2.4.1 First-Order Logic.

Prove the statements:

- $\forall x p(x) \land \forall x q(x) \rightarrow \forall x(p(x) \land q(x))$
- $\forall x(p(x) \rightarrow q(x)) \rightarrow (\forall x p(x) \rightarrow \forall x q(x))$ is a valid formula (but its converse $(\forall x p(x) \rightarrow \forall x q(x)) \rightarrow \forall x(p(x) \rightarrow q(x))$ is not).

2.4.2 First-Order Logic.

Prove that the formula $(\forall x p(x) \rightarrow \forall x q(x)) \rightarrow \forall x(p(x) \rightarrow q(x))$ is not valid by constructing a semantic tableau for its negation.

2.4.3 First-Order Logic.

Prove that the following formulas are valid

- $\exists x(\mathrm{A}(x) \rightarrow \mathrm{B}(x)) \leftrightarrow (\forall x \mathrm{A}(x) \rightarrow \exists x \mathrm{B}(x))$
- $(\exists x \mathrm{A}(x) \rightarrow \forall x \mathrm{B}(x)) \rightarrow \forall x(\mathrm{A}(x) \rightarrow \mathrm{B}(x))$
- $\forall x(\mathrm{A}(x) \lor \mathrm{B}(x)) \rightarrow (\forall x \mathrm{A}(x) \lor \exists x \mathrm{B}(x))$
- $\forall x(\mathrm{A}(x) \rightarrow \mathrm{B}(x)) \rightarrow (\exists x \mathrm{A}(x) \rightarrow \exists x \mathrm{B}(x))$