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# 1 Basic

#### 1.1 vimrc

```
syn on
se ai nu ru mouse=a
se cin et ts=4 sw=4 sts=4
so $VIMRUNTIME/mswin.vim
colo desert
se gfn=Monospace\ 15
execute pathogen#infect()
```

### 1.2 Fast Integer Input

```
#define getchar gtx
inline int gtx() {
   const int N = 1048576;
   static char buffer[N];
   static char *p = buffer, *end = buffer;
   if (p == end) {
      if ((end = buffer + fread(buffer, 1, N, stdin)) ==
      buffer) return EOF;
      p = buffer;
   return *p++;
}
template <typename T>
inline bool rit(T& x) {
  char __c = 0; bool flag = false;
while (__c = getchar(), (__c < '0' && __c != '-') ||
    __c > '9') if (__c == -1) return false;
    __c == '-' ? (flag = true, x = 0) : (x = __c - '0');
while (__c = getchar(), __c >= '0' && __c <= '9') x =
    x * 10 + __c - '0';
if (flag) x = _x'
   if (flag) x = -x;
   return true;
}
template <typename T, typename ...Args>
inline bool rit(T& x, Args& ...args) { return rit(x) &&
        rit(args...); }
```

# 2 Flow

# 2.1 Dinic's

```
struct Dinic {
  int n, s, t;
  vector<int> level;
  struct Edge {
    int to, rev, cap;
    Edge() {}
    Edge(int a, int b, int c): to(a), cap(b), rev(c) {}
  vector<Edge> G[maxn];
  bool bfs() {
    level.assign(n, -1);
    level[s] = 0;
    queue<int> que; que.push(s);
while (que.size()) {
      int tmp = que.front(); que.pop();
      for (auto e : G[tmp]) {
        if (e.cap > 0 \& level[e.to] == -1) {
          level[e.to] = level[tmp] + 1;
          que.push(e.to);
      }
    return level[t] != -1;
  int flow(int now, int low) {
    if (now == t) return low;
    int ret = 0;
```

```
for (auto &e : G[now]) {
      if (e.cap > 0 \& level[e.to] == level[now] + 1) {
         int tmp = flow(e.to, min(e.cap, low - ret));
         e.cap -= tmp; G[e.to][e.rev].cap += tmp;
         ret += tmp;
    if (ret == 0) level[now] = -1;
    return ret;
  Dinic(int _n, int _s, int _t): n(_n), s(_s), t(_t) {
    fill(G, G + maxn, vector<Edge>());
  void add_edge(int a, int b, int c) {
   G[a].push_back(Edge(b, c, G[b].size()));
    G[b].push_back(Edge(a, 0, G[a].size() - 1));
  int maxflow() {
    int ret = 0;
    while (bfs()) ret += flow(s, inf);
     return ret;
};
```

### 2.2 MinCostMaxFlow

```
struct MincostMaxflow {
  struct Edge {
    int to, rev, cap, w;
    Edge() {}
    Edge(int a, int b, int c, int d): to(a), cap(b), w(
c), rev(d) {}
  };
  int n, s, t, p[maxn], id[maxn];
  int d[maxn];
  bool inque[maxn];
  vector<Edge> G[maxn];
  pair<int, int> spfa() {
    memset(p, -1, sizeof(-1));
fill(d, d + maxn, inf);
    memset(id, -1, sizeof(id));
d[s] = 0; p[s] = s;
    queue<int> que; que.push(s); inque[s] = true;
    while (que.size()) {
      int tmp = que.front(); que.pop();
      inque[tmp] = false;
      int i = 0;
      for (auto e : G[tmp]) {
         if (e.cap > 0 \&\& d[e.to] > d[tmp] + e.w) {
           d[e.to] = d[tmp] + e.w;
           p[e.to] = tmp;
           id[e.to] = i;
           if (!inque[e.to]) que.push(e.to), inque[e.to]
      = true:
        }
         ++i;
    if (d[t] == inf) return make_pair(-1, -1);
    int a = inf;
for (int i = t; i != s; i = p[i]) {
      a = min(a, G[p[i]][id[i]].cap);
    for (int i = t; i != s; i = p[i]) {
      Edge &e = G[p[i]][id[i]];
      e.cap -= a; G[e.to][e.rev].cap += a;
    return make_pair(a, d[t]);
  MincostMaxflow(int _n, int _s, int _t): n(_n), s(_s),
     t(_t) {
    fill(G, G + maxn, vector<Edge>());
  void add_edge(int a, int b, int cap, int w) {
    G[a].push_back(Edge(b, cap, w, (int)G[b].size()));
G[b].push_back(Edge(a, 0, -w, (int)G[a].size() - 1)
  pair<int, int> maxflow() {
  int mxf = 0, mnc = 0;
```

```
while (true) {
    pair<int, int> res = spfa();
    if (res.first == -1) break;
    mxf += res.first; mnc += res.first * res.second;
    }
    return make_pair(mxf, mnc);
}
```

```
2.3 Hungarian
struct Hungarian {
  vector<vector<int>> w;
  bitset<maxn>_s, t;
  vector<int> lx, ly, mx, my, slack, prv;
  int n, matched;
  Hungarian() {}
  Hungarian(int _n): n(_n) {
    w = vector<vector<int>>(n, vector<int>(n));
    lx.resize(n); ly.resize(n); mx.assign(n, -1); my.
    assign(n, -1);
    slack.resize(n); prv.resize(n);
  void add_edge(int a, int b, int c) {
   w[a][b] = c;
  void add(int x) {
    s[x] = true;
    for (int i = 0; i < n; ++i) {
      if (lx[x] + ly[i] - w[x][i] < slack[i]) {
        slack[i] = lx[x] + ly[i] - w[x][i];
        prv[i] = x;
    }
  void augment(int now) {
    int x = prv[now], y = now;
    ++matched;
    while (true) {
      int tmp = mx[x]; mx[x] = y; my[y] = x; y = tmp;
      if (y == -1) return;
      x = prv[y];
    }
  void relabel() {
    int delta = inf;
    for (int i = 0; i < n; ++i) if (!t[i]) delta = min(
    delta, slack[i]);
    for (int i = 0; i < n; ++i) if (s[i]) lx[i] -=
    delta;
    for (int i = 0; i < n; ++i) {
      if (t[i]) ly[i] += delta;
      else slack[i] -= delta;
  }
  void go() {
    s.reset(); t.reset();
    fill(slack.begin(), slack.end(), inf);
    int root = 0;
    for (; root < n && mx[root] != -1; ++root);</pre>
    add(root);
    while (true) {
      relabel();
      int y = 0;
      for (; y < n; ++y) if (!t[y] && slack[y] == 0)
    break
      if (my[y] == -1) return augment(y), void();
      add(my[y]); t[y] = true;
    }
  int matching() {
    int ret = 0;
    for (int i = 0; i < n; ++i) {
      for (int j = 0; j < n; ++j) lx[i] = max(lx[i], w[
    i][j]);
    for (int i = 0; i < n; ++i) go();
    for (int i = 0; i < n; ++i) ret += w[i][mx[i]];</pre>
    return ret;
```

|};

### 3 Data Structure

#### 3.1 Disjoint Set

```
struct DisjointSet {
  int p[maxn], sz[maxn], n, cc;
  vector<pair<int*, int>> his;
  vector<int> sh;
  void init(int _n) {
    n = _n; cc = n;
    for (int i = 0; i < n; ++i) sz[i] = 1, p[i] = i;
    sh.clear(); his.clear();
  void assign(int *k, int v) {
    his.emplace_back(k, *k);
    *k = v;
  void save() {
    sh.push_back((int)his.size());
  void undo() {
    int last = sh.back(); sh.pop_back();
    while (his.size() != last) {
      int *k, v;
      tie(k, v) = his.back(); his.pop_back();
      *k = v;
    }
  int find(int x) {
    if (x == p[x]) return x;
    return find(p[x]);
  void merge(int x, int y) {
    x = find(x); y = find(y);
    if (x == y) return;
if (sz[x] > sz[y]) swap(x, y);
    assign(\&sz[y], sz[x] + sz[y]);
    assign(&p[x], y);
    assign(\&cc, cc - 1);
} dsu;
```

### 3.2 Splay Tree

```
struct node {
 static node nil;
 node *ch[2], *fa;
int val, sz, tag;
 node(): sz(0), tag(0), val(-1) { fa = ch[0] = ch[1] = }
     &nil; }
 node(int v): val(v), sz(1), tag(0) { fa = ch[0] = ch}
    [1] = &nil; }
 bool r() { return fa->ch[0] != this && fa->ch[1] !=
    this:
  int dir() { return fa->ch[0] == this ? 0 : 1; }
 void pull() {
    sz = ch[0]->sz + ch[1]->sz + 1;
if (ch[0] != &nil) ch[0]->fa = this;
    if (ch[1] != &nil) ch[1]->fa = this;
  void push() {
    if (tag == 0) return;
    swap(ch[0], ch[1]);
    if (ch[0] != &nil) ch[0]->tag ^= 1;
    if (ch[1] != &nil) ch[1]->tag ^= 1;
    tag = 0;
  void addch(node *c, int d) {
    ch[d] = c;
    if (c != &nil) c->fa = this;
    pull();
} node::nil;
```

```
node *nil = &node::nil;
void rotate(node *s) {
  node *p = s->fa;
  int d = s->dir()
  if (!p->r()) p->fa->addch(s, p->dir());
  else s->fa = p->fa;
  p->addch(s->ch[d ^ 1], d);
  s->addch(p, d ^ 1)
  p->pull(); s->pull();
void splay(node *s) {
  vector<node*> vec;
  for (node *n = s; ; n = n \rightarrow fa) {
    vec.push_back(n);
    if (n->r()) break;
  reverse(vec.begin(), vec.end());
  for (auto it : vec) it->push();
  while (!s->r()) {
    if (s->fa->r()) rotate(s);
    else if (s->dir() == s->fa->dir()) rotate(s->fa),
    rotate(s);
    else rotate(s), rotate(s);
}
```

#### 3.3 Link-Cut Tree

```
node *access(node *s) {
  node *n = nil;
  for (; s != nil; s = s->fa) {
    splav(s):
    s->addch(n, 1);
    n = s;
  return n;
}
void evert(node *s) {
  access(s); splay(s);
  s->tag ^= 1;
  s->push(); s->pull();
void link(node *a, node *b) {
  access(a); splay(a);
  evert(b)
  a->addch(b, 1);
void cut(node *a, node *b) {
  access(b); splay(b);
  b->push();
  b - ch[0] = b - ch[0] - fa = nil;
node *find(node *s) {
  s = access(s)
  while (s\rightarrow ch[0] != nil) s = s\rightarrow ch[0];
  splay(s);
  return s;
int query(node *a, node *b) {
  access(a); access(b);
  splay(a);
  int ret = a->fa->val;
  if (ret == -1) ret = a->val;
  return ret;
```

# 3.4 < ext/pbds >

```
#include <bits/stdc++.h>
#include <bits/extc++.h>
#include <ext/rope>
```

```
using namespace __gnu_pbds;
using namespace __gnu_cxx;
#include <ext/pb_ds/assoc_container.hpp>
typedef tree<int, null_type, std::less<int>,
    rb_tree_tag, tree_order_statistics_node_update>
    tree_set;
typedef cc_hash_table<int, int> umap;
typedef priority_queue<int> heap;
int main() {
  // rb tree
 tree_set s:
  s.insert(71); s.insert(22);
 assert(*s.find_by_order(0) == 22); assert(*s.
    find_by_order(1) == 71);
  assert(s.order\_of\_key(22) == 0); assert(s.
    order_of_key(71) == 1);
  s.erase(22)
 assert(*s.find_by_order(0) == 71); assert(s.
    order_of_key(71) == 0);
  // mergable heap
 heap a, b; a.join(b);
 // persistant
 rope<char> r[2];
 r[1] = r[0];
 std::string st = "abc";
r[1].insert(0, st.c_str());
 r[1].erase(1, 1);
  std::cout << r[1].substr(0, 2) << std::endl;
  return 0;
```

# 4 Graph

### 4.1 Heavy-Light Decomposition

```
struct HeavyLightDecomp {
 vector<int> G[maxn];
  int tin[maxn], top[maxn], dep[maxn], maxson[maxn], sz
    [maxn], p[maxn], n, clk;
  void dfs(int now, int fa, int d) {
   dep[now] = d;
   maxson[now] = -1;
    sz[now] = 1;
   p[now] = fa;
for (int u : G[now]) if (u != fa) {
      dfs(u, now, d + 1);
      sz[now] += sz[u];
      if (maxson[now] == -1 || sz[u] > sz[maxson[now]])
     maxson[now] = u;
 void link(int now, int t) {
    top[now] = t;
    tin[now] = ++clk;
    if (maxson[now] == -1) return;
    link(maxson[now], t);
    for (int u : G[now]) if (u != p[now]) {
      if (u == maxson[now]) continue;
      link(u, u);
   }
 HeavyLightDecomp(int n): n(n) {
   clk = 0;
   memset(tin, 0, sizeof(tin)); memset(top, 0, sizeof(
    top)); memset(dep, 0, sizeof(dep));
   memset(maxson, 0, sizeof(maxson)); memset(sz, 0,
    sizeof(sz)); memset(p, 0, sizeof(p));
 void add_edge(int a, int b) {
    G[a].push_back(b);
    G[b].push_back(a);
  void solve() -
    dfs(0, -1, 0);
    link(0, 0);
 int lca(int a, int b) {
```

```
int ta = top[a], tb = top[b];
    while (ta != tb)
      if (dep[ta] < dep[tb]) {</pre>
        swap(ta, tb); swap(a, b);
      a = p[ta]; ta = top[a];
    if (a == b) return a;
    return dep[a] < dep[b] ? a : b;</pre>
  vector<pair<int, int>> get_path(int a, int b) {
    int ta = top[a], tb = top[b];
    vector<pair<int, int>> ret;
    while (ta != tb) {
  if (dep[ta] < dep[tb]) {</pre>
         swap(ta, tb); swap(a, b);
      ret.push_back(make_pair(tin[ta], tin[a]));
      a = p[ta]; ta = top[a];
    ret.push_back(make_pair(min(tin[a], tin[b]), max(
     tin[a], tin[b])));
    return ret;
};
```

# 4.2 Centroid Decomposition

```
vector<pair<int, int>> G[maxn];
int sz[maxn], mx[maxn];
bool v[maxn];
vector<int> vtx;
void get_center(int now) {
  v[now] = true; vtx.push_back(now);
  sz[now] = 1; mx[now] = 0;
  for (int u : G[now]) if (!v[u]) {
    get_center(u);
    mx[now] = max(mx[now], sz[u]);
    sz[now] += sz[u];
}
void get_dis(int now, int d, int len) {
  dis[d][now] = cnt;
  v[now] = true;
  for (auto u : G[now]) if (!v[u.first]) {
    get_dis(u, d, len + u.second);
}
void dfs(int now, int fa, int d) {
  get_center(now);
  int c = -1;
  for (int i : vtx) {
    if (max(mx[i], (int)vtx.size() - sz[i]) <= (int)vtx</pre>
     .size() / 2) c = i;
    v[i] = false;
  get_dis(c, d, 0);
for (int i : vtx) v[i] = false;
  v[c] = true; vtx.clear();
  dep[c] = d; p[c] = fa;
for (auto u : G[c]) if (u.first != fa && !v[u.first])
    dfs(u.first, c, d + 1);
```

### 4.3 Maximum Clique

```
struct MaxClique {
  int n, deg[maxn], ans;
  bitset<maxn> adj[maxn];
  vector<pair<int, int>> edge;
  void init(int _n) {
    _n = n;
    for (int i = 0; i < n; ++i) adj[i].reset();</pre>
```

```
void add_edge(int a, int b) {
    edge.emplace_back(a, b);
    ++deg[a]; ++deg[b];
  int solve() {
    vector<int> ord;
    for (int i = 0; i < n; ++i) ord.push_back(i);
    sort(ord.begin(), ord.end(), [&](const int &a,
const int &b) { return deg[a] < deg[b]; });</pre>
     vector<int> id(n);
    for (int i = 0; i < n; ++i) id[ord[i]] = i;</pre>
     for (auto e : edge) {
       int u = id[e.first], v = id[e.second];
       adj[u][v] = adj[v][u] = true;
    bitset<maxn> r, p;
for (int i = 0; i < n; ++i) p[i] = true;
    dfs(r, p);
    return ans;
  void go(bitset<maxn> r, bitset<maxn> p) {
    if (1.0 * clock() / CLOCKS_PER_SEC >= time_limit)
    if (p.count() == 0) return ans = max(ans, (int)r.
     count()), void();
     if ((r | p).count() <= ans) return;</pre>
    int now = p._Find_first();
    bitset<maxn> cur = p & ~adj[now];
    for (now = cur._Find_first(); now < n; now = cur.</pre>
     _Find_next(now)) {
       r[now] = true
       go(r, p & adj[now]);
r[now] = false;
       p[now] = false;
  }
};
```

### 4.4 Tarjan's

```
int tin[maxn], low[maxn], t, bccsz;
stack<int> st;
vector<int> bcc[maxn];
void dfs(int now, int fa) {
  tin[now] = ++t; low[now] = tin[now];
  st.push(now);
  for (int u : G[now]) if (u != fa) {
    if (!tin[u]) {
      dfs(u, now);
      low[now] = min(low[now], low[u]);
       if (low[u] >= tin[now]) {
         int v;
         ++bccsz;
         do {
           v = st.top(); st.pop();
           bcc[bccsz].push_back(v);
         } while (v != u);
         bcc[bccsz].push_back(now);
    } else {
      low[now] = min(low[now], tin[u]);
  }
}
```

# 5 String

#### 5.1 KMP

```
int f[maxn];
int kmp(const string& a, const string& b) {
   f[0] = -1; f[1] = 0;
   for (int i = 1, j = 0; i < b.size() - 1; f[++i] = ++j
      ) {</pre>
```

```
if (b[i] == b[j]) f[i] = f[j];
while (j != -1 && b[i] != b[j]) j = f[j];
}
for (int i = 0, j = 0; i - j + b.size() <= a.size();
++i, ++j) {
  while (j != -1 && a[i] != b[j]) j = f[j];
  if (j == b.size() - 1) return i - j;
}
return -1;
}</pre>
```

### 5.2 Z algorithm

```
int z[maxn];

void z_function(const string& s) {
    memset(z, 0, sizeof(z));
    z[0] = (int)s.length();
    int l = 0, r = 0;
    for (int i = 1; i < s.length(); ++i) {
        z[i] = max(0, min(z[i - l], r - i + 1));
        while (i + z[i] < s.length() && s[z[i]] == s[i + z[i]]) {
            l = i; r = i + z[i];
            ++z[i];
        }
    }
}</pre>
```

#### 5.3 Manacher's

#### 5.4 Aho-Corasick

```
struct AC {
  int ptr, ql, qr, root;
  vector<int> cnt, q, ed, el, ch[sigma], f;
  void clear(int p) { for (int i = 0; i < sigma; ++i)
  ch[i][p] = 0; }</pre>
  int newnode() { clear(ptr); ed[ptr] = 0; return ptr
  ++; }
void init() {
    ptr = 1; cnt.resize(maxn); q.resize(maxn);
    ed.resize(maxn); el.resize(maxn); f.resize(maxn);
for (int i = 0; i < sigma; ++i) ch[i].resize(maxn);</pre>
    root = newnode();
  int add(const string &s) {
     int now = root;
     for (int i = 0; i < s.length(); ++i) {</pre>
       if (ch[s[i]][now] == 0) ch[s[i]][now] = newnode()
       now = ch[s[i]][now];
    ed[now] = 1;
    return now;
  void build_fail() {
```

```
ql = qr = 0; q[qr++] = root;
     while (ql < qr) {
       int now = q[ql++];
       for (int i = 0; i < sigma; ++i) if (ch[i][now]) {</pre>
         int p = ch[i][now], fp = f[now];
while (fp && !ch[i][fp]) fp = f[fp];
          int pd = fp ? ch[i][fp] : root;
          f[p] = pd;
          el[p] = ed[pd] ? pd : el[pd];
          q[qr++] = p;
    }
  void build(const string &s) {
    build_fail();
     int now = 1;
     for (int i = 0; i < s.length(); ++i) {
  while (now && !ch[s[i]][now]) now = f[now];</pre>
       now = now ? ch[s[i]][now] : root;
       ++cnt[now];
     for (int i = qr - 1; i >= 0; --i) cnt[f[q[i]]] +=
     cnt[q[i]];
};
```

# 5.5 Suffix Array

```
struct SuffixArray {
  int sa[maxn], tmp[2][maxn], c[maxn], _lcp[maxn], r[
    maxn], n;
  string s
  SparseTable st;
  void suffixarray()
     int* rank = tmp[0];
    int* nRank = tmp[1];
    int A = 128;
    for (int i = 0; i < A; ++i) c[i] = 0;
    for (int i = 0; i < s.length(); ++i) c[rank[i] = s[
    i]]++;
    for (int i = 1; i < A; ++i) c[i] += c[i - 1];
    for (int i = s.length() - 1; i \ge 0; --i) sa[--c[s[
    i]]] = i;
    for (int n = 1; n < s.length(); n *= 2) {
  for (int i = 0; i < A; ++i) c[i] = 0;</pre>
      for (int i = 0; i < s.length(); ++i) c[rank[i</pre>
    ]]++;
      for (int i = 1; i < A; ++i) c[i] += c[i - 1];
      int* sa2 = nRank;
      int r = 0;
      for (int i = s.length() - n; i < s.length(); ++i)</pre>
     sa2[r++] = i;
      for (int i = 0; i < s.length(); ++i) if (sa[i] >=
     n) sa2[r++] = sa[i] - n;
      for (int i = s.length() - 1; i >= 0; --i) sa[--c[
    rank[sa2[i]]] = sa2[i];
      nRank[sa[0]] = r = 0;
      for (int i = 1; i < s.length(); ++i) {</pre>
        if (!(rank[sa[i - 1]] == rank[sa[i]] && sa[i -
    1] + n < s.length() && rank[sa[i - 1] + n] == rank[
    sa[i] + n])) r++;
        nRank[sa[i]] = r;
      swap(rank, nRank);
      if (r == s.length() - 1) break;
      A = r + 1;
  void solve() {
    suffixarray();
    for (int i = 0; i < n; ++i) r[sa[i]] = i;
    int ind = 0; _lcp[0] = 0;
    for (int i = 0; i < n; ++i) {
  if (!r[i]) { ind = 0; continue; }</pre>
      while (i + ind < n \&\& s[i + ind] == s[sa[r[i] -
    1] + ind]) ++ind;
       lcp[r[i]] = ind ? ind-- : 0;
    st = SparseTable(n, _lcp);
```

```
int lcp(int L, int R) {
   if (L == R) return n - L - 1;
   L = r[L]; R = r[R];
   if (L > R) swap(L, R);
   ++L;
   return st.query(L, R);
}
SuffixArray(string s): s(s), n(s.length()) {}
SuffixArray() {}
};
```

### 5.6 Smallest Rotation

```
string rotate(const string &s) {
  int n = s.length();
  string t = s + s;
  int i = 0, j = 1;
  while (i < n && j < n) {
    int k = 0;
    while (k < n && s[i + k] == s[j + k]) ++k;
    if (s[i + k] <= s[j + k]) j += k + 1;
    else i += k + 1;
    if (i == j) ++j;
  }
  int pos = (i < n ? i : j);
  return s.substr(pos, n);
}</pre>
```

# 5.7 Primes (hasing)

```
const int mod[] = { 479001599, 433494437, 1073807359,
    1442968193, 715827883 }
const int p[] = { 101, 233, 457, 173, 211 }
```

# 6 Math

#### 6.1 FFT

```
const double pi = acos(-1);
const complex<double> I(0, 1);
complex<double> omega[maxn + 1];
void prefft() {
 for (int i = 0; i <= maxn; ++i) omega[i] = exp(i * 2
 * pi / maxn * I);</pre>
void fft(vector<complex<double>>& a, int n, bool inv =
    false) {
  int basic = maxn / n;
  int theta = basic;
  for (int m = n; m >= 2; m >>= 1) {
    int h = m >> 1;
for (int i = 0; i < h; ++i) {
      complex < double > w = omega[inv ? maxn - (i * theta)]
     % maxn) : i * theta % maxn];
      for (int j = i; j < n; j += m) {
  int k = j + h;</pre>
         complex<\overline{double}> x = a[j] - a[k];
         a[j] += a[k];
         a[k] = w * x;
    theta = (theta * 2) \% maxn;
  }
  int i = 0;
  for (int j = 1; j < n - 1; ++j) {
    for (int k = n >> 1; k > (i^k = k); k >>= 1);
    if (j < i) swap(a[i], a[j]);</pre>
  if (inv) for (int i = 0; i < n; ++i) a[i] /= (double)
    n;
```

#### 6.2 NTT

```
const long long p = 2013265921, root = 31;
long long omega[maxn + 1];
long long fpow(long long a, long long n) {
  long long ret = 111;
  for (; n; n >>= 1) {
    if (n & 1) ret = ret * a % p;
    a = a * a % p;
  return ret;
void prentt() {
  omega[0] = 1;
  long long r = fpow(root, (p - 1) / maxn);
  for (int i = 1; i \le maxn; ++i) omega[i] = omega[i - maxn]
void ntt(vector<long long>& a, int n, bool inv = false)
  int basic = maxn / n;
  int theta = basic;
  for (int m = n; m >= 2; m >>= 1) {
    int mh = m >> 1;
for (int i = 0; i < mh; ++i) {
  long long w = omega[i * theta % maxn];</pre>
       for (int j = i; j < n; j += m) {
  int k = j + mh;</pre>
          long long x = a[j] - a[k];
         if (x < 0) x += p;
a[j] += a[k];
if (a[j] > p) a[j] -= p;
a[k] = w * x % p;
     theta = theta * 2 % maxn;
  int i = 0:
  for (int j = 1; j < n - 1; ++j) {
  for (int k = n >> 1; k > (i ^= k); k >>= 1);
  if (j < i) swap(a[i], a[j]);</pre>
  if (!inv) return;
  long long ni = fpow(n, p - 2);
  reverse(a.begin() + 1, a.end());
  for (int i = 0; i < n; ++i) a[i] = a[i] * ni % p;
```

#### 6.3 Miller Rabin

```
// n < 4759123141
                         chk = [2, 7, 61]
// n < 1122004669633 chk = [2, 13, 23, 1662803]
// n < 2^64 chk = [2, 325, 9375, 28178, 450775,
     9780504, 1795265022]
long long fpow(long long a, long long n, long long mod)
  long long ret = 1LL;
  for (; n; n >>= 1) {
   if (n & 1) ret = (__int128)ret * (__int128)a % mod;
    a = (__int128)a * (__int128)a % mod;
  }
  return ret;
bool check(long long a, long long u, long long n, int t
  a = fpow(a, u, n);
  if (a == 0) return true;
  if (a == 1 || a == n - 1) return true;
for (int i = 0; i < t; ++i) {
    a = (__int128)a * (__int128)a % n;</pre>
     if (a == 1) return false;
    if (a == n - 1) return true;
  return false;
```

```
bool is_prime(long long n) {
   if (n < 2) return false;
   if (n % 2 == 0) return n == 2;
   long long u = n - 1; int t = 0;
   for (; u & 1; u >>= 1, ++t);
   for (long long i : chk) {
      if (!check(i, u, n, t)) return false;
   }
   return true;
}
```

### 6.4 Guassian Elimination

```
double dat[maxn][maxn];

void guass(int n) {
    for (int i = 0; i < n; ++i) {
        bool ok = false;
        for (int j = i; j < n; ++j) {
            if (fabs(dat[i][j]) > eps) {
                swap(dat[i], dat[j]);
                 ok = true;
                 break;
            }
            if (!ok) continue;
            double f = dat[i][i];
            for (int j = i + 1; j < n; ++j) {
                 double r = dat[j][i] / f;
                 for (int k = i; k < n; ++k) dat[j][k] -= dat[i][k]
            ] * r;
            }
        }
    }
}</pre>
```

#### 6.5 $\mu$ function

### 6.6 $\left| \frac{n}{\epsilon} \right|$ Enumeration

### 6.7 Extended GCD

bc4iaavnedisonihao123

```
template <typename T> tuple<T, T, T> extgcd(T a, T b) {
   if (!b) return make_tuple(a, 1, 0);
   T d, x, y;
   tie(d, x, y) = extgcd(b, a % b);
   return make_tuple(d, y, x - (a / b) * y);
}
double PointToSegDist(Point A, Point B, Point C) {
   if (dis(A, B) < eps) return dis(B, C);
   if (epssgn(Dot(B - A, C - A)) < 0) return dis(A, if (epssgn(Dot(A - B, C - B)) < 0) return dis(B, return fabs(Cross(B - A, C - A)) / dis(B, A);
}

return fabs(Cross(B - A, C - A)) / dis(B, A);
}
</pre>
```

# 7 Geometry

#### 7.1 Basic

```
const double eps = 1e-8;
const double pi = acos(-1);
struct Point {
  double x, y;
  Point(double a = 0, double b = 0): x(a), y(b) {}
typedef Point Vector;
// L:ax+by+c=0
struct Line {
  double a, b, c, angle;
  Point p1, p2;
  Line() {}
  Line(Point s, Point e) {
   a = s.y - e.y, b = e.x - s.x;
c = s.x * e.y - e.x * s.y;
angle = atan2(e.y - s.y, e.x - s.x);
    p1 = s, p2 = e;
};
struct Segment {
  Point s, e;
Segment() {}
  Segment(Point a, Point b): s(a), e(b) {}
  Segment(double x1, double y1, double x2, double y2) {
    s = Point(x1, y1);
    e = Point(x2, y2);
};
Vector operator+(Point a, Point b) { return Vector(a.x
    + b.x, a.y + b.y); }
Vector operator-(Point a, Point b) { return Vector(a.x
    - b.x, a.y - b.y); }
Vector operator*(Point a, double k) { return Vector(a.x
     * k, a.y * k); }
Vector operator/(Point a, double k) { return Vector(a.x
     / k, a.y / k); }
double len(Vector a) { return sqrt(a.x * a.x + a.y * a.
// <0 when ep at opsp clockwise
double Cross(Point &sp, Point &ep, Point &op) { return
    (sp.x - op.x) * (ep.y - op.y) - (ep.x - op.x) * (sp.x - op.x)
    .y - op.y); }
double Cross(Vector a, Vector b) { return a.x * b.y - b
    .x * a.y; }
double Dot(Vector a, Vector b) { return a.x * b.x + a.y
     * b.y; }
int epssgn(double x) {
  if (fabs(x) < eps) return 0;</pre>
  else return x < 0? -1 : 1;
double dis(Point a, Point b) { return sqrt((a.x - b.x)
    * (a.x - b.x) + (a.y - b.y) * (a.y - b.y)); }
bool Parallel(Line l1, Line l2) { return fabs(l1.a * l2
    .b - l2.a * l1.b) < eps;
bool LineEqual(Line 11, Line 12) { return Parallel(l1,
    12) && fabs(l1.a * l2.c - l2.a * l1.c) < eps &&
    fabs(l1.b * l2.c - l2.b * l1.c) < eps; }
```

```
if (dis(A, B) < eps) return dis(B, C);</pre>
  if (epssgn(Dot(B - A, C - A)) < 0) return dis(A, C);
if (epssgn(Dot(A - B, C - B)) < 0) return dis(B, C);</pre>
  return fabs(Cross(B - A, C - A)) / dis(B, A);
double TwoSegMinDist(Point A, Point B, Point C, Point D
     ) { return min(min(PointToSegDist(A, B, C)
    PointToSegDist(A, B, D)), min(PointToSegDist(C, D, A), PointToSegDist(C, D, B))); }
Point SymPoint(Point p, Line l) {
  Point result;
  double a = 1.p2.x - 1.p1.x;
  double b = 1.p2.y - 1.p1.y;
  double t = ((p.x - l.p1.x) * a + (p.y - l.p1.y) * b)
  / (a * a + b * b);
result.x = 2 * l.p1.x + 2 * a * t - p.x;
  result.y = 2 * l.p1.y + 2 * b * t - p.y;
  return result;
// without end points: <= -> <
bool IsSegmentIntersect(Point s1, Point e1, Point s2,
    Point e2) {
  if (min(s1.x, e1.x) \le max(s2.x, e2.x) \&\&
    min(s1.y, e1.y) \le max(s2.y, e2.y) &&
    min(s2.x, e2.x) \le max(s1.x, e1.x) \&\&
    min(s2.y, e2.y) <= max(s1.y, e1.y) &&
Cross(s2, e2, s1) * Cross(s2, e2, e1) <= 0 &&
    Cross(s1, e1, s2) * Cross(s1, e1, e2) \leftarrow 0) return
    1:
  return 0;
}
int IsLineIntersectSegment(Point p1, Point p2, Point s,
      Point e){ return !Cross(p1, p2, s) * Cross(p1, p2,
      e) > eps; }
int IsLineIntersectSegment(Line l1, Point s, Point e) {
      return !Cross(l1.p1, l1.p2, s) * Cross(l1.p1, l1.
     p2, e) > eps; }
Point GetIntersect(Line 11, Line 12) {
  Point res;
  res.x = (1.b * 12.c - 12.b * 11.c) / (11.a * 12.b -
  l2.a * l1.b);
res.y = (l1.c * l2.a - l2.c * l1.a) / (l1.a * l2.b -
    l2.a * l1.b);
  return res;
```

### 7.2 Triangle Center

double lb = len(a - c);

```
Point TriangleCircumCenter(Point a, Point b, Point c) {
  Point res;
  double a1 = atan2(b.y - a.y, b.x - a.x) + pi / 2;
  double a2 = atan2(c.y - b.y, c.x - b.x) + pi / 2;
  double ax = (a.x + b.x) / 2
  double ay = (a.y + b.y) / 2;
  double bx = (c.x + b.x) / 2;
  double by = (c.y + b.y) / 2;
double r1 = (sin(a2) * (ax - bx) + cos(a2) * (by - ay) / (sin(a1) * cos(a2) - sin(a2) * cos(a1));
  return Point(ax + r1 * cos(a1), ay + r1 * sin(a1));
Point TriangleMassCenter(Point a, Point b, Point c) {
  return (a + b + c) / 3.0;
Point TriangleOrthoCenter(Point a, Point b, Point c) {
  return TriangleMassCenter(a, b, c) * 3.0 -
TriangleCircumCenter(a, b, c) * 2.0;
Point TriangleInnerCenter(Point a, Point b, Point c) {
  Point res;
  double la = len(b - c);
```

```
double lc = len(a - b);
res.x = (la * a.x + lb * b.x + lc * c.x) / (la + lb +
    lc);
res.y = (la * a.y + lb * b.y + lc * c.y) / (la + lb +
    lc);
return res;
}
```

#### 7.3 Sector Area

```
// calc area of sector which include a, b
double SectorArea(Point a, Point b, double r) {
  double theta = atan2(a.y, a.x) - atan2(b.y, b.x);
  while (theta <= 0) theta += 2 * pi;
  while (theta >= 2 * pi) theta -= 2 * pi;
  theta = min(theta, 2 * pi - theta);
  return r * r * theta / 2;
}
```

### 7.4 Polygon Area

```
// point sort in counterclockwise
double ConvexPolygonArea(vector<Point> &p, int n) {
  double area = 0;
  for (int i = 1; i < p.size() - 1; i++) area += Cross(
    p[i] - p[0], p[i + 1] - p[0]);
  return area / 2;
}</pre>
```

#### 7.5 Half Plane Intersection

```
int cmp(const Line &l1, const Line &l2) {
  int d = epssgn(l1.angle - l2.angle);
  if (!d) return (epssgn(Cross(l2.p1 - l1.p1, l2.p2 -
    11.p1)) > 0);
  return d < 0;</pre>
void QSort(Line L[], int 1, int r) {
  int i = l, j = r;
Line swap, mid = L[(l+r) / 2];
 while (i <= j) {
  while (cmp(L[i], mid)) ++i;
  while (cmp(mid, L[j])) --j;</pre>
    if (i <= j) {
      swap = L[i];
      L[i] = L[j];
      L[j] = swap;
      ++i, --j;
  if (i < r) QSort(L, i, r);
if (l < j) QSort(L, l, j);</pre>
int IntersectionOutOfHalfPlane(Line &hpl, Line &l1,
    Line &12) {
   Point p = GetIntersect(l1, l2);
   return epssgn(Cross(hpl.p1 - p, hpl.p2 - p)) < 0;</pre>
// move hpl for dis
Line HalfPlaneMoveIn(Line &hpl, double &dis) {
  double dx = hpl.p1.x - hpl.p2.x;
  double dy = hpl.p1.y - hpl.p2.y
  double ll = len(hpl.p1 - hpl.p2);
  Point pa = Point(dis * dy / ll + hpl.p1.x, hpl.p1.y -
      dis * dx / ll)
  Point pb = Point(dis * dy / ll + hpl.p2.x, hpl.p2.y -
     dis * dx / ll);
  return Line(pa, pb);
}
// get intersect of n halfplane l, intersect point in p
void HalfPlaneIntersect(Line l[], int n, Point p[], int
     &pn) {
```

```
int i, j;
int dq[maxn], top = 1, bot = 0;
QSort(l, 0, n-1);
for (i = j = 0; i < n; i++) if (epssgn(l[i].angle - l
  [j].angle) > 0) l[++j] = l[i];
n = j + 1;
dq.push_back(0); dq.push_back(1);
for(i = 2; i < n; i++) {
  while (dq.size() >= 2 && IntersectionOutOfHalfPlane
  (l[i], l[dq[dq.size() - 1]], l[dq[dq.size() - 2]]))
   dq.pop_back();
  while (dq.size() >= 2 && IntersectionOutOfHalfPlane
  (l[i], l[dq[0]], l[dq[1]])) dq.pop_front();
  dq.push_back(i);
while (dq.size() >= 2 && IntersectionOutOfHalfPlane(l
  [dq[0]], l[dq[dq.size() - 1]], l[dq[dq.size() -
  2]])) dq.pop_back();
while (dq.size() >= 2 && IntersectionOutOfHalfPlane(l
  [dq[dq.size() - 1]], l[dq[dq[0]]], l[dq[dq[1]]]))
  dq.pop_front();
dq.push_back(dq.front());
for (pn = 0, i = 0; i < dq.size() - 1; ++i, ++pn) p[
  pn] = GetIntersect(l[dq[i + 1]], l[dq[i]]);
```

### 7.6 Polygon Center

```
Point BaryCenter(vector<Point> &p, int n) {
   Point res(0, 0);
   double s = 0.0, t;
   for (int i = 1; i < p.size() - 1; i++) {
        t = Cross(p[i] - p[0], p[i + 1] - p[0]) / 2;
        s += t;
        res.x += (p[0].x + p[i].x + p[i + 1].x) * t;
        res.y += (p[0].y + p[i].y + p[i + 1].y) * t;
   }
   res.x /= (3 * s);
   res.y /= (3 * s);
   return res;
}</pre>
```

#### 7.7 Maximum Triangle

```
double ConvexHullMaxTriangleArea(Point p[], int res[],
    int chnum) {
  double area = 0, tmp;
  res[chnum] = res[0];
  - p[res[i]], p[res[k]] - p[res[i]])) k = (k + 1) %
    chnum;
    tmp = fabs(Cross(p[res[j]] - p[res[i]], p[res[k]] -
    p[res[i]]));
    if (tmp > area) area = tmp;
   while (fabs(Cross(p[res[(j + 1) % chnum]] - p[res[i
]], p[res[k]] - p[res[i]])) > fabs(Cross(p[res[j]])
    chnum;
    tmp = fabs(Cross(p[res[j]] - p[res[i]], p[res[k]] -
    p[res[i]]));
    if (tmp > area) area = tmp;
  return area / 2;
}
```

#### 7.8 Point in Polygon

```
bool PointInConvexHull(Point p[], int res[], int chnum,
    Point x) {
    Point g = (p[res[0]] + p[res[chnum / 3]] + p[res[2 *
        chnum / 3]]) / 3.0;
    int l = 0, r = chnum, mid;
    while (l + 1 < r) {</pre>
```

```
mid = (l + r) >> 1;
if (epssgn(Cross(p[res[l]] - g, p[res[mid]] - g)) >
    0) {
    if (epssgn(Cross(p[res[l]] - g, x - g)) >= 0 &&
    epssgn(Cross(p[res[mid]] - g, x - g)) < 0) r = mid;
    else l = mid;
} else {
    if (epssgn(Cross(p[res[l]] - g, x - g)) < 0 &&
    epssgn(Cross(p[res[mid]] - g, x - g)) >= 0) l = mid;
;
    else r = mid;
}
r %= chnum;
return epssgn(Cross(p[res[r]] - x, p[res[l]] - x)) ==
    -1;
}
```

#### 7.9 Circle-Line Intersection

```
// remove second level if to get points for line (
     defalut: segment)
void CircleCrossLine(Point a, Point b, Point o, double
  r, Point ret[], int &num) {
double x0 = o.x, y0 = o.y;
  double x1 = a.x, y1 = a.y;
  double x2 = b.x, y2 = b.y;
double dx = x2 - x1, dy = y2 - y1;
  double A = dx * dx + dy * dy;
  double B = 2 * dx * (x1 - x0) + 2 * dy * (y1 - y0);
  double C = (x1 - x0) * (x1 - x0) + (y1 - y0) * (y1 - y0)
     y0) - r * r;
  double delta = B * B - 4 * A * C;
  num = 0:
  if (epssgn(delta) >= 0) {
    double t1 = (-B - sqrt(fabs(delta))) / (2 * A);
double t2 = (-B + sqrt(fabs(delta))) / (2 * A);
    if (epssgn(t1 - 1.0) \le 0 \& epssgn(t1) >= 0) ret[num++] = Point(x1 + t1 * dx, y1 + t1 * dy);
     if (epssgn(t2 - 1.0) \le 0 \& epssgn(t2) >= 0) ret[num++] = Point(x1 + t2 * dx, y1 + t2 * dy);
}
vector<Point> CircleCrossLine(Point a, Point b, Point o
      double r) {
  double x0 = o.x, y0 = o.y;
  double x1 = a.x, y1 = a.y;
  double x2 = b.x, y2 = b.y;
  double dx = x2-x1, dy = y2-y1; double A = dx * dx + dy * dy;
  double B = 2 * dx * (x1 - x0) + 2 * dy * (y1 - y0);
  double C = (x1 - x0) * (x1 - x0) + (y1 - y0) * (y1 - y0)
     y0) - r * r;
  double delta = B * B - 4 * A * C;
  vector<Point> ret;
  if (epssgn(delta) >=0){
     double t1 = (-B - sqrt(fabs(delta))) / (2 * A);
     double t2 = (-B + \sqrt{fabs(delta)}) / (2 * A)
     if (epssgn(t1 - 1.0) \leftarrow 0 \& epssgn(t1) >= 0) ret.
     emplace_back(x1 + t1 * dx, y1 + t1 * dy);
    if (epssgn(t2 - 1.0) \le 0 \& epssgn(t2) >= 0) ret.
emplace_back(x1 + t2 * dx, y1 + t2 * dy);
  return ret;
```

#### 7.10 Circle-Triangle Intersection

```
// calc area intersect by circle with radius r and
    triangle OAB
double Calc(Point a, Point b, double r) {
    Point p[2];
    int num = 0;
    bool ina = epssgn(len(a) - r) < 0, inb = epssgn(len(b) - r) < 0;
    if (ina) {</pre>
```

```
if (inb) return fabs(Cross(a, b)) / 2.0; //
    triangle in circle
    else { // a point inside and another outside: calc
    sector and triangle area
      CircleCrossLine(a, b, Point(0, 0), r, p, num);
      return SectorArea(b, p[0], r) + fabs(Cross(a, p
    [0])) / 2.0;
  } else {
    CircleCrossLine(a, b, Point(0, 0), r, p, num)
    if (inb) return SectorArea(p[0], a, r) + fabs(Cross
    (p[0], b)) / 2.0;
      if (num == 2) return SectorArea(a, p[0], r) +
    SectorArea(p[1], b, r) + fabs(Cross(p[0], p[1])) /
    2.0; // segment ab has 2 point intersect with
    circle
      else return SectorArea(a, b, r); // segment has
    no intersect point with circle
  }
}
```

### 7.11 Polygon Diameter

```
// get diameter of p[res[]] store opposite points in
double Diameter(Point p[], int res[], int chnum, int
    app[][2], int &appnum) {
  double ret = 0, nowlen;
  res[chnum] = res[0];
  appnum = 0;
  1]] - p[res[i + 1]]) < Cross(p[res[i]] - p[res[i +
    1]], p[res[j]] - p[res[i + 1]])) {
      j %= chnum;
    app[appnum][0] = res[i];
    app[appnum][1] = res[j];
    ++appnum;
    nowlen = dis(p[res[i]], p[res[j]]);
    if (nowlen > ret) ret = nowlen;
    nowlen = dis(p[res[i + 1]], p[res[j + 1]]);
    if (nowlen > ret) ret = nowlen;
  return ret;
}
```

### 7.12 Minimun Distance of 2 Polygons

#### 7.13 Convex Hull

```
int Graham(Point p[], int n, int res[]) {
 int len, top;
  top = 1;
  sort(p, p + n, [](const Point &a, const Point &b) {
      return a.y == b.y ? a.x < b.x : a.y < b.y; }
  // QSort(p,0,n-1);
 for (int i = 0; i < 3; i++) res[i] = i;
for (int i = 2; i < n; i++) {
    while (top && epssgn(Cross(p[i], p[res[top]], p[res
    [top - 1]])) >= 0) top--;
    res[++top] = i;
 len = top;
  res[++top] = n - 2;
  for (int i = n-3; i>=0; i--) {
    while (top != len && epssgn(Cross(p[i], p[res[top
    ]], p[res[top - 1]])) >= 0) top--;
    res[++top] = i;
  return top;
```

### 7.14 Rotating Caliper

```
struct pnt {
  int x, y;
pnt(): x(0), y(0) {};
pnt(int xx, int yy): x(xx), y(yy) {};
} p[maxn];
pnt operator-(const pnt &a, const pnt &b) { return pnt(
     b.x - a.x, b.y - a.y); }
int operator^(const pnt &a, const pnt &b) { return a.x
     * b.y - a.y * b.x; } //cross
int operator*(const pnt &a, const pnt &b) { return (a -
      b).x * (a - b).x + (a - b).y * (a - b).y; } //
     distance
int tb[maxn], tbz, rsd;
int dist(int n1, int n2){
  return p[n1] * p[n2];
int cross(int t1, int t2, int n1){
  return (p[t2] - p[t1]) ^ (p[n1] - p[t1]);
bool cmpx(const pnt &a, const pnt &b) { return a.x == b
     .x ? a.y < b.y : a.x < b.x; }
void RotatingCaliper() {
  sort(p, p + n, cmpx);
for (int i = 0; i < n; ++i) {
  while (tbz > 1 && cross(tb[tbz - 2], tb[tbz - 1], i
     ) <= 0) --tbz;
     tb[tbz++] = i;
  rsd = tbz - 1;
  for (int i = n - 2; i >= 0; --i) {
  while (tbz > rsd + 1 && cross(tb[tbz - 2], tb[tbz -
      1], i) <= 0) --tbz;
     tb[tbz++] = i;
  }
   --tbz;
  int lpr = 0, rpr = rsd;
  // tb[lpr], tb[rpr]
  while (lpr < rsd || rpr < tbz - 1) {</pre>
     if (lpr < rsd && rpr < tbz - 1) {
       pnt rvt = p[tb[rpr + 1]] - p[tb[rpr]];
pnt lvt = p[tb[lpr + 1]] - p[tb[lpr]];
if ((lvt ^ rvt) < 0) ++lpr;</pre>
       else ++rpr;
     else if (lpr == rsd) ++rpr;
     else ++lpr;
     // tb[lpr], tb[rpr]
}
```

# 7.15 Min Enclosing Circle

```
pt center(const pt &a, const pt &b, const pt &c) {
  pt p0 = b - a, p1 = c - a;
  double c1 = norm2(p0) * 0.5, c2 = norm2(p1) * 0.5;
  double d = p0 \land p1;
  double x = a.x + (c1 * p1.y - c2 * p0.y) / d;
double y = a.y + (c2 * p0.x - c1 * p1.x) / d;
  return pt(x, y);
}
circle min_enclosing(vector<pt> &p) {
  random_shuffle(p.begin(), p.end());
  double r = 0.0;
  pt cent;
  for (int i = 0; i < p.size(); ++i) {</pre>
     if (norm2(cent - p[i]) <= r) continue;</pre>
     cent = p[i];
     r = 0.0;
     for (int j = 0; j < i; ++j) {
  if (norm2(cent - p[j]) <= r) continue;
  cent = (p[i] + p[j]) / 2;</pre>
       r = norm2(p[j] - cent);
for (int k = 0; k < j;
                                    ++k) {
          if (norm2(cent - p[k]) <= r) continue;</pre>
          cent = center(p[i], p[j], p[k]);
          r = norm2(p[k] - cent);
       }
    }
  return circle(cent, sqrt(r));
```

#### 7.16 Closest Pair

```
pt p[maxn];
double dis(const pt& a, const pt& b) {
  return sqrt((a - b) * (a - b));
}
double closest_pair(int l, int r) {
  if (l == r) return inf;
  if (r - l == 1) return dis(p[l], p[r]);
  int m = (l + r) >> 1;
  double d = min(closest_pair(l, m), closest_pair(m +
    1, r));
  vector<int> vec;
  for (int i = m; i >= l && fabs(p[m].x - p[i].x) < d;
     --i) vec.push_back(i);
  for (int i = m + 1; i \le r \& fabs(p[m].x - p[i].x) <
     d; ++i) vec.push_back(i);
  sort(vec.begin(), vec.end(), [=](const int& a, const
    int& b) { return p[a].y < p[b].y; });</pre>
  for (int i = 0; i < vec.size(); ++i)</pre>
    for (int j = i + 1; j < vec.size() && fabs(p[vec[j
]].y - p[vec[i]].y) < d; ++j) {</pre>
       d = min(d, dis(p[vec[i]], p[vec[j]]));
  }
  return d;
```