Clase 9

Integracion por partes

$$\int u\,dv = uv - \int v\,du$$

donde u y v son funciones no continuas

$$\int \underbrace{x}_{u}\,\underbrace{e^{2x}\,dx}_{dv}$$

$dv = e^{2x}$ $\qquad u = x$

$v = \frac{e^{2x}}{2}$ $\qquad du = dx$

$$\int u\,dv = uv - \int v\,du$$

$$\int x e^{2x} = x\frac{e^{2x}}{2} - \int \frac{e^{2x}}{2}\,dx$$

$$\int x e^{2x} = \frac{1}{2} x e^{2x} - \frac{1}{4} e^{2x} + C$$

$$\int x \sec^2(x)\,dx$$

$$\int u\,dv = uv - \int v\,du$$

$u = x$ $\qquad dv = \sec^2(x)$

$du = dx$ $\qquad v = \tan(x)$

$$\int x\sec^2(x)\,dx = x\tan(x) - \int \tan(x)\,dx$$

$$\int x\sec^2(x)\,dx = x\tan(x) - \ln(\sec(x)) + C$$

$$\int \frac{x^2 e^{2x}}{2}\, dx$$

$$\frac{1}{2} \cdot \int x^2 e^{2x}\, dx \qquad \int u\,dv = uv - \int v\, du$$

$$u = x^2 \qquad v = e^{2x}/2$$

$$du = 2x\, dx \qquad dv = e^{2x}$$

$$\frac{1}{2} \left(x^2 \cdot \frac{e^{2x}}{2} - \int 2x \frac{e^{2x}}{2} \right.$$

$$\frac{1}{2} \left(x^2 \cdot \frac{e^{2x}}{2} - \int e^{2x} x\, dx \right.$$

$$\frac{1}{2} \left(\frac{1}{2} \cdot x^2 \cdot e^{2x} - \int e^{2x} x\, dx \right.$$

$$\int e^{2x}\, dx = \qquad u = 2x \quad du = 2dx \quad dx = \frac{1}{2} du$$

$$x = \frac{u}{2}$$

$$\int e^u \cdot \frac{u}{2} \cdot \frac{du}{2}$$

$$\frac{1}{4} \int e^u \cdot u\, du \quad \therefore \int u\,dv = uv - \int v\,du$$

$$u = u \qquad dv = e^u \qquad \int e^u = e^u$$

$$du = 1 \qquad v = e^u$$

$$\frac{1}{4} \left(e^u u - \int e^u\, du \right) \therefore \frac{1}{4} \left(e^u \cdot u - e^u \right) \therefore \frac{1}{4} \left(e^{2x} \cdot 2x - e^{2x} \right)$$

$$\therefore \frac{1}{2} \left(\frac{1}{2} \cdot x^2 \cdot e^{2x} - \left(\frac{1}{4} \cdot e^{2x} \cdot 2x - e^{2x} \right) + C \right.$$

$\int e^x \cos(x)\, dx$

$u = e^x \quad dv = \cos(x)$
$du = e^x \quad v = sen(x)$

$e^x \cdot sen(x) - \int e^x sen(x)\, dx$

$u = e^x \quad du = e^x$
$dv = sen(x) \quad v = -\cos(x)$

$e^x sen(x) - (-\cos(x) e^x) - \int -e^x \cos(x)\, dx$

$u = \int e^x \cos(x)$

$\int e^x \cos(x)\, dx = e^x sen(x) - (-\cos(x) e^x) + u)$

$u = e^x sen(x) + \cos(x) e^x - u$

$2u = e^x sen(x) + \cos(x) e^x$

$u = \frac{e^x sen(x)}{2} + \frac{\cos(x) e^x}{2}$

$\int e^x \cos(x)\, dx = \frac{e^x sen(x)}{2} + \frac{\cos(x) e^x}{2}$

$\int sec^3(x)\, dx = \int sec^2(x) sec(x)\, dx$

$\int sec^n(x)\, dx = \frac{sec^{n-1}(x) sen(x)}{n-1} + \frac{n-2}{n-1} \int sec^{n-2}(x)\, dx$

$\frac{sec^2(x) sen(x)}{2} + \frac{1}{2} \int sec^2(x)\, dx$

$\int sec^2(x) = \ln|\tan(x) + sec(x)|$

$\frac{sec^2(x) sen(x)}{2} + \frac{1}{2} \ln|\tan(x) + sec(x)|$

$sec^2(x) = \frac{1}{\cos^2(x)} \therefore sec^2(x) sen(x) = \frac{1}{\cos(x)} \cdot \frac{sen(x)}{\cos(x)}$

R// $\frac{sec(x) \tan(x)}{2} + \frac{1}{2} \ln|\tan(x) + sec(x)| + C$