Tarea 5

Deber

Ejercicios 9.1

5. $\int x \cos(5x)\,dx$

$u = 5x \qquad du = 5\,dx \qquad dx = \frac{du}{5}$

$x = \frac{u}{5}$

$\int \frac{u}{5} \cos(u) \frac{du}{5}$

$\frac{1}{25} \int u \cos(u)\,du \qquad u = u \quad du = 1 \qquad dv = \cos(x) \quad v = \operatorname{sen}(u)$

$\frac{1}{25} \left(u \operatorname{sen}(u) - \int \operatorname{sen}(u)\,du \right.$

$\frac{1}{25} \left(u \operatorname{sen}(u) - (-\cos(u)) \right) = \boxed{\frac{1}{25} \left(5x \operatorname{sen}(5x) + \cos(5x)\right) + C}$

10. $\int x^3 \cdot e^{-x}\,dx$

$u = -x \qquad du = -x$

$\int -u^3 \cdot e^u \cdot du \therefore \int u^3 \cdot e^u\,du$

$u = u^3 \quad du = 3u^2 \qquad dv = e^u \quad v = e^u$

$u^3 \cdot e^u - \int e^u \cdot 3u^2\,du \therefore u^3 \cdot e^u - 3 \int e^u \cdot u^2\,du$

$u = u^2 \quad du = 2u \qquad dv = e^u \quad v = e^u$

$u^2 e^u - \int u^2 \cdot e^u\,du \therefore \quad u = u^2 \quad du = 2u \qquad dv = e^u \quad v = e^u$

$u^2 e^u - \int 2u \cdot e^u\,du = u^2 e^u - 2 \int u \cdot e^u\,du$

$u = u \quad du = 1 \qquad dv = e^u \quad v = e^u \qquad / \int e^u\,du = e^u$

$u \cdot e^u - \int e^u\,du \therefore u \cdot e^u - e^u$

$$u^3 \cdot e^u - 3(u^2 e^u - 2(e^u \cdot u - e^u))$$

$$\boxed{-x^3 \cdot e^{-x} - 3(-x^2 e^{-x} - 2(e^{-x} \cdot -x - e^{-x})) + C}$$

(15) $\int x \csc^2(x)\, dx$

$u = x \qquad dv = \csc^2(x)$
$du = 1 \qquad v = -\cot(x)$

$x(-\cot(x)) - \int -\cot(x)\, dx$

$\int -\cot(x) = -\int \frac{\cos(x)}{\operatorname{sen}(x)} \qquad u = \operatorname{sen}(x) \qquad du = \cos(x)\, dx$

$dx = \frac{du}{\cos(x)}$

$-\int \frac{\cancel{\cos(x)}}{u} \frac{du}{\cancel{\cos(x)}} \therefore -\int \frac{1}{u}\, du$

$-\ln(u)$

$-x\cot(x) + \ln(u) + C \rightarrow -x\cot(x) + \ln(\operatorname{sen}(x)) + C$

(20) $\int_0^1 x^3 \cdot e^{-x^2}\, dx$

$u = -x^2 \qquad dx = \frac{du}{-2x}$
$du = -2x\, dx$

$\int_0^1 x^3 \cdot e^u \frac{du}{-2x}$

$-\frac{1}{2}\int_0^1 x^2 \cdot e^u \therefore -\frac{1}{2}\int_0^1 -u \cdot e^u du = \frac{1}{2}\int_0^1 u \cdot e^u du$

$u = u \qquad dv = e^u \qquad \int_0^1 e^u = e^u\Big|_0^1$
$du = 1 \qquad v = e^u$

$\frac{1}{2}\left(u \cdot e^u - \int_0^1 e^u du\right)\Big|_0^1 = \frac{1}{2}(u \cdot e^u - e^u)\Big|_0^1$

$\frac{1}{2}(-x^2 \cdot e^{-x^2} - e^{-x^2})\Big|_0^1 \therefore \frac{1}{2}((-1^2 \cdot e^{-(1^2)} - e^{-(1^2)}) - (0 - e^0))$

$$\boxed{\frac{1}{2}\left(-\frac{2}{e} + 1\right)}$$

(25) $\int_0^{\pi/2} x \operatorname{sen}(2x)\, dx$

$u = x \quad dv = \operatorname{sen}(2x)$
$du = 1 \quad v = -\frac{1}{2}\cos(2x)$

$\int \operatorname{sen}(2x) \quad u = 2x \quad du = 2\,dx$
$dx = du/2$

$\int \operatorname{sen}(u)\,\frac{du}{2} = \frac{1}{2}\int \operatorname{sen} u$

$\frac{1}{2} - \cos(u) = \frac{1}{2} - \cos(2x)$

$x \cdot -\frac{1}{2}\cos(2x) - \int -\frac{1}{2}\cos(2x)$

$-\frac{1}{2} x\cos(2x) - \frac{1}{2}\int -\cos(2x)$

$-\frac{1}{2} x\cos(2x) - \frac{1}{2}\int -\cos(2x)$

$\int \cos(2x) \quad u = 2x \quad du = 2\,dx$
$dx = du/2$

$-\frac{1}{2} x\cos(2x) - \frac{1}{2}\left(-\frac{1}{2}\operatorname{sen}(2x)\right)$

$\int \frac{1}{2}\cos(u) = \frac{1}{2}\int \cos(u)$

$-\frac{1}{2} x\cos(2x) + \frac{1}{4}\operatorname{sen}(2x)\Big|_0^{\pi/2}$

$\frac{1}{2}\operatorname{sen}(u) = \frac{1}{2}\operatorname{sen}(2x)$

$-\frac{1}{2}\left(\frac{\pi}{2}\right)\cos\left(2\left(\pi/2\right)\right) + \frac{1}{4}\operatorname{sen}\left(2\left(\pi/2\right)\right) - \frac{1}{2}(0)\cos(2(0)) + \frac{1}{4}\operatorname{sen}(0)$

$\boxed{\pi/4}$

(30) $\int x^3 \cos(x^2)\, dx$

$dx = \frac{du}{2x} \quad u = x^2$
$du = 2x \quad \therefore \int x^3 \cos(u)\frac{du}{2x} = \frac{1}{2}\int x^2 \cos(u)\, du$

$\frac{1}{2}\int u\cos u\, du \quad u = u \quad dv = \cos(u)$
$du = 1 \quad v = \operatorname{sen}(u)$

$\frac{1}{2}\left[u\operatorname{sen} u - \int \operatorname{sen} u\, du\right] \quad \int \operatorname{sen}(u)\,du = -\cos(u)$

$\frac{1}{2}\left(u\operatorname{sen}(u) - (-\cos(u))\right) = \frac{1}{2}\left(u\operatorname{sen}(u) + \cos(u)\right) + C$

$\boxed{\frac{1}{2}\left(x^2\operatorname{sen}(x^2) + \cos(x^2)\right) + C}$

(35) $\int \cos\sqrt{x}\, dx$

$u = \sqrt{x}$

$du = \frac{1}{2\sqrt{x}}\, dx$

$dx = \frac{du}{\frac{1}{2\sqrt{x}}}$ $\quad dx = du\, 2\sqrt{x}$ $\quad dx = 2u\, du$

$\int \cos(u)\, 2u\, du$

$2\int \cos(u) \cdot u\, du \therefore$ $\quad u = u \quad du = 1$ $\quad dv = \cos(u) \quad v = \text{sen}(u)$

$2 \left[u\,\text{sen}(u) - \int \text{sen}(u)\, du \right.$

$2\left(u\,\text{sen}(u) - (-\cos(u))\right) \therefore 2\left(u\,\text{sen}(u) + \cos(u)\right)$

$2\left(\sqrt{x}\,\text{sen}(\sqrt{x}) + \cos(\sqrt{x})\right) + C$

Ejercicios del 39 al 42 use integracion por partes para deducir la formula de reduccion

(40) ~~$\int x^m \text{sen}(x)\, dx = -x^m \int x$~~

(40) $\int x^m \text{sen}\, x\, dx = -x^m \cos x + m \int x^{m-1} \cos x\, dx$

$u = x^m$ $\quad dv = \text{sen}(x)$

$du = m\, x^{m-1}$ $\quad v = -\cos(x)$

$x^m - \cos(x) - \int m\, x^{m-1} - \cos(x)\, dx$

$-x^m \cos(x) - (-m) \int x^{m-1} - \cos(x)\, dx$

$-x^m \cos(x) + m \int x^{m-1} - \cos(x)\, dx$

49) Sea $f(x) = \text{sen}\sqrt{x}$ Calcule el area de la region bajo la grafica de f entre $x=0$ $x=\pi^2$

$$\int_0^{\pi^2} \text{sen}\sqrt{x}$$

$u=\sqrt{x}$

$du = \frac{1}{2\sqrt{x}}\,dx$ $\quad dx = du\;2\sqrt{x}$

$$\int_0^{\pi^2} \text{sen}\,u \cdot u\,du$$

$dx = du\;u$

$u = u$ $\quad dv = \text{sen}(u)$

$du = 1$ $\quad v = -\cos(u)$

$$u\cdot(-\cos(u)) - \int -\cos(u)\,du$$

$$-u\cos(u) - (-\text{sen}(u))\Big|^{\pi^2}$$

$$-u\cos(u) + \text{sen}(u)\Big|_0^{\pi^2}$$

$$-\sqrt{x}\cos(\sqrt{x}) + \text{sen}\sqrt{x}\Big|_0^{\pi^2}$$

$$-\sqrt{\pi^2}\cos\sqrt{\pi^2} + \text{sen}\sqrt{\pi^2} - 0\cdot\cos(0) + \text{sen}(0)$$

$$-\pi\cos\pi + \text{sen}\,\pi - 0 \quad \boxed{=2\pi}$$

50) La velocidad (al tiempo t) de un punto que se mueve sobre una recta coordenada es t/e^{2t} m/s. Calcule la posicion al tiempo t si el punto se encuentra en el origen $t=0$

$$\int_0^t t/e^{2t}\,dt \therefore \int_0^t t\cdot\frac{1}{e^{2t}}\,dt \therefore \int_0^t t\cdot e^{-2t}\,dt$$

$u = t$ $\quad dv = e^{-2t}$

$du = 1\,dt$ $\quad v = -\frac{1}{2}e^{-2t}$

$\int e^{-2t}\,dt$: $u=-2t$

$du = -2\,dx$

$dx = \frac{du}{-2}$

$$\left[t\cdot -\frac{1}{2}e^{-2t} - \int -\frac{1}{2}e^{-2t}\,dt\right]_0^t$$

$$-\frac{1}{2}\int e^u\,du$$

$$\int e^u\,du = e^u$$

$$\left[-\frac{1}{2}e^{-2t}\cdot t + \frac{1}{2}\int e^{-2t}\,dt\right]_0^t$$

$$-\frac{1}{2}e^u = -\frac{1}{2}e^{-2t}$$

$$-\frac{1}{2}e^{-2t}\cdot t + \frac{1}{2}\left(-\frac{1}{2}e^{-2t}\right)$$

$$-\frac{1}{2}e^{-2t}\cdot t - \frac{1}{4}e^{-2t}\Big|_0^t$$

$$-\frac{1}{2}e^{-2t}\cdot t - \frac{1}{4}e^{-2t} - \left(-\frac{1}{2}e^{0}\cdot 0 - \frac{1}{4}e^{0}\right)$$

$$-\frac{1}{2}e^{-2t}\cdot t - \frac{1}{4}e^{-2t} - \left(-\frac{1}{4}\right)$$

$$\boxed{-\frac{1}{2}e^{-2t}\cdot t - \frac{1}{4}e^{-2t} + \frac{1}{4}}$$