2.5.3. Linear Temporal Logic (LTL)

Prove $\models \neg \diamond \neg p \rightarrow \Box p$ (the converse direction (the sufficiency) of Theorem 13.14 in Ben-Ari, M.).

$$
\begin{aligned}
&\models \neg \diamond \neg p \rightarrow \Box p \\
&\quad \neg \diamond \neg p = True \\
&\quad\quad \diamond \neg p = False \\
&\quad \forall s \in S,\ \forall S_i \in \mathscr{I}(s) \\
&\quad\quad S_i(P) = T \equiv \Box p
\end{aligned}
$$

$$
\begin{array}{c}
\neg(\neg \diamond \neg p \rightarrow \Box p) \\
\Big| \ \alpha \\
\neg \diamond \neg p \wedge \neg \Box p \\
\Big| \ \alpha \\
\neg \diamond \neg p, \neg \Box p \\
\vdots \ \text{instantiation} \\
\exists s \in S,\ S_i \in \mathscr{I}(j) \\
\Big| \\
p, \neg p \\
\times
\end{array}
$$

Exercise 2.5.4. Linear Temporal Logic (LTL)

Prove Theorem 13.15 from Ben-Ari, M.: $\models \Box(p \rightarrow q) \rightarrow (\Box p \rightarrow \Box q)$.

$$
\begin{aligned}
&\models \Box(p \rightarrow q) \rightarrow (\Box p \rightarrow \Box q) \\
&\quad \Box(p \rightarrow q) = True \\
&\quad \forall s \in S,\ \forall S_i \in \mathscr{P}(s)
\end{aligned}
$$

$$
\begin{array}{c}
\neg(\Box(p \rightarrow q) \rightarrow (\Box p \rightarrow \Box q)) \\
\Big| \ \alpha \\
\Box(p \rightarrow q), \neg(\Box p \rightarrow \Box q) \\
\Big| \ \alpha \\
\Box(p \rightarrow q), \Box p, \neg \Box q \\
\vdots \ \text{instantiation} \\
\exists s \in S,\ S_i \in \mathscr{I}(j) \\
\swarrow \qquad \searrow \\
\neg p, p, \neg q \qquad\qquad q, p, \neg q \\
\times \qquad\qquad\qquad \times
\end{array}
$$