Tarea 2

Deber clase 27/10/2020

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① los numeros $[x_0, x_1, x_2 \dots x_n]$ determinan una particpacion P del intervalo $[a, b]$ encuentre $\Delta x_1, \Delta x_2 \dots \Delta x_n$ y la norma de $\|P\|$ de la particion

3 $[-3, 1]$; $[-3, -2.7, -1, 0.4, 0.9, 1]$

$\Delta x_1 = -3 + 2.7 = 0,3$

$\Delta x_2 = -2.7 + 1 = 1.7$

$\Delta x_3 = -1 - 0.4 = -1.4$ $\quad \|P\| = -1.4$

$\Delta x_4 = 0.4 - 0.9 = -0.5$

$\Delta x_5 = 0.9 - 1 = -0.1$

- Calcula la suma de Riemann R_P de f correspondiente a la particpacion uniforme $P\ [1, 5]$ en los cuatros subintervalos determinados por $x_0 = 1$ $x_1 = 2$ $x_2 = 3$ $x_3 = 4$ $x_4 = 5$ eligiendo

a) w_k = como el extremo derecho x_k de $[x_{k-1}, x_k]$

b) w_k como extremo izquierdo x_{k-1} de $[x_{k-1}, x_k]$

c) w_k como el punto medio de $[x_{k-1}, x_k]$

⑥ $f(x) = 3 - 4x$

$$\sum_{k=1}^{5} f(w_k)\,\Delta x$$

$\Delta x_1 = 1$ $\Delta x_2 = 1$ $\Delta x_3 = 1$ $\Delta x_4 = 1$

$\|P\| = 1$

$$\frac{b-a}{\#\ par} = \frac{5-1}{4} = \frac{4}{4} = 1$$

$f(w_1) = 3 - 4(1) = -1 \times 4 = -4$

$f(w_2) = 3 - (4)(2) = -5 \times 4 = -20$

$f(w_3) = 3 - (4)(3) = -9 \times 4 = -36$

$f(w_4) = 3 - 4(4) = -13 \times 4 = -52$

$f(w_5) = 3 - 4(5) = -17 \times 4 = -68$

⑧ Sea $f(x) = x^3$ y P la partición de $[-2, 4]$ en los cuatros subintervalos determinados por $x_0 = -2$ $x_1 = 0$ $x_2 = 1$ $x_3 = 3$ $x_4 = 4$ calcule la suma de Riemann R_P para $w_1 = -1$ $w_2 = 1$ $w_3 = 2$ $w_4 = 4$

$\Delta x_1 = 2$
$\Delta x_2 = 1$
$\Delta x_3 = 2$
$\Delta x_4 = 1$

$$\frac{b-a}{\# \text{pa}} = \frac{4+2}{4} = \frac{6}{4} = \frac{3}{2}$$

$$\begin{aligned}
f(w_1) &= -2^3 = -8 \\
f(w_2) &= -1^3 = -1 \\
f(w_3) &= 0^3 = 0 \\
f(w_4) &= 1^3 \quad 1 \\
f(w_5) &= 2^3 = 8 \\
f(w_6) &= 3^3 = 27 \\
f(w_7) &= 4^3 = 64
\end{aligned}$$

$$\sum_{-2}^{4} x^3 \,|a| = -1(4) + \cancel{1(8/2)} + 8(8/2) + 64(8/2)$$

$$\sum_{-2}^{4} x^3 \Delta x = 268$$

⑬ $$\lim_{\|P\| \to 0} \sum_{k=1}^{n} \pi z\,(w_8^2 - 4) \quad \Delta x = [-1, 2]$$

$$\int_{-1}^{2} \pi (w_8^2 - 4)\, dw$$

⑮ Suponiendo que $\int^{4} \sqrt{x}\, dx = 14/3$ calcule $\int_4^1 \sqrt{x}\, dx$

$$\int_4^1 x^{1/2} dx = \left.\frac{x^{3/2}}{3/2}\right|_4^1 = \frac{1^{3/2}}{3/2} - \frac{4^{3/2}}{3/2} = -14/3$$

⑱ Suponiendo que $\int_{-1}^{8} \sqrt[3]{s}\, ds = 45/4$, calcule $\int_8^{-1} \sqrt[3]{t}\, dt$

$$\int_8^{-1} t^{1/3} dt = \left.\frac{t^{2/3}}{2/3}\right|_8^{-1} = \frac{-1^{2/3}}{2/3} + \frac{8^{2/3}}{2/3} = \frac{45}{4}$$

(21) $\int_0^3 \sqrt{9-x^2}\,dx$ $\qquad x = \frac{\sqrt{a}}{\sqrt{b}}\,\text{sen}(u) \therefore \sqrt{a-bx^2}$

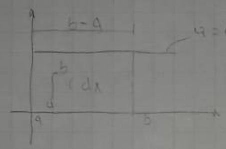
$$\int \sqrt{9-(3\,\text{sen}(u))^2}\cdot 3\cos(u)\,du \therefore \int_0^{\pi/2} 9\cos^2(u)\,du$$

$$9\int_0^{\pi/2} \cos^2(u)\,du = 9\cdot\frac{1}{2}\left[\int_0^{\pi/2} du + \int_0^{\pi/2} \cos(2u)\,du\right.$$

$$9/2\,(\pi/2 + 0) = 9\pi/4$$

(24) Sea c un numero real arbitrario y $f(x) = c$ para todo x sea P una particpacion arbitraria de $[a,b]$. Demuestre que la suma de riemann R_p de f es igual a $c(b-a)$. usando este hecho demuestre $\int_a^b c\,dx = c(b-a)$ interprete geometricamente para el caso en que $c>0$

$$\sum_1 f(w_i)\,\Delta x_i = \sum_N c\,\Delta x_i = c\sum_N \Delta x_i = c(b-a)$$



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① $\int (3x+1)^4\,dx$ $\qquad u = 3x+1 \quad du = 3\,dx \quad dx = \frac{du}{3}$

$$\int (u)^4 \frac{du}{3}$$

$$\frac{u^5}{5}\cdot\frac{1}{3} + C = \frac{u^5}{15} + C = \frac{(3x+1)^5}{15} + C$$

② $\int t^2\cdot\sqrt{t^3-1}\,dt$ $\qquad u = t^3-1 \quad du = t^2\,dt \quad dt = \frac{du}{3t^2}$

$$\int \cancel{t^2}\sqrt{u}\,\frac{du}{3\cancel{t^2}} = \int \sqrt{u}\,\frac{du}{3} = \int u^{1/2}\,\frac{du}{3} = \frac{u^{3/2}}{3/2}\cdot\frac{1}{3} + C$$

$$\frac{1}{3}\,\frac{(t^3-1)^{3/2}}{3/2} + C = \frac{2}{9}(t^3-1)^{3/2} + C$$

③ $\int (2x^2-3)^5 x \, dx$ $\quad u = 2x^2-3 \quad du = 4x \, dx \quad dx = \frac{du}{4x}$

$\int u^5 \cancel{x} \frac{du}{4\cancel{x}} = \frac{1}{4}\int u^5 \, du = \frac{1}{4} \cdot \frac{u^6}{6} + C = \frac{1}{24}(2x^2-3)^6$

④ $\int z\sqrt{9-z^2}\, dz$ $\quad u = 9-z^2 \quad du = -2z \, dx \quad dx = \frac{du}{-2z}$

$\int \cancel{z} \sqrt{u} \frac{du}{-2\cancel{z}} = -\frac{1}{2}\int u^{1/2} = -\frac{1}{2} \cdot \frac{u^{3/2}}{3/2} + C = -\frac{1}{3}(9-z^2)^{3/2} + C$

⑤ $\int \frac{x-2}{(x^2-4x+3)^3} dx$ $\quad u = x^2-4x+3 \quad du = 2x-4 \, dx$

$dx = \frac{du}{2x-4}$

$dx = \frac{du}{2(x-2)}$

$\int \frac{\cancel{x-2}}{(u)^3} \frac{du}{2\cancel{(x-2)}}$

$\frac{1}{2}\int \frac{1}{u^3} du = \frac{1}{2}\int u^{-3} = \frac{1}{2} \cdot \frac{u^{-2}}{-2} + C = \frac{1}{-4} \cdot u^{-2} + C$

$-\frac{1}{4} \cdot (x^2-4x+3)^{-2}$

⑥ $\int \frac{x^2+x}{(4-3x^2-2x^3)^4} dx$ $\quad u = 4-3x^2-2x^3 \quad du = -6x-6x^2 \, dx$

$dx = \frac{du}{-6x-6x^2}$

$dx = \frac{du}{-6(x+x^2)}$

$\int \frac{\cancel{x^2+x}}{u^4} \cdot \frac{du}{-6\cancel{(x+x^2)}}$

$-\frac{1}{6}\int \frac{1}{u^4} du = -\frac{1}{6}\int u^{-4}$

$-\frac{1}{6} \cdot \frac{u^{-3}}{-3} + C = \frac{1}{18} u^{-3} = \frac{1}{18}(4-3x^2-2x^3)^{-3} + C$

⑦ $\int \frac{s}{\sqrt[3]{1-2s^2}}\, ds$ $\quad u = 1-2s^2 \quad du = -4s\, dx \quad dx = \frac{du}{-4s}$

$\int \frac{\cancel{s}}{u^{1/3}} \frac{du}{-4\cancel{s}} = \int u^{-1/3} \cdot \frac{du}{-4} = -\frac{1}{4} \cdot \frac{u^{2/3}}{2/3} + C$

$-\frac{3}{8} (1-s^2)^{2/3} + C$

⑧ $\int \sqrt[3]{t^4-t^2}\, (10t^3-5t)\, dt$ $\quad u = t^4 - t^2 \quad du = 4t^3 - 2t\, dx$

$dx = \frac{du}{2(2t^3-t)}$

$\int \sqrt[3]{u} \cdot 5\cancel{(2t^3-t)} \cdot \frac{du}{2\cancel{(2t^3-t)}}$

$\frac{5}{2} \int u^{1/3}\, du = \frac{1}{3} \cdot \frac{u^{4/3}}{4/3} + C = \frac{1}{4} (t^4-t^2)^{4/3} + C$

⑨ $\int \frac{(\sqrt{u}+3)^4}{\sqrt{u}}\, du$ $\quad v = \sqrt{u} + 3 \quad dv = \frac{1}{2\sqrt{u}}\, du$

$\int \frac{(v)^4}{\cancel{\sqrt{u}}} 2\cancel{\sqrt{u}}\, dv$ $\quad du = 2\sqrt{u}\, dv$

$2 \int (v)^4\, dv = 2 \frac{v^5}{5} = 2 \cdot \frac{(\sqrt{u}+3)^5}{5}$

⑩ $\int \left(1 + \frac{1}{u}\right)^{-3} \cdot \left(\frac{1}{u^2}\right) du$

$u = \left(1 + \frac{1}{x}\right)^{-3} \quad v' = \frac{1}{x^2}$

$u' = \frac{3x^2}{(x+1)^4} \quad v = \int \frac{1}{x^2}\, dx = -\frac{1}{x}$

$\left(1 + \frac{1}{x}\right)^{-3} \left(-\frac{1}{x}\right) - \int \frac{3x^2}{(x+1)^4} \left(-\frac{1}{x}\right) dx$

$\frac{-x^2}{(x+1)^3} - \int -\frac{3x}{(x+1)^4}$

$\int -\frac{3x}{(x+1)^4}\, dx = \frac{3x}{(x+1)^4} - \frac{1}{(x+1)^3}$

$-\frac{x^2}{(x+1)^3} - \left(\frac{3}{2(x+1)^2} - \frac{1}{(x+1)^3}\right)$

$-\frac{x^2}{(x+1)^3} - \frac{3}{2(x+1)^2} + \frac{1}{(x+1)^3} + C$