Tecnológico de Monterrey

Fundamentos de computación

Homework 7

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1 Problems

Solve the following problems:

1. Generate two 4 X 4 matrices and manually apply the Strassen's method to multiply them. Verify that the result is correct by comparing with the one provided by the traditional method. Show the steps.

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \\ 13 & 14 & 15 & 16 \end{bmatrix} \qquad B = \begin{bmatrix} 16 & 15 & 14 & 13 \\ 12 & 11 & 10 & 9 \\ 8 & 7 & 6 & 5 \\ 4 & 3 & 2 & 1 \end{bmatrix}$$

$$A = \left[\begin{array}{cc|cc} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ \hline 9 & 10 & 11 & 12 \\ 13 & 14 & 15 & 16 \end{array}\right] \qquad B = \left[\begin{array}{cc|cc} 16 & 15 & 14 & 13 \\ 12 & 11 & 10 & 9 \\ \hline 8 & 7 & 6 & 5 \\ 4 & 3 & 2 & 1 \end{array}\right]$$

$$A_1 = \begin{bmatrix} 1 & 2 \\ 5 & 6 \end{bmatrix} \quad A_2 = \begin{bmatrix} 3 & 4 \\ 7 & 8 \end{bmatrix} \quad A_3 = \begin{bmatrix} 9 & 10 \\ 13 & 14 \end{bmatrix} \quad A_4 = \begin{bmatrix} 11 & 12 \\ 15 & 16 \end{bmatrix}$$

$$B_1 = \begin{bmatrix} 16 & 15 \\ 12 & 11 \end{bmatrix} \quad B_2 = \begin{bmatrix} 14 & 13 \\ 10 & 9 \end{bmatrix} \quad B_3 = \begin{bmatrix} 8 & 7 \\ 4 & 3 \end{bmatrix} \quad B_4 = \begin{bmatrix} 6 & 5 \\ 2 & 1 \end{bmatrix}$$

$$C_1 = A_1B_1$$
$$C_2 = A_2B_2$$
$$C_3 = A_3B_3$$
$$C_4 = A_4B_4$$

A_1B_1

$$X_1 = (1+6)*(16+11) = 189$$
$$X_2 = 16*(5+6) = 176$$
$$X_3 = 1*(15-11) = 4$$
$$X_4 = 6*(12-16) = -24$$
$$X_5 = 11*(1+2) = 33$$
$$X_6 = (5-1)*(16+15) = 124$$
$$X_7 = (2-6)*(12+11) = -92$$

$$C_1 = \begin{bmatrix} 189 + \text{-}24 - 33 + \text{-}92 & 4 + 33 \\ 176 + \text{-}24 & 189 + 4 - 176 + 124 \end{bmatrix} = \begin{bmatrix} 40 & 37 \\ 152 & 141 \end{bmatrix}$$

A_2B_2

$$X_1 = (3+8)*(14+9) = 253$$
$$X_2 = 14*(7+8) = 210$$
$$X_3 = 3*(13-9) = 12$$
$$X_4 = 8*(10-14) = -32$$
$$X_5 = 9*(3+4) = 63$$
$$X_6 = (7-3)*(14+13) = 108$$
$$X_7 = (4-8)*(10+9) = -76$$

$$C_2 = \begin{bmatrix} 253 + \text{-}32 - 63 + \text{-}76 & 12 + 63 \\ 210 + \text{-}32 & 253 + 12 - 210 + 108 \end{bmatrix} = \begin{bmatrix} 82 & 75 \\ 178 & 163 \end{bmatrix}$$

A_3B_3

$$X_1 = (3+8)*(14+9) = 253$$
$$X_2 = 8*(13+14) = 216$$
$$X_3 = 9*(7-3) = 36$$
$$X_4 = 14*(4-8) = -56$$
$$X_5 = 3*(9+10) = 57$$
$$X_6 = (13-9)*(8+7) = 60$$
$$X_7 = (10-14)*(4+3) = -28$$

$$C_3 = \begin{bmatrix} 253 + \text{-}56 - 57 + \text{-}28 & 36 + 57 \\ 216 + \text{-}56 & 253 + 36 - 216 + 60 \end{bmatrix} = \begin{bmatrix} 112 & 93 \\ 160 & 133 \end{bmatrix}$$

A_4B_4

$$X_1 = (1+6)*(16+11) = 189$$
$$X_2 = 6*(15+16) = 186$$
$$X_3 = 11*(5-1) = 44$$
$$X_4 = 16*(2-6) = -64$$
$$X_5 = 1*(11+12) = 23$$
$$X_6 = (15-11)*(6+5) = 44$$
$$X_7 = (12-16)*(2+1) = -12$$

$$C_4 = \begin{bmatrix} 189 + \text{-}64 - 23 + \text{-}12 & 44 + 23 \\ 186 + \text{-}64 & 189 + 44 - 186 + 44 \end{bmatrix} = \begin{bmatrix} 90 & 67 \\ 122 & 91 \end{bmatrix}$$

2. Generate a monic polynomial with $k = 4$ (that is $n = 15$) and solve it using the recursive algorithm presented in class. show the steps.

$$x^{15} + 2x^{14} + 3x^{13} + 4x^{12} + x^{11} + 2x^{10} + 3x^9 + 4x^8 + x^7 + 2x^6 + 3x^5 + 4x^4 + x^3 + 2x^2 + 3x + 4$$

$$k = 4$$
$$j = 2^{k-1} = 8$$
$$b = a_{2^{k-1}-1} - 1 = a_7 - 1 = 0$$

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$$\begin{array}{r|l} & x^7 + 2x^6 + 3x^5 + 4x^4 + x^3 + 2x^2 + 3x + 4 \\ \hline x^8 & x^{15} + 2x^{14} + 3x^{13} + 4x^{12} + x^{11} + 2x^{10} + 3x^9 + 4x^8 + x^7 + 2x^6 + 3x^5 + 4x^4 + x^3 + 2x^2 + 3x + 4 \\ & \underline{-\,x^{15}} \\ & \quad 2x^{14} \\ & \quad \underline{-\,2x^{14}} \\ & \quad\quad 3x^{13} \\ & \quad\quad \underline{-\,3x^{13}} \\ & \quad\quad\quad 4x^{12} \\ & \quad\quad\quad \underline{-\,4x^{12}} \\ & \quad\quad\quad\quad x^{11} \\ & \quad\quad\quad\quad \underline{-\,x^{11}} \\ & \quad\quad\quad\quad\quad 2x^{10} \\ & \quad\quad\quad\quad\quad \underline{-\,2x^{10}} \\ & \quad\quad\quad\quad\quad\quad 3x^9 \\ & \quad\quad\quad\quad\quad\quad \underline{-\,3x^9} \\ & \quad\quad\quad\quad\quad\quad\quad 4x^8 \\ & \quad\quad\quad\quad\quad\quad\quad \underline{-\,4x^8} \\ & \quad\quad\quad\quad\quad\quad\quad\quad x^7 + 2x^6 + 3x^5 + 4x^4 + x^3 + 2x^2 + 3x + 4 \end{array}$$

$$p(x) = (x^8)(x^7 + 2x^6 + 3x^5 + 4x^4 + x^3 + 2x^2 + 3x + 4) + (x^7 + 2x^6 + 3x^5 + 4x^4 + x^3 + 2x^2 + 3x + 4)$$

$$x^7 + 2x^6 + 3x^5 + 4x^4 + x^3 + 2x^2 + 3x + 4$$

$$k = 3$$
$$j = 2^{k-1} = 4$$
$$b = a_{2^{k-1}-1} - 1 = a_3 - 1 = 0$$

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$$\begin{array}{r|l} & x^3 + 2x^2 + 3x + 4 \\ \hline x^4 & x^7 + 2x^6 + 3x^5 + 4x^4 + x^3 + 2x^2 + 3x + 4 \\ & \underline{-\,x^7} \\ & \quad 2x^6 \\ & \underline{-\,2x^6} \\ & \quad\quad 3x^5 \\ & \quad\quad \underline{-\,3x^5} \\ & \quad\quad\quad 4x^4 \\ & \quad\quad\quad \underline{-\,4x^4} \\ & \quad\quad\quad\quad x^3 + 2x^2 + 3x + 4 \end{array}$$

$$p(x) = (x^8)((x^4)(x^3 + 2x^2 + 3x + 4) + (x^3 + 2x^2 + 3x + 4)) + (x^7 + 2x^6 + 3x^5 + 4x^4 + x^3 + 2x^2 + 3x + 4)$$

$$x^3 + 2x^2 + 3x + 4$$

$$k = 2$$
$$j = 2^{k-1} = 2$$
$$b = a_{2^{k-1}-1} - 1 = a_1 - 1 = 3$$

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$$\begin{array}{r|l} & x + 2 \\ \hline x^2 + 3 & x^3 + 2x^2 + 3x + 4 \\ & \underline{-\,x^3 \quad\quad - 3x} \\ & \quad\quad 2x^2 \quad + 4 \\ & \quad\quad \underline{-\,2x^2 \quad - 6} \\ & \quad\quad\quad\quad -\,2 \end{array}$$

$$p(x) = (x^8)((x^4)((x^2 + 3)(x + 2) + (-2)) + (x^3 + 2x^2 + 3x + 4)) + (x^7 + 2x^6 + 3x^5 + 4x^4 + x^3 + 2x^2 + 3x + 4)$$

$$p(x) = (x^8)[(x^4)((x^2+3)(x+2)+-2)+(x^2+3)(x+2)+-2]+[(x^4)((x^2+3)(x+2)+-2)+(x^2+3)(x+2)+-2]$$
$$p(x) = (x^8)[(x^4)((x^2+3)(x+2)+-2)+(x^2+3)(x+2)+-2]+(x^4)((x^2+3)(x+2)+-2)+(x^2+3)(x+2)+-2$$