Capital markets and the pricing of risk

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What we have learned

- Derive cash flow from accounting information/statements.

What we will learn

- Discount rate, cost of capital, required returns, expected returns
  - Refer to the same concept

- In an equilibrium capital market, required returns on the investor’s investment is the cost of capital of the firm

- Lenders lend $1 today and get $1.05 tomorrow, the return is 5%.
- The cost of borrowing $1 today is 5% from the perspective of the borrower.

What we will learn

Assets	Liabilities
	Equity: R_{Equity}
Investments: R_{Assets}	
	Debt: R_{Debt}

- $R_{assets} = \frac{E}{E+D} R_{Equity} + \frac{D}{E+D} R_{debt} (1-\tau)$, where τ is the tax rate

Related "Big" Question

- What determines stock returns?
  - More broadly, do firm-specific factors determine stock returns?
    - E.g., only Apple's sales/profit/R&D/… determines Apple's stock returns?

A recap on return and risk for a single stock

- Expected return:

$$E(R) = \sum_i p_i R_i$$

  where
  - p_i is the probability of state i happening
  - R_i is return in state i

If p1=50%, R1=3%;
p2=50%, R2=-1%;
E(R)=1%

- Historical average return:

$$\bar{R} = \frac{1}{T}\sum_{t=1}^{T} R_t$$

  where
  - R_t is the return at time t

A recap on return and risk for a single stock

If p1=50%, R1=3%; p2=50%, R2=-1%; E(R)=1%

VAR(R)=$\frac{1}{2}(2\%)^2 + \frac{1}{2}(-2\%)^2 = 0.04\%$

- Variance and standard deviation

$$VAR(R_i) = \sigma_i^2 = E[R_i - E(R_i)]^2 = \sum_i p_i[R_i - E(R_i)]^2$$

$$VAR(R_i) = \sigma_i^2 = \frac{1}{T-1}\sum_{t=1}^{T}(R_t - \bar{R})^2$$

$$SD(R_i) = \sigma_i = \sqrt{\sigma_i^2}$$

- In finance, the standard deviation of a return series is also referred to as its **volatility**.
  - The standard deviation is easier to interpret because it is in the same units as the returns themselves.

Portfolio Return

- The return of a portfolio depends on the weightings we have in each individual asset

$$r_p = \sum_{t=1}^{n} r_{it} w_{it}$$

A portfolio consists of : Stock A: 60 m; Stock B: 40m. The weights are 0.6 and 0.4, respectively

- where portfolio weights are the fraction of the total investment in the portfolio held in each individual investment in the portfolio
  – The portfolio weights must add up to 1.00 or 100%.

$$w_i = \frac{\text{Value of investment } i}{\text{Total value of portfolio}}$$

Portfolio Risk

- What about portfolio risk?
- Can we also do a weighted average?

The Principles of Diversification

Stock	σ	Stock	σ
AT&T	22.6	Exxon	13.7
GE	18.8	Reebok	35.4
Coca-Cola	19.7	McDonald's	20.8
Compaq	42.0	Microsoft	29.4
S&P500	**14.3**		

- The S&P is a wide ranging US market index so why is its risk lower than most of the companies listed ?
- If the index is made up of individual securities, then why is its risk lower ?
- Answer: Diversification

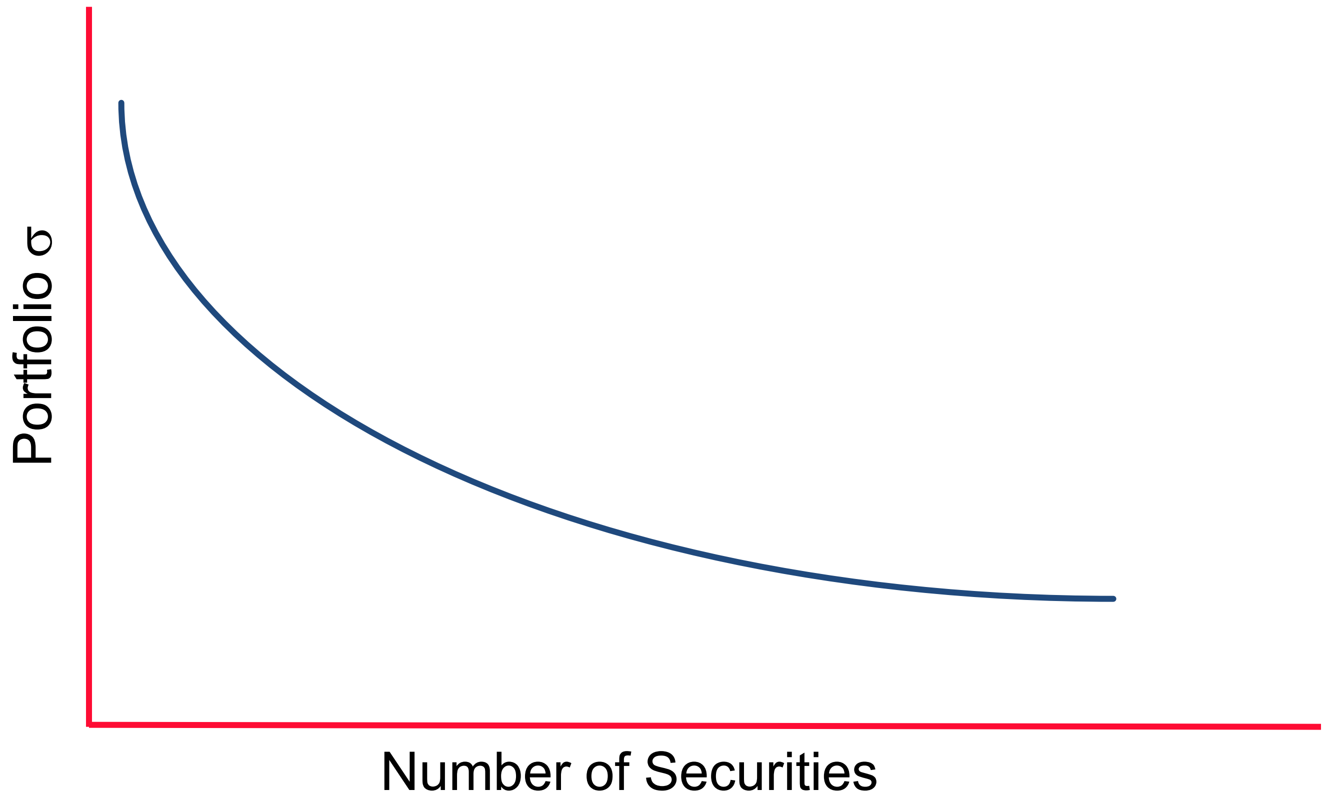
How Do They Do That ?

- Securities tend not to move perfectly together due to factors such as;
  - Industry specific issues
  - Firm specific issues
- Therefore, when firm in the portfolio is doing badly, another will be doing okay
- This means that the risk of the portfolio is reduced

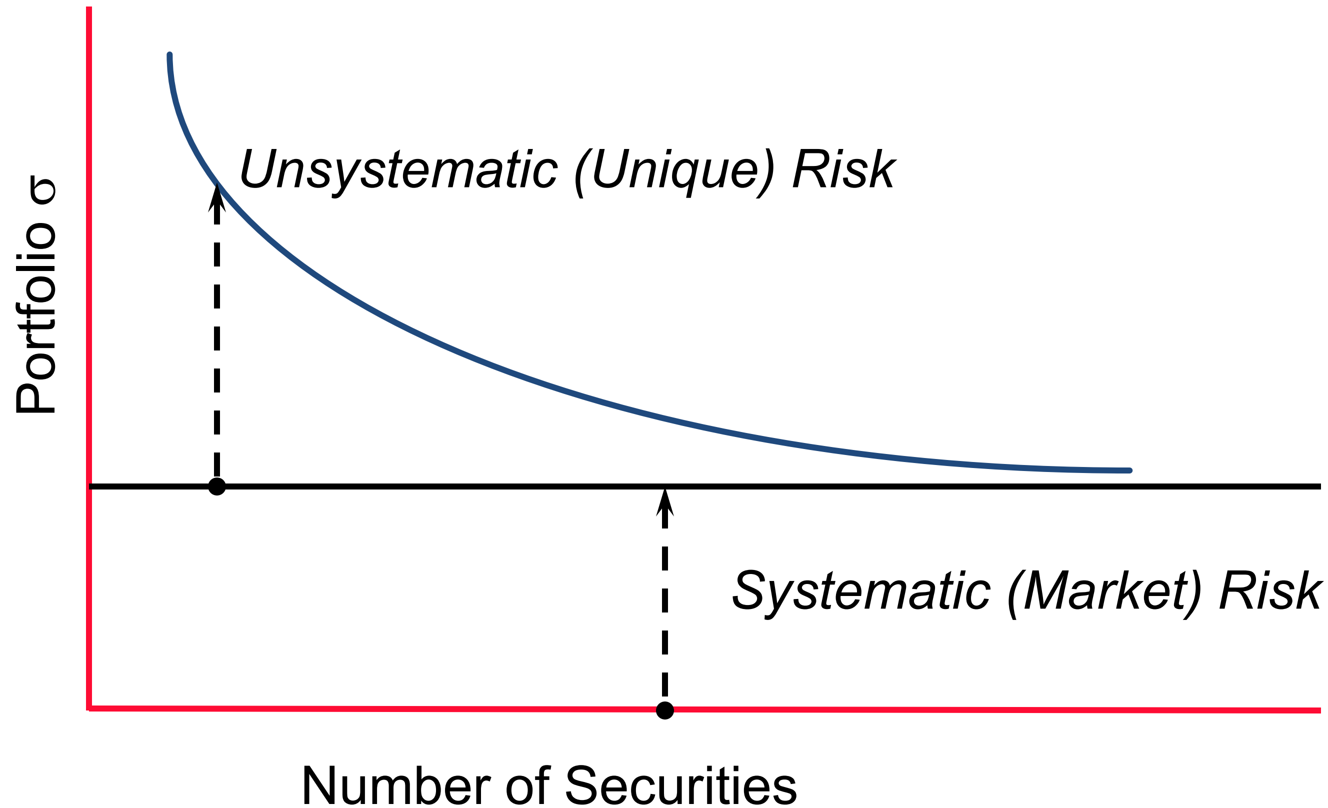
Evans & Archer

- A study by Evans & Archer (1968) examined the risk of various US portfolios
- The average standard deviation of the individual stocks was 21%
- For a two-stock portfolio the average risk was 16%, for three stocks 15%
- Most of the risk reduction benefits were found with the use of 8 stock portfolios

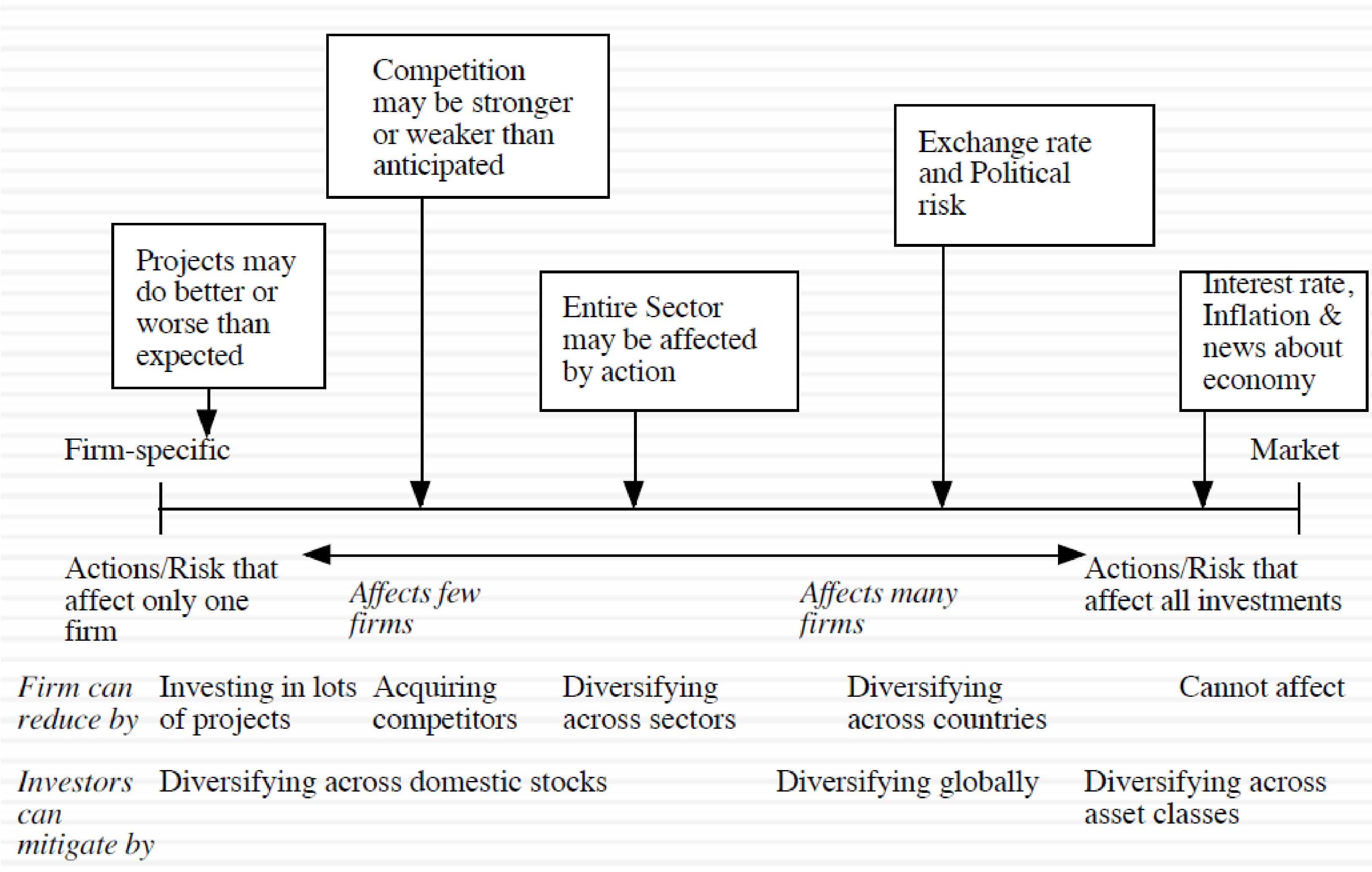
Risk Reduction



Unsystematic & Systematic Risk



Risk types and importance of diversification



Unsystematic & Systematic Risk

- Unsystematic (Unique) Risk: The risk that that can diversified away
  - Risk unique to the individual companies
- Systematic (Market) Risk: Unavoidable risk
  - Risk due to economy wide factors

- The risk premium of a security is determined by its systematic risk and does not depend on its independent risk.
  - Measure of common risk: Beta (β)

Portfolio Risk

- We can't just use the weights
- We must consider how stocks move relative to each other in order to capture the principles of diversification
  - The amount of risk that is eliminated in a portfolio depends on the degree to which the stocks face common risks and their prices move together
- We therefore use the covariance:

$$Cov_{ab} = \frac{1}{n-1}\sum_{t=1}^{n}(r_{at} - \bar{r_a})(r_{bt} - \bar{r_b})$$

Covariance

- As the covariance is the product of two different deviations, it can be positive or negative
- The covariance will be large when the good and bad outcomes for the stocks occur at the same time
- When good outcomes for one stock are accompanied by bad outcomes for the other, the covariance will be negative

Risk of a Two Stock Portfolio

- The **variance** of a two stock portfolio can be calculated as follows:

$$\sigma_p^2 = \sigma_a^2 w_a^2 + \sigma_b^2 w_b^2 + 2Cov_{ab} w_a w_b$$

Stock	a	b
a	$\sigma_a^2 w_a^2$	$Cov_{ab} w_a w_b$
b	$Cov_{ab} w_a w_b$	$\sigma_b^2 w_b^2$

Correlation Coefficient

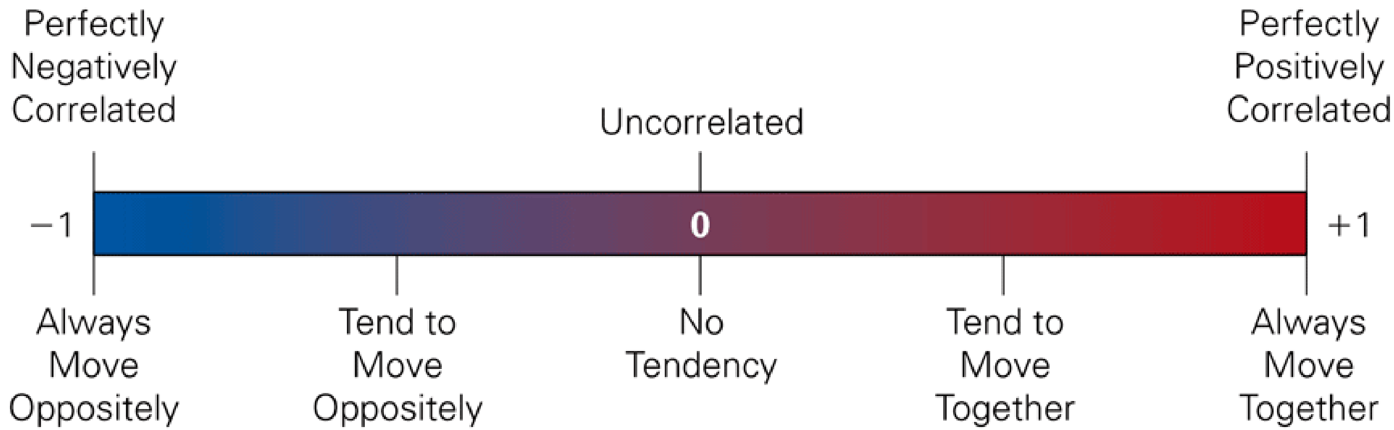
- We can re-write the formula in terms of the correlation coefficient rather than the covariance
- The correlation coefficient measures the extent to which security returns relate to one another

$$\rho_{ab} = \frac{cov_{ab}}{\sigma_a \sigma_b}, \text{ where } \; -1 \leq \rho \leq 1$$

  - Positive correlation means that security returns move together, i.e. if one goes up, so does the other
  - Negative correlation means that security returns move in the opposite direction
  - Zero correlation means that security returns are unrelated to one another

Correlation and Portfolio Risk

- In general, the less positive the correlation among securities in a portfolio, the more the risk-reducing benefit of diversification will be
- Conversely, a portfolio containing highly positive-correlated securities will do little to reduce risk



Portfolio Risk

$$\rho_{ab} = \frac{cov_{ab}}{\sigma_a \sigma_b}$$

$$cov_{ab} = \rho_{ab}\sigma_a\sigma_b$$

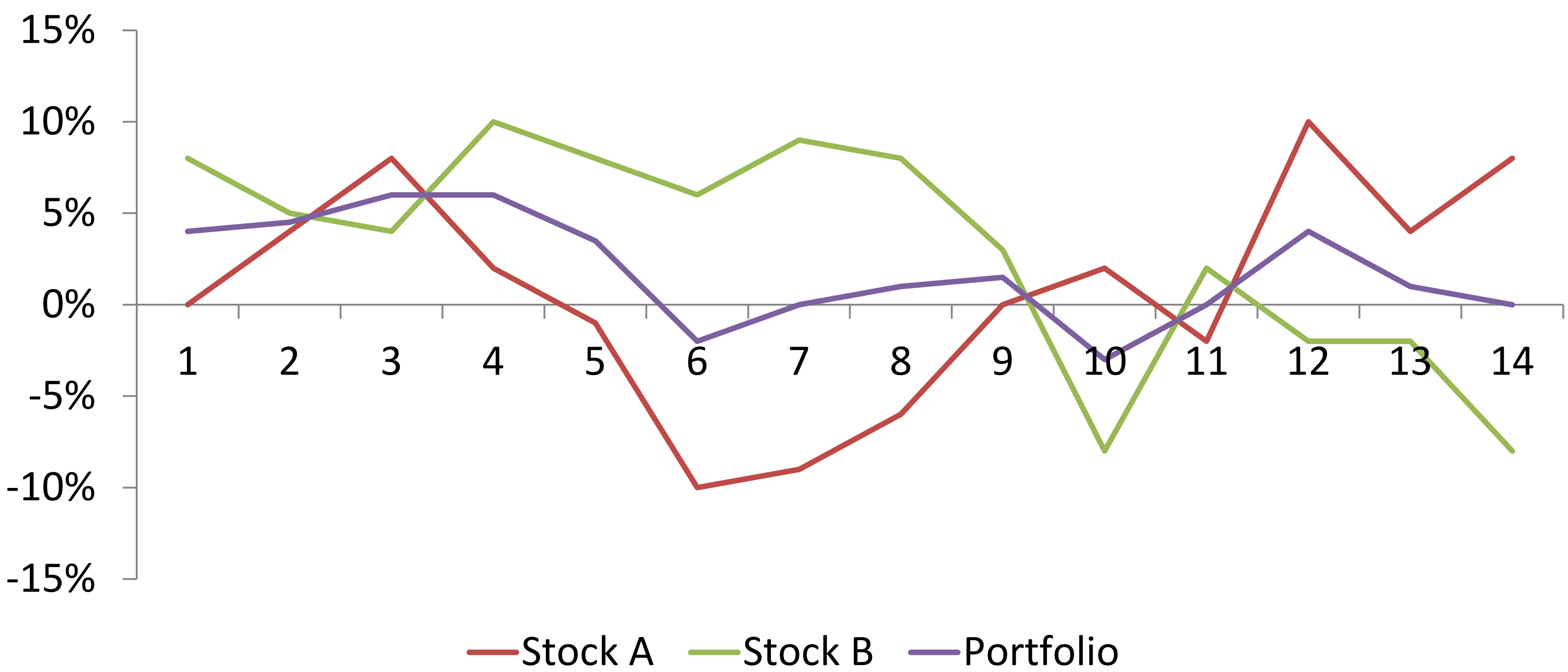
$$\sigma_p^2 = \sigma_a^2 w_a^2 + \sigma_b^2 w_b^2 + 2\rho_{ab}\sigma_a\sigma_b w_a w_b$$

- $\rho_{ab} =$ Correlation between stocks a & b
- Relationship depends on correlation coefficient
- The smaller the correlation, the greater the risk reduction potential
- If $\rho_{ab} = +1.0$, no risk reduction is possible

Two-Asset Portfolio Example

Period	Stock A	Stock B	Portfolio
1	0.0	8.0	4.0
2	4.0	5.0	4.5
3	8.0	4.0	6.0
4	2.0	10.0	6.0
5	-1.0	8.0	3.5
6	-10.0	6.0	-2.0
7	-9.0	9.0	0.0
8	-6.0	8.0	1.0
9	0.0	3.0	1.5
10	2.0	-8.0	-3.0
11	-2.0	2.0	0.0
12	10.0	-2.0	4.0
13	4.0	-2.0	1.0
14	8.0	-8.0	0.0
Average Return	**0.71**	**3.07**	**1.89**
Standard Deviation	**6.09**	**5.99**	**2.82**
Correlation	**-0.56**		

Two Stock Portfolio



Calculation of variance for a large portfolio

- With the above two properties, a 3-stock portfolio's variance can be written as follows:

$$\sigma_P^2 = w_1^2\sigma_1^2(R_1) + w_1w_2COV(R_1,R_2) + w_1w_3COV(R_1,R_3)$$

$$+w_2w_1COV(R_2,R_1) + w_2^2\sigma_2^2(R_2) + w_2w_3COV(R_2,R_3)$$

$$+w_3w_1COV(R_3,R_1) + w_3w_2COV(R_3,R_2) + w_3^2\sigma_3^2(R_3)$$

$$\sigma_P^2 = w_1^2\sigma_1^2(R_1) + w_2^2\sigma_2^2(R_2) + w_3^2\sigma_3^2(R_3) +$$
$$+2w_1w_2COV(R_1,R_2) + 2w_1w_3COV(R_1,R_3)$$
$$+ 2w_2w_3COV(R_2,R_3)$$

Calculation of variance for a large portfolio

Stock	1	2	3
1	$w_1^2\sigma_1^2$	$w_1 w_2 COV(R_1, R_2)$	$w_1 w_3 COV(R_1, R_3)$
2	$w_2 w_1 COV(R_2, R_1)$	$w_2^2\sigma_2^2(R_2)$	$w_2 w_3 COV(R_2, R_3)$
3	$w_3 w_1 COV(R_3, R_1)$	$w_3 w_2 COV(R_3, R_2)$	$w_3^2\sigma_3^2(R_3)$

Example – calculation of variance for a large portfolio

- You have a portfolio that is 50 percent invested in fund A, 25 percent invested in fund B, and 25 percent invested in fund C. Its covariance matrix is shown in the table:

	Covariance Matrix		
Fund	A	B	C
A	0.0400	0.0045	0.0189
B	0.0045	0.0081	0.0038
C	0.0189	0.0038	0.0441

- What is the volatility of the portfolio?

Example – calculation of variance for a large portfolio

$$\sigma_P^2 = 0.50^2 \times 0.04 + 0.25^2 \times 0.0081 + 0.25^2 \times 0.0441 +$$
$$+2 \times 0.50 \times 0.25 \times 0.0045 +$$
$$+2 \times 0.50 \times 0.25 \times 0.0189 +$$
$$+2 \times 0.25 \times 0.25 \times 0.0038 = 0.0196$$

- The volatility is $\sqrt{0.0196} = 14\%$. Suppose the covariance terms were smaller or even negative, portfolio would be smaller.

Extending Concepts to All Securities

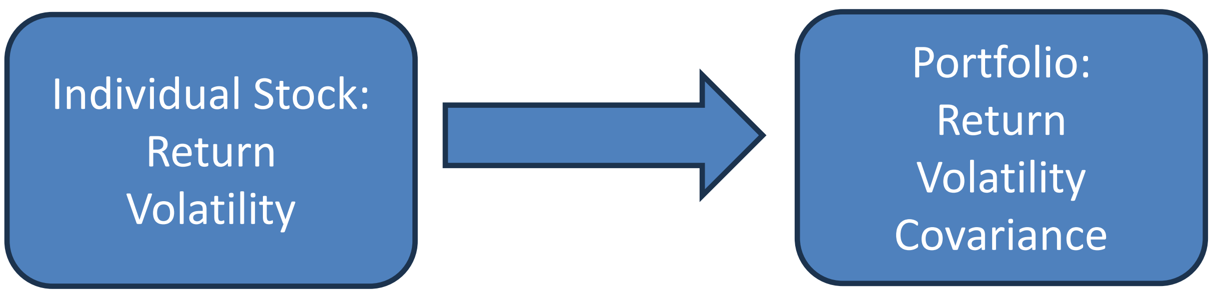
- This formula is used to calculate the portfolio variance for more than two stocks

$$\sigma_p^2 = \sum_{i=1}^{n} \sigma_i^2 w_i^2 + \sum_{i}^{n} \sum_{j \neq i}^{n} w_i w_j \sigma_i \sigma_j \rho_{ij}$$

$$\sigma_p^2 = cov_{p,p} = cov\left(\sum_i w_i r_i, r_p\right) = \sum_i w_i cov_{i,p}$$

$$\sigma_p^2 = \sum_{i=1}^{n} \sum_{j=1}^{n} w_i w_j cov_{i,j}$$

Choosing an Efficient Portfolio



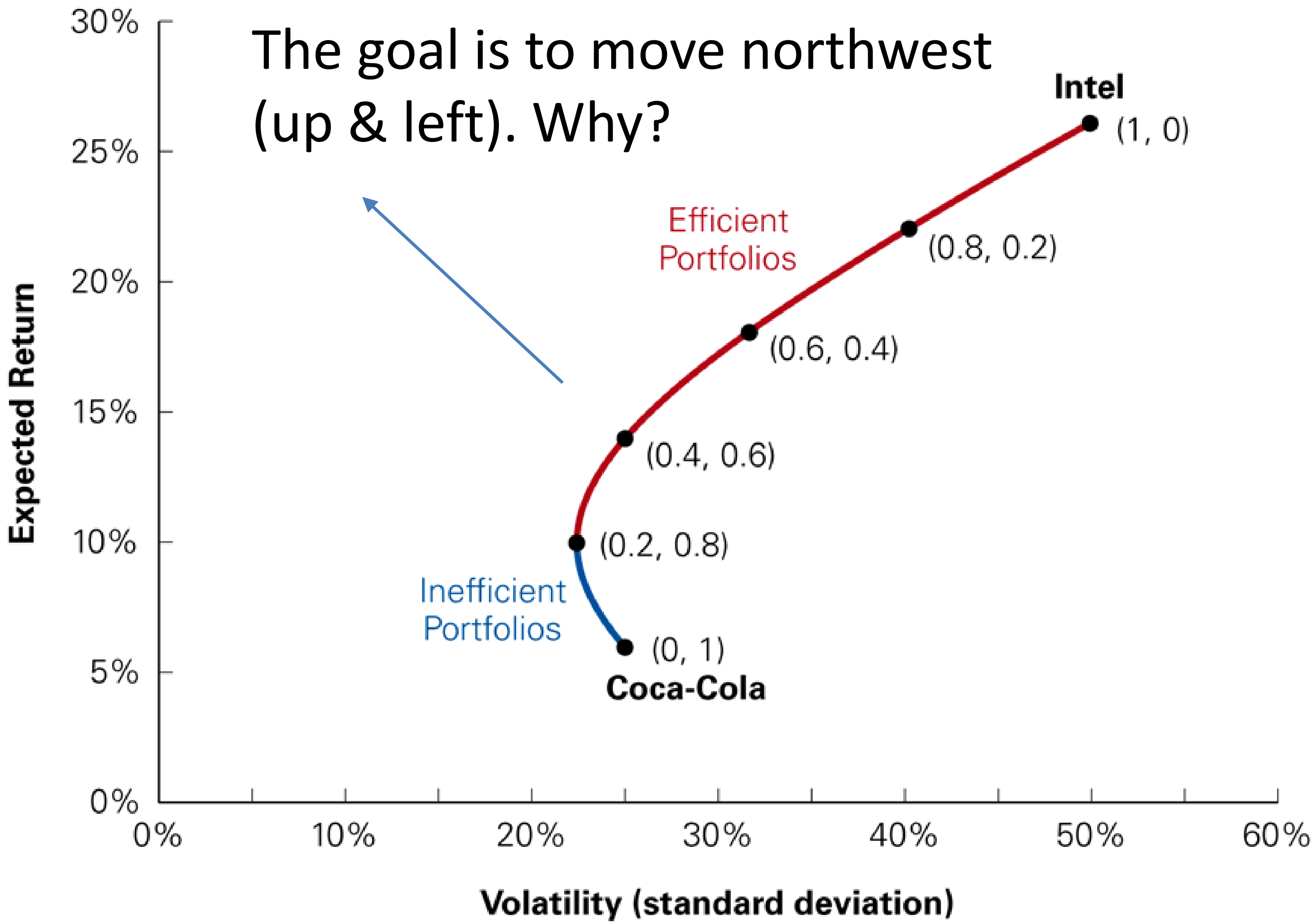
- Question: which is the “best” portfolio?

Choosing an Efficient Portfolio

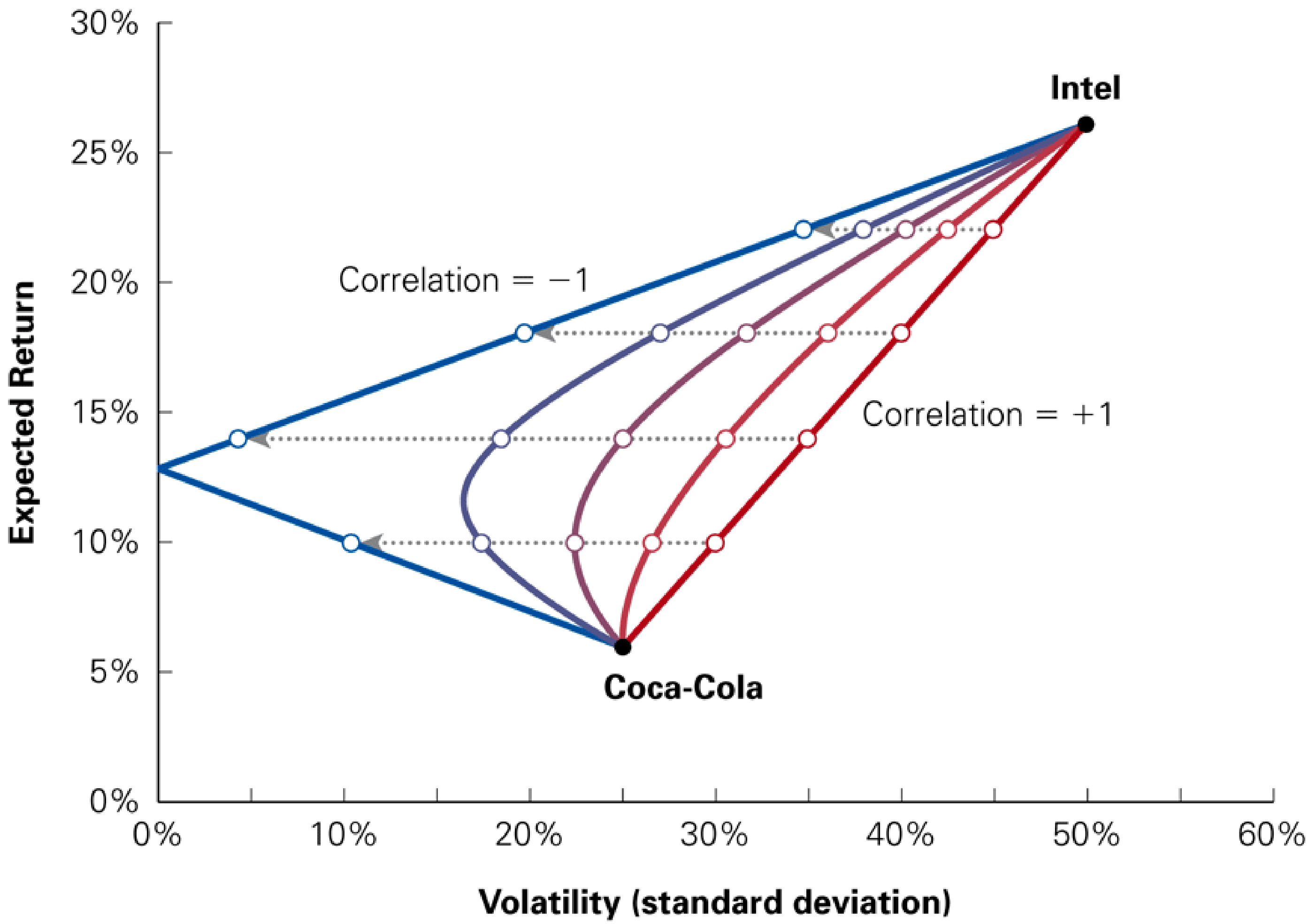
- Consider a portfolio of **uncorrelated** Intel and Coca-Cola

Portfolio Weights		Expected Return (%)	Volatility (%)
x_I	x_C	$E[R_P]$	$SD[R_P]$
1.00	0.00	26.0	50.0
0.80	0.20	22.0	40.3
0.60	0.40	18.0	31.6
0.40	0.60	14.0	25.0
0.20	0.80	10.0	22.3
0.00	1.00	6.0	25.0

Volatility vs. Expected Return for Portfolios of Intel and Coca-Cola Stock



Effect of correlation on risk and expected return of Intel and Coca-Cola stock

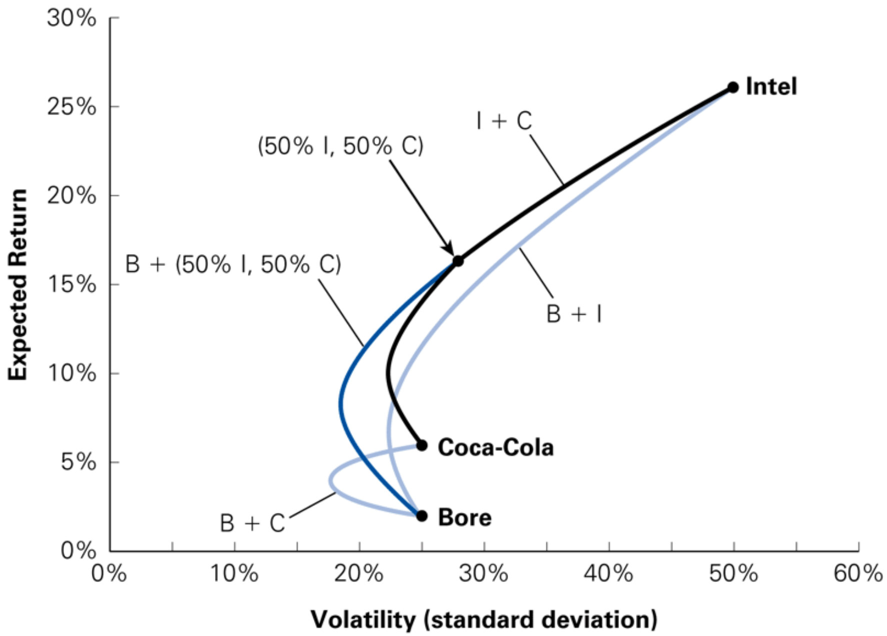


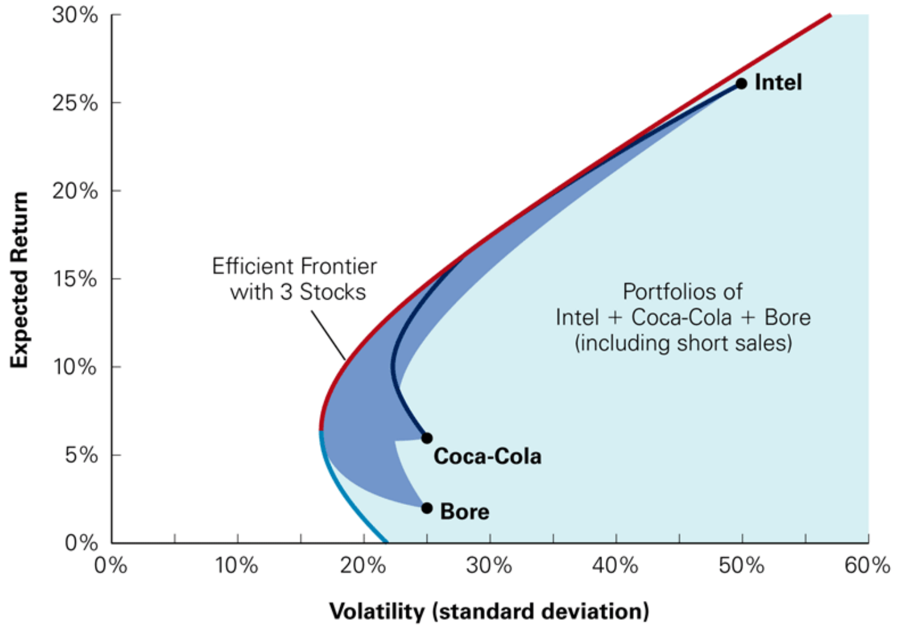
Efficient Portfolios with More Stocks

- Consider adding Bore Industries to the two stock portfolio:
  - Although Bore has a lower return and the same volatility as Coca-Cola, it still may be beneficial to add Bore to the portfolio for the diversification benefits.

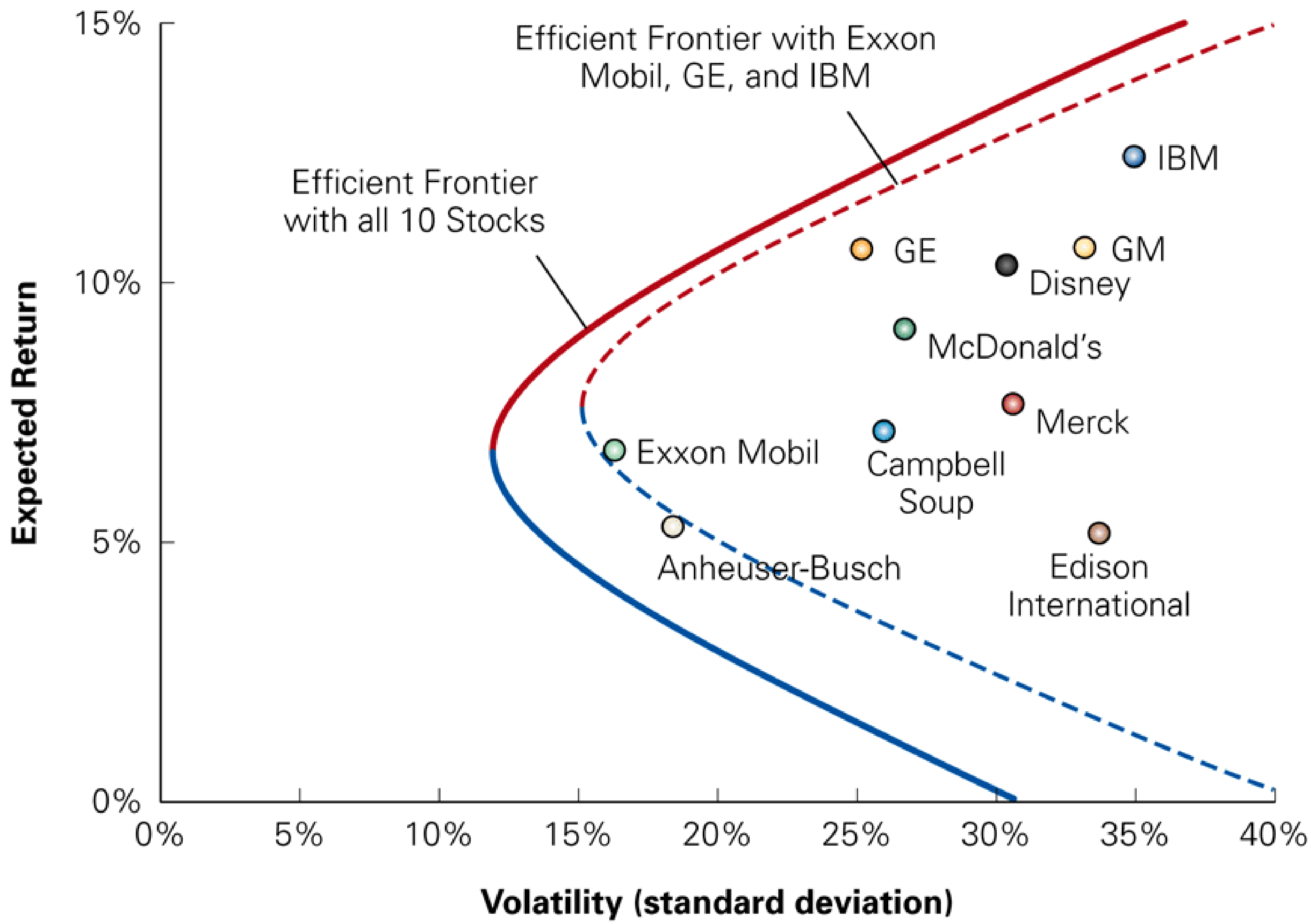
			Correlation with		
Stock	**Expected Return**	**Volatility**	**Intel**	**Coca-Cola**	**Bore Ind.**
Intel	26%	50%	1.0	0.0	0.0
Coca-Cola	6%	25%	0.0	1.0	0.0
Bore Industries	2%	25%	0.0	0.0	1.0

The Volatility and Expected Return for All Portfolios of Intel, Coca-Cola, and Bore Stock





Efficient Frontier with Ten Stocks Versus Three Stocks



Further portfolio enhancement

- Consider an arbitrary risky portfolio and the effect on risk and return of putting a fraction of the money in the portfolio, while leaving the remaining fraction in risk-free Treasury bills.
- The expected return would be:

$$E[R_{wP}] = (1-w)r_f + wE[R_P] = r_f + w\big(E[R_P] - r_f\big)$$

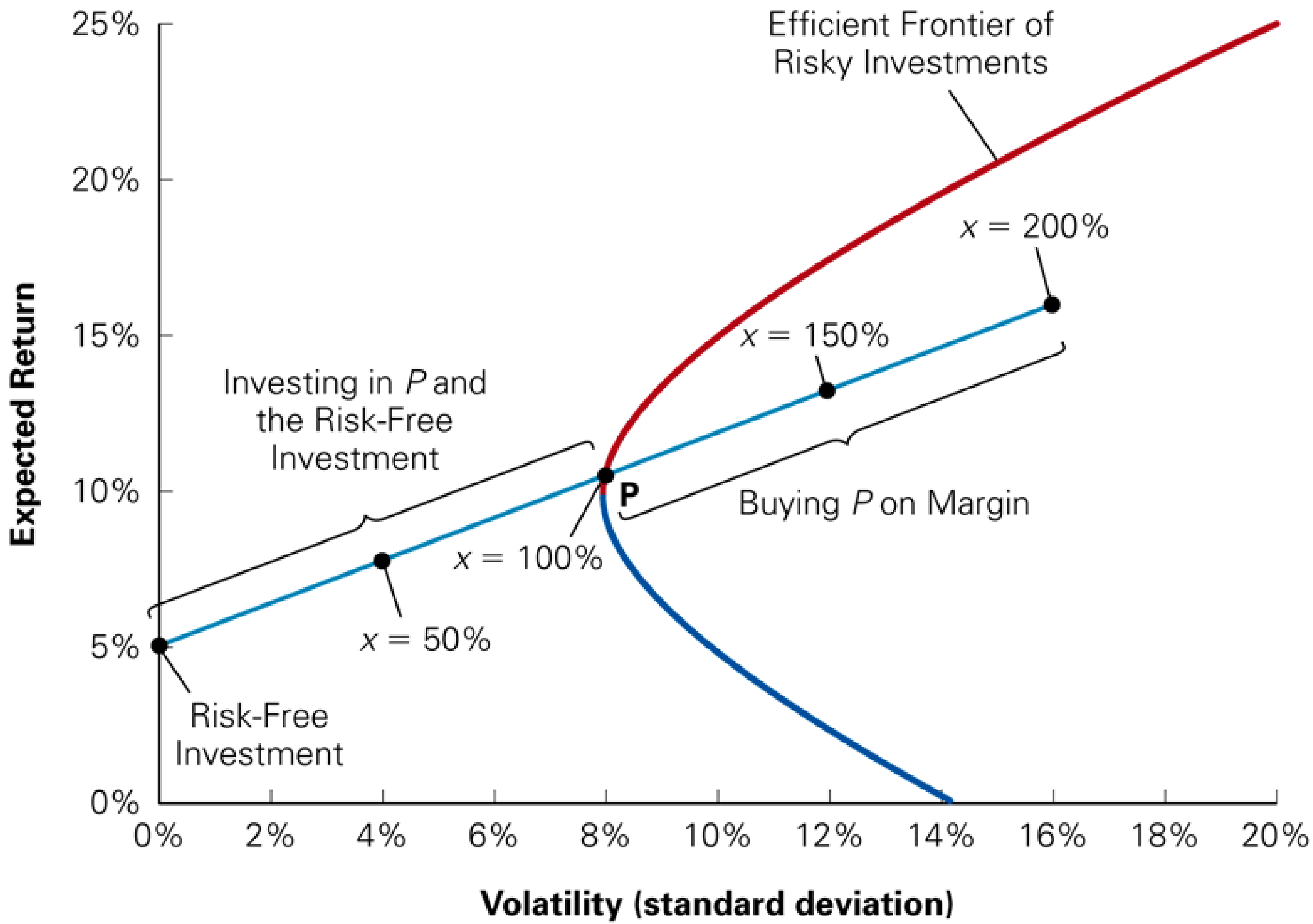
- The standard deviation of the portfolio would be calculated as:

$$\sigma_{wP} = \sqrt{(1-w)^2\sigma_f^2 + w^2\sigma_P^2 + 2(1-w)w\mathrm{cov}_{f,P}}$$

$$\sigma_{wP} = \sqrt{w^2\sigma_P^2} = w\sigma_P \qquad \mathbf{=0}$$

  – Note: The standard deviation is only a fraction of the volatility of the risky portfolio, based on the amount (portion) invested in the risky portfolio.

Borrowing and buying stock on margin



Identifying the Tangent Portfolio

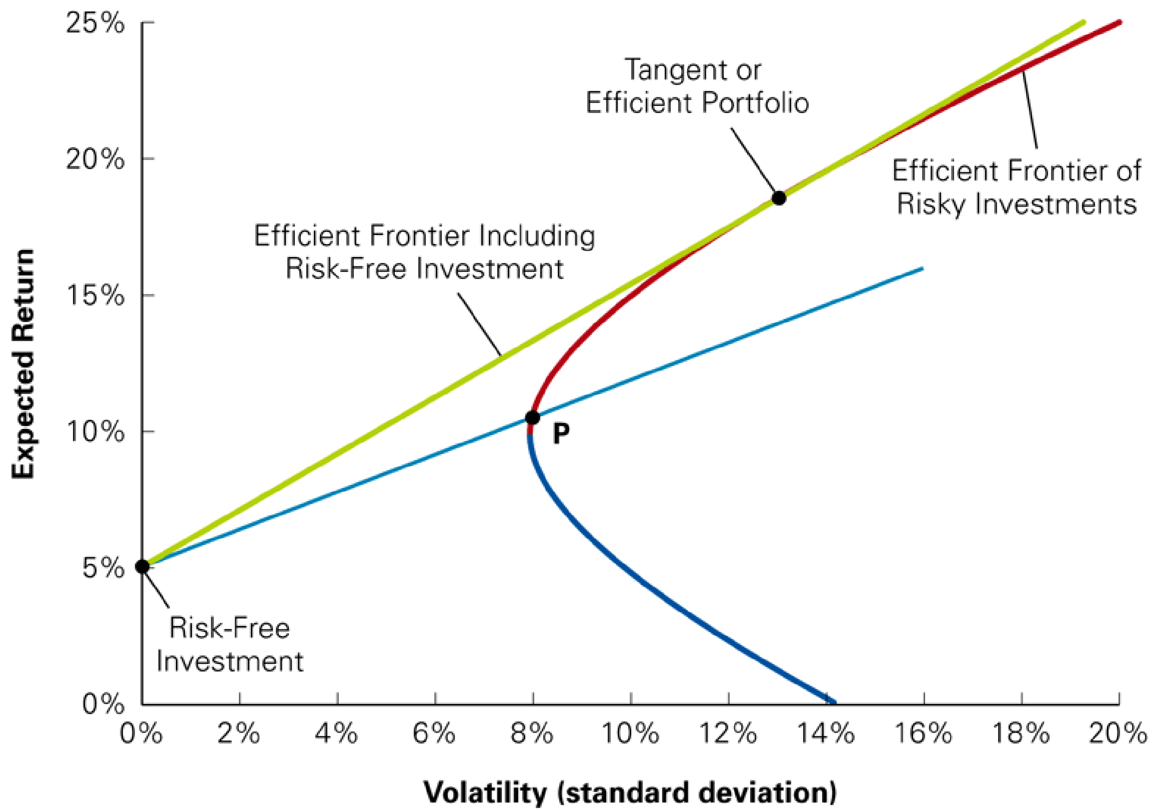
- To earn the highest possible expected return for any level of volatility we must find the portfolio that generates the steepest possible line when combined with the risk-free investment.
- Sharpe Ratio
  - Measures the ratio of reward-to-volatility provided by a portfolio

$$\text{Sharpe Ratio} = \frac{E\left[R_P - r_f\right]}{\sigma_P}$$

  - The portfolio with the highest Sharpe ratio is the portfolio where the line with the risk-free investment is tangent to the efficient frontier of risky investments. The portfolio that generates this tangent line is known as the tangent portfolio.

The Tangent or Efficient Portfolio

The line tangential to efficient frontier gives highest rate of trade-off between risk and return – highest price for risk



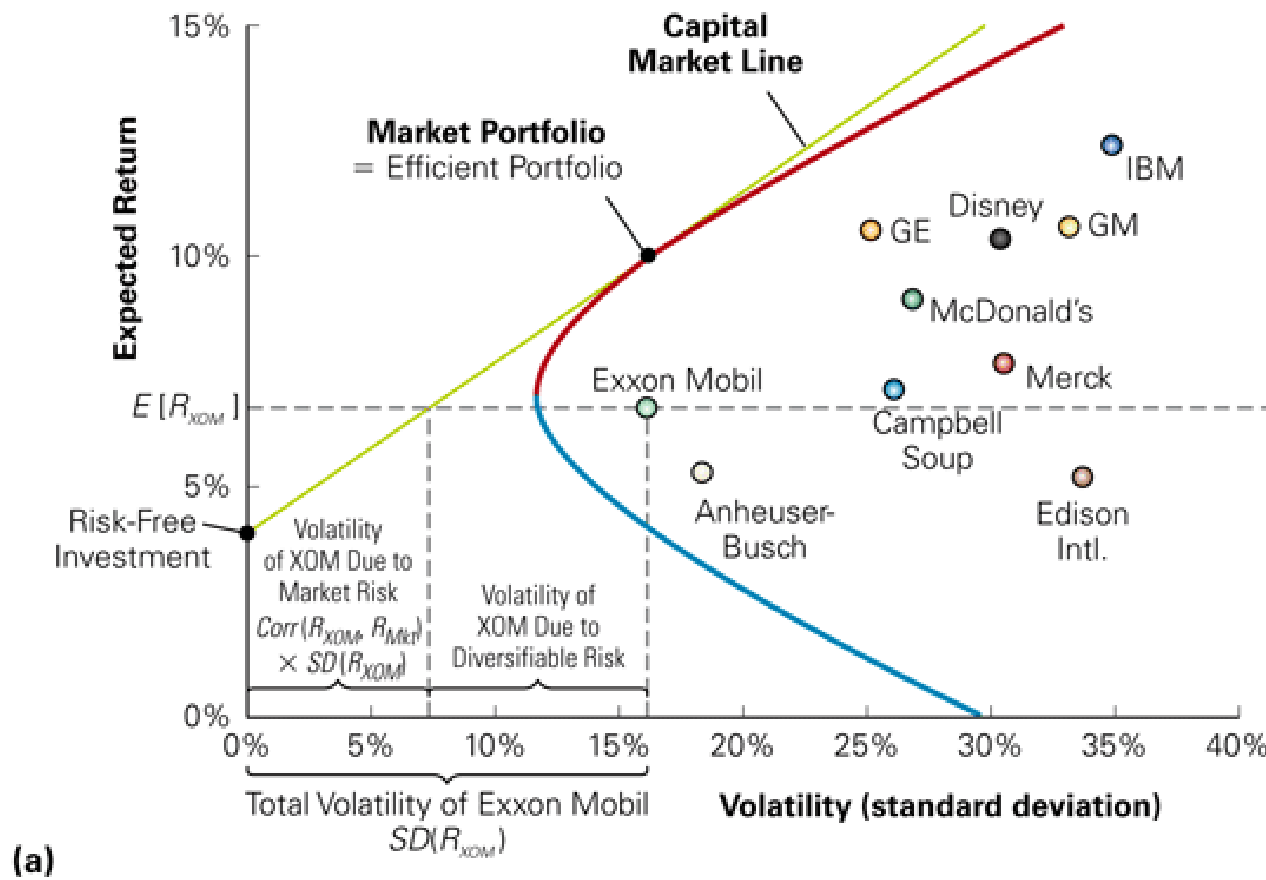
Identifying the Tangent Portfolio (cont'd)

- The tangential portfolio of risky shares that (in combination with the risk-free asset) gives the highest trade-off between the risk and return.
- An investor’s preferences will only determine how much to invest in the tangent portfolio versus the risk-free investment.
  - Conservative investors will invest a small amount in the tangent portfolio.
  - Aggressive investors will invest more in the tangent portfolio.
  - Both types of investors will choose to hold the same portfolio of risky assets, the tangent portfolio, which is the **efficient portfolio**.
    - Assume investors are rational:
      - For a given return, lower volatility is desirable.
      - For a given volatility, higher return is desirable.

The Capital Asset Pricing Model and The Capital Market Line

(a) The CML depicts portfolios combining the risk-free investment and the efficient portfolio, and shows the highest expected return that we can attain for each level of volatility.

According to the CAPM, the market portfolio is on the CML and all other stocks and portfolios contain diversifiable risk and lie to the right of the CML, as illustrated for Exxon Mobil (***XOM***).



The Capital Asset Pricing Model and the risk premium

- Market Risk and Beta
  - Given an efficient market portfolio, the expected return of an investment is:

$$E[R_i] = r_i = r_f + \underbrace{\beta_i^M \left(E[R_M] - r_f\right)}_{\text{Risk premium for security } i}$$

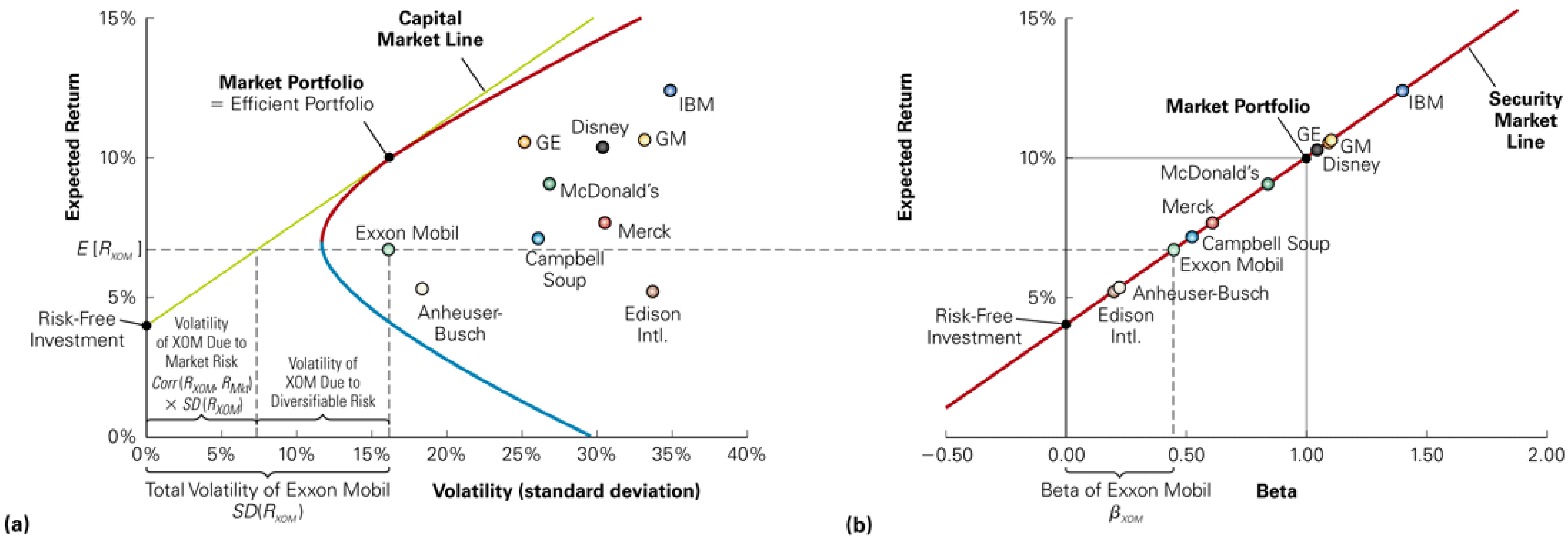
  - The beta is defined as:

$$\beta_i^M \equiv \beta_i = \frac{\overbrace{\sigma_i \times \rho_{i,M}}^{\text{Volatility of } i \text{ that is common with the market}}}{\sigma_M} = \frac{\text{cov}(R_i, R_M)}{\sigma_M^2}$$

The Security Market Line

- There is a linear relationship between a stock's beta and its expected return (See figure on next slide). The **security market line (SML)** is graphed as the line through the risk-free investment and the market.
  - According to the CAPM, if the expected return and beta for individual securities are plotted, they should all fall along the SML.

The Capital Market Line and the Security Market Line



The CAPM Assumptions

- Investors can buy and sell all securities at competitive market prices (*without incurring taxes or transactions costs*) and can borrow and lend at the risk-free interest rate.
- *Investors hold only efficient portfolios of traded securities*—portfolios that yield the maximum expected return for a given level of volatility.
- *Investors have homogeneous expectations* regarding the volatilities, correlations, and expected returns of securities.
  - Homogeneous Expectations
    - All investors have the same estimates concerning future investments and returns.

Capital Asset Pricing Model (CAPM)

- Required return is a linear function of beta.
- Beta is a measure of the extent to which the returns on asset *j* are expected to co-vary with those of the market portfolio.
- Beta is a measure of the market risk of the share.
- This market risk is the only part of risk that is priced. Unique risk is not priced.