Spatial Analysis and Modeling (GIST 4302/5302)

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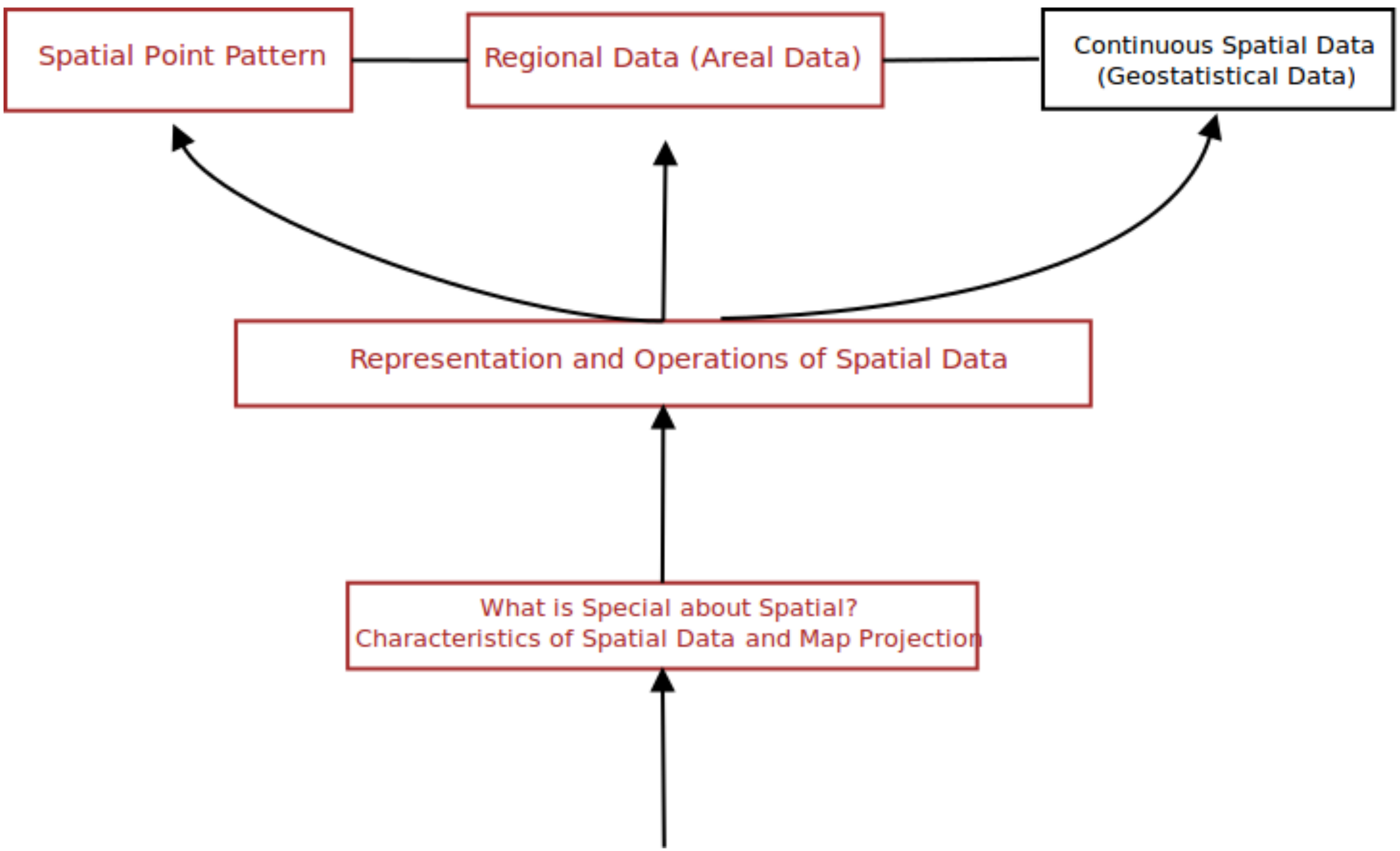
Outline of This Week

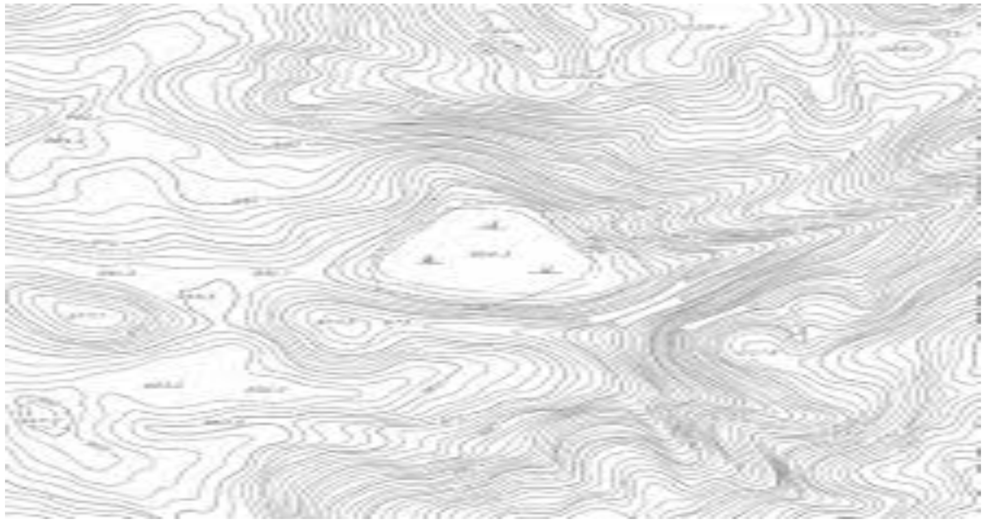
- Last topic, we learned:
  - Spatial autocorrelation of areal data
  - *Spatial regression*
- This topic, we will learn:
  - Spatial fields
  - Interpolation
    - Deterministic interpolation
    - Geostatistics (Kriging family of methods)


Spatial Point Pattern
Regional Data (Areal Data)
Continuous Spatial Data
(Geostatistical Data)
Representation and Operations of Spatial Data
What is Special about Spatial?
Characteristics of Spatial Data and Map Projection

Spatial Fields

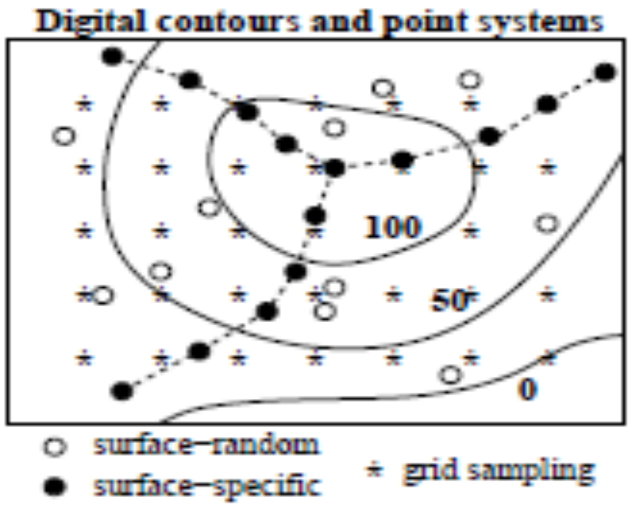
- Scalar versus vector fields:
  - scalar: quantity characterized only by its magnitude
    - *scalar fields* have a single value associated with each location
    - examples: temperature, elevation, precipitation
  - vector: quantity characterized by its magnitude and orientation (e.g., wind speed and direction)
    - *vector fields* have multiple values associated with each location
    - examples: http://hint.fm/wind/

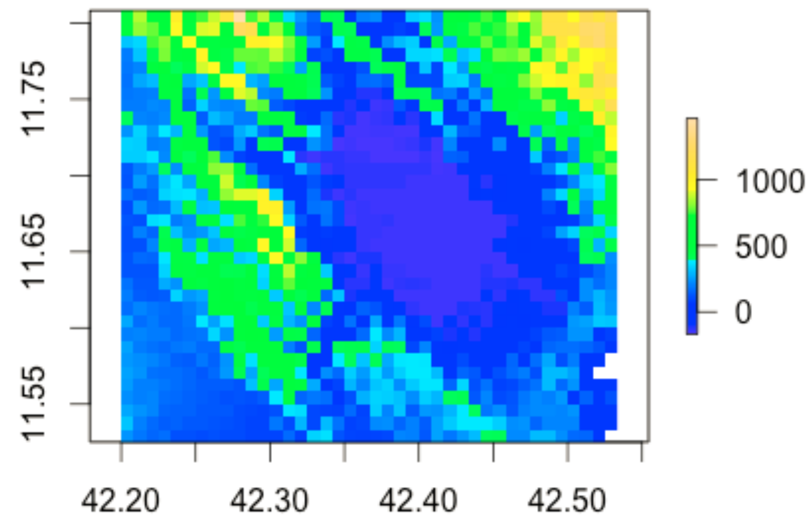
Surface Representation: Contours



- accuracy of digital sample depends on scale and accuracy of source analog map
- details falling between contour lines are lost
- oversampling of steep slopes (many contours) relative to gentle ones (few contours)
- many surface processing operations (e.g., slope calculation or point value determination) are extremely difficult to automate

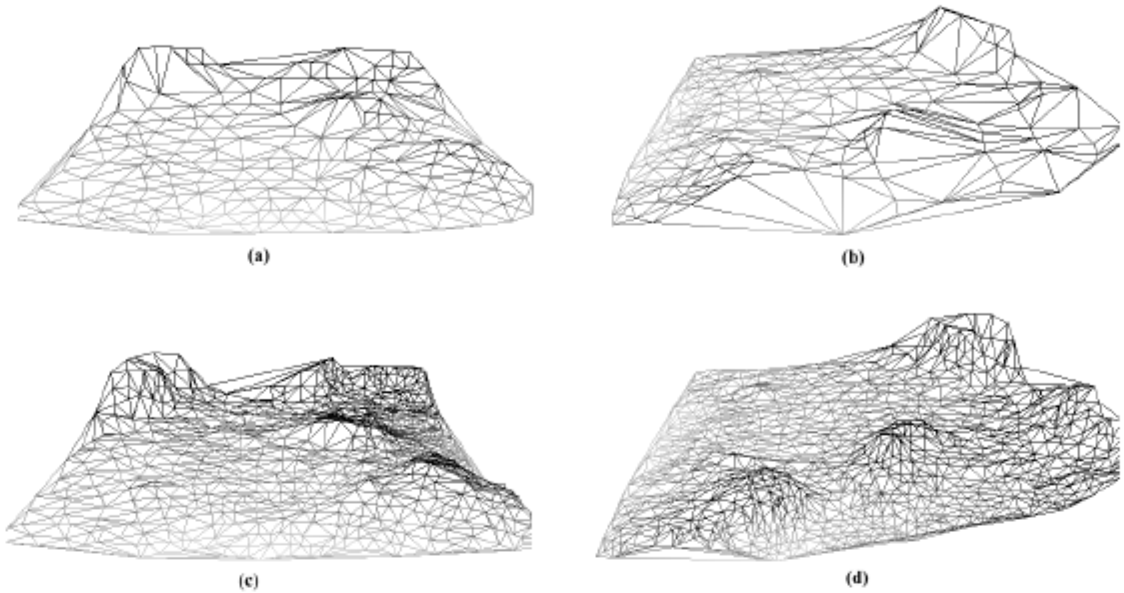
Surface Representation: Point Systems





- uniform data density enables display and surface processing
- no need to store spatial coordinates, just a single point and the grid spacing and orientation
- much larger sample size is required to enhance details (spatial resolution)
- Details/accuracy is controlled by the cell size/resolution
- Value of each cell is homogeneous represented by one single value

Surface Representation: Triangulated Irregular Network



- extremely compact way of storing fields, and their properties (e.g., slope, aspect)
- can capture important surface characteristics
- accuracy depends on accuracy of underlying field (assumed known)

Sampling Spatial Fields

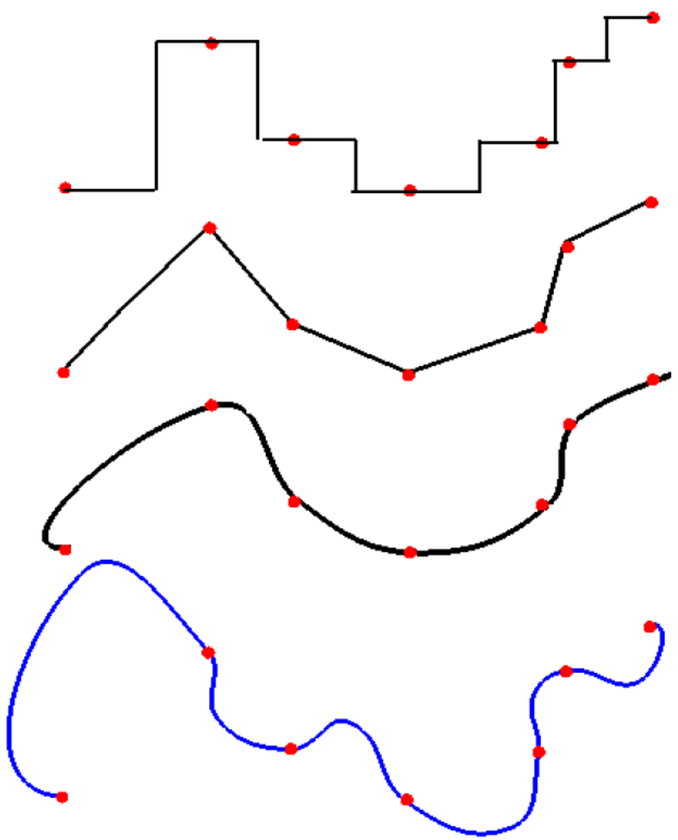
- **Sampling schemes:**
  - collection of measurements at a set of locations(e.g., precipitation at rain gauges, elevation spot heights)
  - regular grids obtained from aerial and/or satellite remote sensing (such measurements are area integrals)
  - digitized contour maps = points from digitized contours derived from analog topographic maps

Sampling Spatial Fields

- **Issues to consider:**
  - data constitute a *sample* of the underlying continuous field (exhaustive sampling is almost always impossible)
  - measurements might have both spatial *and temporal* components
  - often, data are not collected at random $\Rightarrow$ biased and non-random sampling
  - sometime, contours maps are derived from spot heights $\Rightarrow$ digitized contour maps should be treated with caution
  - All measurements are subjective to **<u>uncertainty</u>** (spatial uncertainty, more in the next lecture)

Spatial Interpolation

- Why spatial interpolation:
  - Observations/samples are sparse
- Interpolation: discrete->continuous
- Underline Rationale
  - Again, TFL
- It is difficult



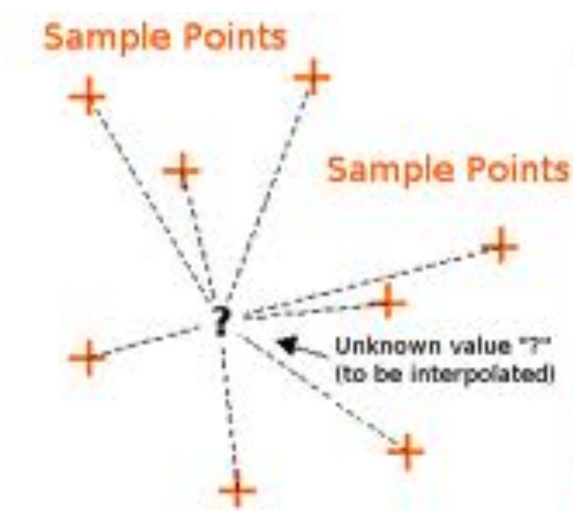
Spatial Interpolation

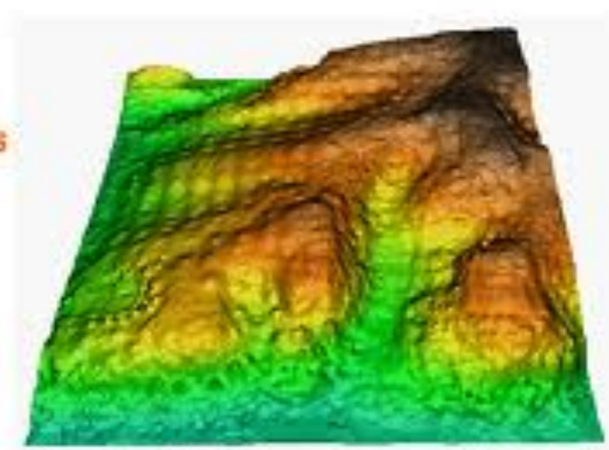
- General formulation of spatial interpolation

unknown value $z(s_i)$ at any non-sampled location s_i expressed as weighted average of n sample data $\{z(s_\alpha),\ \alpha = 1,\ldots \mathrm{n}\}$:

$$z(s_i) = \sum_{\alpha=1}^{n} w_{i\alpha} z(\mathrm{s}_\alpha)$$

$w_{i\alpha}$ denotes weight given to datum $z(s_\alpha)$ for prediction at location s_i



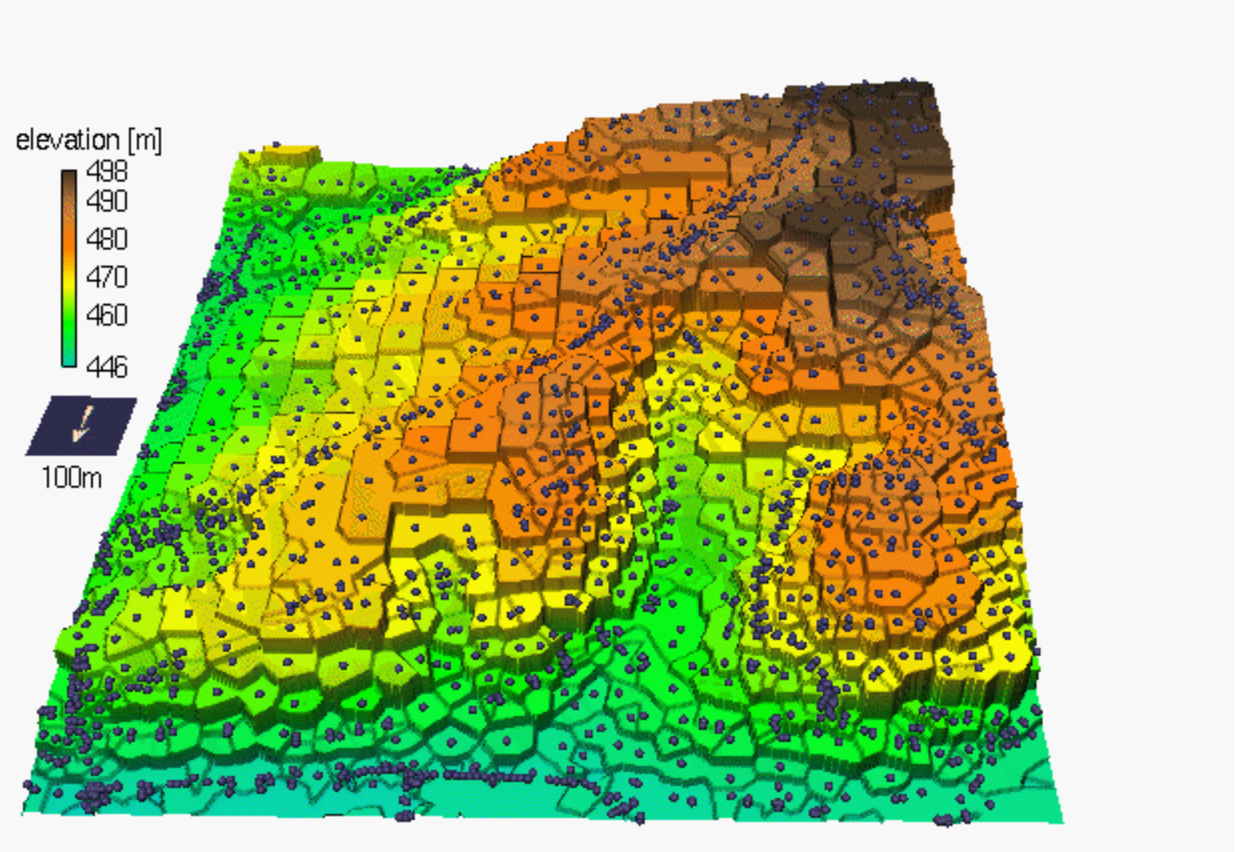


Spatial Interpolation Methods

- Deterministic Interpolators
  - Nearest Neighbor/Natural neighbor
  - Trend Surface
  - Inverse distance weighted method
  - Spatial spline
  - Triangulation
- Stochastic Interpolators
  - Kriging
  - Outcome the credibility information compared to the deterministic interpolators

Spatial Interpolation: Nearest Neighbor

- Assign value of nearest sample point
- Thiessen Polygons/Voronoi diagram



Spatial Interpolation: Nearest Neighbor

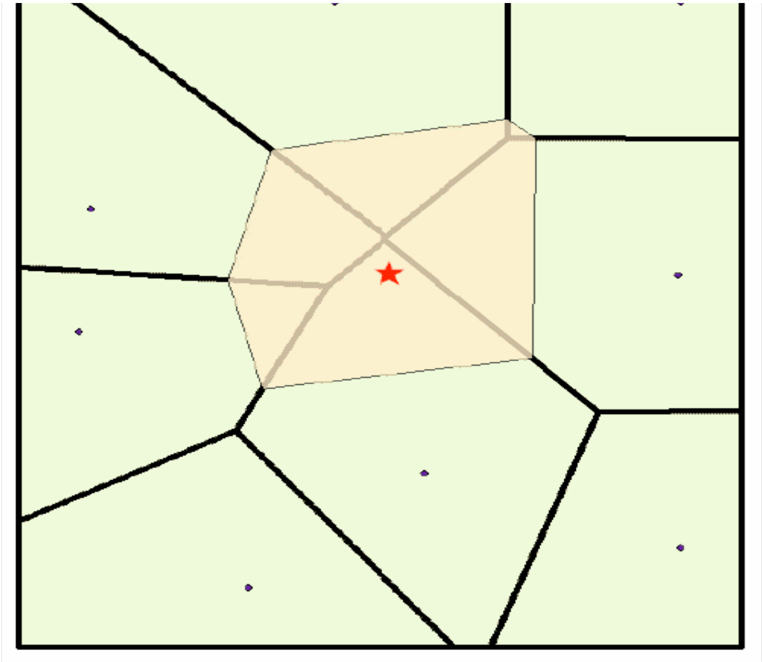
- Datum closest to the prediction location receives all weights

$$z(s_i) = \sum_{\alpha=1}^{n} w_{i\alpha} z(\mathrm{s}_\alpha) = z(\mathrm{s}_\alpha) + \sum_{\alpha=1}^{n-1} 0 * z(\mathrm{s}_\alpha)$$

- Unbiased estimation $\sum_{\alpha=1}^{n} w_{i\alpha} = 1$
- set of predicted values form discontinuous (patchy) surface

Natural Neighbor Interpolation

- Finds the closest subset of input samples to a query point and applies weights to them based on proportionate areas in order to interpolate a value
- “Area-stealing”
- Local interpolation: using only a subset of samples that surround a query point



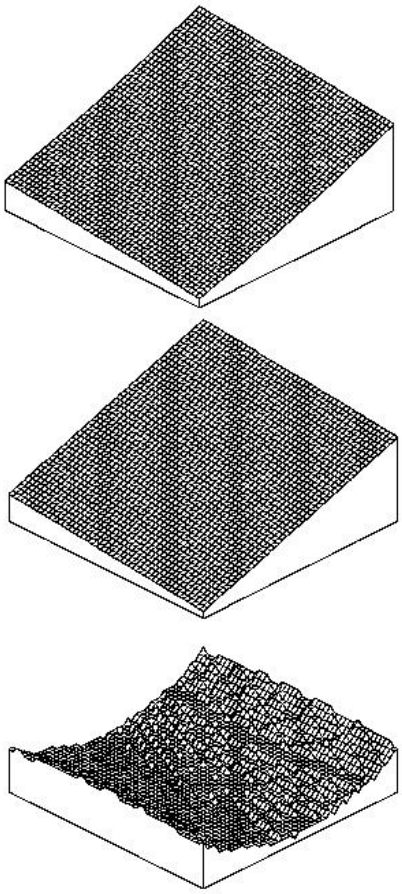
Spatial Interpolation: Trend Surface

- explicit mathematical function(s) of coordinates that interpolates or approximates (smooths) the surface. For example:

$$z(s_i) = a_0 + a_1 * x + a_2 * y$$

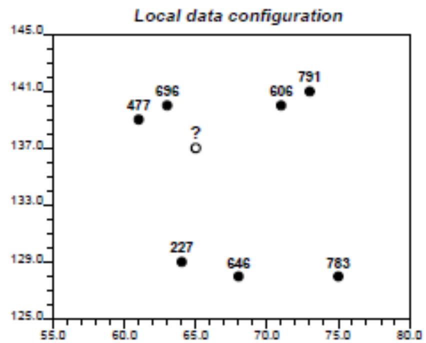
$$or$$

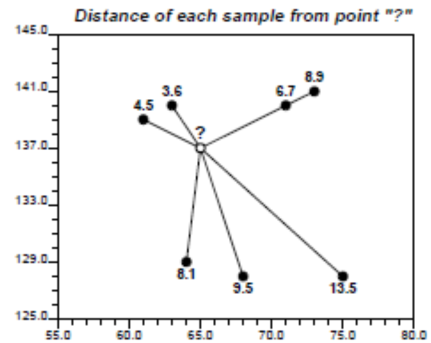
$$z(s_i) = a_0 + a_1 * \mathrm{x}^2 + a_2 * y^2 + a_3 xy$$



- surface operations (e.g., curvature) and values can be analytically computed
- Fit polynomial equation to sample points
- Goal is to minimize deviations between sample points and surface
- arbitrary choice of number and type of functions
- local versus global fitting

Spatial Interpolation: Inverse Distance





Procedure:

- predict unknown value $z(\mathbf{s}_i)$ at any non-sampled location $\mathbf{s}_i$ as weighted linear combination of $n(\mathbf{s}_i)$ nearby data $z(\mathbf{s}_\alpha)$:

$$\hat{z}(\mathbf{s}_i) = \sum_{\alpha=1}^{n(\mathbf{s}_i)} w_{i\alpha} z(\mathbf{s}_\alpha)$$

  where $w_{i\alpha}$ denotes weight received by sample $z(\mathbf{s}_\alpha)$ for prediction at location $\mathbf{s}_i$

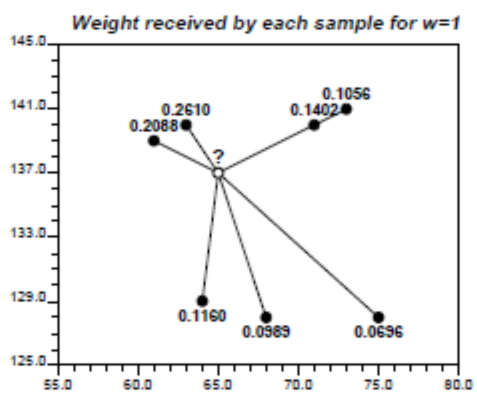
- make weight $w_{i\alpha}$ inversely proportional to power k of distance $h_{i\alpha} = ||\mathbf{s}_i - \mathbf{s}_\alpha||$:

$$w_{i\alpha} = \frac{h_{i\alpha}^{-k}}{\sum_{\alpha=1}^{n(\mathbf{s}_i)} h_{i\alpha}^{-k}} = \frac{1/h_{i\alpha}^{k}}{\sum_{\alpha=1}^{n(\mathbf{s}_i)} 1/h_{i\alpha}^{k}}$$

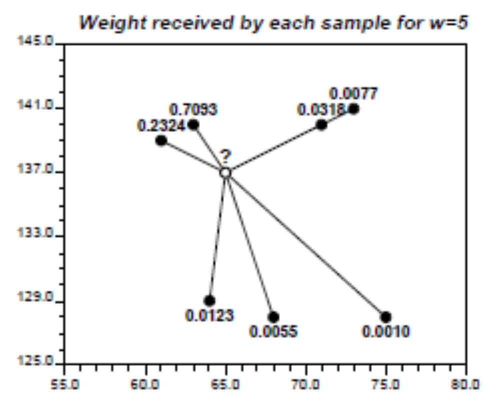
Spatial Interpolation: Inverse Distance

Characteristics:

- unbiased interpolation procedure, since $\sum_{\alpha=1}^{n(\mathbf{s}_i)} w_{i\alpha} = 1$
- "exact" interpolator: $\hat{z}(\mathbf{s}_\alpha) = z(\mathbf{s}_\alpha), \;\forall \alpha$
- exponent k controls importance of data closer to $\mathbf{s}_i$;
  e.g., $k = 2$: inverse distance squared interpolation



$\boxed{\hat{z}(\mathbf{s}) = 598}$



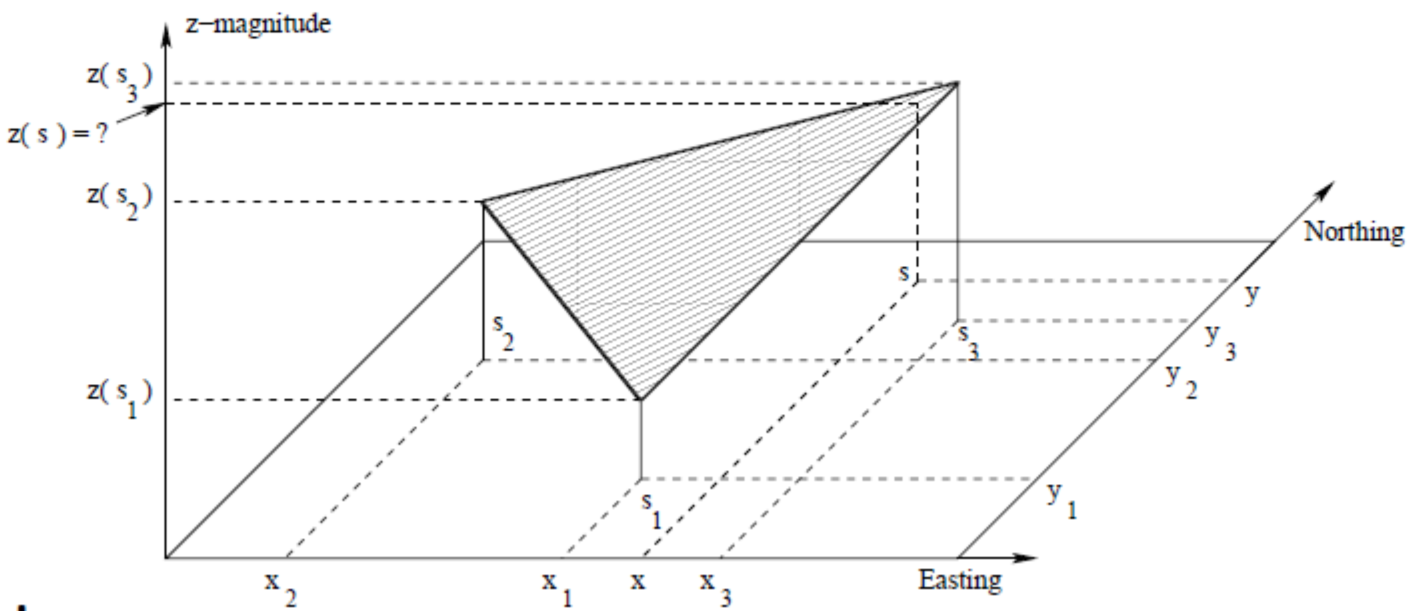
$\boxed{\hat{z}(\mathbf{s}) = 637}$

Spatial Spline

- Estimates values using a mathematical function that minimizes overall surface curvature
  - smooth surface
  - passes exactly through the input points

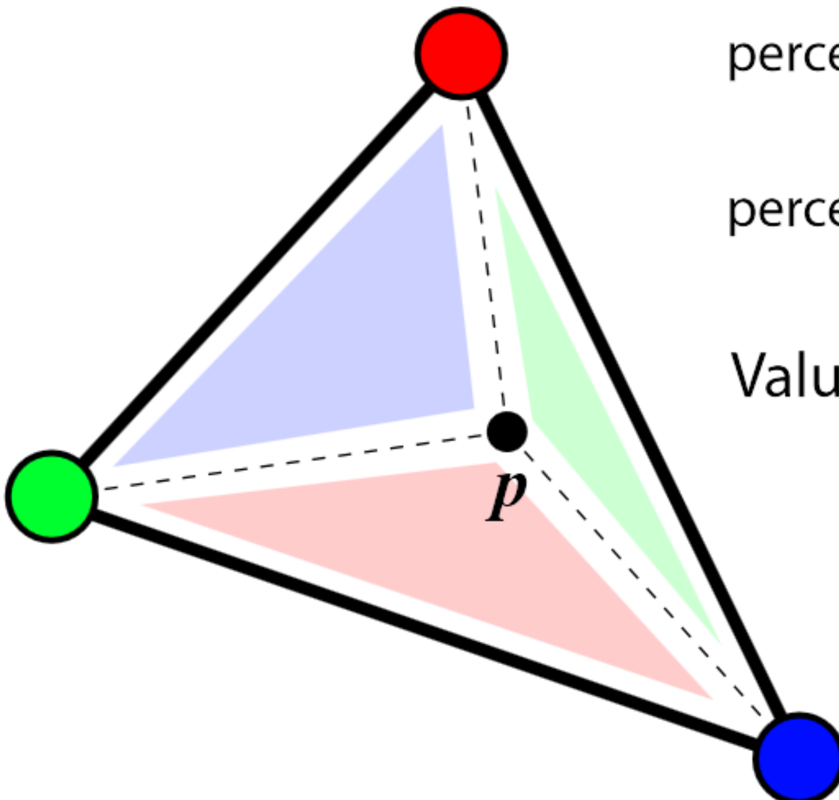
Spatial Interpolation: Triangulation

- Barycentric Interpolation



Spatial Interpolation: Triangulation

- Barycentric Interpolation



percent red $= \dfrac{\text{area of red triangle}}{\text{total area}}$

percent green $= \dfrac{\text{area of green triangle}}{\text{total area}}$

percent blue $= \dfrac{\text{area of blue triangle}}{\text{total area}}$

Value at $\boldsymbol{p}$:

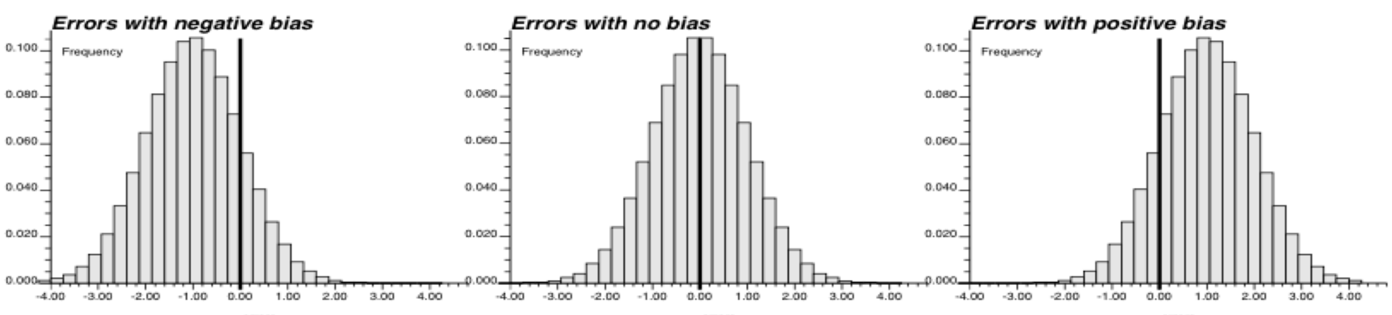
(% red)(value at red) +
(% green)(value at green) +
(% blue)(value at blue)

Evaluating Prediction Performance

- Cross-validation:
  - Loop over sample locations:
  - hide a sample datum
  - predict it from the remaining data using one of the spatial interpolation method
  - repeat until all sample locations are visited and cross-validation predictions are computed

Evaluating Prediction Performance

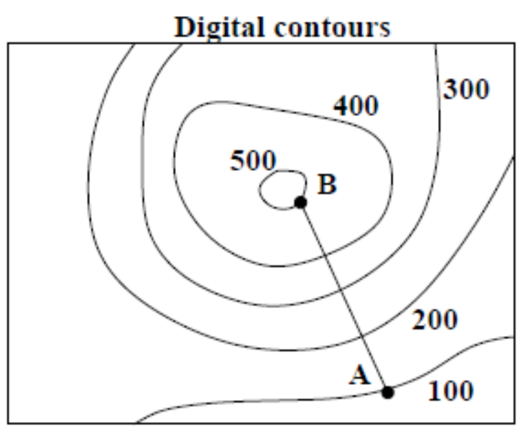
- Compare distribution of predicted values to that of true values for:
  - reproduction of mean (for possible bias), median, variance, and other summary statistics
  - reproduction of entire distribution of true values (QQ plot)

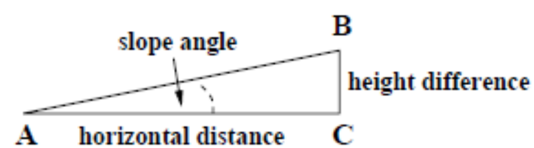


Surface Derivatives: Slope and Gradient

Gradient:

- vector quantity specified by (i) magnitude, and (ii) direction
- gradient magnitude = maximum rate of change of elevation at a point (slope)
- gradient direction = direction of steepest slope trough that point (aspect)





- calculating the tangent of the slope angle:

$$\tan(\theta) = \frac{\text{height difference}}{\text{horizontal distance}} = \frac{BC}{AC} \Rightarrow \theta = \arctan(\frac{BC}{AC})$$

in Matlab: $theta$=rad2deg(atan(BC/AC))

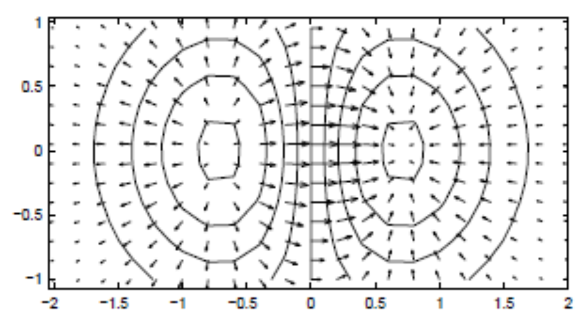
Surface Derivatives: Slope and Gradient

Gradient calculations:

- in TIN surface representation, gradient at s = gradient of containing Delaunay triangle
- in raster surface representation, gradient at s calculated using a square window (typically 9×9) centered at s.
  Slope θ and aspect α are calculated as:

$$\theta = \sqrt{\theta_x^2 + \theta_y^2} \quad \text{and} \quad a = \arctan(\frac{\theta_x^2}{\theta_y^2})$$

  where θ_x and θ_y denote directional derivatives along x and y
  aspect α measured from vertical to direction of steepest slope;
  $\alpha = \alpha + 180$ if $\theta_y > 0$, and $\alpha = \alpha + 360$ if $\theta_x > 0$ and $\theta_y < 0$
- alternatively, a local mathematical surface is fitted within each window, and its derivative is analytically calculated



- End of this topic