Contents

1 Basic

1.1 vimrc

```
syn on
colo desert
se ai nu ru mouse=a
se cin et ts=4 sw=4 sts=4
set backspace=indent,eol,start
set number relativenumber
inoremap {<ENTER> {<ENTER>}<UP><END><ENTER>
```

1.2 Compilation Argument

```
g++ -W -Wall -Wextra -O2 -std=c++14 -fsanitize=address
    -fsanitize=undefined -fsanitize=leak
```

1.3 Checker

```
for ((i = 0; i < 100; i++))
do
  ./gen > in
  ./ac < in > out1
  ./tle < in > out2
  diff out1 out2 || break
done
```

1.4 Fast Integer Input

```
#define getchar gtx

inline int gtx() {
  const int N = 4096;
  static char buffer[N];
  static char *p = buffer, *end = buffer;
  if (p == end) {
    if ((end = buffer + fread(buffer, 1, N, stdin)) ==
    buffer) return EOF;
    p = buffer;
  }
  return *p++;
}

template <typename T>
inline bool rit(T& x) {
  char c = 0; bool flag = false;
  while (c = getchar(), (c < '0' && c != '-') || c > '9
  ') if (c == -1) return false;
  c == '-' ? (flag = true, x = 0) : (x = c - '0');
  while (c = getchar(), c >= '0' && c <= '9') x = x *
    10 + c - '0';
  if (flag) x = -x;
  return true;
}

template <typename T, typename ...Args>
inline bool rit(T& x, Args& ...args) { return rit(x) &&
    rit(args...); }
```

1.5 IncStack

```
const int size = 256 << 20;
register long rsp asm("rsp");
char *p = (char*)malloc(size) + size, *bak = (char*)rsp
    ;
__asm__("movq %0, %%rsp\n"::"r"(p));

// main

__asm__("movq %0, %%rsp\n"::"r"(bak));
```

1.6 Pragma optimization

```
#pragma GCC optimize("Ofast", "no-stack-protector", "no
    -math-errno", "unroll-loops")
#pragma GCC target("sse,sse2,sse3,ssse3,sse4,popcnt,abm
    ,mmx,avx,tune=native")

#pragma warning(disable:4996)
#pragma comment(linker, "/STACK:336777216")

#pragma GCC ivdep
```

2 Flow

2.1 Dinic

```cpp
struct dinic {
  static const int inf = 1e9;
  struct edge {
    int dest, cap, rev;
    edge(int d, int c, int r): dest(d), cap(c), rev(r)
    {}
  };
  vector<edge> g[maxn];
  int qu[maxn], ql, qr;
  int lev[maxn];
  void init() {
    for (int i = 0; i < maxn; ++i)
      g[i].clear();
  }
  void add_edge(int a, int b, int c) {
    g[a].emplace_back(b, c, g[b].size() - 0);
    g[b].emplace_back(a, 0, g[a].size() - 1);
  }
  bool bfs(int s, int t) {
    memset(lev, -1, sizeof(lev));
    lev[s] = 0;
    ql = qr = 0;
    qu[qr++] = s;
    while (ql < qr) {
      int x = qu[ql++];
      for (edge &e : g[x]) if (lev[e.dest] == -1 && e.
    cap > 0) {
        lev[e.dest] = lev[x] + 1;
        qu[qr++] = e.dest;
      }
    }
    return lev[t] != -1;
  }
  int dfs(int x, int t, int flow) {
    if (x == t) return flow;
    int res = 0;
    for (edge &e : g[x]) if (e.cap > 0 && lev[e.dest]
    == lev[x] + 1) {
      int f = dfs(e.dest, t, min(e.cap, flow - res));
      res += f;
      e.cap -= f;
      g[e.dest][e.rev].cap += f;
    }
    if (res == 0) lev[x] = -1;
    return res;
  }
  int operator()(int s, int t) {
    int flow = 0;
    for (; bfs(s, t); flow += dfs(s, t, inf));
    return flow;
  }
};
```

2.2 ISAP

```cpp
struct isap {
  static const int inf = 1e9;
  struct edge {
    int dest, cap, rev;
```

```cpp
    edge(int a, int b, int c): dest(a), cap(b), rev(c)
    {}
  };
  vector<edge> g[maxn];
  int it[maxn], gap[maxn], d[maxn];
  void add_edge(int a, int b, int c) {
    g[a].emplace_back(b, c, g[b].size() - 0);
    g[b].emplace_back(a, 0, g[a].size() - 1);
  }
  int dfs(int x, int t, int tot, int flow) {
    if (x == t) return flow;
    for (int &i = it[x]; i < g[x].size(); ++i) {
      edge &e = g[x][i];
      if (e.cap > 0 && d[e.dest] == d[x] - 1) {
        int f = dfs(e.dest, t, tot, min(flow, e.cap));
        if (f) {
          e.cap -= f;
          g[e.dest][e.rev].cap += f;
          return f;
        }
      }
    }
    if ((--gap[d[x]]) == 0) d[x] = tot;
    else d[x]++, it[x] = 0, ++gap[d[x]];
    return 0;
  }
  int operator()(int s, int t, int tot) {
    memset(it, 0, sizeof(it));
    memset(gap, 0, sizeof(gap));
    memset(d, 0, sizeof(d));
    int r = 0;
    gap[0] = tot;
    for (; d[s] < tot; r += dfs(s, t, tot, inf));
    return r;
  }
};
```

2.3 Minimum-cost flow

```cpp
struct mincost {
  struct edge {
    int dest, cap, w, rev;
    edge(int a, int b, int c, int d): dest(a), cap(b),
    w(c), rev(d) {}
  };
  vector<edge> g[maxn];
  int d[maxn], p[maxn], ed[maxn];
  bool inq[maxn];
  void init() {
    for (int i = 0; i < maxn; ++i) g[i].clear();
  }
  void add_edge(int a, int b, int c, int d) {
    g[a].emplace_back(b, c, +d, g[b].size() - 0);
    g[b].emplace_back(a, 0, -d, g[a].size() - 1);
  }
  bool spfa(int s, int t, int &f, int &c) {
    for (int i = 0; i < maxn; ++i) {
      d[i] = inf;
      p[i] = ed[i] = -1;
      inq[i] = false;
    }
    d[s] = 0;
    queue<int> q;
    q.push(s);
    while (q.size()) {
      int x = q.front(); q.pop();
      inq[x] = false;
      for (int i = 0; i < g[x].size(); ++i) {
        edge &e = g[x][i];
        if (e.cap > 0 && d[e.dest] > d[x] + e.w) {
          d[e.dest] = d[x] + e.w;
          p[e.dest] = x;
          ed[e.dest] = i;
          if (!inq[e.dest]) q.push(e.dest), inq[e.dest]
    = true;
        }
      }
    }
    if (d[t] == inf) return false;
    int dlt = inf;
```

```cpp
    for (int x = t; x != s; x = p[x]) dlt = min(dlt, g[
    p[x]][ed[x]].cap);
    for (int x = t; x != s; x = p[x]) {
      edge &e = g[p[x]][ed[x]];
      e.cap -= dlt;
      g[e.dest][e.rev].cap += dlt;
    }
    f += dlt; c += d[t] * dlt;
    return true;
  }
  pair<int, int> operator()(int s, int t) {
    int f = 0, c = 0;
    while (spfa(s, t, f, c));
    return make_pair(f, c);
  }
};
```

2.4 Gomory-Hu Tree

```cpp
int g[maxn];
vector<edge> GomoryHu(int n){
  vector<edge> rt;
  for(int i=1;i<=n;++i)g[i]=1;
  for(int i=2;i<=n;++i){
    int t=g[i];
    flow.reset(); // clear flows on all edge
    rt.push_back({i,t,flow(i,t)});
    flow.walk(i); // bfs points that connected to i (
    use edges not fully flow)
    for(int j=i+1;j<=n;++j){
      if(g[j]==t && flow.connect(j))g[j]=i; // check if
      i can reach j
    }
  }
  return rt;
}
```

2.5 Hungarian ($O(n^3)$)

```cpp
struct Hungarian {
  vector<vector<int>> w;
  bitset<maxn> s, t;
  vector<int> lx, ly, mx, my, slack, prv;
  int n, matched;
  Hungarian() {}
  Hungarian(int _n): n(_n) {
    w = vector<vector<int>>(n, vector<int>(n));
    lx.resize(n); ly.resize(n); mx.assign(n, -1); my.
    assign(n, -1);
    slack.resize(n); prv.resize(n);
  }
  void add_edge(int a, int b, int c) {
    w[a][b] = c;
  }
  void add(int x) {
    s[x] = true;
    for (int i = 0; i < n; ++i) {
      if (lx[x] + ly[i] - w[x][i] < slack[i]) {
        slack[i] = lx[x] + ly[i] - w[x][i];
        prv[i] = x;
      }
    }
  }
  void augment(int now) {
    int x = prv[now], y = now;
    ++matched;
    while (true) {
      int tmp = mx[x]; mx[x] = y; my[y] = x; y = tmp;
      if (y == -1) return;
      x = prv[y];
    }
  }
  void relabel() {
    int delta = inf;
    for (int i = 0; i < n; ++i) if (!t[i]) delta = min(
    delta, slack[i]);
    for (int i = 0; i < n; ++i) if (s[i]) lx[i] -=
    delta;
```

```cpp
    for (int i = 0; i < n; ++i) {
      if (t[i]) ly[i] += delta;
      else slack[i] -= delta;
    }
  }
  void go() {
    s.reset(); t.reset();
    fill(slack.begin(), slack.end(), inf);
    int root = 0;
    for (; root < n && mx[root] != -1; ++root);
    add(root);
    while (true) {
      relabel();
      int y = 0;
      for (; y < n; ++y) if (!t[y] && slack[y] == 0)
      break;
      if (my[y] == -1) return augment(y), void();
      add(my[y]); t[y] = true;
    }
  }
  int matching() {
    int ret = 0;
    for (int i = 0; i < n; ++i) {
      for (int j = 0; j < n; ++j) lx[i] = max(lx[i], w[
    i][j]);
    }
    for (int i = 0; i < n; ++i) go();
    for (int i = 0; i < n; ++i) ret += w[i][mx[i]];
    return ret;
  }
};
```

2.6 Hungarian ($O(n^4)$)

```cpp
struct hungarian {
  static const int inf = 1e9;
  int lx[maxn], ly[maxn], w[maxn][maxn];
  int match[maxn];
  bool vx[maxn], vy[maxn];
  void init() {
    for (int i = 0; i < maxn; ++i) for (int j = 0; j <
    maxn; ++j) w[i][j] = -inf;
    for (int i = 0; i < maxn; ++i) w[i][i] = 0;
  }
  void add_edge(int a, int b, int c) {
    w[a][b] = max(w[a][b], c);
  }
  bool dfs(int now) {
    vx[now] = true;
    for (int i = 0; i < maxn; ++i) if (lx[now] + ly[i]
    == w[now][i] && !vy[i]) {
      vy[i] = true;
      if (!match[i] || dfs(match[i])) {
        match[i] = now;
        return true;
      }
    }
    return false;
  }
  void relabel() {
    int dlt = inf;
    for (int i = 0; i < maxn; ++i) if (vx[i]) {
      for (int j = 0; j < maxn; ++j) if (!vy[j]) dlt =
    min(dlt, lx[i] + ly[j] - w[i][j]);
    }
    for (int i = 0; i < maxn; ++i) if (vx[i]) lx[i] -=
    dlt;
    for (int i = 0; i < maxn; ++i) if (vy[i]) ly[i] +=
    dlt;
  }
  int operator()() {
    fill(lx, lx + maxn, -inf); fill(ly, ly + maxn, 0);
    for (int i = 0; i < maxn; ++i) {
      for (int j = 0; j < maxn; ++j) lx[i] = max(lx[i],
     w[i][j]);
    }
    memset(match, 0, sizeof(match));
    for (int i = 0; i < maxn; ++i) {
      while (true) {
        memset(vx, false, sizeof(vx));
```

```
        memset(vy, false, sizeof(vy));
        if (dfs(i)) break;
        relabel();
      }
    }
    int r = 0;
    for (int i = 0; i < maxn; ++i) if (w[match[i]][i] >
     0) r += w[match[i]][i];
    return r;
  }
};
```

3 Data Structure

3.1 Disjoint Set

```
struct DisjointSet {
  int p[maxn], sz[maxn], n, cc;
  vector<pair<int*, int>> his;
  vector<int> sh;
  void init(int _n) {
    n = _n; cc = n;
    for (int i = 0; i < n; ++i) sz[i] = 1, p[i] = i;
    sh.clear(); his.clear();
  }
  void assign(int *k, int v) {
    his.emplace_back(k, *k);
    *k = v;
  }
  void save() {
    sh.push_back((int)his.size());
  }
  void undo() {
    int last = sh.back(); sh.pop_back();
    while (his.size() != last) {
      int *k, v;
      tie(k, v) = his.back(); his.pop_back();
      *k = v;
    }
  }
  int find(int x) {
    if (x == p[x]) return x;
    return find(p[x]);
  }
  void merge(int x, int y) {
    x = find(x); y = find(y);
    if (x == y) return;
    if (sz[x] > sz[y]) swap(x, y);
    assign(&sz[y], sz[x] + sz[y]);
    assign(&p[x], y);
    assign(&cc, cc - 1);
  }
} dsu;
```

3.2 <ext/pbds>

```
#include <bits/stdc++.h>
#include <bits/extc++.h>
#include <ext/rope>
using namespace __gnu_pbds;
using namespace __gnu_cxx;
#include <ext/pb_ds/assoc_container.hpp>
typedef tree<int, null_type, std::less<int>,
    rb_tree_tag, tree_order_statistics_node_update>
    tree_set;
typedef cc_hash_table<int, int> umap;
typedef priority_queue<int> heap;

int main() {
  // rb tree
  tree_set s;
  s.insert(71); s.insert(22);
  assert(*s.find_by_order(0) == 22); assert(*s.
    find_by_order(1) == 71);
  assert(s.order_of_key(22) == 0); assert(s.
    order_of_key(71) == 1);
```

```
  s.erase(22);
  assert(*s.find_by_order(0) == 71); assert(s.
    order_of_key(71) == 0);
  // mergable heap
  heap a, b; a.join(b);
  // persistant
  rope<char> r[2];
  r[1] = r[0];
  std::string st = "abc";
  r[1].insert(0, st.c_str());
  r[1].erase(1, 1);
  std::cout << r[1].substr(0, 2) << std::endl;
  return 0;
}
```

3.3 Li Chao Tree

```
namespace lichao {
  struct line {
    long long a, b;
    line(): a(0), b(0) {}
    line(long long a, long long b): a(a), b(b) {}
    long long operator()(int x) const { return a * x +
    b; }
  };
  line st[maxc * 4];
  int sz, lc[maxc * 4], rc[maxc * 4];
  int gnode() {
    st[sz] = line(1e9, 1e9);
    lc[sz] = -1, rc[sz] = -1;
    return sz++;
  }
  void init() {
    sz = 0;
  }
  void add(int l, int r, line tl, int o) {
    bool lcp = st[o](l) > tl(l);
    bool mcp = st[o]((l + r) / 2) > tl((l + r) / 2);
    if (mcp) swap(st[o], tl);
    if (r - l == 1) return;
    if (lcp != mcp) {
      if (lc[o] == -1) lc[o] = gnode();
      add(l, (l + r) / 2, tl, lc[o]);
    } else {
      if (rc[o] == -1) rc[o] = gnode();
      add((l + r) / 2, r, tl, rc[o]);
    }
  }
  long long query(int l, int r, int x, int o) {
    if (r - l == 1) return st[o](x);
    if (x < (l + r) / 2) {
      if (lc[o] == -1) return st[o](x);
      return min(st[o](x), query(l, (l + r) / 2, x, lc[
    o]));
    } else {
      if (rc[o] == -1) return st[o](x);
      return min(st[o](x), query((l + r) / 2, r, x, rc[
    o]));
    }
  }
}
```

4 Graph

4.1 Link-Cut Tree

```
struct node {
  node *ch[2], *fa, *pfa;
  int sum, v, rev;
  node(int s): v(s), sum(s), rev(0), fa(nullptr), pfa(
    nullptr) {
    ch[0] = nullptr;
    ch[1] = nullptr;
  }
  int relation() {
    return this == fa->ch[0] ? 0 : 1;
```

```cpp
  }
  void push() {
    if (!rev) return;
    swap(ch[0], ch[1]);
    if (ch[0]) ch[0]->rev ^= 1;
    if (ch[1]) ch[1]->rev ^= 1;
    rev = 0;
  }
  void pull() {
    sum = v;
    if (ch[0]) sum += ch[0]->sum;
    if (ch[1]) sum += ch[1]->sum;
  }
  void rotate() {
    if (fa->fa) fa->fa->push();
    fa->push(), push();
    swap(pfa, fa->pfa);
    int d = relation();
    node *t = fa;
    if (t->fa) t->fa->ch[t->relation()] = this;
    fa = t->fa;
    t->ch[d] = ch[d ^ 1];
    if (ch[d ^ 1]) ch[d ^ 1]->fa = t;
    ch[d ^ 1] = t;
    t->fa = this;
    t->pull(), pull();
  }
  void splay() {
    while (fa) {
      if (!fa->fa) {
        rotate();
        continue;
      }
      fa->fa->push(), fa->push();
      if (relation() == fa->relation()) fa->rotate(),
  rotate();
      else rotate(), rotate();
    }
  }
  void evert() {
    access();
    splay();
    rev ^= 1;
  }
  void expose() {
    splay(), push();
    if (ch[1]) {
      ch[1]->fa = nullptr;
      ch[1]->pfa = this;
      ch[1] = nullptr;
      pull();
    }
  }
  bool splice() {
    splay();
    if (!pfa) return false;
    pfa->expose();
    pfa->ch[1] = this;
    fa = pfa;
    pfa = nullptr;
    fa->pull();
    return true;
  }
  void access() {
    expose();
    while (splice());
  }
  int query() {
    return sum;
  }
};

namespace lct {
  node *sp[maxn];
  void make(int u, int v) {
    // create node with id u and value v
    sp[u] = new node(v, u);
  }
  void link(int u, int v) {
    // u become v's parent
    sp[v]->evert();
    sp[v]->pfa = sp[u];
```

```cpp
  }
  void cut(int u, int v) {
    // u was v's parent
    sp[u]->evert();
    sp[v]->access(), sp[v]->splay(), sp[v]->push();
    sp[v]->ch[0]->fa = nullptr;
    sp[v]->ch[0] = nullptr;
    sp[v]->pull();
  }
  void modify(int u, int v) {
    sp[u]->splay();
    sp[u]->v = v;
    sp[u]->pull();
  }
  int query(int u, int v) {
    sp[u]->evert(), sp[v]->access(), sp[v]->splay();
    return sp[v]->query();
  }
}
```

4.2 Heavy-Light Decomposition

```cpp
struct HeavyLightDecomp {
  vector<int> G[maxn];
  int tin[maxn], top[maxn], dep[maxn], maxson[maxn], sz
    [maxn], p[maxn], n, clk;
  void dfs(int now, int fa, int d) {
    dep[now] = d;
    maxson[now] = -1;
    sz[now] = 1;
    p[now] = fa;
    for (int u : G[now]) if (u != fa) {
      dfs(u, now, d + 1);
      sz[now] += sz[u];
      if (maxson[now] == -1 || sz[u] > sz[maxson[now]])
    maxson[now] = u;
    }
  }
  void link(int now, int t) {
    top[now] = t;
    tin[now] = ++clk;
    if (maxson[now] == -1) return;
    link(maxson[now], t);
    for (int u : G[now]) if (u != p[now]) {
      if (u == maxson[now]) continue;
      link(u, u);
    }
  }
  HeavyLightDecomp(int n): n(n) {
    clk = 0;
    memset(tin, 0, sizeof(tin)); memset(top, 0, sizeof(
      top)); memset(dep, 0, sizeof(dep));
    memset(maxson, 0, sizeof(maxson)); memset(sz, 0,
      sizeof(sz)); memset(p, 0, sizeof(p));
  }
  void add_edge(int a, int b) {
    G[a].push_back(b);
    G[b].push_back(a);
  }
  void solve() {
    dfs(0, -1, 0);
    link(0, 0);
  }
  int lca(int a, int b) {
    int ta = top[a], tb = top[b];
    while (ta != tb) {
      if (dep[ta] < dep[tb]) {
        swap(ta, tb); swap(a, b);
      }
      a = p[ta]; ta = top[a];
    }
    if (a == b) return a;
    return dep[a] < dep[b] ? a : b;
  }
  vector<pair<int, int>> get_path(int a, int b) {
    int ta = top[a], tb = top[b];
    vector<pair<int, int>> ret;
    while (ta != tb) {
      if (dep[ta] < dep[tb]) {
        swap(ta, tb); swap(a, b);
```

```
        }
        ret.push_back(make_pair(tin[ta], tin[a]));
        a = p[ta]; ta = top[a];
      }
      ret.push_back(make_pair(min(tin[a], tin[b]), max(
      tin[a], tin[b])));
      return ret;
    }
};
```

4.3 Centroid Decomposition

```
vector<pair<int, int>> G[maxn];
int sz[maxn], mx[maxn];
bool v[maxn];
vector<int> vtx;

void get_center(int now) {
  v[now] = true; vtx.push_back(now);
  sz[now] = 1; mx[now] = 0;
  for (int u : G[now]) if (!v[u]) {
    get_center(u);
    mx[now] = max(mx[now], sz[u]);
    sz[now] += sz[u];
  }
}

void get_dis(int now, int d, int len) {
  dis[d][now] = cnt;
  v[now] = true;
  for (auto u : G[now]) if (!v[u.first]) {
    get_dis(u, d, len + u.second);
  }
}

void dfs(int now, int fa, int d) {
  get_center(now);
  int c = -1;
  for (int i : vtx) {
    if (max(mx[i], (int)vtx.size() - sz[i]) <= (int)vtx
    .size() / 2) c = i;
    v[i] = false;
  }
  get_dis(c, d, 0);
  for (int i : vtx) v[i] = false;
  v[c] = true; vtx.clear();
  dep[c] = d; p[c] = fa;
  for (auto u : G[c]) if (u.first != fa && !v[u.first])
    {
    dfs(u.first, c, d + 1);
  }
}
```

4.4 Minimum mean cycle

```
// d[i][j] == 0 if {i,j} !in E
long long d[1003][1003],dp[1003][1003];

pair<long long,long long> MMWC(){
  memset(dp,0x3f,sizeof(dp));
  for(int i=1;i<=n;++i)dp[0][i]=0;
  for(int i=1;i<=n;++i){
    for(int j=1;j<=n;++j){
      for(int k=1;k<=n;++k){
        dp[i][k]=min(dp[i-1][j]+d[j][k],dp[i][k]);
      }
    }
  }
  long long au=1ll<<31,ad=1;
  for(int i=1;i<=n;++i){
    if(dp[n][i]==0x3f3f3f3f3f3f3f3f)continue;
    long long u=0,d=1;
    for(int j=n-1;j>=0;--j){
      if((dp[n][i]-dp[j][i])*d>u*(n-j)){
        u=dp[n][i]-dp[j][i];
        d=n-j;
      }
    }
```

```
    if(u*ad<au*d)au=u,ad=d;
  }
  long long g=__gcd(au,ad);
  return make_pair(au/g,ad/g);
}
```

4.5 Maximum Clique

```
struct MaxClique {
  int n, deg[maxn], ans;
  bitset<maxn> adj[maxn];
  vector<pair<int, int>> edge;
  void init(int _n) {
    n = _n;
    for (int i = 0; i < n; ++i) adj[i].reset();
    for (int i = 0; i < n; ++i) deg[i] = 0;
    edge.clear();
  }
  void add_edge(int a, int b) {
    edge.emplace_back(a, b);
    ++deg[a]; ++deg[b];
  }
  int solve() {
    vector<int> ord;
    for (int i = 0; i < n; ++i) ord.push_back(i);
    sort(ord.begin(), ord.end(), [&](const int &a,
    const int &b) { return deg[a] < deg[b]; });
    vector<int> id(n);
    for (int i = 0; i < n; ++i) id[ord[i]] = i;
    for (auto e : edge) {
      int u = id[e.first], v = id[e.second];
      adj[u][v] = adj[v][u] = true;
    }
    bitset<maxn> r, p;
    for (int i = 0; i < n; ++i) p[i] = true;
    ans = 0;
    dfs(r, p);
    return ans;
  }
  void dfs(bitset<maxn> r, bitset<maxn> p) {
    if (p.count() == 0) return ans = max(ans, (int)r.
    count()), void();
    if ((r | p).count() <= ans) return;
    int now = p._Find_first();
    bitset<maxn> cur = p & ~adj[now];
    for (now = cur._Find_first(); now < n; now = cur.
    _Find_next(now)) {
      r[now] = true;
      dfs(r, p & adj[now]);
      r[now] = false;
      p[now] = false;
    }
  }
};
```

4.6 Tarjan's articulation point

```
vector<pair<int, int>> g[maxn];
int low[maxn], tin[maxn], t;
int bcc[maxn], sz;
int a[maxn], b[maxn], deg[maxn];
bool cut[maxn], ins[maxn];

vector<int> ed[maxn];

stack<int> st;

void dfs(int x, int p) {
  tin[x] = low[x] = ++t;
  int ch = 0;
  for (auto u : g[x]) if (u.first != p) {
    if (!ins[u.second]) st.push(u.second), ins[u.second
    ] = true;
    if (tin[u.first]) {
      low[x] = min(low[x], tin[u.first]);
      continue;
    }
  }
```

```
    ++ch;
    dfs(u.first, x);
    low[x] = min(low[x], low[u.first]);
    if (low[u.first] >= tin[x]) {
      cut[x] = true;
      ++sz;
      while (true) {
        int e = st.top(); st.pop();
        bcc[e] = sz;
        if (e == u.second) break;
      }
    }
  }
  if (ch == 1 && p == -1) cut[x] = false;
}
```

4.7 Tarjan's bridge

```
vector<pair<int, int>> g[maxn];
int tin[maxn], low[maxn], t;
int a[maxn], b[maxn];
int bcc[maxn], sz;
bool br[maxn];

stack<int> st;

void dfs(int x, int p) {
  tin[x] = low[x] = ++t;
  st.push(x);
  for (auto u : g[x]) if (u.first != p) {
    if (tin[u.first]) {
      low[x] = min(low[x], tin[u.first]);
      continue;
    }
    dfs(u.first, x);
    low[x] = min(low[x], low[u.first]);
    if (low[u.first] == tin[u.first]) br[u.second] =
    true;
  }
  if (tin[x] == low[x]) {
    ++sz;
    while (st.size()) {
      int u = st.top(); st.pop();
      bcc[u] = sz;
      if (u == x) break;
    }
  }
}
```

5 String

5.1 KMP

```
int f[maxn];

int kmp(const string& a, const string& b) {
  f[0] = -1; f[1] = 0;
  for (int i = 1, j = 0; i < b.size() - 1; f[++i] = ++j
  ) {
    if (b[i] == b[j]) f[i] = f[j];
    while (j != -1 && b[i] != b[j]) j = f[j];
  }
  for (int i = 0, j = 0; i - j + b.size() <= a.size();
    ++i, ++j) {
    while (j != -1 && a[i] != b[j]) j = f[j];
    if (j == b.size() - 1) return i - j;
  }
  return -1;
}
```

5.2 Z algorithm

```
int z[maxn];
```

```
// z[i] = longest common prefix of suffix i and suffix
   0

void z_function(const string& s) {
  memset(z, 0, sizeof(z));
  z[0] = (int)s.length();
  int l = 0, r = 0;
  for (int i = 1; i < s.length(); ++i) {
    z[i] = max(0, min(z[i - l], r - i + 1));
    while (i + z[i] < s.length() && s[z[i]] == s[i + z[
    i]]) {
      l = i; r = i + z[i];
      ++z[i];
    }
  }
}
```

5.3 Manacher's

```
int z[maxn];

int manacher(const string& s) {
  string t = ".";
  for (int i = 0; i < s.length(); ++i) t += s[i], t +=
    '.';
  int l = 0, r = 0;
  for (int i = 1; i < t.length(); ++i) {
    z[i] = (r > i ? min(z[2 * l - i], r - i) : 1);
    while (i - z[i] >= 0 && i + z[i] < t.length() && t[
    i - z[i]] == t[i + z[i]]) ++z[i];
    if (i + z[i] > r) r = i + z[i], l = i;
  }
  int ans = 0;
  for (int i = 1; i < t.length(); ++i) ans = max(ans, z
    [i] - 1);
  return ans;
}
```

5.4 Aho-Corasick Automaton

```
struct AC {
  static const int maxn = 1e5 + 5;
  int sz, ql, qr, root;
  int cnt[maxn], q[maxn], ed[maxn], el[maxn], ch[maxn
    ][26], f[maxn];
  int gnode() {
    for (int i = 0; i < 26; ++i) ch[sz][i] = -1;
    f[sz] = -1;
    ed[sz] = 0;
    cnt[sz] = 0;
    return sz++;
  }
  void init() {
    sz = 0;
    root = gnode();
  }
  int add(const string &s) {
    int now = root;
    for (int i = 0; i < s.length(); ++i) {
      if (ch[now][s[i] - 'a'] == -1) ch[now][s[i] - 'a'
    ] = gnode();
      now = ch[now][s[i] - 'a'];
    }
    ed[now] = 1;
    return now;
  }
  void build_fail() {
    ql = qr = 0; q[qr++] = root;
    while (ql < qr) {
      int now = q[ql++];
      for (int i = 0; i < 26; ++i) if (ch[now][i] !=
    -1) {
        int p = ch[now][i], fp = f[now];
        while (fp != -1 && ch[fp][i] == -1) fp = f[fp];
        int pd = fp != -1 ? ch[fp][i] : root;
        f[p] = pd;
        el[p] = ed[pd] ? pd : el[pd];
        q[qr++] = p;
```

```
        }
      }
    }
    void build(const string &s) {
      build_fail();
      int now = root;
      for (int i = 0; i < s.length(); ++i) {
        while (now != -1 && ch[now][s[i] - 'a'] == -1)
      now = f[now];
        now = now != -1 ? ch[now][s[i] - 'a'] : root;
        ++cnt[now];
      }
      for (int i = qr - 1; i >= 0; --i) cnt[f[q[i]]] +=
      cnt[q[i]];
    }
};
```

5.5 Suffix Automaton

```
struct SAM {
  static const int maxn = 5e5 + 5;
  int nxt[maxn][26], to[maxn], len[maxn];
  int root, last, sz;
  int gnode(int x) {
    for (int i = 0; i < 26; ++i) nxt[sz][i] = -1;
    to[sz] = -1;
    len[sz] = x;
    return sz++;
  }
  void init() {
    sz = 0;
    root = gnode(0);
    last = root;
  }
  void push(int c) {
    int cur = last;
    last = gnode(len[last] + 1);
    for (; ~cur && nxt[cur][c] == -1; cur = to[cur])
    nxt[cur][c] = last;
    if (cur == -1) return to[last] = root, void();
    int link = nxt[cur][c];
    if (len[link] == len[cur] + 1) return to[last] =
    link, void();
    int tlink = gnode(len[cur] + 1);
    for (; ~cur && nxt[cur][c] == link; cur = to[cur])
    nxt[cur][c] = tlink;
    for (int i = 0; i < 26; ++i) nxt[tlink][i] = nxt[
    link][i];
    to[tlink] = to[link];
    to[link] = tlink;
    to[last] = tlink;
  }
  void add(const string &s) {
    for (int i = 0; i < s.size(); ++i) push(s[i] - 'a')
    ;
  }
  bool find(const string &s) {
    int cur = root;
    for (int i = 0; i < s.size(); ++i) {
      cur = nxt[cur][s[i] - 'a'];
      if (cur == -1) return false;
    }
    return true;
  }
  int solve(const string &t) {
    int res = 0, cnt = 0;
    int cur = root;
    for (int i = 0; i < t.size(); ++i) {
      if (~nxt[cur][t[i] - 'a']) {
        ++cnt;
        cur = nxt[cur][t[i] - 'a'];
      } else {
        for (; ~cur && nxt[cur][t[i] - 'a'] == -1; cur
    = to[cur]);
        if (~cur) cnt = len[cur] + 1, cur = nxt[cur][t[
    i] - 'a'];
        else cnt = 0, cur = root;
      }
      res = max(res, cnt);
    }
```

```
      return res;
    }
};
```

5.6 Suffix Array

```
int sa[maxn], tmp[2][maxn], c[maxn], hi[maxn], r[maxn];
// sa[i]: sa[i]-th suffix is the i-th lexigraphically
    smallest suffix.
// hi[i]: longest common prefix of suffix sa[i] and
    suffix sa[i - 1].
void build(const string &s) {
  int *rnk = tmp[0], *rkn = tmp[1];
  for (int i = 0; i < 256; ++i) c[i] = 0;
  for (int i = 0; i < s.size(); ++i) c[rnk[i] = s[i
    ]]++;
  for (int i = 1; i < 256; ++i) c[i] += c[i - 1];
  for (int i = s.size() - 1; i >= 0; --i) sa[--c[s[i]]]
    = i;
  int sigma = 256;
  for (int n = 1; n < s.size(); n *= 2) {
    for (int i = 0; i < sigma; ++i) c[i] = 0;
    for (int i = 0; i < s.size(); ++i) c[rnk[i]]++;
    for (int i = 1; i < sigma; ++i) c[i] += c[i - 1];
    int *sa2 = rkn;
    int r = 0;
    for (int i = s.size() - n; i < s.size(); ++i) sa2[r
    ++] = i;
    for (int i = 0; i < s.size(); ++i) {
      if (sa[i] >= n) sa2[r++] = sa[i] - n;
    }
    for (int i = s.size() - 1; i >= 0; --i) sa[--c[rnk[
    sa2[i]]]] = sa2[i];
    rkn[sa[0]] = r = 0;
    for (int i = 1; i < s.size(); ++i) {
      if (!(rnk[sa[i - 1]] == rnk[sa[i]] && sa[i - 1] +
    n < s.size() && rnk[sa[i - 1] + n] == rnk[sa[i] +
    n])) r++;
      rkn[sa[i]] = r;
    }
    swap(rnk, rkn);
    if (r == s.size() - 1) break;
    sigma = r + 1;
  }
  for (int i = 0; i < s.size(); ++i) r[sa[i]] = i;
  int ind = 0; hi[0] = 0;
  for (int i = 0; i < s.size(); ++i) {
    if (!r[i]) { ind = 0; continue; }
    while (i + ind < s.size() && s[i + ind] == s[sa[r[i
    ] - 1] + ind]) ++ind;
    hi[r[i]] = ind ? ind-- : 0;
  }
}
```

5.7 SAIS

```
namespace SAIS {
  enum type { L, S, LMS };
  const int maxn = 1e5 + 5;
  int bkt[maxn], cnt[maxn], lptr[maxn], rptr[maxn],
    tptr[maxn];
  int rev[maxn];
  void pre(const vector<int> &s, int sigma) {
    fill(bkt, bkt + s.size(), -1);
    fill(cnt, cnt + sigma, 0);
    for (int i = 0; i < s.size(); ++i) ++cnt[s[i]];
    int last = 0;
    for (int i = 0; i < sigma; ++i) {
      lptr[i] = last;
      last += cnt[i];
      rptr[i] = tptr[i] = last - 1;
    }
  }
  void induce(const vector<int> &s, const vector<type>
    &v) {
    for (int i = 0; i < s.size(); ++i) if (bkt[i] > 0)
    {
```

```cpp
      if (v[bkt[i] - 1] == L) bkt[lptr[s[bkt[i] -
1]]++] = bkt[i] - 1;
    }
    for (int i = s.size() - 1; i >= 0; --i) if (bkt[i]
> 0) {
      if (v[bkt[i] - 1] != L) bkt[rptr[s[bkt[i] -
1]]--] = bkt[i] - 1;
    }
  }
  bool equal(int l, int r, const vector<int> &s, const
    vector<type> &v) {
    do { if (s[l] != s[r]) return false; ++l, ++r; }
    while (v[l] != LMS && v[r] != LMS);
    return s[l] == s[r];
  }
  vector<int> radix_sort(const vector<int> &lms, const
    vector<int> &s, const vector<type> &v, int sigma) {
    pre(s, sigma);
    for (int i = 0; i < lms.size(); ++i) bkt[tptr[s[lms
[i]]]--] = lms[i];
    induce(s, v);
    vector<int> rt(lms.size());
    for (int i = 0; i < lms.size(); ++i) rev[lms[i]] =
i;
    int prv = -1, rnk = 0;
    for (int i = 0; i < s.size(); ++i) {
      int x = bkt[i];
      if (v[x] != LMS) continue;
      if (prv == -1) {
        rt[rev[x]] = rnk;
        prv = x;
        continue;
      }
      if (!equal(prv, x, s, v)) ++rnk;
      rt[rev[x]] = rnk;
      prv = x;
    }
    return rt;
  }
  vector<int> counting_sort(const vector<int> &s) {
    vector<int> o(s.size());
    for (int i = 0; i < s.size(); ++i) o[s[i]] = i;
    return o;
  }
  vector<int> reconstruct(const vector<int> &sa, const
    vector<int> &s, const vector<type> &v) {
    vector<int> pos;
    for (int i = 0; i < s.size(); ++i) if (v[i] == LMS)
     pos.push_back(i);
    vector<int> rev(sa.size());
    for (int i = 0; i < sa.size(); ++i) rev[i] = pos[sa
[i]];
    return rev;
  }
  vector<int> sais(const vector<int> &s, int sigma) {
    vector<type> v(s.size());
    v[s.size() - 1] = S;
    for (int i = s.size() - 2; i >= 0; --i) {
      if (s[i] < s[i + 1] || s[i] == s[i + 1] && v[i +
1] == S) v[i] = S;
      else v[i] = L;
    }
    for (int i = s.size() - 1; i >= 1; --i) {
      if (v[i] == S && v[i - 1] == L) v[i] = LMS;
    }
    vector<int> lms;
    for (int i = 0; i < s.size(); ++i) if (v[i] == LMS)
     lms.push_back(i);
    vector<int> r = radix_sort(lms, s, v, sigma);
    vector<int> sa;
    if (*max_element(r.begin(), r.end()) == r.size() -
1) sa = counting_sort(r);
    else sa = sais(r, *max_element(r.begin(), r.end())
+ 1);
    sa = reconstruct(sa, s, v);
    pre(s, sigma);
    for (int i = sa.size() - 1; i >= 0; --i) bkt[tptr[s
[sa[i]]]--] = sa[i];
    induce(s, v);
    return vector<int>(bkt, bkt + s.size());
  }
  vector<int> build(const string &s) {
```

```cpp
    vector<int> v(s.size() + 1);
    for (int i = 0; i < s.size(); ++i) v[i] = s[i];
    v[v.size() - 1] = 0;
    vector<int> sa = sais(v, 256);
    return vector<int>(sa.begin() + 1, sa.end());
  }
}
```

5.8 DC3

```cpp
namespace DC3{
#pragma GCC diagnostic push
#pragma GCC diagnostic ignored "-Wsign-compare"

#define SG(v,i) ((i)>=int(v.size())?0:v[i])
  inline bool smaller(int a, int b, vector<int> &r){
    if(SG(r,a+0) != SG(r,b+0)) return SG(r,a+0)<SG(r,b
+0);
    if(SG(r,a+1) != SG(r,b+1)) return SG(r,a+1)<SG(r,b
+1);
    return SG(r,a+2)<SG(r,b+2);
  }

  int cc[100005];
  inline vector<int> sort(vector<int> &r, int o, vector
    <int> &ix, int m){
    vector<int> rt(ix.size());
    for(int z=0;z<o;++z) r.push_back(0);
    for(int i=0;i<=m;++i) cc[i] = 0;
    for(int i=0;i<ix.size();++i) ++cc[r[ix[i]+o]];
    for(int i=0;i<=m;++i) cc[i+1] += cc[i];
    for(int i=ix.size()-1;i>=0;--i) rt[--cc[r[ix[i]+o
]]] = ix[i];
    for(int z=0;z<o;++z) r.pop_back();
    return rt;
  }

  vector<int> dc3(vector<int> &v, int n, int m){
    int c1 = (n+1)/3;
    vector<int> i12;
    for(int i=0;i<n;++i){
      if(i%3==0)continue;
      i12.push_back(i);
    }
    i12 = sort(v, 2, i12, m);
    i12 = sort(v, 1, i12, m);
    i12 = sort(v, 0, i12, m);

    int nr = 1;
    vector<int> r12(i12.size());
#define GRI(x) ((x)/3 + ((x)%3==2?c1:0))
    r12[GRI(i12[0])] = 1;
    for(int i=1;i<i12.size();++i){
      if(smaller(i12[i-1], i12[i], v)) r12[GRI(i12[i])]
 = ++nr;
      else r12[GRI(i12[i])] = nr;
    }

#define GEI(x) ((x)<c1?(x)*3+1:(x-c1)*3+2)
    if(nr != i12.size()){
      i12 = dc3(r12, i12.size(), nr);

      for(int i=0;i<i12.size();++i) r12[i12[i]] = i+1;
      for(int &i: i12) i = GEI(i);
    }

    vector<int> i0;
    if(n%3==1) i0.push_back(n-1);
    for(int i=0;i<i12.size();++i) if(i12[i]%3 == 1) i0.
    push_back(i12[i]-1);
    i0 = sort(v, 0, i0, m);

    vector<int> ret(v.size());
    int ptr12=0, ptr0=0, ptr=0;
    while(ptr12<i12.size() && ptr0<i0.size()){
      if(i12[ptr12]%3 == 1){
        if([&](int i, int j) -> bool{
          if(SG(v,i) != SG(v,j)) return SG(v,i)<SG(v,j)
;
          return SG(r12,GRI(i+1))<SG(r12,GRI(j+1));
```

```
        }(i12[ptr12], i0[ptr0]))ret[ptr++] = i12[ptr12
    ++];
        else ret[ptr++] = i0[ptr0++];
      }
    else{
      if([&](int i, int j) -> bool{
        if(SG(v,i+0) != SG(v,j+0)) return SG(v,i+0)<
    SG(v,j+0);
        if(SG(v,i+1) != SG(v,j+1)) return SG(v,i+1)<
    SG(v,j+1);
        return SG(r12,GRI(i+2))<SG(r12,GRI(j+2));
      }(i12[ptr12], i0[ptr0]))ret[ptr++] = i12[ptr12
    ++];
        else ret[ptr++] = i0[ptr0++];
      }
    }
    while(ptr12<i12.size()) ret[ptr++] = i12[ptr12++];
    while(ptr0<i0.size()) ret[ptr++] = i0[ptr0++];

    return ret;
  }
  vector<int> build(string str){
    vector<int> val(str.size()+1, 0);
    for(int i=0;i<str.size();++i) val[i] = str[i];
    return dc3(val, val.size(), 255);
  }
#pragma GCC diagnostic pop
}
```

5.9 Smallest Rotation

```
string rotate(const string &s) {
  int n = s.length();
  string t = s + s;
  int i = 0, j = 1;
  while (i < n && j < n) {
    int k = 0;
    while (k < n && t[i + k] == t[j + k]) ++k;
    if (t[i + k] <= t[j + k]) j += k + 1;
    else i += k + 1;
    if (i == j) ++j;
  }
  int pos = (i < n ? i : j);
  return t.substr(pos, n);
}
```

6 Math

6.1 Fast Fourier transform

```
struct cplx {
  double re, im;
  cplx(): re(0), im(0) {}
  cplx(double r, double i): re(r), im(i) {}
  cplx operator+(const cplx &rhs) const { return cplx(
    re + rhs.re, im + rhs.im); }
  cplx operator-(const cplx &rhs) const { return cplx(
    re - rhs.re, im - rhs.im); }
  cplx operator*(const cplx &rhs) const { return cplx(
    re * rhs.re - im * rhs.im, re * rhs.im + im * rhs.
    re); }
  cplx conj() const { return cplx(re, -im); }
};

const int maxn = 262144;
const double pi = acos(-1);
cplx omega[maxn + 1];

void prefft() {
  for (int i = 0; i <= maxn; ++i)
    omega[i] = cplx(cos(2 * pi * i / maxn), sin(2 * pi
    * i / maxn));
}

void bitrev(vector<cplx> &v, int n) {
  int z = __builtin_ctz(n) - 1;
```

```
  for (int i = 0; i < n; ++i) {
    int x = 0;
    for (int j = 0; (1 << j) < n; ++j) x ^= (((i >> j &
    1)) << (z - j));
    if (x > i) swap(v[x], v[i]);
  }
}

void fft(vector<cplx> &v, int n) {
  bitrev(v, n);
  for (int s = 2; s <= n; s <<= 1) {
    int z = s >> 1;
    for (int i = 0; i < n; i += s) {
      for (int k = 0; k < z; ++k) {
        cplx x = v[i + z + k] * omega[maxn / s * k];
        v[i + z + k] = v[i + k] - x;
        v[i + k] = v[i + k] + x;
      }
    }
  }
}

void ifft(vector<cplx> &v, int n) {
  fft(v, n);
  reverse(v.begin() + 1, v.end());
  for (int i = 0; i < n; ++i) v[i] = v[i] * cplx(1. / n
    , 0);
}

vector<int> conv(const vector<int> &a, const vector<int
    > &b) {
  int sz = 1;
  while (sz < a.size() + b.size() - 1) sz <<= 1;
  vector<cplx> v(sz);
  for (int i = 0; i < sz; ++i) {
    double re = i < a.size() ? a[i] : 0;
    double im = i < b.size() ? b[i] : 0;
    v[i] = cplx(re, im);
  }
  fft(v, sz);
  for (int i = 0; i <= sz / 2; ++i) {
    int j = (sz - i) & (sz - 1);
    cplx x = (v[i] + v[j].conj()) * (v[i] - v[j].conj()
    ) * cplx(0, -0.25);
    if (j != i) v[j] = (v[j] + v[i].conj()) * (v[j] - v
    [i].conj()) * cplx(0, -0.25);
    v[i] = x;
  }
  ifft(v, sz);
  vector<int> c(sz);
  for (int i = 0; i < sz; ++i) c[i] = round(v[i].re);
  while (c.size() && c.back() == 0) c.pop_back();
  return c;
}
```

6.2 Number theoretic transform

```
const int maxn = 262144;
const long long mod = 2013265921, root = 31;
long long omega[maxn + 1];

long long fpow(long long a, long long n) {
  (n += mod - 1) %= mod - 1;
  long long r = 1;
  for (; n; n >>= 1) {
    if (n & 1) (r *= a) %= mod;
    (a *= a) %= mod;
  }
  return r;
}

void prentt() {
  long long x = fpow(root, (mod - 1) / maxn);
  omega[0] = 1;
  for (int i = 1; i <= maxn; ++i)
    omega[i] = omega[i - 1] * x % mod;
}

void bitrev(vector<long long> &v, int n) {
  int z = __builtin_ctz(n) - 1;
```

```cpp
  for (int i = 0; i < n; ++i) {
    int x = 0;
    for (int j = 0; j <= z; ++j) x ^= ((i >> j & 1) <<
    (z - j));
    if (x > i) swap(v[x], v[i]);
  }
}

void ntt(vector<long long> &v, int n) {
  bitrev(v, n);
  for (int s = 2; s <= n; s <<= 1) {
    int z = s >> 1;
    for (int i = 0; i < n; i += s) {
      for (int k = 0; k < z; ++k) {
        long long x = v[i + k + z] * omega[maxn / s * k
    ] % mod;
        v[i + k + z] = (v[i + k] + mod - x) % mod;
        (v[i + k] += x) %= mod;
      }
    }
  }
}

void intt(vector<long long> &v, int n) {
  ntt(v, n);
  reverse(v.begin() + 1, v.end());
  long long inv = fpow(n, mod - 2);
  for (int i = 0; i < n; ++i) (v[i] *= inv) %= mod;
}

vector<long long> conv(vector<long long> a, vector<long
    long> b) {
  int sz = 1;
  while (sz < a.size() + b.size() - 1) sz <<= 1;
  vector<long long> c(sz);
  while (a.size() < sz) a.push_back(0);
  while (b.size() < sz) b.push_back(0);
  ntt(a, sz), ntt(b, sz);
  for (int i = 0; i < sz; ++i) c[i] = a[i] * b[i] % mod
    ;
  intt(c, sz);
  while (c.size() && c.back() == 0) c.pop_back();
  return c;
}
```

6.2.1 NTT Prime List

Prime	Root
97	5
193	5
257	3
7681	17
12289	11
40961	3
65537	3
786433	10
5767169	3
7340033	3
23068673	3
104857601	3
167772161	3
469762049	3
605028353	3
1107296257	10
2013265921	31
2810183681	11
2885681153	3

6.3 Fast Walsh-Hadamard transform

```cpp
void xorfwt(int v[], int l, int r) {
  if (r - l == 1) return;
  int m = l + r >> 1;
  xorfwt(v, l, m), xorfwt(v, m, r);
```

```cpp
  for (int i = l, j = m; i < m; ++i, ++j) {
    int x = v[i] + v[j];
    v[j] = v[i] - v[j], v[i] = x;
  }
}

void xorifwt(int v[], int l, int r) {
  if (r - l == 1) return;
  int m = l + r >> 1;
  for (int i = l, j = m; i < m; ++i, ++j) {
    int x = (v[i] + v[j]) / 2;
    v[j] = (v[i] - v[j]) / 2, v[i] = x;
  }
  xorifwt(v, l, m), xorifwt(v, m, r);
}

void andfwt(int v[], int l, int r) {
  if (r - l == 1) return;
  int m = l + r >> 1;
  andfwt(v, l, m), andfwt(v, m, r);
  for (int i = l, j = m; i < m; ++i, ++j) v[i] += v[j];
}

void andifwt(int v[], int l, int r) {
  if (r - l == 1) return;
  int m = l + r >> 1;
  andifwt(v, l, m), andifwt(v, m, r);
  for (int i = l, j = m; i < m; ++i, ++j) v[i] -= v[j];
}

void orfwt(int v[], int l, int r) {
  if (r - l == 1) return;
  int m = l + r >> 1;
  orfwt(v, l, m), orfwt(v, m, r);
  for (int i = l, j = m; i < m; ++i, ++j) v[j] += v[i];
}

void orifwt(int v[], int l, int r) {
  if (r - l == 1) return;
  int m = l + r >> 1;
  orifwt(v, l, m), orifwt(v, m, r);
  for (int i = l, j = m; i < m; ++i, ++j) v[j] -= v[i];
}
```

6.4 Lagrange Interpolation

```cpp
namespace lagrange {
  long long pf[maxn], nf[maxn];
  void init() {
    pf[0] = nf[0] = 1;
    for (int i = 1; i < maxn; ++i) {
      pf[i] = pf[i - 1] * i % mod;
      nf[i] = nf[i - 1] * (mod - i) % mod;
    }
  }
  // given y: value of f(a), a = [0, n], find f(x)
  long long solve(int n, vector<long long> y, long long
     x) {
    if (x <= n) return y[x];
    long long all = 1;
    for (int i = 0; i <= n; ++i) (all *= (x - i + mod))
     %= mod;
    long long ans = 0;
    for (int i = 0; i <= n; ++i) {
      long long z = all * fpow(x - i, -1) % mod;
      long long l = pf[i], r = nf[n - i];
      (ans += y[i] * z % mod * fpow(l * r, -1)) %= mod;
    }
    return ans;
  }
}
```

6.5 Miller Rabin

```cpp
// n < 4759123141    chk = [2, 7, 61]
// n < 1122004669633 chk = [2, 13, 23, 1662803]
// n < 2^64          chk = [2, 325, 9375, 28178, 450775,
    9780504, 1795265022]
```

```cpp
//
vector<long long> chk = { 2, 325, 9375, 28178, 450775,
    9780504, 1795265022 };

long long fmul(long long a, long long n, long long mod)
    {
  long long ret = 0;
  for (; n; n >>= 1) {
    if (n & 1) (ret += a) %= mod;
    (a += a) %= mod;
  }
  return ret;
}

long long fpow(long long a, long long n, long long mod)
    {
  long long ret = 1LL;
  for (; n; n >>= 1) {
    if (n & 1) ret = fmul(ret, a, mod);
    a = fmul(a, a, mod);
  }
  return ret;
}
bool check(long long a, long long u, long long n, int t
    ) {
  a = fpow(a, u, n);
  if (a == 0) return true;
  if (a == 1 || a == n - 1) return true;
  for (int i = 0; i < t; ++i) {
    a = fmul(a, a, n);
    if (a == 1) return false;
    if (a == n - 1) return true;
  }
  return false;
}
bool is_prime(long long n) {
  if (n < 2) return false;
  if (n % 2 == 0) return n == 2;
  long long u = n - 1; int t = 0;
  for (; u & 1; u >>= 1, ++t);
  for (long long i : chk) {
    if (!check(i, u, n, t)) return false;
  }
  return true;
}
```

6.6　Pollard's rho

```cpp
long long f(long long x, long long n, int p) { return (
    fmul(x, x, n) + p) % n; }

map<long long, int> cnt;

void pollard_rho(long long n) {
  if (n == 1) return;
  if (prime(n)) return ++cnt[n], void();
  if (n % 2 == 0) return pollard_rho(n / 2), ++cnt[2],
    void();
  long long x = 2, y = 2, d = 1, p = 1;
  while (true) {
    if (d != n && d != 1) {
      pollard_rho(n / d);
      pollard_rho(d);
      return;
    }
    if (d == n) ++p;
    x = f(x, n, p); y = f(f(y, n, p), n, p);
    d = __gcd(abs(x - y), n);
  }
}
```

6.7　Prime counting

```cpp
int prc[maxn];
long long phic[msz][nsz];

void sieve() {
  bitset<maxn> v;
```

```cpp
  pr.push_back(0);
  for (int i = 2; i < maxn; ++i) {
    if (!v[i]) pr.push_back(i);
    for (int j = 1; i * pr[j] < maxn; ++j) {
      v[i * pr[j]] = true;
      if (i % pr[j] == 0) break;
    }
  }
  for (int i = 1; i < pr.size(); ++i) prc[pr[i]] = 1;
  for (int i = 1; i < maxn; ++i) prc[i] += prc[i - 1];
}

long long p2(long long, long long);

long long phi(long long m, long long n) {
  if (m < msz && n < nsz && phic[m][n] != -1) return
    phic[m][n];
  if (n == 0) return m;
  if (pr[n] >= m) return 1;
  long long ret = phi(m, n - 1) - phi(m / pr[n], n - 1)
    ;
  if (m < msz && n < nsz) phic[m][n] = ret;
  return ret;
}

long long pi(long long m) {
  if (m < maxn) return prc[m];
  long long n = pi(cbrt(m));
  return phi(m, n) + n - 1 - p2(m, n);
}

long long p2(long long m, long long n) {
  long long ret = 0;
  long long lim = sqrt(m);
  for (int i = n + 1; pr[i] <= lim; ++i) ret += pi(m /
    pr[i]) - pi(pr[i]) + 1;
  return ret;
}
```

6.8　Gaussian Elimination

```cpp
void gauss(vector<vector<double>> &d) {
  int n = d.size(), m = d[0].size();
  for (int i = 0; i < m; ++i) {
    int p = -1;
    for (int j = i; j < n; ++j) {
      if (fabs(d[j][i]) < eps) continue;
      if (p == -1 || fabs(d[j][i]) > fabs(d[p][i])) p =
    j;
    }
    if (p == -1) continue;
    for (int j = 0; j < m; ++j) swap(d[p][j], d[i][j]);
    for (int j = 0; j < n; ++j) {
      if (i == j) continue;
      double z = d[j][i] / d[i][i];
      for (int k = 0; k < m; ++k) d[j][k] -= z * d[i][k
    ];
    }
  }
}
```

6.9　Linear Equations (full pivoting)

```cpp
void linear_equation(vector<vector<double>> &d, vector<
    double> &aug, vector<double> &sol) {
  int n = d.size(), m = d[0].size();
  vector<int> r(n), c(m);
  iota(r.begin(), r.end(), 0);
  iota(c.begin(), c.end(), 0);
  for (int i = 0; i < m; ++i) {
    int p = -1, z = -1;
    for (int j = i; j < n; ++j) {
      for (int k = i; k < m; ++k) {
        if (fabs(d[r[j]][c[k]]) < eps) continue;
        if (p == -1 || fabs(d[r[j]][c[k]]) > fabs(d[r[p
    ]][c[z]])) p = j, z = k;
      }
    }
```

```
      if (p == -1) continue;
      swap(r[p], r[i]), swap(c[z], c[i]);
      for (int j = 0; j < n; ++j) {
        if (i == j) continue;
        double z = d[r[j]][c[i]] / d[r[i]][c[i]];
        for (int k = 0; k < m; ++k) d[r[j]][c[k]] -= z *
      d[r[i]][c[k]];
        aug[r[j]] -= z * aug[r[i]];
      }
    }
    vector<vector<double>> fd(n, vector<double>(m));
    vector<double> faug(n), x(n);
    for (int i = 0; i < n; ++i) {
      for (int j = 0; j < m; ++j) fd[i][j] = d[r[i]][c[j
    ]];
      faug[i] = aug[r[i]];
    }
    d = fd, aug = faug;
    for (int i = n - 1; i >= 0; --i) {
      double p = 0.0;
      for (int j = i + 1; j < n; ++j) p += d[i][j] * x[j
    ];
      x[i] = (aug[i] - p) / d[i][i];
    }
    for (int i = 0; i < n; ++i) sol[c[i]] = x[i];
}
```

6.10 μ function

```
int mu[maxn], pi[maxn];
vector<int> prime;

void sieve() {
  mu[1] = pi[1] = 1;
  for (int i = 2; i < maxn; ++i) {
    if (!pi[i]) {
      pi[i] = i;
      prime.push_back(i);
      mu[i] = -1;
    }
    for (int j = 0; i * prime[j] < maxn; ++j) {
      pi[i * prime[j]] = prime[j];
      mu[i * prime[j]] = -mu[i];
      if (i % prime[j] == 0) {
        mu[i * prime[j]] = 0;
        break;
      }
    }
  }
}
```

6.11 $\lfloor \frac{n}{i} \rfloor$ Enumeration

```
vector<int> solve(int n) {
  vector<int> vec;
  for (int t = 1; t < n; t = (n / (n / (t + 1)))) vec.
    push_back(t);
  vec.push_back(n);
  vec.resize(unique(vec.begin(), vec.end()) - vec.begin
    ());
  return vec;
}
```

6.12 Extended GCD

```
template <typename T> tuple<T, T, T> extgcd(T a, T b) {
  if (!b) return make_tuple(a, 1, 0);
  T d, x, y;
  tie(d, x, y) = extgcd(b, a % b);
  return make_tuple(d, y, x - (a / b) * y);
}
```

6.13 Chinese remainder theorem

Given $x \equiv a_i \mod n_i \forall 1 \le i \le k$, where n_i are pairwise coprime, find x.
Let $N = \prod_{i=1}^{k} n_i$ and $N_i = N/n_i$, there exist integer M_i and m_i such that $M_i N_i + m_i n_i = 1$.
A solution to the system of congruence is $x = \sum_{i=1}^{k} a_i M_i N_i$.

6.14 Lucas's theorem

For non-negative integers m and n and prime p,
$\binom{m}{n} = \prod_{i=0}^{k} \binom{m_i}{n_i} \mod p$
where
$m = m_k p^k + m_{k-1} p^{k-1} + \ldots + m_1 p + m_0,$
$m = n_k p^k + n_{k-1} p^{k-1} + \ldots + n_1 p + n_0.$

6.15 Primes

$97, 101, 131, 487, 593, 877, 1087, 1187, 1487, 1787, 3187, 12721,$
$13331, 14341, 75577, 123457, 222557, 556679, 999983,$
$1097774749, 1076767633, 100102021, 999997771,$
$1001010013, 1000512343, 987654361, 999991231,$
$999888733, 98789101, 987777733, 999991921, 1000000007,$
$1000000087, 1000000123, 1010101333, 1010102101,$
$1000000000039, 1000000000000037, 2305843009213693951,$
$4611686018427387847, 9223372036854775783,$
18446744073709551557

7 Dynamic Programming

7.1 Convex Hull (monotone)

```
struct line {
  double a, b;
  inline double operator()(const double &x) const {
    return a * x + b; }
  inline bool checkfront(const line &l, const double &x
    ) const { return (*this)(x) < l(x); }
  inline double intersect(const line &l) const { return
    (l.b - b) / (a - l.a); }
  inline bool checkback(const line &l, const line &
    pivot) const { return pivot.intersect((*this)) <=
    pivot.intersect(l); }
};

void solve() {
  for (int i = 1; i < maxn; ++i) dp[0][i] = inf;
  for (int i = 1; i <= k; ++i) {
    deque<line> dq; dq.push_back((line){ 0.0, dp[i -
    1][0] });
    for (int j = 1; j <= n; ++j) {
      while (dq.size() >= 2 && dq[1].checkfront(dq[0],
    invt[j])) dq.pop_front();
      dp[i][j] = st[j] + dq.front()(invt[j]);
      line nl = (line){ -s[j], dp[i - 1][j] - st[j] + s
    [j] * invt[j] };
      while (dq.size() >= 2 && nl.checkback(dq[dq.size
    () - 1], dq[dq.size() - 2])) dq.pop_back();
      dq.push_back(nl);
    }
  }
}
```

7.2 Convex Hull (non-monotone)

```
struct line {
  int m, y;
  int l, r;
  line(int m = 0,int y = 0, int l = -5, int r =
    1000000009): m(m), y(y), l(l), r(r) {}
  int get(int x) const { return m * x + y; }
```

```cpp
  int useful(line le) const {
    return (int)(get(l) >= le.get(l)) + (int)(get(r) >=
    le.get(r));
  }
};

int magic;
bool operator < (const line &a, const line &b) {
  if (magic) return a.m < b.m;
  return a.l < b.l;
}

set<line> st;

void addline(line l) {
  magic = 1;
  auto it = st.lower_bound(l);
  if (it != st.end() && it->useful(l) == 2) return;
  while (it != st.end() && it->useful(l) == 0) it = st.
    erase(it);
  if (it != st.end() && it->useful(l) == 1) {
    int L = it->l, R = it->r, M;
    while (R > L) {
      M = (L + R + 1) >> 1;
      if (it->get(M) >= l.get(M)) R = M - 1;
      else L = M;
    }
    line cp = *it;
    st.erase(it);
    cp.l = L + 1;
    if (cp.l <= cp.r) st.insert(cp);
    l.r = L;
  }
  else if (it != st.end()) l.r = it->l - 1;
  it = st.lower_bound(l);
  while (it != st.begin() && prev(it)->useful(l) == 0)
    it = st.erase(prev(it));
  if (it != st.begin() && prev(it)->useful(l) == 1) {
    --it;
    int L = it->l, R = it->r, M;
    while (R > L) {
      M = (L + R) >> 1;
      if (it->get(M) >= l.get(M)) L = M + 1;
      else R = M;
    }
    line cp = *it;
    st.erase(it);
    cp.r = L - 1;
    if (cp.l <= cp.r) st.insert(cp);
    l.l = L;
  }
  else if (it != st.begin()) l.l = prev(it)->r + 1;
  if (l.l <= l.r) st.insert(l);
}

int getval(int d) {
  magic = 0;
  return (--st.upper_bound(line(0, 0, d, 0)))->get(d);
}
```

7.3 $1D/1D$ Convex Optimization

```cpp
struct segment {
  int i, l, r;
  segment() {}
  segment(int a, int b, int c): i(a), l(b), r(c) {}
};

inline long long f(int l, int r) {
  return dp[l] + w(l + 1, r);
}

void solve() {
  dp[0] = 0ll;
  deque<segment> deq; deq.push_back(segment(0, 1, n));
  for (int i = 1; i <= n; ++i) {
    dp[i] = f(deq.front().i, i);
    while (deq.size() && deq.front().r < i + 1) deq.
    pop_front();
    deq.front().l = i + 1;
```

```cpp
    segment seg = segment(i, i + 1, n);
    while (deq.size() && f(i, deq.back().l) < f(deq.
    back().i, deq.back().l)) deq.pop_back();
    if (deq.size()) {
      int d = 1048576, c = deq.back().l;
      while (d >>= 1) if (c + d <= deq.back().r) {
        if (f(i, c + d) > f(deq.back().i, c + d)) c +=
    d;
      }
      deq.back().r = c; seg.l = c + 1;
    }
    if (seg.l <= n) deq.push_back(seg);
  }
}
```

7.4 Conditon

7.4.1 concave totally monotone

$$\forall i < i', j < j',\ B[i][j] \le B[i'][j] \implies B[i][j'] \le B[i'][j']$$

7.4.2 convex totally monotone

$$\forall i < i', j < j',\ B[i][j] \ge B[i'][j] \implies B[i][j'] \ge B[i'][j']$$

7.4.3 concave monge condition

$$\forall i < i', j < j',\ B[i][j] + B[i'][j'] \ge B[i][j'] + B[i'][j]$$

7.4.4 convex monge condition

$$\forall i < i', j < j',\ B[i][j] + B[i'][j'] \le B[i][j'] + B[i'][j]$$

8   Geometry

8.1   Basic

```cpp
const double eps = 1e-8;
const double pi = acos(-1);

struct Point {
  double x, y;
  Point(double a = 0, double b = 0): x(a), y(b) {}
};

typedef Point Vector;

// L:ax+by+c=0
struct Line {
  double a, b, c, angle;
  Point p1, p2;
  Line() {}
  Line(Point s, Point e) {
    a = s.y - e.y, b = e.x - s.x;
    c = s.x * e.y - e.x * s.y;
    angle = atan2(e.y - s.y, e.x - s.x);
    p1 = s, p2 = e;
  }
};

struct Segment {
  Point s, e;
  Segment() {}
  Segment(Point a, Point b): s(a), e(b) {}
  Segment(double x1, double y1, double x2, double y2) {
    s = Point(x1, y1);
    e = Point(x2, y2);
  }
};

Vector operator+(Point a, Point b) { return Vector(a.x
    + b.x, a.y + b.y); }
Vector operator-(Point a, Point b) { return Vector(a.x
    - b.x, a.y - b.y); }
```

```
Vector operator*(Point a, double k) { return Vector(a.x
    * k, a.y * k); }
Vector operator/(Point a, double k) { return Vector(a.x
    / k, a.y / k); }
double len(Vector a) { return sqrt(a.x * a.x + a.y * a.
    y); }

// <0 when ep at opsp clockwise
double Cross(Point &sp, Point &ep, Point &op) { return
    (sp.x - op.x) * (ep.y - op.y) - (ep.x - op.x) * (sp
    .y - op.y); }
double Cross(Vector a, Vector b) { return a.x * b.y - b
    .x * a.y; }
double Dot(Vector a, Vector b) { return a.x * b.x + a.y
    * b.y; }

int epssgn(double x) {
  if (fabs(x) < eps) return 0;
  else return x < 0 ? -1 : 1;
}

double dis(Point a, Point b) { return sqrt((a.x - b.x)
    * (a.x - b.x) + (a.y - b.y) * (a.y - b.y)); }

bool Parallel(Line l1, Line l2) { return fabs(l1.a * l2
    .b - l2.a * l1.b) < eps; }
bool LineEqual(Line l1, Line l2) { return Parallel(l1,
    l2) && fabs(l1.a * l2.c - l2.a * l1.c) < eps &&
    fabs(l1.b * l2.c - l2.b * l1.c) < eps; }

double PointToSegDist(Point A, Point B, Point C) {
  if (dis(A, B) < eps) return dis(B, C);
  if (epssgn(Dot(B - A, C - A)) < 0) return dis(A, C);
  if (epssgn(Dot(A - B, C - B)) < 0) return dis(B, C);
  return fabs(Cross(B - A, C - A)) / dis(B, A);
}

double TwoSegMinDist(Point A, Point B, Point C, Point D
    ) { return min(min(PointToSegDist(A, B, C),
    PointToSegDist(A, B, D)), min(PointToSegDist(C, D,
    A), PointToSegDist(C, D, B))); }

Point SymPoint(Point p, Line l) {
  Point result;
  double a = l.p2.x - l.p1.x;
  double b = l.p2.y - l.p1.y;
  double t = ((p.x - l.p1.x) * a + (p.y - l.p1.y) * b)
    / (a * a + b * b);
  result.x = 2 * l.p1.x + 2 * a * t - p.x;
  result.y = 2 * l.p1.y + 2 * b * t - p.y;
  return result;
}

// without end points: <= -> <
bool IsSegmentIntersect(Point s1, Point e1, Point s2,
    Point e2) {
  if (min(s1.x, e1.x) <= max(s2.x, e2.x) &&
    min(s1.y, e1.y) <= max(s2.y, e2.y) &&
    min(s2.x, e2.x) <= max(s1.x, e1.x) &&
    min(s2.y, e2.y) <= max(s1.y, e1.y) &&
    Cross(s2, e2, s1) * Cross(s2, e2, e1) <= 0 &&
    Cross(s1, e1, s2) * Cross(s1, e1, e2) <= 0) return
    1;
  return 0;
}

int IsLineIntersectSegment(Point p1, Point p2, Point s,
    Point e){ return !Cross(p1, p2, s) * Cross(p1, p2,
    e) > eps; }
int IsLineIntersectSegment(Line l1, Point s, Point e) {
    return !Cross(l1.p1, l1.p2, s) * Cross(l1.p1, l1.
    p2, e) > eps; }

Point GetIntersect(Line l1, Line l2) {
  Point res;
  res.x = (l1.b * l2.c - l2.b * l1.c) / (l1.a * l2.b -
    l2.a * l1.b);
  res.y = (l1.c * l2.a - l2.c * l1.a) / (l1.a * l2.b -
    l2.a * l1.b);
  return res;
}
```

8.2 Triangle Center

```
Point TriangleCircumCenter(Point a, Point b, Point c) {
  Point res;
  double a1 = atan2(b.y - a.y, b.x - a.x) + pi / 2;
  double a2 = atan2(c.y - b.y, c.x - b.x) + pi / 2;
  double ax = (a.x + b.x) / 2;
  double ay = (a.y + b.y) / 2;
  double bx = (c.x + b.x) / 2;
  double by = (c.y + b.y) / 2;
  double r1 = (sin(a2) * (ax - bx) + cos(a2) * (by - ay
    )) / (sin(a1) * cos(a2) - sin(a2) * cos(a1));
  return Point(ax + r1 * cos(a1), ay + r1 * sin(a1));
}

Point TriangleMassCenter(Point a, Point b, Point c) {
  return (a + b + c) / 3.0;
}

Point TriangleOrthoCenter(Point a, Point b, Point c) {
  return TriangleMassCenter(a, b, c) * 3.0 -
    TriangleCircumCenter(a, b, c) * 2.0;
}

Point TriangleInnerCenter(Point a, Point b, Point c) {
  Point res;
  double la = len(b - c);
  double lb = len(a - c);
  double lc = len(a - b);
  res.x = (la * a.x + lb * b.x + lc * c.x) / (la + lb +
    lc);
  res.y = (la * a.y + lb * b.y + lc * c.y) / (la + lb +
    lc);
  return res;
}
```

8.3 Sector Area

```
// calc area of sector which include a, b
double SectorArea(Point a, Point b, double r) {
  double theta = atan2(a.y, a.x) - atan2(b.y, b.x);
  while (theta <= 0) theta += 2 * pi;
  while (theta >= 2 * pi) theta -= 2 * pi;
  theta = min(theta, 2 * pi - theta);
  return r * r * theta / 2;
}
```

8.4 Polygon Area

```
// point sort in counterclockwise
double ConvexPolygonArea(vector<Point> &p, int n) {
  double area = 0;
  for (int i = 1; i < p.size() - 1; i++) area += Cross(
    p[i] - p[0], p[i + 1] - p[0]);
  return area / 2;
}
```

8.5 Half Plane Intersection

```
const double eps = 1e-9;

struct plane {
  // points t are in this plane if (q - t) ^ (p - t) >=
    0
  point p, q;
  double ang;
  plane(point p, point q): p(p), q(q), ang(atan2(q.y -
    p.y, q.x - p.x)) {}
};

point inter(plane a, plane b) {
  if (fabs(a.q.x - a.p.x) < eps) {
    double mb = (b.q.y - b.p.y) / (b.q.x - b.p.x);
    double kb = b.p.y - mb * b.p.x;
    return point(a.q.x, mb * a.q.x + kb);
```

```
  }
  if (fabs(b.q.x - b.p.x) < eps) {
    double ma = (a.q.y - a.p.y) / (a.q.x - a.p.x);
    double ka = a.p.y - ma * a.p.x;
    return point(b.q.x, ma * b.q.x + ka);
  }
  double ma = (a.q.y - a.p.y) / (a.q.x - a.p.x);
  double mb = (b.q.y - b.p.y) / (b.q.x - b.p.x);
  double ka = a.p.y - ma * a.p.x;
  double kb = b.p.y - mb * b.p.x;
  double x = (kb - ka) / (ma - mb);
  double y = ma * x + ka;
  return point(x, y);
}

bool check(point p, plane l) {
  return ((l.q - p) ^ (l.p - p)) > eps || fabs((l.q - p
    ) ^ (l.p - p)) < eps;
}

vector<point> hpi(vector<plane> l) {
  sort(l.begin(), l.end(), [](const plane &a, const
    plane &b) {
    if (fabs(a.ang - b.ang) > eps) return a.ang < b.ang
      ;
    return ((a.q - a.p) ^ (b.q - a.p)) > eps;
  });
  vector<plane> tl;
  for (int i = 0; i < l.size(); ++i) {
    if (tl.size() && fabs(l[i].ang - tl.back().ang) <
      eps) continue;
    tl.push_back(l[i]);
  }
  for (int i = 0; i < tl.size(); ++i) debug(tl[i]);
  deque<plane> dq;
  for (int i = 0; i < tl.size(); ++i) {
    while (dq.size() >= 2 && !check(inter(dq[dq.size()
      - 1], dq[dq.size() - 2]), tl[i])) dq.pop_back();
    while (dq.size() >= 2 && !check(inter(dq[0], dq[1])
      , tl[i])) dq.pop_front();
    dq.push_back(tl[i]);
  }
  while (dq.size() >= 2 && !check(inter(dq[dq.size() -
    1], dq[dq.size() - 2]), dq[0])) dq.pop_back();
  while (dq.size() >= 2 && !check(inter(dq[0], dq[1]),
    dq[dq.size() - 1])) dq.pop_front();
  for (int i = 0; i < dq.size(); ++i) debug(dq[i]);
  vector<point> res;
  for (int i = 0; i < dq.size(); ++i) res.push_back(
    inter(dq[i], dq[(i + 1) % dq.size()]));
  return res;
}
```

8.6 Rotating Sweep Line

```
void rotatingSweepLine(vector<pair<int,int>> &ps){
  int n=int(ps.size());
  vector<int> id(n),pos(n);
  vector<pair<int,int>> line(n*(n-1)/2);
  int m=-1;
  for(int i=0;i<n;++i)for(int j=i+1;j<n;++j)line[++m]=
    make_pair(i,j); ++m;
  sort(line.begin(),line.end(),[&](const pair<int,int>
    &a,const pair<int,int> &b)->bool{
    if(ps[a.first].first==ps[a.second].first)return 0;
    if(ps[b.first].first==ps[b.second].first)return 1;
    return (double)(ps[a.first].second-ps[a.second].
      second)/(ps[a.first].first-ps[a.second].first) < (
      double)(ps[b.first].second-ps[b.second].second)/(ps
      [b.first].first-ps[b.second].first);
  });
  for(int i=0;i<n;++i)id[i]=i;
  sort(id.begin(),id.end(),[&](const int &a,const int &
    b){ return ps[a]<ps[b]; });
  for(int i=0;i<n;++i)pos[id[i]]=i;

  for(int i=0;i<m;++i){
    auto l=line[i];
    // meow
```

```
    tie(pos[l.first],pos[l.second],id[pos[l.first]],id[
      pos[l.second]])=make_tuple(pos[l.second],pos[l.
      first],l.second,l.first);
  }
}
```

8.7 Polygon Center

```
Point BaryCenter(vector<Point> &p, int n) {
  Point res(0, 0);
  double s = 0.0, t;
  for (int i = 1; i < p.size() - 1; i++) {
    t = Cross(p[i] - p[0], p[i + 1] - p[0]) / 2;
    s += t;
    res.x += (p[0].x + p[i].x + p[i + 1].x) * t;
    res.y += (p[0].y + p[i].y + p[i + 1].y) * t;
  }
  res.x /= (3 * s);
  res.y /= (3 * s);
  return res;
}
```

8.8 Maximum Triangle

```
double ConvexHullMaxTriangleArea(Point p[], int res[],
    int chnum) {
  double area = 0, tmp;
  res[chnum] = res[0];
  for (int i = 0, j = 1, k = 2; i < chnum; i++) {
    while (fabs(Cross(p[res[j]] - p[res[i]], p[res[(k +
      1) % chnum]] - p[res[i]])) > fabs(Cross(p[res[j]]
      - p[res[i]], p[res[k]] - p[res[i]]))) k = (k + 1) %
      chnum;
    tmp = fabs(Cross(p[res[j]] - p[res[i]], p[res[k]] -
      p[res[i]]));
    if (tmp > area) area = tmp;
    while (fabs(Cross(p[res[(j + 1) % chnum]] - p[res[i
      ]], p[res[k]] - p[res[i]])) > fabs(Cross(p[res[j]]
      - p[res[i]], p[res[k]] - p[res[i]]))) j = (j + 1) %
      chnum;
    tmp = fabs(Cross(p[res[j]] - p[res[i]], p[res[k]] -
      p[res[i]]));
    if (tmp > area) area = tmp;
  }
  return area / 2;
}
```

8.9 Point in Polygon

```
bool PointInConvexHull(Point p[], int res[], int chnum,
    Point x) {
  Point g = (p[res[0]] + p[res[chnum / 3]] + p[res[2 *
    chnum / 3]]) / 3.0;
  int l = 0, r = chnum, mid;
  while (l + 1 < r) {
    mid = (l + r) >> 1;
    if (epssgn(Cross(p[res[l]] - g, p[res[mid]] - g)) >
      0) {
      if (epssgn(Cross(p[res[l]] - g, x - g)) >=0 &&
      epssgn(Cross(p[res[mid]] - g ,x - g)) < 0) r = mid;
      else l = mid;
    } else {
      if (epssgn(Cross(p[res[l]] - g, x - g)) < 0 &&
      epssgn(Cross(p[res[mid]] - g, x - g)) >=0 ) l = mid
      ;
      else r = mid;
    }
  }
  r %= chnum;
  return epssgn(Cross(p[res[r]] - x, p[res[l]] - x)) ==
    -1;
}
```

8.10   Circle-Line Intersection

```cpp
// remove second level if to get points for line (
    defalut: segment)
void CircleCrossLine(Point a, Point b, Point o, double
    r, Point ret[], int &num) {
  double x0 = o.x, y0 = o.y;
  double x1 = a.x, y1 = a.y;
  double x2 = b.x, y2 = b.y;
  double dx = x2 - x1, dy = y2 - y1;
  double A = dx * dx + dy * dy;
  double B = 2 * dx * (x1 - x0) + 2 * dy * (y1 - y0);
  double C = (x1 - x0) * (x1 - x0) + (y1 - y0) * (y1 -
    y0) - r * r;
  double delta = B * B - 4 * A * C;
  num = 0;
  if (epssgn(delta) >= 0) {
    double t1 = (-B - sqrt(fabs(delta))) / (2 * A);
    double t2 = (-B + sqrt(fabs(delta))) / (2 * A);
    if (epssgn(t1 - 1.0) <= 0 && epssgn(t1) >= 0) ret[
    num++] = Point(x1 + t1 * dx, y1 + t1 * dy);
    if (epssgn(t2 - 1.0) <= 0 && epssgn(t2) >= 0) ret[
    num++] = Point(x1 + t2 * dx, y1 + t2 * dy);
  }
}

vector<Point> CircleCrossLine(Point a, Point b, Point o
    , double r) {
  double x0 = o.x, y0 = o.y;
  double x1 = a.x, y1 = a.y;
  double x2 = b.x, y2 = b.y;
  double dx = x2- x1, dy = y2 - y1;
  double A = dx * dx + dy * dy;
  double B = 2 * dx * (x1 - x0) + 2 * dy * (y1 - y0);
  double C = (x1 - x0) * (x1 - x0) + (y1 - y0) * (y1 -
    y0) - r * r;
  double delta = B * B - 4 * A * C;
  vector<Point> ret;
  if (epssgn(delta) >= 0) {
    double t1 = (-B - sqrt(fabs(delta))) / (2 * A);
    double t2 = (-B + sqrt(fabs(delta))) / (2 * A);
    if (epssgn(t1 - 1.0) <= 0 && epssgn(t1) >= 0) ret.
    emplace_back(x1 + t1 * dx, y1 + t1 * dy);
    if (epssgn(t2 - 1.0) <= 0 && epssgn(t2) >= 0) ret.
    emplace_back(x1 + t2 * dx, y1 + t2 * dy);
  }
  return ret;
}
```

8.11   Circle-Triangle Intersection

```cpp
// calc area intersect by circle with radius r and
    triangle OAB
double Calc(Point a, Point b, double r) {
  Point p[2];
  int num = 0;
  bool ina = epssgn(len(a) - r) < 0, inb = epssgn(len(b
    ) - r) < 0;
  if (ina) {
    if (inb) return fabs(Cross(a, b)) / 2.0; //
    triangle in circle
    else { // a point inside and another outside: calc
    sector and triangle area
      CircleCrossLine(a, b, Point(0, 0), r, p, num);
      return SectorArea(b, p[0], r) + fabs(Cross(a, p
    [0])) / 2.0;
    }
  } else {
    CircleCrossLine(a, b, Point(0, 0), r, p, num);
    if (inb) return SectorArea(p[0], a, r) + fabs(Cross
    (p[0], b)) / 2.0;
    else {
      if (num == 2) return SectorArea(a, p[0], r) +
    SectorArea(p[1], b, r) + fabs(Cross(p[0], p[1])) /
    2.0; // segment ab has 2 point intersect with
    circle
      else return SectorArea(a, b, r); // segment has
    no intersect point with circle
    }
```

```cpp
  }
}
```

8.12   Polygon Diameter

```cpp
// get diameter of p[res[]] store opposite points in
    app
double Diameter(Point p[], int res[], int chnum, int
    app[][2], int &appnum) {
  double ret = 0, nowlen;
  res[chnum] = res[0];
  appnum = 0;
  for (int i = 0, j = 1; i < chnum; ++i) {
    while (Cross(p[res[i]] - p[res[i + 1]], p[res[j +
    1]] - p[res[i + 1]]) < Cross(p[res[i]] - p[res[i +
    1]], p[res[j]] - p[res[i + 1]])) {
      ++j;
      j %= chnum;
    }
    app[appnum][0] = res[i];
    app[appnum][1] = res[j];
    ++appnum;
    nowlen = dis(p[res[i]], p[res[j]]);
    if (nowlen > ret) ret = nowlen;
    nowlen = dis(p[res[i + 1]], p[res[j + 1]]);
    if (nowlen > ret) ret = nowlen;
  }
  return ret;
}
```

8.13   Minimun Distance of 2 Polygons

```cpp
// p, q is convex
double TwoConvexHullMinDist(Point P[], Point Q[], int n
    , int m) {
  int YMinP = 0, YMaxQ = 0;
  double tmp, ans = 999999999;
  for (i = 0; i < n; ++i) if(P[i].y < P[YMinP].y) YMinP
    = i;
  for (i = 0; i < m; ++i) if(Q[i].y > Q[YMaxQ].y) YMaxQ
    = i;
  P[n] = P[0], Q[m] = Q[0];
  for (int i = 0; i < n; ++i) {
    while (tmp = Cross(Q[YMaxQ + 1] - P[YMinP + 1], P[
    YMinP] - P[YMinP + 1]) > Cross(Q[YMaxQ] - P[YMinP +
    1], P[YMinP] - P[YMinP + 1])) YMaxQ = (YMaxQ + 1)
    % m;
    if (tmp < 0) ans = min(ans, PointToSegDist(P[YMinP
    ], P[YMinP + 1], Q[YMaxQ]));
    else ans = min(ans, TwoSegMinDist(P[YMinP], P[YMinP
    + 1], Q[YMaxQ], Q[YMaxQ + 1]));
    YMinP = (YMinP + 1) % n;
  }
  return ans;
}
```

8.14   Convex Hull

```cpp
vector<point> convex(vector<point> p) {
  sort(p.begin(), p.end());
  vector<point> ch;
  for (int i = 0; i < n; ++i) {
    while (ch.size() >= 2 && ((p[i] - ch[ch.size() -
    2]) ^ (ch[ch.size() - 1] - ch[ch.size() - 2])) >=
    0) ch.pop_back();
    ch.push_back(p[i]);
  }
  int t = ch.size();
  for (int i = n - 2; i >= 0; --i) {
    while (ch.size() > t && ((p[i] - ch[ch.size() - 2])
     ^ (ch[ch.size() - 1] - ch[ch.size() - 2])) >= 0)
    ch.pop_back();
    ch.push_back(p[i]);
  }
  ch.pop_back();
  return ch;
```

```
}
```

8.15   Rotating Caliper

```cpp
struct pnt {
  int x, y;
  pnt(): x(0), y(0) {};
  pnt(int xx, int yy): x(xx), y(yy) {};
} p[maxn];

pnt operator-(const pnt &a, const pnt &b) { return pnt(
    b.x - a.x, b.y - a.y); }
int operator^(const pnt &a, const pnt &b) { return a.x
    * b.y - a.y * b.x; } //cross
int operator*(const pnt &a, const pnt &b) { return (a -
    b).x * (a - b).x + (a - b).y * (a - b).y; } //
    distance
int tb[maxn], tbz, rsd;

int dist(int n1, int n2){
  return p[n1] * p[n2];
}
int cross(int t1, int t2, int n1){
  return (p[t2] - p[t1]) ^ (p[n1] - p[t1]);
}
bool cmpx(const pnt &a, const pnt &b) { return a.x == b
    .x ? a.y < b.y : a.x < b.x; }

void RotatingCaliper() {
  sort(p, p + n, cmpx);
  for (int i = 0; i < n; ++i) {
    while (tbz > 1 && cross(tb[tbz - 2], tb[tbz - 1], i
    ) <= 0) --tbz;
    tb[tbz++] = i;
  }
  rsd = tbz - 1;
  for (int i = n - 2; i >= 0; --i) {
    while (tbz > rsd + 1 && cross(tb[tbz - 2], tb[tbz -
    1], i) <= 0) --tbz;
    tb[tbz++] = i;
  }
  --tbz;
  int lpr = 0, rpr = rsd;
  // tb[lpr], tb[rpr]
  while (lpr < rsd || rpr < tbz - 1) {
    if (lpr < rsd && rpr < tbz - 1) {
      pnt rvt = p[tb[rpr + 1]] - p[tb[rpr]];
      pnt lvt = p[tb[lpr + 1]] - p[tb[lpr]];
      if ((lvt ^ rvt) < 0) ++lpr;
      else ++rpr;
    }
    else if (lpr == rsd) ++rpr;
    else ++lpr;
    // tb[lpr], tb[rpr]
  }
}
```

8.16   Min Enclosing Circle

```cpp
pt center(const pt &a, const pt &b, const pt &c) {
  pt p0 = b - a, p1 = c - a;
  double c1 = norm2(p0) * 0.5, c2 = norm2(p1) * 0.5;
  double d = p0 ^ p1;
  double x = a.x + (c1 * p1.y - c2 * p0.y) / d;
  double y = a.y + (c2 * p0.x - c1 * p1.x) / d;
  return pt(x, y);
}

circle min_enclosing(vector<pt> &p) {
  random_shuffle(p.begin(), p.end());
  double r = 0.0;
  pt cent;
  for (int i = 0; i < p.size(); ++i) {
    if (norm2(cent - p[i]) <= r) continue;
    cent = p[i];
    r = 0.0;
    for (int j = 0; j < i; ++j) {
      if (norm2(cent - p[j]) <= r) continue;
```

```cpp
      cent = (p[i] + p[j]) / 2;
      r = norm2(p[j] - cent);
      for (int k = 0; k < j; ++k) {
        if (norm2(cent - p[k]) <= r) continue;
        cent = center(p[i], p[j], p[k]);
        r = norm2(p[k] - cent);
      }
    }
  }
  return circle(cent, sqrt(r));
}
```

8.17   Closest Pair

```cpp
pt p[maxn];

double dis(const pt& a, const pt& b) {
  return sqrt((a - b) * (a - b));
}

double closest_pair(int l, int r) {
  if (l == r) return inf;
  if (r - l == 1) return dis(p[l], p[r]);
  int m = (l + r) >> 1;
  double d = min(closest_pair(l, m), closest_pair(m +
    1, r));
  vector<int> vec;
  for (int i = m; i >= l && fabs(p[m].x - p[i].x) < d;
    --i) vec.push_back(i);
  for (int i = m + 1; i <= r && fabs(p[m].x - p[i].x) <
    d; ++i) vec.push_back(i);
  sort(vec.begin(), vec.end(), [=](const int& a, const
    int& b) { return p[a].y < p[b].y; });
  for (int i = 0; i < vec.size(); ++i) {
    for (int j = i + 1; j < vec.size() && fabs(p[vec[j
    ]].y - p[vec[i]].y) < d; ++j) {
      d = min(d, dis(p[vec[i]], p[vec[j]]));
    }
  }
  return d;
}
```

9   Problems

9.1   Manhattan distance minimum spanning tree

```cpp
#include <bits/stdc++.h>
using namespace std;

const int maxn = 1e5 + 5;
int x[maxn], y[maxn], fa[maxn];
pair<int, int> bit[maxn];
vector<tuple<int, int, int>> ed;

void init() {
  for (int i = 0; i < maxn; ++i)
    bit[i] = make_pair(1e9, -1);
}

void add(int p, pair<int, int> v) {
  for (; p < maxn; p += p & -p)
    bit[p] = min(bit[p], v);
}

pair<int, int> query(int p) {
  pair<int, int> res = make_pair(1e9, -1);
  for (; p; p -= p & -p)
    res = min(res, bit[p]);
  return res;
}

void add_edge(int u, int v) {
  ed.emplace_back(u, v, abs(x[u] - x[v]) + abs(y[u] - y
    [v]));
}
```

```cpp
void solve(int n) {
  init();
  vector<int> v(n), ds;
  for (int i = 0; i < n; ++i) {
    v[i] = i;
    ds.push_back(x[i] - y[i]);
  }
  sort(ds.begin(), ds.end());
  ds.resize(unique(ds.begin(), ds.end()) - ds.begin());
  sort(v.begin(), v.end(), [&](int i, int j) { return x
    [i] == x[j] ? y[i] > y[j] : x[i] > x[j]; });
  int j = 0;
  for (int i = 0; i < n; ++i) {
    int p = lower_bound(ds.begin(), ds.end(), x[v[i]] -
      y[v[i]]) - ds.begin() + 1;
    pair<int, int> q = query(p);
    if (~q.second) add_edge(v[i], q.second);
    add(p, make_pair(x[v[i]] + y[v[i]], v[i]));
  }
}

int find(int x) {
  if (x == fa[x]) return x;
  return fa[x] = find(fa[x]);
}

void merge(int x, int y) {
  fa[find(x)] = find(y);
}

int main() {
  int n; scanf("%d", &n);
  for (int i = 0; i < n; ++i) scanf("%d %d", &x[i], &y[
    i]);
  solve(n);
  for (int i = 0; i < n; ++i) swap(x[i], y[i]);
  solve(n);
  for (int i = 0; i < n; ++i) x[i] = -x[i];
  solve(n);
  for (int i = 0; i < n; ++i) swap(x[i], y[i]);
  solve(n);
  sort(ed.begin(), ed.end(), [](const tuple<int, int,
    int> &a, const tuple<int, int, int> &b) {
    return get<2>(a) < get<2>(b);
  });
  for (int i = 0; i < n; ++i) fa[i] = i;
  long long ans = 0;
  for (int i = 0; i < ed.size(); ++i) {
    int x, y, w; tie(x, y, w) = ed[i];
    if (find(x) == find(y)) continue;
    merge(x, y);
    ans += w;
  }
  printf("%lld\n", ans);
  return 0;
}
```

9.2 "Dynamic" kth element (parallel binary search)

```cpp
#include <bits/stdc++.h>
using namespace std;

const int maxn = 1e5 + 5;
int a[maxn], ans[maxn], tmp[maxn];

struct query { int op, l, r, k, qid; };

struct fenwick {
  int dat[maxn];
  void init() { memset(dat, 0, sizeof(dat)); }
  void add(int p, int v) { for (; p < maxn; p += p & -p
    ) dat[p] += v; }
  int qry(int p, int v = 0) { for (; p; p -= p & -p) v
    += dat[p]; return v; }
} bit;

void bs(vector<query> &qry, int l, int r) {
  if (l == r) {
    for (int i = 0; i < qry.size(); ++i) {
      if (qry[i].op == 3) ans[qry[i].qid] = l;
    }
    return;
  }
  if (qry.size() == 0) return;
  int m = l + r >> 1;
  for (int i = 0; i < qry.size(); ++i) {
    if (qry[i].op == 1 && qry[i].r <= m) bit.add(qry[i
      ].l, 1);
    else if (qry[i].op == 2 && qry[i].r <= m) bit.add(
      qry[i].l, -1);
    else if (qry[i].op == 3) tmp[qry[i].qid] += bit.qry
      (qry[i].r) - bit.qry(qry[i].l - 1);
  }
  vector<query> ql, qr;
  for (int i = 0; i < qry.size(); ++i) {
    if (qry[i].op == 3) {
      if (qry[i].k - tmp[qry[i].qid] > 0) qry[i].k -=
      tmp[qry[i].qid], qr.push_back(qry[i]);
      else ql.push_back(qry[i]);
      tmp[qry[i].qid] = 0;
      continue;
    }
    if (qry[i].r <= m) ql.push_back(qry[i]);
    else qr.push_back(qry[i]);
  }
  for (int i = 0; i < qry.size(); ++i) {
    if (qry[i].op == 1 && qry[i].r <= m) bit.add(qry[i
      ].l, -1);
    else if (qry[i].op == 2 && qry[i].r <= m) bit.add(
      qry[i].l, 1);
  }
  bs(ql, l, m), bs(qr, m + 1, r);
}

int main() {
  int t; scanf("%d", &t);
  while (t--) {
    int n, q; scanf("%d %d", &n, &q);
    vector<query> qry;
    vector<int> ds;
    bit.init();
    for (int i = 1; i <= n; ++i) {
      scanf("%d", a + i); ds.push_back(a[i]);
      qry.push_back({ 1, i, a[i], -1, -1 });
    }
    int qid = 0;
    for (int i = 0; i < q; ++i) {
      int t; scanf("%d", &t);
      if (t == 1) {
        int l, r, k; scanf("%d %d %d", &l, &r, &k);
        qry.push_back({ 3, l, r, k, qid }); ++qid;
      }
      if (t == 2) {
        int c, v; scanf("%d %d", &c, &v);
        ds.push_back(v);
        qry.push_back({ 2, c, a[c], -1, -1 });
        qry.push_back({ 1, c, v, -1, -1 });
        a[c] = v;
      }
      if (t == 3) {
        int x, v; scanf("%d %d", &x, &v);
        ans[qid] = -1, ++qid;
      }
    }
    sort(ds.begin(), ds.end()); ds.resize(unique(ds.
    begin(), ds.end()) - ds.begin());
    for (int i = 0; i < qry.size(); ++i) {
      if (qry[i].op == 3) continue;
      qry[i].r = lower_bound(ds.begin(), ds.end(), qry[
    i].r) - ds.begin();
    }
    bs(qry, 0, ds.size() - 1);
    for (int i = 0; i < qid; ++i) {
      if (ans[i] == -1) puts("7122");
      else assert(ans[i] < ds.size()), printf("%d\n",
    ds[ans[i]]);
    }
  }
  return 0;
```

```
|  }
```

9.3 Dynamic kth element (persistent segment tree)

```cpp
#include <bits/stdc++.h>
using namespace std;

const int maxn = 1e5 + 5;
int a[maxn], bit[maxn];
vector<int> ds;
vector<vector<int>> qr;

namespace segtree {
  int st[maxn * 97], lc[maxn * 97], rc[maxn * 97], sz;
  int gnode() {
    st[sz] = 0;
    lc[sz] = rc[sz] = 0;
    return sz++;
  }
  int gnode(int z) {
    st[sz] = st[z];
    lc[sz] = lc[z], rc[sz] = rc[z];
    return sz++;
  }
  int build(int l, int r) {
    int z = gnode();
    if (r - l == 1) return z;
    lc[z] = build(l, (l + r) / 2), rc[z] = build((l + r
    ) / 2, r);
    return z;
  }
  int modify(int l, int r, int p, int v, int o) {
    int z = gnode(o);
    if (r - l == 1) return st[z] += v, z;
    if (p < (l + r) / 2) lc[z] = modify(l, (l + r) / 2,
     p, v, lc[o]);
    else rc[z] = modify((l + r) / 2, r, p, v, rc[o]);
    st[z] = st[lc[z]] + st[rc[z]];
    return z;
  }
  int query(int l, int r, int ql, int qr, int o) {
    if (l >= qr || ql >= r) return 0;
    if (l >= ql && r <= qr) return st[o];
    return query(l, (l + r) / 2, ql, qr, lc[o]) +
        query((l + r) / 2, r, ql, qr, rc[o]);
  }
}

void init(int n) {
  segtree::sz = 0;
  bit[0] = segtree::build(0, ds.size());
  for (int i = 1; i <= n; ++i) bit[i] = bit[0];
}

void add(int p, int n, int x, int v) {
  for (; p <= n; p += p & -p)
    bit[p] = segtree::modify(0, ds.size(), x, v, bit[p
    ]);
}

vector<int> query(int p) {
  vector<int> z;
  for (; p; p -= p & -p)
    z.push_back(bit[p]);
  return z;
}

int dfs(int l, int r, vector<int> lz, vector<int> rz,
    int k) {
  if (r - l == 1) return l;
  int ls = 0, rs = 0;
  for (int i = 0; i < lz.size(); ++i) ls += segtree::st
    [segtree::lc[lz[i]]];
  for (int i = 0; i < rz.size(); ++i) rs += segtree::st
    [segtree::lc[rz[i]]];
  if (rs - ls >= k) {
    for (int i = 0; i < lz.size(); ++i) lz[i] = segtree
    ::lc[lz[i]];
    for (int i = 0; i < rz.size(); ++i) rz[i] = segtree
    ::lc[rz[i]];
    return dfs(l, (l + r) / 2, lz, rz, k);
  } else {
    for (int i = 0; i < lz.size(); ++i) lz[i] = segtree
    ::rc[lz[i]];
    for (int i = 0; i < rz.size(); ++i) rz[i] = segtree
    ::rc[rz[i]];
    return dfs((l + r) / 2, r, lz, rz, k - (rs - ls));
  }
}

int main() {
  int t; scanf("%d", &t);
  while (t--) {
    int n, q; scanf("%d %d", &n, &q);
    for (int i = 1; i <= n; ++i) scanf("%d", &a[i]), ds
    .push_back(a[i]);
    for (int i = 0; i < q; ++i) {
      int a, b, c; scanf("%d %d %d", &a, &b, &c);
      vector<int> v = { a, b, c };
      if (a == 1) {
        int d; scanf("%d", &d);
        v.push_back(d);
      }
      qr.push_back(v);
    }
    for (int i = 0; i < q; ++i) if (qr[i][0] == 2) ds.
    push_back(qr[i][2]);
    sort(ds.begin(), ds.end()), ds.resize(unique(ds.
    begin(), ds.end()) - ds.begin());
    for (int i = 1; i <= n; ++i) a[i] = lower_bound(ds.
    begin(), ds.end(), a[i]) - ds.begin();
    for (int i = 0; i < q; ++i) if (qr[i][0] == 2) qr[i
    ][2] = lower_bound(ds.begin(), ds.end(), qr[i][2])
    - ds.begin();
    init(n);
    for (int i = 1; i <= n; ++i) add(i, n, a[i], 1);
    for (int i = 0; i < q; ++i) {
      if (qr[i][0] == 3) {
        puts("7122");
        continue;
      }
      if (qr[i][0] == 1) {
        vector<int> lz = query(qr[i][1] - 1);
        vector<int> rz = query(qr[i][2]);
        int ans = dfs(0, ds.size(), lz, rz, qr[i][3]);
        printf("%d\n", ds[ans]);
      } else {
        add(qr[i][1], n, a[qr[i][1]], -1);
        add(qr[i][1], n, qr[i][2], 1);
        a[qr[i][1]] = qr[i][2];
      }
    }
    ds.clear(), qr.clear();
  }
  return 0;
}
```

9.4 Hilbert's curve (faster MO's algorithm)

```cpp
long long hilbert(int n, int x, int y) {
  long long res = 0;
  for (int s = n / 2; s; s >>= 1) {
    int rx = (x & s) > 0;
    int ry = (y & s) > 0;
    res += s * 1ll * s * ((3 * rx) ^ ry);
    if (ry == 0) {
      if (rx == 1) {
        x = s - 1 - x;
        y = s - 1 - y;
      }
      swap(x, y);
    }
  }
  return res;
}
```