ME424 Design Project-1

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Purpose

Designing a planetary gear that will transmit the maximum amount of power with the given specifications and constraints.

Givens

- Rotation speed of the sun: 3700 rpm
- Pressure angle: 20°
- Life of the arm: 10^8
- Maximum inner diameter of the housing: 520 mm (5 mm clearance)
- Maximum rotation speed of the arm: 230 rpm
- Minimum rotation speed of the arm: 115 rpm
- Maximum hardness value: 400 HB
- Minimum hardness value: 200 HB
- Minimum safety factor against bending fatigue failure: 1.5
- Minimum safety factor against surface fatigue failure: 1.2

Design Parameters

1. Hardness
2. Face width
3. Module
4. Number of teeth of the sun
5. Number of teeth of the planet 1
6. Number of teeth of the planet 2

Constraints

- Gears must fit into the housing with the given clearance.
- Rotational speed of the arm must be in the given range.
- Maximum pitch line velocity must be smaller than 25 m/s.
- Hardness value of the steel must be within the given range.
- Minimum safety factors are specified.
- Contact ratio must be greater than 1.4.
- Interference must not be occurred.

Assumptions and Decisions:

- Gears have minimum 12 teeth.
- Planet 1 is rotating around the sun clockwise when the sun rotates clockwise.
- Maximum pitch line velocity occurs between the sun and planet 1.
- Only the sun and planet 2 will be investigated against failure. This is because the sun and planet 2 have more contacts than their mating gears planet 1 and ring. (It is assumed that radius of planet 2 is smaller than planet 1, and radius of the ring is at least 3 times greater than planet 2's.)
- Temperature does not exceed 70 °C.
- **Face width is taken as 14m.** Increasing face width decreases bending and surface fatigue stresses which are beneficial. It increases mounting factor less.
- **HB is taken as 400.** Harder the material the more strength it has if its ultimate strength is less than 1400 MPa which suits our case. It decreases surface factor less.

Procedure:

There are 6 design parameters to be decided, thus 6 for loops are needed. To speed up the iteration process, loops for hardness and face width are removed from the code after we checked our assumptions regarding to hardness and face width that is explained in the above section.

1. First, geometric and kinematic variables are calculated according to the design parameters that are chosen in the loop.

2. All the checks are done before starting fatigue analysis: pitch line velocity, contact ratio, interference. If the checks fail, loop starts from the beginning.

3. Fatigue analysis: Strengths against bending and surface fatigue failure are calculated. From these and minimum safety factors, maximum possible force that will not fail the gear is determined for the sun planet 2 by the following equations.

$$\sigma_b = \frac{F_t}{\text{mbJ}} K_v K_o K_m \ , \ S_n = S_n' C_L C_G C_S k_r k_t k_{\text{ms}} \ , \ \text{SF} = \frac{S_n}{\sigma_b} \ , \ \sigma_b = \frac{S_n}{\text{SF}} \ , \ \boxed{\boldsymbol{F_t} = \frac{\sigma_b m b J}{K_v K_o K_m}}$$

$$\sigma_H = C_p \sqrt{\frac{F_t}{b d_p I} K_v K_o K_m} \ , \ S_H = S_{\text{fe}} C_{\text{Li}} C_R \ , \ \text{SF} = \left(\frac{S_H}{\sigma_H}\right)^2 \ , \ \sigma_H = \frac{S_H}{\sqrt{\text{SF}}} \ , \ \boxed{\boldsymbol{F_t} = \frac{\sigma_H^2 \, b \, d_p I}{C_p^2 K_v K_o K_m}}$$

4. 4 force data is achieved: sun-bending, sun-surface, planet2-bending, planet2-surface. The forces of the planet 2 is converted to the forces in the sun enabling us to compare them with the given equation.

$$F_{sun} = F_{p2} \frac{r_{p2}}{r_{p1}}$$

5. Finally, the maximum possible tangential at the sun is achieved. To find the maximum power, this force is multiplied by the pitch line velocity of the sun and three as three planets are connected.

$$\dot{W} = 3 * F_t * v$$

6. The computed powers of every iteration are compared with each other to find the maximum transmitted power.

Additional Notes:

- To calculate the values of 'J' and 'C_s' MATLAB's curve fitting tool is used to create the calculator function. Interpolation is made by this method.
- The equations that give the number of contacts that sun and planet 2 make until the arm completes 10^8 revolutions.

$$\text{Contact}_{\text{sun}} = 3 * \text{Life}_{\text{arm}} * \left(\frac{(N_{\text{sun}} + N_{\text{p1}}) * (N_{\text{p1}} + N_{\text{p2}})}{N_{\text{sun}} * N_{\text{p2}}} - 1\right) , \ \text{Contact}_{\text{p2}} = \text{Life}_{\text{arm}} * \frac{N_{\text{ring}}}{N_{\text{p2}}}$$

Finding Number of Contact: From part a, we know the revolutions that the sun will make in one revolution of arm. Also, planet 1 will complete one revolution around the sun and will reduce the number of contacts that the sun will make with it, because they are rotating in same clockwise direction. This reduction is one contact in each revolution of arm.

Optimum Design Parameters

1. Hardness: 400 HB
2. Face width: 70 mm
3. Module: 5 mm
4. Number of teeth of the sun: 16
5. Number of teeth of the planet 1: 42
6. Number of teeth of the planet 2: 12

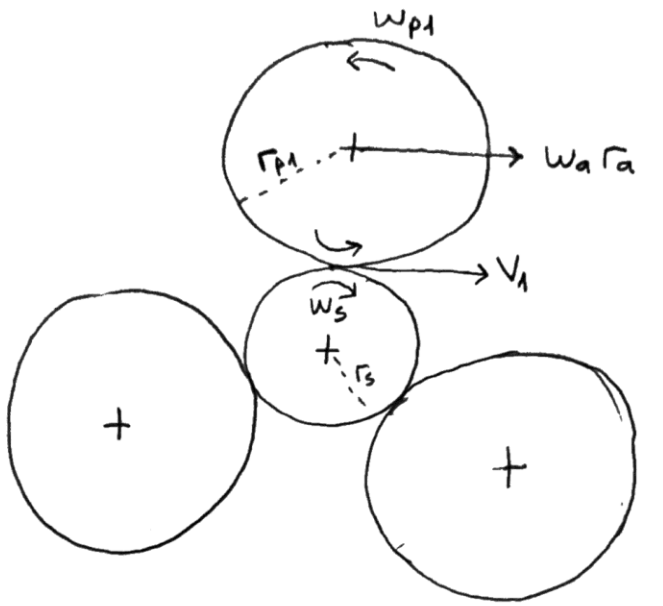
Power: 33.393 kW

Hand Solution

Scanned handwritten solutions to parts a and b are provided after this point. Also, the analysis of the optimum design is carried out.

(a)

Sun and Planet 1 schematic



ω_a: angular velocity of arm

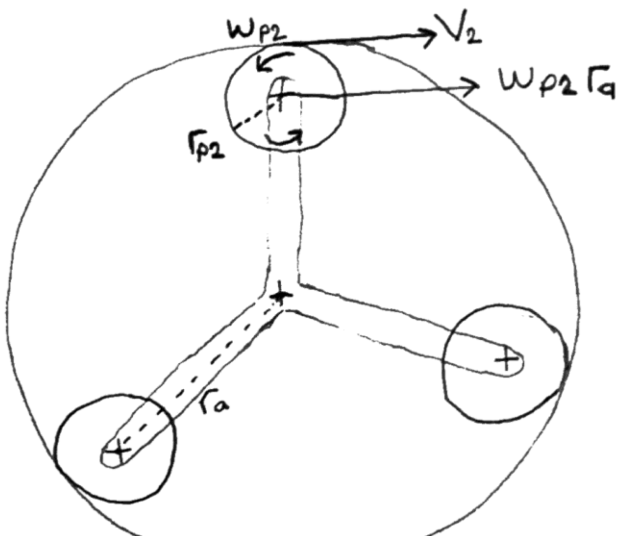
$d_{arm} = d_{sun} + d_{p1}$

$r_a = \frac{d_{arm}}{2}$

Pitch line velocity between sun and planet 1:

$$V_1 = \omega_s \cdot r_s = \omega_a r_a + \omega_{p1} r_{p1} \quad (1)$$

Ring and Planet 2 schematic



Pitch line velocity between sun and planet 2:

$$V_2 = \omega_r r_r = \omega_a r_a - \omega_{p2} r_{p2} \quad (2)$$

We know $\omega_{p1} = \omega_{p2}$ because they are mounted

Solving equations (1) and (2) simultaneously gives

$$r_{p2} / \quad w_s r_s r_{p2} = w_a r_a r_{p2} + w_{p1} r_{p1} r_{p2}$$

$$r_{p1} / \quad w_r r_r r_{p1} = w_a r_a r_{p1} - w_{p1} r_{p2} r_{p2}$$

$$+ \quad \overline{\hphantom{w_r r_r r_{p1} = w_a r_a r_{p1} - w_{p1} r_{p2} r_{p2}}}$$

$$w_s r_s r_{p2} + w_r r_r r_{p1} = (w_a r_a)(r_{p1} + r_{p2})$$

We know radius and number of teeth have linear relationship, so

$$w_s N_s N_{p2} + w_r N_r N_{p1} = w_a N_a (N_{p1} + N_{p2})$$

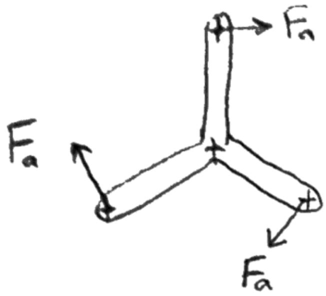
$$\Rightarrow \quad w_a = \frac{w_r N_r N_{p1} + w_s N_s N_{p2}}{N_a (N_{p1} + N_{p2})}$$

We know $N_a = N_s + N_{p1}$, so

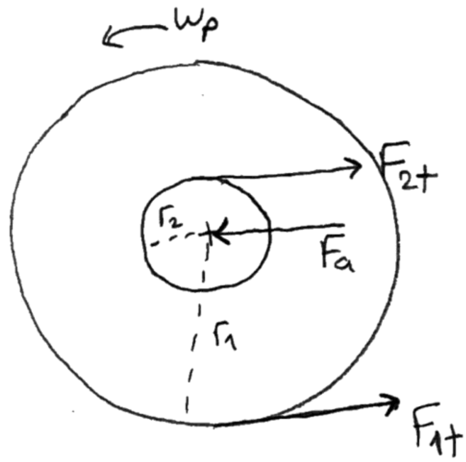
$$\Rightarrow \quad \boxed{w_a = \frac{w_r N_r N_{p1} + w_s N_s N_{p2}}{(N_s + N_{p1})(N_{p1} + N_{p2})}}$$

(b)

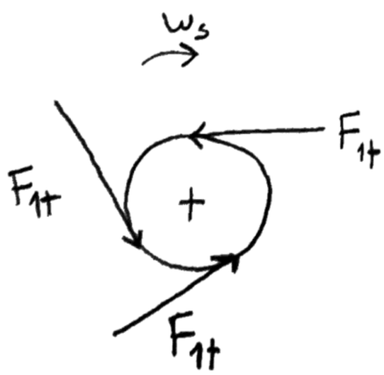
FBD of arm:



FBD of planet 1 and 2:



FBD of sun:



F_{1t}: force between sun and planet 1
F_{2t}: force between ring and planet 2
F_a: force applied to planet 2

$$\dot{W} = F_t V \quad \text{where} \quad V = \frac{\pi d n}{60}$$

so that

for sun:

$$\dot{W} = 3F_{1t} \cdot w_s r_s \Rightarrow \boxed{F_{1t} = \frac{\dot{W}}{3 w_s r_s}}$$

From FBD of planet 1 and planet 2, we have

$$\Sigma M_o = 0 \Rightarrow F_{2t} \cdot r_2 = F_{1t} r_1 \Rightarrow F_{2t} = \frac{F_{1t} r_1}{r_2} \quad (1)$$

$$\Sigma F_x = 0 \Rightarrow F_{2t} + F_{1t} = F_a \Rightarrow F_a = \left(1 + \frac{r_1}{r_2}\right) F_{1t} \quad (2)$$

we substitute

$F_{1t} = \dfrac{\dot{W}}{3 w_s r_s}$ into transmitted forces

$$\Rightarrow \boxed{F_{2t} = \frac{\dot{W} r_1}{3 w_s r_s r_2}}$$

$$\Rightarrow \boxed{F_a = \left(1 + \frac{r_1}{r_2}\right) \frac{\dot{W}}{3 w_s r_s}}$$

Analysis of the Optimum Design

$n_{sun} = 3700$ rev/min , $\phi = 20$ deg. , revolution of arm : 10^8

Selected and Calculated Design Parameters :

$HB = 400$, $b = 70$ mm , $m = 5$ mm, $d_{P2} = 60$ mm ; $N_{P2} = 12$

$d_r = 350$ mm, $N_r = 70$, $\omega_{P2} = 147.6$ rad/s , $\dot{W} = 34393$ J/s

We specify ring and planet 2 as most critical gear.

Contact ratio : $$CR = \frac{\sqrt{r_{ap2}^2 - R_{ring}^2} - \sqrt{r_{ring}^2 - R_{p2}^2} + c\sin\phi}{\pi m \cos\phi}$$

$$r_{ap2} = \frac{d_{p2}}{2} + m$$

$$R_{ring} = \frac{d_r}{2}\cos\phi$$

$$c = \frac{d_r}{2} - \frac{d_{p2}}{2}.$$

$$CR = 1.845$$

Checking interference : $N_r - N_{P2} = 58 > 10$ satisfied

Then, no interference

Bending Fatigue Strength Analysis

Calculating bending stress

$P = \frac{\dot{W}}{3}$ (For one planet 2 from 3 gears)

$P = 11464$ W

Pitch line velocity : $V = w_{p2} \frac{d_{p2}}{2} \frac{1}{1000}$

$V = 4.43$ m/s $<$ pitch line velocit is smaller than 25 m/s

Transmitted force: $F_t = \frac{P}{V} = 2588$ N

Geometry factor J by interpolation
for non precision gears, we may assume no sharing.

$J = 0.2136$

Velocity factor, for the chosen manufacturing method, the range of values for K_v factor is given by

$$K_v = \frac{1200 + 200 V}{1200} = 1.738$$

$K_o = 1.75$ Uniform source of power and heavy shock driven machinery

Mounting conditions are average, less then accurate ; then we use less rigid mountings, less accurate gears, contact across the full face for $b = 70$mm

$K_m = 1.6200$

Bending stress is then given by

$$\sigma_b = \frac{F_t}{m \cdot b \cdot J} \cdot K_v \cdot K_o \cdot K_m \qquad \sigma_b = 170.62 \text{ MPa}$$

Calculating bending fatigue strength

$C_L = 1$ Load factor for bending loads

$$C_G = \begin{cases} 1 \text{ if } & m \leq 5 \text{ mm} \\ 0.85 & \text{otherwise} \end{cases} \qquad C_G = 1$$

$C_S = 0.6261$ cold formed or machined, from Fig. 813 for HB = 400

$k_r = 0.814$ for $Rel = 99$

$k_t = 1$ for $T < 70°C$

$k_{ms} = 1.4$ for driver or driven gear

$S_n' = 0.5\, S_u$ for $S_u < 1400 \text{MPa}$

$S_u = 3.45\, HB$

$S_u = 1380 \text{ MPa} \Rightarrow S_n' = 690 \text{ MPa}$

$$S_n = S_n' \cdot C_L \cdot C_G \cdot C_S \cdot k_r \cdot k_t \cdot k_{ms}$$

$S_n = 492.32 \text{ MPa}$

$$SF_{bending} = \frac{S_n}{\sigma_b} \qquad SF_{bending} = 2.885 \geqslant 1.5$$

Surface Fatigue Strength Analysis

Calculating Surface contact stresses

$C_p = 191 \sqrt{MPa}$ for pinion and gear material steel

$$R = \frac{d_g}{d_p} \qquad I = \frac{\cos\phi \cdot \sin\phi}{2} \cdot \frac{R}{R-1} \qquad I = 0.1939$$

$$\sigma_H = C_p \cdot \sqrt{\frac{F_t}{b \cdot d_p \cdot I} \cdot K_v \cdot K_o \cdot K_M} \qquad \sigma_H = 755.86$$

$$S_{fe} = (2.76\,HB - 69)\,MPa \qquad S_{fe} = 1035\,MPa$$

$C_{Li} = 0.80$ from table of life factor for intervals

$C_R = 1$ for Rel $= 99$

$$S_H = S_{fe} \cdot C_{Li} \cdot C_R \qquad S_H = 828\,MPa$$

$$SF_{surface} = \left(\frac{S_H}{\sigma_H}\right)^2 \qquad SF_{surface} = 1.200 \geqslant 1.2$$