2.3.1 Propositional Logic.

Prove the following logical equivalences making use of semantic tableaux:

a) $A \wedge (B \vee C) \equiv (A \wedge B) \vee (A \wedge C)$

$$\begin{tikzcd}
& A \wedge (B \vee C) \arrow[d] & \\
& A, B \vee C \arrow[dl] \arrow[dr] & \\
A, B & & A, C \\
\odot & & \odot
\end{tikzcd}$$

$$\begin{tikzcd}
& (A \wedge B) \vee (A \wedge C) \arrow[dl] \arrow[dr] & \\
A \wedge B \arrow[d] & & A \wedge C \arrow[d] \\
A, B & & A, C \\
\odot & & \odot
\end{tikzcd}$$

b) $A \vee B \equiv \neg(\neg A \wedge \neg B)$

$$\begin{tikzcd}
& A \vee B \arrow[dl] \arrow[dr] & \\
A & & B \\
\odot & & \odot
\end{tikzcd}$$

$$\begin{tikzcd}
& \neg(\neg A \wedge \neg B) \arrow[dl] \arrow[dr] & \\
\neg(\neg A) \arrow[d] & & \neg(\neg B) \arrow[d] \\
A & & B \\
\odot & & \odot
\end{tikzcd}$$

c) $A \wedge B \equiv \neg(\neg A \vee \neg B)$

$$\begin{tikzcd}
A \wedge B \arrow[d] \\
A, B \\
\odot
\end{tikzcd}$$

$$\begin{tikzcd}
\neg(\neg A \vee \neg B) \arrow[d] \\
\neg(\neg A), \neg(\neg B) \arrow[d] \\
A, B \\
\odot
\end{tikzcd}$$

d) $A \rightarrow B \equiv \neg A \vee B$

$$\begin{tikzcd}
& A \rightarrow B \arrow[dl] \arrow[dr] & \\
\neg A & & B \\
\odot & & \odot
\end{tikzcd}$$

$$\begin{tikzcd}
& \neg A \vee B \arrow[dl] \arrow[dr] & \\
\neg A & & B \\
\odot & & \odot
\end{tikzcd}$$

e) $A \rightarrow B \equiv \neg(A \wedge \neg B)$

$$\begin{tikzcd}
& A \rightarrow B \arrow[dl] \arrow[dr] & \\
\neg A & & B \\
\odot & & \odot
\end{tikzcd}$$

$$\begin{tikzcd}
& \neg(A \wedge \neg B) \arrow[dl] \arrow[dr] & \\
\neg A & & \neg(\neg B) \arrow[d] \\
\odot & & B \\
& & \odot
\end{tikzcd}$$

2.3.2 Propositional Logic.

Prove or disprove making use of semantic tableaux:

a) $\models (A \to B) \lor (B \to A)$

$$\begin{tikzcd}
 & & & (A \to B) \lor (B \to A) \arrow[dll] \arrow[drr] & & & \\
 & A \to B \arrow[dl] \arrow[dr] & & & & B \to A \arrow[dl] \arrow[dr] & \\
\neg A & & B & & \neg B & & A \\
\odot & & \odot & & \odot & & \odot
\end{tikzcd}$$

As each item is represented only one, we can assume that this is always true

b) $\models ((A \to B) \to B) \to B$

$$\begin{tikzcd}
 & & ((A \to B) \to B) \to B \arrow[dl] \arrow[dr] & \\
 & \neg((A \to B) \to B) \arrow[d] & & B \\
 & A \to B, \neg B \arrow[dl] \arrow[dr] & & \odot \\
\neg A, \neg B & & B, \neg B & \\
\odot & & \times &
\end{tikzcd}$$

As we found a contradiction, this is not always true