2.5.3. Linear Temporal Logic (LTL)

Prove $\models \neg \diamond \neg p \to \Box p$ (the converse direction (the sufficiency) of Theorem 13.14 in Ben-Ari, M.).

$$\models \neg \diamond \neg p \to \Box p$$
$$\neg \diamond \neg p = True$$
$$\diamond \neg p = False$$
$$\forall s \in S,\ \forall S_i \in \mathscr{I}(s)$$
$$S_i(P) = True \equiv \Box p$$

$$\neg(\neg \diamond \neg p \to \Box p)$$
$$\Big|\ \alpha$$
$$\neg \diamond \neg p, \neg \Box p$$
$$\vdots\ \text{instantiation}$$
$$\exists s \in S,\ S_i \in \mathscr{I}(j)$$
$$\Big|$$
$$p, \neg p$$
$$\times$$

By contradiction of the inverse, it's proved true

Exercise 2.5.4. Linear Temporal Logic (LTL)

Prove Theorem 13.15 from Ben-Ari, M.: $\models \Box(p \to q) \to (\Box p \to \Box q)$.

$$\models \Box(p \to q) \to (\Box p \to \Box q)$$
$$\Box(p \to q) = True$$
$$\forall s \in S,\ \forall S_i \in \mathscr{P}(s)$$

$$\neg(\Box(p \to q) \to (\Box p \to \Box q))$$
$$\Big|\ \alpha$$
$$\Box(p \to q), \neg(\Box p \to \Box q)$$
$$\Big|\ \alpha$$
$$\Box(p \to q), \Box p, \neg \Box q$$
$$\vdots\ \text{instantiation}$$
$$\exists s \in S,\ S_i \in \mathscr{I}(j)$$
$$\swarrow \qquad \searrow$$
$$\neg p, p, \neg q \qquad\qquad q, p, \neg q$$
$$\times \qquad\qquad\qquad \times$$

By contradiction of the inverse, it's proved true