Chair of Applied Mechanics
Department of Mechanical Engineering
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The Spring Mass Walker

Assignment 6

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Introduction

Modelling of walking was a challenge for decades starting from the mid 19th century. There are several different approaches in this topic. Scientists created different models but in the end it is seen that the most suitable model without being too complex is the mass spring model. The results of this model is close to the experimental values of the spring forces as well as trajectory of the mass.

There are some challenges that they faced during the modeling of bipedal walking. There are lot's of muscles that are activated during bipedal walking motion. Taking everything into account is not feasible while solving the problem. That is why this spring-mass model is created. (Geyer, 2006)

Problem statement

In this assignment, a SIMULINK model will be created by using a spring-mass system that will represent bipedal walking. From that model, several values like speed, position and spring forces can be extracted to MATLAB. With these values, plots can be produced which will give use the opportunity to investigate patterns in bipedal walking, energy transformation during this motion, stability of the system, failure modes and spring forces. Also maximum and instantaneous speeds can be calculated easily.

Normally, stable solutions are achieved by applying a stability analysis to the model. By this way, a solution space can be found where the system is stable. But it is not in the scope of this assignment. Thus, only one stable solution will be achieved by using the given initial conditons which will be provided in the next section (Definitions). Limit cycle will be created to show the solution is stable.

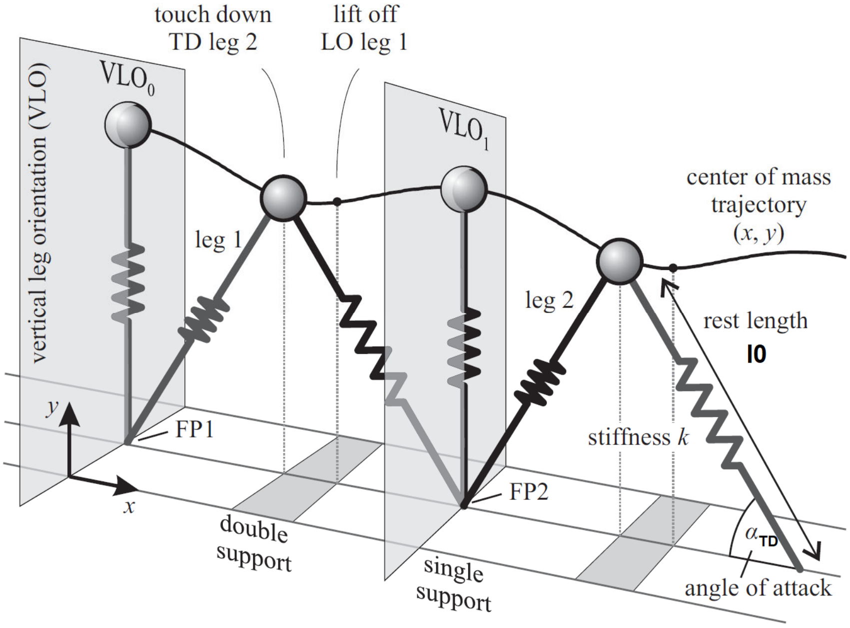

Figure 1: The system that will be modelled is provided in the figure. Our system consists of one point mass, two massless springs. At the initial condition, the mass has a positive horizontal velocity, and the spring is in a compressed position. The motion of the mass and springs can be seen during one step of bipedal walking. It has two phases as single support and double support. **(J. Rummel, 2010)**

Figure 1 shows the system as well as the phases in the single step of bipedal walking. The system includes a mass and two massless springs as shown. System has two different phases as single support and double support.

Definitions

Below, initial conditions, constants and parameters are shown:

Initial conditions:

- Horizontal position: x0 = 0 m
- Horizontal velocity: dx0 ~ 1.05 m/s
- Vertical position: y0 = 0.97 m
- Vertical velocity: dy0 = 0 m/s
- Foot point pos.: FP1 = 0 m
- Phase: Single support

Parameters:

- Mass: m = 80 kg
- Unsprung length of springs: l0 = 1 m
- Spring stiffness: k = 10 kN/m
- Gravitational acceleration: g = -9.81 m/s^2
- Energy of the system: E = 810 J
- Touch down angle: α_{TD}= 68 deg

In the Table 1 below, variables and some of the shortcuts that are used in this report are provided.

Table 1: Variables and shortcuts are provided in this table.

Symbol	**Property**	**Value**	**Unit**
FP1	Foot point of first spring	-	m
FP2	Foot point of second spring	-	m
TD	Touch down	-	-
TO	Take off	-	-
x	Horizontal position	-	m
y	Vertical position	-	m
dx	Horizontal speed	-	m/s
dy	Vertical speed	-	m/s
l1	Length of first spring	-	m
l2	Length of second spring	-	m
Fs1	Spring force of spring 1	-	N
Fs2	Spring force of spring 2	-	N
KE	Kinetic energy	-	J
PE	Potential energy	-	J
GPE	Gravitational potential	-	J
SPE	Spring potential	-	J

Assumptions:

- No air drag.
- No frictional energy losses.
- Springs are massless.
- Touch down angle is constant.

Approach and Implementation

The problem will be solved in SIMULINK as told in the problem statement. So, the process of creating the model will be discussed step by step in this section.

Firstly, free body diagrams regarding the two phases of the model are drawn, and equations of motion are created according to the Newton's second law.

Free body diagrams:

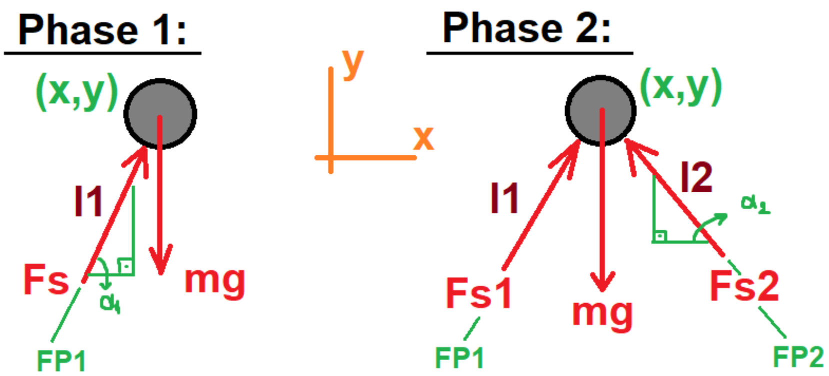

Figure 2: Free body diagrams regarding to the single support and double support cases. Red arrows show the forces acting on the mass.

Trigonometric values of the springs:

- $\sin(\alpha_1) = \frac{y}{l1}$
- $\cos(\alpha_1) = \frac{x-FP1}{l1}$
- $\sin(\alpha_2) = \frac{y}{l2}$
- $\cos(\alpha_2) = \frac{x-FP2}{l2}$

Lengths of the springs:

- $l1 = \sqrt{(x - FP1)^2 + y^2}$
- $l2 = \sqrt{(FP2 - X)^2 + y^2}$

Equations of motion:

$$\sum \boldsymbol{F_x} = \boldsymbol{m\ddot{x}} \quad => \quad \boldsymbol{k(l0 - l1)\cos(\alpha_1) + k(l0 - l2)\cos(\alpha_2) = m\ddot{x}} \tag{1}$$

$$\sum \boldsymbol{F_y} = \boldsymbol{m\ddot{y}} \quad => \quad \boldsymbol{-mg + k(l0 - l1)\sin(\alpha_1) + k(l0 - l2)\sin(\alpha_2) = m\ddot{y}} \tag{2}$$

It is obvious that the system is a hybrid dynamical system. It is dynamic because the states defining the system changes with time and it is hybrid because it has two phases. Therefore, it is needed to use "trigger" and "enable" blocks in SIMULINK according to the "touch down" and "take off" conditions.

Brief summary of the algorithm: Foot points are calculated according to the touch down condition. From these foot points, lengths of the springs are calculated. Using the take off condition and the lengths of the springs, the springs that are touching the ground are found. Forces of the contacting springs are calculated and given as input to the differential equations. The differential equations are solved.

To implement this algorithm, let's start with defining the triggering conditions.

- Touch down occurs when vertical position falls under the touch down height:

$$\mathrm{y} < \mathrm{yTD}$$

where $$yTD = l0 \times \sin(\alpha_{TD})$$

Touch down triggers calculation of FP2. It is calculated by the following equation:

$$\mathrm{FP2} = x + l0 \times \cos(\alpha_{TD})$$

- Take off occurs when the spring reaches its unsprung length.

$$\mathrm{l} \geq \mathrm{l0}$$

If the length of any spring is less than the unsprung length, it means that it touches the gorund.

So, foot point positions give use the information about the current phase. It is checked by a “relational operator”.

Finally, the differential equations are solved by the built-in tool of SIMULINK. Absolute and relative tolerance values are chosen as 1e-6, and maximum step size is chosen as 1e-3 to create a smooth curve. In conclusion, velocity and position values of the mass at any given time are achieved.

Results

Position and spring forces in each leg:

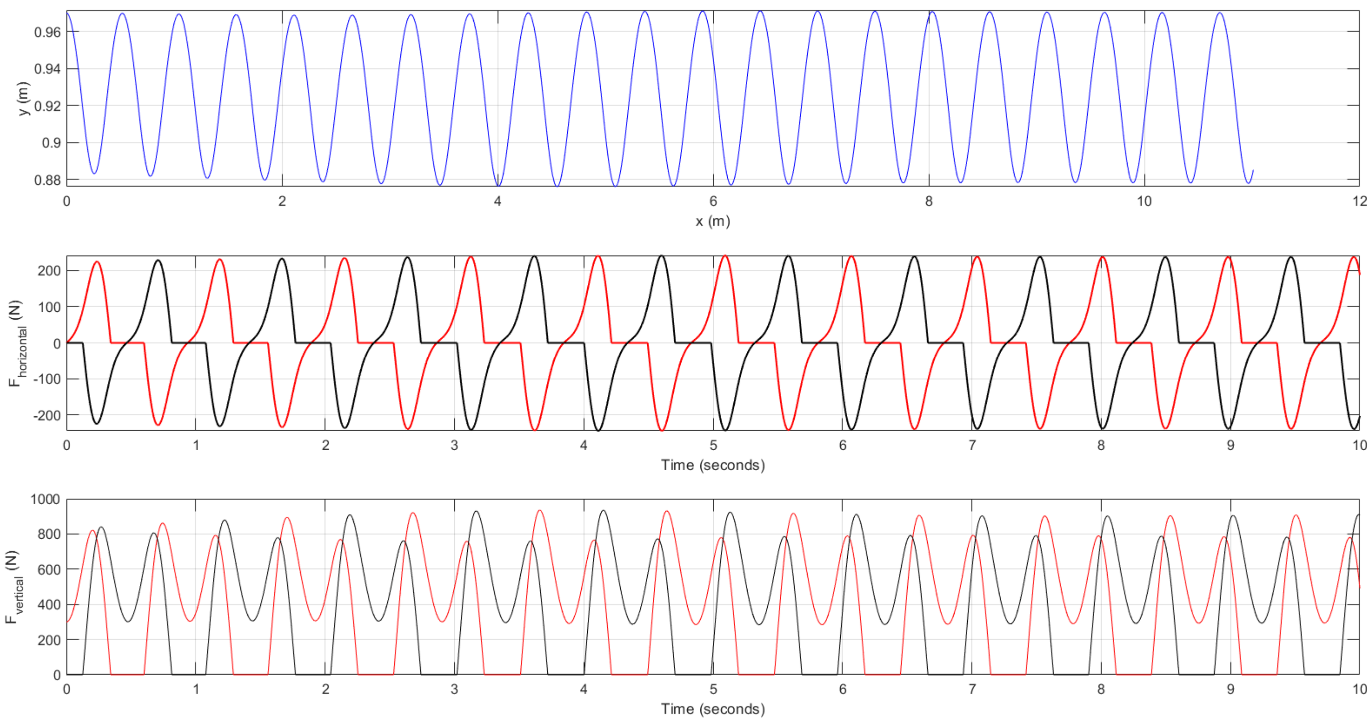

Figure 3: Trajectory of the mass in 2D space (x,y). Simulation duration is 10 seconds.

In Figure 3, there are 3 plots that shows trajectory of the mass, horizontal and vertical forces acting on the mass respectively. Red colored lines show the spring forces belonging to the first leg. On the contrary, the black lines show the spring forces belonging to the second leg. By plotting them together in the same figure, we can deduce the movement of the mass when touch down or take off occurs. Thus, it is seen that Touch down always occurs when the mass is moving downwards and take off always occurs when the mass is moving upwards. Also it can be said that from the repeating pattern of the graphs, The solution is stable. If we look at the plots that show forces in each leg, we can say that the period of the pattern is close to 1. It is noticed that there is only a phase difference between the right leg and the left leg. The black line is following taking the similar values with the red one but there is one step between them.

Forward speed of the mass:

Forward speed of the mass can be investigated from Figure 4. The simulation time is taken 100 seconds which is longer than the previous plots. It is due to the variation in speed in the first 10 seconds. It is seen that after sufficient amount of time the forward speed is oscillating between same values which also shows that the solution is stable.

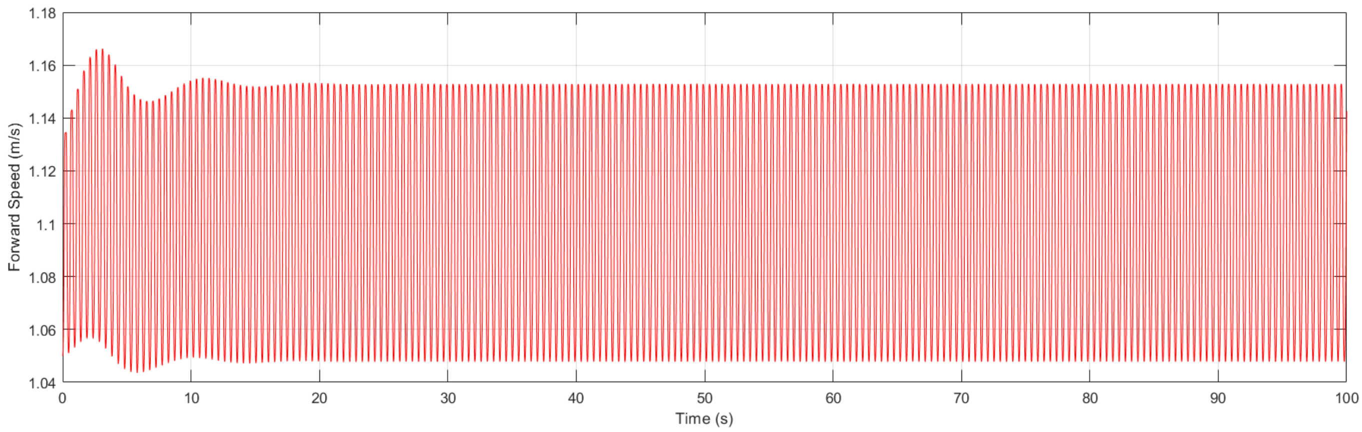

Figure 4: Forward speed of the mass. Simulation duration is 100 seconds. It is seen that after some time, the system reaches a stable state.

Below speeds are found by MATLAB script using max(velocity) and mean(velocity) commands. These speeds verifies the Figure 4. Starting speed is higher but after some time it starts to oscillate around its average speed.

Maximum speed = 1.1662 m/s

Average speed = 1.1002 m/s

Total spring forces:

Total spring force that acts on the mass in the horizontal direction is provided in Figure 5. We again see that there is continuous pattern in that force. Also there are sharp points which point out the touch down and take off situations. The average horizontal speed is constant as we saw in the previous figure. So, the sum of the forces in the horizontal direction must be zero to ensure that the mass moves at the same speed. This is why the horizontal force becomes negative and positive repeatatively.

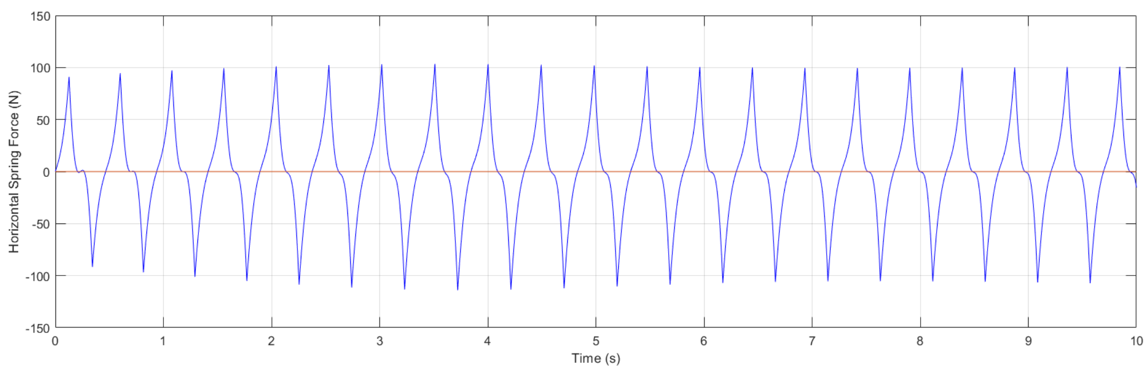

Figure 5: Total spring force acting on the mass in the horizontal direction.

In Figure 6, the total spring force that acts on the mass in the vertical direction are provided. Again we see a continuously repeating pattern. It is seen that the period of the motion is close to 0.5 from both of the graphs. But we said the period was close to 1 by looking at the Figure 3. So, each leg's motion and forces should be similar to each other. Apart from that, the one can infer that the vertical spring force is mininmum when the horizontal spring force is zero. It corresponds to the situation where the mass is directly on top of one leg.

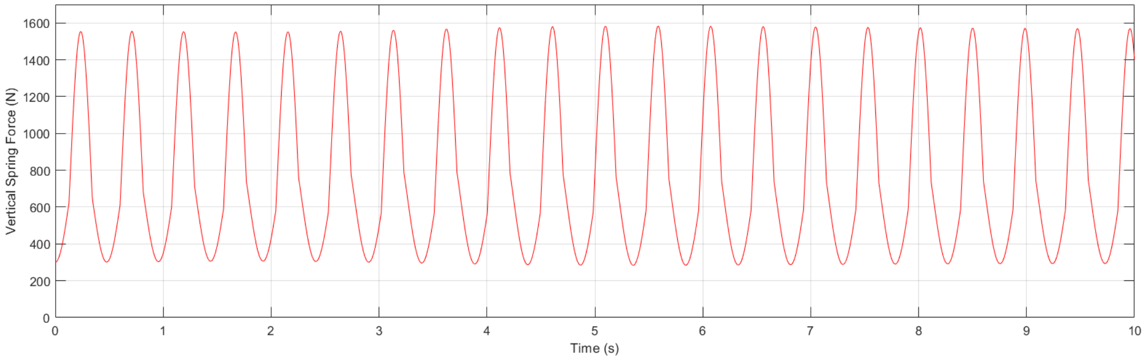

Figure 6: Total spring force acting on the mass in the vertical direction.

Limit cycle:

We have seen several clues that show us the solution is stable with the given initial conditions and parameters. But to be sure, it is checked whether the phase plot converges to a limit cycle or not. Vertical velocity and position are plotted against each other. As expected, the model converged to a limit cycle which can be seen in the Figure 7.

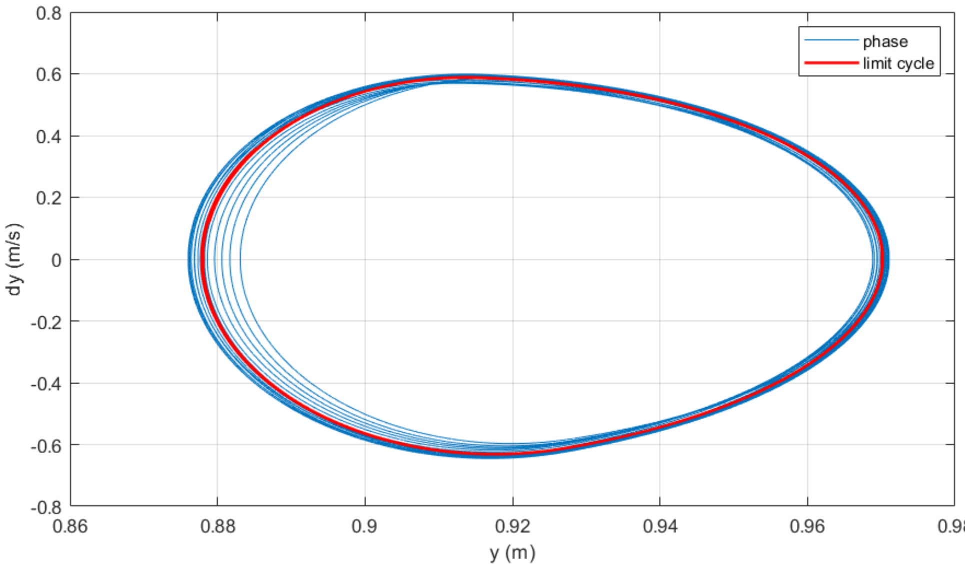

Figure 7: Phase plot of the bipedal walking model with the given initial conditions. The model converges towards the red limit cycle over time which shows the system is stable with the given initial conditions and parameters.

Energy transformations:

Figure 8 is created to check if the model has any errors. This check is made by summing kinetic and potential energies and looking if the mechanical energy is constant or not. It should be constant because we assumed that there is no losses in the system. As expected, their sum which is equal to the mechanical energy is always constant during the motion. Also, it is equal to 809.86 joules which is approximately same as the given initial condition.

Kinetic energy is calculated by the following equation as we know the horizontal speed.

$$KE = \frac{1}{2}m(dx)^2$$

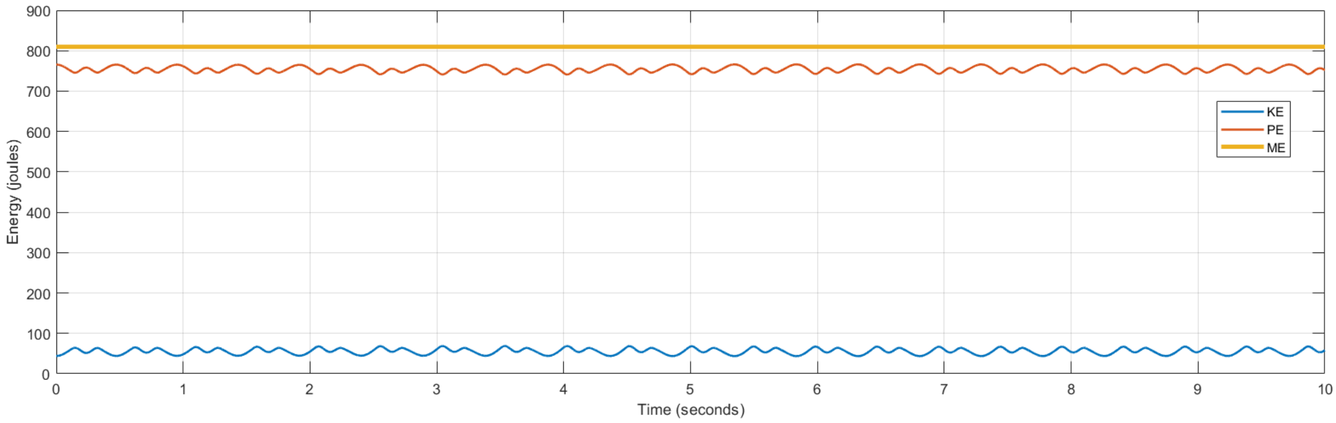

Figure 8: Kinetic and potential energy changes during the bipedal walking motion are plotted. Their sum is constant and equal to 809.86 joules which is same as the given initial condition. Potential energy is calculated by summing gravitational potential and spring potential energies.

Potential energy is found by summing gravitational and spring potential energies at each time step. Below equations are used in the calculations.

$$GPE = mgy$$

$$SPE = \frac{1}{2}k(l0 - l)^2$$

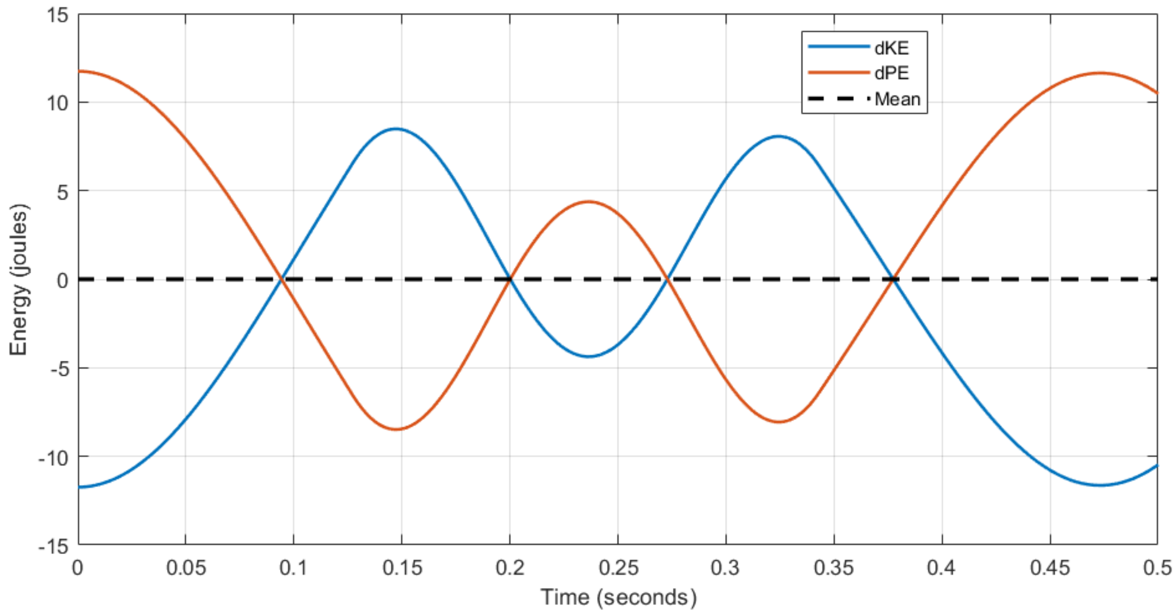

Figure 9: Deviations of the kinetic and potential energies from their mean values are plotted. Simulation time is short because the aim of this plot is to make a closer look the repeating pattern of the energy change. Energy transformation during one single step is shown.

Figure 9 is created to investigate closely the energy transformation during one step. It is created by substracting the energy values in each time step from their mean values.

Discussion

In this report, we introduced the mass-spring model for bipedal walking by using SIMULINK. Initial conditions were determined as well as all the parameters. There are only three parameters which can be selected randomly. They are the touch down angle, spring stiffness, and initial energy. They all were defined before solving the problem. So, the goal in this assignment was to create the model, extract the desired values from it, create the plots and investigate them as whether they are suitable for human walking logically and numerically. Also checking whether the solution is stable by creating a phase plot and looking for the limit cycle. Different plots are created throughout the report. They all included some continuous patterns which are also compatible with each other.

Average forward speed is found as approximately 1.1 m/s which is equal to 3.96 km/h. A human can walk in that speed which shows that this model is compatible with the real world. Note that, the mass and the spring lengths are chosen such that they are similar to the measures of a human. This enables us to comment on the validity of the forward speed of the model.

Mechanical energy is conserved as expected which contributes to the fact that model is well. Change in the kinetic energy and potential energy are always in the opposite directions which is also epected.

In conclusion, the mass-spring model is easy to implement and gives similar results to the experimental values.

Bibliography

Geyer, Hartmut & Seyfarth, Andre & Blickhan, Reinhard. 2006. *Compliant leg behavior explains basic dynamics of walking and running.* s.l. : Proceedings of the Royal Society B: Biological Sciences, 2006.
J. Rummel, Y. Blum, H. M. Maus, C. Rode and A. Seyfarth. 2010. *"Stable and robust walking with compliant legs,".* s.l. : IEEE International Conference on Robotics and Automation, 2010.

Required time (hh:mm): Minimum 35 hours.