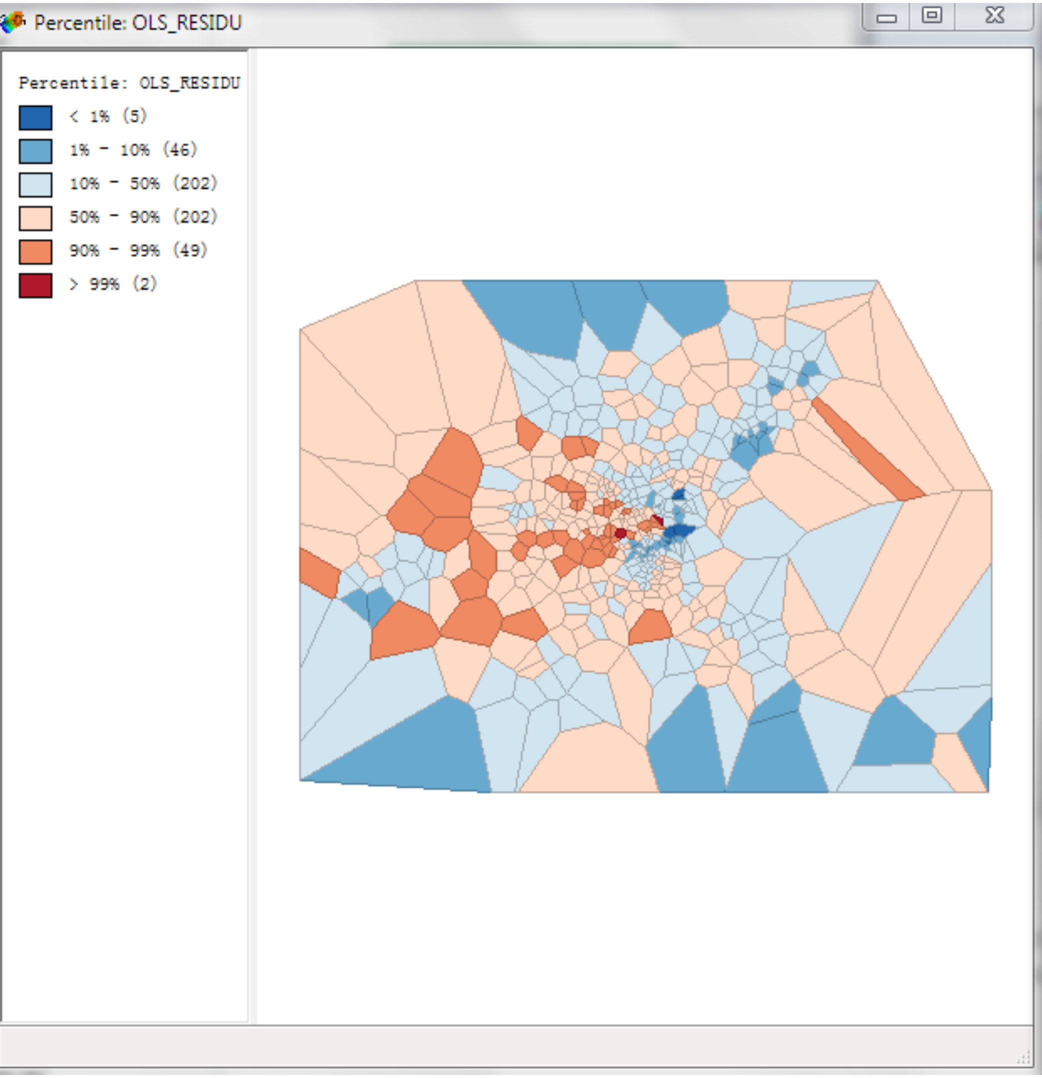
Consequences of Ignoring Spatial Autocorrelation

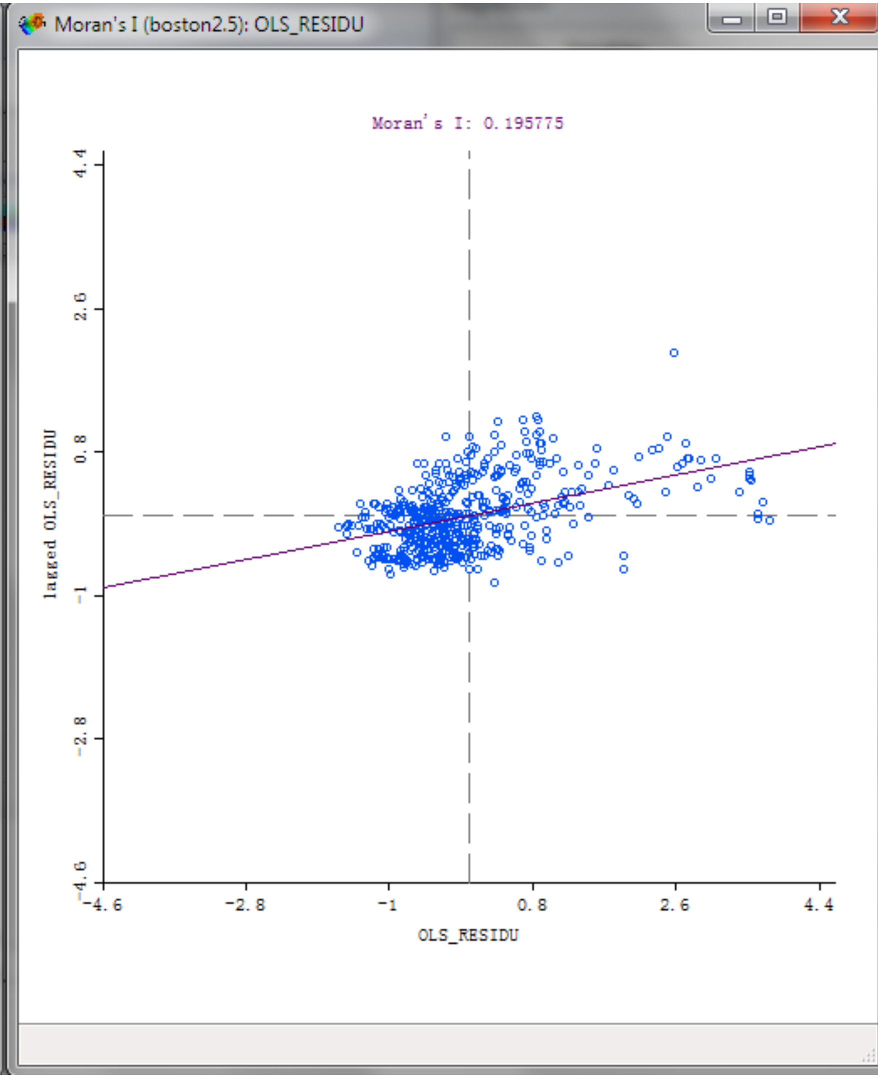
- correlation coefficients and coefficients of determination appear bigger than they really are
  - You think the relationship is stronger than it really is
  - the variables in nearby areas affect each other
- Standard errors appear smaller than they really are
  - *exaggerated precision*
  - You think your predictions are better than they really are since standard errors measure *predictive accuracy*
  - More likely to conclude relationship is *statistically significant*.

Diagnostic of Spatial Dependence

- **For correlation**
  - calculate Moran’s I for each variable and test its statistical significance
  - If Moran’s I is significant, you may have a problem!
- **For regression**
  - calculate the residuals

    map the residuals: do you see any spatial patterns?
  - Calculate Moran’s I for the residuals: is it statistically significant?


Percentile: OLS_RESIDU
Percentile: OLS_RESIDU
< 1% (5)
1% - 10% (46)
10% - 50% (202)
50% - 90% (202)
90% - 99% (49)
> 99% (2)


Moran's I (boston2.5): OLS_RESIDU
Moran's I: 0.195775
lagged OLS_RESIDU
4.4
2.6
0.8
-1
-2.8
-4.6
-4.6
-2.8
-1
0.8
2.6
4.4
OLS_RESIDU

When (spatial) correlation happens

- Try to think of omitted variables and include them in a multiple regression.
  - Missing (omitted) variables may cause spatial autocorrelation
- Regression assumes all relevant variables influencing the dependent variable are included
  - If relevant variables are missing, model is *misspecified*

Spatial Regression Methods

- Spatial Econometrics Approaches (available in GeoDa)
  - Lag model
  - Error model

- Spatial Statistics Approaches
  - Simultaneous Autoregressive Models (SAR)
    - A more general case of Spatial Econometrics
  - Conditional Autoregressive Models (CAR)

- Other methods:
  - Bayesian spatiotemporal methods

- End of this topic