

DEBER.

Deber

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- 1) Los números $[x_0, x_1, x_2, \dots, x_n]$ determinan una partición p de intervalo $[a, b]$ encuentra $\Delta x_1, \Delta x_2, \dots, \Delta x_n$ y la norma de $\|p\|$ de la partición.

$$[-3, 1] : [-3, -2.7, -1, 0.4, 0.9, 1]$$

$$\Delta x_1 = -3, -2.7 = 0.3$$

$$\Delta x_2 = -2.7 + 1 = 1.7$$

$$\Delta x_3 = -1 - 0.4 = -1.4 \quad \|\Delta x_3\| = 1.4$$

$$\Delta x_4 = 0.4 - 0.9 = -0.5$$

$$\Delta x_5 = 0.9 - 1 = -0.1$$

- 2) Calcula la suma de Riemann R_p de f correspondiente a la partición uniforme $PEI.67$ en los cuales subintervalos determinados por $x_0=1$ $x_1=2$ $x_2=3$ $x_3=4$ $x_4=5$ eligiendo

- a) w_k = Como el extremo derecho x_k de $[x_{k-1}, x_k]$
b) w_k = Como extremo izquierdo x_{k-1} de $[x_{k-1}, x_k]$
c) w_k = Como el punto medio de $[x_{k-1}, x_k]$

$$f(x) = 3 - 4x$$
$$\sum_{k=1}^n f(w_k) \Delta x \quad \Delta x = 1 \quad \Delta x = 1 \quad \Delta x = 1 \quad \Delta x = 1$$

$$\frac{4-1}{1-3} = \frac{3-3}{1} = \frac{3}{1} = 3$$

$$f(w_1) = 3 - 4(1) = -1 \quad x_1 = 1$$

$$f(w_2) = 3 - 4(2) = -5 \quad x_2 = 2$$

$$f(w_3) = 3 - 4(3) = -9 \quad x_3 = 3$$

$$f(w_4) = 3 - 4(4) = -13 \quad x_4 = 4$$

$$f(w_4) = 3 - 4(3) = -17 \times 4 = -68$$

3) Sea $f(x) = x^3$ y P la partición de $[2, 4]$ en los cuatro subintervalos determinados por $x_0 = 2$ $x_1 = 0$ $x_2 = 1$ $x_3 = 3$ $x_4 = 4$ calcule la suma de Riemann R_4 para $w_1 = -1$ $w_2 = 1$ $w_3 = 2$ $w_4 = 4$

$$\Delta x_1 = 2$$

$$\Delta x_2 = 1$$

$$\Delta x_3 = 2$$

$$\Delta x_4 = -1$$

$$\#_{\text{par}} = \frac{b-a}{4} = \frac{4-2}{4} = \frac{6}{4} = \frac{3}{2}$$

$$f(w_1) = -2 = -8$$

$$f(w_2) = -1 = -1$$

$$f(w_3) = 0^3 = 0$$

$$f(w_4) = 1^3 = 1$$

$$f(w_5) = 2^3 = 8$$

$$f(w_6) = 3^3 = 27$$

$$f(w_7) = 4^3 = 64$$

$$\sum_{i=1}^4 x^3(w_i) = -1(2) + 1(1/2) + 8(1/1) + 64(1/2)$$

$$x^3 R_4 = 268.$$

$$u) \lim_{n \rightarrow \infty} \sum_{k=1}^n w_k (w_k^2 - u) \Delta x = [-1, 2]$$

$$\int_{-1}^2 \pi (w^2/8 - u) dw$$

5) Suponiendo que $\int_0^u \sqrt{x} dx = 14/3$ calcule $\int_u^1 \sqrt{x} dx$

$$\int_0^1 x^{1/2} dx = \frac{x^{3/2}}{3/2} \Big|_0^1 = \frac{1^{3/2}}{3/2} - \frac{0^{3/2}}{3/2} = -14/3$$

6) Suponiendo que $\int_0^8 \sqrt[3]{s} ds = 45/4$ calcule $\int_0^1 \sqrt[3]{t} dt$

$$\int_0^1 t^{1/3} dt = \frac{t^{3/3+1}}{2/3} \Big|_0^1 = \frac{1^{2/3}}{2/3} - \frac{0^{2/3}}{2/3} = \frac{8^{2/3}}{2/3} = \frac{45}{8}$$

$$7) \int_0^3 \sqrt{9-x^2} dx$$

$$x = \frac{\sqrt{a}}{\sqrt{b}} \sin(u) \cdot \sqrt{a-bx^2}$$

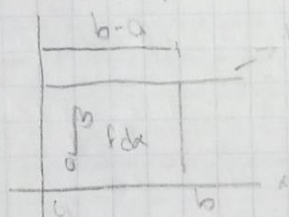
$$\int \sqrt{a - b \sin^2(u)} \cdot \frac{1}{\sqrt{b}} \cos(u) du = \int_0^{\pi/2} \cos^2(u) du$$

$$\int_0^{\pi/2} \cos^2(u) du = \frac{1}{2} \left(\int_0^{\pi/2} du + \int_0^{\pi/2} \cos(2u) du \right)$$

$$\frac{1}{2} \left(\frac{\pi}{2} + 0 \right) = \frac{\pi}{4}$$

8) Sea c un número real arbitrario y $f(x) = c$ para todo x sea p una partición arbitraria de $[a, b]$. Demuestre que la suma de Riemann R_p de f es igual a $c(b-a)$. Usando este hecho demuestre $\int_a^b c dx = c(b-a)$ interprete geométricamente para el caso en que $c > 0$.

$$\sum_{i=1}^n f(x_i) \Delta x_i = \sum_{i=1}^n (c \Delta x_i) = c \sum_{i=1}^n \Delta x_i = c(b-a)$$



$$9) \int_0^3 \sqrt{9-x^2} dx$$

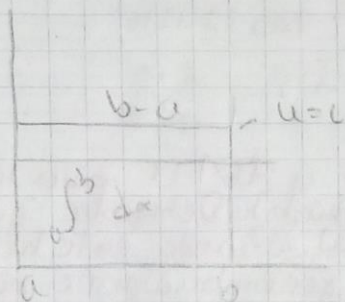
$$\int \sqrt{9 - 1 \sin^2(u)} \cdot \frac{1}{1} \cos(u) du = \int_0^{\pi/2} \cos^2(u) du$$

$$\cos^2(u) dx = \frac{1}{2} \left(\int_0^{\pi/2} du + \int_0^{\pi/2} \cos(2u) du \right)$$

$$\frac{1}{2} \left(\frac{\pi}{2} + 0 \right) = \frac{\pi}{4}$$

5) Un número real arbitrario c , $f(x) = c$ para todo x y una partición arbitraria de $[a, b]$. Demuestre que $\int_a^b c \, dx = c(b-a)$ interprete y comente brevemente el caso en que $c=0$.

$$f(x) \Delta x = \sum_{i=1}^n (A x_i) = c \Delta x = c(b-a)$$



Ejercicios 5.5

Evaluar los siguientes integrales

1. $\int (3x+1)^4 \, dx$ $u = 3x+1 \quad du = 3 \, dx \quad \frac{du}{3}$

$$(u)^4 = \frac{du}{3}$$

$$\frac{u^5}{5} \cdot \frac{1}{3} = \frac{u^5}{15} + C = \frac{(3x+1)^5}{15} + C$$

2. $\int \sqrt{x^3-1} \, dx$ $u = x^3-1 \quad du = 3x^2 \, dx$

$$\sqrt{u} \frac{du}{3} = \int u^{1/2} \frac{du}{3} = \frac{u^{3/2}}{3 \cdot 1/2} \cdot \frac{1}{3} + C$$

$$\frac{1}{3} \frac{(x^3-1)^{3/2}}{3/2} + C = \frac{2}{9} (x^3-1)^{3/2} + C = \frac{u^{3/2}}{3/2} \cdot \frac{1}{3} + C$$

$$3) \int \sqrt{1-25x^2} dx \quad u = 1-25x^2 \quad du = -50 dx \quad dx = \frac{du}{-50}$$

$$\int \frac{8}{u^{1/3}} \cdot \frac{du}{-2/5} = \int u^{1/3} \cdot \frac{du}{-2} = \frac{1}{u} \cdot \frac{u^{2/3}}{2/3} + C$$

$$= -\frac{3}{8} (1-25x^2)^{2/3} + C$$

$$u) \int \sqrt[3]{t^4 - t^2} (10t^3 - 5t) dt \quad u = t^4 - t^2 \quad du = 4t^3 - 2t dt$$

$$\int \sqrt[3]{u} \cdot 5(2t^3 - 1) \cdot \frac{2/3}{2(2t^3 - 1)} du$$

$$\frac{5}{2} \int u^{1/3} du = \frac{5}{2} \cdot \frac{u^{4/3}}{4/3} + C = \frac{15}{8} (t^4 - t^2)^{4/3} + C$$

$$5) \int \frac{\sqrt{u+3}^4}{\sqrt{u}} du \quad u = \sqrt{u+3} \quad du = \frac{1}{2\sqrt{u}} du$$

$$\int \frac{(u)^4}{\sqrt{u}} \cdot 2\sqrt{u} du$$

$$\int (u)^4 = 2 \frac{u^5}{5} = \frac{2}{5} (\sqrt{u+3})^5$$

$$6) \int \left(1 + \frac{1}{x}\right)^3 \cdot \left(\frac{1}{x^2}\right) dx$$

$$u = \left(1 + \frac{1}{x}\right)^3 \quad u' = -\frac{3}{x^2}$$

$$u' = \frac{3x^2}{(x+1)^4} = \int \frac{1}{x^2} dx = -\frac{1}{x}$$

$$\left(1 + \frac{1}{x}\right)^3 \left(-\frac{1}{3}\right) = \int \frac{3x^2}{(x+1)^4} \left(-\frac{1}{2}\right) dx$$

$$\frac{-x^2}{(x+1)^3} = \int \frac{3x}{(x+1)^4}$$

$$\int \frac{3x}{(x+1)^4} dx = \frac{3x}{(x+1)^4} - \frac{1}{(x+1)^3}$$

$$= \frac{x^2}{(x+1)^3} - \left(\frac{3}{2(x+1)^2} - \frac{1}{(x+1)^3} \right)$$

$$= \frac{x^2}{(x+1)^2} - \frac{3}{2(x+1)^2} + \frac{1}{(x+1)^3} + C$$

$$\int (2x^2 - 3)^5 dx \quad u = 2x^2 - 3 \quad du = 4x \quad dx = \frac{du}{4x}$$

$$\int u^5 \cdot \frac{du}{4x} = \frac{1}{4} \int u^5 du = \frac{1}{4} \cdot \frac{u^6}{6} + C = \frac{1}{24} (2x^2 - 3)^6 + C$$

$$8) \int 7\sqrt{9-7x} dx \quad u = 9-7x \quad du = -7 dx \quad dx = \frac{du}{-7}$$

$$\int 7\sqrt{u} \cdot \frac{du}{-7} = - \int u^{1/2} du = - \frac{1}{2} \cdot \frac{u^{3/2}}{3/2} + C = - \frac{1}{3} (9-7x)^{3/2} + C$$

$$9) \int \frac{x-2}{(x^2-4x+3)} dx \quad u = x^2-4x+3 \quad du = 2x-4 dx$$

$$dx = \frac{du}{2x-4} = \frac{du}{2(x-2)}$$

$$\int \frac{1 \cdot 2}{(u)^3} \cdot \frac{du}{2(x-2)} = \frac{1}{2} \int \frac{1}{u^3} du = \frac{1}{2} \cdot \frac{u^{-2}}{-2} + C = - \frac{1}{4} \cdot u^{-2} + C$$

$$= - \frac{1}{4} \cdot (x^2 - 4x + 3)^{-2}$$

$$10) \int \frac{x^2}{(u-9x^2-2x^3)^4} dx \quad u = u-9x^2-2x^3 \quad du = -6x-6x^2 dx$$

$$du = \frac{du}{-6x-6x^2}$$

$$- \frac{1}{6} \int \frac{1}{u^4} du = - \frac{1}{6} \int u^{-4} du$$

$$= - \frac{1}{6} \cdot \frac{u^{-3}}{-3} + C = \frac{1}{18} u^{-3} = \frac{1}{18} (u-9x^2-2x^3)^{-3} + C$$