3.2.1

In an effort to estimate the mean amount spent per customer for dinner at a major Atlanta restaurant, data were collected for a sample of 49 customers. Assume a population standard deviation of \$5.

a) At 95% confidence, what is the margin of error?

$$\begin{aligned}\sigma &= 5\\ n &= 49\\ \text{Error Margin} &= Z_{a/2}(\sigma/\sqrt{n})\\ &= Z_{.05/2}(5/\sqrt{49})\\ &= Z_{.025}(5/7)\\ &= 1.96(5/7)\\ &= 1.4\end{aligned}$$

b) If the sample mean is \$24.80, what is the 95% confidence interval for the population mean?

$$\begin{aligned}\bar{x} &= 24.80\\ interval &= \bar{x} \pm \text{Error Margin}\\ &= 24.80 \pm 1.4\\ &= (23.40, 26.20)\end{aligned}$$

3.2.2

The Wall Street Journal reported that automobile crashes cost the United States \$162 billion annually (The Wall Street Journal, March 5, 2008). The average cost per person for crashes in the Tampa, Florida, area was reported to be \$1,599. Suppose this average cost was based on a sample of 50 persons who had been involved in car crashes and that the population standard deviation is $\sigma = 600$

a) What is the margin of error for a 95% confidence interval?

$$\begin{aligned}\sigma &= 600\\ n &= 50\\ \text{Error Margin} &= Z_{a/2}(\sigma/\sqrt{n})\\ &= Z_{.05/2}(600/\sqrt{50})\\ &= Z_{.025}(84.8528137423)\\ &= 1.96(84.8528137423)\\ &= 166.311514\end{aligned}$$

b) What would you recommend if the study required a margin of error of \$150 or less?

We should reduce the confidence interval to get a smaller error margin

3.2.3

The National Quality Research Center at the University of Michigan provides a quarterly measure of consumer opinions about products and services (The Wall Street Journal, February 18, 2003). A survey of 10 restaurants in the Fast Food/Pizza group showed a sample mean customer satisfaction index of 71. Past data indicate that the population standard deviation of the index has been relatively stable with $\sigma = 5$.

a) What assumption should the researcher be willing to make if a margin of error is desired?

   That the data has a normal distribution and that you can based the margins in that

b) Using 95% confidence, what is the margin of error?

$$\begin{aligned}
\sigma &= 5 \\
n &= 10 \\
\text{Error Margin} &= Z_{a/2}(\sigma/\sqrt{n}) \\
&= Z_{.05/2}(5/\sqrt{10}) \\
&= Z_{.025}(1.58113883) \\
&= 1.96(1.58113883) \\
&= 3.0990321
\end{aligned}$$

c) What is the margin of error if 99% confidence is desired?

$$\begin{aligned}
\sigma &= 5 \\
n &= 10 \\
\text{Error Margin} &= Z_{a/2}(\sigma/\sqrt{n}) \\
&= Z_{.01/2}(5/\sqrt{10}) \\
&= Z_{.005}(1.58113883) \\
&= 2.576(1.58113883) \\
&= 4.07301362
\end{aligned}$$

3.1.4

Playbill magazine reported that the mean annual household income of its readers is \$119,155 (Playbill, January 2006). Assume this estimate of the mean annual household income is based on a sample of 80 households, and based on past studies, the population standard deviation is known to be $\sigma = \$30,000$.

a) Develop a 90% confidence interval estimate of the population mean.

$$\begin{aligned}
\sigma &= 30,000 \\
n &= 80 \\
\bar{x} &= 119,155 \\
\text{Error Margin} &= Z_{a/2}(\sigma/\sqrt{n}) \\
&= Z_{.1/2}(30,000/\sqrt{80}) \\
&= Z_{.05}(3354.101966) \\
&= 1.645(3354.101966) \\
&= 5,517.4977344 \\
interval &= \bar{x} \pm \text{Error Margin} \\
&= 119,155 \pm 5,517.4977344 \\
&= (2113637.5022656, 124672.4977344)
\end{aligned}$$

b) Develop a 95% confidence interval estimate of the population mean.

$$\begin{aligned}
\sigma &= 30,000 \\
n &= 80 \\
\bar{x} &= 119,155 \\
\text{Error Margin} &= Z_{a/2}(\sigma/\sqrt{n}) \\
&= Z_{.05/2}(30,000/\sqrt{80}) \\
&= Z_{.025}(3354.101966) \\
&= 1.96(3354.101966) \\
&= 6,574.0398538 \\
interval &= \bar{x} \pm \text{Error Margin} \\
&= 119,155 \pm 6,574.0398538 \\
&= (112580.9601462, 125729.0398538)
\end{aligned}$$

c) Develop a 99% confidence interval estimate of the population mean.

$$\begin{aligned}
\sigma &= 30,000 \\
n &= 80 \\
\bar{x} &= 119,155 \\
\text{Error Margin} &= Z_{a/2}(\sigma/\sqrt{n}) \\
&= Z_{.01/2}(30,000/\sqrt{80}) \\
&= Z_{.005}(3354.101966) \\
&= 2.576(3354.101966) \\
&= 8,640.166665 \\
interval &= \bar{x} \pm \text{Error Margin} \\
&= 119,155 \pm 8,640.166665 \\
&= (110514.833335, 127795.166665)
\end{aligned}$$

d) Discuss what happens to the width of the confidence interval as the confidence level is increased. Does this result seem reasonable? Explain.

We can see that the width of the confidence interval grows as the confidence level is increased. It seems reasonable because a confidence interval of 99 must include 99% of the results, which logically must be a wider part than a 95% or 90%.