2.5.3. Linear Temporal Logic (LTL)

Prove $\models \neg \diamond \neg p \rightarrow \Box p$ (the converse direction (the sufficiency) of Theorem 13.14 in Ben-Ari, M.).

$$
\begin{aligned}
\models \neg \diamond \neg p \rightarrow \Box p \\
\neg \diamond \neg p = True \\
\diamond \neg p = False \\
\forall s \in S,\ \forall S_i \in \mathscr{I}(s) \\
S_i(P) = T \equiv \Box p \\
\neg(\neg \diamond \neg p \rightarrow \Box p) \equiv \neg \diamond \neg p \wedge \Box p \\
\equiv \neg \diamond \neg p, \Box p \\
\exists s \in S,\ S_i \in \mathscr{I}(j) \\
p, \neg p
\end{aligned}
$$

Exercise 2.5.4. Linear Temporal Logic (LTL)

Prove Theorem 13.15 from Ben-Ari, M.: $\models \Box(p \rightarrow q) \rightarrow (\Box p \rightarrow \Box q)$.

$$
\begin{aligned}
\models \Box(p \rightarrow q) \rightarrow (\Box p \rightarrow \Box q) \\
\Box(p \rightarrow q) = True \\
\forall s \in S,\ \forall S_i \in \mathscr{P}(s)
\end{aligned}
$$