

Clase 6

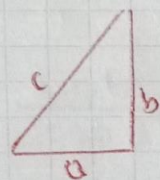
3/11/2020.

Plan

- Repaso
- Integrales trigonometricas
- Tarea.

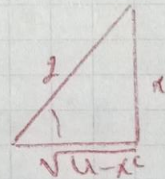
Integrales Trigonometricas

- No se desarrollan de manera directa
- No es un cambio de variable
- No es por partes



$$c^2 = a^2 + b^2$$

$$\int \sqrt{u-x^2} dx$$



$$a=2$$

$$b=x$$

$$c^2 = u$$

$$b^2 = x^2$$

$$a = \sqrt{u-x^2}$$

- Encontrando la relación entre c y θ

$$\cos \theta = \frac{\sqrt{u-x^2}}{2}$$

$$2 \cos \theta = \sqrt{u-x^2}$$

- Encontrando la relación entre c y b

$$\sin \theta = \frac{x}{2}$$

$$dx = 2 \cos \theta$$

$$2 \sin \theta = x$$

$$dx = 2 \cos \theta d\theta$$

reemplazando queda.

$$\int 2 \cos \theta (2 \cos \theta) d\theta$$

u $\cos^2 \theta$ de usando la identidad

$$u \cos^2 \theta = \frac{1 + \cos(2\theta)}{2}$$

$$u \int \frac{1 + \cos(2\theta)}{2} d\theta = \frac{1}{2} \int (1 + \cos(2\theta)) d\theta$$

$$2 \left(\theta + \frac{\sin 2\theta}{2} \right) + c$$

$$2 \left(\sin^{-1} \left(\frac{x}{2} \right) + \sin \frac{(2\theta)}{2} \right) + c$$

$$2 \left(\sin^{-1} \left(\frac{x}{2} \right) + 2 \sin(\theta) \cos(\theta) \right)$$

$$2 \left[\sin^{-1} \left(\frac{x}{2} \right) + \frac{x}{2} \frac{\sqrt{u-x^2}}{1} \right] + c$$

$$2 \left[\sin^{-1} \left(\frac{x}{2} \right) + \frac{x \sqrt{u-x^2}}{u} \right] + c$$

$$2 \left[\sin^{-1} \left[\frac{x}{2} \right] \left(1 + \frac{1}{2} \sqrt{u-x^2} \right) \right] //$$

$$\int \frac{\sqrt{u-x^2}}{x^2}$$

$$\cos \theta = \frac{\sqrt{u-x^2}}{2}$$

$$\cos \theta = \frac{\sqrt{u-x^2}}{3}$$

$$\sin \theta = \frac{x}{3}$$

$$x = 3 \sin \theta$$

$$\frac{dx}{d\theta} = 3 \cos \theta$$

$$dx = 3 \cos \theta d\theta$$

$$x^2 = 9 \sin^2 \theta$$

$$\int \frac{3 \cos \theta}{x^2} [3 \cos \theta d\theta]$$

$$\int \frac{u \cos^2 \theta}{u \sin^2 \theta} d\theta = \int \cot^2 \theta d\theta$$

$$\cot^2 \theta = -1 + \csc^2 \theta$$

$$\cot = \frac{\sqrt{u-x^2}}{x}$$

$$\int -1 d\theta \int \sec^2(\theta)$$

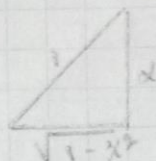
$$-\theta = \cot(\theta) + C$$

$$\theta = \sec^{-1}\left(\frac{x}{3}\right)$$

$$\sec^{-1}\left(\frac{x}{3}\right) + \frac{\sqrt{9-x^2}}{x}$$

TAREA

$$\int \frac{x^2}{\sqrt{1-x^2}}$$



$$A = x^2$$

$$b = \sqrt{1-x^2}$$

$$1 = 1$$

$$\sec \theta = 1$$

$$\cos \theta = \sqrt{1-x^2}$$

$$\sec \theta = x^2$$

$$\frac{dx}{d\theta} = \cos \theta$$

$$dx = \cos \theta d\theta$$

$$\int \frac{\sec^2 \theta}{\cos \theta} = \cos \theta d\theta$$

$$\int \sec^2 \theta d\theta = \sec^2 \theta = \frac{1 - \cos(2\theta)}{2}$$

$$\int \frac{1}{2} \cos(2\theta) d\theta = \frac{1}{2} \int 1 - \cos(2\theta)$$

$$\frac{1}{2} (9 - \sec(2\theta))$$

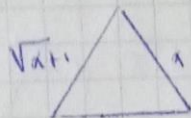
$$\frac{1}{2} \left[\sec^{-1}(x) + \frac{1}{2} \sec^{-1} \sec^{-1}(x) \right]$$

17/11/2020

Plan

Ejercicios

1) $\int \frac{x}{\sqrt{x+1}} dx$



$$z = x+1$$

$$z^2 = x+1$$

$$x = z^2 - 1$$

$$dx = 2z dz$$

$$c^2 = a^2 + b^2$$

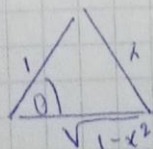
$$\int \frac{z^2 - 1}{\sqrt{z}} 2z dz$$

$$\int \frac{2z^3 - 2z}{2} dz$$

$$2 \int (z^2 - 1) dz$$

$$2 \left(\frac{z^3}{3} - z \right) + C$$

2) $\int x \cdot \sqrt{1-x^2} dx$



$$\cos(\theta) = \frac{1}{\sqrt{1-x^2}}$$

$$\cos(\theta) = \frac{1}{\sqrt{1-x^2}}$$

$$\sin(\theta) = x$$

$$\frac{dx}{d\theta} = \cos(\theta)$$

$$dx = \cos(\theta)$$

$$\int \sin(\theta) \cos^2(\theta) d\theta$$

$$\int \sin(\theta) \left(\frac{1 + \cos(2\theta)}{2} \right)$$

$$\int \frac{\sin(\theta)}{2} + \frac{\sin(\theta) \cos(2\theta)}{2}$$

$$3) \int x \cdot \sqrt{1-x^2} dx$$

$$z^2 = 1-x^2$$

$$x^2 = 1-z^2$$

$$x^2 = 1-z^2$$

$$x = \sqrt{1-z^2}$$

$$dx = \frac{-2z}{2\sqrt{1-z^2}} dz$$

$$\frac{-(\sqrt{1-z^2})^3}{3}$$

$$1-z^2$$

$$dx = \frac{-2z}{\sqrt{1-z^2}} dz$$

TAREA

$$1) \int \frac{e^x}{\sqrt{1-e^{2x}}} dx$$

$$e^x = u$$

$$e^{2x} = (e^x)^2 = u^2$$

$$\int \frac{u}{\sqrt{1-u^2}} du$$

$$du = e^x dx$$

$$dx = \frac{du}{u}$$

$$\int \frac{u}{\sqrt{1-u^2}} \cdot \frac{1}{u} du$$

$$\int \frac{1}{\sqrt{1-u^2}} du = \arcsin(u)$$

$$\arcsin(u) = \arcsin(e^x)$$

$$2) \int \frac{\cos(x)}{2-3\sin(x)} dx$$

$$2-3\sin(x) = u$$

$$du = -3\cos(x) dx$$

$$\int \frac{\cos(x)}{u} \cdot \frac{1}{-3\cos(x)} du$$

$$dx = \frac{du}{-3\cos(x)}$$

$$\int -\frac{1}{3u} du$$

$$-\frac{1}{3} \int \frac{1}{u} du = -\frac{1}{3} \ln(u) + C$$

$$-\frac{1}{3} \ln(2-3\sin(x)) + C$$

$$3) \int \sin^3(x) \cos^4(x) dx$$

$$\sin^3(x) = \sin u \cdot \sin^2(x)$$

$$\sin^2(x) = 1 - \cos^2(x) \quad (\sin^2 u)$$

$$\int (1 - \cos^2(x)) \sin(x) \cdot \cos^4(x) dx$$

$$\cos(x) = u$$

$$du = -\sin x \, dx \quad dx = \frac{du}{-\sin x}$$

$$\int (1 - u^2) \sin(x) \cdot u^4 \cdot \frac{du}{-\sin(x)}$$

$$\int (1 - u^2) \cdot u^4 \cdot -du$$

$$\int -u^4 \cdot (1 - u^2) du = \int -u^4 + u^6 du$$

$$-\frac{u^5}{5} + \frac{u^7}{7} + C = -\frac{(\cos(x))^5}{5} + \frac{(\cos(x))^7}{7} + C$$