Clase 6

22/10/2021

- Operaciones vectoriales
  - Producto mixto de 3 vectores: $abc = (\vec{a} \times \vec{b}) \cdot \vec{c}$

$\vec{a} = \frac{1}{2}\vec{i} + 2\vec{j} + 3\vec{k}$

$\vec{b} = -2\vec{i} + 3\vec{j} - 5\vec{k}$

$\vec{c} = 6\vec{i} - 1\vec{k}$

* $\vec{a} \times \vec{b} = (-10-9)\vec{i} - \left(-\frac{5}{2} - 6\right)\vec{j} + \left(\frac{3}{2} + 4\right)\vec{k}$

$\vec{a} \times \vec{b} = -19\vec{i} - \frac{17}{2}\vec{j} + \frac{11}{2}\vec{k}$ //

* $(\vec{a} \times \vec{b}) \cdot \vec{c} = (-19 \cdot 6)\vec{i} - \left(\frac{11}{2} \cdot 1\right)\vec{k} = 104\vec{i} - \frac{11}{2}\vec{k}$ //

- Doble producto vectorial: $(a \times b) \times c = (a \cdot c)b - (b \cdot c)a$
- Doble escalar de dos productos vectoriales: $(a \times b) \cdot (c \times d) = (a \cdot c)(b \cdot d) - (b \cdot c)(a \cdot d)$
- Producto vectorial de dos productos vectoriales: $(a \times b) \times (c \times d) = (abc)c - (abc)d$
- Rotacional: $\text{rot}\,\vec{\beta} = \nabla \times \vec{\beta}$

• $\nabla = \left(\frac{\partial}{\partial x}\vec{i} + \frac{\partial}{\partial y}\vec{j} + \frac{\partial}{\partial z}\vec{k}\right)$

$$\nabla \times \vec{\beta} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \vec{\beta}_x & \vec{\beta}_y & \vec{\beta}_z \end{vmatrix}$$

• $\text{rot}\,\vec{\beta} = \left(\frac{\partial B_z}{\partial y} - \frac{\partial B_y}{\partial z}\right)\vec{i} + \left(\frac{\partial B_x}{\partial z} - \frac{\partial B_z}{\partial x}\right)\vec{j} + \left(\frac{\partial B_y}{\partial x} - \frac{\partial B_x}{\partial y}\right)\vec{k}$

* Ejemplo: Calcular el rotacional de $\vec{\beta} = 2x^2yz\vec{i} + x^2z^2\vec{j}$

• $\text{rot}\,\vec{\beta} = \nabla \times \vec{\beta} = (0 - x^2)\vec{i} + (0 - 2x^2y)\vec{j} + (2xz - 2x^2z)\vec{k}$

$\text{rot}\,\vec{\beta} = -x^2\vec{i} - 2x^2y\vec{j} + (2xz - 2x^2z)\vec{k}$

* Ejemplo: Calcular la divergencia de $\vec{\beta} = x^2z^2\vec{i} - 2y^2z^2\vec{j} + xy^2z\vec{k}$

• $\text{div}\,\vec{\beta} = \nabla \cdot \vec{B} = \frac{\partial}{\partial x}(x^2z^2) + \frac{\partial}{\partial y}(-2y^2z^2) + \frac{\partial}{\partial z}(xy^2z) = 2xz^2 - 4yz^2 + xy^2$