Group Project EDA and Cross-validation MLR Workflow for Power Plant Data

In [1]:
```r
# If some libraries are missing, start the R interactive command line and run install.packages(<missing-package-na

library(tidyverse)
library(tidymodels)
library(repr)
library(readxl)
library(ggplot2)
library(GGally)
library(Metrics)
```

```
── Attaching packages ──────────────────────────────────────────────────────────────────────────────────
─────────────────────────────────── tidyverse 1.3.1 ──

✔ ggplot2 3.3.5     ✔ purrr   0.3.4
✔ tibble  3.1.6     ✔ dplyr   1.0.8
✔ tidyr   1.1.4     ✔ stringr 1.4.0
✔ readr   2.1.1     ✔ forcats 0.5.1

── Conflicts ──────────────────────────────────────────────────────────────────────────────────────────
───────────────────────────── tidyverse_conflicts() ──
✖ dplyr::filter() masks stats::filter()
✖ dplyr::lag()    masks stats::lag()

Registered S3 method overwritten by 'tune':
  method                   from
  required_pkgs.model_spec parsnip

── Attaching packages ──────────────────────────────────────────────────────────────────────────────────
────────────────────────────────── tidymodels 0.1.4 ──

✔ broom        0.7.11     ✔ rsample      0.1.1
✔ dials        0.1.0      ✔ tune         0.1.6
✔ infer        1.0.0      ✔ workflows    0.2.4
✔ modeldata    0.1.1      ✔ workflowsets 0.1.0
✔ parsnip      0.2.0      ✔ yardstick    0.0.9
✔ recipes      0.2.0

── Conflicts ──────────────────────────────────────────────────────────────────────────────────────────
──────────────────────────── tidymodels_conflicts() ──
✖ scales::discard() masks purrr::discard()
✖ dplyr::filter()   masks stats::filter()
✖ recipes::fixed()  masks stringr::fixed()
✖ dplyr::lag()      masks stats::lag()
✖ yardstick::spec() masks readr::spec()
✖ recipes::step()   masks stats::step()
✖ tune::tune()      masks parsnip::tune()
• Search for functions across packages at https://www.tidymodels.org/find/

Registered S3 method overwritten by 'GGally':
  method from
  +.gg   ggplot2


Attaching package: ‘Metrics’


The following objects are masked from ‘package:yardstick’:

    accuracy, mae, mape, mase, precision, recall, rmse, smape
```

In [2]:
```r
# Mallows Cp Statistic (from lecture 15)
mallowsCp <- function(model) {
    rss <- sum(model$residuals^2)
    rms_full <-
    p <- length(model$coefficients)
    n <- length(model$fitted.values)

    return (rss / rms_full) - (n - 2*p)
}
```

In [3]:
```r
# set the random seed for this notebook
set.seed(4533)
```

```
# load in the data and update column names
power <- read_excel("data/powerplant-fulload-data.xlsx")
colnames(power) <- c("AvgAmbientTemperatureC",
                     "ExhaustVacuum_cm_Hg",
                     "AvgAmbientPressureMilibars",
                     "RelativeHumidity",
                     "NetEnergyOutputMegaWatts")

# save to csv for easier loading to RStudio
# write_csv(power, "data/powerplant-fulload-data-rstudio.csv")

power %>% head(8)
```

A tibble: 8 × 5

AvgAmbientTemperatureC	ExhaustVacuum_cm_Hg	AvgAmbientPressureMilibars	RelativeHumidity	NetEnergyOutputMegaWatts
<dbl>	<dbl>	<dbl>	<dbl>	<dbl>
14.96	41.76	1024.07	73.17	463.26
25.18	62.96	1020.04	59.08	444.37
5.11	39.40	1012.16	92.14	488.56
20.86	57.32	1010.24	76.64	446.48
10.82	37.50	1009.23	96.62	473.90
26.27	59.44	1012.23	58.77	443.67
15.89	43.96	1014.02	75.24	467.35
9.48	44.71	1019.12	66.43	478.42

In [4]:

```
min_df <- power %>%
            map_df(min)
max_df <- power %>%
            map_df(max)
mean_df <- power %>%
            map_df(mean)
sd_df <- power %>%
            map_df(sd)
var_df <- power %>%
            map_df(var)
count_nan_df <- power %>%
                map_df(~sum(is.na(.)))

# get a vector of all the columns to join by
join_cols = colnames(power)

# join the stats tables together
stats_table <- full_join(min_df, max_df, by=join_cols) %>%
                full_join(., mean_df, by=join_cols) %>%
                    full_join(., var_df, by=join_cols) %>%
                        full_join(., sd_df, by=join_cols) %>%
                            full_join(., count_nan_df, by=join_cols) %>%
                mutate(stat_type = c("min", "max", "mean", "var", "st_dev", "nan_count")) %>%
                mutate(stat_type = as_factor(stat_type)) %>%
                relocate(stat_type)

print("Unscaled Training Data Statistics Summary")
stats_table
```

```
[1] "Unscaled Training Data Statistics Summary"
```

A tibble: 6 × 6

stat_type	AvgAmbientTemperatureC	ExhaustVacuum_cm_Hg	AvgAmbientPressureMilibars	RelativeHumidity	NetEnergyOutputMegaWatts
<fct>	<dbl>	<dbl>	<dbl>	<dbl>	<dbl>
min	1.810000	25.36000	992.890000	25.56000	420.26000
max	37.110000	81.56000	1033.300000	100.16000	495.76000
mean	19.651231	54.30580	1013.259078	73.30898	454.36501
var	55.539357	161.49054	35.269152	213.16785	291.28232
st_dev	7.452473	12.70789	5.938784	14.60027	17.06699
nan_count	0.000000	0.00000	0.000000	0.00000	0.00000

The below pair plot provides an quick visual data overview. I think we are in good shape for a simple MLR model on this dataset.

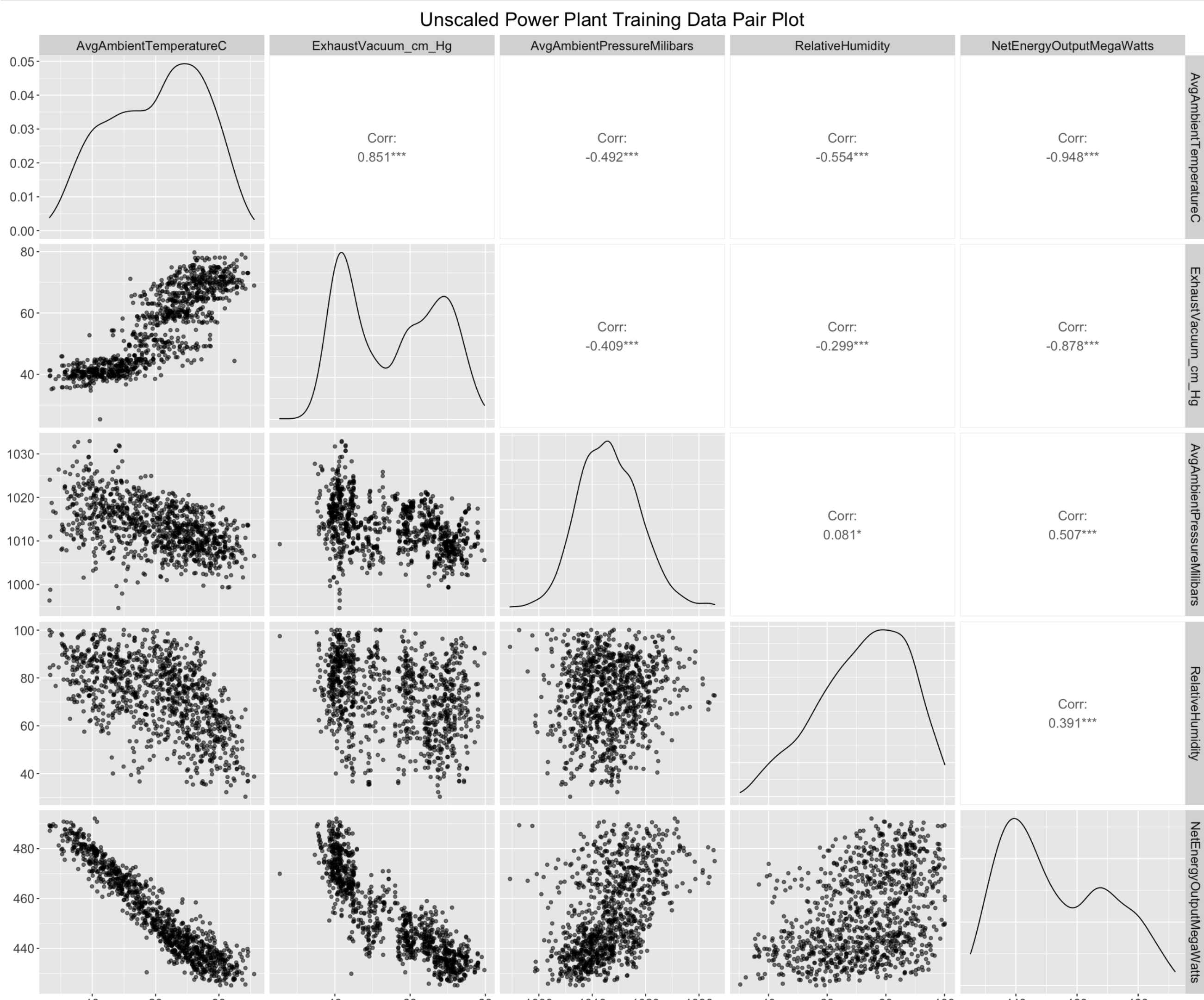
Multiple Linear Regressoin using Test/Train Split as Described in Lecture 15

We will split into test and train data, and run cross validation minimizing RMSE on the training/validation splits. Finally we will test the linear model on the hold-out testing data.

There will be no data scaling in this workflow as we have not discussed it in class.

In [5]:
```
# 70% training data, 30% test data to start
power_split <- initial_split(power, prop = 0.7)
power_train <- training(power_split)
power_test <- testing(power_split)
```

In [6]:
```
# set plot size options
options(repr.plot.width = 18, repr.plot.height = 15)

# subset the data for plotting (just looks like black blobs otherwise)
power_train_subset <- power_train %>% sample_frac(0.15)

# pair plot for concrete data
ggpairs(power_train_subset,
            mapping = aes(alpha = 0.2),
            lower=list(combo=wrap("facethist", binwidth=0.5)),
            upper=list(continuous = wrap("cor", size=5))) +
        ggtitle("Unscaled Power Plant Training Data Pair Plot") +
        theme(text = element_text(size = 17),
              plot.title = element_text(hjust = 0.5))
```



Calculate Variance Inflation Factors for All Predictors

In the pair plot above, some of the predictors appear highly correlated (e.g. AvgAmbientTemperatureC vs ExhaustVacuum_cm_Hg). Below we calculate the VIF for all predictors to determine if any variables should be removed in order to stabalize parameter estimation. We will keep any variables with VIF < 5 (arbitrary cuttoff from class).

In [7]:
```
predictor_names <- colnames(power_train %>% select(-NetEnergyOutputMegaWatts))
vif_all_predictors <- list()
runs <- length(predictor_names)
precision = 4

for (i in 1:runs) {
    curr_pred = predictor_names[1]
    predictor_names = predictor_names[-1] # drop the current pred so we don't calc VIF against itself (divide by z

    for (other_pred in predictor_names){
        rsq <- cor(power_train[curr_pred],
                   power_train[other_pred],
                   method='pearson')^2

        vif_all_predictors[paste(curr_pred, other_pred , sep = " vs ")] = round(1 / (1 - rsq), precision)
    }
}
```

Although AmbientTemperatureC vs ExhaustVacuum_cm_Hg are highly correlated, the VIF suggests that it is likely not high enough to be problematic, so we will keep all predictors in model contention at this stage.

In [8]:
```
vif_df <- data.frame(Variables=names(vif_all_predictors), VIF=as.double(vif_all_predictors))
vif_df
```

A data.frame: 6 × 2

Variables	VIF
<chr>	<dbl>
AvgAmbientTemperatureC vs ExhaustVacuum_cm_Hg	3.4810
AvgAmbientTemperatureC vs AvgAmbientPressureMilibars	1.3486
AvgAmbientTemperatureC vs RelativeHumidity	1.4374
ExhaustVacuum_cm_Hg vs AvgAmbientPressureMilibars	1.2056
ExhaustVacuum_cm_Hg vs RelativeHumidity	1.1132
AvgAmbientPressureMilibars vs RelativeHumidity	1.0113

Try forward selection of parameters? (probably won't do this)

Cross-validation

Generate all possible model strings (no including interactions)

TODO: Interactions

In [9]:
```
power_train_5fold_cv <- vfold_cv(power_train, v = 5)
```

In [10]:
```
vars <- colnames(power_test)
N <- list(1,2,3,4)

# use combn to generate all variable combinations
all_combos <- sapply(N, function(v) combn(x=vars[1:4], v))
model_strings <- c()

for(combo_set in all_combos){
    for(j in seq(ncol(combo_set))){
        model_strings <- append(model_strings, paste("NetEnergyOutputMegaWatts", "~", paste(combo_set[,j], collaps

        # don't double add the single term main effect models
        if (length(combo_set[,j]) > 1) {
            model_strings <- append(model_strings, paste("NetEnergyOutputMegaWatts", "~", paste(combo_set[,j], col
        }
    }
}

# add the null model
model_strings <- append(model_strings, "NetEnergyOutputMegaWatts ~ 1")
```

The list below is all the models that will be tested in cross-validation workflow

Models that only show the interaction terms also include main effects, but the model string is written in shorthand.

In [11]:
```
print(model_strings)
```

```
 [1] "NetEnergyOutputMegaWatts ~ AvgAmbientTemperatureC"
 [2] "NetEnergyOutputMegaWatts ~ ExhaustVacuum_cm_Hg"
 [3] "NetEnergyOutputMegaWatts ~ AvgAmbientPressureMilibars"
 [4] "NetEnergyOutputMegaWatts ~ RelativeHumidity"
 [5] "NetEnergyOutputMegaWatts ~ AvgAmbientTemperatureC+ExhaustVacuum_cm_Hg"
 [6] "NetEnergyOutputMegaWatts ~ AvgAmbientTemperatureC*ExhaustVacuum_cm_Hg"
 [7] "NetEnergyOutputMegaWatts ~ AvgAmbientTemperatureC+AvgAmbientPressureMilibars"
 [8] "NetEnergyOutputMegaWatts ~ AvgAmbientTemperatureC*AvgAmbientPressureMilibars"
 [9] "NetEnergyOutputMegaWatts ~ AvgAmbientTemperatureC+RelativeHumidity"
[10] "NetEnergyOutputMegaWatts ~ AvgAmbientTemperatureC*RelativeHumidity"
[11] "NetEnergyOutputMegaWatts ~ ExhaustVacuum_cm_Hg+AvgAmbientPressureMilibars"
[12] "NetEnergyOutputMegaWatts ~ ExhaustVacuum_cm_Hg*AvgAmbientPressureMilibars"
[13] "NetEnergyOutputMegaWatts ~ ExhaustVacuum_cm_Hg+RelativeHumidity"
[14] "NetEnergyOutputMegaWatts ~ ExhaustVacuum_cm_Hg*RelativeHumidity"
[15] "NetEnergyOutputMegaWatts ~ AvgAmbientPressureMilibars+RelativeHumidity"
[16] "NetEnergyOutputMegaWatts ~ AvgAmbientPressureMilibars*RelativeHumidity"
[17] "NetEnergyOutputMegaWatts ~ AvgAmbientTemperatureC+ExhaustVacuum_cm_Hg+AvgAmbientPressureMilibars"
[18] "NetEnergyOutputMegaWatts ~ AvgAmbientTemperatureC*ExhaustVacuum_cm_Hg*AvgAmbientPressureMilibars"
[19] "NetEnergyOutputMegaWatts ~ AvgAmbientTemperatureC+ExhaustVacuum_cm_Hg+RelativeHumidity"
[20] "NetEnergyOutputMegaWatts ~ AvgAmbientTemperatureC*ExhaustVacuum_cm_Hg*RelativeHumidity"
[21] "NetEnergyOutputMegaWatts ~ AvgAmbientTemperatureC+AvgAmbientPressureMilibars+RelativeHumidity"
[22] "NetEnergyOutputMegaWatts ~ AvgAmbientTemperatureC*AvgAmbientPressureMilibars*RelativeHumidity"
[23] "NetEnergyOutputMegaWatts ~ ExhaustVacuum_cm_Hg+AvgAmbientPressureMilibars+RelativeHumidity"
[24] "NetEnergyOutputMegaWatts ~ ExhaustVacuum_cm_Hg*AvgAmbientPressureMilibars*RelativeHumidity"
[25] "NetEnergyOutputMegaWatts ~ AvgAmbientTemperatureC+ExhaustVacuum_cm_Hg+AvgAmbientPressureMilibars+RelativeHum
idity"
[26] "NetEnergyOutputMegaWatts ~ AvgAmbientTemperatureC*ExhaustVacuum_cm_Hg*AvgAmbientPressureMilibars*RelativeHum
idity"
[27] "NetEnergyOutputMegaWatts ~ 1"
```

Next we run 5 fold cross-validation for each of the above models, recording the mean RMSE on the validation sets.

In [12]:
```
cv_results <- tibble(model_string = character(), val_rmse = numeric(), adj_rsq = numeric())

# extract test and train data into separate columns in the fold_cv tibble
power_train_5fold_cv <- power_train_5fold_cv %>%
  mutate(
    train = map(splits, ~training(.x)),
    validate = map(splits, ~testing(.x))
  )


for (model_string in model_strings) {

    # generate the models for all cross-val splits
    evaluated_models <- power_train_5fold_cv %>%
        mutate(model = map(train, ~lm(formula = as.formula(model_string), data = .x))) %>%

    # map the real response validation set response and predicted response to validate_actual & validate_predicted
        mutate(validate_actual = map(validate, ~.x$NetEnergyOutputMegaWatts),
               validate_predicted = map2(model, validate, ~predict(.x, .y))) %>%

    # calculate the RMSE between the real and predicted on the validation set (simplfied with Metrics library)
        mutate(validate_rmse = map2_dbl(validate_actual, validate_predicted,
               ~rmse(actual = .x, predicted = .y)))


    cv_results <- cv_results %>% add_row(model_string = model_string,
                                         val_rmse = mean(evaluated_models$validate_rmse),
                                         adj_rsq = evaluated_models$model %>% map(., ~(summary(.x))$adj.r.squared)


}


print("All models with interaction terms also have the main effects added in, but the model_string is shorthanded
cv_results %>% arrange(val_rmse)
```

```
[1] "All models with interaction terms also have the main effects added in, but the model_string is shorthanded to
exclude them"
```

A tibble: 27 × 3

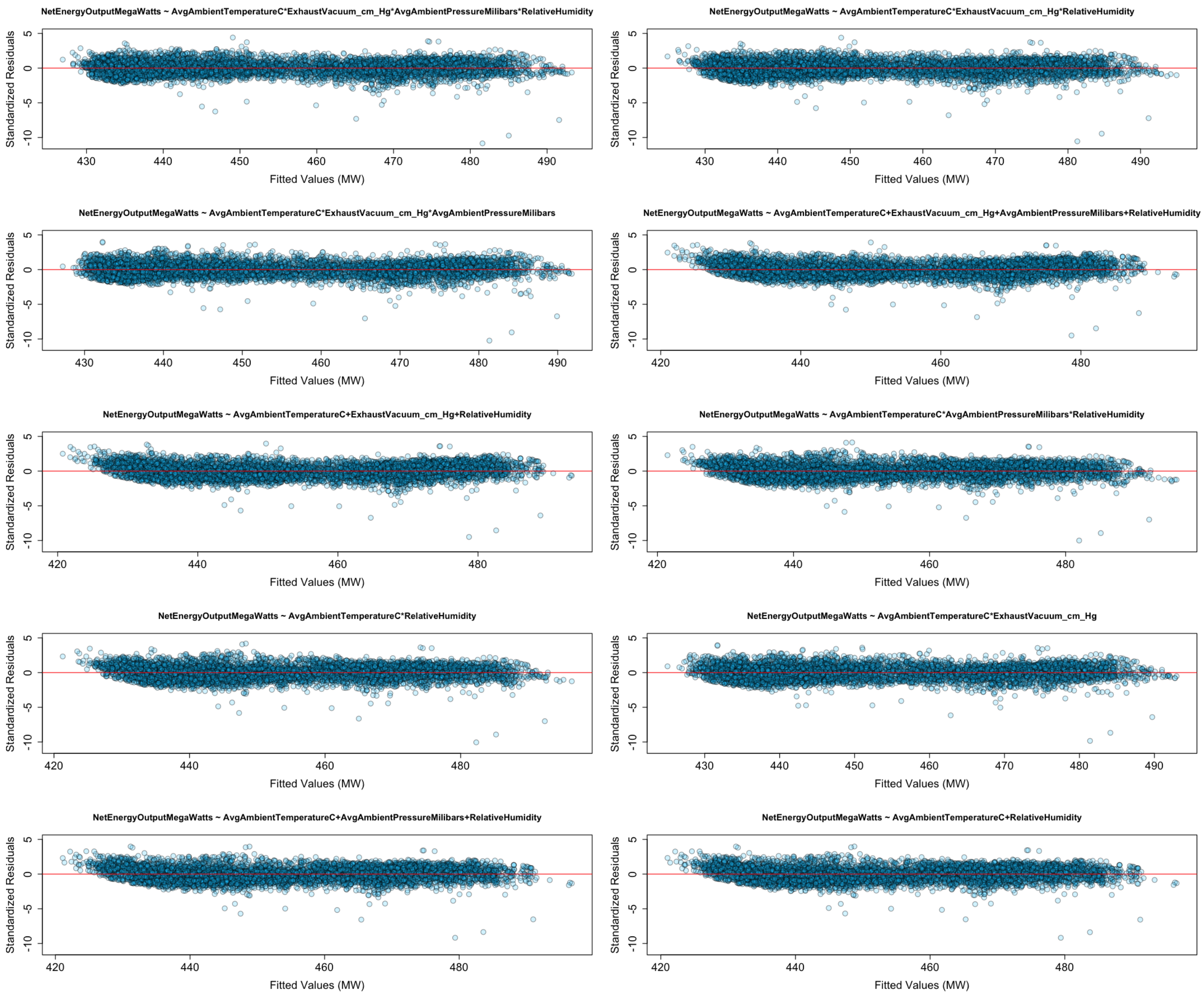
model_string	val_rmse	adj_rsq
<chr>	<dbl>	<dbl>
NetEnergyOutputMegaWatts ~ AvgAmbientTemperatureC*ExhaustVacuum_cm_Hg*AvgAmbientPressureMilibars*RelativeHumidity	4.252300	0.9376846
NetEnergyOutputMegaWatts ~ AvgAmbientTemperatureC*ExhaustVacuum_cm_Hg*RelativeHumidity	4.337507	0.9350726
NetEnergyOutputMegaWatts ~ AvgAmbientTemperatureC*ExhaustVacuum_cm_Hg*AvgAmbientPressureMilibars	4.477524	0.9309139
NetEnergyOutputMegaWatts ~ AvgAmbientTemperatureC+ExhaustVacuum_cm_Hg+AvgAmbientPressureMilibars+RelativeHumidity	4.539478	0.9288965
NetEnergyOutputMegaWatts ~ AvgAmbientTemperatureC+ExhaustVacuum_cm_Hg+RelativeHumidity	4.548293	0.9286080
NetEnergyOutputMegaWatts ~ AvgAmbientTemperatureC*AvgAmbientPressureMilibars*RelativeHumidity	4.642084	0.9256203
NetEnergyOutputMegaWatts ~ AvgAmbientTemperatureC*RelativeHumidity	4.652311	0.9252913
NetEnergyOutputMegaWatts ~ AvgAmbientTemperatureC*ExhaustVacuum_cm_Hg	4.660164	0.9250306
NetEnergyOutputMegaWatts ~ AvgAmbientTemperatureC+AvgAmbientPressureMilibars+RelativeHumidity	4.778304	0.9212202
NetEnergyOutputMegaWatts ~ AvgAmbientTemperatureC+RelativeHumidity	4.779221	0.9211936
NetEnergyOutputMegaWatts ~ AvgAmbientTemperatureC+ExhaustVacuum_cm_Hg+AvgAmbientPressureMilibars	4.874054	0.9180320
NetEnergyOutputMegaWatts ~ AvgAmbientTemperatureC+ExhaustVacuum_cm_Hg	4.937977	0.9158152
NetEnergyOutputMegaWatts ~ AvgAmbientTemperatureC+AvgAmbientPressureMilibars	5.367306	0.9005822
NetEnergyOutputMegaWatts ~ AvgAmbientTemperatureC*AvgAmbientPressureMilibars	5.367543	0.9005673
NetEnergyOutputMegaWatts ~ AvgAmbientTemperatureC	5.414960	0.8987876
NetEnergyOutputMegaWatts ~ ExhaustVacuum_cm_Hg*AvgAmbientPressureMilibars*RelativeHumidity	7.353529	0.8134968
NetEnergyOutputMegaWatts ~ ExhaustVacuum_cm_Hg+AvgAmbientPressureMilibars+RelativeHumidity	7.507611	0.8055207
NetEnergyOutputMegaWatts ~ ExhaustVacuum_cm_Hg*AvgAmbientPressureMilibars	7.807791	0.7895986
NetEnergyOutputMegaWatts ~ ExhaustVacuum_cm_Hg+AvgAmbientPressureMilibars	7.837476	0.7880090
NetEnergyOutputMegaWatts ~ ExhaustVacuum_cm_Hg*RelativeHumidity	8.027155	0.7775308
NetEnergyOutputMegaWatts ~ ExhaustVacuum_cm_Hg+RelativeHumidity	8.098942	0.7735822
NetEnergyOutputMegaWatts ~ ExhaustVacuum_cm_Hg	8.382168	0.7574510
NetEnergyOutputMegaWatts ~ AvgAmbientPressureMilibars*RelativeHumidity	13.313389	0.3882294
NetEnergyOutputMegaWatts ~ AvgAmbientPressureMilibars+RelativeHumidity	13.316066	0.3879397
NetEnergyOutputMegaWatts ~ AvgAmbientPressureMilibars	14.548903	0.2689029
NetEnergyOutputMegaWatts ~ RelativeHumidity	15.612984	0.1583468
NetEnergyOutputMegaWatts ~ 1	17.017159	0.0000000

Take the top 10 models and look at their residual plots.

In [13]:
```
top_10_model_strings <- cv_results %>%
        arrange(val_rmse)%>%
        head(10) %>%
        select(model_string)

models = list()

idx = 1
for (mod in top_10_model_strings$model_string) {
    models[[idx]] = lm(as.formula(mod), data=power_train)
    idx = idx + 1
}
```

In [14]:
```
par(mfrow=c(5,2))

name_idx = 1
for (model in models) {
    plot(model$fitted.values, rstandard(model),
         pch=21,
         bg=rgb(red = 0, green = 0.8, blue = 1, alpha = 0.2),
         col=rgb(red = 0, green = 0, blue = 0, alpha = 0.4),
         cex=1.5,
         cex.sub = 1.5,  # Subtitle size
         cex.lab = 1.5,    # X-axis and Y-axis labels size
         cex.axis = 1.5,
         xlab="Fitted Values (MW)",
         ylab="Standardized Residuals",
```

```
        ylim=c(-11,5),
        main=top_10_model_strings$model_string[name_idx])
    abline(h=0, col="red")
    name_idx = name_idx + 1

}
```



The standardized residual plots above show that while each of these models have similar adjusted R^2 and RMSE, most models have some systematic bias to underestimate low power output (as evidenced by the upward tail on the LHS of these plots for many models). The only models which remove this are the models which contain interaction terms between exhuast vacuum pressure and average ambient temperature (models at the top of the plot).

In [15]:

```
# Chosen manually based on the top 10 models and their residual plots
best_model <- lm(NetEnergyOutputMegaWatts ~ . + AvgAmbientTemperatureC*ExhaustVacuum_cm_Hg*AvgAmbientPressureMilib

test_prediction <- predict(best_model, power_test)
test_pred_interval <- predict(best_model, power_test, interval = 'prediction')

results <- data.frame(pred=test_prediction, actual=power_test$NetEnergyOutputMegaWatts)
test_rmse <- round(rmse(results$actual, results$pred), precision)

print(paste("The RMSE for the test data is:",
            test_rmse,
            ", compared to",
            (cv_results %>% arrange(val_rmse))$val_rmse[1] %>% round(precision),
            "in cross-validation."))
```

```
[1] "The RMSE for the test data is: 4.3143 , compared to 4.2523 in cross-validation."
```

In [16]:

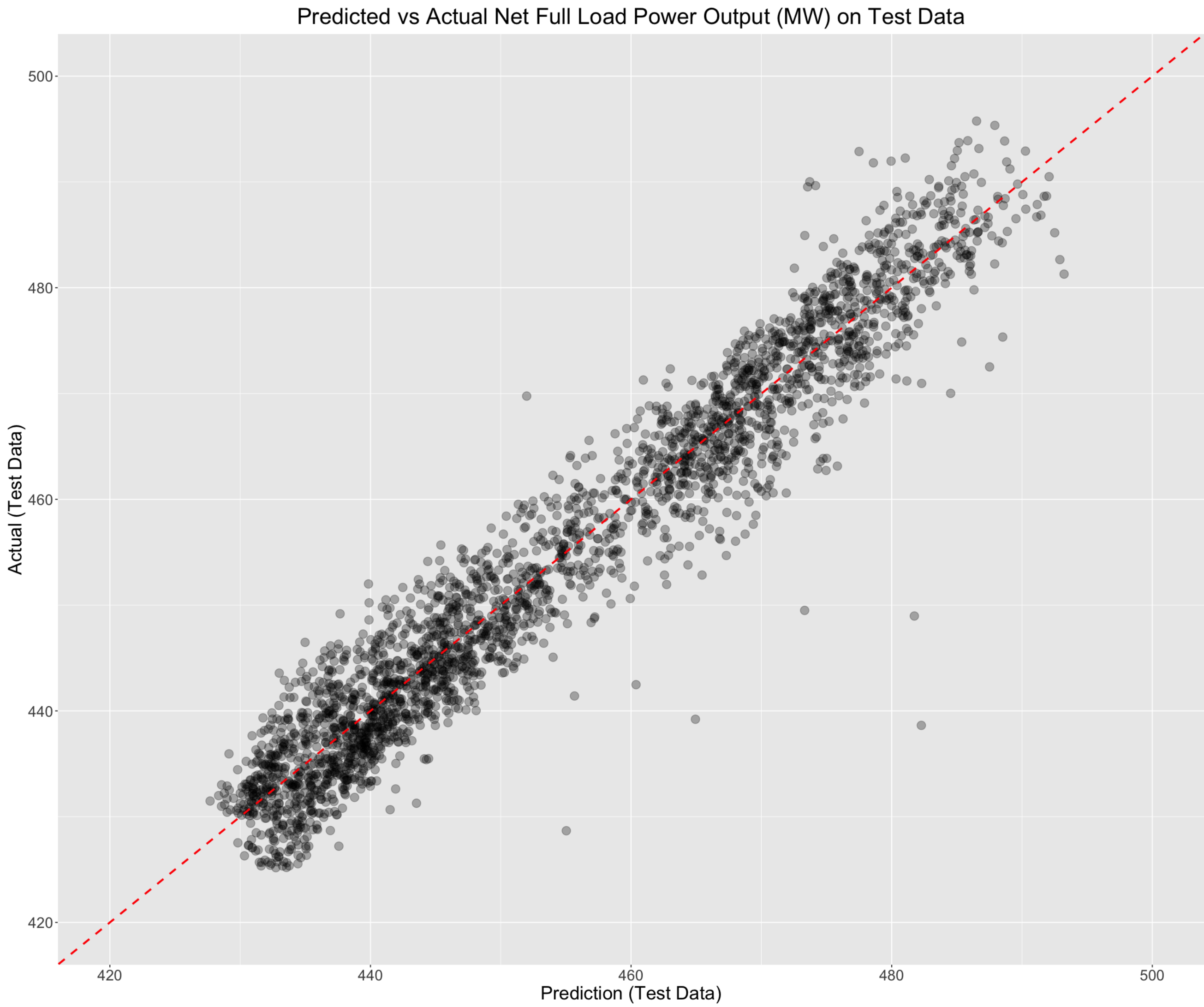
```
summary(best_model)$coefficients
```

A matrix: 16 × 4 of type dbl

	Estimate	Std. Error	t value
(Intercept)	4.318106e+03	9.648068e+02	4.475617
AvgAmbientTemperatureC	-2.055893e+02	3.938971e+01	-5.219366
ExhaustVacuum_cm_Hg	-8.450710e+01	2.178940e+01	-3.878358
AvgAmbientPressureMilibars	-3.744898e+00	9.524628e-01	-3.931805
RelativeHumidity	-2.533337e+01	1.239626e+01	-2.043631
AvgAmbientTemperatureC:ExhaustVacuum_cm_Hg	3.831101e+00	7.806775e-01	4.907406
AvgAmbientTemperatureC:AvgAmbientPressureMilibars	2.009862e-01	3.891050e-02	5.165347
ExhaustVacuum_cm_Hg:AvgAmbientPressureMilibars	8.275193e-02	2.152220e-02	3.844957
AvgAmbientTemperatureC:RelativeHumidity	1.782479e+00	5.215546e-01	3.417626
ExhaustVacuum_cm_Hg:RelativeHumidity	5.303901e-01	2.827966e-01	1.875518
AvgAmbientPressureMilibars:RelativeHumidity	2.502893e-02	1.224026e-02	2.044803
AvgAmbientTemperatureC:ExhaustVacuum_cm_Hg:AvgAmbientPressureMilibars	-3.769729e-03	7.712845e-04	-4.887598
AvgAmbientTemperatureC:ExhaustVacuum_cm_Hg:RelativeHumidity	-3.070894e-02	1.043899e-02	-2.941753
AvgAmbientTemperatureC:AvgAmbientPressureMilibars:RelativeHumidity	-1.771729e-03	5.153060e-04	-3.438207
ExhaustVacuum_cm_Hg:AvgAmbientPressureMilibars:RelativeHumidity	-5.237237e-04	2.793713e-04	-1.874651
AvgAmbientTemperatureC:ExhaustVacuum_cm_Hg:AvgAmbientPressureMilibars:RelativeHumidity	3.041164e-05	1.031523e-05	2.948227

In [17]:

```
prediction_plot <- results %>% ggplot(aes(x=pred, y=actual)) +
                        geom_point(size=4, alpha=0.3) +
                        labs(x = "Prediction (Test Data)", y = "Actual (Test Data)") +
                        ggtitle("Predicted vs Actual Net Full Load Power Output (MW) on Test Data") +
                                theme(text = element_text(size=20),
                                      plot.title = element_text(hjust = 0.5)) +
                        geom_abline(slope=1, intercept=0, col='red', linetype='dashed', size=1) +
                        xlim(420, 500) + ylim(420, 500)
prediction_plot
```

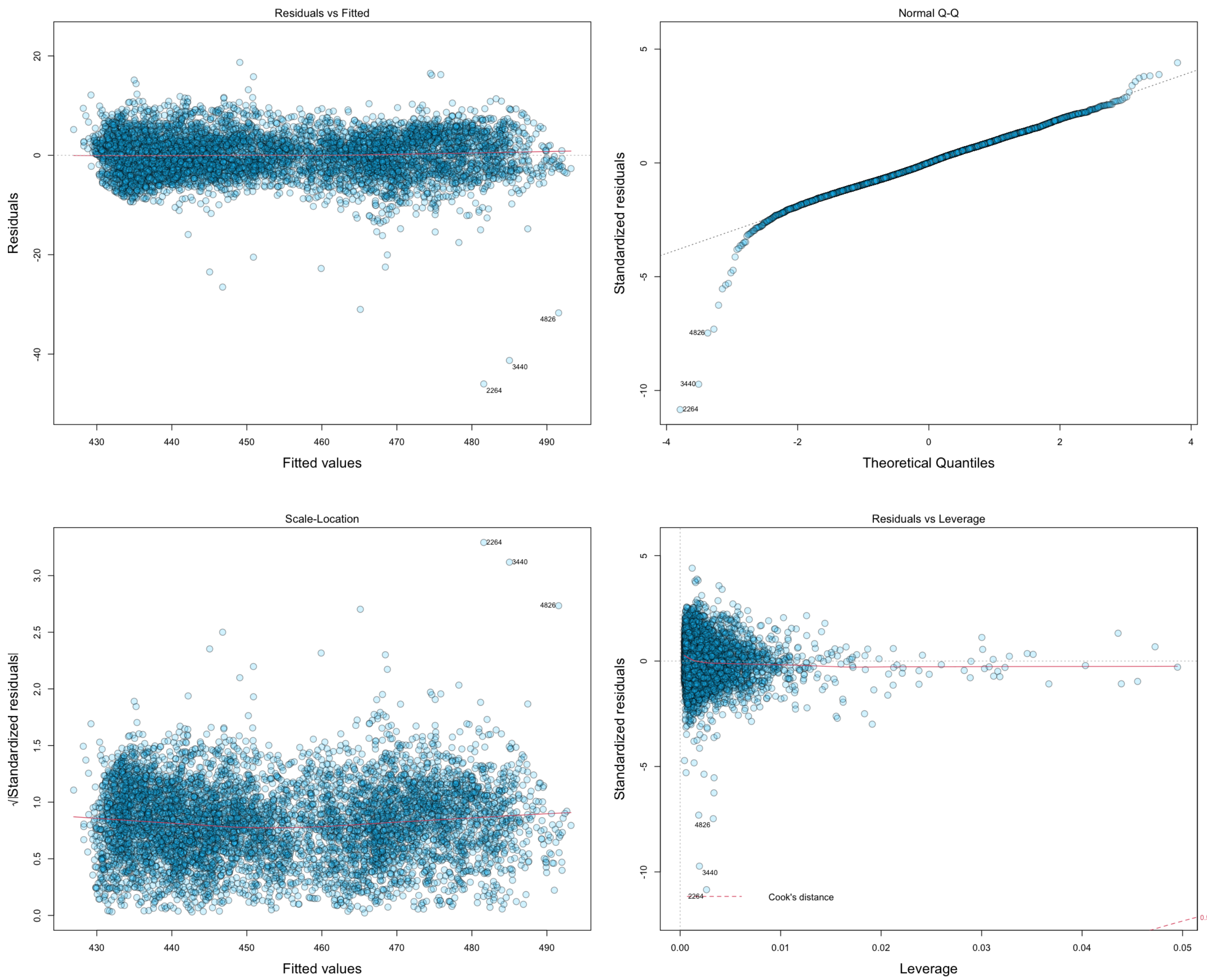


Thoughts on prediction goal

We want to predict power output, so ideally we want to choose a model which contains significant terms, and which also does not needlessly include additional terms at little benefit to model performance. I can try a C_p matrix for each model similar to Activity 15 as another way to select a model. Ideally we have as few terms as possible so that when predicting power output we can collect as little data as possible and still have a good result.

From experimentataion it seems like the best trade off model for few predictors is a simple interaction model between ambient temperature and exhuast vacuum pressure. This is probably because a majority of the power generated is going towards heating, additionally the exhuast vacuum pressure is directly related to the power generation of the gas turbine.

In [18]:
```
par(mfrow=c(2,2))
plot(best_model,
     pch=21,
     bg=rgb(red = 0, green = 0.8, blue = 1, alpha = 0.2),
     col=rgb(red = 0, green = 0, blue = 0, alpha = 0.4),
     cex=1.5,
     cex.sub = 1.5,  # Subtitle size
     cex.lab = 1.5,    # X-axis and Y-axis labels size
     cex.axis = 1) # Axis labels size)
```


Residuals vs Fitted
Residuals
Fitted values
4826
3440
2264
Normal Q-Q
Standardized residuals
Theoretical Quantiles
4826
3440
2264
Scale-Location
√|Standardized residuals|
Fitted values
2264
3440
4826
Residuals vs Leverage
Standardized residuals
Leverage
4826
3440
2264
Cook's distance
0.5