Report 2: Restricted 3-Body Problem

Transforming second order system to the first order system:

Second order differential equations:

- y1'' = y1 + 2*y2' - m2*(y1 + m1)/D1 - m1*(y1 - m2)/D2
- y2'' = y2 - 2*y1' - m2*y2/D1 - m1*y2/D2
- D1 = ((y1 + m1)^2 + y2^2)^(3/2)
- D2 = ((y1 - m2)^2 + y2^2)^(3/2)
- y1(0) = 0.994
- y1'(0) = 0
- y2(0) = 0
- y2'(0) = -2.0015851063790825

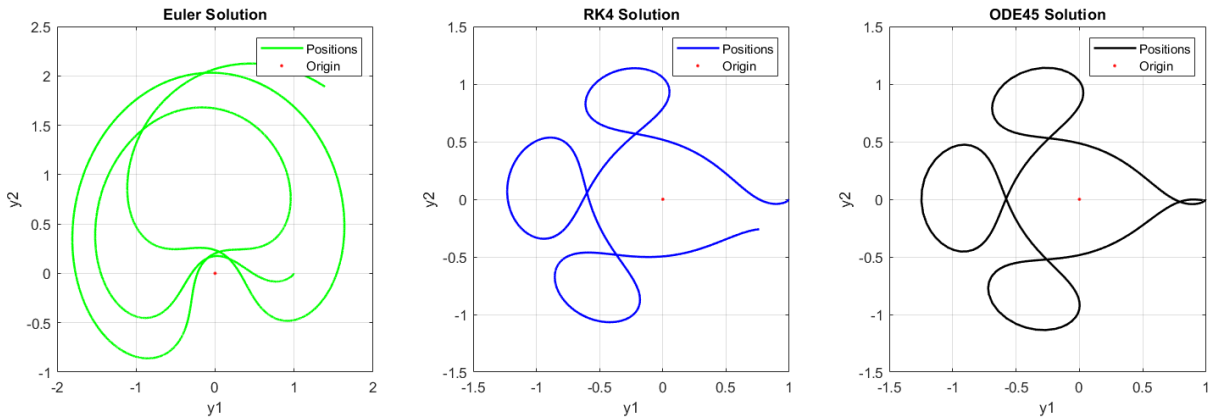
Transform these second order equations to first order equations:

Note that: I created y(1), y(2), y(3), y(4) row vectors for new system of first order differential equations.

- y(1) = y1, y(2) = y1', y(3) = y2, y(4) = y2'
- y(2)' = y(1) + 2*y(4) - m2*(y(1) + m1)/D1 - m1*(y(1) - m2)/D2;
- y(4)' = y(3) - 2*y(2) - m2*y(3)/D1 - m1*y(3)/D2;
- y(1)' = y(2);
- y(3)' = y(4);
- D1 = @(y(1),y(3)) ((y(1) + m1)^2 + y(3)^2)^(3/2);
- D2 = @(y(1),y(3)) ((y(1) - m2)^2 + y(3)^2)^(3/2);
- y(1)(0) = 0.994;
- y(2)(0) = 0;
- y(3)(0) = 0;
- y(4)(0) = -2.0015851063790825;

Discussion about methods:

We talked about specialities of each method in the first report. But there are some differences in both problems and we can investigate these differences.



We can see from the graphs, Euler didn't give the accurate solution this time due to the fact that in this example derivative of trajectories aren't zero near the end points. As we stated in the previous report, Euler causes large truncation errors and these errors mostly come from the end points. So, it would be a bad choice to choose Euler method fort his problem.

Other than that we infer from the graphs that this is a nonstiff first order differential system. So, using ode45 would be the most suitable choice from both point of views of accuracy and efficiency.

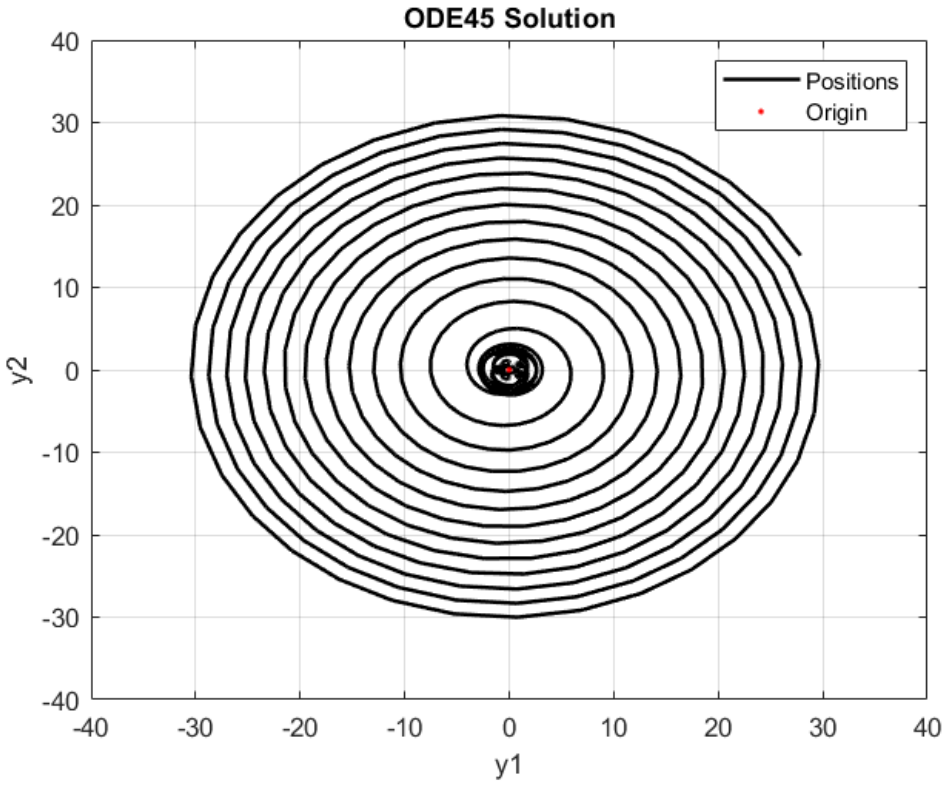
Besides, the graph already shows that 4th order Runge Kutta solution has some error even though it takes more time to compute RK4 than ode45.

- `elapsedTime_ode = 0.014331100000000`
- `elapsedTime_rk4 = 0.429451500000000`

Increase the Time:

I will use ode45 for this part of the report since it is the only method which gives accurate results.

In the above graphs we can only see one full round of the astreoid. Let's increase tmax to the 10 times of the period (T)



As we see from the graph, after 10*T seconds, the astroid would be far away from the Earth-Moon system.