Clase 8

17/11/2020

Plan

- Ejercicios de repaso

(A) $\int \frac{x}{\sqrt{x+1}}$ $\qquad x+1 = u^2$

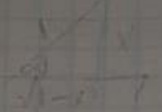
$x = u^2 - 1$

$\int \frac{u^2-1}{\cancel{u}} \cdot 2\cancel{u}\,du \qquad dx = 2u\,du$

$2\int u^2 - 1\,du = 2\left(\frac{u^3}{3} - u\right) + C$

$2\left(\frac{(x+1)^3}{3} - (x+1)\right) + C$

② $\int x \cdot \sqrt{1-x^2}\, dx$



$\operatorname{sen}\theta = x$ $\qquad \frac{dx}{d\theta} = \cos\theta$

$\cos\theta = \sqrt{1-x^2}$

$dx = \cos\theta \, d\theta$

$\int \operatorname{sen}\theta \cdot \cos\theta \cdot \cos\theta \, d\theta$

$\int \operatorname{sen}\theta \cdot \cos^2\theta \, d\theta$

$\int \operatorname{sen}\theta \left(\frac{1 + \cos(2\theta)}{2} \right) d\theta$

$\int \frac{\operatorname{sen}\theta}{2} + \frac{\operatorname{sen}\theta \cos(2\theta)}{2} \, d\theta$

$\frac{1}{2} \cos\theta \int \operatorname{sen}\theta \cos(2\theta) \, d\theta$

$z^2 = 1 - x^2$

$x = \sqrt{1 - z^2}$

$dx = \frac{-z}{\sqrt{1-z^2}} \, dz$

$\int \sqrt{1-z^2} \cdot z \cdot -\frac{z}{\sqrt{1-z^2}} \, dz$

$\int -z^2 \, dz$

$-\frac{z^3}{3} + C \quad \therefore \quad \boxed{-\frac{(\sqrt{1-x^2})^3}{3} + C}$

③ $\int \frac{e^x}{\sqrt{1-e^{2x}}} dx$ ④ $\int sen^3(x) \cos^4(x)\, dx$

⑤ $\int \frac{\cos x}{2-3\,sen(x)}\, dx$

③ $\int \frac{e^x}{\sqrt{1-e^{2x}}} dx$ $e^x = u$ $e^{2x} = (e^x)^2 \therefore (u)^2$

$du = e^x\, dx$

$\int \frac{u}{\sqrt{1-e^{2x}}}$ $dx = \frac{du}{e^x}$ $e^x = u$

$\int \frac{\cancel{x}}{\sqrt{1-u^2}} \cdot \frac{1}{\cancel{x}}\, du$

$\int \frac{1}{\sqrt{1-u^2}}\, du \therefore \int \frac{1}{\sqrt{1-u^2}}\, du = arc\ sen\ (u)$

$arc\ sen\ (u) + C \therefore arc\ sen\ (e^x) + C$

④ $\int sen^3(x) \cos^4(x)\, dx$

$sen^3(x) = (sen^2(x))\, sen(x)$

$sen^3(x) = (1-\cos^2(x))\, sen(x)$

$\int (1-\cos^2(x))\, sen(x) \cdot \cos^4(x)\, dx$

$du = -sen(x)\, dx \therefore dx = \frac{du}{-sen\ x}$

$\cos(x) = u$

$\int (1-u^2)\, sen(x) \cdot u^4\, \frac{du}{-sen(x)}$

$\int (1-u^2) \cdot u^4 \cdot -du$

$\int -u^4 \cdot (1-u^2)\, du \therefore \int -u^4 + u^6\, du \therefore$

$\frac{-u^5}{5} + \frac{u^7}{7} + C \therefore \frac{-(\cos(x))^5}{5} + \frac{(\cos(x))^7}{7} + C$

$$\int \frac{\cos(x)}{2 - 3\,\text{sen}(x)}\,dx$$

$$2 - 3\,\text{sen}\,x = u$$

$$du = -3\cos(x)\,dx$$

$$dx = \frac{du}{-3\cos(x)}$$

$$\int \frac{\cancel{\cos(x)}}{u} \cdot -\frac{1}{3\,\cancel{\cos(x)}}\,du$$

$$\int -\frac{1}{3u}\,du$$

$$-\frac{1}{3}\int \frac{1}{u}\,du \therefore -\frac{1}{3}\ln(u) + C \therefore$$

$$-\frac{1}{3}\ln(2 - 3\,\text{sen}(x)) + C$$