Problems

1. Use the substitution method to solve the following recurrences and determine their corresponding complexity. Establish the proper initial conditions for each problem.

   (a) $T(n) = 4T(n/2) + n^4$

   $$T(n) = \begin{cases} b & \text{if } n = 1. \\ aT(\frac{n}{c}) + bn^x, & \text{otherwise.} \end{cases}$$

   $$\begin{aligned} T(n) &= 4T(n/2) + n^4 \\ &= 4(4T(\frac{n}{2^2}) + (\frac{n}{2^2})^4) + n^4 \\ &= 4^2(4T(\frac{n}{2^3}) + (\frac{n}{2^3})^4) + \frac{n^4}{4^3} + n^4 \\ &= 4^3(4T(\frac{n}{2^4}) + (\frac{n}{2^4})^4) + \frac{n^4}{4^4} + \frac{n^4}{4^3} + n^4 \\ &= 4^4(4T(\frac{n}{2^5}) + (\frac{n}{2^5})^4) + \frac{n^4}{4^5} + \frac{n^4}{4^4} + \frac{n^4}{4^3} + n^4 \end{aligned}$$

   (b) $T(n) = 3T(n/3) + n \; log \; n$

   (c) $T(n) = 3T(n/2) + \frac{\sqrt{n}}{log \; n}$

   (d) $T(n) = T(n-3) + n$