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2017405087

Question 2:

Part a: Effect of Mass Flow Rate on Major Head Loss by Colebrook Formula

Calculate the friction factor by using Colebrook formula and an iterative method

$$\frac{1}{\sqrt{f}} = -2 * \log_{10}\left(\frac{\varepsilon}{3.7 * D} + \frac{2.51}{\mathrm{Re}\sqrt{f}}\right)$$

As we can see, this formula is a transcendental equation. Hence, we should use a numeric method to find its solution.

Let's create the function g:

$$g(f) = \frac{1}{\sqrt{f}} + 2 * \log_{10}\left(\frac{\varepsilon}{3.7 * D} + \frac{2.51}{\mathrm{Re}\sqrt{f}}\right)$$

The roots of this function will give us the friction factor.

Secant Method

Secant method is used to find roots of the function g(f).

With 10^{-1} relative error, we found friction factors for each mass flow rate from 15 kg/s to 30 kg/s. (h=0.5 kg/s)

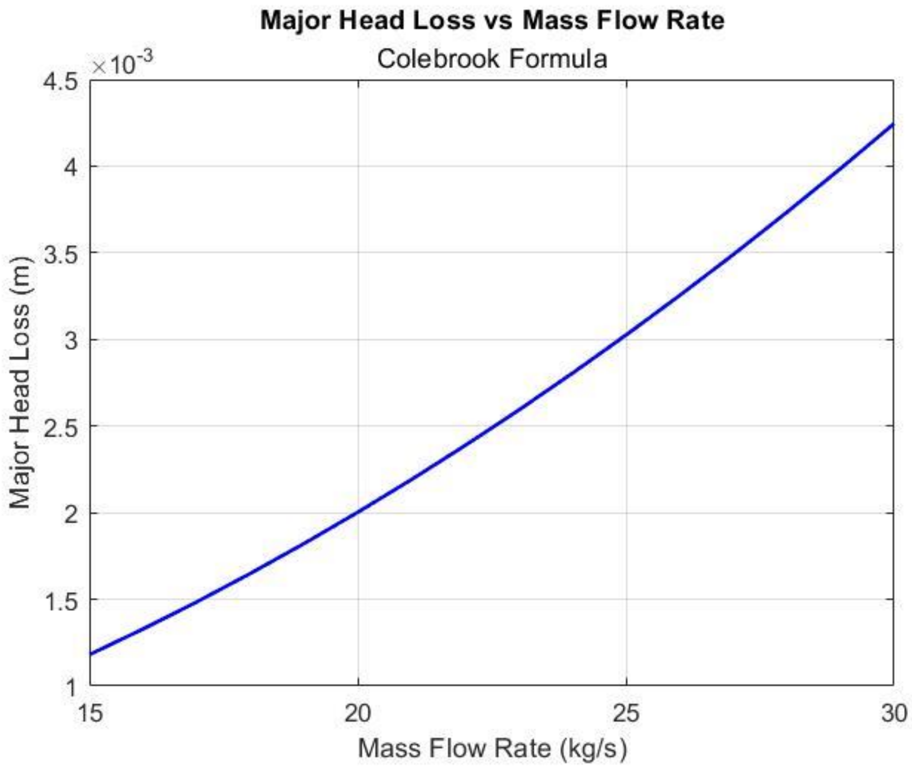
f = [0.0214939708542736 0.0212588946382272... 0.0210445335408819 0.0208480626261322... 0.0206671700947701 0.0204999329293745... 0.0203447718759493 0.0202003285067112... 0.0200654463754950 0.0199391633042891... 0.0198206128520902 0.0197090802793064... 0.0196039050409735 0.0195045449433351... 0.0194104850481655 0.0193213068294344]

Major Head Loss

Darcy-Weisbach equation is used to calculate major head loss for each mass flow rate and friction factor.

$$h_{L\,\text{major}} = f * \frac{l * V^2}{D * 2g}$$

Now, we can plot the major head loss vs. mass flow rate.



Part b: Effect of Mass Flow Rate on Major Head Loss by Haaland Formula

Calculate the friction factor by using Haaland formula

$$\frac{1}{\sqrt{f}} = -1.8 * \log_{10}\left(\left(\frac{\varepsilon}{3.7 * D}\right)^{1.11} + \frac{6.9}{\mathrm{Re}}\right)$$

This formula approximates the friction factor.

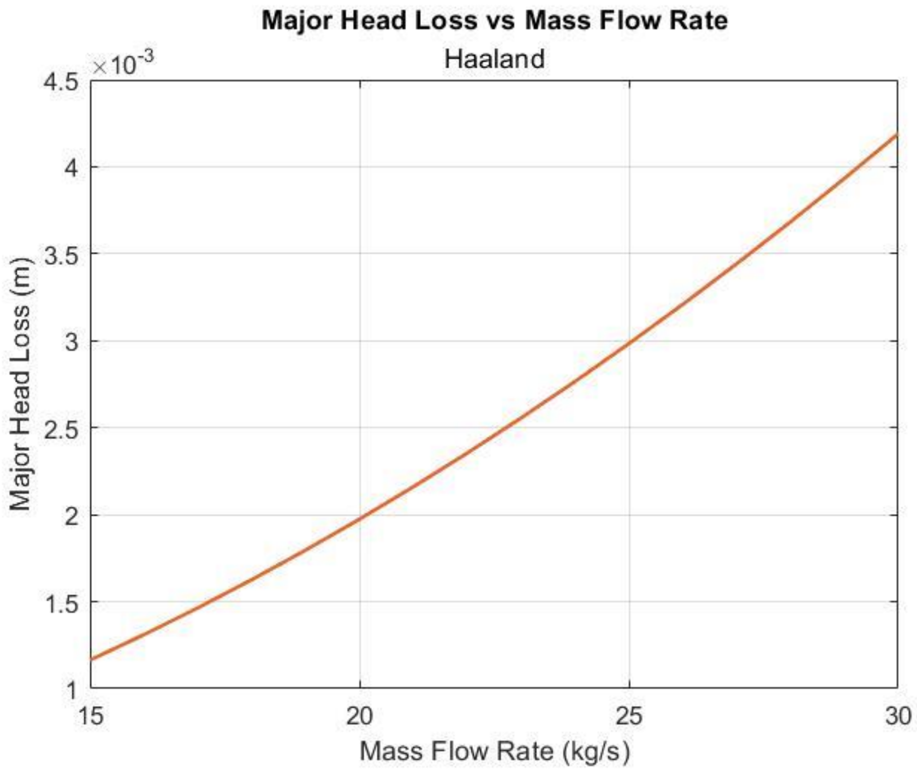
f = [0.0211905396085055 0.0209582350535331 0.0207468323144582 0.0205534518809528 0.0203757405070234 0.0202117533049765 0.0200598666864594 0.0199187130250888 0.0197871308996048 0.0196641266995854 0.0195488446428384 0.0194405431053383 0.0193385757476723 0.0192423763277949 0.0191514463766901 0.0190653451190537]

Major Head Loss

Darcy-Weisbach equation is used to calculate major head loss for each mass flow rate and friction factor.

$$h_{L\,\mathrm{major}} = f * \frac{l * V^2}{D * 2g}$$

Now, we can plot the major head loss vs. mass flow rate.

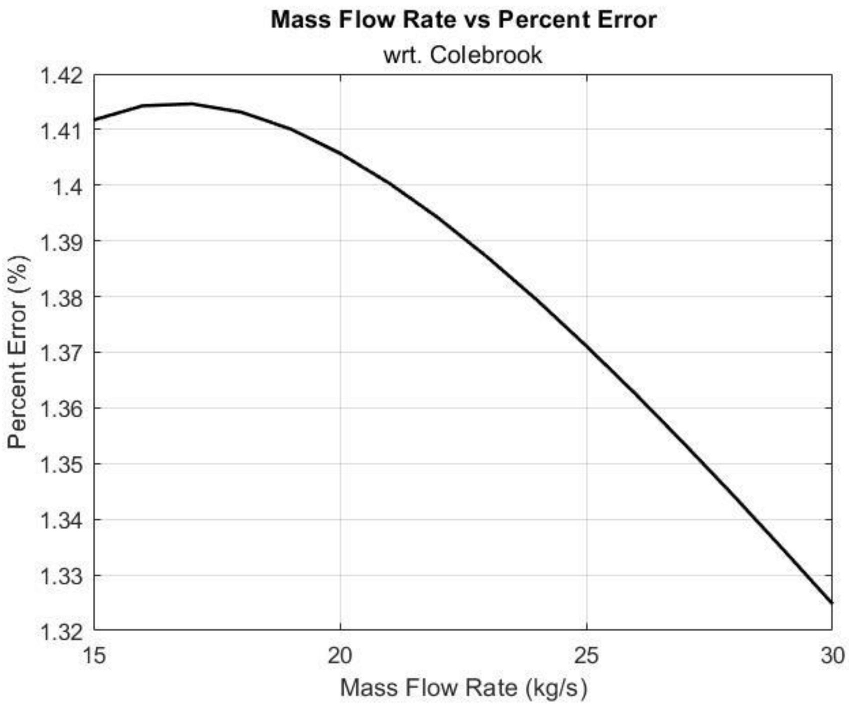


Part c: Percent Error

Percent error of the results of Haaland formula with respect to the results of Colebrook formula is calculated.

$$\%\ \text{error} = \frac{|\text{approx} - \text{exact}|}{\text{exact}} * 100$$

The graph of percent error vs mass flow rate:



Codes:

```
% FLUID MECHANICS COMPUTATIONAL HOMEWORK QUESTION-2
clear all, close all, clc
format long

% Assumptions
% -Steady
% -Incompressible
% -Fully-developed
% -Non-laminar
% -Pipe flow

% PIPE PARAMETERS
epsilon = 0.00015; % surface roughness (m)
D = 0.4; % diameter (m)
L = 30; % length (m)

gravity = 9.81; % gravity (m/s^2)

% FLUID PARAMETERS 1
d = 995.7; % density (kg/m^3)
nu = 0.801*10^(-3); % dynamic viscosity (N*s/m)

% FLUID PARAMETERS 2
mflow = 15:1:30; % mass flow rate (kg/s)
Q = mflow/d; % volumetric flow rate (m^3/s)
v = Q/(pi/4*D^2); % velocity (m/s)
Re = d*v*D/nu; % Reynold's number
f = zeros(size(mflow)); % friction factor
N = length(f);


%%
%========= COLEBROOK FORMULA =========%
cb = @(f,Re) f.^(-1/2) + 2*log10(epsilon/D/3.7 + 2.51./Re.*f^(-1/2));

% Secant method to find the roots
delta_abs = 1e-8;
delta_rel = 1e-1;
maxI = 10000;
for j=1:N
    err = 100;
    relerr = 100;
    p0 = 0.1;
    p1 = 0.2;
    for i=1:maxI
        p2 = p1 - cb(p1,Re(j))*(p1-p0)/(cb(p1,Re(j))-cb(p0,Re(j)))/100;
        p0 = p1;
        p1 = p2;
        err = abs(p1 - p0);
        relerr = 2*abs(p1-p0)/(abs(p1)+abs(p0));
        if err < delta_abs && relerr < delta_rel
            break;
        end
    end
    f(j) = p2;
end

% Print the f values calculated from Colebrook formula
f
```

```matlab
% Major head loss for Colebrook: Darcy-Weisbach equation
headLoss_cb = f.*(L/D).*v.^2/2/gravity;

% PLOT
figure
plot(mflow,headLoss_cb,'b','linewidth',1.5)
grid on
%ylim([2 10]*1e-4)
xlabel('Mass Flow Rate (kg/s)')
ylabel('Major Head Loss (m)')
title('Major Head Loss vs Mass Flow Rate','Colebrook Formula')

%%
%========== HAALAND ==========%
f_ha = (-1.8*log10((epsilon/D/3.7)^1.11 + 6.9./Re)).^(-2)

% Major head loss for Haaland: Darcy-Weisbach equation
headLoss_ha = f_ha.*(L/D).*v.^2/2/gravity;

% PLOT
figure
plot(mflow,headLoss_ha,'color',[0.9 0.4 0.17],'linewidth',1.5)
grid on
%ylim([2 10]*1e-4)
xlabel('Mass Flow Rate (kg/s)')
ylabel('Major Head Loss (m)')
title('Major Head Loss vs Mass Flow Rate','Haaland')

%%
%=========== PERCENT ERROR ===========%
error = abs(headLoss_cb-headLoss_ha);
percentError = error./headLoss_cb*100;

% PLOT
figure
plot(mflow,percentError,'k','linewidth',1.5)
grid on
xlabel('Mass Flow Rate (kg/s)')
ylabel('Percent Error (%)')
title('Mass Flow Rate vs Percent Error','wrt. Colebrook')
```