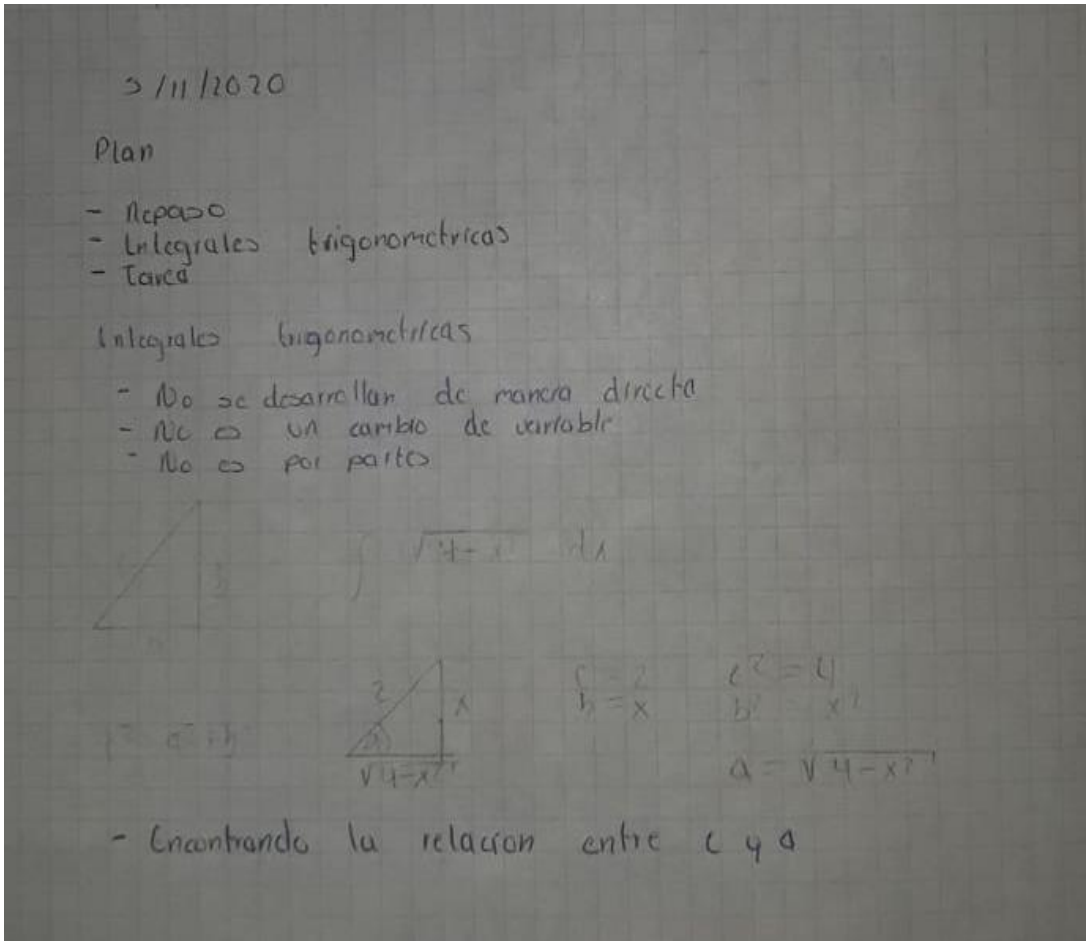
Clase 6


3/11/2020
Plan
- Repaso
- Integrales trigonometricas
- Tarea
Integrales trigonometricas
- No se desarrollan de manera directa
- No es un cambio de variable
- No es por partes
$\int \sqrt{4-x^2}\,dx$
2
x
θ
$\sqrt{4-x^2}$
$c^2 = 4$
$b^2 = x^2$
$a = \sqrt{4-x^2}$
- Encontrando la relacion entre c y a

$$\cos\theta = \frac{\sqrt{4-x^2}}{2}$$

$$\boxed{2\cos\theta = \sqrt{4-x^2}}$$

Encontrando la relación entre c y b

$$\operatorname{sen}\theta = \frac{x}{2} \qquad \frac{dx}{d\theta} = 2\cos\theta$$

$$\boxed{2\operatorname{sen}\theta = x} \qquad \boxed{dx = 2\cos\theta\, d\theta}$$

reemplazando queda:

$$\int 2\cos\theta\,[2\cos\theta]\,d\theta$$

$$4\int \cos^2\theta\, d\theta \quad \therefore \text{ usando la identidad}$$

$$\cos^2\theta = \frac{1+\cos(2\theta)}{2}$$

$$4\int \frac{1+\cos(2\theta)}{2}\,d\theta = 2\int 1+\cos(2\theta)\,d\theta$$

$$2\left(\theta + \frac{\operatorname{sen}(2\theta)}{2}\right) + C$$

$$2\left(\operatorname{sen}^{-1}\left(\frac{x}{2}\right) + \operatorname{sen}(2\theta)\right) + C$$

$$2\left[\operatorname{sen}^{-1}\left(\frac{x}{2}\right) + 2\operatorname{sen}\theta\cos\theta\right]$$

$$2\left[\operatorname{sen}^{-1}\left(\frac{x}{2}\right) + \frac{x}{2}\cdot\frac{\sqrt{4-x^2}}{2}\right] + C$$

$$2\left[\operatorname{sen}^{-1}\left(\frac{x}{2}\right) + \frac{x\sqrt{4-x^2}}{4}\right] + C$$

$$2\left[\operatorname{sen}^{-1}\left[\frac{x}{2}\left(1+\frac{1}{2}\sqrt{4-x^2}\right)\right]\right]$$

$$\int \frac{\sqrt{9-x^2}}{x^2}$$

$$\cos\theta = \sqrt{9-x^2}$$

$$\frac{dx}{d\theta} = 3\cos\theta$$

$$\sin\theta = \frac{x}{3}$$

$$dx = 3\cos\theta\, d\theta$$

$$x = 3\sin\theta \qquad x^2 = 9\sin^2\theta$$

$$\int \frac{3\cos\theta}{x^2}\, 3\cos\theta\, d\theta$$

$$\int \frac{\cos^2\theta}{\sin^2\theta}\, d\theta = \int \cot^2\theta\, d\theta$$

$$\cot^2\theta = -1 + \csc^2(\theta)$$

$$\int -1 + \int \csc^2(\theta)\, d\theta$$

$$-\theta - \cot(\theta) + C$$