

# Spatial Analysis and Modeling (GIST 4302/5302)

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# Outline of This Week

- Last week, we learned:
  - spatial point pattern analysis (PPA)
  - focus on location distribution of ‘events’
  - Measure the cluster (spatial autocorrelation) in point pattern
- This week, we will learn:
  - How to measure and detect clusters/spatial autocorrelation in areal data (regional data)

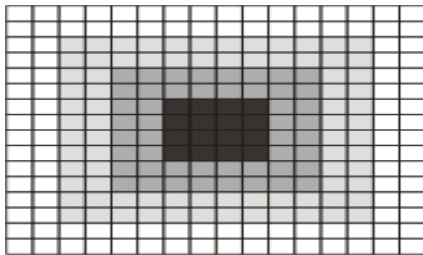
# Spatial Autocorrelation

- Spatial autocorrelation is everywhere
  - Spatial point pattern
    - K, G functions
    - Kernel functions
  - Areal/lattice (this topic)
  - Geostatistical data (next topic)

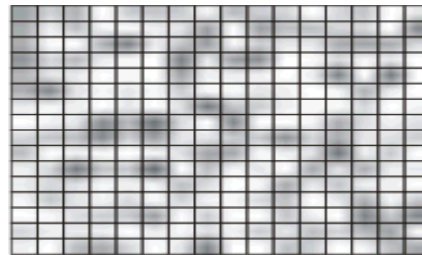
# Spatial Autocorrelation of Areal Data

# Spatial Autocorrelation

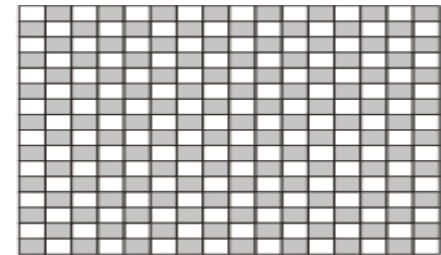
- Tobler's first law of geography
- Spatial auto/cross correlation



If like values  
tend to cluster  
together,  
then the field  
exhibits  
high **positive  
spatial  
autocorrelation**



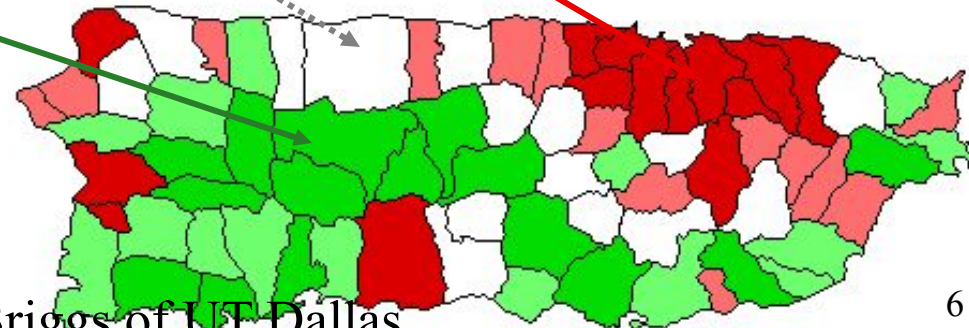
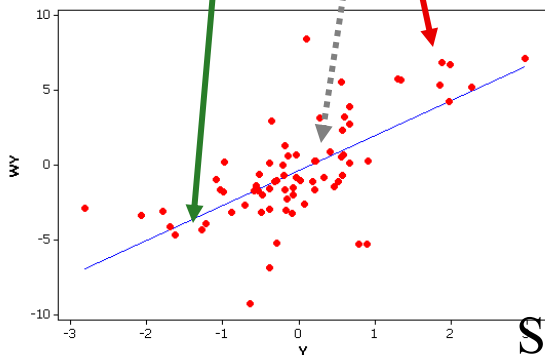
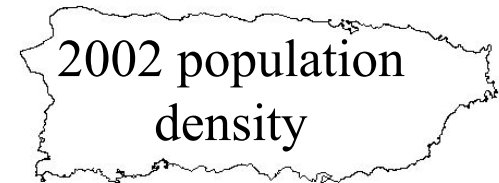
If there is no apparent  
relationship between  
attribute value and  
location then there is  
**zero spatial  
autocorrelation**



If like values tend  
to be located  
away from each  
other, then there  
is **negative  
spatial  
autocorrelation**

## *Positive spatial autocorrelation*

- high values surrounded by nearby high values
- intermediate values surrounded by nearby intermediate values
- low values surrounded by nearby low values

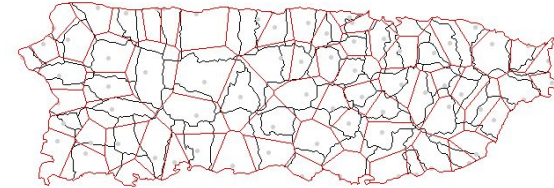


Source: Ron Briggs of UT Dallas

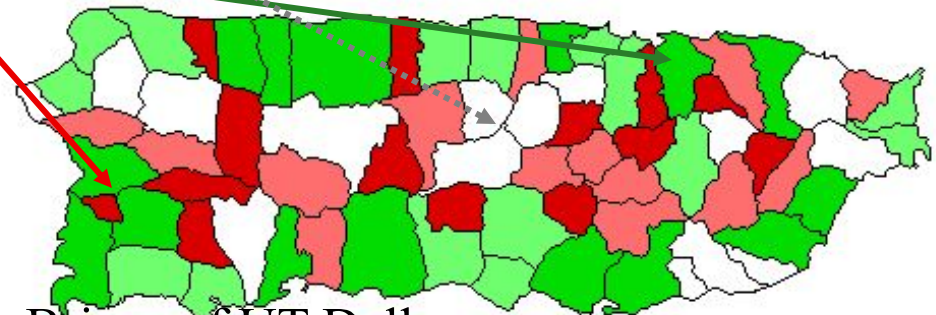
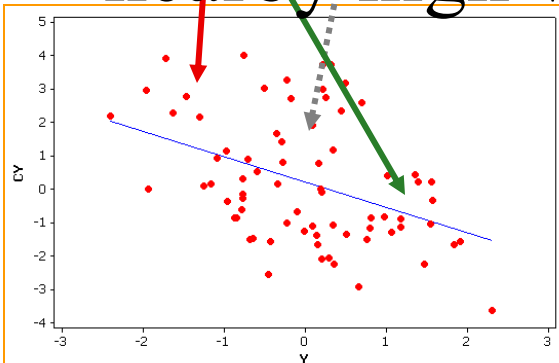
# *Negative spatial autocorrelation*

- high values surrounded by nearby low values
- intermediate values surrounded by nearby intermediate values
- low values surrounded by nearby high values

competition for space



Grocery store density



Source: Ron Briggs of UT Dallas

# Measuring Spatial Autocorrelation: the problem of measuring “nearness”

**To measure spatial autocorrelation, we must know the “nearness” of our observations as we did for point pattern case**

- Which points or polygons are “near” or “next to” other points or polygons?

– *Which states are near Texas?*

– How to measure this?

Seems simple and obvious,  
but it is not!





# Spatial Weight Matrix

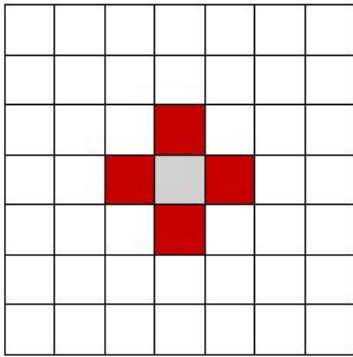
- **Core** concept in statistical analysis of areal data
- Two steps involved:
  - define which relationships between observations are to be given a nonzero weight, i.e., define spatial neighbors
  - assign weights to the neighbors

# Spatial Neighbors

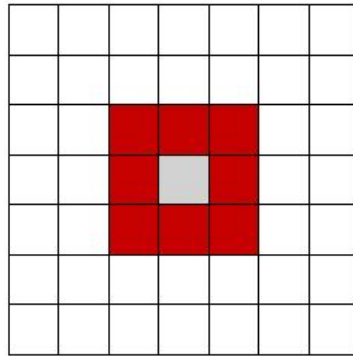
- **Contiguity-based neighbors**
  - Zone  $i$  and  $j$  are neighbors if zone  $i$  is contiguity or adjacent to zone  $j$
  - But what constitutes contiguity?
- **Distance-based neighbors**
  - Zone  $i$  and  $j$  are neighbors if the distance between them are less than the threshold distance
  - But what distance do we use?

# Contiguity-based Spatial Neighbors

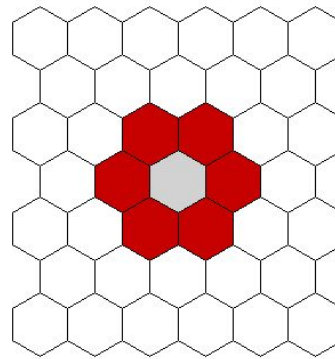
- Sharing a border or boundary
  - Rook: sharing a border
  - Queen: sharing a border or a point



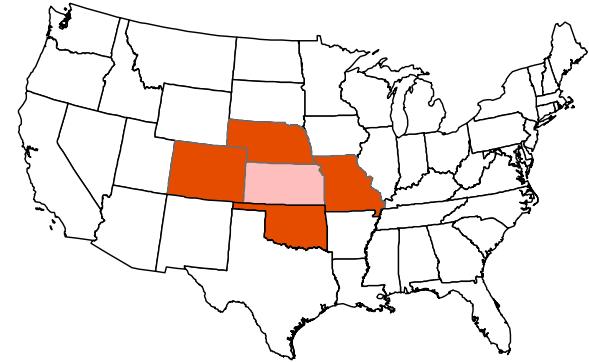
rook



queen



Hexagons



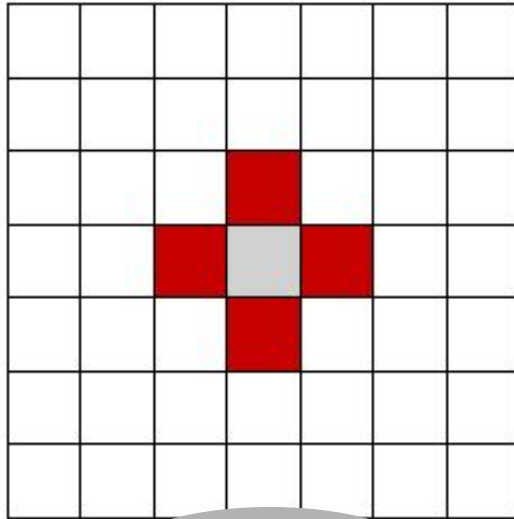
Irregular

Which use?

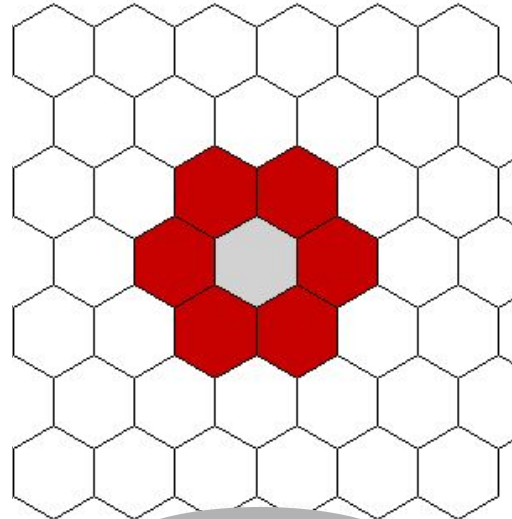
# Higher-Order Contiguity

1<sup>st</sup>  
order

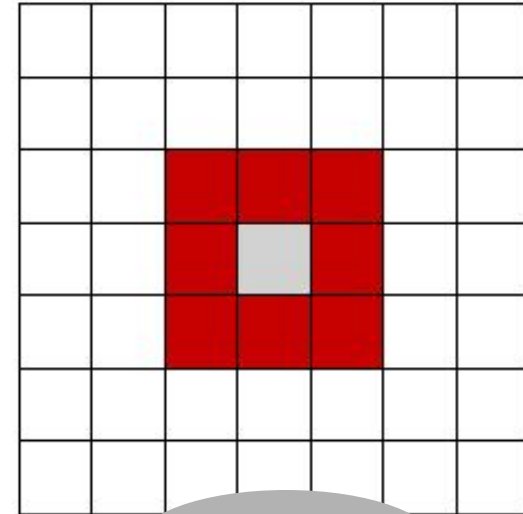
Nearest  
neighbor



rook



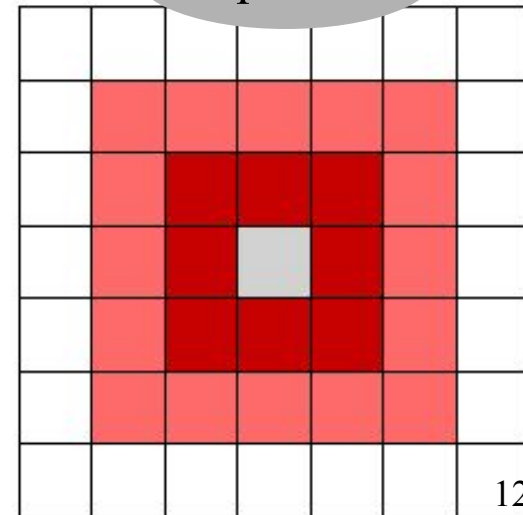
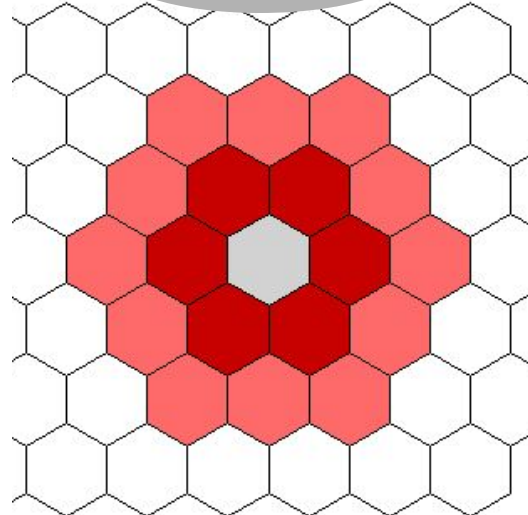
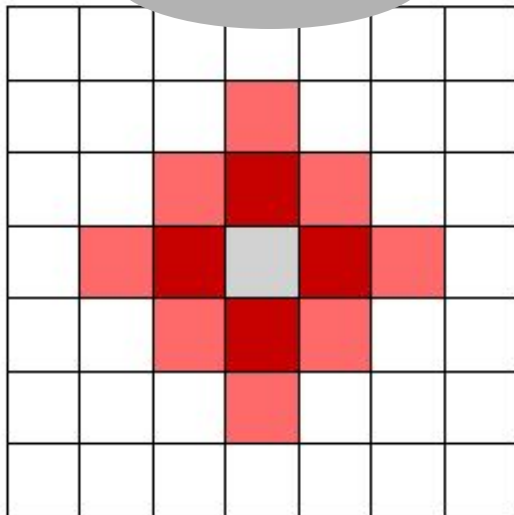
hexagon



queen

2<sup>nd</sup>  
order

Next  
nearest  
neighbor



# Distance-based Neighbors

- How to measure distance between polygons?
- Distance metrics
  - 2D Cartesian distance (projected data)
  - 3D spherical distance/great-circle distance (lat/long data)
    - Haversine formula

*Haversine*  $a = \sin^2(\Delta\phi/2) + \cos(\phi_1) \cdot \cos(\phi_2) \cdot \sin^2(\Delta\lambda/2)$

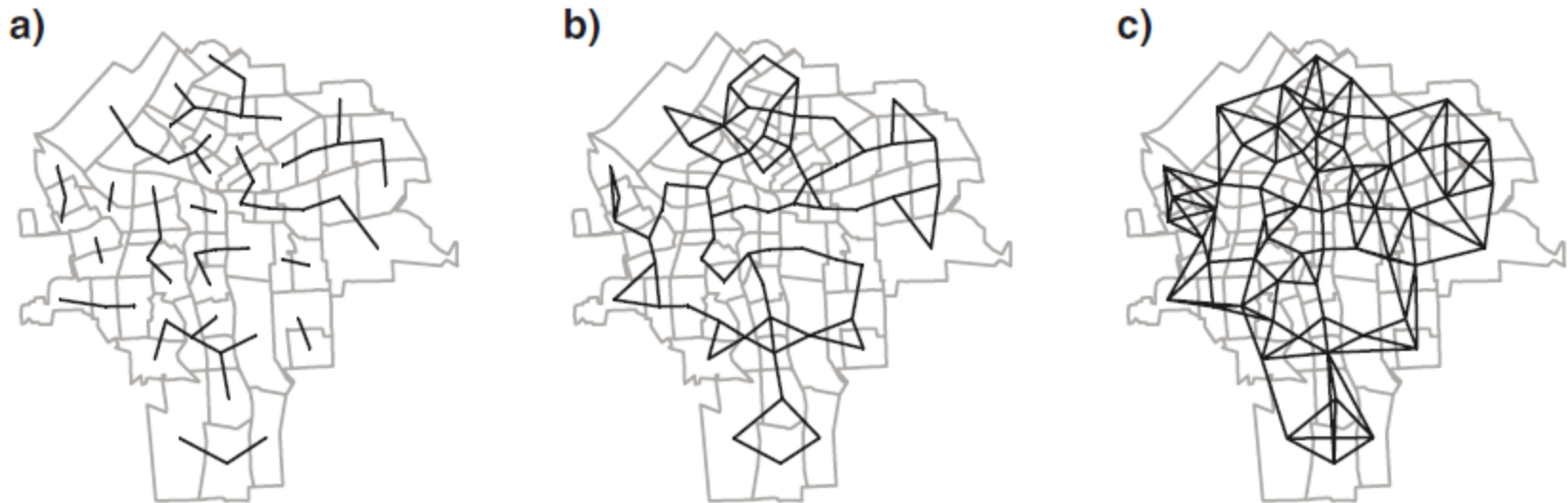
*formula:*  $c = 2 \cdot \text{atan2}(\sqrt{a}, \sqrt{1-a})$

$d = R \cdot c$

*where  $\phi$  is latitude,  $\lambda$  is longitude,  $R$  is earth's radius (mean radius = 6,371km)*

# Distance-based Neighbors

- $k$ -nearest neighbors

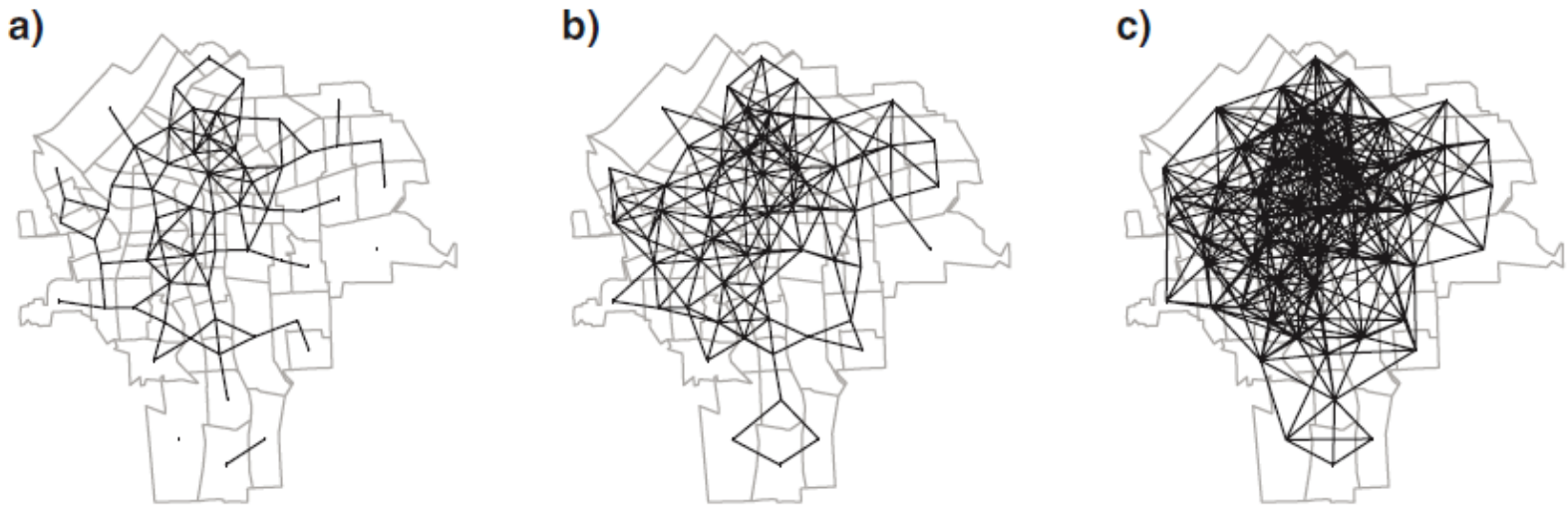


**Fig. 9.5.** (a)  $k = 1$  neighbours; (b)  $k = 2$  neighbours; (c)  $k = 4$  neighbours

Source: Bivand and Pebesma and Gomez-Rubio

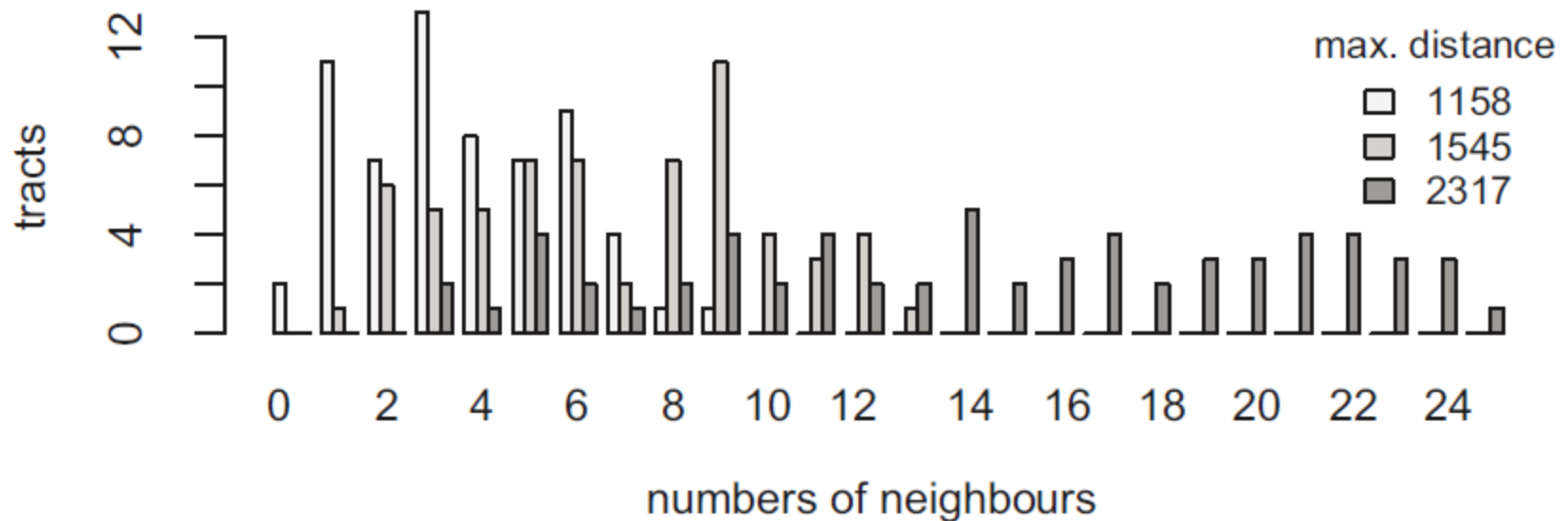
# Distance-based Neighbors

- thresh-hold distance (buffer)



**Fig. 9.6.** (a) Neighbours within 1,158 m; (b) neighbours within 1,545 m; (c) neighbours within 2,317 m

# Neighbor/Connectivity Histogram

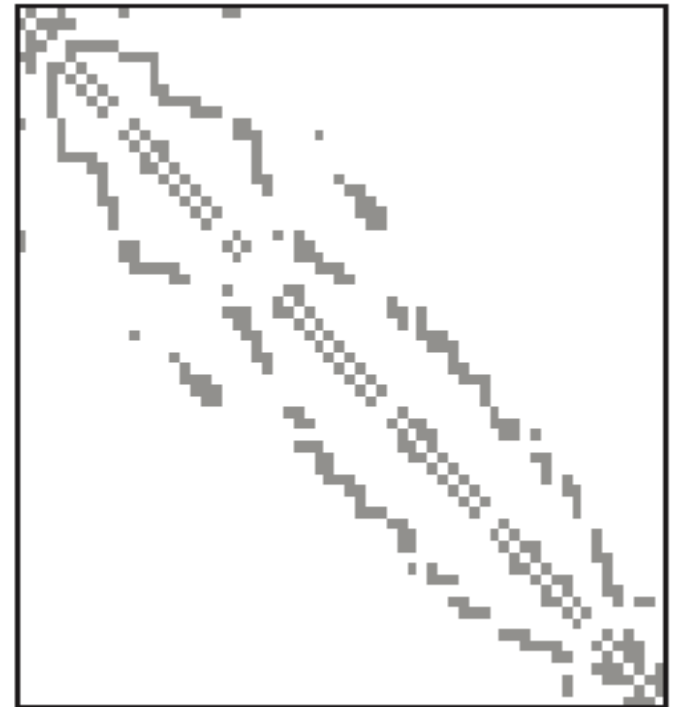


Source: Bivand and Pebesma and Gomez-Rubio



# Spatial Weight Matrix

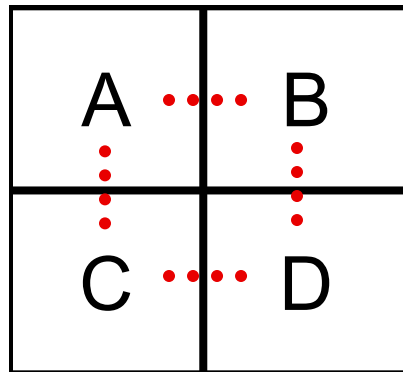
- Spatial weights can be seen as a list of weights indexed by a list of neighbors
- If zone  $j$  is not a neighbor of zone  $i$ , weights  $W_{ij}$  will set to zero
  - The weight matrix can be illustrated as an image
  - Sparse matrix



# A Simple Example for Rook case

- Matrix contains a:
  - 1 if share a border
  - 0 if do not share a border

4 areal units

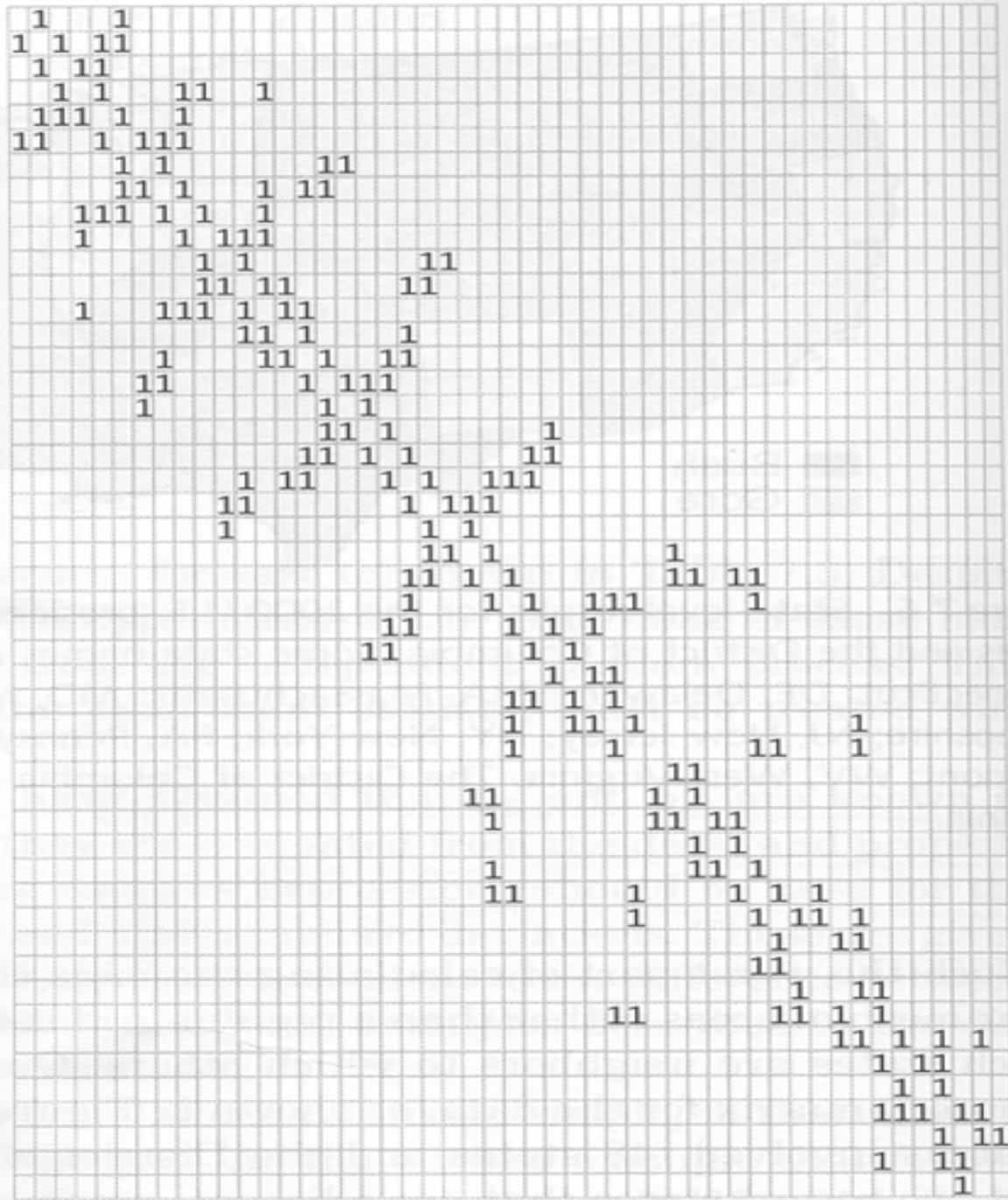


Common border

4x4 matrix

	A	B	C	D
A	0	1	1	0
B	1	0	0	1
C	1	0	0	1
D	0	1	1	0

- 1 Washington
- 2 Oregon
- 3 California
- 4 Arizona
- 5 Nevada
- 6 Idaho
- 7 Montana
- 8 Wyoming
- 9 Utah
- 10 New Mexico
- 11 Texas
- 12 Oklahoma
- 13 Colorado
- 14 Kansas
- 15 Nebraska
- 16 South Dakota
- 17 North Dakota
- 18 Minnesota
- 19 Iowa
- 20 Missouri
- 21 Arkansas
- 22 Louisiana
- 23 Mississippi
- 24 Tennessee
- 25 Kentucky
- 26 Illinois
- 27 Wisconsin
- 28 Michigan
- 29 Indiana
- 30 Ohio
- 31 West Virginia
- 32 Florida
- 33 Alabama
- 34 Georgia
- 35 South Carolina
- 36 North Carolina
- 37 Virginia
- 38 Maryland
- 39 Delaware
- 40 District of Columbia
- 41 New Jersey
- 42 Pennsylvania
- 43 New York
- 44 Connecticut
- 45 Rhode Island
- 46 Massachusetts
- 47 New Hampshire
- 48 Vermont
- 49 Maine



**Sparse Contiguity Matrix for US States -- obtained from Anselin's web site (see powerpoint for link)**

<b>Name</b>	<b>Fips</b>	<b>Ncount</b>	<b>N1</b>	<b>N2</b>	<b>N3</b>	<b>N4</b>	<b>N5</b>	<b>N6</b>	<b>N7</b>	<b>N8</b>
Alabama	1	4	28	13	12	47				
Arizona	4	5	35	8	49	6	32			
Arkansas	5	6	22	28	48	47	40	29		
California	6	3	4	32	41					
Colorado	8	7	35	4	20	40	31	49	56	
Connecticut	9	3	44	36	25					
Delaware	10	3	24	42	34					
District of Columbia	11	2	51	24						
Florida	12	2	13	1						
Georgia	13	5	12	45	37	1	47			
Idaho	16	6	32	41	56	49	30	53		
Illinois	17	5	29	21	18	55	19			
Indiana	18	4	26	21	17	39				
Iowa	19	6	29	31	17	55	27	46		
Kansas	20	4	40	29	31	8				
Kentucky	21	7	47	29	18	39	54	51	17	
Louisiana	22	3	28	48	5					
Maine	23	1	33							
Maryland	24	5	51	10	54	42	11			
Massachusetts	25	5	44	9	36	50	33			
Michigan	26	3	18	39	55					
Minnesota	27	4	19	55	46	38				
Mississippi	28	4	22	5	1	47				
Missouri	29	8	5	40	17	21	47	20	19	31
Montana	30	4	16	56	38	46				
Nebraska	31	6	29	20	8	19	56	46		
Nevada	32	5	6	4	49	16	41			
New Hampshire	33	3	25	23	50					
New Jersey	34	3	10	36	42					
New Mexico	35	5	48	40	8	4	49			
New York	36	5	34	9	42	50	25			
North Carolina	37	4	45	13	47	51				
North Dakota	38	3	46	27	30					
Ohio	39	5	26	21	54	42	18			
Oklahoma	40	6	5	35	48	29	20	8		
Oregon	41	4	6	32	16	53				
Pennsylvania	42	6	24	54	10	39	36	34		
Rhode Island	44	2	25	9						
South Carolina	45	2	13	37						
South Dakota	46	6	56	27	19	31	38	30		
Tennessee	47	8	5	28	1	37	13	51	21	29
Texas	48	4	22	5	35	40				
Utah	49	6	4	8	35	56	32	16		
Vermont	50	3	36	25	33					
Virginia	51	6	47	37	24	54	11	21		
Washington	53	2	41	16						
West Virginia	54	5	51	21	24	39	42			
Wisconsin	55	4	26	17	19	27				
Wyoming	56	6	49	16	31	8	46	30		

# Style of Spatial Weight Matrix

- Row
  - a weight of unity for each neighbor relationship
- Row standardization
  - Symmetry not guaranteed
  - can be interpreted as allowing the calculation of average values across neighbors
- General spatial weights based on distances

# Row vs. Row standardization

A	B	C
D	E	F

Divide each  
number by the  
**row sum**

Total number of neighbors  
--some have more than others

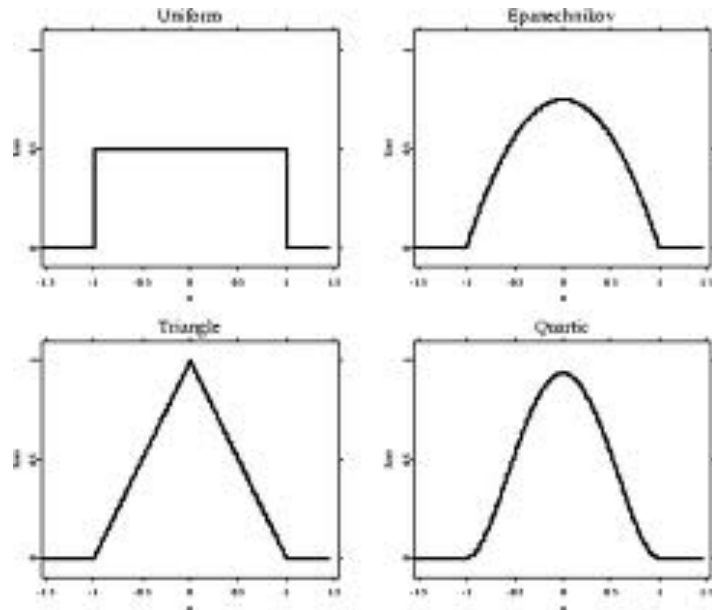
	A	B	C	D	E	F	Row Sum
A	0	1	0	1	0	0	2
B	1	0	1	0	1	0	3
C	0	1	0	0	0	1	2
D	1	0	0	0	1	0	2
E	0	1	0	1	0	1	3
F	0	0	1	0	1	0	2

Row standardized  
--usually use this

	A	B	C	D	E	F	Row Sum
A	0.0	0.5	0.0	0.5	0.0	0.0	1
B	0.3	0.0	0.3	0.0	0.3	0.0	1
C	0.0	0.5	0.0	0.0	0.0	0.5	1
D	0.5	0.0	0.0	0.0	0.5	0.0	1
E	0.0	0.3	0.0	0.3	0.0	0.3	1
F	0.0	0.0	0.5	0.0	0.5	0.0	1

# General Spatial Weights Based on Distance

- Decay functions of distance
  - Most common choice is the inverse (reciprocal) of the distance between locations  $i$  and  $j$  ( $w_{ij} = 1/d_{ij}$ )
  - Other functions also used
    - inverse of squared distance ( $w_{ij} = 1/d_{ij}^2$ ), or
    - negative exponential ( $w_{ij} = e^{-d}$  or  $w_{ij} = e^{-d^2}$ )



# Measure of Spatial Autocorrelation



# Global Measures and Local Measures

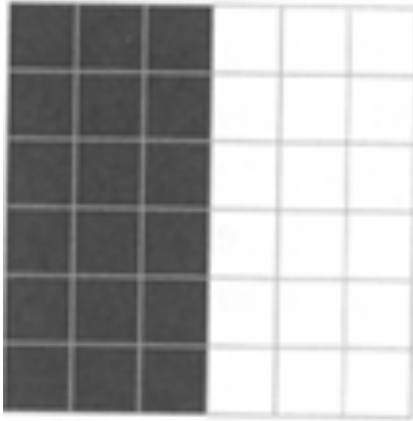
- Global Measures
  - A single value which applies to the entire data set
    - The same pattern or process occurs over the entire geographic area
    - An average for the entire area
- Local Measures
  - A value calculated for each observation unit
    - Different patterns or processes may occur in different parts of the region
    - A unique number for each location
- Global measures usually can be decomposed into a combination of local measures

# Global Measures and Local Measures

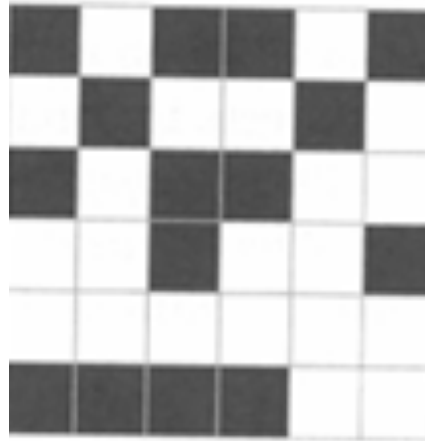
- Global Measures
  - Join Count
  - Moran's I
- Local Measures
  - Local Moran's I

# Join (or Joint or Joins) Count Statistic

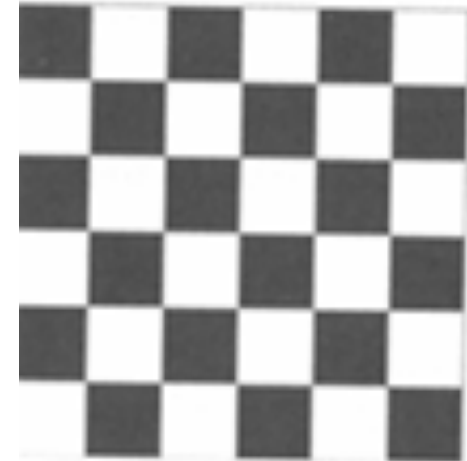
Positive autocorrelation



No autocorrelation



Negative autocorrelation



Rook's case

$$J_{BB} = 27$$

$$J_{WW} = 27$$

$$J_{BW} = 6$$

Queen's case

$$J_{BB} = 47$$

$$J_{WW} = 47$$

$$J_{BW} = 16$$

$$J_{BB} = 6$$

$$J_{WW} = 19$$

$$J_{BW} = 35$$

$$J_{BB} = 14$$

$$J_{WW} = 40$$

$$J_{BW} = 56$$

$$J_{BB} = 0$$

$$J_{WW} = 0$$

$$J_{BW} = 60$$

$$J_{BB} = 25$$

$$J_{WW} = 25$$

$$J_{BW} = 60$$

- 60 for Rook Case
- 110 for Queen Case

# Join Count: Test Statistic

Test Statistic given by:  $Z = \frac{\text{Observed} - \text{Expected}}{\text{SD of Expected}}$

**Expected** = random pattern generated by tossing a coin in each cell.

Expected given by: Standard Deviation of Expected (standard error) given by:

$$E(J_{BB}) = kp_B^2$$

$$E(s_{BB}) = \sqrt{kp_B^2 + 2mp_B^3 - (k + 2m)p_B^4}$$

$$E(J_{WW}) = kp_W^2$$

$$E(s_{WW}) = \sqrt{kp_W^2 + 2mp_W^3 - (k + 2m)p_W^4}$$

$$E(J_{BW}) = 2kp_Bp_W$$

$$E(s_{BW}) = \sqrt{2(k + m)p_Bp_W - 4(k + 2m)p_B^2p_W^2}$$

Where: k is the total number of joins (neighbors)

$p_B$  is the expected proportion Black, if random

$p_W$  is the expected proportion White

m is calculated from k according to:

$$m = \frac{1}{2} \sum_{i=1}^n k_i(k_i - 1)$$

# Gore/Bush Presidential Election 2000



	Actual
Jbb	60
Jgg	21
Jbg	28
Total	109

# Join Count Statistic for Gore/Bush 2000 by State

<b>candidates</b>	probability
Bush	0.49885
Gore	0.50115

	Actual	Expected	Stan Dev	Z-score
Jbb	60	27.125	8.667	3.7930
Jgg	21	27.375	8.704	-0.7325
Jbg	28	54.500	5.220	-5.0763
Total	109	109.000		

- The expected number of joins is calculated based on the proportion of votes each received in the election (for Bush =  $109 * .499 * .499 = 27.125$ )
- There are far more Bush/Bush joins (actual = 60) than would be expected (27)
  - Positive autocorrelation
- There are far fewer Bush/Gore joins (actual = 28) than would be expected (54)
  - Positive autocorrelation
- No strong clustering evidence for Gore (actual = 21 slightly less than 27.375)

# Moran's I

- The most common measure of Spatial Autocorrelation
- Use for points or polygons
  - Join Count statistic only for polygons
- Use for a continuous variable (any value)
  - Join Count statistic only for binary variable (1,0)



Patrick Alfred Pierce Moran (1917-1988)

# Formula for Moran's I

$$I = \frac{N \sum_{i=1}^n \sum_{j=1}^n w_{ij} (x_i - \bar{x})(x_j - \bar{x})}{(\sum_{i=1}^n \sum_{j=1}^n w_{ij}) \sum_{i=1}^n (x_i - \bar{x})^2}$$

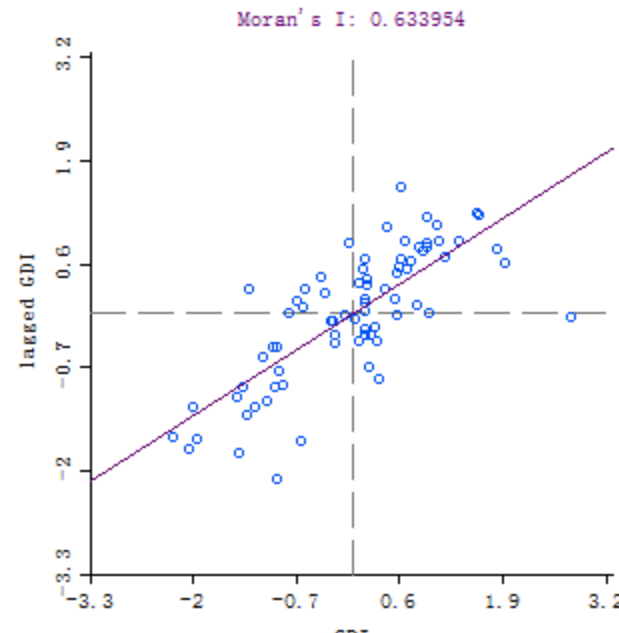
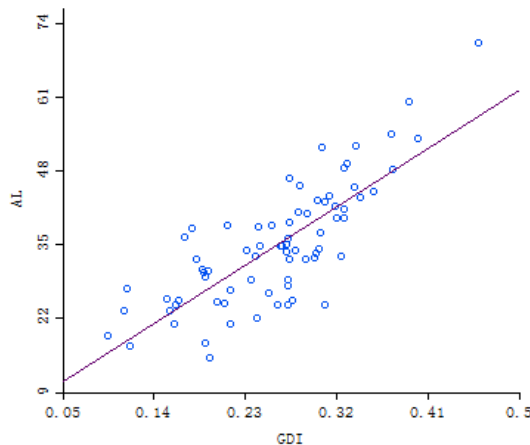
- Where:

$N$  is the number of observations (points or polygons)  
 $\bar{x}$  is the mean of the variable  
 $x_i$  is the variable value at a particular location  
 $x_j$  is the variable value at another location  
 $w_{ij}$  is a weight indexing location of  $i$  relative to  $j$



# Moran's $I$ and Correlation Coefficient

- **Correlation Coefficient [-1, 1]**
  - Relationship between two different variables
- **Moran's  $I$  [-1, 1]**
  - Spatial autocorrelation and often involves one (spatially indexed) variable only
  - Correlation between observations of a spatial variable at location  $X$  and “spatial lag” of  $X$  formed by averaging all the observation at neighbors of  $X$



# Correlation Coefficient

Note the similarity of the numerator (top) to the measures of spatial association discussed earlier if we view  $Y_i$  as being the  $X_i$  for the neighboring polygon

**(see next slide)**

$$\frac{N \sum_{i=1}^n \sum_{j=1}^n w_{ij} (x_i - \bar{x})(x_j - \bar{x})}{\left( \sum_{i=1}^n \sum_{j=1}^n w_{ij} \right) \sum_{i=1}^n (x_i - \bar{x})^2}$$

**Spatial  
auto-correlation**

$$\frac{\sum_{i=1}^n \sum_{j=1}^n w_{ij} (x_i - \bar{x})(x_j - \bar{x}) / \sum_{i=1}^n \sum_{j=1}^n w_{ij}}{\sqrt{\frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n}} \sqrt{\frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n}}}$$

# Correlation Coefficient

$$\frac{\sum_{i=1}^n 1(y_i - \bar{y})(x_i - \bar{x})/n}{\sqrt{\frac{\sum_{i=1}^n (y_i - \bar{y})^2}{n}} \sqrt{\frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n}}}$$

Spatial weights

Yi is the Xi for the neighboring polygon →

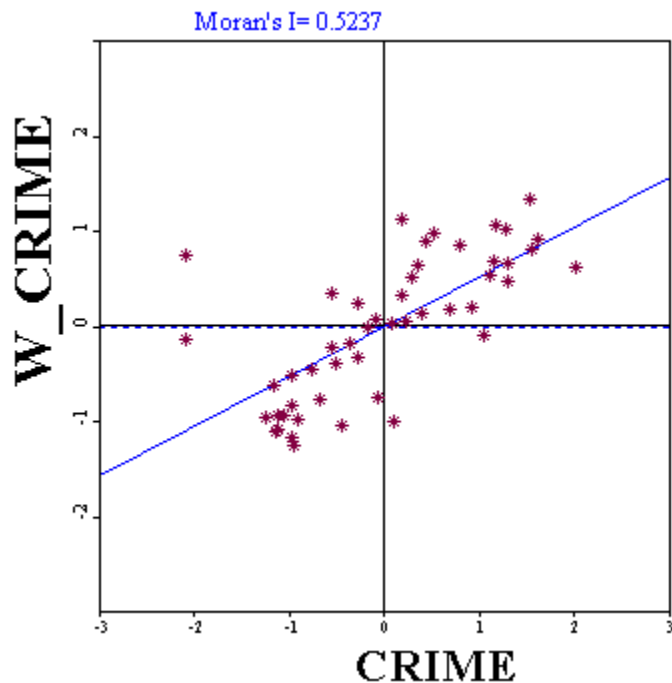
$$\frac{N \sum_{i=1}^n \sum_{j=1}^n w_{ij} (x_i - \bar{x})(x_j - \bar{x})}{(\sum_{i=1}^n \sum_{j=1}^n w_{ij}) \sum_{i=1}^n (x_i - \bar{x})^2}$$

Moran's I

$$\frac{\sum_{i=1}^n \sum_{j=1}^n w_{ij} (x_i - \bar{x})(x_j - \bar{x}) / \sum_{i=1}^n \sum_{j=1}^n w_{ij}}{\sqrt{\frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n}} \sqrt{\frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n}}}$$

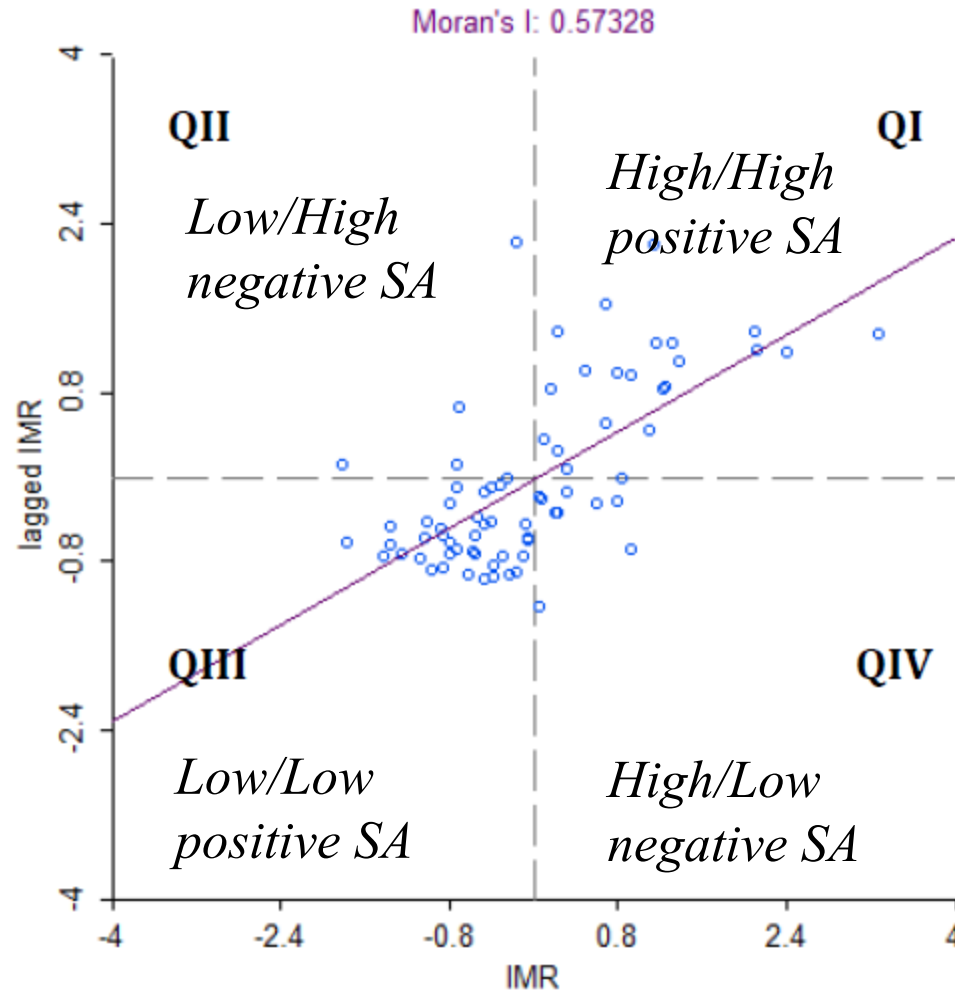
# Moran Scatter Plots

We can draw a scatter diagram between these two variables (in standardized form):  $\mathbf{X}$  and  $\mathbf{lag-X}$  (or  $\mathbf{W\_X}$ )

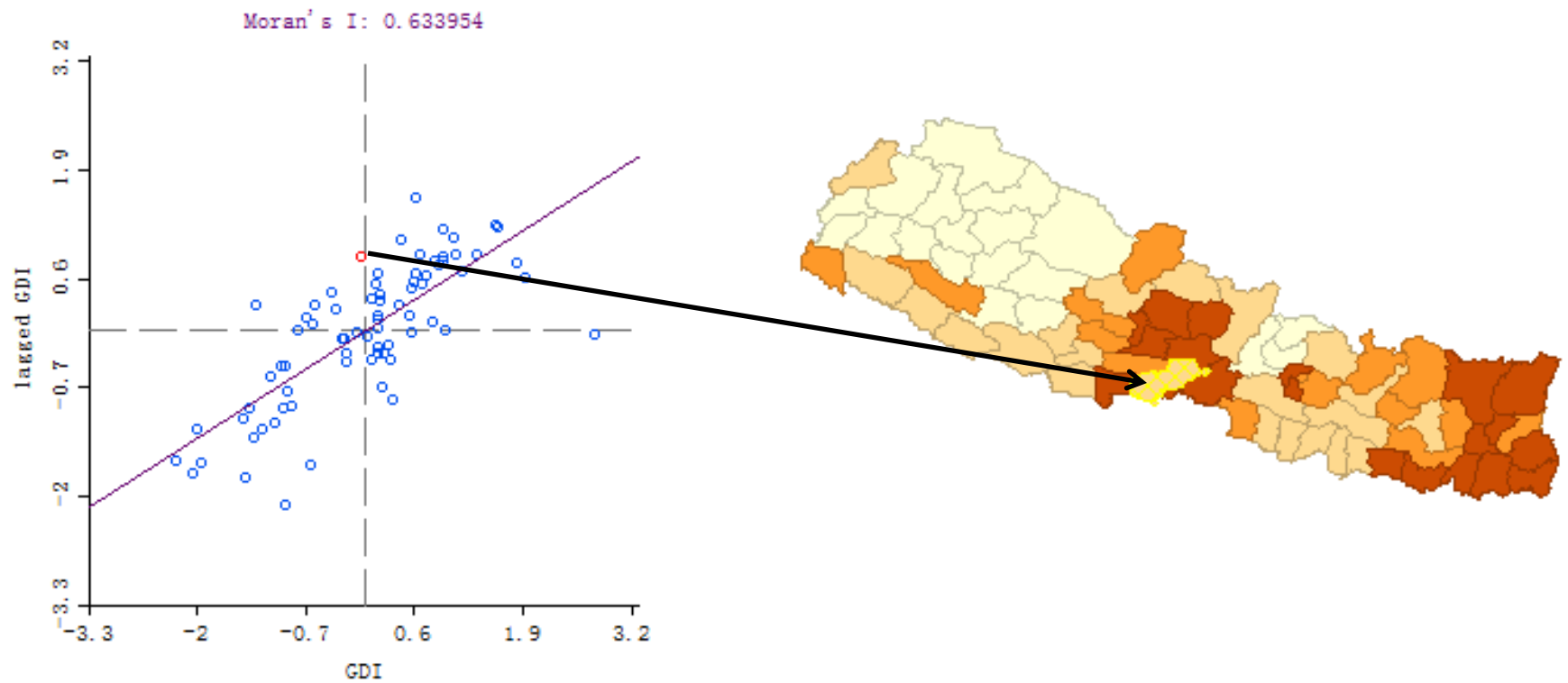


The slope of this *regression line* is  
Moran's I

# Moran Scatter Plots



# Moran Scatterplot: Example



# Statistical Significance Tests for Moran's I

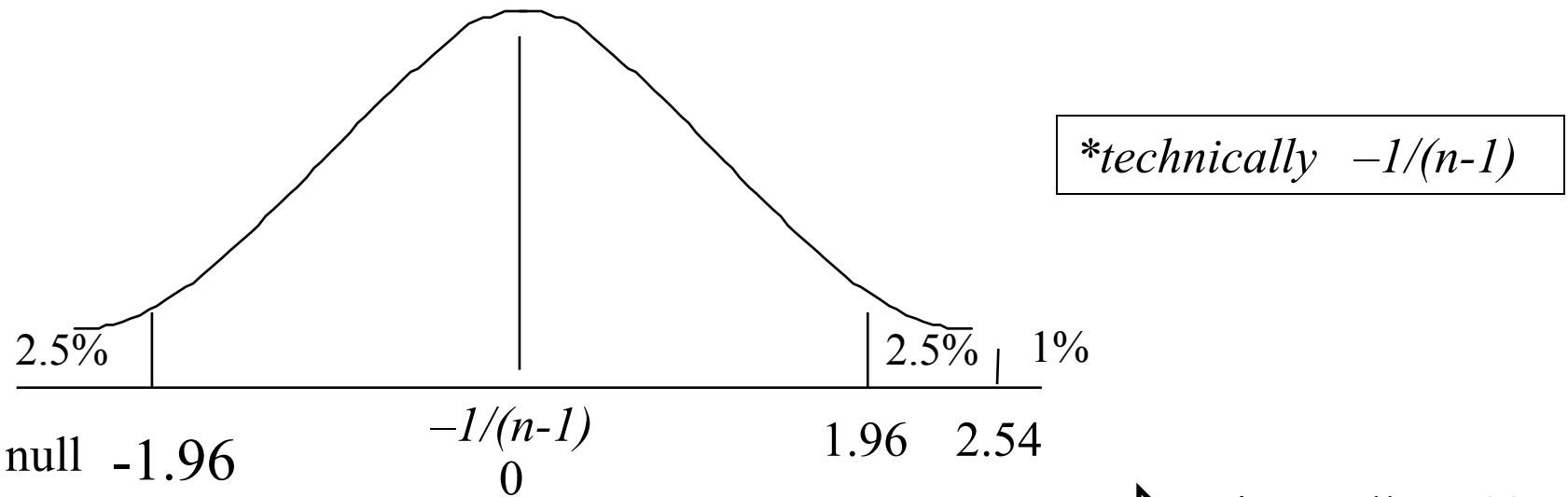
- Based on the normal frequency distribution with

$$Z = \frac{I - E(I)}{S_{error}(I)}$$

Where: I is the calculated value for Moran's I  
from the sample  
E(I) is the expected value if random  
S is the standard error

- Statistical significance test
  - Monte Carlo test, as we did for spatial pattern analysis
  - Permutation test
    - Non-parametric
    - Data-driven, no assumption of the data
    - Implemented in GeoDa

# Test Statistic for Normal Frequency Distribution



*Null Hypothesis*: no spatial autocorrelation

\*Moran's  $I = 0$

*Alternative Hypothesis*: spatial autocorrelation exists

\*Moran's  $I > 0$

Reject *Null Hypothesis* if Z test statistic  $> 1.96$  (or  $< -1.96$ )

---less than a 5% chance that, in the population, there is no spatial autocorrelation

---95% confident that spatial auto correlation exists



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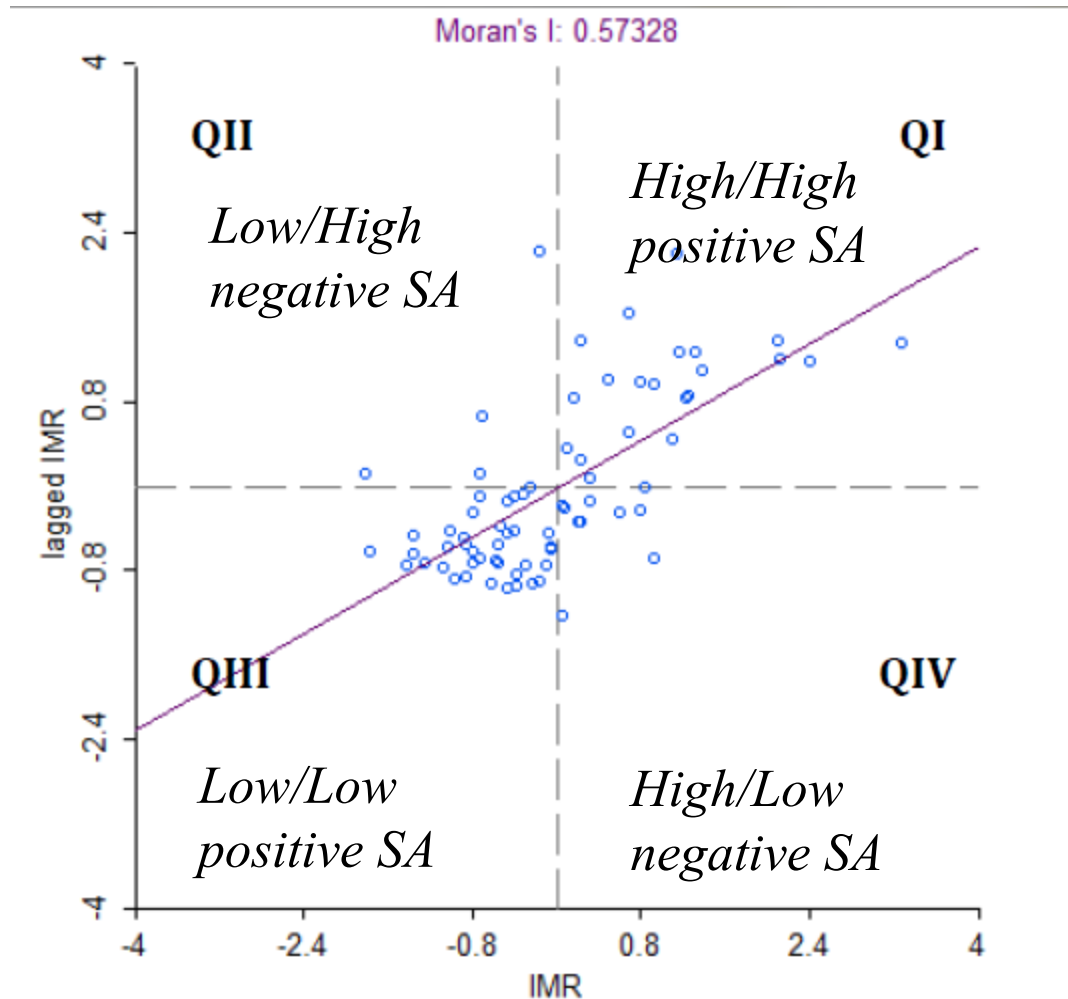
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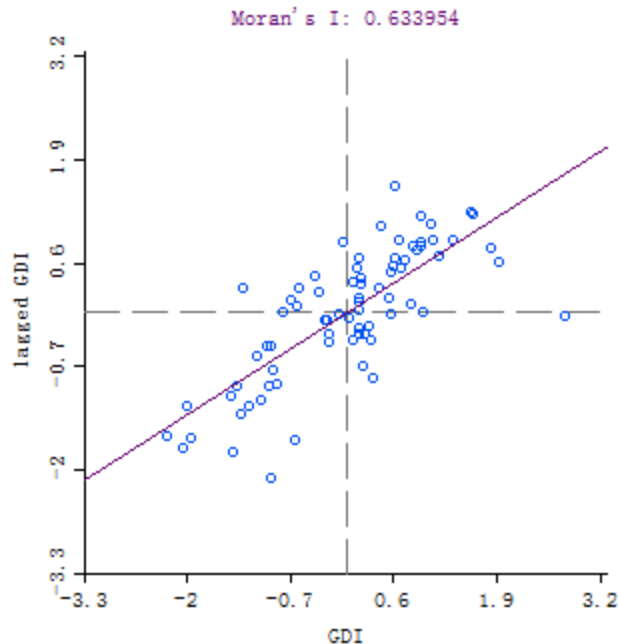
# Bivariate Moran Scatter Plot



# Spatial Autocorrelation vs Correlation

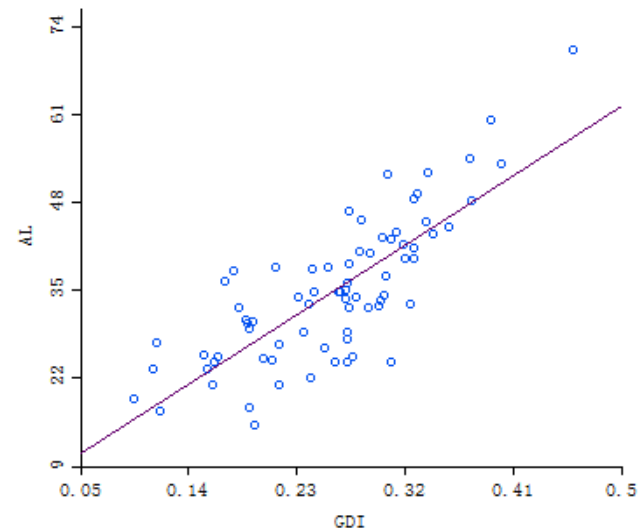
## Spatial Autocorrelation:

shows the association or relationship between the same variable in “near-by” areas.



## Standard Correlation

shows the association or relationship between two different variables



# Consequences of Ignoring Spatial Autocorrelation

- correlation coefficients and coefficients of determination appear bigger than they really are
  - You think the relationship is stronger than it really is
  - the variables in nearby areas affect each other
- Standard errors appear smaller than they really are
  - *exaggerated precision*
  - You think your predictions are better than they really are  
since standard errors measure *predictive accuracy*
  - More likely to conclude  
relationship is *statistically significant*.

# Diagnostic of Spatial Dependence

- **For correlation**
  - calculate Moran's I for each variable and test its statistical significance
  - If Moran's I is significant, you may have a problem!
- **For regression**
  - calculate the residuals
    - map the residuals: do you see any spatial patterns?
  - Calculate Moran's I for the residuals: is it statistically significant?

# Summary

- Spatial autocorrelation of areal data
- Spatial weight matrix
- Measures of spatial autocorrelation
- Global Measure
  - Moran's I
- Consequences of ignoring spatial autocorrelation
- Significance test

- End of this topic