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8.5 Divide and Conquer Optimization

1 Basic

1.1 vimrc

```
syn on
se ai nu ru mouse=a
se cin et ts=4 sw=4 sts=4
so $VIMRUNTIME/mswin.vim
colo desert
se gfn=Monospace\ 15
execute pathogen#infect()
```

1.2 Fast Integer Input

```
#define getchar gtx
 5
 5
     inline int gtx() {
        const int N = 1048576;
 6
        static char buffer[N];
 6
        static char *p = buffer, *end = buffer;
        if (p == end) {
           if ((end = buffer + fread(buffer, 1, N, stdin)) ==
           buffer) return EOF;
           p = buffer;
        return *p++;
 8
     }
 8
     template <typename T>
     inline bool rit(T& x) {
       char __c = 0; bool flag = false;
while (__c = getchar(), (__c < '0' && __c != '-') ||
    __c > '9') if (__c == -1) return false;
    __c == '-' ? (flag = true, x = 0) : (x = __c - '0');
while (__c = getchar(), __c >= '0' && __c <= '9') x =
    x * 10 + __c - '0';
if (flag) x = -x'
 9
 9
        if (flag) x = -x;
        return true;
10
     }
10
10
     template <typename T, typename ...Args>
10
     inline bool rit(T& x, Args& ...args) { return rit(x) &&
11
             rit(args...); }
```

1.3 Big Integer

11

11

```
#include <bits/stdc++.h>
struct Int {
  static const int inf = 1e9;
  std::vector<int> dig;
  bool sgn;
  Int() {
    dig.push_back(0);
    sgn = true;
  Int(int n) {
    sgn = n >= 0;
    while (n) {
      dig.push_back(n % 10);
      n /= 10;
    if (dig.size() == 0) dig.push_back(0);
  Int(std::string s) {
    int i = 0; sgn = true;
if (s[i] == '-') sgn = false, ++i;
    for (;i < s.length(); ++i) dig.push_back(s[i] - '0'</pre>
    reverse(dig.begin(), dig.end());
    if (dig.size() == 1 && dig[0] == '0') sgn = true;
  Int(const std::vector<int>& d, const bool& s = true)
    dig = std::vector<int>(d.begin(), d.end());
    sgn = s;
```

```
Int(const Int& n) {
  sgn = n.sgn;
  dig = n.dig;
bool operator<(const Int& rhs) const {
  if (sgn && !rhs.sgn) return true;
if (!sgn && rhs.sgn) return false
  if (!sgn && !rhs.sgn) return Int(dig) > Int(rhs.dig
  if (dig.size() < rhs.dig.size()) return true;</pre>
  if (dig.size() > rhs.dig.size()) return false;
  for (int i = dig.size() - 1; i >= 0; --i) {
    if (dig[i] != rhs.dig[i]) return dig[i] < rhs.dig</pre>
  [i];
  return false;
bool operator==(const Int& rhs) const {
  if (sgn != rhs.sgn) return false;
  return dig == rhs.dig;
bool operator>(const Int& rhs) const {
  return !(*this < rhs) && !(*this == rhs);</pre>
bool operator<(const int& n) const {</pre>
  return *this < Int(n);</pre>
bool operator>(const int& n) const {
  return *this > Int(n);
bool operator==(const int& n) const {
  return *this == Int(n);
Int operator-() const {
  return Int(dig, !sgn);
Int operator+(const Int& rhs) const {
  bool res = true;
  if (!sgn && !rhs.sgn) res = false;
  else if (!sqn && rhs.sqn) return rhs - (-*this);
  else if (sgn && !rhs.sgn) return *this - -rhs;
  std::vector<int> v1 = dig, v2 = rhs.dig;
if (v2.size() > v1.size()) swap(v1, v2);
  int car = 0;
  std::vector<int> nvec;
  for (int i = 0; i < v2.size(); ++i) {</pre>
    int k = v1[i] + v2[i] + car;
    nvec.push_back(k % 10);
    car = k / 10;
  for (int i = v2.size(); i < v1.size(); ++i) {</pre>
    int k = v1[i] + car
    nvec.push_back(k % 10);
    car = k / 10;
  return Int(nvec, res);
Int operator-(const Int& rhs) const {
  if (*this < rhs) {</pre>
    std::vector<int> nvec = (rhs - *this).dig;
    return Int(nvec, false);
  if (*this == rhs) return Int(0);
  std::vector<int> v1 = dig, v2 = rhs.dig;
  std::vector<int> nvec;
  for (int i = 0; i < v2.size(); ++i) {
    int k = v1[i] - v2[i];
    if (k < 0)^{-}
      for (int j = i + 1; j < v1.size(); ++j) if (v1[</pre>
  j] > 0) {
          -v1[j]; k += 10;
        break;
      }
    nvec.push_back(k);
  int rind = v1.size() - 1;
  while (rind >= v2.size() && v1[rind] == 0) --rind;
  for (int i = v2.size(); i <= rind; ++i) {</pre>
    nvec.push_back(v1[i]);
```

```
return Int(nvec);
Int operator*(const Int& rhs) const {
  if (sgn && !rhs.sgn || !sgn && rhs.sgn) return -(
Int(dig, true) * Int(rhs.dig, true));
  if (*this == 0) return Int();
  if (rhs == 0) return Int();
  std::vector<int> v1 = dig, v2 = rhs.dig;
if (v1.size() < v2.size()) swap(v1, v2);
std::vector<int> res(v1.size() * v2.size(), 0);
  for (int i = 0; i < v2.size(); ++i) {
    int car = 0;
    for (int j = 0; j < v1.size(); ++j) {</pre>
       int k = car + v1[j] * v2[i];
       res[j + i] += k \% 10;
       car = k / 10;
    }
  int car = 0;
  for (int i = 0; i < res.size(); ++i) {</pre>
    int k = car + res[i];
    res[i] = k % 10;
    car = k / 10;
  while (car) {
    res.push_back(car % 10);
    car \neq 10;
  int ind = res.size() - 1;
  while (ind >= 0 && res[ind] == 0) --ind;
  std::vector<int> nvec;
  for (int i = 0; i <= ind; ++i) nvec.push_back(res[i</pre>
  1):
  return Int(nvec);
Int operator+(const int& n) const {
  return *this + Int(n);
Int operator-(const int& n) const {
  return *this - Int(n);
Int& operator+=(const Int& n) {
  *this = (*this + n);
  return *this;
Int& operator-=(const Int& n) {
  *this = (*this - n);
  return *this;
Int& operator+=(const int& n) {
  *this += Int(n);
  return *this;
Int& operator-=(const int& n) {
  *this -= Int(n);
  return *this;
Int& operator*=(const Int& n) {
  *this = *this * n;
  return *this;
Int& operator*=(const int& n) {
  *this *= Int(n);
  return *this;
Int& operator++(int) {
  *this += 1;
  return *this;
Int& operator--(int) {
  *this -= 1;
  return *this;
friend std::istream& operator>>(std::istream& in, Int
  std::string s; in >> s;
  n = Int(s);
  return in;
friend std::ostream& operator<<(std::ostream& out,</pre>
  const Int& n) {
  if (!n.sgn) out << "-";</pre>
```

```
for (int i = n.dig.size() - 1; i >= 0; --i) out <<
    n.dig[i];
    return out;
}
</pre>
```

2 Flow

2.1 Dinic's Algorithm

```
struct Dinic {
  int n, s, t;
  vector<int> level;
  struct Edge {
    int to, rev, cap;
    Edge() {}
    Edge(int a, int b, int c): to(a), cap(b), rev(c) {}
  vector<Edge> G[maxn];
  bool bfs() {
    level.assign(n, -1);
    level[s] = 0;
    queue<int> que; que.push(s);
    while (que.size()) {
      int tmp = que.front(); que.pop();
      for (auto e : G[tmp]) {
        if (e.cap > 0 && level[e.to] == -1) {
          level[e.to] = level[tmp] + 1;
          que.push(e.to);
        }
      }
    return level[t] != -1;
  int flow(int now, int low) {
    if (now == t) return low;
    int ret = 0;
    for (auto &e : G[now]) {
      if (e.cap > 0 \& level[e.to] == level[now] + 1) {
        int tmp = flow(e.to, min(e.cap, low - ret));
        e.cap -= tmp; G[e.to][e.rev].cap += tmp;
        ret += tmp;
    if (ret == 0) level[now] = -1;
    return ret;
  Dinic(int _n, int _s, int _t): n(_n), s(_s), t(_t) {
    fill(G, G + maxn, vector<Edge>());
  void add_edge(int a, int b, int c) {
    G[a].push_back(Edge(b, c, G[b].size()));
G[b].push_back(Edge(a, 0, G[a].size() - 1));
  int maxflow() {
    int ret = 0;
    while (bfs()) ret += flow(s, inf);
    return ret;
};
```

2.2 Min cost Max flow

```
struct MCMF {
   struct Edge {
     int to, rev, cap, w;
     Edge() {}
     Edge(int a, int b, int c, int d): to(a), cap(b), w(
        c), rev(d) {}
   };
   int n, s, t;
   vector<int> p, id, d;
   bitset<maxn> inque;
   vector<Edge> G[maxn];
   pair<int, int> spfa() {
     p.assign(n, -1);
}
```

```
d.assign(n, inf);
     id.assign(n, -1);
     d[s] = 0; p[s] = s;
     queue<int> que; que.push(s); inque[s] = true;
     while (que.size()) {
       int tmp = que.front(); que.pop();
       inque[tmp] = false;
       int i = 0;
       for (auto e : G[tmp]) {
         if (e.cap > 0 \& d[e.to] > d[tmp] + e.w) {
           d[e.to] = d[tmp] + e.w;
           p[e.to] = tmp;
           id[e.to] = i;
           if (!inque[e.to]) que.push(e.to), inque[e.to]
      = true;
         ++i:
       }
     if (d[t] == inf) return make_pair(-1, -1);
     int a = inf;
     for (int i = t; i != s; i = p[i]) {
       a = min(a, G[p[i]][id[i]].cap);
     for (int i = t; i != s; i = p[i]) {
       Edge &e = G[p[i]][id[i]];
       e.cap -= a; G[e.to][e.rev].cap += a;
     return make_pair(a, d[t]);
   MCMF(int _n, int _s, int _t): n(_n), s(_s), t(_t) {
     fill(G, G + maxn, vector<Edge>());
   void add_edge(int a, int b, int cap, int w) {
     G[a].push_back(Edge(b, cap, w, (int)G[b].size()));
G[b].push_back(Edge(a, 0, -w, (int)G[a].size() - 1)
  pair<int, int> maxflow() {
  int mxf = 0, mnc = 0;
     while (true) {
       pair<int, int> res = spfa();
       if (res.first == -1) break;
       mxf += res.first; mnc += res.first * res.second;
     return make_pair(mxf, mnc);
};
```

2.3 Maximum Bipartite Matching

```
struct MaximumMatching {
  vector<int> G[maxn], mt;
  bitset<maxn> v;
  MaximumMatching(int n): n(n) {
    fill(G, G + maxn, vector<int>());
    v.reset();
  void add_edge(int a, int b) {
    G[a].push_back(b);
  bool dfs(int now) {
    v[now] = true;
     for (int u : G[now]) {
      if (mt[u] == -1 \mid \mid !v[mt[u]] \&\& dfs(mt[u])) {
         mt[u] = now;
         return true;
      }
    }
    return false;
  int solve() {
    mt.assign(n, -1);
    int ret = 0;
    for (int i = 0; i < n; ++i) {
  memset(v, false, sizeof(v));</pre>
       if (dfs(i)) ++ret;
    return ret;
```

| |};

2.4 Maximum Weighted Bipartite Matching

```
struct Hungarian {
  vector<vector<int>> w;
  bitset<maxn> s, t;
vector<int> lx, ly, mx, my, slack, prv;
  int n, matched;
  Hungarian() {}
  Hungarian(int _n): n(_n) {
    w = vector<vector<int>>(n, vector<int>(n));
    lx.resize(n); ly.resize(n); mx.assign(n, -1); my.
    assign(n, -1);
    slack.resize(n); prv.resize(n);
  void add_edge(int a, int b, int c) {
    w[a][b] = c;
  void add(int x) {
    s[x] = true;
    for (int i = 0; i < n; ++i) {
  if (lx[x] + ly[i] - w[x][i] < slack[i]) {</pre>
        slack[i] = lx[x] + ly[i] - w[x][i];
        prv[i] = x;
    }
  void augment(int now) {
    int x = prv[now], y = now;
    ++matched;
    while (true) {
      int tmp = mx[x]; mx[x] = y; my[y] = x; y = tmp;
      if (y == -1) return;
      x = prv[y];
    }
  }
  void relabel() {
    int delta = inf;
    for (int i = 0; i < n; ++i) if (!t[i]) delta = min(</pre>
    delta, slack[i]);
    for (int i = 0; i < n; ++i) if (s[i]) lx[i] -=
    delta;
    for (int i = 0; i < n; ++i) {
      if (t[i]) ly[i] += delta;
      else slack[i] -= delta;
  void go() {
    s.reset(); t.reset();
    fill(slack.begin(), slack.end(), inf);
    int root = 0;
    for (; root < n && mx[root] != -1; ++root);</pre>
    add(root);
    while (true) {
      relabel();
      int y = 0;
      for (; y < n; ++y) if (!t[y] && slack[y] == 0)
      if (my[y] == -1) return augment(y), void();
      add(my[y]); t[y] = true;
  int matching() {
    int ret = 0;
    for (int i = 0; i < n; ++i) {
      for (int j = 0; j < n; ++j) lx[i] = max(lx[i], w[
    i][j]);
    for (int i = 0; i < n; ++i) go();
    for (int i = 0; i < n; ++i) ret += w[i][mx[i]];</pre>
    return ret;
};
```

3 Math

```
3.1 FFT
```

```
const double pi = acos(-1);
const complex<double> I(0, 1)
complex<double> omega[maxn + 1];
void prefft() {
  for (int i = 0; i <= maxn; ++i) omega[i] = exp(i * 2
 * pi / maxn * I);</pre>
void fft(vector<complex<double>>& a, int n, bool inv=
  int basic = maxn / n;
  int theta = basic;
  for (int m = n; m >= 2; m >>= 1) {
    int h = m >> 1;
    for (int i = 0; i < h; ++i)_{
      complex<double> w = omega[inv ? maxn - (i * theta
     % maxn) : i * theta % maxn];
      for (int j = i; j < n; j += m) {
        int k = j + h;

complex<double> x = a[j] - a[k];
         a[j] += a[k];
        a[k] = w * x;
      }
    theta = (theta * 2) % maxn;
  int i = 0:
  for (int j = 1; j < n - 1; ++j) {
  for (int k = n >> 1; k > (i ^= k); k >>= 1);
    if (j < i) swap(a[i], a[j]);
  if (inv) for (int i = 0; i < n; ++i) a[i] /= (double)
void invfft(vector<complex<double>>& a, int n) {
  fft(a, n, true);
}
3.2 NTT
const long long p = 2013265921, root = 31;
long long omega[maxn + 1];
long long fpow(long long a, long long n) {
  long long ret = 1ll;
  for (; n; n >>= 1) {
    if (n & 1) ret = ret * a % p;
    a = a * a % p;
  return ret;
}
void prentt() {
  omega[0] = 1;
  long long r = fpow(root, (p - 1) / maxn);
  for (int i = 1; i <= maxn; ++i) omega[i] = omega[i -</pre>
    1] * r % p;
void ntt(vector<long long>& a, int n, bool inv = false)
  int basic = maxn / n;
  int theta = basic;
  for (int m = n; m >= 2; m >>= 1) {
    int mh = m >> 1;
    for (int i = 0; i < mh; ++i) {
  long long w = omega[i * theta % maxn];</pre>
```

for (int j = i; j < n; j += m) {

long long x = a[j] - a[k];

if (a[j] > p) a[j] -= p; a[k] = w * x % p;

int k = j + mh;

if (x < 0) x += p;a[j] += a[k];

3.3 Miller-Rabin

```
// n < 4759123141
                        chk = [2, 7, 61]
// n < 1122004669633 chk = [2, 13, 23, 1662803]
// n < 2^64 chk = [2, 325, 9375, 28178, 450775,
     9780504, 1795265022]
long long fpow(long long a, long long n, long long mod)
  long long ret = 1LL;
  for (; n; n >>= 1) {
    if (n & 1) ret = (__int128)ret * (__int128)a % mod;
    a = (__int128)a * (__int128)a % mod;
  return ret;
bool check(long long a, long long u, long long n, int t
  a = fpow(a, u, n);
  if (a == 0) return true;
  if (a == 1 \mid \mid a == n - 1) return true;
  for (int i = 0; i < t; ++i) {
    a = (__int128)a * (__int128)a % n;
     if (a == 1) return false;
    if (a == n - 1) return true;
  return false;
bool is_prime(long long n) {
  if (n < 2) return false;
if (n % 2 == 0) return n == 2;</pre>
  long long u = n - 1; int t = 0;
for (; u & 1; u >>= 1, ++t);
for (long long i : chk) {
    if (!check(i, u, n, t)) return false;
  return true;
```

3.4 Pollard's Rho

```
long long f(long long x, long long mod) {
    return add(mul(x, x, mod), 1, mod);
}

long long pollard_rho(long long n) {
    if (n % 2 == 0) return 2;
    while (true) {
        long long y = 2, x = rand() % (n - 1) + 1, res = 1;
        for (int sz = 2; res == 1; sz <<= 1) {
            for (int i = 0; i < sz && res <= 1; ++i) {
                 x = f(x, n);
                 res = __gcd(abs(x - y), n);
            }
            y = x;
        }
        if (res && res != n) return res;
    }
}</pre>
```

3.5 $\lfloor \frac{n}{i} \rfloor$ Enumeration

```
vector<int> solve(int n) {
  vector<int> vec;
  for (int t = 1; t < n; t = (n / (n / (t + 1)))) vec.
    push_back(t);
  vec.push_back(n);
  vec.resize(unique(vec.begin(), vec.end()) - vec.begin
        ());
  return vec;
}</pre>
```

3.6 μ function

3.7 Extended GCD

```
template <typename T> tuple<T, T, T> extgcd(T a, T b) {
  if (!b) return make_tuple(a, 1, 0);
  T d, x, y;
  tie(d, x, y) = extgcd(b, a % b);
  return make_tuple(d, y, x - (a / b) * y);
}
```

4 Graph

4.1 Strongly Connected Components

```
struct SCC {
  vector<int> G[maxn], R[maxn], topo;
  int n, nscc;
  vector<int> scc, sz;
  bitset<maxn> v;
  void dfs(int now) {
    v[now] = true;
    for (int u : G[now]) if (!v[u]) {
      dfs(u);
    topo.push_back(now);
  void rdfs(int now) {
    v[now] = true;
    scc[now] = nscc;
    ++sz[nscc];
    for (int u : R[now]) if (!v[u]) {
      rdfs(u);
    }
  SCC(): {}
  SCC(int n): n(n) {
    scc.assign(n, 0); sz.assign(n, 0);
  void add_edge(int a, int b) {
```

4.2 Heavy-Light Decomposition

```
struct HeavyLightDecomp {
 vector<int> G[maxn];
  vector<int> tin, top, dep, maxson, sz, p;
  int n, t;
  void dfs(int now, int fa, int d) {
    dep[now] = d;
    maxson[now] = -1;
    sz[now] = 1;
    p[now] = fa;
    for (int u : G[now]) if (u != fa) {
      dfs(u, now, d + 1);
      sz[now] += sz[u];
      if (\max son[now] == -1 \mid \mid sz[u] > sz[\max son[now]])
     maxson[now] = u;
  void link(int now, int tp) {
    top[now] = tp;
    tin[now] = ++t;
    if (maxson[now] == -1) return;
    link(maxson[now], tp);
    for (int u : G[now]) if (u != p[now]) {
      if (u == maxson[now]) continue;
      link(u, u);
    }
 HeavyLightDecomp(int n): n(n) {
    tin.assign(n, 0); top.assign(n, 0); dep.assign(n,
    maxson.assign(n, 0); sz.assign(n, 0); p.assign(n,
    0);
  void add_edge(int a, int b) {
    G[a].push_back(b);
    G[b].push_back(a);
  void build() {
    dfs(0, -1, 0);
    link(0, 0);
  int lca(int a, int b) {
  int ta = top[a], tb = top[b];
    while (ta != tb) {
      if (dep[ta] < dep[tb]) {
   swap(ta, tb); swap(a, b);</pre>
      a = p[ta]; ta = top[a];
    if (a == b) return a;
    return dep[a] < dep[b] ? a : b;</pre>
  vector<pair<int, int>> get_path(int a, int b) {
    int ta = top[a], tb = top[b];
    vector<pair<int, int>> ret;
    while (ta != tb) {
      if (dep[ta] < dep[tb]) {</pre>
        swap(ta, tb); swap(a, b);
      ret.push_back(make_pair(tin[ta], tin[a]));
      a = p[ta]; ta = top[a];
```

```
ret.push_back(make_pair(min(tin[a], tin[b]), max(
    tin[a], tin[b])));
    return ret;
}
};
```

4.3 Centroid Decomposition

```
vector<pair<int, int>> G[maxn];
int sz[maxn], mx[maxn];
bool v[maxn];
vector<int> vtx;
void get_center(int now) {
  v[now] = true; vtx.push_back(now);
sz[now] = 1; mx[now] = 0;
  for (int u : G[now]) if (!v[u]) {
    get_center(u);
    mx[now] = max(mx[now], sz[u]);
    sz[now] += sz[u];
}
void get_dis(int now, int d, int len) {
  dis[d][now] = cnt;
  v[now] = true;
  for (auto u : G[now]) if (!v[u.first]) {
    get_dis(u, d, len + u.second);
  }
}
void dfs(int now, int fa, int d) {
  get_center(now);
  int c = -1;
for (int i : vtx) {
    if (max(mx[i], (int)vtx.size() - sz[i]) <= (int)vtx</pre>
     .size() / 2) c = i;
    v[i] = false;
  get_dis(c, d, 0);
  for (int i : vtx) v[i] = false;
  v[c] = true; vtx.clear();
  dep[c] = d; p[c] = fa;
for (auto u : G[c]) if (u.first != fa && !v[u.first])
    dfs(u.first, c, d + 1);
}
```

4.4 Maximum Clique

```
struct MaxClique {
  static const int maxv = 64;
  unsigned long long adj[maxv];
  int n, ans;
  void init(int _n): n(_n) {
    for (int i = 0; i < n; ++i) adj[i] = Oull;</pre>
  void add_edge(int a, int b) {
    adj[a] = (1ull \ll a);
    adj[b] = (1ull \ll b);
  void go(unsigned long long r, unsigned long long p,
    unsigned long long x, int c, int res) {
if (c + res < ans) return;</pre>
    if (p == 0ull \&\& x == 0ull) return ans = max(ans, c
    ), void();
    unsigned long long y = p \mid x; y \&= -y
    unsigned long long q = p & (~adj[(int)log2(y)]);
    while (q) {
      int i = (int)log2(q \& -q);
      go(r \mid (1ull \ll i), p \& adj[i], x \& adj[i], cnt +
     1, __builtin_popcountll(p & nb[i]));
      q &= ~(1ull << i);
      p &= ~(1ull << i);
      x |= (1ull << i);
  }
```

```
int maxclique() {
    ans = 0;
    unsigned long long s = 0;
    if (n < maxv) s = (1ull << n) - 1;
    else for (int i = 0; i < n; ++i) s |= (1ull << i);
    go(0ull, s, 0ull, 0ull, n);
    return ans;
}
</pre>
```

4.5 Biconnected Component

```
int tin[maxn], low[maxn], t, bccsz;
stack<int> st;
vector<int> bcc[maxn];
void dfs(int now, int fa) {
  tin[now] = ++t; low[now] = tin[now];
  st.push(now);
for (int u : G[now]) if (u != fa) {
    if (!tin[u]) {
      dfs(u, now);
      low[now] = min(low[now], low[u]);
      if (low[u] >= tin[now]) {
        int v:
        ++bccsz;
        do {
          v = st.top(); st.pop();
          bcc[bccsz].push_back(v);
        } while (v != u);
        bcc[bccsz].push_back(now);
    } else {
      low[now] = min(low[now], tin[u]);
  }
}
```

4.6 2-Satisfiability

```
struct TwoSat {
  vector<int> G[maxn << 1];</pre>
  bitset<maxn << 1> v;
  vector<int> s;
  int c;
  bool dfs(int now) {
     if (v[now ^ 1]) return false;
     if (v[now]) return true;
     v[now] = true;
     s[c++] = now;
     for (int u : G[now]) if (!dfs(u)) return false;
     return true;
  TwoSat() {
     s.assign(maxn << 1, 0);
     v.reset();
  void add_edge(int a, int b) {
     G[a].push_back(b);
  bool solve() {
  for (int i = 0; i < maxn << 1; i += 2) {
    if (!v[i] && !v[i + 1]) {</pre>
         if (!dfs(i)) {
  while (c) v[s[--c]] = false;
            if (!dfs(i + 1)) return false;
       }
     return true;
};
```

5 Data Structures

5.1 Dark Magic

```
#include <bits/stdc++.h>
#include <bits/extc++.h>
#include <ext/rope>
using namespace __gnu_pbds;
using namespace __gnu_cxx;
#include <ext/pb_ds/assoc_container.hpp>
typedef tree<int, null_type, std::less<int>,
    rb_tree_tag, tree_order_statistics_node_update>
     tree_set;
typedef cc_hash_table<int, int> umap;
typedef priority_queue<int> heap;
int main() {
  // rb tree
  tree_set s
  s.insert(71); s.insert(22);
  assert(*s.find_by_order(0) == 22); assert(*s.
     find_by_order(1) == 71);
  assert(s.order_of_key(22) == 0); assert(s.
  order_of_key(71) == 1);
  s.erase(22);
  assert(*s.find_by_order(0) == 71); assert(s.
     order_of_key(71) == 0);
  // mergable heap
  heap a, b; a.join(b);
  // persistant
  rope<char> r[2];
  r[1] = r[0];
  std::string st = "abc";
r[1].insert(0, st.c_str());
r[1].erase(1, 1);
  std::cout << r[1].substr(0, 2) << std::endl;</pre>
  return 0;
```

5.2 Treap

```
#define tsize(t) (t ? t->size : 0)
struct treap {
  treap *lc, *rc;
  int val, pri, sz;
  treap(int v): val(v) {
    lc = rc = nullptr;
    pri = rand();
    sz = 1;
  void pull() {
    sz = tsize(lc) + tsize(rc) + 1;
};
treap *merge(treap *a, treap *b) {
  if (!a | | !b) return a ? a : b;
  if (a->pri > b->pri) {
    a \rightarrow rc = merge(a \rightarrow rc, b);
    a->pull();
    return a;
  } else {
    b \rightarrow lc = merge(a, b \rightarrow lc);
    b->pull();
    return b;
  }
}
void split_by_val(treap *t, int k, treap *&a, treap *&b
  if (!t) {
    a = b = nullptr;
    return;
  if (t->val <= k) {
    split(t->rc, k, a->rc, b);
```

```
a->pull();
  } else {
   b = t;
    split(t->lc, k, a, b->lc);
    b->pull();
}
void split_by_size(treap *t, int k, treap *&a, treap *&
   b) {
  if (!t) {
   a = b = nullptr;
    return;
  if (tsize(t->lc) + 1 <= k) {</pre>
    split(t->rc, k - tsize(t->lc) - 1, a->rc, b);
    a->pull();
 } else {
   b = t;
    split(t->lc, k, a, b->lc);
    b->pull();
}
```

5.3 Persistant Disjoint Set

```
struct DisjointSet {
  int p[maxn], sz[maxn], n;
  vector<pair<int*, int>> h;
  vector<int> sp;
  void init(int size) {
    n = size;
    for (int i = 0; i < n; ++i) p[i] = i, sz[i] = 1;
    sp.clear(); h.clear();
  void assign(int *k, int v) {
  h.push_back(make_pair(k, *k));
    *k = v;
  void save() {
    sp.push_back(h.size());
  void undo() {
    int last = sp.back(); sp.pop_back();
    while (h.size() != last) {
      pair<int*, int> pi = h.back(); h.pop_back();
       *pi.first = pi.second;
    }
  int find(int x) {
    if (x == p[x]) return x;
    return p[x] = find(p[x]);
  void merge(int x, int y) {
    x = find(x); y = find(y);
    if (x == y) return;
    if (sz[x] < sz[y]) swap(x, y);
    assign(\&sz[x], sz[x] + sz[y]);
    assign(&p[y], x);
};
```

5.4 Leftlist Tree

```
namespace LeftlistTree {
    struct Node {
        T val;
        int s;
        Node *lc, *rc;
        Node(T v): val(v) {
            lc = rc = nullptr;
            s = 1;
        }
    };
    inline int rank(Node* t) {
        return t ? t->s : 0;
    }
}
```

```
Node *merge(Node *a, Node *b) {
    if (!a | | !b) return a ? a : b;
    if (a->val < b->val) swap(a, b);
    a->rc = merge(a->rc, b);
    if (rank(a->lc) < rank(a->rc)) swap(a->lc, a->rc);
    a->s = rank(a->rc) + 1;
    return a;
}
void clear(Node *t) {
    if (!t) return;
    if (t->lc) clear(t->lc);
    if (t->rc) clear(t->rc);
    delete t;
}
```

6 Geometry

6.1 Points

```
using type = double;
struct pt {
  type x, y
  pt(): x(0), y(0) {}
  pt(type x, type y): x(x), y(y) {}
  pt operator+(const pt& a) const { return pt(x + a.x,
    y + a.y);
  pt operator-(const pt& a) const { return pt(x - a.x,
    y - a.y);
  pt operator*(const type& t) const { return pt(x * t,
    y * t); }
  pt operator/(const type& t) const { return pt(x / t,
    y / t); }
  type operator*(const pt& a) const { return x * a.x +
    y * a.y; }
  type operator^(const pt& a) const { return x * a.y -
     y * a.x; }
  bool operator<(const pt& a) const { return x == a.x ?
      y < a.y : x < a.x; }
  bool operator>(const pt& a) const { return x == a.x ?
     y > a.y : x > a.x; }
  bool operator==(const pt& a) const { return x == a.x
  && y == a.y; } bool operator!=(const pt& a) const { return x != a.x
     | | y != a.y; 
  double dist(const pt &p) const { return sqrt((p - *
     this) * (p - *this)); }
  friend double norm2(const pt &p) { return p * p; }
  friend double norm(const pt &p) { return sqrt(norm2(p
     )); }
|};
```

6.2 Segment Intersection

```
type orient(const pt& o, const pt& a, const pt& b) {
    type t = (a - o) ^ (b - o);
    return t / max(1, abs(t));
}

// if parallel and min(p1, p2) != max(q1, q2) and p1 !=
        p2 and q1 != q2 ==> segment

// else ==> point

bool intersect(const pt& p1, const pt& p2, const pt& q1
        , const pt& q2) {
    if (q1 == q2 && p1 == p2) return p1 == q1;
    if (q1 == q2) return ((p2 - q1) ^ (p1 - q1)) == 0 &&
        q1.x >= min(p1.x, p2.x) && q1.x <= max(p1.x, p2.x);
    if (p1 == p2) return ((q1 - p1) ^ (q2 - p1)) == 0 &&
        p1.x >= min(q1.x, q2.x) && p1.x <= max(q1.x, q2.x);
    int c = (p2 - p1) ^ (q2 - q1);
    if (c == 0) {
        if (orient(p1, p2, q1)) return false;
        return (p1 - q1) * (p2 - q1) <= 0 || (p1 - q2) * (
        p2 - q2) <= 0 || (q1 - p1) * (q2 - p1) <= 0 || (q1 - p2) * (q2 - p2) <= 0;</pre>
```

```
}
return (orient(p1, p2, q1) * orient(p1, p2, q2) <= 0)
    && (orient(q1, q2, p1) * orient(q1, q2, p2) <= 0);
}

else ++lpr;
// ch[lpr], ch[rpr]
}
</pre>
```

6.3 Line Intersection

```
pt intersection(const pt& p1, const pt& p2, const pt&
    q1, const pt& q2) {
    double f1 = (p2 - p1) ^ (q1 - p1);
    double f2 = (p2 - p1) ^ (p1 - q2);
    double f = f1 + f2;
    if (fabs(f) < eps) return pt(nan(""), nan(""));
    return q1 * (f2 / f) + q2 * (f1 / f);
}</pre>
```

6.4 Circle Intersection

```
vector<pt> circle_intersection(const pt& 01, const
    double& r1, const pt& 02, const double& r2) {
    double d2 = (o1 - o2) * (o1 - o2);
    double d = sqrt(d2);
    if (d > r1 + r2) return {};
    pt u = (o1 + o2) * 0.5 + (o1 - o2) * ((r2 * r2 - r1 *
        r1) / (2 * d2));
    double a = sqrt((r1 + r2 + d) * (r1 - r2 + d) * (r1+
        r2 - d) * (-r1 + r2 + d));
    pt v = pt(o1.y - o2.y, -o1.x + o2.x) * a / (2 * d2);
    return { u + v, u - v };
}
```

6.5 Convex Hull

```
type cross(const pt& o, const pt& a, const pt& b) {
  return (a - o) ^ (b - o);
int rsd;
vector<pt> convex_hull(vector<pt> p) {
  sort(p.begin(), p.end());
  int m = 0:
  vector<pt> ret(2 * p.size());
  for (int i = 0; i < p.size(); ++i) {
  while (m >= 2 && cross(ret[m - 2], ret[m - 1], p[i
    ]) < 0) --m;
    ret[m++] = p[i];
 for (int i = p.size() - 2, t = m + 1; i \ge 0; --i) {
    while (m >= t && cross(ret[m - 2], ret[m - 1], p[i
    ]) < 0) --m;
    ret[m++] = p[i];
 ret.resize(m - 1);
  return ret;
```

6.6 Rotating Caliper

```
void rotating_caliper(vector<pt> p) {
  vector<pt> ch = convex_hull(p);
  int tbz = ch.size();
  int lpr = 0, rpr = rsd;
  // ch[lpr], ch[rpr]
  while (lpr < rsd || rpr < tbz - 1) {
    if (lpr < rsd && rpr < tbz - 1) {
      pt rvt = ch[rpr + 1] - ch[rpr];
      pt lvt = ch[lpr + 1] - ch[lpr];
      if ((lvt ^ rvt) < 0) ++lpr;
      else ++rpr;
    }
    else if (lpr == rsd) ++rpr;</pre>
```

6.7 Closest Pair

```
pt p[maxn];
double dis(const pt& a, const pt& b) {
  return sqrt((a - b) * (a - b));
double closest_pair(int l, int r) {
  if (l == r) return inf;
if (r - l == 1) return dis(p[l], p[r]);
  int m = (l + r) >> 1;
  double d = min(closest_pair(l, m), closest_pair(m +
     1, r));
  vector<int> vec;
  for (int i = m; i >= l && fabs(p[m].x - p[i].x) < d;
     --i) vec.push_back(i);
  for (int i = m + 1; i \le r \& fabs(p[m].x - p[i].x) <
      d; ++i) vec.push_back(i);
  sort(vec.begin(), vec.end(), [=](const int& a, const
     int& b) { return p[a].y < p[b].y; });</pre>
  for (int i = 0; i < vec.size(); ++i) {
  for (int j = i + 1; j < vec.size() && fabs(p[vec[j
]].y - p[vec[i]].y) < d; ++j) {</pre>
       d = min(d, dis(p[vec[i]], p[vec[j]]));
  }
  return d;
}
```

6.8 Circle

```
struct circle {
  pt o;
  type r;
  circle() {}
  circle(const pt &p, const type &d): o(p), r(d) {}
  circle(const type &x, const type &y, const type &d):
      o(pt(x, y)), r(d) {}
  double operator()() const { return pi * r * r; }
  bool overlap(const pt &p) const { return p.dist(o) <=
      r; }
  bool on(const pt &p) const { return p.dist(o) == r; }
};</pre>
```

6.9 Minimum Enclosing Circle

```
pt center(const pt &a, const pt &b, const pt &c) {
  pt p0 = b - a, p1 = c - a;
  double c1 = norm2(p0) * 0.5, c2 = norm2(p1) * 0.5;
  double d = p0 \land p1;
  double x = a.x + (c1 * p1.y - c2 * p0.y) / d;
  double y = a.y + (c2 * p0.x - c1 * p1.x) / d;
  return pt(x, y);
circle min_enclosing(vector<pt> &p) {
  random_shuffle(p.begin(), p.end());
  double r = 0.0;
  pt cent;
  for (int i = 0; i < p.size(); ++i) {</pre>
     if (norm2(cent - p[i]) <= r) continue;</pre>
     cent = p[i];
     r = 0.0;
     for (int j = 0; j < i; ++j) {
  if (norm2(cent - p[j]) <= r) continue;
  cent = (p[i] + p[j]) / 2;
  r = norm2(p[j] - cent);
  for (int b) (int b) (int b) (int b)</pre>
        for (int k = 0; k < j; ++k) {
          if (norm2(cent - p[k]) <= r) continue;</pre>
          cent = center(p[i], p[j], p[k]);
```

```
r = norm2(p[k] - cent);
}
}
return circle(cent, sqrt(r));
}
```

6.10 Triangle Center

```
pt incenter(const pt &pa, const pt &pb, const pt &pc) {
 double a = norm(pb - pc), b = norm(pc - pa), c = norm
 (pa - pb);
return (pa * a + pb * b + pc * c) / (a + b + c);
pt circumcenter(const pt &pa, const pt &pb, const pt &
 pt ba = b - a, ca = c - a;
  double db = norm2(ba), dc = norm2(ca), d = 2.0 * (ba)
    ^ ca);
  return a + pt(ba.y * dc - ca.y * db, ca.x * db - ba.x
     * dc) / 2;
pt othrocenter(const pt &pa, const pt &pb, const pt &pc
    ) {
 pt pa = pb - pa, ca = pc - pa, bc = pb - pc;
 double yp = ca.y * ca.y * bc.y;
 double a = (ca \wedge ba);
 double x = (yp + ca.x * ba.y * pb.x - ba.x * ca.y *
    pc.x) / a;
  double y = -ba.x * (x - pc.x) / ba.y + ca.y;
  return pt(x, y);
```

7 String

7.1 KMP

```
int f[maxn];
int kmp(const string& a, const string& b) {
    f[0] = -1; f[1] = 0;
    for (int i = 1, j = 0; i < b.size() - 1; f[++i] = ++j
      ) {
        if (b[i] == b[j]) f[i] = f[j];
        while (j != -1 && b[i] != b[j]) j = f[j];
    }
    for (int i = 0, j = 0; i - j + b.size() <= a.size();
        ++i, ++j) {
        while (j != -1 && a[i] != b[j]) j = f[j];
        if (j == b.size() - 1) return i - j;
    }
    return -1;
}</pre>
```

7.2 Suffix Array

```
struct SuffixArray {
 int sa[maxn], tmp[2][maxn], c[maxn], _lcp[maxn], r[
    maxn], n;
  string s;
  SparseTable st;
  void suffixarray() -
     int* rank = tmp[0]
    int* nRank = tmp[1];
    int A = 128;
    for (int i = 0; i < A; ++i) c[i] = 0;
    for (int i = 0; i < s.length(); ++i) c[rank[i] = s[
    i]]++;
    for (int i = 1; i < A; ++i) c[i] += c[i - 1];
    for (int i = s.length() - 1; i \ge 0; --i) sa[--c[s[
    i]]] = i;
    for (int n = 1; n < s.length(); n *= 2) {</pre>
```

```
for (int i = 0; i < A; ++i) c[i] = 0;
for (int i = 0; i < s.length(); ++i) c[rank[i</pre>
  ]]++;
    for (int i = 1; i < A; ++i) c[i] += c[i - 1];
    int* sa2 = nRank;
    int r = 0;
    for (int i = s.length() - n; i < s.length(); ++i)
   sa2[r++] = i;
    for (int i = 0; i < s.length(); ++i) if (sa[i] >=
   n) sa2[r++] = sa[i] - n;
    for (int i = s.length() - 1; i >= 0; --i) sa[--c[
  rank[sa2[i]]] = sa2[i];
    nRank[sa[0]] = r = 0;
    for (int i = 1; i < s.length(); ++i) {</pre>
      if (!(rank[sa[i - 1]] == rank[sa[i]] && sa[i -
  1] + n < s.length() && rank[sa[i - 1] + n] == rank[
  sa[i] + n]) r++;
      nRank[sa[i]] = r;
    swap(rank, nRank);
    if (r == s.length() - 1) break;
    A = r + 1;
void solve() {
  suffixarray();
  for (int i = 0; i < n; ++i) r[sa[i]] = i;
  int ind = 0; _1cp[0] = 0;
  for (int i = 0; i < n; ++i) {
  if (!r[i]) { ind = 0; continue; }</pre>
    while (i + ind < n \&\& s[i + ind] == s[sa[r[i] -
  1] + ind]) ++ind;
    _lcp[r[i]] = ind ? ind-- : 0;
  st = SparseTable(n, _lcp);
int lcp(int L, int R) {
  if (L == R) return n - L - 1;
  L = r[L]; R = r[R];
  if (L > R) swap(L, R);
  ++1:
  return st.query(L, R);
SuffixArray(string s): s(s), n(s.length()) {}
SuffixArray() {}
```

7.3 Z Algorithm

7.4 Manacher's Algorithm

```
if (i + z[i] > r) r = i + z[i], l = i;
}
int ans = 0;
for (int i = 1; i < t.length(); ++i) ans = max(ans, z
   [i] - 1);
return ans;
}</pre>
```

7.5 Smallest Rotation

```
string rotate(const string &s) {
  int n = s.length();
  string t = s + s;
  int i = 0, j = 1;
  while (i < n && j < n) {
    int k = 0;
    while (k < n && s[i + k] == s[j + k]) ++k;
    if (s[i + k] <= s[j + k]) j += k + 1;
    else i += k + 1;
    if (i == j) ++j;
  }
  int pos = (i < n ? i : j);
  return s.substr(pos, n);
}</pre>
```

7.6 Rolling Hash Primes

```
const int mod[] = { 479001599, 433494437, 1073807359,
    1442968193, 715827883 }
const int p[] = { 101, 233, 457, 173, 211 }
```

8 Dynamic Programming

8.1 Concept

- 1. Convex Hull Optimization: $dp[i] = \max_{j < i} \{a[j]x[i] + b[j]\}$
- 2. Concave totally Montone: $\forall i < i', j < j', f[i][j] \leq f[i'][j] \Rightarrow f[i][j'] \leq f[i'][j']$
- 3. Convex totally Montone: $\forall i < i', j < j', f[i][j] \ge f[i'][j] \Rightarrow f[i][j'] \ge f[i'][j']$

8.2 Linear Convex Hull Optimization

```
struct ConvexHull {
  // Max convex hull
  deque<pair<int, int>> dq;
  bool check(const pair<int, int>& l1, const pair<int,
    int>& 12, int x) {
    // for min case, replace <= with >=
return l1.first * x + l1.second <= l2.first * x +</pre>
    12.second:
  bool elim(const pair<int, int>& l1, const pair<int,</pre>
    int>& 12, const pair<int, int>& 1) {
    return (double)(l1.second - l2.second) / (double)(
    12.first - 11.first) \leftarrow (double)(1.second - 12.
    second) / (double)(12.first - l.first);
  int query(int x) {
    while (dq.size() \ge 2 \& check(dq[0], dq[1], x)) dq
     .pop_front();
    return dq.front().first * x + dq.front().second;
  void add(int a, int b) {
    while (dq.size() >= 2 && elim(dq[dq.size() - 1],
     [dq.size() - 2], make_pair(a, b))) dq.pop_back();
    dq.push_back(make_pair(a, b));
};
```

8.3 Convex Hull Optimization

```
int dp[maxn], type;
struct line {
  int a, b, l, r;
  int operator()(const int& d) const { return a * d + b
  int intersect(const line& rhs) const { return (rhs.b
    - b) / (a - rhs.a); }
  bool operator<(const line& rhs) const { return type ?</pre>
     r < rhs.l : a < rhs.a; }
void solve() {
  set<line> st; st.insert((line){ a[0], b[0], -inf, inf
  for (int i = 1; i \le n + 1; ++i)
    dp[i] = (*st.lower_bound((line){0, 0, x[i], x[i]})
    }));
    line nl = (line){a[i], b[i], -inf, inf};
    auto it = st.lower_bound(nl);
    bool ins = true;
    while (it != st.end()) {
      if ((*it)(it->l) > nl(it->l) && (*it)(it->r) > nl
    (it->r)) {
        ins = false;
        break;
      if ((*it)(it->l) <= nl(it->l) && (*it)(it->r) <=
    nl(it->r)) it = st.erase(it);
      else break;
    if (!ins) continue;
    while (it != st.begin()) {
      --it;
      if ((*it)(it->l) <= nl(it->l) && (*it)(it->r) <=
    nl(it->r)) it = st.erase(it);
      else { ++it; break; }
    if (it != st.begin()) {
      --it;
      line tmp = *it; it = st.erase(it);
      long long t = nl.intersect(tmp);
      tmp.r = t; nl.l = t + 1;
      st.insert(tmp);
    if (it != st.end()) {
      line tmp = *it; it = st.erase(it);
      long long t = nl.intersect(tmp);
      tmp.l = t + 1; nl.r = t;
      st.insert(tmp);
    st.insert(nl);
  }
}
```

8.4 Convex Montone Optimization

```
void convex() {
  deque<segment> dq; dq.push_back({ 0, 1, n });
  for (int i = 1; i <= n; ++i) {
  dp[i] = f(dq.front().i, i);</pre>
    while (dq.size() && dq.front().r < i + 1)</pre>
       dq.pop_front();
    dq.front().l = i + 1
    while (dq.size() && f(i, dq.back().1) < f(dq.back()</pre>
     .i, dq.back().l))
       dq.pop_back();
    segment new_seg = \{ i, i + 1, n \};
    if (dq.size()) {
       int c = bs();
       // c is the maximal k such that f(i, k) > f(dq.
    back().i, k)
       dq.back().r = c;
       new_seg.l = c + 1;
     if (new_seg.l <= n) dq.push_back(new_seg);</pre>
```

|}

8.5 Divide and Conquer Optimization

```
int dp[maxk][maxn], f[maxk][maxn];

void go(int k, int l, int r, int fl, int fr) {
    if (l > r) return;
    int m = (l + r) >> 1;
    f[k][m] = -1;
    for (int i = fl; i <= min(m - 1, fr); ++i) {
        int t = dp[k - 1][i] + f(i + 1, m);
        if (t > dp[k][m]) {
            dp[k][m] = t;
            f[k][m] = i;
        }
    }
    go(k, l, m - 1, fl, f[k][m]);
    go(k, m + 1, r, f[k][m], fr);
}

void solve() {
    for (int i = 1; i <= k; ++i) go(i, 1, n, 0, n - 1);
}</pre>
```