1.2.1

(a) Show that $A \times (B \cap C) = (A \times B) \cap (A \times C)$

$$x = 1$$

(b) Show that $A \times (B \cup C) = (A \times B) \cup (A \times C)$

1.2.2

(a) Consider the relations $R = \{(1,7),(3,3),(13,11)\}$ and $S = \{(1,1),(1,7),(3,11),(13,12),(15,1)\}$ over the positive integers. Identify $dom(R \cap S)$, $range(R \cap S)$, $dom(R \cup S)$, $range(R \cup S)$

$$dom(R \cap S) = \{1\}$$

$$range(R \cap S) = \{7\}$$

$$dom(R \cup S) = \{1, 3, 13, 15\}$$

$$range(R \cup S) = \{7, 3, 11, 1, 12\}$$

(b) In the same example, identify $join(R, S)$, $join(S, R)$, $S \circ R$, $R \circ S$, $R \circ R$, $S \circ S$. (c) In the same example, identify $R(X)$ and $S(X)$ for $X = 1, 3, 11$ and $X = \emptyset$. (d) Explain how to carry out composition by means of join and projection.

1.2.3

(a) Show that R is reflexive over A iff $I_A \subseteq R$. Here I_A is the identity relation over A, defined in an exercise in Sect. 2.1.3. (b) Show that the converse of a relation R that is reflexive over a set A is also reflexive over A. (c) Show that R is transitive iff $R \circ R \subseteq R$.

1.2.4

(a) Show that the following three conditions are equivalent: (i) R is symmetric, (ii) $R \subseteq R^{-}1$, (iii) $R = R^{-}1$. (b) Show that if R is reflexive over A and also transitive, then the relation S defined by $(a, b) \in S$ iff both $(a, b) \in R$ and $(b, a) \in R$ is an equivalence relation. (c) Enumerate all the partitions of $A = \{1, 2, 3\}$ and draw a Hasse diagram for them under fineness.

1.2.5

Let R be any transitive relation over a set A. Define S over A by putting $(a, b) \in S$ iff either $a = b$ or both $(a, b) \in R$ and $\neg(b, a) \in R$. Show that S partially orders A.