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ME353 Computational Homework Q1

Question 1:

Part a)

Dimension Analysis

1. **Dimensions:**

Quantity:	*Symbol:*	*M-L-T:*
Dynamic viscosity	μ	$ML^{-1}T^{-1}$
Density	ρ	ML^{-3}
Mass	m	M
Area	A	A^2
Velocity	v	LT^{-1}

2. **Repeating Variables:**

- Dynamic viscosity
- Mass
- Area

3. **Pi Terms:**

- $\pi_1 = \frac{m * v}{\mu * A}$
- $\pi_2 = \frac{A^{\frac{3}{2}} * \rho}{m}$

4. **The Function which Connects the two Pi Terms:**

- $\pi_1 = f(\pi_2)$

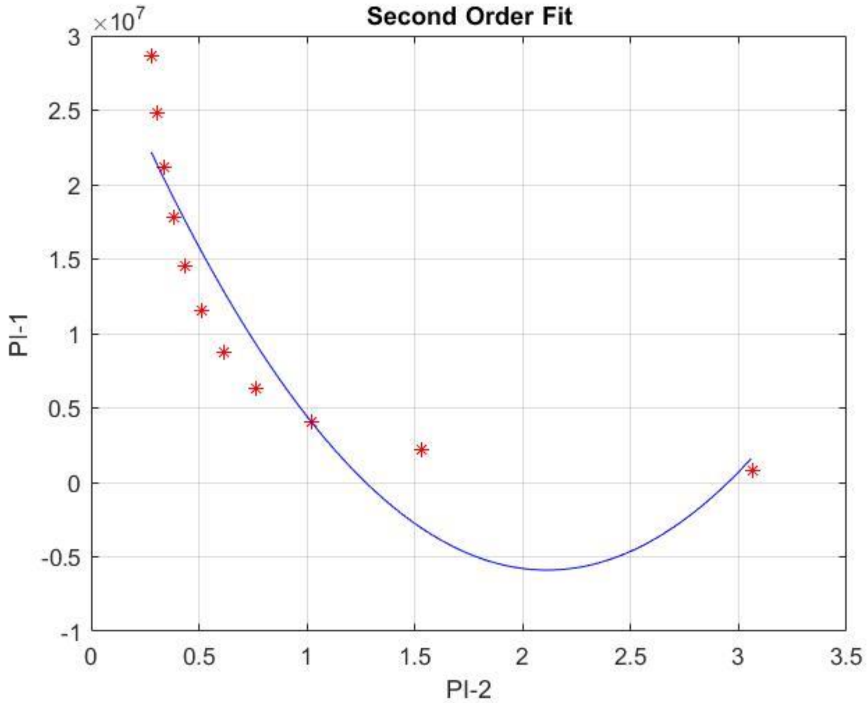
5. **The Velocity**

- $v = \frac{\mu * A}{m} * f(\pi_2)$

Curve Fitting

To find the suitable f function which connects the two pi terms, we will apply three different curve fitting technics by using the data given in the problem.

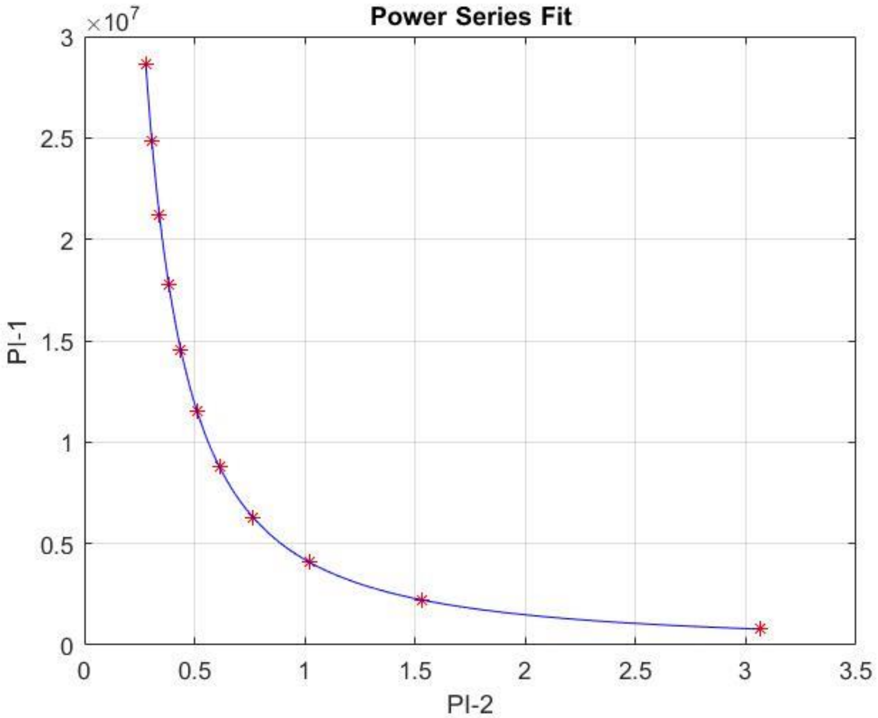
1. Second Order Fit:



As we see from the graph, second degree polynomial curve fitting doesn't work well in our problem. Apart from the graph, very high rms value also leads us to the same conclusion.

RMS Value: 3.5139e+06 $f(x) = ax^2 + bx + c$ *(a=8.36e+06, b=-3.53e+07, c=3.14e+07)*

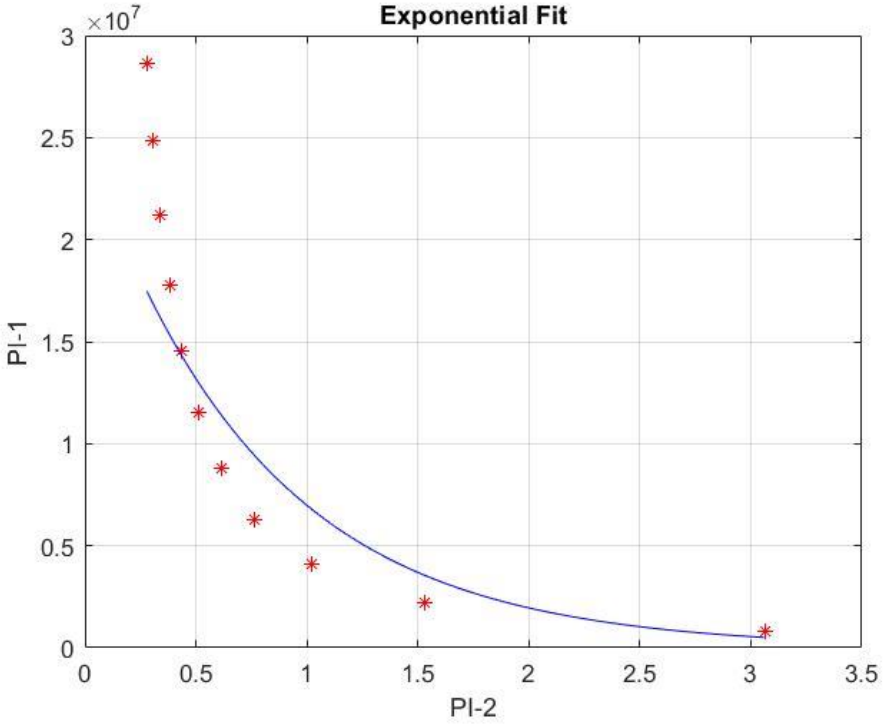
2. Power Series Fit:



The graph shows us how good the power series fitting technic fits the data in this range. Also the rms value is lower than second degree fit.

RMS Value: 1.1315e+03 $f(x) = ax^b$ *(a=4.21e+06, b=-1.5)*

3. Exponential Fit:



This fitting technic looks better than second degree fit from a conceptual perspective but still it has very high rms errors. Even slightly bigger than second degree fit as we can see from the rms error results.

RMS Value: 4.7491e+06 $f(x) = ae^{bx}$ *(a=2.49e+07, b=-1.27)*

Compare the Graphs and RMS Errors:

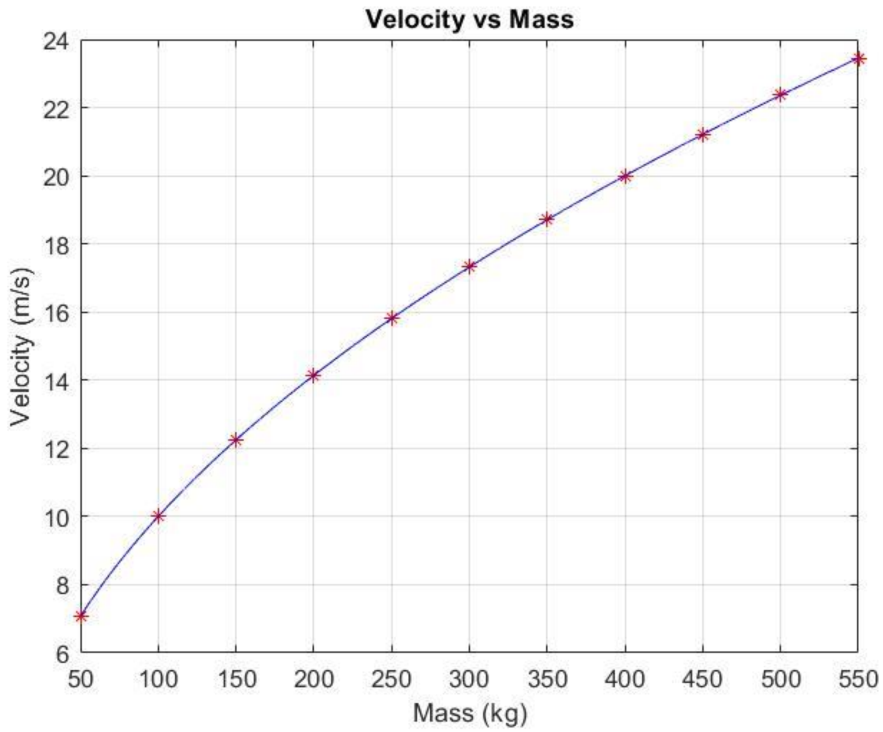
rms_sec = 4.2106e+06	Here, it can be seen that rms value of the
rms_pow = 1.1315e+03	power series fit is much less than second degree
rms_exp = 4.7491e+06	polynomial fit and the exponential fit.

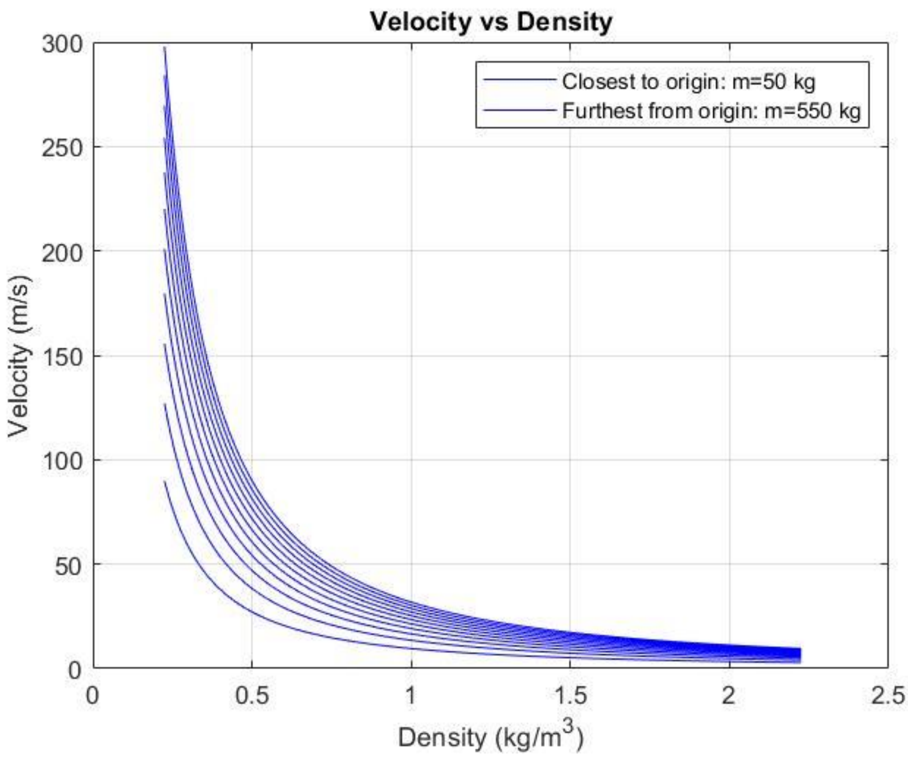
So, we will continue with the function that we found with power series fit for the rest of the problem, since it gives more accurate results.

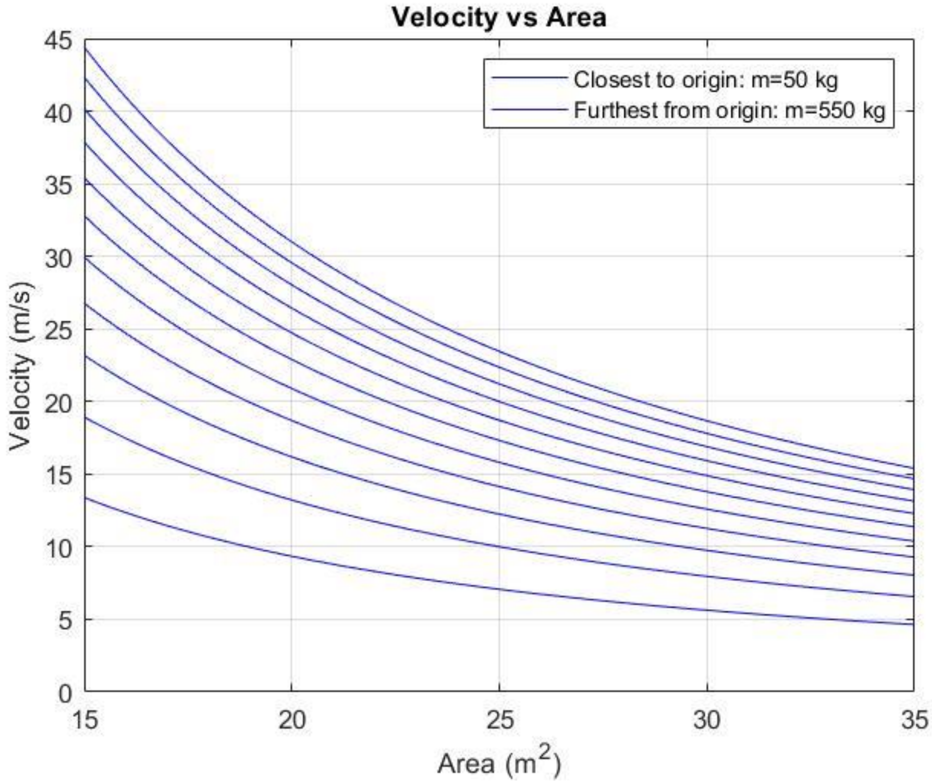
The Graphs of Velocity vs. Density, Mass and Area

These graphs are created by using the function we got from power series fit.

$$v = \frac{a * \mu * A * \left(A^{1.5} * \frac{\rho}{m}\right)^{b}}{m}$$







Part b)

Leave area term alone in the function that relates pi_1 to pi_2 which we found by power series fitting technique.

$$\text{area} = \left(\frac{m^{b+1} * v}{a * \mu * d^{b}}\right)^{\left(\frac{2}{3*b+2}\right)}$$

The result is 65.4986 m^2 which is a reasonable result if we look from an conceptual perspective. On the other hand, we should note that an extrapolation far beyond the data points we have is needed for the given mass and velocity of the prototype. So, we can't be sure the area calculated from the relationship would be enough to carry the prototype.

Codes:

```
% FLUID MECHANICS COMPUTATIONAL HOMEWORK QUESTION-1
clear all, close all, clc

%=========== PARAMETERS ===========%
% VELOCITY (Column vector)
v = [7.07 10.00 12.25 14.14 15.81 17.32 18.71...
    20.00 21.21 22.36 23.45]'; % m/s
S = size(v); % size of data
N = length(v);
% MASS (Column vector)
m = [50 100 150 200 250 300 350 400 450 500 550]';
% DYNAMIC VISCOSITY
nu = 1.8*10^(-5); % Pa*s
% DENSITY
d = 1.225; % kg/m^3
% AREA
area = 25; % m^2

%=========== PI TERMS ===========%
pi_1 = m.*v/(nu*area); % 11x1
pi_2 = area^(3/2)*d./m; % 11x1


%=========== SECOND ORDER FIT ===========%
% y=ax^2+bx+c, y=pi_1, x=pi_2
A_sec = [pi_2.^2 pi_2 ones(size(pi_2))];
y_sec = pi_1;
x_sec = (A_sec'*A_sec)^(-1)*A_sec'*y_sec;
a_sec = x_sec(1);
b_sec = x_sec(2);
c_sec = x_sec(3);

% THE FUNCTION f(pi_2) = pi_1
pi_1_sec = @(pi_2) a_sec*pi_2.^2 + b_sec*pi_2 +c_sec;
t_sec = min(pi_2):0.01:max(pi_2);

% Pi-2 vs Pi-1 graph
figure
plot(pi_2,pi_1,'*r')                    % As we see from the graph, second
hold on, grid on                        % degree polynomial curve fitting
plot(t_sec,pi_1_sec(t_sec'),'b')        % doesn't work well in our problem.
xlabel('PI-2')                          % Later, the rms values also will
ylabel('PI-1')                          % lead us to the same conclusion.
title('Second Order Fit')

rms_sec = sqrt(sum((pi_1_sec(pi_2) - pi_1).^2)/N) % RMS VALUE FOR SECOND D.


%%
%=========== POWER SERIES FIT ===========%
% y=a*x^b, y=pi_1, x=pi_2
A_pow = [log(pi_2) ones(size(pi_2))]; % lineerization done
y_pow = log(pi_1);
x_pow = (A_pow'*A_pow)^(-1)*A_pow'*y_pow;
b_pow = x_pow(1);
a_pow = exp(x_pow(2));

% THE FUNCTION f(pi_2) = pi_1
pi_1_pow = @(pi_2) a_pow*pi_2.^b_pow;
```

```
t_pow = min(pi_2):0.01:max(pi_2);

% Pi-2 vs Pi-1 graph
figure
plot(pi_2,pi_1,'*r')
hold on, grid on
plot(t_pow,pi_1_pow(t_pow'),'b')
xlabel('PI-2')
ylabel('PI-1')
title('Power Series Fit')

rms_pow = sqrt(sum((pi_1_pow(pi_2) - pi_1).^2)/N) % RMS VALUE FOR POWER S.

%%
%============== EXPONENTIAL FIT ==============%
% y=a*e^(b*x), y=pi_1, x=pi_2
A_exp = [pi_2 ones(size(pi_2))]; % lineerization done
y_exp = log(pi_1);
x_exp = (A_exp'*A_exp)^(-1)*A_exp'*y_exp;
b_exp = x_exp(1);
a_exp = exp(x_exp(2));

% THE FUNCTION f(pi_2) = pi_1
pi_1_exp = @(pi_2) a_exp*exp(b_exp*pi_2);
t_exp = min(pi_2):0.01:max(pi_2);

% Pi-2 vs Pi-1 graph
figure
plot(pi_2,pi_1,'*r')
hold on, grid on
plot(t_exp,pi_1_exp(t_exp'),'b')
xlabel('PI-2')
ylabel('PI-1')
title('Exponential Fit')

rms_exp = sqrt(sum((pi_1_exp(pi_2) - pi_1).^2)/N) % RMS VALUE FOR POWER S.



% COMMENT ON RMS VALUES:
% rms_sec = 4.2106e+06   Here, it can be seen that rms value of the
% rms_pow = 1.1315e+03   power series fit is much less than second degree
% rms_exp = 4.7491e+06             polynomial fit and the exponential fit.
%
%                        So, we will continue with the function that we
%                        found with power series fit for the rest of the
%                        problem, since it gives more accurate results.


%%
% VELOCITY VS MASS graph
v_m = @(m) nu.*area./m.*a_pow.*(area.^(3/2).*d./m).^b_pow;
tm=min(m):0.01:max(m);

figure
plot(m,v,'*r')
hold on
plot(tm,v_m(tm),'b')
grid on
xlabel('Mass (kg)')
ylabel('Velocity (m/s)')
```

```
title('Velocity vs Mass')

% VELOCITY VS DENSITY graph
v_d = @(d) nu.*area./m.*a_pow.*(area.^(3/2).*d./m).^b_pow;
td=d-1:0.01:d+1;

figure
plot(td,v_d(td),'b')
grid on
xlabel('Density (kg/m^3)')
ylabel('Velocity (m/s)')
title('Velocity vs Density')
legend('Closest to origin: m=50 kg','Furthest from origin: m=550 kg')

% VELOCITY VS AREA graph
v_area = @(area) nu.*area./m.*a_pow.*(area.^(3/2).*d./m).^b_pow;
tarea=area-10:0.01:area+10;

figure
plot(tarea,v_area(tarea),'b')
grid on
xlabel('Area (m^2)')
ylabel('Velocity (m/s)')
title('Velocity vs Area')
legend('Closest to origin: m=50 kg','Furthest from origin: m=550 kg')


%%
% PART-B OF THE PROBLEM:
m_b = 10000; % (kg)
v_b = 30; % (m/s)

% Leave area term alone in the function that relates pi_1 to pi_2 which we
% found by power series fitting technique.

area_b = (m_b^(b_pow+1)*v_b/nu/a_pow/d^b_pow)^(1/((3*b_pow+2)/2));
```