Contents

1 Basic

1.1 vimrc

```
se nu rnu
syn on
colo desert
se bs=2 ai ru mouse=a cin et ts=4 sw=4 sts=4
inoremap {<CR> {<CR>}<Esc>O
```

1.2 Compilation Argument

```
g++ -W -Wall -Wextra -O2 -std=c++14 -fsanitize=address
    -fsanitize=undefined -fsanitize=leak
```

1.3 Checker

```
for ((i = 0; i < 100; i++))
do
  ./gen > in
  ./ac < in > out1
  ./tle < in > out2
  diff out1 out2 || break
done
```

1.4 Fast Integer Input

```
#define getchar gtx
inline int gtx() {
  const int N = 4096;
  static char buffer[N];
  static char *p = buffer, *end = buffer;
  if (p == end) {
    if ((end = buffer + fread(buffer, 1, N, stdin)) ==
    buffer) return EOF;
    p = buffer;
  }
  return *p++;
}
template <typename T>
inline bool rit(T& x) {
  char c = 0; bool flag = false;
  while (c = getchar(), (c < '0' && c != '-') || c > '9
    ') if (c == -1) return false;
  c == '-' ? (flag = true, x = 0) : (x = c - '0');
  while (c = getchar(), c >= '0' && c <= '9') x = x *
    10 + c - '0';
  if (flag) x = -x;
  return true;
}
template <typename T, typename ...Args>
inline bool rit(T& x, Args& ...args) { return rit(x) &&
     rit(args...); }
```

1.5 Increase stack size

```
const int size = 256 << 20;
register long rsp asm("rsp");
char *p = (char*)malloc(size) + size, *bak = (char*)rsp
    ;
__asm__("movq %0, %%rsp\n"::"r"(p));
// main
__asm__("movq %0, %%rsp\n"::"r"(bak));
```

1.6 Pragma optimization

```
#pragma GCC optimize("Ofast", "no-stack-protector", "no
    -math-errno", "unroll-loops")
#pragma GCC target("sse,sse2,sse3,ssse3,sse4,sse4.2,
    popcnt,abm,mmx,avx,tune=native,arch=core-avx2,tune=
    core-avx2")
#pragma warning(disable:4996)
#pragma GCC ivdep
```

1.7 Java

```
import java.io.*;
import java.util.*;
import java.lang.*;
import java.math.*;

public class filename{
  static Scanner in = new Scanner(System.in);
  public static void main(String[] args) {
    int t = 7122;
    while(in.hasNext()) {
      t = in.nextInt();
      float b = in.nextFloat();
      String str = in.nextLine(); // getline
      String stu = in.next(); // string
    }
    System.out.println("Case #" + t);
    System.out.printf("%d\n", 7122);
    int[] c = new int[5];
    int[][] d = {{7,1,2,2},{8,7}};
    int g = Integer.parseInt("-123");

    long f = (long)d[0][2];

    List<Integer> l = new ArrayList<>();
    Random rg = new Random();
    for (int i = 9; i >= 0; --i) {
      l.add(Integer.valueOf(rg.nextInt(100) + 1));
      l.add(Integer.valueOf((int)(Math.random() * 100)
    + 1));
    }
    Collections.sort(l, new Comparator<Integer>() {
      public int compare(Integer a, Integer b) {
        return a - b;
      }
    });
    for (int i = 0; i < l.size(); ++i) {
      System.out.print(l.get(i));
    }

    Set<String> s = new HashSet<String>(); // TreeSet
    s.add("jizz");
    System.out.println(s);
    System.out.println(s.contains("jizz"));

    Map<String, Integer> m = new HashMap<String,
    Integer>();
    m.put("lol", 7122);
    System.out.println(m);
    for(String key: m.keySet()) {
      System.out.println(key + " : " + m.get(key));
    }
    System.out.println(m.containsKey("lol"));
    System.out.println(m.containsValue(7122));

    System.out.println(Math.PI);
    System.out.println(Math.acos(-1));

    BigInteger bi = in.nextBigInteger(), bj = new
    BigInteger("-7122"), bk = BigInteger.valueOf(17171)
    ;
    bi = bi.add(bj);
    bi = bi.subtract(BigInteger.ONE);
    bi = bi.multiply(bj);
    bi = bi.divide(bj);
    bi = bi.and(bj);
    bi = bi.gcd(bj);
    bi = bi.max(bj);
    bi = bi.pow(10);
    int meow = bi.compareTo(bj); // -1 0 1
    String stz = "f5abd69150";
```

```
    BigInteger b16 = new BigInteger(stz, 16);
    System.out.println(b16.toString(2));
  }
}
```

2 Flow

2.1 Dinic

```
struct dinic {
  static const int inf = 1e9;
  struct edge {
    int dest, cap, rev;
    edge(int d, int c, int r): dest(d), cap(c), rev(r)
    {}
  };
  vector<edge> g[maxn];
  int qu[maxn], ql, qr;
  int lev[maxn];
  void init() {
    for (int i = 0; i < maxn; ++i)
      g[i].clear();
  }
  void add_edge(int a, int b, int c) {
    g[a].emplace_back(b, c, g[b].size() - 0);
    g[b].emplace_back(a, 0, g[a].size() - 1);
  }
  bool bfs(int s, int t) {
    memset(lev, -1, sizeof(lev));
    lev[s] = 0;
    ql = qr = 0;
    qu[qr++] = s;
    while (ql < qr) {
      int x = qu[ql++];
      for (edge &e : g[x]) if (lev[e.dest] == -1 && e.
    cap > 0) {
        lev[e.dest] = lev[x] + 1;
        qu[qr++] = e.dest;
      }
    }
    return lev[t] != -1;
  }
  int dfs(int x, int t, int flow) {
    if (x == t) return flow;
    int res = 0;
    for (edge &e : g[x]) if (e.cap > 0 && lev[e.dest]
    == lev[x] + 1) {
      int f = dfs(e.dest, t, min(e.cap, flow - res));
      res += f;
      e.cap -= f;
      g[e.dest][e.rev].cap += f;
    }
    if (res == 0) lev[x] = -1;
    return res;
  }
  int operator()(int s, int t) {
    int flow = 0;
    for (; bfs(s, t); flow += dfs(s, t, inf));
    return flow;
  }
};
```

2.2 ISAP

```
struct isap {
  static const int inf = 1e9;
  struct edge {
    int dest, cap, rev;
    edge(int a, int b, int c): dest(a), cap(b), rev(c)
    {}
  };
  vector<edge> g[maxn];
  int it[maxn], gap[maxn], d[maxn];
  void add_edge(int a, int b, int c) {
    g[a].emplace_back(b, c, g[b].size() - 0);
    g[b].emplace_back(a, 0, g[a].size() - 1);
  }
  int dfs(int x, int t, int tot, int flow) {
    if (x == t) return flow;
    for (int &i = it[x]; i < g[x].size(); ++i) {
      edge &e = g[x][i];
      if (e.cap > 0 && d[e.dest] == d[x] - 1) {
        int f = dfs(e.dest, t, tot, min(flow, e.cap));
        if (f) {
          e.cap -= f;
          g[e.dest][e.rev].cap += f;
          return f;
        }
      }
    }
    if ((--gap[d[x]]) == 0) d[x] = tot;
    else d[x]++, it[x] = 0, ++gap[d[x]];
    return 0;
  }
  int operator()(int s, int t, int tot) {
    memset(it, 0, sizeof(it));
    memset(gap, 0, sizeof(gap));
    memset(d, 0, sizeof(d));
    int r = 0;
    gap[0] = tot;
    for (; d[s] < tot; r += dfs(s, t, tot, inf));
    return r;
  }
};
```

2.3 Minimum-cost flow

```
struct mincost {
  struct edge {
    int dest, cap, w, rev;
    edge(int a, int b, int c, int d): dest(a), cap(b),
    w(c), rev(d) {}
  };
  vector<edge> g[maxn];
  int d[maxn], p[maxn], ed[maxn];
  bool inq[maxn];
  void init() {
    for (int i = 0; i < maxn; ++i) g[i].clear();
  }
  void add_edge(int a, int b, int c, int d) {
    g[a].emplace_back(b, c, +d, g[b].size() - 0);
    g[b].emplace_back(a, 0, -d, g[a].size() - 1);
  }
  bool spfa(int s, int t, int &f, int &c) {
    for (int i = 0; i < maxn; ++i) {
      d[i] = inf;
      p[i] = ed[i] = -1;
      inq[i] = false;
    }
    d[s] = 0;
    queue<int> q;
    q.push(s);
    while (q.size()) {
      int x = q.front(); q.pop();
      inq[x] = false;
      for (int i = 0; i < g[x].size(); ++i) {
        edge &e = g[x][i];
        if (e.cap > 0 && d[e.dest] > d[x] + e.w) {
          d[e.dest] = d[x] + e.w;
          p[e.dest] = x;
          ed[e.dest] = i;
          if (!inq[e.dest]) q.push(e.dest), inq[e.dest]
     = true;
        }
      }
    }
    if (d[t] == inf) return false;
    int dlt = inf;
    for (int x = t; x != s; x = p[x]) dlt = min(dlt, g[
    p[x]][ed[x]].cap);
    for (int x = t; x != s; x = p[x]) {
      edge &e = g[p[x]][ed[x]];
      e.cap -= dlt;
      g[e.dest][e.rev].cap += dlt;
    }
    f += dlt; c += d[t] * dlt;
```

```
    return true;
  }
  pair<int, int> operator()(int s, int t) {
    int f = 0, c = 0;
    while (spfa(s, t, f, c));
    return make_pair(f, c);
  }
};
```

2.4 Gomory-Hu Tree

```
int g[maxn];
vector<edge> GomoryHu(int n){
  vector<edge> rt;
  for(int i=1;i<=n;++i)g[i]=1;
  for(int i=2;i<=n;++i){
    int t=g[i];
    flow.reset(); // clear flows on all edge
    rt.push_back({i,t,flow(i,t)});
    flow.walk(i); // bfs points that connected to i (
    use edges not fully flow)
    for(int j=i+1;j<=n;++j){
      if(g[j]==t && flow.connect(j))g[j]=i; // check if
     i can reach j
    }
  }
  return rt;
}
```

2.5 Stoer–Wagner Minimum Cut

```
const int maxn = 500 + 5;
int w[maxn][maxn], g[maxn];
bool v[maxn], del[maxn];

void add_edge(int x, int y, int c) {
  w[x][y] += c;
  w[y][x] += c;
}

pair<int, int> phase(int n) {
  memset(v, false, sizeof(v));
  memset(g, 0, sizeof(g));
  int s = -1, t = -1;
  while (true) {
    int c = -1;
    for (int i = 0; i < n; ++i) {
      if (del[i] || v[i]) continue;
      if (c == -1 || g[i] > g[c]) c = i;
    }
    if (c == -1) break;
    v[c] = true;
    s = t, t = c;
    for (int i = 0; i < n; ++i) {
      if (del[i] || v[i]) continue;
      g[i] += w[c][i];
    }
  }
  return make_pair(s, t);
}

int mincut(int n) {
  int cut = 1e9;
  memset(del, false, sizeof(del));
  for (int i = 0; i < n - 1; ++i) {
    int s, t; tie(s, t) = phase(n);
    del[t] = true;
    cut = min(cut, g[t]);
    for (int j = 0; j < n; ++j) {
      w[s][j] += w[t][j];
      w[j][s] += w[j][t];
    }
  }
  return cut;
}
```

2.6 Kuhn–Munkres Algorithm

```
int w[maxn][maxn], lx[maxn], ly[maxn];
int match[maxn], slack[maxn];
bool vx[maxn], vy[maxn];

bool dfs(int x) {
  vx[x] = true;
  for (int i = 0; i < n; ++i) {
    if (vy[i]) continue;
    if (lx[x] + ly[i] > w[x][i]) {
      slack[i] = min(slack[i], lx[x] + ly[i] - w[x][i])
    ;
      continue;
    }
    vy[i] = true;
    if (match[i] == -1 || dfs(match[i])) {
      match[i] = x;
      return true;
    }
  }
  return false;
}

int solve() {
  fill_n(match, n, -1);
  fill_n(lx, n, -inf);
  fill_n(ly, n, 0);
  for (int i = 0; i < n; ++i) {
    for (int j = 0; j < n; ++j) lx[i] = max(lx[i], w[i
    ][j]);
  }
  for (int i = 0; i < n; ++i) {
    fill_n(slack, n, inf);
    while (true) {
      fill_n(vx, n, false);
      fill_n(vy, n, false);
      if (dfs(i)) break;
      int dlt = inf;
      for (int j = 0; j < n; ++j) if (!vy[j]) dlt = min
    (dlt, slack[j]);
      for (int j = 0; j < n; ++j) {
        if (vx[j]) lx[j] -= dlt;
        if (vy[j]) ly[j] += dlt;
        else slack[j] -= dlt;
      }
    }
  }
  int res = 0;
  for (int i = 0; i < n; ++i) res += w[match[i]][i];
  return res;
}
```

2.7 Flow Model

- Maximum flow with lower/upper bound from s to t
  1. Construct super source S and sink T
  2. For each edge (x, y, l, u), connect $x \to y$ with capacity $u - l$
  3. For each vertex v, denote $in(v)$ as the difference between the sum of incoming lower bounds and the sum of outgoing lower bounds
  4. If $in(v) > 0$, connect $S \to v$ with capacity $in(v)$, otherwise, connect $v \to T$ with capacity $-in(v)$
  5. Denote f as the maximum flow of the current graph from S to T
  6. Connect $t \to s$ with capacity ∞, increment f by the maximum flow from S to T
  7. If $f \neq \sum_{v \in V, in(v) > 0} in(v)$, there's no solution
  8. Otherwise, the solution of each edge e is $l_e + f_e$, where f_e corresponds to the flow on the graph
- Construct minimum vertex cover from maximum matching M on bipartite graph (X, Y)
  1. Redirect every edge ($y \to x$ if $(x, y) \in M$, $x \to y$ otherwise)
  2. DFS from unmatched vertices in X
  3. $x \in X$ is chosen iff x is unvisited
  4. $y \in Y$ is chosen iff y is visited
- Minimum cost cyclic flow

1. Consruct super source S and sink T
2. For each edge (x, y, c), connect $x \to y$ with $(cost, cap) = (c, 1)$ if $c > 0$, otherwise connect $y \to x$ with $(cost, cap) = (-c, 1)$
3. For each edge with $c < 0$, sum these cost as K, then increase $d(y)$ by 1, decrease $d(x)$ by 1
4. For each vertex v with $d(v) > 0$, connect $S \to v$ with $(cost, cap) = (0, d(v))$
5. For each vertex v with $d(v) < 0$, connect $v \to T$ with $(cost, cap) = (0, -d(v))$
6. Flow from S to T, the answer is the cost of the flow $C + K$

- Maximum density induced subgraph

1. Binary search on answer, suppose we're checking answer T
2. Construct a max flow model, let K be the sum of all weights
3. Connect source $s \to v$, $v \in G$ with capacity K
4. For each edge (u, v, w) in G, connect $u \to v$ and $v \to u$ with capacity w
5. For $v \in G$, connect it with sink $v \to t$ with capacity $K + 2T - (\sum_{e \in E(v)} w(e)) - 2w(v)$
6. T is a valid answer if the maximum flow $f < T|V|$

3 Data Structure

3.1 Disjoint Set

```
struct DisjointSet {
  int p[maxn], sz[maxn], n, cc;
  vector<pair<int*, int>> his;
  vector<int> sh;
  void init(int _n) {
    n = _n; cc = n;
    for (int i = 0; i < n; ++i) sz[i] = 1, p[i] = i;
    sh.clear(); his.clear();
  }
  void assign(int *k, int v) {
    his.emplace_back(k, *k);
    *k = v;
  }
  void save() {
    sh.push_back((int)his.size());
  }
  void undo() {
    int last = sh.back(); sh.pop_back();
    while (his.size() != last) {
      int *k, v;
      tie(k, v) = his.back(); his.pop_back();
      *k = v;
    }
  }
  int find(int x) {
    if (x == p[x]) return x;
    return find(p[x]);
  }
  void merge(int x, int y) {
    x = find(x); y = find(y);
    if (x == y) return;
    if (sz[x] > sz[y]) swap(x, y);
    assign(&sz[y], sz[x] + sz[y]);
    assign(&p[x], y);
    assign(&cc, cc - 1);
  }
} dsu;
```

3.2 <ext/pbds>

```
#include <bits/stdc++.h>
#include <bits/extc++.h>
#include <ext/rope>
using namespace __gnu_pbds;
using namespace __gnu_cxx;
#include <ext/pb_ds/assoc_container.hpp>
typedef tree<int, null_type, std::less<int>,
    rb_tree_tag, tree_order_statistics_node_update>
    tree_set;
typedef cc_hash_table<int, int> umap;
typedef priority_queue<int> heap;

int main() {
  // rb tree
  tree_set s;
  s.insert(71); s.insert(22);
  assert(*s.find_by_order(0) == 22); assert(*s.
    find_by_order(1) == 71);
  assert(s.order_of_key(22) == 0); assert(s.
    order_of_key(71) == 1);
  s.erase(22);
  assert(*s.find_by_order(0) == 71); assert(s.
    order_of_key(71) == 0);
  // mergable heap
  heap a, b; a.join(b);
  // persistant
  rope<char> r[2];
  r[1] = r[0];
  std::string st = "abc";
  r[1].insert(0, st.c_str());
  r[1].erase(1, 1);
  std::cout << r[1].substr(0, 2) << std::endl;
  return 0;
}
```

3.3 Li Chao Tree

```
namespace lichao {
  struct line {
    long long a, b;
    line(): a(0), b(0) {}
    line(long long a, long long b): a(a), b(b) {}
    long long operator()(int x) const { return a * x +
    b; }
  };
  line st[maxc * 4];
  int sz, lc[maxc * 4], rc[maxc * 4];
  int gnode() {
    st[sz] = line(1e9, 1e9);
    lc[sz] = -1, rc[sz] = -1;
    return sz++;
  }
  void init() {
    sz = 0;
  }
  void add(int l, int r, line tl, int o) {
    bool lcp = st[o](l) > tl(l);
    bool mcp = st[o]((l + r) / 2) > tl((l + r) / 2);
    if (mcp) swap(st[o], tl);
    if (r - l == 1) return;
    if (lcp != mcp) {
      if (lc[o] == -1) lc[o] = gnode();
      add(l, (l + r) / 2, tl, lc[o]);
    } else {
      if (rc[o] == -1) rc[o] = gnode();
      add((l + r) / 2, r, tl, rc[o]);
    }
  }
  long long query(int l, int r, int x, int o) {
    if (r - l == 1) return st[o](x);
    if (x < (l + r) / 2) {
      if (lc[o] == -1) return st[o](x);
      return min(st[o](x), query(l, (l + r) / 2, x, lc[
    o]));
    } else {
      if (rc[o] == -1) return st[o](x);
      return min(st[o](x), query((l + r) / 2, r, x, rc[
    o]));
    }
  }
}
```

4 Graph

4.1 Link-Cut Tree

```
struct node {
  node *ch[2], *fa, *pfa;
  int sum, v, rev;
  node(int s): v(s), sum(s), rev(0), fa(nullptr), pfa(
    nullptr) {
    ch[0] = nullptr;
    ch[1] = nullptr;
  }
  int relation() {
    return this == fa->ch[0] ? 0 : 1;
  }
  void push() {
    if (!rev) return;
    swap(ch[0], ch[1]);
    if (ch[0]) ch[0]->rev ^= 1;
    if (ch[1]) ch[1]->rev ^= 1;
    rev = 0;
  }
  void pull() {
    sum = v;
    if (ch[0]) sum += ch[0]->sum;
    if (ch[1]) sum += ch[1]->sum;
  }
  void rotate() {
    if (fa->fa) fa->fa->push();
    fa->push(), push();
    swap(pfa, fa->pfa);
    int d = relation();
    node *t = fa;
    if (t->fa) t->fa->ch[t->relation()] = this;
    fa = t->fa;
    t->ch[d] = ch[d ^ 1];
    if (ch[d ^ 1]) ch[d ^ 1]->fa = t;
    ch[d ^ 1] = t;
    t->fa = this;
    t->pull(), pull();
  }
  void splay() {
    while (fa) {
      if (!fa->fa) {
        rotate();
        continue;
      }
      fa->fa->push(), fa->push();
      if (relation() == fa->relation()) fa->rotate(),
    rotate();
      else rotate(), rotate();
    }
  }
  void evert() {
    access();
    splay();
    rev ^= 1;
  }
  void expose() {
    splay(), push();
    if (ch[1]) {
      ch[1]->fa = nullptr;
      ch[1]->pfa = this;
      ch[1] = nullptr;
      pull();
    }
  }
  bool splice() {
    splay();
    if (!pfa) return false;
    pfa->expose();
    pfa->ch[1] = this;
    fa = pfa;
    pfa = nullptr;
    fa->pull();
    return true;
  }
  void access() {
    expose();
    while (splice());
  }
  int query() {
    return sum;
  }
};
```

```
namespace lct {
  node *sp[maxn];
  void make(int u, int v) {
    // create node with id u and value v
    sp[u] = new node(v, u);
  }
  void link(int u, int v) {
    // u become v's parent
    sp[v]->evert();
    sp[v]->pfa = sp[u];
  }
  void cut(int u, int v) {
    // u was v's parent
    sp[u]->evert();
    sp[v]->access(), sp[v]->splay(), sp[v]->push();
    sp[v]->ch[0]->fa = nullptr;
    sp[v]->ch[0] = nullptr;
    sp[v]->pull();
  }
  void modify(int u, int v) {
    sp[u]->splay();
    sp[u]->v = v;
    sp[u]->pull();
  }
  int query(int u, int v) {
    sp[u]->evert(), sp[v]->access(), sp[v]->splay();
    return sp[v]->query();
  }
}
```

4.2 Heavy-Light Decomposition

```
struct HeavyLightDecomp {
  vector<int> G[maxn];
  int tin[maxn], top[maxn], dep[maxn], maxson[maxn], sz
    [maxn], p[maxn], n, clk;
  void dfs(int now, int fa, int d) {
    dep[now] = d;
    maxson[now] = -1;
    sz[now] = 1;
    p[now] = fa;
    for (int u : G[now]) if (u != fa) {
      dfs(u, now, d + 1);
      sz[now] += sz[u];
      if (maxson[now] == -1 || sz[u] > sz[maxson[now]])
     maxson[now] = u;
    }
  }
  void link(int now, int t) {
    top[now] = t;
    tin[now] = ++clk;
    if (maxson[now] == -1) return;
    link(maxson[now], t);
    for (int u : G[now]) if (u != p[now]) {
      if (u == maxson[now]) continue;
      link(u, u);
    }
  }
  HeavyLightDecomp(int n): n(n) {
    clk = 0;
    memset(tin, 0, sizeof(tin)); memset(top, 0, sizeof(
    top)); memset(dep, 0, sizeof(dep));
    memset(maxson, 0, sizeof(maxson)); memset(sz, 0,
    sizeof(sz)); memset(p, 0, sizeof(p));
  }
  void add_edge(int a, int b) {
    G[a].push_back(b);
    G[b].push_back(a);
  }
  void solve() {
    dfs(0, -1, 0);
    link(0, 0);
  }
  int lca(int a, int b) {
    int ta = top[a], tb = top[b];
    while (ta != tb) {
      if (dep[ta] < dep[tb]) {
        swap(ta, tb); swap(a, b);
      }
      a = p[ta]; ta = top[a];
```

```
    }
    if (a == b) return a;
    return dep[a] < dep[b] ? a : b;
  }
  vector<pair<int, int>> get_path(int a, int b) {
    int ta = top[a], tb = top[b];
    vector<pair<int, int>> ret;
    while (ta != tb) {
      if (dep[ta] < dep[tb]) {
        swap(ta, tb); swap(a, b);
      }
      ret.push_back(make_pair(tin[ta], tin[a]));
      a = p[ta]; ta = top[a];
    }
    ret.push_back(make_pair(min(tin[a], tin[b]), max(
    tin[a], tin[b])));
    return ret;
  }
};
```

4.3 Centroid Decomposition

```
vector<pair<int, int>> G[maxn];
int sz[maxn], mx[maxn];
bool v[maxn];
vector<int> vtx;

void get_center(int now) {
  v[now] = true; vtx.push_back(now);
  sz[now] = 1; mx[now] = 0;
  for (int u : G[now]) if (!v[u]) {
    get_center(u);
    mx[now] = max(mx[now], sz[u]);
    sz[now] += sz[u];
  }
}

void get_dis(int now, int d, int len) {
  dis[d][now] = cnt;
  v[now] = true;
  for (auto u : G[now]) if (!v[u.first]) {
    get_dis(u, d, len + u.second);
  }
}

void dfs(int now, int fa, int d) {
  get_center(now);
  int c = -1;
  for (int i : vtx) {
    if (max(mx[i], (int)vtx.size() - sz[i]) <= (int)vtx
    .size() / 2) c = i;
    v[i] = false;
  }
  get_dis(c, d, 0);
  for (int i : vtx) v[i] = false;
  v[c] = true; vtx.clear();
  dep[c] = d; p[c] = fa;
  for (auto u : G[c]) if (u.first != fa && !v[u.first])
     {
    dfs(u.first, c, d + 1);
  }
}
```

4.4 Minimum Mean Cycle

```
// d[i][j] == 0 if {i,j} !in E
long long d[1003][1003],dp[1003][1003];

pair<long long,long long> MMWC(){
  memset(dp,0x3f,sizeof(dp));
  for(int i=1;i<=n;++i)dp[0][i]=0;
  for(int i=1;i<=n;++i){
    for(int j=1;j<=n;++j){
      for(int k=1;k<=n;++k){
        dp[i][k]=min(dp[i-1][j]+d[j][k],dp[i][k]);
      }
    }
  }
  long long au=1ll<<31,ad=1;
  for(int i=1;i<=n;++i){
    if(dp[n][i]==0x3f3f3f3f3f3f3f3f)continue;
    long long u=0,d=1;
    for(int j=n-1;j>=0;--j){
      if((dp[n][i]-dp[j][i])*d>u*(n-j)){
        u=dp[n][i]-dp[j][i];
        d=n-j;
      }
    }
    if(u*ad<au*d)au=u,ad=d;
  }
  long long g=__gcd(au,ad);
  return make_pair(au/g,ad/g);
}
```

4.5 Minimum Steiner Tree

```
namespace steiner {
  const int maxn = 64, maxk = 10;
  const int inf = 1e9;
  int w[maxn][maxn], dp[1 << maxk][maxn], off[maxn];
  void init(int n) {
    for (int i = 0; i < n; ++i) {
      for (int j = 0; j < n; ++j) w[i][j] = inf;
      w[i][i] = 0;
    }
  }
  void add_edge(int x, int y, int d) {
    w[x][y] = min(w[x][y], d);
    w[y][x] = min(w[y][x], d);
  }
  int solve(int n, vector<int> mark) {
    for (int k = 0; k < n; ++k) {
      for (int i = 0; i < n; ++i) {
        for (int j = 0; j < n; ++j) w[i][j] = min(w[i][
    j], w[i][k] + w[k][j]);
      }
    }
    int k = (int)mark.size();
    assert(k < maxk);
    for (int s = 0; s < (1 << k); ++s) {
      for (int i = 0; i < n; ++i) dp[s][i] = inf;
    }
    for (int i = 0; i < n; ++i) dp[0][i] = 0;
    for (int s = 1; s < (1 << k); ++s) {
      if (__builtin_popcount(s) == 1) {
        int x = __builtin_ctz(s);
        for (int i = 0; i < n; ++i) dp[s][i] = w[mark[x
    ]][i];
        continue;
      }
      for (int i = 0; i < n; ++i) {
        for (int sub = s & (s - 1); sub; sub = s & (sub
     - 1)) {
          dp[s][i] = min(dp[s][i], dp[sub][i] + dp[s ^
    sub][i]);
        }
      }
      for (int i = 0; i < n; ++i) {
        off[i] = inf;
        for (int j = 0; j < n; ++j) off[i] = min(off[i
    ], dp[s][j] + w[j][i]);
      }
      for (int i = 0; i < n; ++i) dp[s][i] = min(dp[s][
    i], off[i]);
    }
    int res = inf;
    for (int i = 0; i < n; ++i) res = min(res, dp[(1 <<
     k) - 1][i]);
    return res;
  }
}
```

4.6 Directed Minimum Spanning Tree

```
template <typename T> struct DMST {
  T g[maxn][maxn], fw[maxn];
```

```
  int n, fr[maxn];
  bool vis[maxn], inc[maxn];
  void clear() {
    for(int i = 0; i < maxn; ++i) {
      for(int j = 0; j < maxn; ++j) g[i][j] = inf;
      vis[i] = inc[i] = false;
    }
  }
  void addedge(int u, int v, T w) {
    g[u][v] = min(g[u][v], w);
  }
  double operator()(int root, int _n) {
    n = _n;
    if (dfs(root) != n) return -1;
    T ans = 0;
    while (true) {
      for (int i = 1; i <= n; ++i) fw[i] = inf, fr[i] =
       i;
      for (int i = 1; i <= n; ++i) if (!inc[i]) {
        for (int j = 1; j <= n; ++j) {
          if (!inc[j] && i != j && g[j][i] < fw[i]) {
            fw[i] = g[j][i];
            fr[i] = j;
          }
        }
      }
      int x = -1;
      for (int i = 1; i <= n; ++i) if (i != root && !
    inc[i]) {
        int j = i, c = 0;
        while (j != root && fr[j] != i && c <= n) ++c,
    j = fr[j];
        if (j == root || c > n) continue;
        else { x = i; break; }
      }
      if (!~x) {
        for (int i = 1; i <= n; ++i) if (i != root && !
    inc[i]) ans += fw[i];
        return ans;
      }
      int y = x;
      for (int i = 1; i <= n; ++i) vis[i] = false;
      do { ans += fw[y]; y = fr[y]; vis[y] = inc[y] =
    true; } while (y != x);
      inc[x] = false;
      for (int k = 1; k <= n; ++k) if (vis[k]) {
        for (int j = 1; j <= n; ++j) if (!vis[j]) {
          if (g[x][j] > g[k][j]) g[x][j] = g[k][j];
          if (g[j][k] < inf && g[j][k] - fw[k] < g[j][x
    ]) g[j][x] = g[j][k] - fw[k];
        }
      }
    }
    return ans;
  }
  int dfs(int now) {
    int r = 1;
    vis[now] = true;
    for (int i = 1; i <= n; ++i) if (g[now][i] < inf &&
     !vis[i]) r += dfs(i);
    return r;
  }
};
```

4.7 Maximum Matching on General Graph

```
namespace matching {
  int fa[maxn], match[maxn], aux[maxn], orig[maxn], v[
    maxn], tk;
  vector<int> g[maxn];
  queue<int> q;
  void init() {
    for (int i = 0; i < maxn; ++i) {
      g[i].clear();
      match[i] = -1;
      fa[i] = -1;
      aux[i] = 0;
    }
    tk = 0;
  }
  void add_edge(int x, int y) {
    g[x].push_back(y);
    g[y].push_back(x);
  }
  void augment(int x, int y) {
    int a = y, b = -1;
    do {
      a = fa[y], b = match[a];
      match[y] = a, match[a] = y;
      y = b;
    } while (x != a);
  }
  int lca(int x, int y) {
    ++tk;
    while (true) {
      if (~x) {
        if (aux[x] == tk) return x;
        aux[x] = tk;
        x = orig[fa[match[x]]];
      }
      swap(x, y);
    }
  }
  void blossom(int x, int y, int a) {
    while (orig[x] != a) {
      fa[x] = y, y = match[x];
      if (v[y] == 1) q.push(y), v[y] = 0;
      orig[x] = orig[y] = a;
      x = fa[y];
    }
  }
  bool bfs(int s) {
    for (int i = 0; i < maxn; ++i) {
      v[i] = -1;
      orig[i] = i;
    }
    q = queue<int>();
    q.push(s);
    v[s] = 0;
    while (q.size()) {
      int x = q.front(); q.pop();
      for (const int &u : g[x]) {
        if (v[u] == -1) {
          fa[u] = x, v[u] = 1;
          if (!~match[u]) return augment(s, u), true;
          q.push(match[u]);
          v[match[u]] = 0;
        } else if (v[u] == 0 && orig[x] != orig[u]) {
          int a = lca(orig[x], orig[u]);
          blossom(u, x, a);
          blossom(x, u, a);
        }
      }
    }
    return false;
  }
  int solve(int n) {
    int ans = 0;
    vector<int> z(n);
    iota(z.begin(), z.end(), 0);
    random_shuffle(z.begin(), z.end());
    for (int x : z) if (!~match[x]) {
      for (int y : g[x]) if (!~match[y]) {
        match[y] = x;
        match[x] = y;
        ++ans;
        break;
      }
    }
    for (int i = 0; i < n; ++i) if (!~match[i] && bfs(i
    )) ++ans;
    return ans;
  }
}
```

4.8 Maximum Weighted Matching on General Graph

```
struct WeightGraph {
```

```
static const int INF = INT_MAX;
static const int N = 514;
struct edge{
 int u,v,w; edge(){}
 edge(int ui,int vi,int wi)
  :u(ui),v(vi),w(wi){}
};
int n,n_x;
edge g[N*2][N*2];
int lab[N*2];
int match[N*2],slack[N*2],st[N*2],pa[N*2];
int flo_from[N*2][N+1],S[N*2],vis[N*2];
vector<int> flo[N*2];
queue<int> q;
int e_delta(const edge &e){
 return lab[e.u]+lab[e.v]-g[e.u][e.v].w*2;
}
void update_slack(int u,int x){
 if(!slack[x]||e_delta(g[u][x])<e_delta(g[slack[x]][x
   ]))slack[x]=u;
}
void set_slack(int x){
 slack[x]=0;
 for(int u=1;u<=n;++u)
  if(g[u][x].w>0&&st[u]!=x&&S[st[u]]==0)
   update_slack(u,x);
}
void q_push(int x){
 if(x<=n)q.push(x);
 else for(size_t i=0;i<flo[x].size();i++)
  q_push(flo[x][i]);
}
void set_st(int x,int b){
 st[x]=b;
 if(x>n)for(size_t i=0;i<flo[x].size();++i)
  set_st(flo[x][i],b);
}
int get_pr(int b,int xr){
 int pr=find(flo[b].begin(),flo[b].end(),xr)-flo[b].
   begin();
 if(pr%2==1){
  reverse(flo[b].begin()+1,flo[b].end());
  return (int)flo[b].size()-pr;
 }else return pr;
}
void set_match(int u,int v){
 match[u]=g[u][v].v;
 if(u<=n) return;
 edge e=g[u][v];
 int xr=flo_from[u][e.u],pr=get_pr(u,xr);
 for(int i=0;i<pr;++i)set_match(flo[u][i],flo[u][i^1])
   ;
 set_match(xr,v);
 rotate(flo[u].begin(),flo[u].begin()+pr,flo[u].end())
   ;
}
void augment(int u,int v){
 for(;;){
  int xnv=st[match[u]];
  set_match(u,v);
  if(!xnv)return;
  set_match(xnv,st[pa[xnv]]);
  u=st[pa[xnv]],v=xnv;
 }
}
int get_lca(int u,int v){
 static int t=0;
 for(++t;u||v;swap(u,v)){
  if(u==0)continue;
  if(vis[u]==t)return u;
  vis[u]=t;
  u=st[match[u]];
  if(u)u=st[pa[u]];
 }
 return 0;
}
void add_blossom(int u,int lca,int v){
 int b=n+1;
 while(b<=n_x&&st[b])++b;
 if(b>n_x)++n_x;
 lab[b]=0,S[b]=0;
 match[b]=match[lca];
 flo[b].clear();
 flo[b].push_back(lca);
 for(int x=u,y;x!=lca;x=st[pa[y]])
  flo[b].push_back(x),flo[b].push_back(y=st[match[x]])
   ,q_push(y);
 reverse(flo[b].begin()+1,flo[b].end());
 for(int x=v,y;x!=lca;x=st[pa[y]])
  flo[b].push_back(x),flo[b].push_back(y=st[match[x]])
   ,q_push(y);
 set_st(b,b);
 for(int x=1;x<=n_x;++x)g[b][x].w=g[x][b].w=0;
 for(int x=1;x<=n;++x)flo_from[b][x]=0;
 for(size_t i=0;i<flo[b].size();++i){
  int xs=flo[b][i];
  for(int x=1;x<=n_x;++x)
   if(g[b][x].w==0||e_delta(g[xs][x])<e_delta(g[b][x])
   )
    g[b][x]=g[xs][x],g[x][b]=g[x][xs];
  for(int x=1;x<=n;++x)
   if(flo_from[xs][x])flo_from[b][x]=xs;
 }
 set_slack(b);
}
void expand_blossom(int b){
 for(size_t i=0;i<flo[b].size();++i)
  set_st(flo[b][i],flo[b][i]);
 int xr=flo_from[b][g[b][pa[b]].u],pr=get_pr(b,xr);
 for(int i=0;i<pr;i+=2){
  int xs=flo[b][i],xns=flo[b][i+1];
  pa[xs]=g[xns][xs].u;
  S[xs]=1,S[xns]=0;
  slack[xs]=0,set_slack(xns);
  q_push(xns);
 }
 S[xr]=1,pa[xr]=pa[b];
 for(size_t i=pr+1;i<flo[b].size();++i){
  int xs=flo[b][i];
  S[xs]=-1,set_slack(xs);
 }
 st[b]=0;
}
bool on_found_edge(const edge &e){
 int u=st[e.u],v=st[e.v];
 if(S[v]==-1){
  pa[v]=e.u,S[v]=1;
  int nu=st[match[v]];
  slack[v]=slack[nu]=0;
  S[nu]=0,q_push(nu);
 }else if(S[v]==0){
  int lca=get_lca(u,v);
  if(!lca)return augment(u,v),augment(v,u),true;
  else add_blossom(u,lca,v);
 }
 return false;
}
bool matching(){
 memset(S+1,-1,sizeof(int)*n_x);
 memset(slack+1,0,sizeof(int)*n_x);
 q=queue<int>();
 for(int x=1;x<=n_x;++x)
  if(st[x]==x&&!match[x])pa[x]=0,S[x]=0,q_push(x);
 if(q.empty())return false;
 for(;;){
  while(q.size()){
   int u=q.front();q.pop();
   if(S[st[u]]==1)continue;
   for(int v=1;v<=n;++v)
    if(g[u][v].w>0&&st[u]!=st[v]){
     if(e_delta(g[u][v])==0){
      if(on_found_edge(g[u][v]))return true;
     }else update_slack(u,st[v]);
    }
  }
  int d=INF;
  for(int b=n+1;b<=n_x;++b)
   if(st[b]==b&&S[b]==1)d=min(d,lab[b]/2);
  for(int x=1;x<=n_x;++x)
   if(st[x]==x&&slack[x]){
    if(S[x]==-1)d=min(d,e_delta(g[slack[x]][x]));
    else if(S[x]==0)d=min(d,e_delta(g[slack[x]][x])/2)
   ;
  }
```

```
    for(int u=1;u<=n;++u){
     if(S[st[u]]==0){
      if(lab[u]<=d)return 0;
      lab[u]-=d;
     }else if(S[st[u]]==1)lab[u]+=d;
    }
    for(int b=n+1;b<=n_x;++b)
     if(st[b]==b){
      if(S[st[b]]==0)lab[b]+=d*2;
      else if(S[st[b]]==1)lab[b]-=d*2;
     }
    q=queue<int>();
    for(int x=1;x<=n_x;++x)
     if(st[x]==x&&slack[x]&&st[slack[x]]!=x&&e_delta(g[
     slack[x]][x])==0)
      if(on_found_edge(g[slack[x]][x]))return true;
    for(int b=n+1;b<=n_x;++b)
     if(st[b]==b&&S[b]==1&&lab[b]==0)expand_blossom(b);
   }
   return false;
  }
  pair<long long,int> solve(){
   memset(match+1,0,sizeof(int)*n);
   n_x=n;
   int n_matches=0;
   long long tot_weight=0;
   for(int u=0;u<=n;++u)st[u]=u,flo[u].clear();
   int w_max=0;
   for(int u=1;u<=n;++u)
    for(int v=1;v<=n;++v){
     flo_from[u][v]=(u==v?u:0);
     w_max=max(w_max,g[u][v].w);
    }
   for(int u=1;u<=n;++u)lab[u]=w_max;
   while(matching())++n_matches;
   for(int u=1;u<=n;++u)
    if(match[u]&&match[u]<u)
     tot_weight+=g[u][match[u]].w;
   return make_pair(tot_weight,n_matches);
  }
  void add_edge( int ui , int vi , int wi ){
   g[ui][vi].w = g[vi][ui].w = wi;
  }
  void init( int _n ){
   n = _n;
   for(int u=1;u<=n;++u)
    for(int v=1;v<=n;++v)
     g[u][v]=edge(u,v,0);
  }
} graph;
```

4.9 Maximum Clique

```
struct MaxClique {
  int n, deg[maxn], ans;
  bitset<maxn> adj[maxn];
  vector<pair<int, int>> edge;
  void init(int _n) {
    n = _n;
    for (int i = 0; i < n; ++i) adj[i].reset();
    for (int i = 0; i < n; ++i) deg[i] = 0;
    edge.clear();
  }
  void add_edge(int a, int b) {
    edge.emplace_back(a, b);
    ++deg[a]; ++deg[b];
  }
  int solve() {
    vector<int> ord;
    for (int i = 0; i < n; ++i) ord.push_back(i);
    sort(ord.begin(), ord.end(), [&](const int &a,
    const int &b) { return deg[a] < deg[b]; });
    vector<int> id(n);
    for (int i = 0; i < n; ++i) id[ord[i]] = i;
    for (auto e : edge) {
      int u = id[e.first], v = id[e.second];
      adj[u][v] = adj[v][u] = true;
    }
    bitset<maxn> r, p;
    for (int i = 0; i < n; ++i) p[i] = true;
    ans = 0;
    dfs(r, p);
    return ans;
  }
  void dfs(bitset<maxn> r, bitset<maxn> p) {
    if (p.count() == 0) return ans = max(ans, (int)r.
    count()), void();
    if ((r | p).count() <= ans) return;
    int now = p._Find_first();
    bitset<maxn> cur = p & ~adj[now];
    for (now = cur._Find_first(); now < n; now = cur.
    _Find_next(now)) {
      r[now] = true;
      dfs(r, p & adj[now]);
      r[now] = false;
      p[now] = false;
    }
  }
};
```

4.10 Tarjan's Articulation Point

```
vector<pair<int, int>> g[maxn];
int low[maxn], tin[maxn], t;
int bcc[maxn], sz;
int a[maxn], b[maxn], deg[maxn];
bool cut[maxn], ins[maxn];

vector<int> ed[maxn];

stack<int> st;

void dfs(int x, int p) {
  tin[x] = low[x] = ++t;
  int ch = 0;
  for (auto u : g[x]) if (u.first != p) {
    if (!ins[u.second]) st.push(u.second), ins[u.second
    ] = true;
    if (tin[u.first]) {
      low[x] = min(low[x], tin[u.first]);
      continue;
    }
    ++ch;
    dfs(u.first, x);
    low[x] = min(low[x], low[u.first]);
    if (low[u.first] >= tin[x]) {
      cut[x] = true;
      ++sz;
      while (true) {
        int e = st.top(); st.pop();
        bcc[e] = sz;
        if (e == u.second) break;
      }
    }
  }
  if (ch == 1 && p == -1) cut[x] = false;
}
```

4.11 Tarjan's Bridge

```
vector<pair<int, int>> g[maxn];
int tin[maxn], low[maxn], t;
int a[maxn], b[maxn];
int bcc[maxn], sz;
bool br[maxn];

stack<int> st;

void dfs(int x, int p) {
  tin[x] = low[x] = ++t;
  st.push(x);
  for (auto u : g[x]) if (u.first != p) {
    if (tin[u.first]) {
      low[x] = min(low[x], tin[u.first]);
      continue;
    }
    dfs(u.first, x);
```

```
    low[x] = min(low[x], low[u.first]);
    if (low[u.first] == tin[u.first]) br[u.second] =
    true;
  }
  if (tin[x] == low[x]) {
    ++sz;
    while (st.size()) {
      int u = st.top(); st.pop();
      bcc[u] = sz;
      if (u == x) break;
    }
  }
}
```

4.12 Dominator Tree

```
namespace dominator {
  vector<int> g[maxn], r[maxn], rdom[maxn];
  int dfn[maxn], rev[maxn], fa[maxn], sdom[maxn], dom[
    maxn], val[maxn], rp[maxn], tk;
  void add_edge(int x, int y) {
    g[x].push_back(y);
  }
  void dfs(int x) {
    rev[dfn[x] = tk] = x;
    fa[tk] = sdom[tk] = val[tk] = tk;
    tk++;
    for (const int &u : g[x]) {
      if (dfn[u] == -1) dfs(u), rp[dfn[u]] = dfn[x];
      r[dfn[u]].push_back(dfn[x]);
    }
  }
  void merge(int x, int y) {
    fa[x] = y;
  }
  int find(int x, int c = 0) {
    if (fa[x] == x) return x;
    int p = find(fa[x], 1);
    if (p == -1) return c ? fa[x] : val[x];
    if (sdom[val[x]] > sdom[val[fa[x]]]) val[x] = val[
    fa[x]];
    fa[x] = p;
    return c ? p : val[x];
  }
  vector<int> build(int s) {
    memset(dfn, -1, sizeof(dfn));
    memset(rev, -1, sizeof(rev));
    memset(fa, -1, sizeof(fa));
    memset(val, -1, sizeof(val));
    memset(sdom, -1, sizeof(sdom));
    memset(rp, -1, sizeof(rp));
    memset(dom, -1, sizeof(dom));
    tk = 0, dfs(s);
    for (int i = tk - 1; i >= 0; --i) {
      for (const int &u : r[i]) sdom[i] = min(sdom[i],
    sdom[find(u)]);
      if (i) rdom[sdom[i]].push_back(i);
      for (const int &u : rdom[i]) {
        int p = find(u);
        if (sdom[p] == i) dom[u] = i;
        else dom[u] = p;
      }
      if (i) merge(i, rp[i]);
    }
    vector<int> p(maxn, -1);
    for (int i = 1; i < tk; ++i) if (sdom[i] != dom[i])
     dom[i] = dom[dom[i]];
    for (int i = 1; i < tk; ++i) p[rev[i]] = rev[dom[i
    ]];
    return p;
  }
}
```

4.13 System of Difference Constraints

Given m constrains on n variables $x_1, x_2, \ldots, x_n$ of form $x_i - x_j \leq w$ (resp, $x_i - x_j \geq w$), connect $i \to j$ with weight w. Then connect $0 \to i$ for all i with weight 0 and find the shortest path (resp, longest path) on the graph. $dis(i)$ will be the maximum (resp, minimum) solution to x_i.

5 String

5.1 Knuth–Morris–Pratt Algorithm

```
int f[maxn];

int kmp(const string& a, const string& b) {
  f[0] = -1; f[1] = 0;
  for (int i = 1, j = 0; i < b.size() - 1; f[++i] = ++j
    ) {
    if (b[i] == b[j]) f[i] = f[j];
    while (j != -1 && b[i] != b[j]) j = f[j];
  }
  for (int i = 0, j = 0; i - j + b.size() <= a.size();
    ++i, ++j) {
    while (j != -1 && a[i] != b[j]) j = f[j];
    if (j == b.size() - 1) return i - j;
  }
  return -1;
}
```

5.2 Z Algorithm

```
int z[maxn];
// z[i] = longest common prefix of suffix i and suffix
    0

void z_function(const string& s) {
  memset(z, 0, sizeof(z));
  z[0] = (int)s.length();
  int l = 0, r = 0;
  for (int i = 1; i < s.length(); ++i) {
    z[i] = max(0, min(z[i - l], r - i + 1));
    while (i + z[i] < s.length() && s[z[i]] == s[i + z[
    i]]) {
      l = i; r = i + z[i];
      ++z[i];
    }
  }
}
```

5.3 Manacher's Algorithm

```
int z[maxn];

int manacher(const string& s) {
  string t = ".";
  for (int i = 0; i < s.length(); ++i) t += s[i], t +=
    '.';
  int l = 0, r = 0;
  for (int i = 1; i < t.length(); ++i) {
    z[i] = (r > i ? min(z[2 * l - i], r - i) : 1);
    while (i - z[i] >= 0 && i + z[i] < t.length() && t[
    i - z[i]] == t[i + z[i]]) ++z[i];
    if (i + z[i] > r) r = i + z[i], l = i;
  }
  int ans = 0;
  for (int i = 1; i < t.length(); ++i) ans = max(ans, z
    [i] - 1);
  return ans;
}
```

5.4 Aho-Corasick Automaton

```
struct AC {
  static const int maxn = 1e5 + 5;
  int sz, ql, qr, root;
  int cnt[maxn], q[maxn], ed[maxn], el[maxn], ch[maxn
    ][26], f[maxn];
  int gnode() {
    for (int i = 0; i < 26; ++i) ch[sz][i] = -1;
    f[sz] = -1;
    ed[sz] = 0;
    cnt[sz] = 0;
```

```
    return sz++;
  }
  void init() {
    sz = 0;
    root = gnode();
  }
  int add(const string &s) {
    int now = root;
    for (int i = 0; i < s.length(); ++i) {
      if (ch[now][s[i] - 'a'] == -1) ch[now][s[i] - 'a'
    ] = gnode();
      now = ch[now][s[i] - 'a'];
    }
    ed[now] = 1;
    return now;
  }
  void build_fail() {
    ql = qr = 0; q[qr++] = root;
    while (ql < qr) {
      int now = q[ql++];
      for (int i = 0; i < 26; ++i) if (ch[now][i] !=
    -1) {
        int p = ch[now][i], fp = f[now];
        while (fp != -1 && ch[fp][i] == -1) fp = f[fp];
        int pd = fp != -1 ? ch[fp][i] : root;
        f[p] = pd;
        el[p] = ed[pd] ? pd : el[pd];
        q[qr++] = p;
      }
    }
  }
  void build(const string &s) {
    build_fail();
    int now = root;
    for (int i = 0; i < s.length(); ++i) {
      while (now != -1 && ch[now][s[i] - 'a'] == -1)
    now = f[now];
      now = now != -1 ? ch[now][s[i] - 'a'] : root;
      ++cnt[now];
    }
    for (int i = qr - 1; i >= 0; --i) cnt[f[q[i]]] +=
    cnt[q[i]];
  }
};
```

5.5 Suffix Automaton

```
struct SAM {
  static const int maxn = 5e5 + 5;
  int nxt[maxn][26], to[maxn], len[maxn];
  int root, last, sz;
  int gnode(int x) {
    for (int i = 0; i < 26; ++i) nxt[sz][i] = -1;
    to[sz] = -1;
    len[sz] = x;
    return sz++;
  }
  void init() {
    sz = 0;
    root = gnode(0);
    last = root;
  }
  void push(int c) {
    int cur = last;
    last = gnode(len[last] + 1);
    for (; ~cur && nxt[cur][c] == -1; cur = to[cur])
    nxt[cur][c] = last;
    if (cur == -1) return to[last] = root, void();
    int link = nxt[cur][c];
    if (len[link] == len[cur] + 1) return to[last] =
    link, void();
    int tlink = gnode(len[cur] + 1);
    for (; ~cur && nxt[cur][c] == link; cur = to[cur])
    nxt[cur][c] = tlink;
    for (int i = 0; i < 26; ++i) nxt[tlink][i] = nxt[
    link][i];
    to[tlink] = to[link];
    to[link] = tlink;
    to[last] = tlink;
  }
  void add(const string &s) {
    for (int i = 0; i < s.size(); ++i) push(s[i] - 'a')
    ;
  }
  bool find(const string &s) {
    int cur = root;
    for (int i = 0; i < s.size(); ++i) {
      cur = nxt[cur][s[i] - 'a'];
      if (cur == -1) return false;
    }
    return true;
  }
  int solve(const string &t) {
    int res = 0, cnt = 0;
    int cur = root;
    for (int i = 0; i < t.size(); ++i) {
      if (~nxt[cur][t[i] - 'a']) {
        ++cnt;
        cur = nxt[cur][t[i] - 'a'];
      } else {
        for (; ~cur && nxt[cur][t[i] - 'a'] == -1; cur
    = to[cur]);
        if (~cur) cnt = len[cur] + 1, cur = nxt[cur][t[
    i] - 'a'];
        else cnt = 0, cur = root;
      }
      res = max(res, cnt);
    }
    return res;
  }
};
```

5.6 Suffix Array

```
int sa[maxn], tmp[2][maxn], c[maxn], hi[maxn], r[maxn];
// sa[i]: sa[i]-th suffix is the i-th lexigraphically
    smallest suffix.
// hi[i]: longest common prefix of suffix sa[i] and
    suffix sa[i - 1].
void build(const string &s) {
  int *rnk = tmp[0], *rkn = tmp[1];
  for (int i = 0; i < 256; ++i) c[i] = 0;
  for (int i = 0; i < s.size(); ++i) c[rnk[i] = s[i
    ]]++;
  for (int i = 1; i < 256; ++i) c[i] += c[i - 1];
  for (int i = s.size() - 1; i >= 0; --i) sa[--c[s[i]]]
     = i;
  int sigma = 256;
  for (int n = 1; n < s.size(); n *= 2) {
    for (int i = 0; i < sigma; ++i) c[i] = 0;
    for (int i = 0; i < s.size(); ++i) c[rnk[i]]++;
    for (int i = 1; i < sigma; ++i) c[i] += c[i - 1];
    int *sa2 = rkn;
    int r = 0;
    for (int i = s.size() - n; i < s.size(); ++i) sa2[r
    ++] = i;
    for (int i = 0; i < s.size(); ++i) {
      if (sa[i] >= n) sa2[r++] = sa[i] - n;
    }
    for (int i = s.size() - 1; i >= 0; --i) sa[--c[rnk[
    sa2[i]]]] = sa2[i];
    rkn[sa[0]] = r = 0;
    for (int i = 1; i < s.size(); ++i) {
      if (!(rnk[sa[i - 1]] == rnk[sa[i]] && sa[i - 1] +
     n < s.size() && rnk[sa[i - 1] + n] == rnk[sa[i] +
    n])) r++;
      rkn[sa[i]] = r;
    }
    swap(rnk, rkn);
    if (r == s.size() - 1) break;
    sigma = r + 1;
  }
  for (int i = 0; i < s.size(); ++i) r[sa[i]] = i;
  int ind = 0; hi[0] = 0;
  for (int i = 0; i < s.size(); ++i) {
    if (!r[i]) { ind = 0; continue; }
    while (i + ind < s.size() && s[i + ind] == s[sa[r[i
    ] - 1] + ind]) ++ind;
    hi[r[i]] = ind ? ind-- : 0;
  }
```

```
}
```

5.7 Lexicographically Smallest Rotation

```
string rotate(const string &s) {
  int n = s.length();
  string t = s + s;
  int i = 0, j = 1;
  while (i < n && j < n) {
    int k = 0;
    while (k < n && t[i + k] == t[j + k]) ++k;
    if (t[i + k] <= t[j + k]) j += k + 1;
    else i += k + 1;
    if (i == j) ++j;
  }
  int pos = (i < n ? i : j);
  return t.substr(pos, n);
}
```

6 Math

6.1 Fast Fourier Transform

```
struct cplx {
  double re, im;
  cplx(): re(0), im(0) {}
  cplx(double r, double i): re(r), im(i) {}
  cplx operator+(const cplx &rhs) const { return cplx(
    re + rhs.re, im + rhs.im); }
  cplx operator-(const cplx &rhs) const { return cplx(
    re - rhs.re, im - rhs.im); }
  cplx operator*(const cplx &rhs) const { return cplx(
    re * rhs.re - im * rhs.im, re * rhs.im + im * rhs.
    re); }
  cplx conj() const { return cplx(re, -im); }
};

const int maxn = 262144;
const double pi = acos(-1);
cplx omega[maxn + 1];

void prefft() {
  for (int i = 0; i <= maxn; ++i)
    omega[i] = cplx(cos(2 * pi * i / maxn), sin(2 * pi
    * i / maxn));
}

void bitrev(vector<cplx> &v, int n) {
  int z = __builtin_ctz(n) - 1;
  for (int i = 0; i < n; ++i) {
    int x = 0;
    for (int j = 0; (1 << j) < n; ++j) x ^= (((i >> j &
     1)) << (z - j));
    if (x > i) swap(v[x], v[i]);
  }
}

void fft(vector<cplx> &v, int n) {
  bitrev(v, n);
  for (int s = 2; s <= n; s <<= 1) {
    int z = s >> 1;
    for (int i = 0; i < n; i += s) {
      for (int k = 0; k < z; ++k) {
        cplx x = v[i + z + k] * omega[maxn / s * k];
        v[i + z + k] = v[i + k] - x;
        v[i + k] = v[i + k] + x;
      }
    }
  }
}

void ifft(vector<cplx> &v, int n) {
  fft(v, n);
  reverse(v.begin() + 1, v.end());
  for (int i = 0; i < n; ++i) v[i] = v[i] * cplx(1. / n
    , 0);
}

vector<int> conv(const vector<int> &a, const vector<int
    > &b) {
  int sz = 1;
  while (sz < a.size() + b.size() - 1) sz <<= 1;
  vector<cplx> v(sz);
  for (int i = 0; i < sz; ++i) {
    double re = i < a.size() ? a[i] : 0;
    double im = i < b.size() ? b[i] : 0;
    v[i] = cplx(re, im);
  }
  fft(v, sz);
  for (int i = 0; i <= sz / 2; ++i) {
    int j = (sz - i) & (sz - 1);
    cplx x = (v[i] + v[j].conj()) * (v[i] - v[j].conj()
    ) * cplx(0, -0.25);
    if (j != i) v[j] = (v[j] + v[i].conj()) * (v[j] - v
    [i].conj()) * cplx(0, -0.25);
    v[i] = x;
  }
  ifft(v, sz);
  vector<int> c(sz);
  for (int i = 0; i < sz; ++i) c[i] = round(v[i].re);
  while (c.size() && c.back() == 0) c.pop_back();
  return c;
}
```

6.2 Number Theoretic Transform

```
const int maxn = 262144;
const long long mod = 2013265921, root = 31;
long long omega[maxn + 1];

long long fpow(long long a, long long n) {
  (n += mod - 1) %= mod - 1;
  long long r = 1;
  for (; n; n >>= 1) {
    if (n & 1) (r *= a) %= mod;
    (a *= a) %= mod;
  }
  return r;
}

void prentt() {
  long long x = fpow(root, (mod - 1) / maxn);
  omega[0] = 1;
  for (int i = 1; i <= maxn; ++i)
    omega[i] = omega[i - 1] * x % mod;
}

void bitrev(vector<long long> &v, int n) {
  int z = __builtin_ctz(n) - 1;
  for (int i = 0; i < n; ++i) {
    int x = 0;
    for (int j = 0; j <= z; ++j) x ^= ((i >> j & 1) <<
    (z - j));
    if (x > i) swap(v[x], v[i]);
  }
}

void ntt(vector<long long> &v, int n) {
  bitrev(v, n);
  for (int s = 2; s <= n; s <<= 1) {
    int z = s >> 1;
    for (int i = 0; i < n; i += s) {
      for (int k = 0; k < z; ++k) {
        long long x = v[i + k + z] * omega[maxn / s * k
    ] % mod;
        v[i + k + z] = (v[i + k] + mod - x) % mod;
        (v[i + k] += x) %= mod;
      }
    }
  }
}

void intt(vector<long long> &v, int n) {
  ntt(v, n);
  reverse(v.begin() + 1, v.end());
  long long inv = fpow(n, mod - 2);
```

```
  for (int i = 0; i < n; ++i) (v[i] *= inv) %= mod;
}

vector<long long> conv(vector<long long> a, vector<long
     long> b) {
  int sz = 1;
  while (sz < a.size() + b.size() - 1) sz <<= 1;
  vector<long long> c(sz);
  while (a.size() < sz) a.push_back(0);
  while (b.size() < sz) b.push_back(0);
  ntt(a, sz), ntt(b, sz);
  for (int i = 0; i < sz; ++i) c[i] = a[i] * b[i] % mod
    ;
  intt(c, sz);
  while (c.size() && c.back() == 0) c.pop_back();
  return c;
}
```

6.2.1 NTT Prime List

Prime	Root	Prime	Root
7681	17	167772161	3
12289	11	104857601	3
40961	3	985661441	3
65537	3	998244353	3
786433	10	1107296257	10
5767169	3	2013265921	31
7340033	3	2810183681	11
23068673	3	2885681153	3
469762049	3	605028353	3

6.3 Polynomial Division

```
vector<int> inverse(const vector<int> &v, int n) {
  vector<int> q(1, fpow(v[0], mod - 2));
  for (int i = 2; i <= n; i <<= 1) {
    vector<int> fv(v.begin(), v.begin() + i);
    vector<int> fq(q.begin(), q.end());
    fv.resize(2 * i), fq.resize(2 * i);
    ntt(fq, 2 * i), ntt(fv, 2 * i);
    for (int j = 0; j < 2 * i; ++j) {
      fv[j] = fv[j] * 1ll * fq[j] % mod * 1ll * fq[j] %
     mod;
    }
    intt(fv, 2 * i);
    vector<int> res(i);
    for (int j = 0; j < i; ++j) {
      res[j] = mod - fv[j];
      if (j < (i >> 1)) (res[j] += 2 * q[j] % mod) %=
    mod;
    }
    q = res;
  }
  return q;
}

vector<int> divide(const vector<int> &a, const vector<
    int> &b) {
  // leading zero should be trimmed
  int n = (int)a.size(), m = (int)b.size();
  int k = 2;
  while (k < n - m + 1) k <<= 1;
  vector<int> ra(k), rb(k);
  for (int i = 0; i < min(n, k); ++i) ra[i] = a[n - i -
     1];
  for (int i = 0; i < min(m, k); ++i) rb[i] = b[m - i -
     1];
  vector<int> rbi = inverse(rb, k);
  vector<int> res = conv(rbi, ra);
  res.resize(n - m + 1);
  reverse(res.begin(), res.end());
  return res;
}
```

6.4 Fast Walsh-Hadamard Transform

```
void xorfwt(int v[], int l, int r) {
  if (r - l == 1) return;
  int m = l + r >> 1;
  xorfwt(v, l, m), xorfwt(v, m, r);
  for (int i = l, j = m; i < m; ++i, ++j) {
    int x = v[i] + v[j];
    v[j] = v[i] - v[j], v[i] = x;
  }
}

void xorifwt(int v[], int l, int r) {
  if (r - l == 1) return;
  int m = l + r >> 1;
  for (int i = l, j = m; i < m; ++i, ++j) {
    int x = (v[i] + v[j]) / 2;
    v[j] = (v[i] - v[j]) / 2, v[i] = x;
  }
  xorifwt(v, l, m), xorifwt(v, m, r);
}

void andfwt(int v[], int l, int r) {
  if (r - l == 1) return;
  int m = l + r >> 1;
  andfwt(v, l, m), andfwt(v, m, r);
  for (int i = l, j = m; i < m; ++i, ++j) v[i] += v[j];
}

void andifwt(int v[], int l, int r) {
  if (r - l == 1) return;
  int m = l + r >> 1;
  andifwt(v, l, m), andifwt(v, m, r);
  for (int i = l, j = m; i < m; ++i, ++j) v[i] -= v[j];
}

void orfwt(int v[], int l, int r) {
  if (r - l == 1) return;
  int m = l + r >> 1;
  orfwt(v, l, m), orfwt(v, m, r);
  for (int i = l, j = m; i < m; ++i, ++j) v[j] += v[i];
}

void orifwt(int v[], int l, int r) {
  if (r - l == 1) return;
  int m = l + r >> 1;
  orifwt(v, l, m), orifwt(v, m, r);
  for (int i = l, j = m; i < m; ++i, ++j) v[j] -= v[i];
}
```

6.5 Simplex Algorithm

```
namespace simplex {
  // maximize c^Tx under Ax <= B
  // return vector<double>(n, -inf) if the solution
    doesn't exist
  // return vector<double>(n, +inf) if the solution is
    unbounded
  const double eps = 1e-9;
  const double inf = 1e+9;
  int n, m;
  vector<vector<double>> d;
  vector<int> p, q;
  void pivot(int r, int s) {
    double inv = 1.0 / d[r][s];
    for (int i = 0; i < m + 2; ++i) {
      for (int j = 0; j < n + 2; ++j) {
        if (i != r && j != s) d[i][j] -= d[r][j] * d[i
    ][s] * inv;
      }
    }
    for (int i = 0; i < m + 2; ++i) if (i != r) d[i][s]
     *= -inv;
    for (int j = 0; j < n + 2; ++j) if (j != s) d[r][j]
     *= +inv;
    d[r][s] = inv;
    swap(p[r], q[s]);
  }
  bool phase(int z) {
    int x = m + z;
    while (true) {
      int s = -1;
      for (int i = 0; i <= n; ++i) {
        if (!z && q[i] == -1) continue;
        if (s == -1 || d[x][i] < d[x][s]) s = i;
```

```
    }
    if (d[x][s] > -eps) return true;
    int r = -1;
    for (int i = 0; i < m; ++i) {
      if (d[i][s] < eps) continue;
      if (r == -1 || d[i][n + 1] / d[i][s] < d[r][n + 1] / d[r][s]) r = i;
    }
    if (r == -1) return false;
    pivot(r, s);
  }
}
vector<double> solve(const vector<vector<double>> &a, const vector<double> &b, const vector<double> &c) {
  m = b.size(), n = c.size();
  d = vector<vector<double>>(m + 2, vector<double>(n + 2));
  for (int i = 0; i < m; ++i) {
    for (int j = 0; j < n; ++j) d[i][j] = a[i][j];
  }
  p.resize(m), q.resize(n + 1);
  for (int i = 0; i < m; ++i) p[i] = n + i, d[i][n] = -1, d[i][n + 1] = b[i];
  for (int i = 0; i < n; ++i) q[i] = i, d[m][i] = -c[i];
  q[n] = -1, d[m + 1][n] = 1;
  int r = 0;
  for (int i = 1; i < m; ++i) if (d[i][n + 1] < d[r][n + 1]) r = i;
  if (d[r][n + 1] < -eps) {
    pivot(r, n);
    if (!phase(1) || d[m + 1][n + 1] < -eps) return vector<double>(n, -inf);
    for (int i = 0; i < m; ++i) if (p[i] == -1) {
      int s = min_element(d[i].begin(), d[i].end() - 1) - d[i].begin();
      pivot(i, s);
    }
  }
  if (!phase(0)) return vector<double>(n, inf);
  vector<double> x(n);
  for (int i = 0; i < n; ++i) if (p[i] < n) x[p[i]] = d[i][n + 1];
  return x;
  }
}
```

6.5.1 Construction

Standard form: maximize $\sum_{1\le i\le n} c_i x_i$ such that for all $1 \le j \le m$, $\sum_{1\le i\le n} A_{ji}x_i \le b_j$.and $x_i \ge 0$ for all $1 \le i \le n$.

1. In case of minimization, let $c'_i = -c_i$
2. $\sum_{1\le i\le n} A_{ji}x_i \ge b_j \to \sum_{1\le i\le n} -A_{ji}x_i \le -b_j$
3. $\sum_{1\le i\le n} A_{ji}x_i = b_j$
   - $\sum_{1\le i\le n} A_{ji}x_i \le b_j$
   - $\sum_{1\le i\le n} A_{ji}x_i \ge b_j$
4. If x_i has no lower bound, replace x_i with $x_i - x'_i$

6.6 Lagrange Interpolation

```
namespace lagrange {
  long long pf[maxn], nf[maxn];
  void init() {
    pf[0] = nf[0] = 1;
    for (int i = 1; i < maxn; ++i) {
      pf[i] = pf[i - 1] * i % mod;
      nf[i] = nf[i - 1] * (mod - i) % mod;
    }
  }
  // given y: value of f(a), a = [0, n], find f(x)
  long long solve(int n, vector<long long> y, long long x) {
    if (x <= n) return y[x];
    long long all = 1;
    for (int i = 0; i <= n; ++i) (all *= (x - i + mod)) %= mod;
    long long ans = 0;
    for (int i = 0; i <= n; ++i) {
      long long z = all * fpow(x - i, -1) % mod;
      long long l = pf[i], r = nf[n - i];
      (ans += y[i] * z % mod * fpow(l * r, -1)) %= mod;
    }
    return ans;
  }
}
```

6.7 Miller Rabin

```
// n < 4759123141    chk = [2, 7, 61]
// n < 1122004669633 chk = [2, 13, 23, 1662803]
// n < 2^64       chk = [2, 325, 9375, 28178, 450775, 9780504, 1795265022]
//
vector<long long> chk = { 2, 325, 9375, 28178, 450775, 9780504, 1795265022 };

bool check(long long a, long long u, long long n, int t) {
  a = fpow(a, u, n);
  if (a == 0) return true;
  if (a == 1 || a == n - 1) return true;
  for (int i = 0; i < t; ++i) {
    a = fmul(a, a, n);
    if (a == 1) return false;
    if (a == n - 1) return true;
  }
  return false;
}

bool is_prime(long long n) {
  if (n < 2) return false;
  if (n % 2 == 0) return n == 2;
  long long u = n - 1; int t = 0;
  for (; u & 1; u >>= 1, ++t);
  for (long long i : chk) {
    if (!check(i, u, n, t)) return false;
  }
  return true;
}
```

6.8 Pollard's Rho

```
long long f(long long x, long long n, int p) { return (fmul(x, x, n) + p) % n; }

map<long long, int> cnt;

void pollard_rho(long long n) {
  if (n == 1) return;
  if (prime(n)) return ++cnt[n], void();
  if (n % 2 == 0) return pollard_rho(n / 2), ++cnt[2], void();
  long long x = 2, y = 2, d = 1, p = 1;
  while (true) {
    if (d != n && d != 1) {
      pollard_rho(n / d);
      pollard_rho(d);
      return;
    }
    if (d == n) ++p;
    x = f(x, n, p); y = f(f(y, n, p), n, p);
    d = __gcd(abs(x - y), n);
  }
}
```

6.9 Meissel–Lehmer Algorithm

```
int prc[maxn];
long long phic[msz][nsz];

void sieve() {
  bitset<maxn> v;
  pr.push_back(0);
  for (int i = 2; i < maxn; ++i) {
    if (!v[i]) pr.push_back(i);
    for (int j = 1; i * pr[j] < maxn; ++j) {
      v[i * pr[j]] = true;
      if (i % pr[j] == 0) break;
    }
  }
  for (int i = 1; i < pr.size(); ++i) prc[pr[i]] = 1;
  for (int i = 1; i < maxn; ++i) prc[i] += prc[i - 1];
}

long long p2(long long, long long);

long long phi(long long m, long long n) {
  if (m < msz && n < nsz && phic[m][n] != -1) return
    phic[m][n];
  if (n == 0) return m;
  if (pr[n] >= m) return 1;
  long long ret = phi(m, n - 1) - phi(m / pr[n], n - 1)
    ;
  if (m < msz && n < nsz) phic[m][n] = ret;
  return ret;
}

long long pi(long long m) {
  if (m < maxn) return prc[m];
  long long n = pi(cbrt(m));
  return phi(m, n) + n - 1 - p2(m, n);
}

long long p2(long long m, long long n) {
  long long ret = 0;
  long long lim = sqrt(m);
  for (int i = n + 1; pr[i] <= lim; ++i) ret += pi(m /
    pr[i]) - pi(pr[i]) + 1;
  return ret;
}
```

6.10 Gaussian Elimination

```
void gauss(vector<vector<double>> &d) {
  int n = d.size(), m = d[0].size();
  for (int i = 0; i < m; ++i) {
    int p = -1;
    for (int j = i; j < n; ++j) {
      if (fabs(d[j][i]) < eps) continue;
      if (p == -1 || fabs(d[j][i]) > fabs(d[p][i])) p =
        j;
    }
    if (p == -1) continue;
    for (int j = 0; j < m; ++j) swap(d[p][j], d[i][j]);
    for (int j = 0; j < n; ++j) {
      if (i == j) continue;
      double z = d[j][i] / d[i][i];
      for (int k = 0; k < m; ++k) d[j][k] -= z * d[i][k
    ];
    }
  }
}
```

6.11 Linear Equations (full pivoting)

```
void linear_equation(vector<vector<double>> &d, vector<
    double> &aug, vector<double> &sol) {
  int n = d.size(), m = d[0].size();
  vector<int> r(n), c(m);
  iota(r.begin(), r.end(), 0);
  iota(c.begin(), c.end(), 0);
  for (int i = 0; i < m; ++i) {
    int p = -1, z = -1;
    for (int j = i; j < n; ++j) {
      for (int k = i; k < m; ++k) {
        if (fabs(d[r[j]][c[k]]) < eps) continue;
        if (p == -1 || fabs(d[r[j]][c[k]]) > fabs(d[r[p
    ]][c[z]])) p = j, z = k;
      }
    }
    if (p == -1) continue;
    swap(r[p], r[i]), swap(c[z], c[i]);
    for (int j = 0; j < n; ++j) {
      if (i == j) continue;
      double z = d[r[j]][c[i]] / d[r[i]][c[i]];
      for (int k = 0; k < m; ++k) d[r[j]][c[k]] -= z *
    d[r[i]][c[k]];
      aug[r[j]] -= z * aug[r[i]];
    }
  }
  vector<vector<double>> fd(n, vector<double>(m));
  vector<double> faug(n), x(n);
  for (int i = 0; i < n; ++i) {
    for (int j = 0; j < m; ++j) fd[i][j] = d[r[i]][c[j
    ]];
    faug[i] = aug[r[i]];
  }
  d = fd, aug = faug;
  for (int i = n - 1; i >= 0; --i) {
    double p = 0.0;
    for (int j = i + 1; j < n; ++j) p += d[i][j] * x[j
    ];
    x[i] = (aug[i] - p) / d[i][i];
  }
  for (int i = 0; i < n; ++i) sol[c[i]] = x[i];
}
```

6.12 μ function

```
int mu[maxn], pi[maxn];
vector<int> prime;

void sieve() {
  mu[1] = pi[1] = 1;
  for (int i = 2; i < maxn; ++i) {
    if (!pi[i]) {
      pi[i] = i;
      prime.push_back(i);
      mu[i] = -1;
    }
    for (int j = 0; i * prime[j] < maxn; ++j) {
      pi[i * prime[j]] = prime[j];
      mu[i * prime[j]] = -mu[i];
      if (i % prime[j] == 0) {
        mu[i * prime[j]] = 0;
        break;
      }
    }
  }
}
```

6.13 $\lfloor \frac{n}{i} \rfloor$ Enumeration

```
vector<int> solve(int n) {
  vector<int> vec;
  for (int t = 1; t < n; t = (n / (n / (t + 1)))) vec.
    push_back(t);
  vec.push_back(n);
  vec.resize(unique(vec.begin(), vec.end()) - vec.begin
    ());
  return vec;
}
```

6.14 Extended GCD

```
template <typename T> tuple<T, T, T> extgcd(T a, T b) {
  if (!b) return make_tuple(a, 1, 0);
  T d, x, y;
  tie(d, x, y) = extgcd(b, a % b);
  return make_tuple(d, y, x - (a / b) * y);
}
```

6.15 Chinese Remainder Theorem

Given $x \equiv a_i \mod n_i \forall 1 \le i \le k$, where n_i are pairwise coprime, find x.
Let $N = \prod_{i=1}^{k} n_i$ and $N_i = N/n_i$, there exist integer M_i and m_i such that $M_i N_i + m_i n_i = 1$.
A solution to the system of congruence is $x = \sum_{i=1}^{k} a_i M_i N_i$.

6.16 Kirchhoff's Theorem

Denote L be a $n \times n$ matrix as the Laplacian matrix of graph G, where $L_{ii} = d(i)$, $L_{ij} = -c$ where c is the number of edge (i, j) in G.

- The number of undirected spanning in G is $|\det(L^*)|$, where L^* is the $(n-1) \times (n-1)$ matrix by removing row x and column x for some arbitrary x in L
- The number of directed spanning tree rooted at r in G is $|\det(L_r)|$, where L_r is the $(n-1) \times (n-1)$ matrix by removing row r and column r in L

6.17 Tutte Matrix

Let D be a $n \times n$ matrix, where $d_{ij} = x_{ij}$ (x_{ij} is chosen uniform randomly) if $i < j$ and $(i, j) \in E$, otherwise $d_{ij} = -d_{ji}$. $\frac{rank(D)}{2}$ is the maximum matching on G.

6.18 Primes

97, 101, 131, 487, 593, 877, 1087, 1187, 1487, 1787, 3187, 12721, 13331, 14341, 75577, 123457, 222557, 556679, 999983, 1097774749, 1076767633, 100102021, 999997771, 1001010013, 1000512343, 987654361, 999991231, 999888733, 98789101, 987777733, 999991921, 1000000007, 1000000087, 1000000123, 1010101333, 1010102101, 1000000000039, 1000000000000037, 2305843009213693951, 4611686018427387847, 9223372036854775783, 18446744073709551557

7 Dynamic Programming

7.1 Convex Hull (monotone)

```
struct line {
  double a, b;
  inline double operator()(const double &x) const {
    return a * x + b; }
  inline bool checkfront(const line &l, const double &x
    ) const { return (*this)(x) < l(x); }
  inline double intersect(const line &l) const { return
      (l.b - b) / (a - l.a); }
  inline bool checkback(const line &l, const line &
    pivot) const { return pivot.intersect((*this)) <=
    pivot.intersect(l); }
};

void solve() {
  for (int i = 1; i < maxn; ++i) dp[0][i] = inf;
  for (int i = 1; i <= k; ++i) {
    deque<line> dq; dq.push_back((line){ 0.0, dp[i -
    1][0] });
    for (int j = 1; j <= n; ++j) {
      while (dq.size() >= 2 && dq[1].checkfront(dq[0],
    invt[j])) dq.pop_front();
      dp[i][j] = st[j] + dq.front()(invt[j]);
      line nl = (line){ -s[j], dp[i - 1][j] - st[j] + s
    [j] * invt[j] };
      while (dq.size() >= 2 && nl.checkback(dq[dq.size
    () - 1], dq[dq.size() - 2])) dq.pop_back();
      dq.push_back(nl);
    }
  }
}
```

7.2 Convex Hull (non-monotone)

```
struct line {
  int m, y;
  int l, r;
  line(int m = 0,int y = 0, int l = -5, int r =
    1000000009): m(m), y(y), l(l), r(r) {}
  int get(int x) const { return m * x + y; }
  int useful(line le) const {
    return (int)(get(l) >= le.get(l)) + (int)(get(r) >=
      le.get(r));
  }
};

int magic;
bool operator < (const line &a, const line &b) {
  if (magic) return a.m < b.m;
  return a.l < b.l;
}

set<line> st;

void addline(line l) {
  magic = 1;
  auto it = st.lower_bound(l);
  if (it != st.end() && it->useful(l) == 2) return;
  while (it != st.end() && it->useful(l) == 0) it = st.
    erase(it);
  if (it != st.end() && it->useful(l) == 1) {
    int L = it->l, R = it->r, M;
    while (R > L) {
      M = (L + R + 1) >> 1;
      if (it->get(M) >= l.get(M)) R = M - 1;
      else L = M;
    }
    line cp = *it;
    st.erase(it);
    cp.l = L + 1;
    if (cp.l <= cp.r) st.insert(cp);
    l.r = L;
  }
  else if (it != st.end()) l.r = it->l - 1;
  it = st.lower_bound(l);
  while (it != st.begin() && prev(it)->useful(l) == 0)
    it = st.erase(prev(it));
  if (it != st.begin() && prev(it)->useful(l) == 1) {
    --it;
    int L = it->l, R = it->r, M;
    while (R > L) {
      M = (L + R) >> 1;
      if (it->get(M) >= l.get(M)) L = M + 1;
      else R = M;
    }
    line cp = *it;
    st.erase(it);
    cp.r = L - 1;
    if (cp.l <= cp.r) st.insert(cp);
    l.l = L;
  }
  else if (it != st.begin()) l.l = prev(it)->r + 1;
  if (l.l <= l.r) st.insert(l);
}

int getval(int d) {
  magic = 0;
  return (--st.upper_bound(line(0, 0, d, 0)))->get(d);
}
```

7.3 $1D/1D$ Convex Optimization

```
struct segment {
  int i, l, r;
  segment() {}
  segment(int a, int b, int c): i(a), l(b), r(c) {}
};

inline long long f(int l, int r) {
  return dp[l] + w(l + 1, r);
}

void solve() {
  dp[0] = 0ll;
  deque&lt;segment&gt; deq; deq.push_back(segment(0, 1, n));
  for (int i = 1; i <= n; ++i) {
    dp[i] = f(deq.front().i, i);
```

```
    while (deq.size() && deq.front().r < i + 1) deq.
    pop_front();
    deq.front().l = i + 1;
    segment seg = segment(i, i + 1, n);
    while (deq.size() && f(i, deq.back().l) < f(deq.
    back().i, deq.back().l)) deq.pop_back();
    if (deq.size()) {
      int d = 1048576, c = deq.back().l;
      while (d >>= 1) if (c + d <= deq.back().r) {
        if (f(i, c + d) > f(deq.back().i, c + d)) c +=
    d;
      }
      deq.back().r = c; seg.l = c + 1;
    }
    if (seg.l <= n) deq.push_back(seg);
  }
}
```

7.4 Conditon

7.4.1 totally monotone (concave/convex)

$\forall i < i', j < j', B[i][j] \leq B[i'][j] \implies B[i][j'] \leq B[i'][j']$
$\forall i < i', j < j', B[i][j] \geq B[i'][j] \implies B[i][j'] \geq B[i'][j']$

7.4.2 monge condition (concave/convex)

$\forall i < i', j < j', B[i][j] + B[i'][j'] \geq B[i][j'] + B[i'][j]$
$\forall i < i', j < j', B[i][j] + B[i'][j'] \leq B[i][j'] + B[i'][j]$

8 Geometry

8.1 Basic

```
bool same(const double a,const double b){ return abs(a-
    b)<1e-9; }

struct Point{
  double x,y;
  Point():x(0),y(0){}
  Point(double x,double y):x(x),y(y){}
};
Point operator+(const Point a,const Point b){ return
    Point(a.x+b.x,a.y+b.y); }
Point operator-(const Point a,const Point b){ return
    Point(a.x-b.x,a.y-b.y); }
Point operator*(const Point a,const double b){ return
    Point(a.x*b,a.y*b); }
Point operator/(const Point a,const double b){ return
    Point(a.x/b,a.y/b); }
double operator^(const Point a,const Point b){ return a
    .x*b.y-a.y*b.x; }
double abs(const Point a){ return sqrt(a.x*a.x+a.y*a.y)
    ; }


struct Line{
  // ax+by+c=0
  double a,b,c;
  double angle;
  Point pa,pb;
  Line():a(0),b(0),c(0),angle(0),pa(),pb(){}
  Line(Point pa,Point pb):a(pa.y-pb.y),b(pb.x-pa.x),c(
    pa^pb),angle(atan2(-a,b)),pa(pa),pb(pb){}
};

Point intersect(Line la,Line lb){
  if(same(la.a*lb.b,la.b*lb.a))return Point(7122,7122);
  double bot=-la.a*lb.b+la.b*lb.a;
  return Point(-la.b*lb.c+la.c*lb.b,la.a*lb.c-la.c*lb.a
    )/bot;
}
```

8.2 KD Tree

```
namespace kdt {
  int root, lc[maxn], rc[maxn], xl[maxn], xr[maxn], yl[
    maxn], yr[maxn];
  point p[maxn];
  int build(int l, int r, int dep = 0) {
    if (l == r) return -1;
    function<bool(const point &, const point &)> f = [
    dep](const point &a, const point &b) {
      if (dep & 1) return a.x < b.x;
      else return a.y < b.y;
    };
    int m = (l + r) >> 1;
    nth_element(p + l, p + m, p + r, f);
    xl[m] = xr[m] = p[m].x;
    yl[m] = yr[m] = p[m].y;
    lc[m] = build(l, m, dep + 1);
    if (~lc[m]) {
      xl[m] = min(xl[m], xl[lc[m]]);
      xr[m] = max(xr[m], xr[lc[m]]);
      yl[m] = min(yl[m], yl[lc[m]]);
      yr[m] = max(yr[m], yr[lc[m]]);
    }
    rc[m] = build(m + 1, r, dep + 1);
    if (~rc[m]) {
      xl[m] = min(xl[m], xl[rc[m]]);
      xr[m] = max(xr[m], xr[rc[m]]);
      yl[m] = min(yl[m], yl[rc[m]]);
      yr[m] = max(yr[m], yr[rc[m]]);
    }
    return m;
  }
  bool bound(const point &q, int o, long long d) {
    double ds = sqrt(d + 1.0);
    if (q.x < xl[o] - ds || q.x > xr[o] + ds ||
      q.y < yl[o] - ds || q.y > yr[o] + ds) return
    false;
    return true;
  }
  long long dist(const point &a, const point &b) {
    return (a.x - b.x) * 1ll * (a.x - b.x) +
        (a.y - b.y) * 1ll * (a.y - b.y);
  }
  void dfs(const point &q, long long &d, int o, int dep
     = 0) {
    if (!bound(q, o, d)) return;
    long long cd = dist(p[o], q);
    if (cd != 0) d = min(d, cd);
    if ((dep & 1) && q.x < p[o].x || !(dep & 1) && q.y
    < p[o].y) {
      if (~lc[o]) dfs(q, d, lc[o], dep + 1);
      if (~rc[o]) dfs(q, d, rc[o], dep + 1);
    } else {
      if (~rc[o]) dfs(q, d, rc[o], dep + 1);
      if (~lc[o]) dfs(q, d, lc[o], dep + 1);
    }
  }
  void init(const vector<point> &v) {
    for (int i = 0; i < v.size(); ++i) p[i] = v[i];
    root = build(0, v.size());
  }
  long long nearest(const point &q) {
    long long res = 1e18;
    dfs(q, res, root);
    return res;
  }
}
```

8.3 Delaunay Triangulation

```
namespace triangulation {
  static const int maxn = 1e5 + 5;
  vector<point> p;
  set<int> g[maxn];
  int o[maxn];
  set<int> s;
  void add_edge(int x, int y) {
    s.insert(x), s.insert(y);
    g[x].insert(y);
    g[y].insert(x);
  }
```

```
bool inside(point a, point b, point c, point p) {
  if (((b - a) ^ (c - a)) < 0) swap(b, c);
  function<long long(int)> sqr = [](int x) { return x
   * 1ll * x; };
  long long k11 = a.x - p.x, k12 = a.y - p.y, k13 =
  sqr(a.x) - sqr(p.x) + sqr(a.y) - sqr(p.y);
  long long k21 = b.x - p.x, k22 = b.y - p.y, k23 =
  sqr(b.x) - sqr(p.x) + sqr(b.y) - sqr(p.y);
  long long k31 = c.x - p.x, k32 = c.y - p.y, k33 =
  sqr(c.x) - sqr(p.x) + sqr(c.y) - sqr(p.y);
  long long det = k11 * (k22 * k33 - k23 * k32) - k12
   * (k21 * k33 - k23 * k31) + k13 * (k21 * k32 - k22
   * k31);
  return det > 0;
}
bool intersect(const point &a, const point &b, const
  point &c, const point &d) {
  return ((b - a) ^ (c - a)) * ((b - a) ^ (d - a)) <
  0 &&
      ((d - c) ^ (a - c)) * ((d - c) ^ (b - c)) < 0;
}
void dfs(int l, int r) {
  if (r - l <= 3) {
    for (int i = l; i < r; ++i) {
      for (int j = i + 1; j < r; ++j) add_edge(i, j);
    }
    return;
  }
  int m = (l + r) >> 1;
  dfs(l, m), dfs(m, r);
  int pl = l, pr = r - 1;
  while (true) {
    int z = -1;
    for (int u : g[pl]) {
      long long c = ((p[pl] - p[pr]) ^ (p[u] - p[pr])
);
      if (c > 0 || c == 0 && abs(p[u] - p[pr]) < abs(
p[pl] - p[pr])) {
        z = u;
        break;
      }
    }
    if (z != -1) {
      pl = z;
      continue;
    }
    for (int u : g[pr]) {
      long long c = ((p[pr] - p[pl]) ^ (p[u] - p[pl])
);
      if (c < 0 || c == 0 && abs(p[u] - p[pl]) < abs(
p[pr] - p[pl])) {
        z = u;
        break;
      }
    }
    if (z != -1) {
      pr = z;
      continue;
    }
    break;
  }
  add_edge(pl, pr);
  while (true) {
    int z = -1;
    bool b = false;
    for (int u : g[pl]) {
      long long c = ((p[pl] - p[pr]) ^ (p[u] - p[pr])
);
      if (c < 0 && (z == -1 || inside(p[pl], p[pr], p
[z], p[u]))) z = u;
    }
    for (int u : g[pr]) {
      long long c = ((p[pr] - p[pl]) ^ (p[u] - p[pl])
);
      if (c > 0 && (z == -1 || inside(p[pl], p[pr], p
[z], p[u]))) z = u, b = true;
    }
    if (z == -1) break;
    int x = pl, y = pr;
    if (b) swap(x, y);
    for (auto it = g[x].begin(); it != g[x].end(); )
  {
      int u = *it;
      if (intersect(p[x], p[u], p[y], p[z])) {
        it = g[x].erase(it);
        g[u].erase(x);
      } else {
        ++it;
      }
    }
    if (b) add_edge(pl, z), pr = z;
    else add_edge(pr, z), pl = z;
  }
}
vector<vector<int>> solve(vector<point> v) {
  int n = v.size();
  for (int i = 0; i < n; ++i) g[i].clear();
  for (int i = 0; i < n; ++i) o[i] = i;
  sort(o, o + n, [&](int i, int j) { return v[i] < v[
  j]; });
  p.resize(n);
  for (int i = 0; i < n; ++i) p[i] = v[o[i]];
  dfs(0, n);
  vector<vector<int>> res(n);
  for (int i = 0; i < n; ++i) {
    for (int j : g[i]) res[o[i]].push_back(o[j]);
  }
  return res;
}
}
```

8.4 Sector Area

```
// calc area of sector which include a, b
double SectorArea(Point a, Point b, double r) {
  double theta = atan2(a.y, a.x) - atan2(b.y, b.x);
  while (theta <= 0) theta += 2 * pi;
  while (theta >= 2 * pi) theta -= 2 * pi;
  theta = min(theta, 2 * pi - theta);
  return r * r * theta / 2;
}
```

8.5 Polygon Area

```
// point sort in counterclockwise
double ConvexPolygonArea(vector<Point> &p, int n) {
  double area = 0;
  for (int i = 1; i < p.size() - 1; i++) area += Cross(
    p[i] - p[0], p[i + 1] - p[0]);
  return area / 2;
}
```

8.6 Half Plane Intersection

```
bool jizz(Line l1,Line l2,Line l3){
  Point p=intersect(l2,l3);
  return ((l1.pb-l1.pa)^(p-l1.pa))<-eps;
}

bool cmp(const Line &a,const Line &b){
  return same(a.angle,b.angle)?(((b.pb-b.pa)^(a.pb-b.pa
    ))>eps):a.angle<b.angle;
}

// availble area for Line l is (l.pb-l.pa)^(p-l.pa)>0
vector<Point> HPI(vector<Line> &ls){
  sort(ls.begin(),ls.end(),cmp);
  vector<Line> pls(1,ls[0]);
  for(unsigned int i=0;i<ls.size();++i)if(!same(ls[i].
    angle,pls.back().angle))pls.push_back(ls[i]);
  deque<int> dq; dq.push_back(0); dq.push_back(1);
  for(unsigned int i=2u;i<pls.size();++i){
    while(dq.size()>1u && jizz(pls[i],pls[dq.back()],
    pls[dq[dq.size()-2]]))dq.pop_back();
    while(dq.size()>1u && jizz(pls[i],pls[dq[0]],pls[dq
    [1]]))dq.pop_front();
    dq.push_back(i);
```

```
  }
  while(dq.size()>1u && jizz(pls[dq.front()],pls[dq.
    back()],pls[dq[dq.size()-2]]))dq.pop_back();
  while(dq.size()>1u && jizz(pls[dq.back()],pls[dq[0]],
    pls[dq[1]]))dq.pop_front();
  if(dq.size()<3u)return vector<Point>(); // no
    solution or solution is not a convex
  vector<Point> rt;
  for(unsigned int i=0u;i<dq.size();++i)rt.push_back(
    intersect(pls[dq[i]],pls[dq[(i+1)%dq.size()]]));
  return rt;
}
```

8.7 Rotating Sweep Line

```
void rotatingSweepLine(vector<pair<int,int>> &ps){
  int n=int(ps.size());
  vector<int> id(n),pos(n);
  vector<pair<int,int>> line(n*(n-1)/2);
  int m=-1;
  for(int i=0;i<n;++i)for(int j=i+1;j<n;++j)line[++m]=
    make_pair(i,j); ++m;
  sort(line.begin(),line.end(),[&](const pair<int,int>
    &a,const pair<int,int> &b)->bool{
    if(ps[a.first].first==ps[a.second].first)return 0;
    if(ps[b.first].first==ps[b.second].first)return 1;
    return (double)(ps[a.first].second-ps[a.second].
    second)/(ps[a.first].first-ps[a.second].first) < (
    double)(ps[b.first].second-ps[b.second].second)/(ps
    [b.first].first-ps[b.second].first);
  });
  for(int i=0;i<n;++i)id[i]=i;
  sort(id.begin(),id.end(),[&](const int &a,const int &
    b){ return ps[a]<ps[b]; });
  for(int i=0;i<n;++i)pos[id[i]]=i;

  for(int i=0;i<m;++i){
    auto l=line[i];
    // meow
    tie(pos[l.first],pos[l.second],id[pos[l.first]],id[
    pos[l.second]])=make_tuple(pos[l.second],pos[l.
    first],l.second,l.first);
  }
}
```

8.8 Triangle Center

```
Point TriangleCircumCenter(Point a, Point b, Point c) {
  Point res;
  double a1 = atan2(b.y - a.y, b.x - a.x) + pi / 2;
  double a2 = atan2(c.y - b.y, c.x - b.x) + pi / 2;
  double ax = (a.x + b.x) / 2;
  double ay = (a.y + b.y) / 2;
  double bx = (c.x + b.x) / 2;
  double by = (c.y + b.y) / 2;
  double r1 = (sin(a2) * (ax - bx) + cos(a2) * (by - ay
    )) / (sin(a1) * cos(a2) - sin(a2) * cos(a1));
  return Point(ax + r1 * cos(a1), ay + r1 * sin(a1));
}

Point TriangleMassCenter(Point a, Point b, Point c) {
  return (a + b + c) / 3.0;
}

Point TriangleOrthoCenter(Point a, Point b, Point c) {
  return TriangleMassCenter(a, b, c) * 3.0 -
    TriangleCircumCenter(a, b, c) * 2.0;
}

Point TriangleInnerCenter(Point a, Point b, Point c) {
  Point res;
  double la = len(b - c);
  double lb = len(a - c);
  double lc = len(a - b);
  res.x = (la * a.x + lb * b.x + lc * c.x) / (la + lb +
     lc);
  res.y = (la * a.y + lb * b.y + lc * c.y) / (la + lb +
     lc);
  return res;
}
```

8.9 Polygon Center

```
Point BaryCenter(vector<Point> &p, int n) {
  Point res(0, 0);
  double s = 0.0, t;
  for (int i = 1; i < p.size() - 1; i++) {
    t = Cross(p[i] - p[0], p[i + 1] - p[0]) / 2;
    s += t;
    res.x += (p[0].x + p[i].x + p[i + 1].x) * t;
    res.y += (p[0].y + p[i].y + p[i + 1].y) * t;
  }
  res.x /= (3 * s);
  res.y /= (3 * s);
  return res;
}
```

8.10 Maximum Triangle

```
double ConvexHullMaxTriangleArea(Point p[], int res[],
    int chnum) {
  double area = 0, tmp;
  res[chnum] = res[0];
  for (int i = 0, j = 1, k = 2; i < chnum; i++) {
    while (fabs(Cross(p[res[j]] - p[res[i]], p[res[(k +
     1) % chnum]] - p[res[i]])) > fabs(Cross(p[res[j]]
    - p[res[i]], p[res[k]] - p[res[i]]))) k = (k + 1) %
     chnum;
    tmp = fabs(Cross(p[res[j]] - p[res[i]], p[res[k]] -
     p[res[i]]));
    if (tmp > area) area = tmp;
    while (fabs(Cross(p[res[(j + 1) % chnum]] - p[res[i
    ]], p[res[k]] - p[res[i]])) > fabs(Cross(p[res[j]]
    - p[res[i]], p[res[k]] - p[res[i]]))) j = (j + 1) %
     chnum;
    tmp = fabs(Cross(p[res[j]] - p[res[i]], p[res[k]] -
     p[res[i]]));
    if (tmp > area) area = tmp;
  }
  return area / 2;
}
```

8.11 Point in Polygon

```
bool on(point a, point b, point c) {
  if (a.x == b.x) {
    if (c.x != a.x) return false;
    if (c.y >= min(a.y, b.y) && c.y <= max(a.y, b.y))
    return true;
    return false;
  }
  if (((a - c) ^ (b - c)) != 0) return false;
  if (a.x > b.x) swap(a, b);
  if (c.x < min(a.x, b.x) || c.x > max(a.x, b.x))
    return false;
  return ((a - b) ^ (a - c)) == 0;
}

int sgn(long long x) {
  if (x > 0) return 1;
  if (x < 0) return -1;
  return 0;
}

bool in(const vector<point> &c, point p) {
  int last = -2;
  int n = c.size();
  for (int i = 0; i < c.size(); ++i) {
    if (on(c[i], c[(i + 1) % n], p)) return true;
    int g = sgn((c[i] - p) ^ (c[(i + 1) % n] - p));
    if (last == -2) last = g;
    else if (last != g) return false;
```

```
  }
  return true;
}

bool in(point a, point b, point c, point p) {
  return in({ a, b, c }, p);
}

bool inside(const vector<point> &ch, point t) {
  point p = ch[1] - ch[0];
  point q = t - ch[0];
  if ((p ^ q) < 0) return false;
  if ((p ^ q) == 0) {
    if (p * q < 0) return false;
    if (q.len() > p.len()) return false;
    return true;
  }
  p = ch[ch.size() - 1] - ch[0];
  if ((p ^ q) > 0) return false;
  if ((p ^ q) == 0) {
    if (p * q < 0) return false;
    if (q.len() > p.len()) return false;
    return true;
  }
  p = ch[1] - ch[0];
  double ang = acos(1.0 * (p * q) / p.len() / q.len());
  int d = 20, z = ch.size() - 1;
  while (d--) {
    if (z - (1 << d) < 1) continue;
    point p1 = ch[1] - ch[0];
    point p2 = ch[z - (1 << d)] - ch[0];
    double tang = acos(1.0 * (p1 * p2) / p1.len() / p2.
    len());
    if (tang >= ang) z -= (1 << d);
  }
  return in(ch[0], ch[z - 1], ch[z], t);
}
```

8.12 Circle-Line Intersection

```
// remove second level if to get points for line (
    defalut: segment)
void CircleCrossLine(Point a, Point b, Point o, double
    r, Point ret[], int &num) {
  double x0 = o.x, y0 = o.y;
  double x1 = a.x, y1 = a.y;
  double x2 = b.x, y2 = b.y;
  double dx = x2 - x1, dy = y2 - y1;
  double A = dx * dx + dy * dy;
  double B = 2 * dx * (x1 - x0) + 2 * dy * (y1 - y0);
  double C = (x1 - x0) * (x1 - x0) + (y1 - y0) * (y1 -
    y0) - r * r;
  double delta = B * B - 4 * A * C;
  num = 0;
  if (epssgn(delta) >= 0) {
    double t1 = (-B - sqrt(fabs(delta))) / (2 * A);
    double t2 = (-B + sqrt(fabs(delta))) / (2 * A);
    if (epssgn(t1 - 1.0) <= 0 && epssgn(t1) >= 0) ret[
    num++] = Point(x1 + t1 * dx, y1 + t1 * dy);
    if (epssgn(t2 - 1.0) <= 0 && epssgn(t2) >= 0) ret[
    num++] = Point(x1 + t2 * dx, y1 + t2 * dy);
  }
}

vector<Point> CircleCrossLine(Point a, Point b, Point o
    , double r) {
  double x0 = o.x, y0 = o.y;
  double x1 = a.x, y1 = a.y;
  double x2 = b.x, y2 = b.y;
  double dx = x2- x1, dy = y2 - y1;
  double A = dx * dx + dy * dy;
  double B = 2 * dx * (x1 - x0) + 2 * dy * (y1 - y0);
  double C = (x1 - x0) * (x1 - x0) + (y1 - y0) * (y1 -
    y0) - r * r;
  double delta = B * B - 4 * A * C;
  vector<Point> ret;
  if (epssgn(delta) >= 0) {
    double t1 = (-B - sqrt(fabs(delta))) / (2 * A);
    double t2 = (-B + sqrt(fabs(delta))) / (2 * A);
    if (epssgn(t1 - 1.0) <= 0 && epssgn(t1) >= 0) ret.
    emplace_back(x1 + t1 * dx, y1 + t1 * dy);
    if (epssgn(t2 - 1.0) <= 0 && epssgn(t2) >= 0) ret.
    emplace_back(x1 + t2 * dx, y1 + t2 * dy);
  }
  return ret;
}
```

8.13 Circle-Triangle Intersection

```
// calc area intersect by circle with radius r and
    triangle OAB
double Calc(Point a, Point b, double r) {
  Point p[2];
  int num = 0;
  bool ina = epssgn(len(a) - r) < 0, inb = epssgn(len(b
    ) - r) < 0;
  if (ina) {
    if (inb) return fabs(Cross(a, b)) / 2.0; //
    triangle in circle
    else { // a point inside and another outside: calc
    sector and triangle area
      CircleCrossLine(a, b, Point(0, 0), r, p, num);
      return SectorArea(b, p[0], r) + fabs(Cross(a, p
    [0])) / 2.0;
    }
  } else {
    CircleCrossLine(a, b, Point(0, 0), r, p, num);
    if (inb) return SectorArea(p[0], a, r) + fabs(Cross
    (p[0], b)) / 2.0;
    else {
      if (num == 2) return SectorArea(a, p[0], r) +
    SectorArea(p[1], b, r) + fabs(Cross(p[0], p[1])) /
    2.0; // segment ab has 2 point intersect with
    circle
      else return SectorArea(a, b, r); // segment has
    no intersect point with circle
    }
  }
}
```

8.14 Polygon Diameter

```
// get diameter of p[res[]] store opposite points in
    app
double Diameter(Point p[], int res[], int chnum, int
    app[][2], int &appnum) {
  double ret = 0, nowlen;
  res[chnum] = res[0];
  appnum = 0;
  for (int i = 0, j = 1; i < chnum; ++i) {
    while (Cross(p[res[i]] - p[res[i + 1]], p[res[j +
    1]] - p[res[i + 1]]) < Cross(p[res[i]] - p[res[i +
    1]], p[res[j]] - p[res[i + 1]])) {
      ++j;
      j %= chnum;
    }
    app[appnum][0] = res[i];
    app[appnum][1] = res[j];
    ++appnum;
    nowlen = dis(p[res[i]], p[res[j]]);
    if (nowlen > ret) ret = nowlen;
    nowlen = dis(p[res[i + 1]], p[res[j + 1]]);
    if (nowlen > ret) ret = nowlen;
  }
  return ret;
}
```

8.15 Minimun Distance of 2 Polygons

```
// p, q is convex
double TwoConvexHullMinDist(Point P[], Point Q[], int n
    , int m) {
  int YMinP = 0, YMaxQ = 0;
  double tmp, ans = 999999999;
```

```
  for (i = 0; i < n; ++i) if(P[i].y < P[YMinP].y) YMinP
     = i;
  for (i = 0; i < m; ++i) if(Q[i].y > Q[YMaxQ].y) YMaxQ
     = i;
  P[n] = P[0], Q[m] = Q[0];
  for (int i = 0; i < n; ++i) {
    while (tmp = Cross(Q[YMaxQ + 1] - P[YMinP + 1], P[
    YMinP] - P[YMinP + 1]) > Cross(Q[YMaxQ] - P[YMinP +
     1], P[YMinP] - P[YMinP + 1])) YMaxQ = (YMaxQ + 1)
    % m;
    if (tmp < 0) ans = min(ans, PointToSegDist(P[YMinP
    ], P[YMinP + 1], Q[YMaxQ]));
    else ans = min(ans, TwoSegMinDist(P[YMinP], P[YMinP
     + 1], Q[YMaxQ], Q[YMaxQ + 1]));
    YMinP = (YMinP + 1) % n;
  }
  return ans;
}
```

8.16 2D Convex Hull

```
vector<point> convex(vector<point> p) {
  sort(p.begin(), p.end());
  vector<point> ch;
  for (int i = 0; i < n; ++i) {
    while (ch.size() >= 2 && ((p[i] - ch[ch.size() -
    2]) ^ (ch[ch.size() - 1] - ch[ch.size() - 2])) >=
    0) ch.pop_back();
    ch.push_back(p[i]);
  }
  int t = ch.size();
  for (int i = n - 2; i >= 0; --i) {
    while (ch.size() > t && ((p[i] - ch[ch.size() - 2])
     ^ (ch[ch.size() - 1] - ch[ch.size() - 2])) >= 0)
    ch.pop_back();
    ch.push_back(p[i]);
  }
  ch.pop_back();
  return ch;
}
```

8.17 3D Convex Hull

```
double absvol(const Point a,const Point b,const Point c
    ,const Point d){
  return abs(((b-a)^(c-a))*(d-a))/6;
}

struct convex3D{
static const int maxn=1010;
struct Triangle{
  int a,b,c;
  bool res;
  Triangle(){}
  Triangle(int a,int b,int c,bool res=1):a(a),b(b),c(c)
    ,res(res){}
};
int n,m;
Point p[maxn];
Triangle f[maxn*8];
int id[maxn][maxn];
bool on(Triangle &t,Point &pt){
  return ((p[t.c]-p[t.b])^(p[t.a]-p[t.b]))*(pt-p[t.a])>
    eps;
}
void meow(int pi,int a,int b){
  int f2=id[a][b];
  if(f[f2].res){
    if(on(f[f2],p[pi]))dfs(pi,f2);
    else{
      id[pi][b]=id[a][pi]=id[b][a]=m;
      f[m++]=Triangle(b,a,pi,1);
    }
  }
}
void dfs(int pi,int now){
  f[now].res=0;
  meow(pi,f[now].b,f[now].a);
  meow(pi,f[now].c,f[now].b);
  meow(pi,f[now].a,f[now].c);
}
void operator()(){
  if(n<4)return;
  if([&]()->int{
    for(int i=1;i<n;++i){
      if(abs(p[0]-p[i])>eps){
        swap(p[1],p[i]);
        return 0;
      }
    }
    return 1;
  }())return;
  if([&]()->int{
    for(int i=2;i<n;++i){
      if(abs((p[0]-p[i])^(p[1]-p[i]))>eps){
        swap(p[2],p[i]);
        return 0;
      }
    }
    return 1;
  }())return;
  if([&]()->int{
    for(int i=3;i<n;++i){
      if(abs(((p[1]-p[0])^(p[2]-p[0]))*(p[i]-p[0]))>eps
    ){
        swap(p[3],p[i]);
        return 0;
      }
    }
    return 1;
  }())return;
  for(int i=0;i<4;++i){
    Triangle tmp((i+1)%4,(i+2)%4,(i+3)%4,1);
    if(on(tmp,p[i]))swap(tmp.b,tmp.c);
    id[tmp.a][tmp.b]=id[tmp.b][tmp.c]=id[tmp.c][tmp.a]=
    m;
    f[m++]=tmp;
  }
  for(int i=4;i<n;++i){
    for(int j=0;j<m;++j){
      if(f[j].res && on(f[j],p[i])){
        dfs(i,j);
        break;
      }
    }
  }
  int mm=m; m=0;
  for(int i=0;i<mm;++i){
    if(f[i].res)f[m++]=f[i];
  }
}
bool same(int i,int j){
  return !(absvol(p[f[i].a],p[f[i].b],p[f[i].c],p[f[j].
    a])>eps || absvol(p[f[i].a],p[f[i].b],p[f[i].c],p[f
    [j].b])>eps || absvol(p[f[i].a],p[f[i].b],p[f[i].c
    ],p[f[j].c])>eps);
}
int faces(){
  int rt=0;
  for(int i=0;i<m;++i){
    int iden=1;
    for(int j=0;j<i;++j){
      if(same(i,j))iden=0;
    }
    rt+=iden;
  }
  return rt;
}
} tb;
```

8.18 Rotating Caliper

```
struct pnt {
  int x, y;
  pnt(): x(0), y(0) {};
  pnt(int xx, int yy): x(xx), y(yy) {};
} p[maxn];
```

```
pnt operator-(const pnt &a, const pnt &b) { return pnt(
    b.x - a.x, b.y - a.y); }
int operator^(const pnt &a, const pnt &b) { return a.x
    * b.y - a.y * b.x; } //cross
int operator*(const pnt &a, const pnt &b) { return (a -
    b).x * (a - b).x + (a - b).y * (a - b).y; } //
    distance
int tb[maxn], tbz, rsd;

int dist(int n1, int n2){
  return p[n1] * p[n2];
}
int cross(int t1, int t2, int n1){
  return (p[t2] - p[t1]) ^ (p[n1] - p[t1]);
}
bool cmpx(const pnt &a, const pnt &b) { return a.x == b
    .x ? a.y < b.y : a.x < b.x; }

void RotatingCaliper() {
  sort(p, p + n, cmpx);
  for (int i = 0; i < n; ++i) {
    while (tbz > 1 && cross(tb[tbz - 2], tb[tbz - 1], i
    ) <= 0) --tbz;
    tb[tbz++] = i;
  }
  rsd = tbz - 1;
  for (int i = n - 2; i >= 0; --i) {
    while (tbz > rsd + 1 && cross(tb[tbz - 2], tb[tbz -
     1], i) <= 0) --tbz;
    tb[tbz++] = i;
  }
  --tbz;
  int lpr = 0, rpr = rsd;
  // tb[lpr], tb[rpr]
  while (lpr < rsd || rpr < tbz - 1) {
    if (lpr < rsd && rpr < tbz - 1) {
      pnt rvt = p[tb[rpr + 1]] - p[tb[rpr]];
      pnt lvt = p[tb[lpr + 1]] - p[tb[lpr]];
      if ((lvt ^ rvt) < 0) ++lpr;
      else ++rpr;
    }
    else if (lpr == rsd) ++rpr;
    else ++lpr;
    // tb[lpr], tb[rpr]
  }
}
```

8.19 Minimum Enclosing Circle

```
pt center(const pt &a, const pt &b, const pt &c) {
  pt p0 = b - a, p1 = c - a;
  double c1 = norm2(p0) * 0.5, c2 = norm2(p1) * 0.5;
  double d = p0 ^ p1;
  double x = a.x + (c1 * p1.y - c2 * p0.y) / d;
  double y = a.y + (c2 * p0.x - c1 * p1.x) / d;
  return pt(x, y);
}

circle min_enclosing(vector<pt> &p) {
  random_shuffle(p.begin(), p.end());
  double r = 0.0;
  pt cent;
  for (int i = 0; i < p.size(); ++i) {
    if (norm2(cent - p[i]) <= r) continue;
    cent = p[i];
    r = 0.0;
    for (int j = 0; j < i; ++j) {
      if (norm2(cent - p[j]) <= r) continue;
      cent = (p[i] + p[j]) / 2;
      r = norm2(p[j] - cent);
      for (int k = 0; k < j; ++k) {
        if (norm2(cent - p[k]) <= r) continue;
        cent = center(p[i], p[j], p[k]);
        r = norm2(p[k] - cent);
      }
    }
  }
  return circle(cent, sqrt(r));
}
```

8.20 Closest Pair

```
pt p[maxn];

double dis(const pt& a, const pt& b) {
  return sqrt((a - b) * (a - b));
}

double closest_pair(int l, int r) {
  if (l == r) return inf;
  if (r - l == 1) return dis(p[l], p[r]);
  int m = (l + r) >> 1;
  double d = min(closest_pair(l, m), closest_pair(m +
    1, r));
  vector<int> vec;
  for (int i = m; i >= l && fabs(p[m].x - p[i].x) < d;
    --i) vec.push_back(i);
  for (int i = m + 1; i <= r && fabs(p[m].x - p[i].x) <
     d; ++i) vec.push_back(i);
  sort(vec.begin(), vec.end(), [=](const int& a, const
    int& b) { return p[a].y < p[b].y; });
  for (int i = 0; i < vec.size(); ++i) {
    for (int j = i + 1; j < vec.size() && fabs(p[vec[j
    ]].y - p[vec[i]].y) < d; ++j) {
      d = min(d, dis(p[vec[i]], p[vec[j]]));
    }
  }
  return d;
}
```

9 Problems

9.1 Manhattan Distance Minimum Spanning Tree

```
#include <bits/stdc++.h>
using namespace std;

const int maxn = 1e5 + 5;
int x[maxn], y[maxn], fa[maxn];
pair<int, int> bit[maxn];
vector<tuple<int, int, int>> ed;

void init() {
  for (int i = 0; i < maxn; ++i)
    bit[i] = make_pair(1e9, -1);
}

void add(int p, pair<int, int> v) {
  for (; p < maxn; p += p & -p)
    bit[p] = min(bit[p], v);
}

pair<int, int> query(int p) {
  pair<int, int> res = make_pair(1e9, -1);
  for (; p; p -= p & -p)
    res = min(res, bit[p]);
  return res;
}

void add_edge(int u, int v) {
  ed.emplace_back(u, v, abs(x[u] - x[v]) + abs(y[u] - y
    [v]));
}

void solve(int n) {
  init();
  vector<int> v(n), ds;
  for (int i = 0; i < n; ++i) {
    v[i] = i;
    ds.push_back(x[i] - y[i]);
  }
  sort(ds.begin(), ds.end());
  ds.resize(unique(ds.begin(), ds.end()) - ds.begin());
  sort(v.begin(), v.end(), [&](int i, int j) { return x
    [i] == x[j] ? y[i] > y[j] : x[i] > x[j]; });
  int j = 0;
```

```
  for (int i = 0; i < n; ++i) {
    int p = lower_bound(ds.begin(), ds.end(), x[v[i]] -
     y[v[i]]) - ds.begin() + 1;
    pair<int, int> q = query(p);
    if (~q.second) add_edge(v[i], q.second);
    add(p, make_pair(x[v[i]] + y[v[i]], v[i]));
  }
}

int find(int x) {
  if (x == fa[x]) return x;
  return fa[x] = find(fa[x]);
}

void merge(int x, int y) {
  fa[find(x)] = find(y);
}

int main() {
  int n; scanf("%d", &n);
  for (int i = 0; i < n; ++i) scanf("%d %d", &x[i], &y[
    i]);
  solve(n);
  for (int i = 0; i < n; ++i) swap(x[i], y[i]);
  solve(n);
  for (int i = 0; i < n; ++i) x[i] = -x[i];
  solve(n);
  for (int i = 0; i < n; ++i) swap(x[i], y[i]);
  solve(n);
  sort(ed.begin(), ed.end(), [](const tuple<int, int,
    int> &a, const tuple<int, int, int> &b) {
    return get<2>(a) < get<2>(b);
  });
  for (int i = 0; i < n; ++i) fa[i] = i;
  long long ans = 0;
  for (int i = 0; i < ed.size(); ++i) {
    int x, y, w; tie(x, y, w) = ed[i];
    if (find(x) == find(y)) continue;
    merge(x, y);
    ans += w;
  }
  printf("%lld\n", ans);
  return 0;
}
```

9.2 "Dynamic" Kth Element (parallel binary search)

```
#include <bits/stdc++.h>
using namespace std;

const int maxn = 1e5 + 5;
int a[maxn], ans[maxn], tmp[maxn];

struct query { int op, l, r, k, qid; };

struct fenwick {
  int dat[maxn];
  void init() { memset(dat, 0, sizeof(dat)); }
  void add(int p, int v) { for (; p < maxn; p += p & -p
    ) dat[p] += v; }
  int qry(int p, int v = 0) { for (; p; p -= p & -p) v
    += dat[p]; return v; }
} bit;

void bs(vector<query> &qry, int l, int r) {
  if (l == r) {
    for (int i = 0; i < qry.size(); ++i) {
      if (qry[i].op == 3) ans[qry[i].qid] = l;
    }
    return;
  }
  if (qry.size() == 0) return;
  int m = l + r >> 1;
  for (int i = 0; i < qry.size(); ++i) {
    if (qry[i].op == 1 && qry[i].r <= m) bit.add(qry[i
    ].l, 1);
    else if (qry[i].op == 2 && qry[i].r <= m) bit.add(
    qry[i].l, -1);
    else if (qry[i].op == 3) tmp[qry[i].qid] += bit.qry
    (qry[i].r) - bit.qry(qry[i].l - 1);
  }
  vector<query> ql, qr;
  for (int i = 0; i < qry.size(); ++i) {
    if (qry[i].op == 3) {
      if (qry[i].k - tmp[qry[i].qid] > 0) qry[i].k -=
    tmp[qry[i].qid], qr.push_back(qry[i]);
      else ql.push_back(qry[i]);
      tmp[qry[i].qid] = 0;
      continue;
    }
    if (qry[i].r <= m) ql.push_back(qry[i]);
    else qr.push_back(qry[i]);
  }
  for (int i = 0; i < qry.size(); ++i) {
    if (qry[i].op == 1 && qry[i].r <= m) bit.add(qry[i
    ].l, -1);
    else if (qry[i].op == 2 && qry[i].r <= m) bit.add(
    qry[i].l, 1);
  }
  bs(ql, l, m), bs(qr, m + 1, r);
}

int main() {
  int t; scanf("%d", &t);
  while (t--) {
    int n, q; scanf("%d %d", &n, &q);
    vector<query> qry;
    vector<int> ds;
    bit.init();
    for (int i = 1; i <= n; ++i) {
      scanf("%d", a + i); ds.push_back(a[i]);
      qry.push_back({ 1, i, a[i], -1, -1 });
    }
    int qid = 0;
    for (int i = 0; i < q; ++i) {
      int t; scanf("%d", &t);
      if (t == 1) {
        int l, r, k; scanf("%d %d %d", &l, &r, &k);
        qry.push_back({ 3, l, r, k, qid }); ++qid;
      }
      if (t == 2) {
        int c, v; scanf("%d %d", &c, &v);
        ds.push_back(v);
        qry.push_back({ 2, c, a[c], -1, -1 });
        qry.push_back({ 1, c, v, -1, -1 });
        a[c] = v;
      }
      if (t == 3) {
        int x, v; scanf("%d %d", &x, &v);
        ans[qid] = -1, ++qid;
      }
    }
    sort(ds.begin(), ds.end()); ds.resize(unique(ds.
    begin(), ds.end()) - ds.begin());
    for (int i = 0; i < qry.size(); ++i) {
      if (qry[i].op == 3) continue;
      qry[i].r = lower_bound(ds.begin(), ds.end(), qry[
    i].r) - ds.begin();
    }
    bs(qry, 0, ds.size() - 1);
    for (int i = 0; i < qid; ++i) {
      if (ans[i] == -1) puts("7122");
      else assert(ans[i] < ds.size()), printf("%d\n",
    ds[ans[i]]);
    }
  }
  return 0;
}
```

9.3 Dynamic Kth Element (persistent segment tree)

```
#include <bits/stdc++.h>
using namespace std;

const int maxn = 1e5 + 5;
int a[maxn], bit[maxn];
```

```cpp
vector<int> ds;
vector<vector<int>> qr;

namespace segtree {
  int st[maxn * 97], lc[maxn * 97], rc[maxn * 97], sz;
  int gnode() {
    st[sz] = 0;
    lc[sz] = rc[sz] = 0;
    return sz++;
  }
  int gnode(int z) {
    st[sz] = st[z];
    lc[sz] = lc[z], rc[sz] = rc[z];
    return sz++;
  }
  int build(int l, int r) {
    int z = gnode();
    if (r - l == 1) return z;
    lc[z] = build(l, (l + r) / 2), rc[z] = build((l + r
    ) / 2, r);
    return z;
  }
  int modify(int l, int r, int p, int v, int o) {
    int z = gnode(o);
    if (r - l == 1) return st[z] += v, z;
    if (p < (l + r) / 2) lc[z] = modify(l, (l + r) / 2,
     p, v, lc[o]);
    else rc[z] = modify((l + r) / 2, r, p, v, rc[o]);
    st[z] = st[lc[z]] + st[rc[z]];
    return z;
  }
  int query(int l, int r, int ql, int qr, int o) {
    if (l >= qr || ql >= r) return 0;
    if (l >= ql && r <= qr) return st[o];
    return query(l, (l + r) / 2, ql, qr, lc[o]) +
        query((l + r) / 2, r, ql, qr, rc[o]);
  }
}

void init(int n) {
  segtree::sz = 0;
  bit[0] = segtree::build(0, ds.size());
  for (int i = 1; i <= n; ++i) bit[i] = bit[0];
}

void add(int p, int n, int x, int v) {
  for (; p <= n; p += p & -p)
    bit[p] = segtree::modify(0, ds.size(), x, v, bit[p
    ]);
}

vector<int> query(int p) {
  vector<int> z;
  for (; p; p -= p & -p)
    z.push_back(bit[p]);
  return z;
}

int dfs(int l, int r, vector<int> lz, vector<int> rz,
    int k) {
  if (r - l == 1) return l;
  int ls = 0, rs = 0;
  for (int i = 0; i < lz.size(); ++i) ls += segtree::st
    [segtree::lc[lz[i]]];
  for (int i = 0; i < rz.size(); ++i) rs += segtree::st
    [segtree::lc[rz[i]]];
  if (rs - ls >= k) {
    for (int i = 0; i < lz.size(); ++i) lz[i] = segtree
    ::lc[lz[i]];
    for (int i = 0; i < rz.size(); ++i) rz[i] = segtree
    ::lc[rz[i]];
    return dfs(l, (l + r) / 2, lz, rz, k);
  } else {
    for (int i = 0; i < lz.size(); ++i) lz[i] = segtree
    ::rc[lz[i]];
    for (int i = 0; i < rz.size(); ++i) rz[i] = segtree
    ::rc[rz[i]];
    return dfs((l + r) / 2, r, lz, rz, k - (rs - ls));
  }
}

int main() {
  int t; scanf("%d", &t);
  while (t--) {
    int n, q; scanf("%d %d", &n, &q);
    for (int i = 1; i <= n; ++i) scanf("%d", &a[i]), ds
    .push_back(a[i]);
    for (int i = 0; i < q; ++i) {
      int a, b, c; scanf("%d %d %d", &a, &b, &c);
      vector<int> v = { a, b, c };
      if (a == 1) {
        int d; scanf("%d", &d);
        v.push_back(d);
      }
      qr.push_back(v);
    }
    for (int i = 0; i < q; ++i) if (qr[i][0] == 2) ds.
    push_back(qr[i][2]);
    sort(ds.begin(), ds.end()), ds.resize(unique(ds.
    begin(), ds.end()) - ds.begin());
    for (int i = 1; i <= n; ++i) a[i] = lower_bound(ds.
    begin(), ds.end(), a[i]) - ds.begin();
    for (int i = 0; i < q; ++i) if (qr[i][0] == 2) qr[i
    ][2] = lower_bound(ds.begin(), ds.end(), qr[i][2])
    - ds.begin();
    init(n);
    for (int i = 1; i <= n; ++i) add(i, n, a[i], 1);
    for (int i = 0; i < q; ++i) {
      if (qr[i][0] == 3) {
        puts("7122");
        continue;
      }
      if (qr[i][0] == 1) {
        vector<int> lz = query(qr[i][1] - 1);
        vector<int> rz = query(qr[i][2]);
        int ans = dfs(0, ds.size(), lz, rz, qr[i][3]);
        printf("%d\n", ds[ans]);
      } else {
        add(qr[i][1], n, a[qr[i][1]], -1);
        add(qr[i][1], n, qr[i][2], 1);
        a[qr[i][1]] = qr[i][2];
      }
    }
    ds.clear(), qr.clear();
  }
  return 0;
}
```

9.4 Hilbert's Curve (faster Mo's algorithm)

```cpp
long long hilbert(int n, int x, int y) {
  long long res = 0;
  for (int s = n / 2; s; s >>= 1) {
    int rx = (x & s) > 0;
    int ry = (y & s) > 0;
    res += s * 1ll * s * ((3 * rx) ^ ry);
    if (ry == 0) {
      if (rx == 1) {
        x = s - 1 - x;
        y = s - 1 - y;
      }
      swap(x, y);
    }
  }
  return res;
}
```