Edi Isvernari

Temă

$v_1=(1,2,3) \quad v_2=(2,3,7) \quad v_3=(4,1,a)$

$$\alpha_1 v_1+\alpha_2 v_2+\alpha_3 v_3=\theta_{\mathbb{R}_3}$$

$$\alpha_1(1;2,3)+\alpha_2(2;3;7)+\alpha_3(4;1;a)=(0;0;0)$$

$$(\alpha_1;2\alpha_1;3\alpha_1)+(2\alpha_2;3\alpha_2;7\alpha_2)+(4\alpha_3;\alpha_3;a\alpha_3)=(0;0;0)$$

$$(\alpha_1+2\alpha_2+4\alpha_3;\,2\alpha_1+3\alpha_2+\alpha_3;\,3\alpha_1+7\alpha_2+a\alpha_3)=(0;0;0)$$

$$\begin{cases}\alpha_1+2\alpha_2+4\alpha_3=0\\ 2\alpha_1+3\alpha_2+\alpha_3=0\\ 3\alpha_1+7\alpha_2+a\alpha_3=0\end{cases}$$

v_1, v_2 și v_3 - liniari dependenți $\Rightarrow$ rang $A<3$, $A=\begin{pmatrix}1&2&4\\2&3&1\\3&7&a\end{pmatrix}$

$$\begin{array}{ccc} \uparrow & \uparrow & \uparrow \\ v_1 & v_2 & v_3 \end{array}$$

$$\text{rang}A=\text{rang}\begin{pmatrix}1&2&4\\2&3&1\\3&7&a\end{pmatrix}\xrightarrow[L_3=L_3-3L_1]{L_2=L_2-2L_1}\text{rang}\begin{pmatrix}1&2&4\\0&-1&-7\\0&1&a-12\end{pmatrix}\xrightarrow{L_3=L_3+L_2}$$

$$=\text{rang}\begin{pmatrix}1&2&4\\0&-1&-7\\0&0&a-19\end{pmatrix}\xrightarrow{L_2=-L_2}\text{rang}\begin{pmatrix}1&2&4\\0&1&7\\0&0&a-19\end{pmatrix}\xrightarrow{L_1=L_1-2L_2}\text{rang}\begin{pmatrix}1&0&-10\\0&1&7\\0&0&a-19\end{pmatrix}$$

$$\left.\begin{array}{l} \text{rang } A = \text{nr pivoților} \\ \text{rang } A < 3 \end{array}\right\} \Rightarrow a = 19 \in \mathbb{R}$$

pentru $a = 19 \Rightarrow v_1, v_2$ și v_3 sunt liniar independenți

$$\begin{pmatrix} 1 & 0 & -10 \\ 0 & 1 & 7 \\ 0 & 0 & 0 \end{pmatrix} \Rightarrow C_3 = -10C_1 + 7C_2$$

$$V_3 = -10V_1 + 7V_2$$