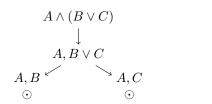
2.3.1 Propositional Logic.

Prove the following logical equivalences making use of semantic tableaux:

a)
$$A \wedge (B \vee C) \equiv (A \wedge B) \vee (A \wedge C)$$



$$\begin{array}{ccc} A \wedge B) \vee (A \wedge C) \\ A \wedge B & A \wedge C \\ \downarrow & \downarrow \\ A, B & A, C \\ \odot & \odot \end{array}$$

b)
$$A \lor B \equiv \neg(\neg A \land \neg B)$$

$$\begin{array}{ccc} A \vee B \\ A & \stackrel{\checkmark}{\searrow} B \\ \odot & \odot \end{array}$$

$$\neg(\neg A \land \neg B)$$

$$\neg(\neg A) \quad \neg(\neg B)$$

$$\downarrow \qquad \downarrow$$

$$A \qquad B$$

$$\odot \qquad \odot$$

c)
$$A \wedge B \equiv \neg(\neg A \vee \neg B)$$

$$\begin{array}{c} A \wedge B \\ \downarrow \\ A, B \\ \odot \end{array}$$

$$\neg(\neg A \lor \neg B)$$

$$\downarrow$$

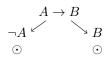
$$\neg(\neg A), \neg(\neg B)$$

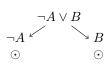
$$\downarrow$$

$$A, B$$

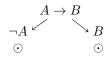
$$\odot$$

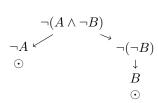
d)
$$A \to B \equiv \neg A \lor B$$





e)
$$A \to B \equiv \neg (A \land \neg B)$$

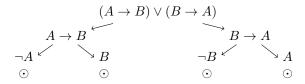




2.3.2 Propositional Logic.

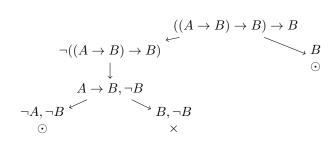
Prove or disprove making use of semantic tableaux:

a)
$$\models (A \rightarrow B) \lor (B \rightarrow A)$$



As each item is represented only one, we can assume that this is always true

b)
$$\models ((A \rightarrow B) \rightarrow B) \rightarrow B$$



As we found a contradiction, this is not always true