Large-scale Geospatial Analytics: Problems, Challenges, and Opportunities

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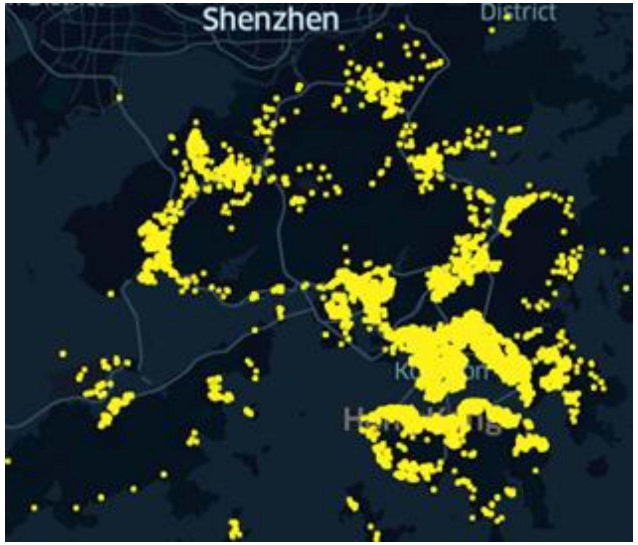
Tutorial Outline

1. Background of Geospatial Analytics
2. Overview of Different Geospatial Analysis Tools
3. Kernel Density Visualization (KDV)
4. K-function
5. Future Opportunities

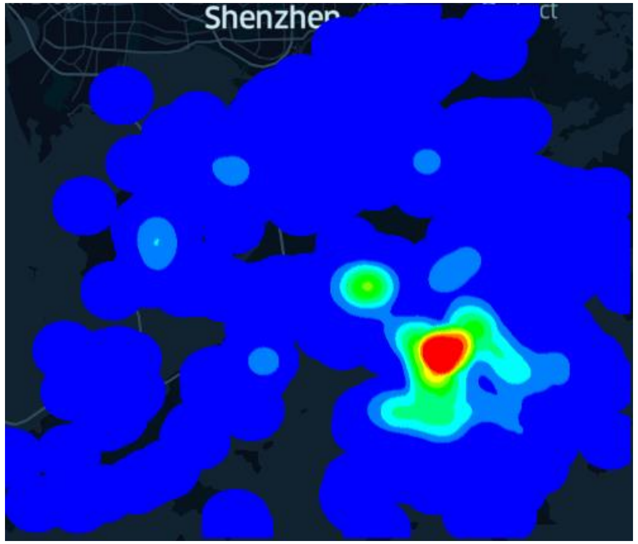
Background of Geospatial Analytics

Why Geospatial Analytics?

- Epidemiologists analyze disease outbreak in different regions.



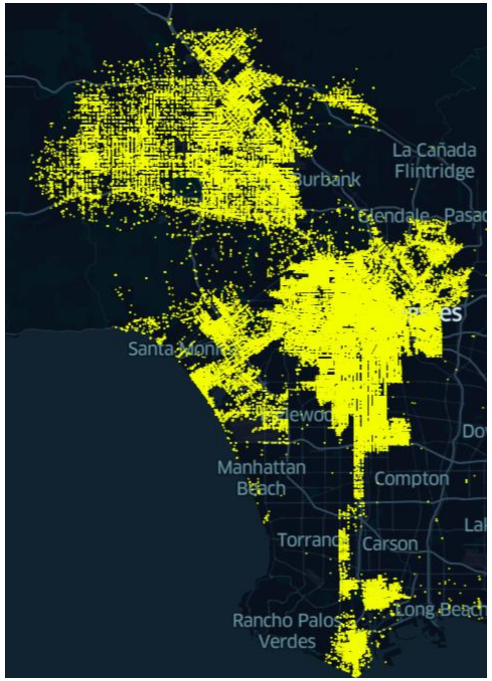
(a) Hong Kong COVID-19 cases



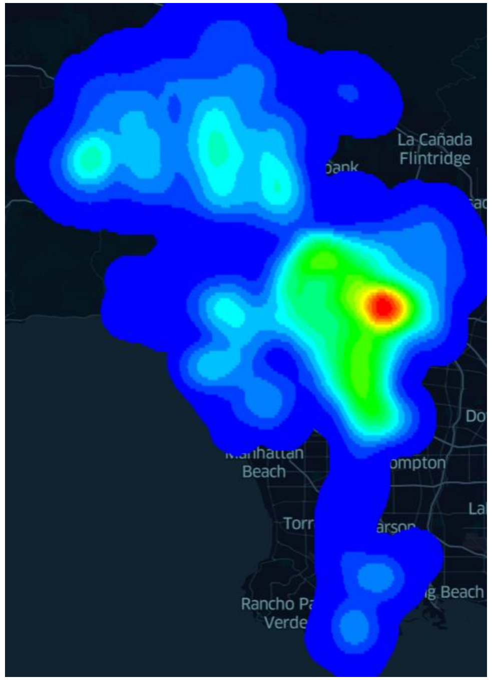
(b) Hotspot map

Why Geospatial Analytics?

- Criminologists/Transportation experts need to detect the crime/traffic accident hotspots in different regions.



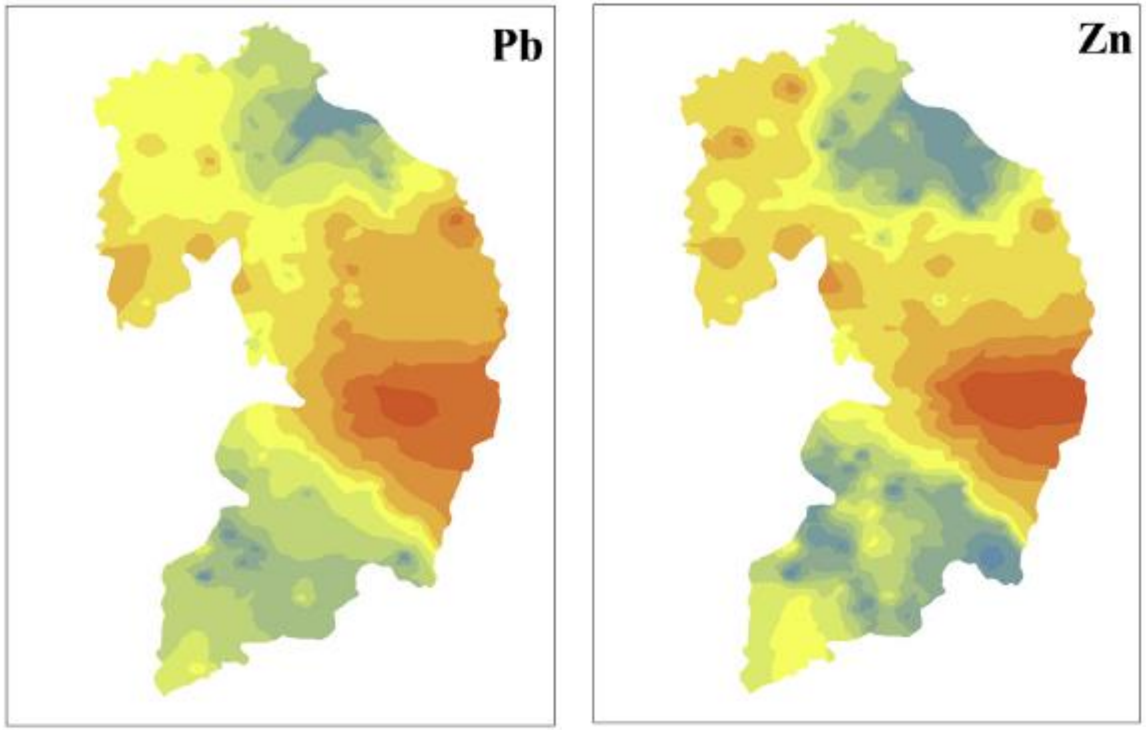
(a) Crime events in Los Angeles



(b) Crime hotspots

Why Geospatial Analytics?

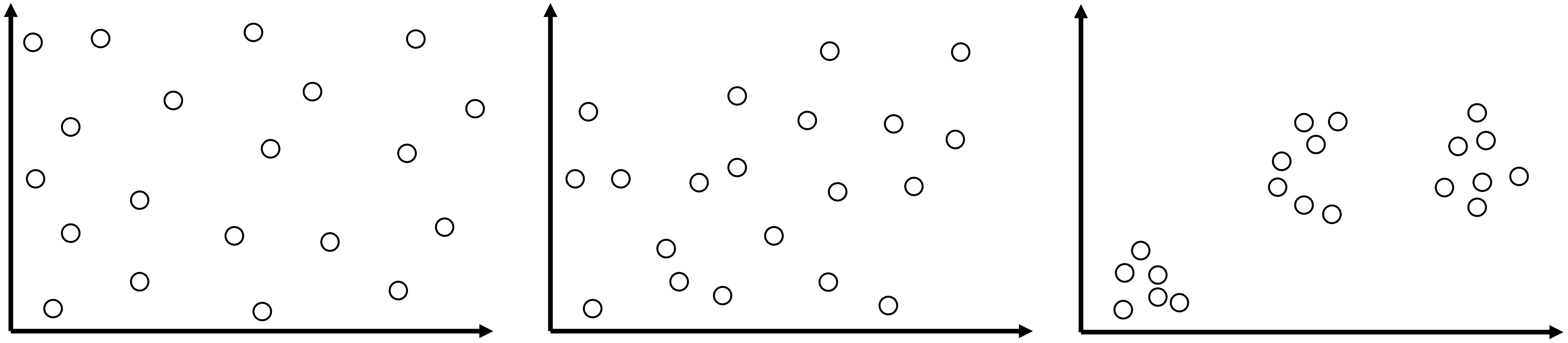
- Ecologists need to analyze the air pollution levels in different geographical regions.



[JEM18] Q. Ding, Y. Wang, D. Zhuang. Comparison of the common spatial interpolation methods used to analyze potentially toxic elements surrounding mining regions. Journal of Environmental Management 2018.

Why Geospatial Analytics?

- Geographical researchers need to analyze the cluster properties of a location dataset.



(a) Dispersed (b) Random (c) Clustered

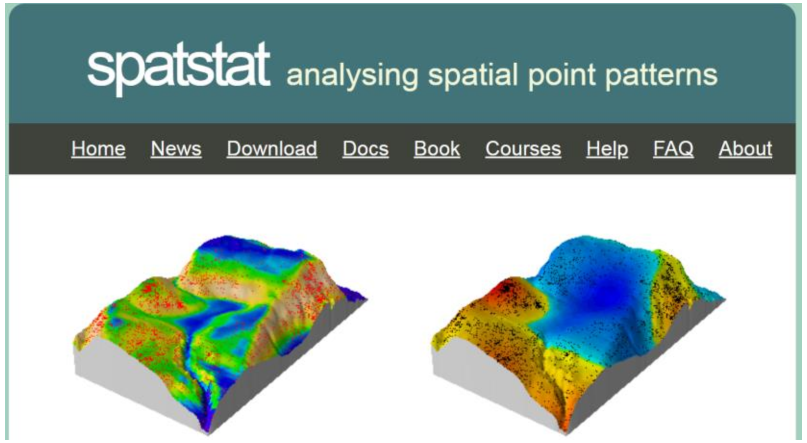
Representative Tools in Geospatial Analytics

Application type	Geospatial analytic tool
Hotspot detection	Kernel density visualization (KDV)
	Inverse distance weighting (IDW)
	Kriging
Correlation analysis	K-function
	Moran's I
	Getis-Ord General G

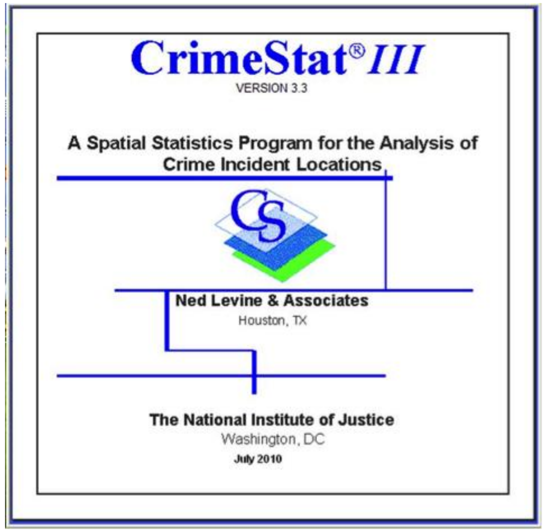
Software Packages for Supporting Geospatial Analysis Tools

ArcGIS

PySAL



spNetwork



CrimeStat® III
VERSION 3.3

A Spatial Statistics Program for the Analysis of Crime Incident Locations

Ned Levine & Associates
Houston, TX

The National Institute of Justice
Washington, DC
July 2010

Geospatial Analysis Tools are Slow!

- At least quadratic time complexity for these tools ☹
- Large-scale location datasets are available ☹
  - San Francisco 311-call location dataset contains more than 8 million data points.
  - New York taxi location dataset contains nearly 14 million data points.
- Lack of efficient algorithms for handling these tools ☹
- Lack of efficient software packages for handling these tools ☹

Geospatial Analysis Tools are Slow!

- Many complaints from domain experts ☹
  - Gramacki et al. [SIP17] “However, many (or even most) of the practical algorithms and solutions designed in the context of KDE are very time-consuming with quadratic computational complexity being a commonplace.”
  - Zhang et al. [IJGIS16] “Given what we have seen above, conducting this type of analysis using a sequential Ripley’s K function is extremely time-consuming, even to the level which prohibits this comprehensive analysis.”
  - Hohl et al. [SSE16] “The detection of space-time clusters can be computationally demanding, and this issue is exacerbated with spatiotemporal datasets of increasing size, diversity and availability (Grubesic et al., 2014; Robertson et al., 2010).”

[SIP17] A. Gramacki. Nonparametric Kernel Density Estimation and Its Computational Aspects. Springer International Publishing, 2017.
[IJGIS16] G. Zhang, Q. Huang, A. X. Zhu, J. H. Keel. Enabling point pattern analysis on spatial big data using cloud computing: optimizing and accelerating Ripley’s K function. International Journal of Geographical Information Science 2016.
[SSE16] A. Hohl, E. Delmelle, W. Tang, I. Casas. Accelerating the discovery of space-time patterns of infectious diseases using parallel computing. Spatial and Spatio-temporal Epidemiology 2016.

What Should Database Researchers Do?

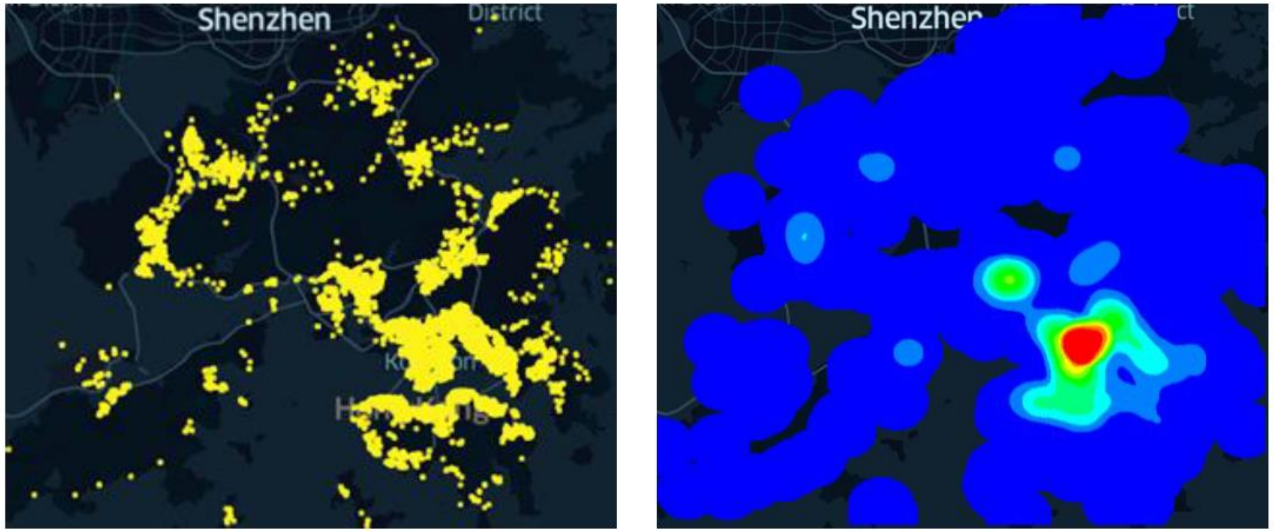
- Regard different tools as the spatial query processing problems.

Application type	Geospatial analytic tool
Hotspot detection	Kernel density visualization (KDV)
	Inverse distance weighting (IDW)
	Kriging
Correlation analysis	*K*-function
	Moran's I
	Getis-Ord General G

- Develop efficient algorithms (based on some techniques in database (e.g., indexing)) for these spatiotemporal query processing problems.

Overview of Different Geospatial Analysis Tools

Kernel Density Visualization (KDV)



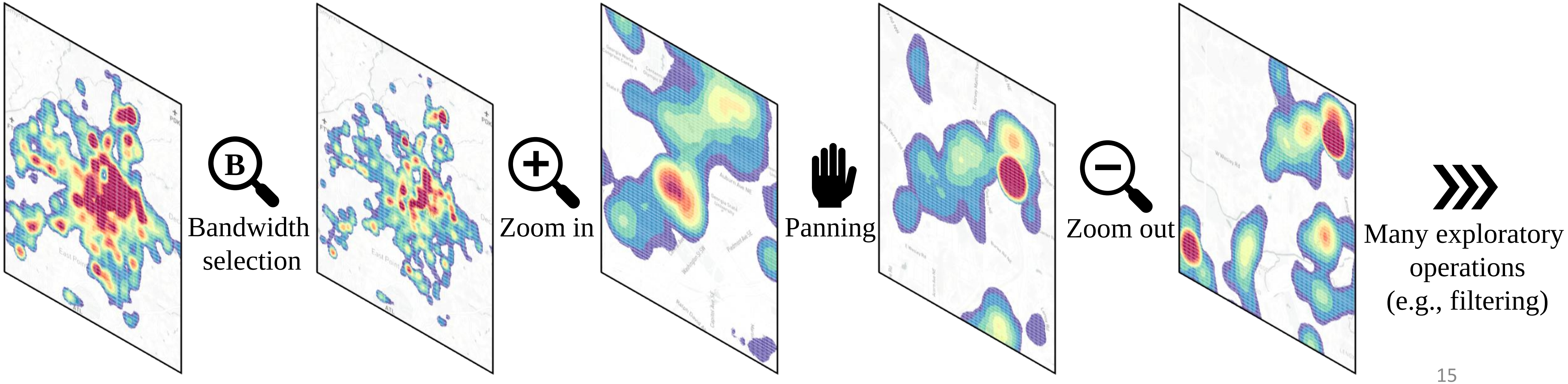
- Each $\mathbf{p}$ (yellow dot) represents the location of a COVID-19 case.
- Predict the risk of a given location $\mathbf{q}$ by computing the ***kernel density function*** $\mathcal{F}_P(\mathbf{q})$.

$$\underbrace{\mathcal{F}_P}_{\text{dataset}}(\overbrace{\mathbf{q}}^{\text{2D pixel}}) = \sum_{\mathbf{p}\in P} \overbrace{w}^{\text{weighting}} \cdot \begin{cases} 1 - \dfrac{1}{\underbrace{b^2}_{\text{bandwidth}}} \overbrace{dist(\mathbf{q},\mathbf{p})}^{\text{Euclidean distance}}{}^2 & \text{If } dist(\mathbf{q},\mathbf{p}) \le b \\ 0 & \text{Otherwise} \end{cases}$$

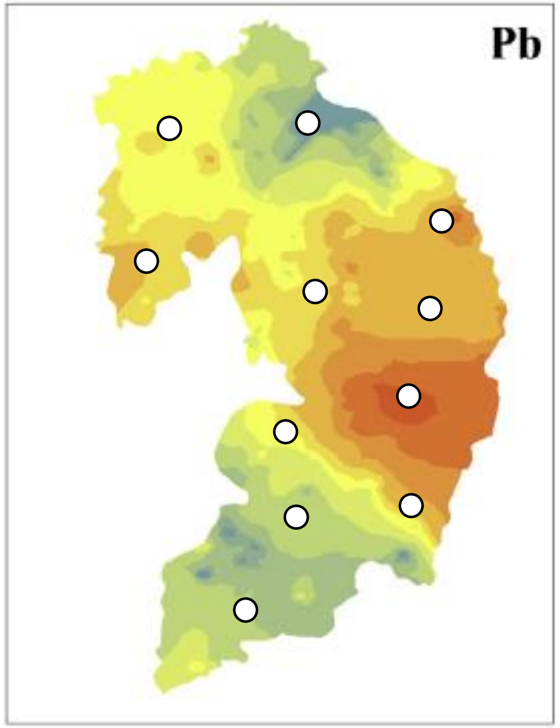
Challenges of KDV

- The time complexity is $O(XYn)$ ☹
  - $X \times Y$ denotes the number of pixels.
  - n denotes the number of location data points.

- Domain experts need to generate multiple KDVs ☹



Inverse Distance Weighting (IDW)



- Each white point $\mathbf{p}$ denotes the location of a sensor, which has the value $v_{\mathbf{p}}$ for measuring the level of Pb.
- Predict the level of Pb for the location $\mathbf{q}$ based on the inverse distance weighting function $I_P(\mathbf{q})$.

$$I_P(\mathbf{q}) = \begin{cases} \dfrac{\sum_{\mathbf{p}\in P}\left(\dfrac{v_{\mathbf{p}}}{dist(\mathbf{q},\mathbf{p})^{deg}}\right)}{\sum_{\mathbf{p}\in P}\left(\dfrac{1}{dist(\mathbf{q},\mathbf{p})^{deg}}\right)} & \text{If } dist(\mathbf{q},\mathbf{p}) \neq 0 \text{ for all } \mathbf{p} \\ v_{\mathbf{p}} & \text{Otherwise} \end{cases}$$

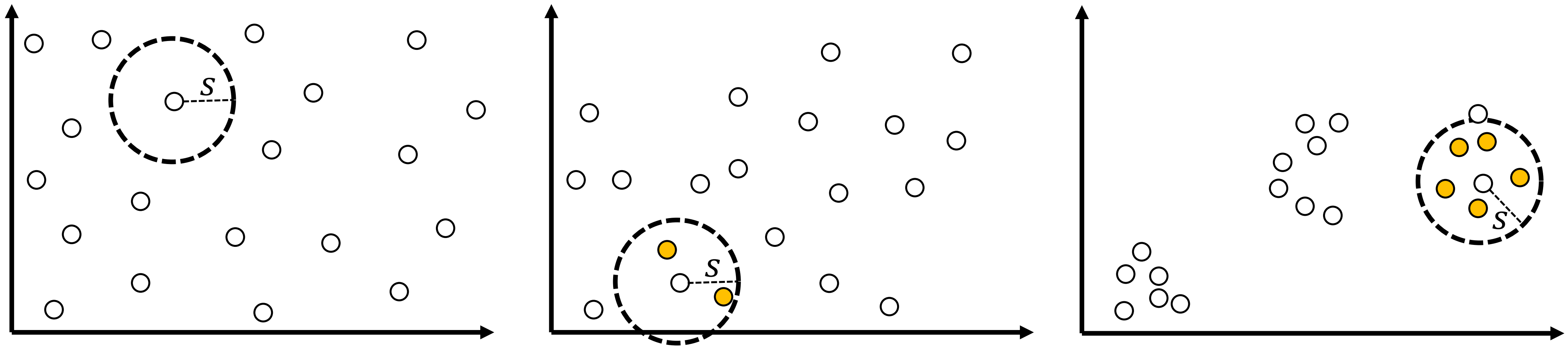
Challenges of IDW

- The time complexity is $O(XYn)$ ☹
  - $X \times Y$ denotes the number of pixels
  - n denotes the number of sensors.

- Need to achieve real-time performance (< 0.5 sec) ☹
  - Liang et al. [TGIS18] "With very large numbers of concurrent observation streams, novel algorithms are necessary that integrate streams into rasters, or other continuous representations, continuously in real time."

[TGIS18] Q. Liang, S. Nittel, J. C. Whittier, S. Bruin. Real-time inverse distance weighting interpolation for streaming sensor data. Transactions in GIS. 2018.

K-function



(a) Dispersed (b) Random (c) Clustered

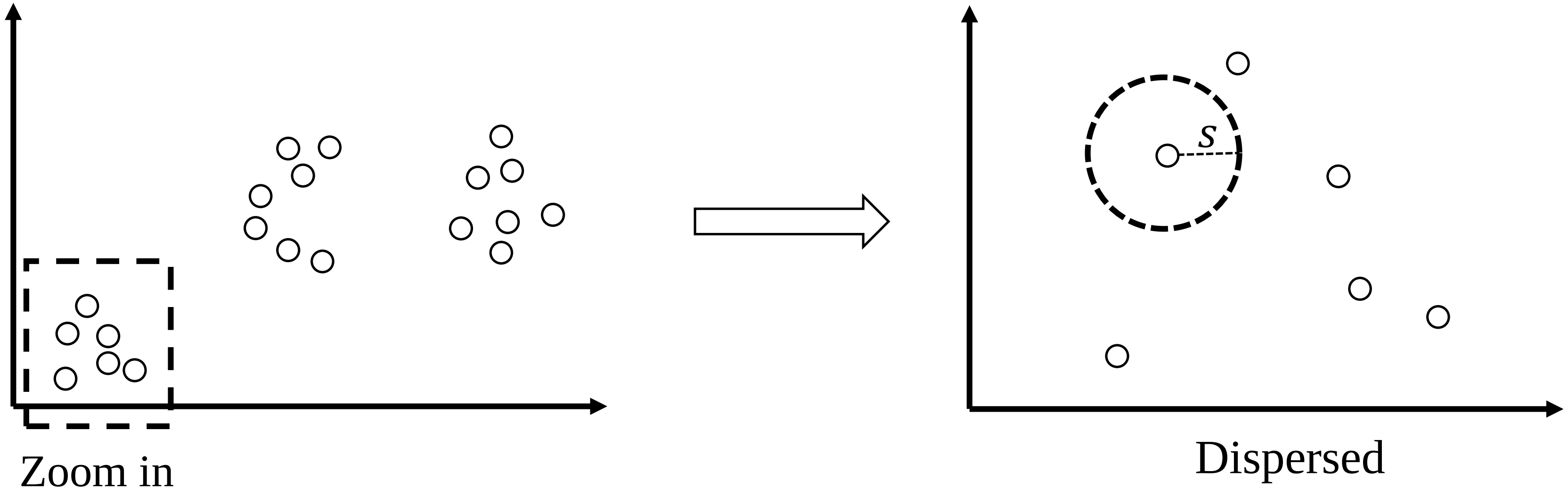
- Each **p** (white dot) represents the location of a geographical event (e.g., COVID-19 case or traffic accident).

- Domain experts need to know the cluster property of each dataset for a given spatial threshold s using the K-function.

$$K_P(s) = \sum_{\mathbf{p}_i \in P} \sum_{\substack{\mathbf{p}_j \in P \\ \mathbf{p}_j \neq \mathbf{p}_i}} \mathbb{1}(dist(\mathbf{p}_i, \mathbf{p}_j) \leq s) \quad \text{where } \mathbb{1}(dist(\mathbf{p}_i, \mathbf{p}_j) \leq s) = \begin{cases} 1 & \text{if } dist(\mathbf{p}_i, \mathbf{p}_j) \leq s \\ 0 & \text{otherwise} \end{cases}$$

K-function

- A location dataset may exhibit different cluster properties under different thresholds.



- Domain experts need to know the cluster properties under different spatial thresholds.

K-function Plot

- Provide a location dataset $P = \{\mathbf{p}_1, \mathbf{p}_2, \ldots, \mathbf{p}_n\}$ and D thresholds, which are $s_1, s_2, \ldots, s_D$.

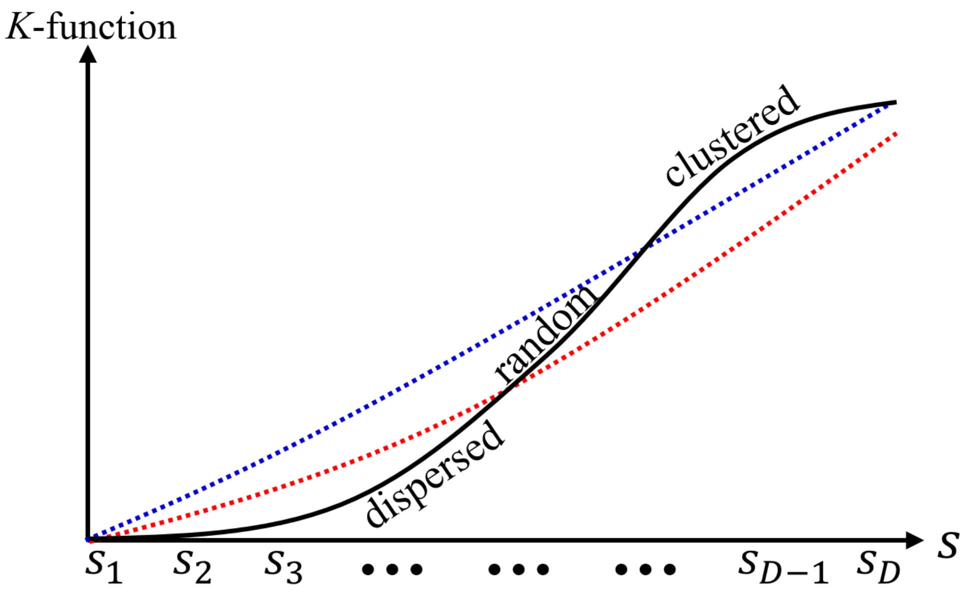
- Randomly generate L datasets, which are $R_1, R_2, \ldots, R_L$.

- For each threshold s_d $(1 \leq d \leq D)$, compute the following three terms.

  (1) $K_P(s_d)$

  (2) $\mathcal{L}(s_d) = \min\left(K_{R_1}(s_d), K_{R_2}(s_d), \ldots, K_{R_L}(s_d)\right)$

  (3) $U(s_d) = \max\left(K_{R_1}(s_d), K_{R_2}(s_d), \ldots, K_{R_L}(s_d)\right)$



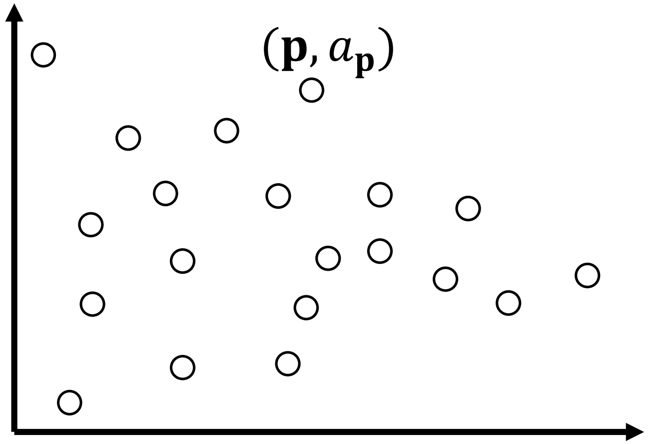
Challenges of K-function

- The time complexity of K-function is $O(n^2)$ ☹

- Need to compute multiple K-functions in order to generate a K-function plot, which takes $O(LDn^2)$ time ☹

Moran's I

- Each white data point $(\mathbf{p}, a_{\mathbf{p}})$ is represented by the location $\mathbf{p}$ (e.g., location of a traffic accident) and one attribute $a_{\mathbf{p}}$ (e.g., age and number of injuries).



- Analyze the autocorrelation between the attributes of these data points based on the Moran's I function.

$$M_P = \frac{\sum_{(\mathbf{p}_i, a_{\mathbf{p}_i}) \in P} \sum_{(\mathbf{p}_k, a_{\mathbf{p}_k}) \in P, k \neq i} \frac{(a_{\mathbf{p}_i} - \mu_a)(a_{\mathbf{p}_k} - \mu_a)}{dist(\mathbf{p}_i, \mathbf{p}_k)^{deg}}}{\left(\sum_{(\mathbf{p}_i, a_{\mathbf{p}_i}) \in P} \sum_{(\mathbf{p}_k, a_{\mathbf{p}_k}) \in P, k \neq i} \frac{1}{dist(\mathbf{p}_i, \mathbf{p}_k)^{deg}}\right) \sum_{(\mathbf{p}_i, a_{\mathbf{p}_i}) \in P} (a_{\mathbf{p}_i} - \mu_a)^2} \quad \text{where} \quad \mu_a = \frac{\sum_{(\mathbf{p}_i, a_{\mathbf{p}_i}) \in P} a_{\mathbf{p}_i}}{|P|}$$

Challenges of Moran's I

- The time complexity of Moran's I is $O(n^2)$ ☹

- Cannot be scalable to moderate-scale datasets ☹
  - Amgalan et al. [ICDM20] "Although statistics like Moran's I and Geary's C are widely used to measure spatial autocorrelation, they are slow: all popular methods run in $\Omega(n^2)$ time, rendering them unusable for large data sets, or long time-courses with moderate numbers of points."

[ICDM20] A. Amgalan, L. Mujica-Parodi, S. S. Skiena. Fast Spatial Autocorrelation. ICDM 2020.

Kernel Density Visualization (KDV)

State-of-the-art Solutions for Generating KDV

- Function approximation [**TKDE22**, **SIGMOD20**, **ICDE19**, SIGMOD17, SDM03]

- Data sampling [SOCG18, SODA18, SODA13, SIGMOD13]

- Computational sharing [**SIGMOD22**, **VLDB22a**, AISTATS03]

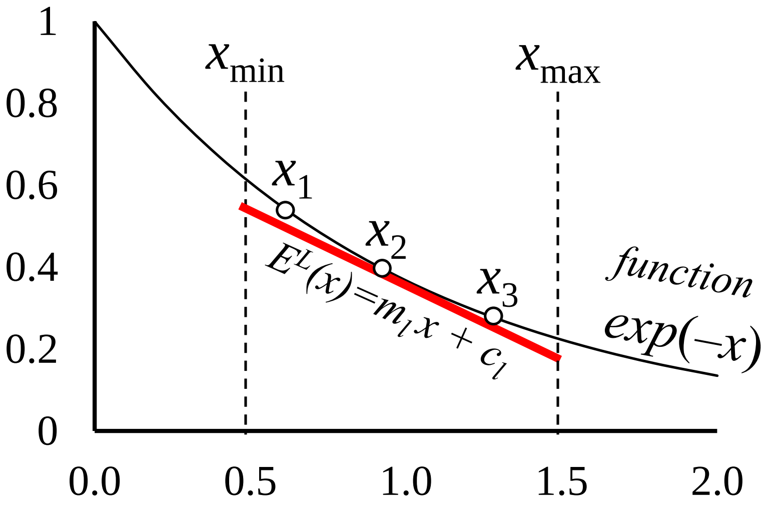
[TKDE22] T. N. Chan, L. H. U, R. Cheng, M. L. Yiu, Shivansh Mittal. Efficient Algorithms for Kernel Aggregation Queries. TKDE 2022.
[SIGMOD22] T. N. Chan, L. H. U, B. Choi, J. Xu. SLAM: Efficient Sweep Line Algorithms for Kernel Density Visualization. SIGMOD 2022.
[VLDB22a] T. N. Chan, P. L. Ip, L. H. U, B. Choi, J. Xu. SAFE: A Share-and-Aggregate Bandwidth Exploration Framework for Kernel Density Visualization. VLDB 2022.
[SIGMOD20] T. N. Chan, R. Cheng, M. L. Yiu. QUAD: Quadratic-Bound-based Kernel Density Visualization. SIGMOD 2020.
[ICDE19] T. N. Chan, M. L. Yiu, L. H. U. KARL: Fast Kernel Aggregation Queries. ICDE 2019.
[SOCG18] J. M. Phillips and W. M. Tai. Near-Optimal Coresets of Kernel Density Estimates. SOCG 2018.
[SODA18] J. M. Phillips and W. M. Tai. Improved Coresets for Kernel Density Estimates. SODA 2018.
[SIGMOD17] E. Gan and P. Bailis. Scalable Kernel Density Classification via Threshold-Based Pruning. SIGMOD 2017.
[SODA13] J. M. Phillips. ϵ-Samples for Kernels. In SODA 2013.
[SIGMOD13] Y. Zheng, J. Jestes, J. M. Phillips, F. Li. Quality and Efficiency for Kernel Density Estimates in Large Data. SIGMOD 2013.
[SDM03] A. G. Gray and A. W. Moore. Nonparametric Density Estimation: Toward Computational Tractability. SDM 2003.
[AISTATS03] A. G. Gray and A. W. Moore. Rapid Evaluation of Multiple Density Models. AISTATS 2003.

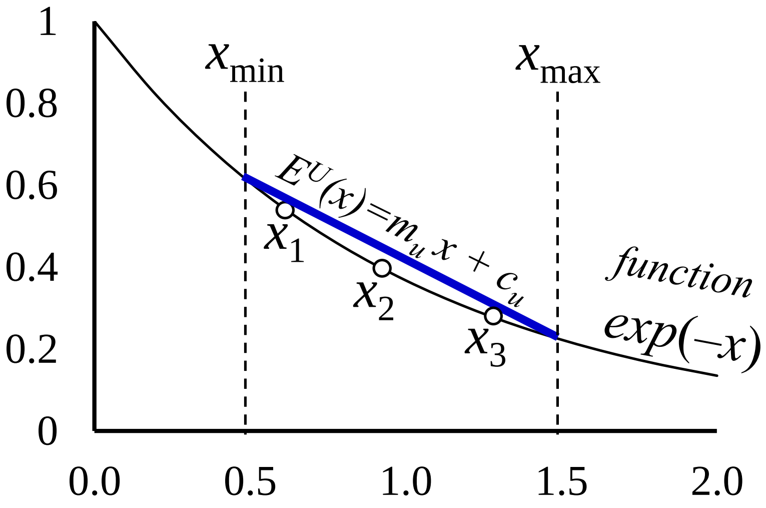
Function Approximation

- Consider the kernel density function (with the Gaussian kernel).

$$\mathcal{F}_P(\mathbf{q}) = \sum_{\mathbf{p}\in P} w \cdot \exp\left(\underbrace{-\frac{1}{b^2} dist(\mathbf{q},\mathbf{p})^2}_{x}\right)$$

- Use some simple functions (e.g., linear functions) to approximate the exponential function so that we can obtain the lower and upper bounds of $\mathcal{F}_P(\mathbf{q})$.





Function Approximation

$$\mathcal{F}_P(\mathbf{q}) = \sum_{\mathbf{p}\in P} w \cdot \exp\Bigg(\underbrace{-\frac{1}{b^2} dist(\mathbf{q},\mathbf{p})^2}_{x}\Bigg) \qquad O(n) \text{ time}$$

We have $LB_P(\mathbf{q}) \leq \mathcal{F}_P(\mathbf{q}) \leq UB_P(\mathbf{q})$.

Lower bound of $\mathcal{F}_P(\mathbf{q})$:

$$LB_P(\mathbf{q}) = \sum_{\mathbf{p_i}\in P} w\left(m_l \underbrace{\left(\frac{1}{b^2} dist(\mathbf{q},\mathbf{p})^2\right)}_{x} + c_l\right)$$

$$= wm\frac{1}{b^2}(|P|\underbrace{\|\mathbf{q}\|^2}_{\mathrm{O}(1)} - 2\underbrace{\mathbf{q}\cdot\mathbf{a_P}}_{\mathrm{O}(1)} + b_P) + wc|P| \qquad O(1) \text{ time}$$

We can further tighten these bound values using some index structures (e.g., kd-tree) until they fulfill the relative error guarantees.

Advantages and Disadvantages of Function Approximation

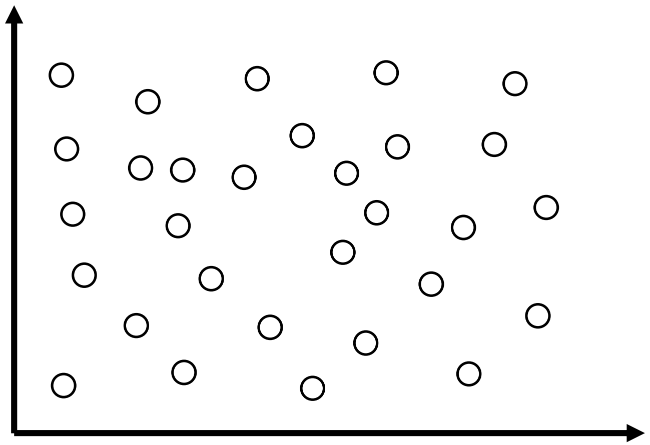
- Advantages ☺
  - Achieve better practical performance.
  - Can handle all kernel functions.
  - Can achieve approximation guarantees for generating KDV.

- Disadvantages ☹
  - Cannot reduce the worst-case time complexity for generating KDV.
  - Cannot achieve exact solution.
  - Can still be slow for generating KDV with some famous kernel functions (Epanechnikov and quartic kernels).

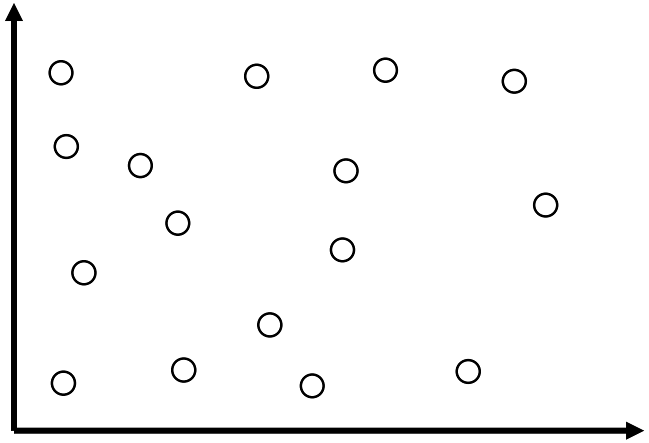
Data Sampling

- Consider the kernel density function (with the Gaussian kernel).

$$\mathcal{F}_P(\mathbf{q}) = \sum_{\mathbf{p} \in P} w \cdot \exp\left(-\frac{1}{b^2} dist(\mathbf{q}, \mathbf{p})^2\right)$$



Sampling



- Compute the modified kernel density function based on the sampled dataset S.

$$\mathcal{F}_S^{(M)}(\mathbf{q}) = \sum_{\mathbf{p}_i \in S} w_i \cdot \exp\left(-\frac{1}{b^2} dist(\mathbf{q}, \mathbf{p}_i)^2\right)$$

Advantages and Disadvantages of Data Sampling

- Advantages ☺
  - Can achieve probabilistic approximation guarantees for generating KDV.
  - Can reduce the worst-case time complexity for generating KDV.
  - Can handle all kernel functions.

- Disadvantages ☹
  - Cannot achieve exact solution.
  - Can still be slow for generating KDV.
  - Can degrade the practical visualization quality.

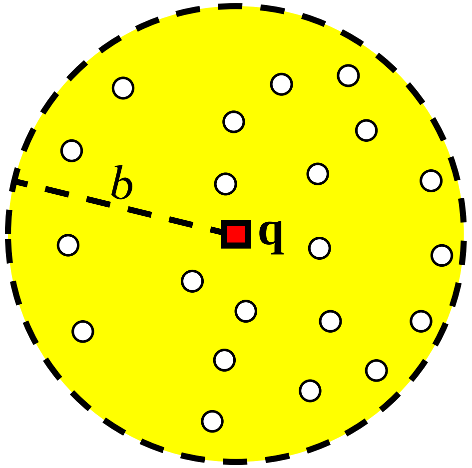
Computational Sharing

- Consider the kernel density function (with the Epanechnikov kernel).

$$\mathcal{F}_P(\mathbf{q}) = \sum_{\mathbf{p} \in P} w \cdot \begin{cases} 1 - \frac{1}{b^2} dist(\mathbf{q}, \mathbf{p})^2 & \text{If } dist(\mathbf{q}, \mathbf{p}) \leq b \\ 0 & \text{Otherwise} \end{cases}$$

- Only those white data points can contribute to $\mathcal{F}_P(\mathbf{q})$.

$$\mathcal{F}_P(\mathbf{q}) = \sum_{\mathbf{p} \in R(\mathbf{q})} w \cdot \left(1 - \frac{1}{b^2} dist(\mathbf{q}, \mathbf{p})^2\right)$$

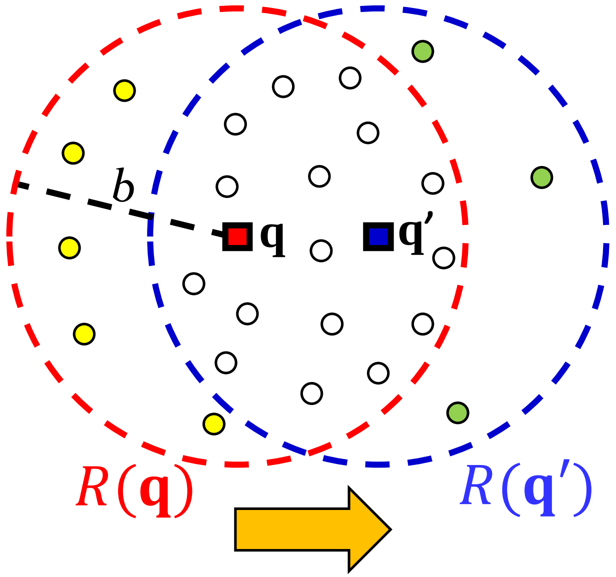


$R(\mathbf{q})$

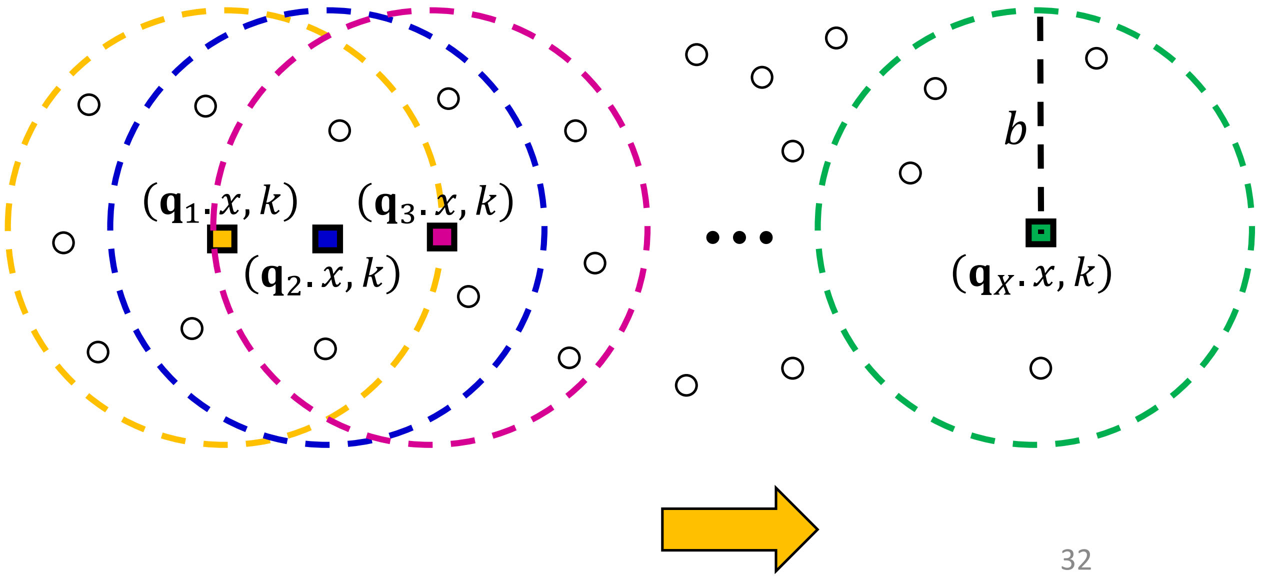
- Efficiently maintaining $R(\mathbf{q})$ for each pixel $\mathbf{q}$ can improve the efficiency.

Computational Sharing

- Two consecutive pixels can share many data points (white circles) in the range set.



- Consider a row of pixels. If we can efficiently share the computations of $R(\mathbf{q})$ between these pixels $\mathbf{q}$, we can improve the efficiency of generating KDV.



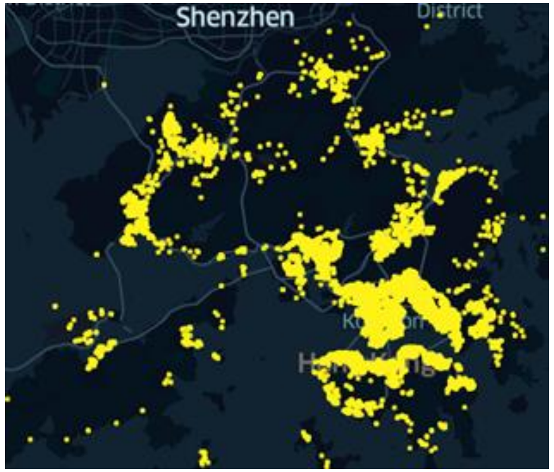
Advantages and Disadvantages of Computational Sharing

- Advantages ☺
  - Can achieve the exact solution
  - Can reduce the worst-case time complexity
  - Can achieve the best practical efficiency
  - Can combine with data sampling methods

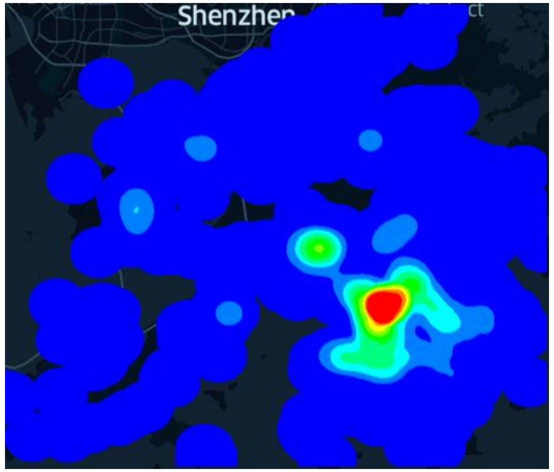
- Disadvantages ☹
  - Cannot support all kernel functions (e.g., cannot support Gaussian kernel).
  - Cannot achieve optimal worst-case time complexity.

Variant 1: Spatiotemporal Kernel Density Visualization (STKDV)

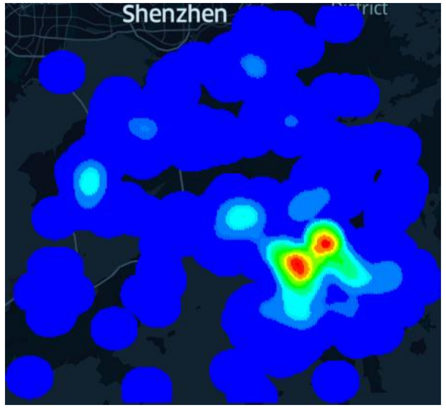
- KDV does not consider the occurrence time of each geographical event, which may provide misleading visualization results.



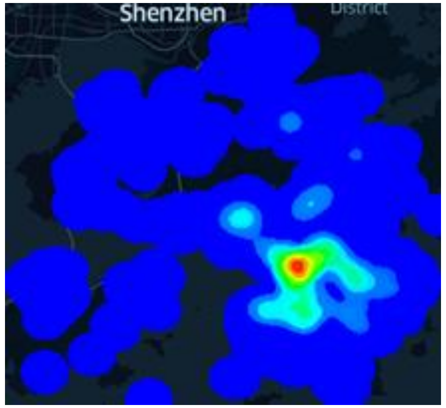
Hong Kong COVID-19 cases



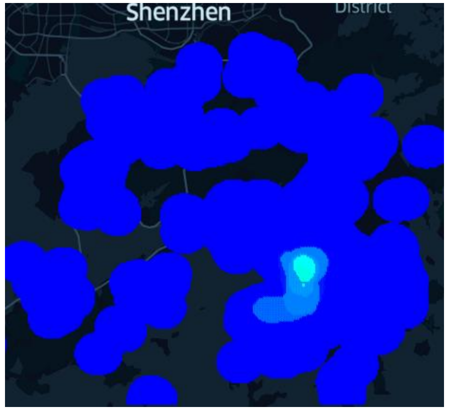
Hotspot map (based on KDV)



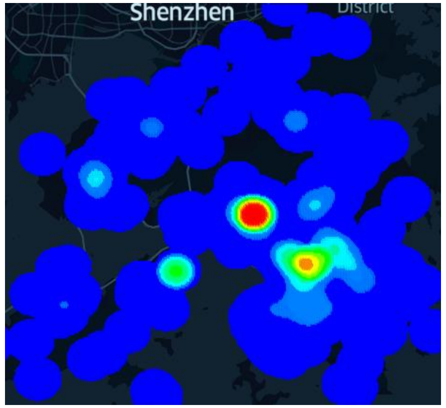
2nd August 2020



6th December 2020

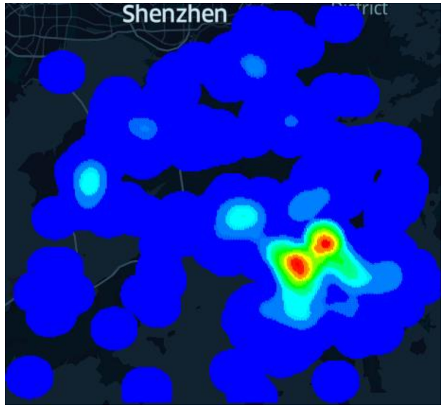


28th February 2021

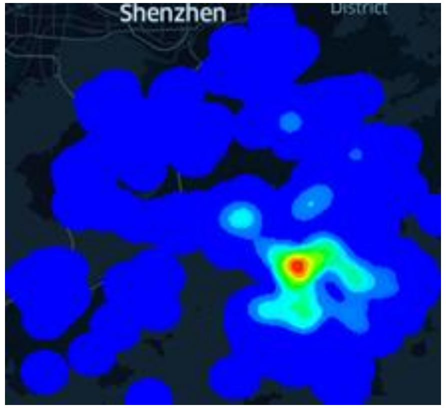


28th January 2022

Variant 1: Spatiotemporal Kernel Density Visualization (STKDV)



2^{nd} August 2020



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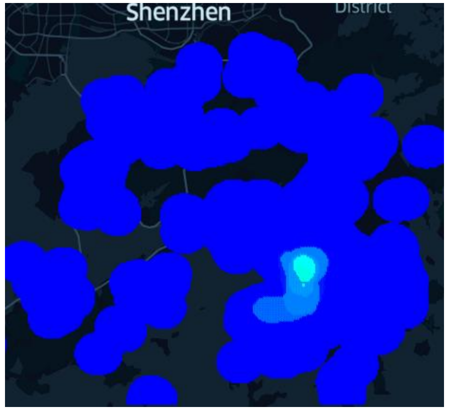

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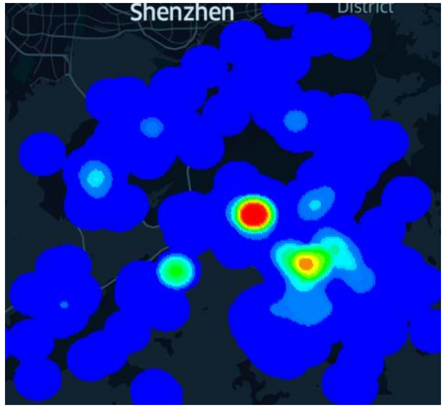


28^{th} January 2022

- Consider a location dataset $\hat{P} = \{(\mathbf{p}_1, t_{\mathbf{p}_1}), (\mathbf{p}_2, t_{\mathbf{p}_2}), \dots, (\mathbf{p}_n, t_{\mathbf{p}_n})\}$ with size n.

- Color each pixel $\mathbf{q}$ with the timestamp $t_{\mathbf{q}}$ based on the spatial-temporal kernel density function $\mathcal{F}_{\hat{P}}(\mathbf{q}, t_{\mathbf{q}})$.

$$\mathcal{F}_{\hat{P}}(\mathbf{q}, t_{\mathbf{q}}) = \sum_{(\mathbf{p}, t_{\mathbf{p}}) \in \hat{P}} w \cdot K_{\text{space}}(\mathbf{q}, \mathbf{p}) \cdot K_{\text{time}}(t_{\mathbf{q}}, t_{\mathbf{p}})$$

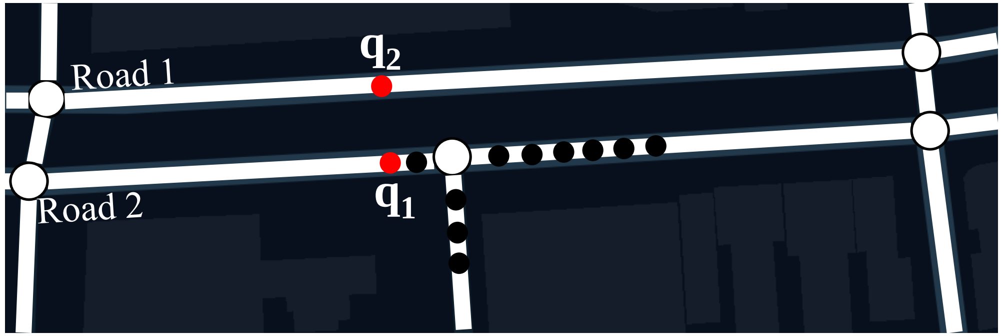
Variant 1: Spatiotemporal Kernel Density Visualization (STKDV)

- Time complexity of a naïve solution is $O(XYTn)$ (Very slow!) ☹
- The time complexity of the best solution, called SWS [**VLDB22b**], is $O(XY(T+n))$ ☺

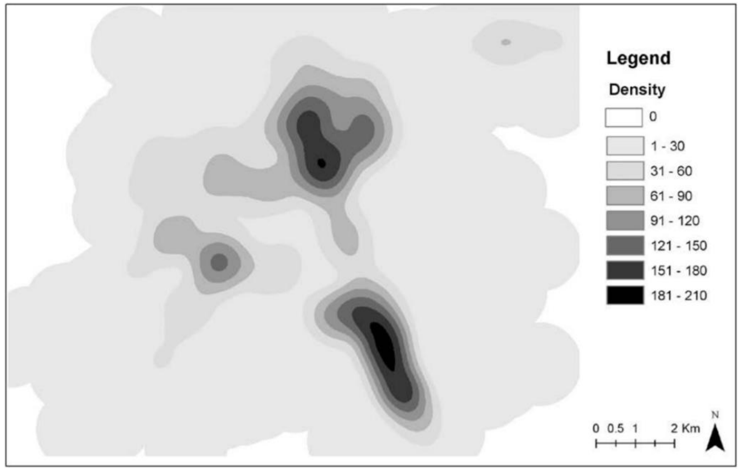
[VLDB22b] T. N. Chan, P. L. Ip, L. H. U, B. Choi, J. Xu. SWS: A Complexity-Optimized Solution for Spatial-Temporal Kernel Density Visualization. VLDB 2022.

Variant 2: Network Kernel Density Visualization (NKDV)

- KDV ignores the road network
  1. Can overestimate the density value of some regions (e.g., $\mathbf{q}_2$)



  2. Cannot correctly identify which road segments are the hotspot.



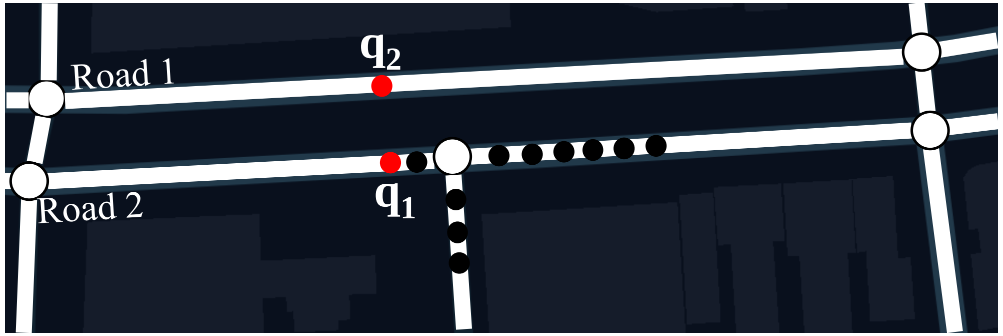
Kernel Density Visualization (KDV)

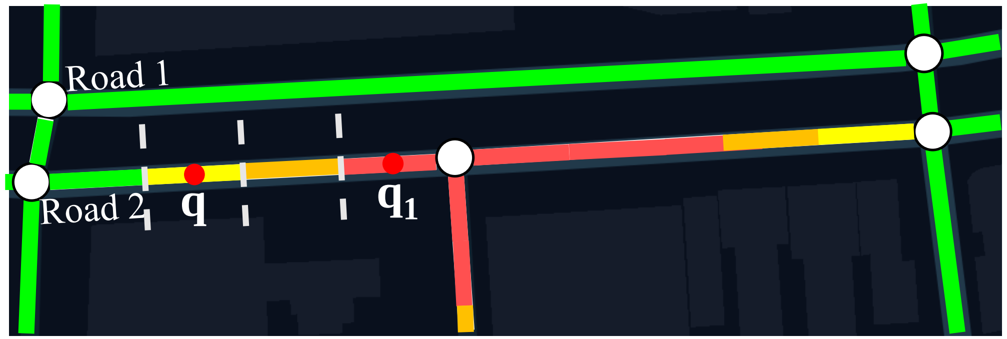
Network Kernel Density Visualization (NKDV)

Variant 2: Network Kernel Density Visualization (NKDV)

- Divide each road in the road network $G = (V, E)$ into a set of lixels.
- Color each lixel **q**, based on the network kernel density function.

$$\underbrace{\mathcal{F}_P}_{\text{dataset}}(\overbrace{\mathbf{q}}^{\text{lixel}}) = \sum_{\mathbf{p_i} \in P} \overbrace{w}^{\text{weighting}} \cdot \begin{cases} 1 - \frac{1}{\underbrace{b^2}_{\text{bandwidth}}} \overbrace{dist_G(\mathbf{q}, \mathbf{p_i})}^{\text{shortest path distance}}{}^2 & \text{if } dist_G(\mathbf{q}, \mathbf{p_i}) \le b \\ 0 & \text{otherwise} \end{cases}$$





Variant 2: Network Kernel Density Visualization (NKDV)

- Time complexity of a naïve solution is $O(L(T_{\text{SP}} + n))$ (Very slow!) ☹
  - L is the number of lixels.
  - T_{SP} is the time complexity of a shortest path algorithm.
  - n is the number of data points.

- Time complexity of the best solution, ADA [**VLDB21a**], is $O\left(|E|\left(T_{\text{SP}} + L\log\left(\frac{n}{|E|}\right)\right)\right)$ time (Why?).

$$O\left(\log\left(\frac{n}{|E|}\right)\right) \quad < \quad O\left(\frac{n}{|E|}\right)$$

$$O\left(|E|L\log\left(\frac{n}{|E|}\right)\right) \quad < \quad O(nL)$$

[VLDB21a] T. N. Chan, Z. Li, L. H. U, J. Xu, R. Cheng. Fast Augmentation Algorithms for Network Kernel Density Visualization. VLDB 2021.

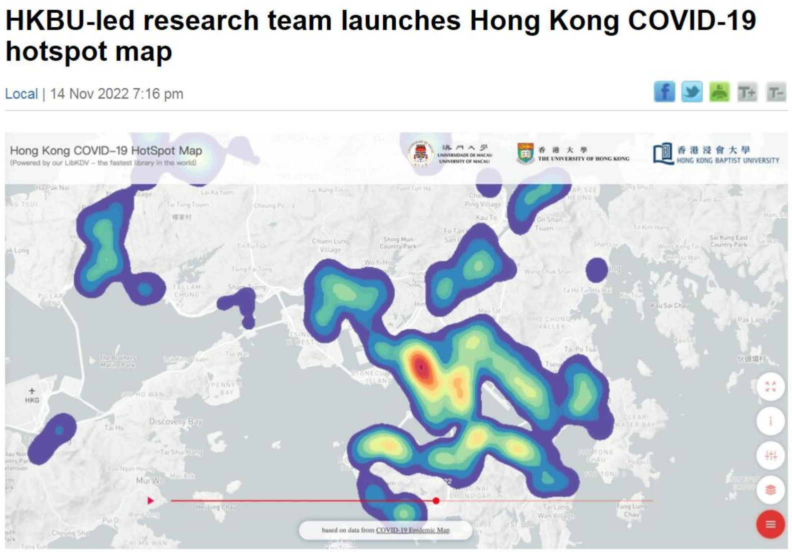
Software Development of KDV and its Variants

- KDV-Explorer (an online system for KDV) [**VLDB21b**]
- LIBKDV (a python library for KDV and STKDV) [**VLDB22c**]
- PyNKDV (a python library for NKDV) [**SIGMOD23**]

[VLDB21b] T. N. Chan, P. L. Ip, L. H. U, W. H. Tong, S. Mittal, Y. Li, R. Cheng. KDV-Explorer: A Near Real-Time Kernel Density Visualization System for Spatial Analysis. VLDB 2021.
[VLDB22c] T. N. Chan, P. L. Ip, K. Zhao, L. H. U, B. Choi, J. Xu. LIBKDV: A Versatile Kernel Density Visualization Library for Geospatial Analytics. VLDB 2022.
[SIGMOD23] T. N. Chan, R. Zang, P. L. Ip, L. H. U, J. Xu. PyNKDV: An Efficient Network Kernel Density Visualization Library for Geospatial Analytic Systems. SIGMOD 2023.

Software Development of KDV and its Variants

- Hong Kong COVID-19 hotspot map (based on LIBKDV and KDV-Explorer)
- Macau COVID-19 hotspot map (based on LIBKDV and KDV-Explorer)



K-function

State-of-the-art Solutions for Computing K-function

- Range-query-based methods [Springer08, UAI00, ACM75]
- Parallel/distributed and hardware-based methods [IJGIS16, IJGIS15]

[IJGIS16] G. Zhang, Q. Huang, A. X. Zhu, J. H. Keel. 2016. Enabling Point Pattern Analysis on Spatial Big Data using Cloud Computing: Optimizing and Accelerating Ripley's K function. International Journal of Geographical Information Science 2016.
[IJGIS15] W. Tang, W. Feng, M. Jia. Massively Parallel Spatial Point Pattern Analysis: Ripley's K function Accelerated using Graphics Processing Units. International Journal of Geographical Information Science 2015.
[Springer08] M. Berg, O. Cheong, M. J. Kreveld, and M. H. Overmars. Computational Geometry: Algorithms and Applications, 3rd Edition. Springer 2008.
[UAI00] A. W. Moore. The Anchors Hierarchy: Using the Triangle Inequality to Survive High Dimensional Data. UAI 2000.
[ACM75] J. L. Bentley. Multidimensional Binary Search Trees Used for Associative Searching. Commun. ACM 1975.

Range-Query-based Methods

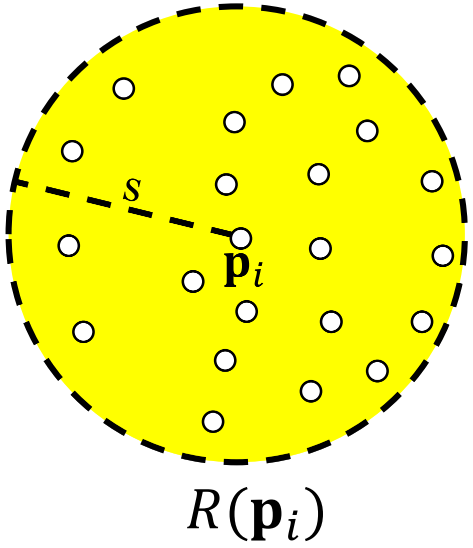
- Consider the K-function.

$$K_P(s) = \sum_{\mathbf{p}_i \in P} \sum_{\substack{\mathbf{p}_j \in P \\ \mathbf{p}_j \neq \mathbf{p}_i}} \mathbb{I}(dist(\mathbf{p}_i, \mathbf{p}_j) \leq s) \quad \text{where } \mathbb{I}(dist(\mathbf{p}_i, \mathbf{p}_j) \leq s) = \begin{cases} 1 & \text{if } dist(\mathbf{p}_i, \mathbf{p}_j) \leq s \\ 0 & \text{otherwise} \end{cases}$$

- Only those white data points that are within the spatial threshold s (i.e., $R(\mathbf{p}_i)$) can contribute to $K_P(s)$.

$$R(\mathbf{p}_i) = \{\mathbf{p}_j \in P: dist(\mathbf{p}_i, \mathbf{p}_j) \leq s, \mathbf{p}_j \neq \mathbf{p}_i\}$$

$$K_P(s) = \sum_{\mathbf{p}_i \in P} |R(\mathbf{p}_i)|$$



Range-Query-based Methods

- Many index structures can be adopted for improving the efficiency of finding $R(\mathbf{p}_i)$.
  - kd-tree [ACM75]
  - Ball-tree [UAI00]
  - Range-tree [Springer08]

[Springer08] M. Berg, O. Cheong, M. J. Kreveld, and M. H. Overmars. Computational Geometry: Algorithms and Applications, 3rd Edition. Springer 2008.
[UAI00] A. W. Moore. The Anchors Hierarchy: Using the Triangle Inequality to Survive High Dimensional Data. UAI 2000.
[ACM75] J. L. Bentley. Multidimensional Binary Search Trees Used for Associative Searching. Commun. ACM 1975.

Advantages and Disadvantages of Range-Query-based Methods

- Advantages ☺
  - Can practically improve the efficiency for computing K-function.
  - Many index structures are available for improving the efficiency of computing $R(\mathbf{p}_i)$.
  - Can achieve exact solution.

- Disadvantages ☹
  - Cannot reduce the worst-case time complexity for computing K-function (remains in $O(n^2)$ time).
  - Do not investigate the optimization opportunity for computing multiple K-functions (generating K-function plot).

Parallel/Distributed and Hardware-based Methods

- Aim to assign computations into different computers/GPUs/threads.
- Based on the naïve implementation of K-function.

Advantages and Disadvantages of Parallel/Distributed and Hardware-based Methods

- Advantages ☺
  - Significantly improve the efficiency of K-function, given many resources.
  - Simple (No new algorithm)
  - Can retain exact results.

- Disadvantages ☹
  - Domain experts may not have enough computational resources (32 CPUs and 96 GPUs are used in [IJGIS15]).
  - Can still not be scalable for large-scale datasets.
  - Cannot reduce the time complexity of this problem.

[IJGIS15] W. Tang, W. Feng, M. Jia. Massively Parallel Spatial Point Pattern Analysis: Ripley's K function Accelerated using Graphics Processing Units. International Journal of Geographical Information Science 2015.

Variant 1: Spatiotemporal K-function

- Many geographical events (e.g., COVID-19 cases) depend on both space and time.

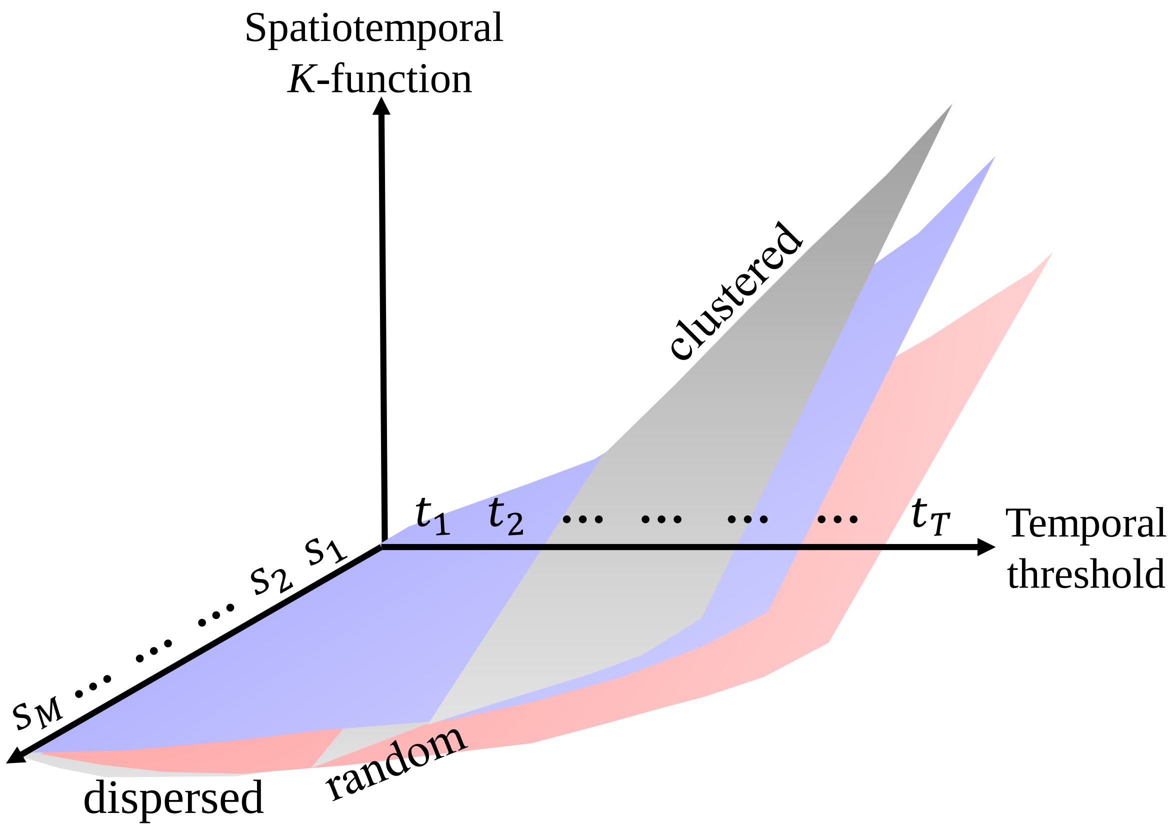
- Domain experts need to understand the spatiotemporal cluster properties of a location dataset.

- Given a location dataset $\hat{P} = \{(\mathbf{p}_1, t_{\mathbf{p}_1}), (\mathbf{p}_2, t_{\mathbf{p}_2}), \dots, (\mathbf{p}_n, t_{\mathbf{p}_n})\}$ with size n, the spatial threshold s, and the temporal threshold t, the spatiotemporal K-function is:

$$K_{\hat{P}}(s,t) = \sum_{(\mathbf{p}_i, t_{\mathbf{p}_i}) \in \hat{P}} \sum_{\substack{(\mathbf{p}_j, t_{\mathbf{p}_j}) \in \hat{P} \\ j \neq i}} \mathbb{I}(dist(\mathbf{p}_i, \mathbf{p}_j) \leq s, dist(t_{\mathbf{p}_i}, t_{\mathbf{p}_j}) \leq t)$$

Variant 1: Spatiotemporal K-function

- Generate a spatiotemporal K-function plot.



Variant 1: Spatiotemporal K-function

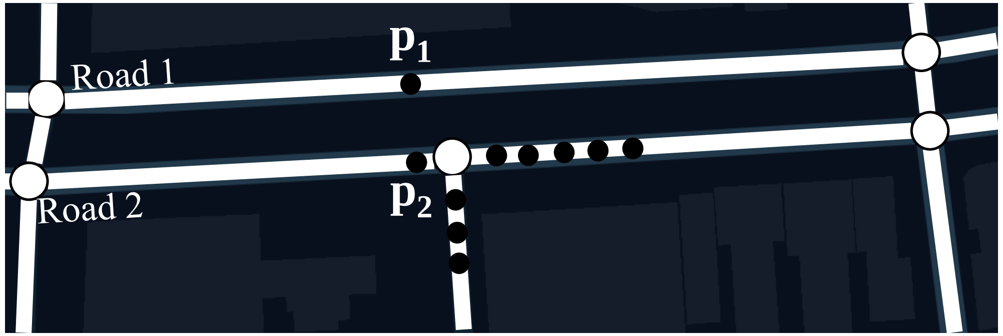
- The naïve solution for computing spatiotemporal K-function is $O(n^2)$ ☹

- The naïve solution for generating spatiotemporal K-function plot is $O(LMTn^2)$ ☹
  - L is the number of random datasets.
  - M is the number of spatial thresholds.
  - T is the number of temporal thresholds.

- There is no complexity-reduced solution for supporting spatiotemporal K-function and generating spatiotemporal K-function plot ☹

Variant 2: Network K-function

- Many geographical events (e.g., traffic accidents) may be in/along with a road network.



- Two data points, which are close to each other in terms of Euclidean distance, may be far away from each other in a road network.

- Domain experts propose to adopt the network K-function.

$$K_P(s) = \sum_{\mathbf{p}_i \in P} \sum_{\substack{\mathbf{p}_j \in P \\ \mathbf{p}_j \neq \mathbf{p}_i}} \mathbb{I}(dist_G(\mathbf{p}_i, \mathbf{p}_j) \leq s) \text{ where } \mathbb{I}(dist_G(\mathbf{p}_i, \mathbf{p}_j) \leq s) = \begin{cases} 1 & \text{if } dist_G(\mathbf{p}_i, \mathbf{p}_j) \leq s \\ 0 & \text{otherwise} \end{cases}$$

Variant 2: Network K-function

- The naïve solution for computing network K-function is $O(n(T_{SP} + n))$ ☹

- The naïve solution for computing network K-function plot is $O(LDn(T_{SP} + n))$ ☹

- The best solution for computing network K-function is $O(|E|T_{SP} + n|E| + n\log n)$ [**VLDB22d**] ☺

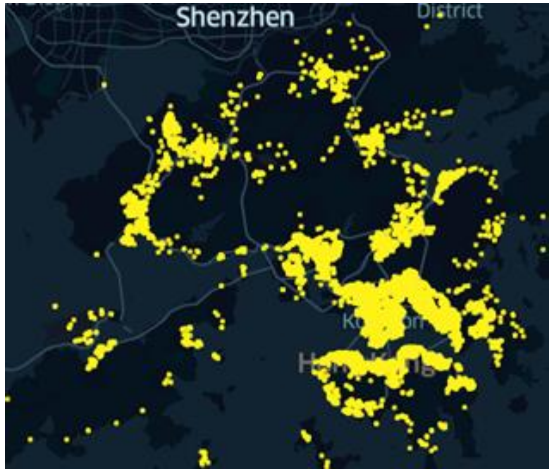
- The best solution for generating network K-function plot is $O(|E|T_{SP} + nLD|E| + Ln\log n)$ [**VLDB22d**] ☺

[VLDB22d] T. N. Chan, L. H. U, Y. Peng, B. Choi, J. Xu. Fast Network K-function-based Spatial Analysis. VLDB 2022.

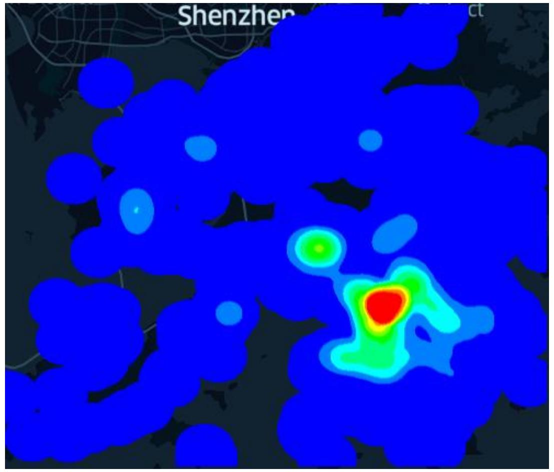
Future Opportunities

KDV and its Variants

- The time complexity of the state-of-the-art method for generating KDV is $O(Y(X+n))$.
- The current lower bound time complexity is $O(XY+n)$.
- Can be further achieve the optimal solution for generating KDV?
- This question applies to NKDV and STKDV.



Hong Kong COVID-19 cases



Hotspot map (based on KDV)

KDV and its Variants

- Complexity-reduced solutions for KDV [**SIGMOD22**], NKDV [**VLDB21a**], and STKDV [**VLDB22b**], can only support polynomial-based kernel functions, which cannot support all kernel functions (e.g., Gaussian kernel).

Kernel	$\mathcal{K}(\mathbf{q}, \mathbf{p})$
Uniform	$\begin{cases} \frac{1}{b} & \text{if } dist(\mathbf{q}, \mathbf{p}) \leq b \\ 0 & \text{otherwise} \end{cases}$
Epanechnikov	$\begin{cases} 1 - \frac{1}{b^2} dist(\mathbf{q}, \mathbf{p})^2 & \text{if } dist(\mathbf{q}, \mathbf{p}) \leq b \\ 0 & \text{otherwise} \end{cases}$
Quartic	$\begin{cases} \left(1 - \frac{1}{b^2} dist(\mathbf{q}, \mathbf{p})^2\right)^2 & \text{if } dist(\mathbf{q}, \mathbf{p}) \leq b \\ 0 & \text{otherwise} \end{cases}$
Gaussian	$\exp\left(-\frac{1}{b^2} dist(\mathbf{q}, \mathbf{p})^2\right)$

- Can we develop complexity-reduced algorithms for generating KDV with all kernel functions with non-trivial accuracy guarantees?

[SIGMOD22] T. N. Chan, L. H. U, B. Choi, J. Xu. SLAM: Efficient Sweep Line Algorithms for Kernel Density Visualization. SIGMOD 2022.
[VLDB22b] T. N. Chan, P. L. Ip, L. H. U, B. Choi, J. Xu. SWS: A Complexity-Optimized Solution for Spatial-Temporal Kernel Density Visualization. VLDB 2022.
[VLDB21a] T. N. Chan, Z. Li, L. H. U, J. Xu, R. Cheng. Fast Augmentation Algorithms for Network Kernel Density Visualization. VLDB 2021.

K-function and its Variants

- There is no advanced solution for improving the efficiency of computing K-function and spatiotemporal K-function.
  - Remain in $O(n^2)$ time.
  - Cannot be scalable to support the K-function plot and spatiotemporal K-function plot.

- Can we develop complexity-reduced algorithms for supporting these tools with exact guarantees?

- Can we further develop optimal solutions for all K-function-based tools?

K-function and its Variants

- No approximation solution has been proposed for these tools.

- Many approximation solutions have been proposed for supporting KDV and its variants.
    - Function approximation
    - Data sampling

- Can we extend these techniques for supporting all K-function-based tools with non-trivial accuracy guarantees?

Other Geospatial Analysis Tools

- No complexity-reduced solution has been developed for supporting other geospatial analysis tools.

- Many complexity-reduced solutions have been developed for generating KDV and its variants.
  - Computational sharing
  - Data sampling

- Can we extend these solutions for supporting other geospatial analysis tools?

Other Geospatial Analysis Tools

- No researcher has investigated the lower-bound time complexity of these geospatial analysis tools.

- Without this knowledge, it is hard to develop optimal solutions for supporting these geospatial analysis tools.

- Can we tighten the lower-bound time complexity for different geospatial analysis tools?

Other Geospatial Analysis Tools

- Many parallel/distributed/hardware-based solutions are based on naïve implementation (e.g., [IJGIS15]).
  - Can consume many computational resources ☹
  - Can still be not scalable to large-scale datasets ☹

- Can we combine parallel/distributed/hardware-based approaches with (new) complexity-reduced solutions?

[IJGIS15] W. Tang, W. Feng, M. Jia. Massively Parallel Spatial Point Pattern Analysis: Ripley's K function Accelerated using Graphics Processing Units. International Journal of Geographical Information Science 2015.

Software Development

- Existing software packages are based on naïve solutions for supporting geospatial analysis tools.

- Goal: Replace all these naïve solutions with efficient solutions.

- Target users:
  - GIS researchers with some basic programming skills: Can call some python and R libraries (e.g., spatstat, spNetwork, and PySAL) for using geospatial analysis tools.
  - Laymen: Only use some well-known GIS software packages with UI (e.g., QGIS, ArcGIS, QGIS Cloud, and ArcGIS Online).

Software Development

- Can we develop new python and R libraries, based on new solutions, for supporting all geospatial analysis tools?
- Can we develop new QGIS and ArcGIS plugins, based on new solutions, for supporting all geospatial analysis tools?
- Can we integrate new solutions into web-based (online) GIS systems (e.g., QGIS Cloud and ArcGIS Online)?