2.2.1 Propositional Logic.

Prove the following logical equivalences making use of truth tables:

a) $A \wedge (B \vee C) \equiv (A \wedge B) \vee (A \wedge C)$

A	**B**	**C**	$A \wedge (B \vee C)$	$(A \wedge B) \vee (A \wedge C)$
1	1	1	1	1
1	1	0	1	1
1	0	1	1	1
1	0	0	0	0
0	1	1	0	0
0	1	0	0	0
0	0	1	0	0
0	0	0	0	0

b) $A \vee B \equiv \neg(\neg A \wedge \neg B)$

A	**B**	$A \vee B$	$\neg(\neg A \wedge \neg B)$
1	1	1	1
1	0	1	1
0	1	1	1
0	0	0	0

c) $A \wedge B \equiv \neg(\neg A \vee \neg B)$

A	**B**	$A \wedge B$	$\neg(\neg A \vee \neg B)$
1	1	1	1
1	0	0	0
0	1	0	0
0	0	0	0

d) $A \rightarrow B \equiv \neg A \vee B$

A	**B**	$A \rightarrow B$	$\neg A \vee B$
1	1	1	1
1	0	0	0
0	1	1	1
0	0	1	1

e) $A \rightarrow B \equiv \neg(A \wedge \neg B)$

A	**B**	$A \rightarrow B$	$\neg(A \wedge \neg B)$
1	1	1	1
1	0	0	0
0	1	1	1
0	0	1	1

f) $((A \oplus B) \oplus B) \equiv A$

A	**B**	$((A \oplus B) \oplus B)$	A
1	1	1	1
1	0	1	1
0	1	0	0
0	0	0	0

g) $((A \leftrightarrow B) \leftrightarrow B) \equiv A$

A	**B**	$((A \leftrightarrow B) \leftrightarrow B)$	A
1	1	1	1
1	0	1	1
0	1	0	0
0	0	0	0

2.2.2 Propositional Logic.

Prove or disprove making use of truth tables:

a) $\models (A \rightarrow B) \lor (B \rightarrow A)$

A	**B**	$(A \rightarrow B) \lor (B \rightarrow A)$
1	1	1
1	0	1
0	1	1
0	0	1

As we can see, this is a Tautology, so is proved

b) $\models ((A \rightarrow B) \rightarrow B) \rightarrow B$

A	**B**	$((A \rightarrow B) \rightarrow B) \rightarrow B$
1	1	1
1	0	0
0	1	1
0	0	1

As we can see, there is a combination that it's not true, so this fails

c) $\models (A \leftrightarrow B) \leftrightarrow (A \leftrightarrow (B \leftrightarrow A))$

A	**B**	$(A \leftrightarrow B) \leftrightarrow (A \leftrightarrow (B \leftrightarrow A))$
1	1	1
1	0	1
0	1	0
0	0	0

As we can see, there is two combinations that are not true, so this fails

d) $\models ((A \wedge B) \rightarrow C) \rightarrow ((A \rightarrow C) \vee (B \rightarrow C))$

A	**B**	**C**	$((A \wedge B) \rightarrow C) \rightarrow ((A \rightarrow C) \vee (B \rightarrow C))$
1	1	1	1
1	1	0	1
1	0	1	1
1	0	0	1
0	1	1	1
0	1	0	1
0	0	1	1
0	0	0	1

As we can see, this is a Tautology, so is proved