23

Electric Fields

CHAPTER OUTLINE

ANSWERS TO QUESTIONS

Q23.1 A neutral atom is one that has no net charge. This means that it has the same number of electrons orbiting the nucleus as it has protons in the nucleus. A negatively charged atom has one or more excess electrons.

Q23.2 When the comb is nearby, molecules in the paper are polarized, similar to the molecules in the wall in Figure 23.5a, and the paper is attracted. During contact, charge from the comb is transferred to the paper by conduction. Then the paper has the same charge as the comb, and is repelled.

Q23.3 The clothes dryer rubs dissimilar materials together as it tumbles the clothes. Electrons are transferred from one kind of molecule to another. The charges on pieces of cloth, or on nearby objects charged by induction, can produce strong electric fields that promote the ionization process in the surrounding air that is necessary for a spark to occur. Then you hear or see the sparks.

Q23.4 To avoid making a spark. Rubber-soled shoes acquire a charge by friction with the floor and could discharge with a spark, possibly causing an explosion of any flammable material in the oxygen-enriched atmosphere.

Q23.5 Electrons are less massive and more mobile than protons. Also, they are more easily detached from atoms than protons.

Q23.6 The electric field due to the charged rod induces charges on near and far sides of the sphere. The attractive Coulomb force of the rod on the dissimilar charge on the close side of the sphere is larger than the repulsive Coulomb force of the rod on the like charge on the far side of the sphere. The result is a net attraction of the sphere to the rod. When the sphere touches the rod, charge is conducted between the rod and the sphere, leaving both the rod and the sphere like-charged. This results in a repulsive Coulomb force.

Q23.7 All of the constituents of air are nonpolar except for water. The polar water molecules in the air quite readily "steal" charge from a charged object, as any physics teacher trying to perform electrostatics demonstrations in the summer well knows. As a result—it is difficult to accumulate large amounts of excess charge on an object in a humid climate. During a North American winter, the cold, dry air allows accumulation of significant excess charge, giving the potential (pun intended) for a shocking (pun also intended) introduction to static electricity sparks.

Q23.8 Similarities: A force of gravity is proportional to the product of the intrinsic properties (masses) of two particles, and inversely proportional to the square of the separation distance. An electrical force exhibits the same proportionalities, with charge as the intrinsic property.

Differences: The electrical force can either attract or repel, while the gravitational force as described by Newton's law can only attract. The electrical force between elementary particles is vastly stronger than the gravitational force.

Q23.9 No. The balloon induces polarization of the molecules in the wall, so that a layer of positive charge exists near the balloon. This is just like the situation in Figure 23.5a, except that the signs of the charges are reversed. The attraction between these charges and the negative charges on the balloon is stronger than the repulsion between the negative charges on the balloon and the negative charges in the polarized molecules (because they are farther from the balloon), so that there is a net attractive force toward the wall. Ionization processes in the air surrounding the balloon provide ions to which excess electrons in the balloon can transfer, reducing the charge on the balloon and eventually causing the attractive force to be insufficient to support the weight of the balloon.

Q23.10 The electric field due to the charged rod induces a charge in the aluminum foil. If the rod is brought towards the aluminum from above, the top of the aluminum will have a negative charge induced on it, while the parts draping over the pencil can have a positive charge induced on them. These positive induced charges on the two parts give rise to a repulsive Coulomb force. If the pencil is a good insulator, the net charge on the aluminum can be zero.

Q23.11 So the electric field created by the test charge does not distort the electric field you are trying to measure, by moving the charges that create it.

Q23.12 With a very high budget, you could send first a proton and then an electron into an evacuated region in which the field exists. If the field is gravitational, both particles will experience a force in the same direction, while they will experience forces in opposite directions if the field is electric.

On a more practical scale, stick identical pith balls on each end of a toothpick. Charge one pith ball + and the other –, creating a large-scale dipole. Carefully suspend this dipole about its center of mass so that it can rotate freely. When suspended in the field in question, the dipole will rotate to align itself with an electric field, while it will not for a gravitational field. If the test device does not rotate, be sure to insert it into the field in more than one orientation in case it was aligned with the electric field when you inserted it on the first trial.

Q23.13 The student standing on the insulating platform is held at the same electrical potential as the generator sphere. Charge will only flow when there is a difference in potential. The student who unwisely touches the charged sphere is near zero electrical potential when compared to the charged sphere. When the student comes in contact with the sphere, charge will flow from the sphere to him or her until they are at the same electrical potential.

Q23.14 An electric field once established by a positive or negative charge extends in all directions from the charge. Thus, it can exist in empty space if that is what surrounds the charge. There is no material at point A in Figure 23.23(a), so there is no charge, nor is there a force. There would be a force if a charge were present at point A, however. A field does exist at point A.

Q23.15 If a charge distribution is small compared to the distance of a field point from it, the charge distribution can be modeled as a single particle with charge equal to the net charge of the distribution. Further, if a charge distribution is spherically symmetric, it will create a field at exterior points just as if all of its charge were a point charge at its center.

Q23.16 The direction of the electric field is the direction in which a positive test charge would feel a force when placed in the field. A charge will not experience two electrical forces at the same time, but the vector sum of the two. If electric field lines crossed, then a test charge placed at the point at which they cross would feel a force in two directions. Furthermore, the path that the test charge would follow if released at the point where the field lines cross would be indeterminate.

Q23.17 Both figures are drawn correctly. $\mathbf{E}_1$ and $\mathbf{E}_2$ are the electric fields separately created by the point charges q_1 and q_2 in Figure 23.14 or q and $-q$ in Figure 23.15, respectively. The net electric field is the vector sum of $\mathbf{E}_1$ and $\mathbf{E}_2$, shown as $\mathbf{E}$. Figure 23.21 shows only one electric field line at each point away from the charge. At the point location of an object modeled as a point charge, the direction of the field is undefined, and so is its magnitude.

Q23.18 The electric forces on the particles have the same magnitude, but are in opposite directions. The electron will have a much larger acceleration (by a factor of about 2 000) than the proton, due to its much smaller mass.

Q23.19 The electric field around a point charge approaches infinity as r approaches zero.

Q23.20 Vertically downward.

Q23.21 Four times as many electric field lines start at the surface of the larger charge as end at the smaller charge. The extra lines extend away from the pair of charges. They may never end, or they may terminate on more distant negative charges. Figure 23.24 shows the situation for charges $+2q$ and $-q$.

Q23.22 At a point exactly midway between the two changes.

Q23.23 Linear charge density, λ, is charge per unit length. It is used when trying to determine the electric field created by a charged rod.

Surface charge density, σ, is charge per unit area. It is used when determining the electric field above a charged sheet or disk.

Volume charge density, ρ, is charge per unit volume. It is used when determining the electric field due to a uniformly charged sphere made of insulating material.

Q23.24 Yes, the path would still be parabolic. The electrical force on the electron is in the downward direction. This is similar to throwing a ball from the roof of a building horizontally or at some angle with the vertical. In both cases, the acceleration due to gravity is downward, giving a parabolic trajectory.

Q23.25 No. Life would be no different if electrons were + charged and protons were – charged. Opposite charges would still attract, and like charges would repel. The naming of + and – charge is merely a convention.

Q23.26 If the antenna were not grounded, electric charges in the atmosphere during a storm could place the antenna at a high positive or negative potential. The antenna would then place the television set inside the house at the high voltage, to make it a shock hazard. The wire to the ground keeps the antenna, the television set, and even the air around the antenna at close to zero potential.

Q23.27 People are all attracted to the Earth. If the force were electrostatic, people would all carry charge with the same sign and would repel each other. This repulsion is not observed. When we changed the charge on a person, as in the chapter-opener photograph, the person's weight would change greatly in magnitude or direction. We could levitate an airplane simply by draining away its electric charge. The failure of such experiments gives evidence that the attraction to the Earth is not due to electrical forces.

Q23.28 In special orientations the force between two dipoles can be zero or a force of repulsion. In general each dipole will exert a torque on the other, tending to align its axis with the field created by the first dipole. After this alignment, each dipole exerts a force of attraction on the other.

SOLUTIONS TO PROBLEMS

Section 23.1 **Properties of Electric Charges**

***P23.1** (a) The mass of an average neutral hydrogen atom is 1.007 9u. Losing one electron reduces its mass by a negligible amount, to

$$1.007\,9\left(1.660\times10^{-27}\text{ kg}\right)-9.11\times10^{-31}\text{ kg}=\boxed{1.67\times10^{-27}\text{ kg}}.$$

Its charge, due to loss of one electron, is

$$0-1\left(-1.60\times10^{-19}\text{ C}\right)=\boxed{+1.60\times10^{-19}\text{ C}}.$$

(b) By similar logic, charge $=\boxed{+1.60\times10^{-19}\text{ C}}$

mass $=22.99\left(1.66\times10^{-27}\text{ kg}\right)-9.11\times10^{-31}\text{ kg}=\boxed{3.82\times10^{-26}\text{ kg}}$

(c) charge of $Cl^-=\boxed{-1.60\times10^{-19}\text{ C}}$

mass $=35.453\left(1.66\times10^{-27}\text{ kg}\right)+9.11\times10^{-31}\text{ kg}=\boxed{5.89\times10^{-26}\text{ kg}}$

(d) charge of $Ca^{++}=-2\left(-1.60\times10^{-19}\text{ C}\right)=\boxed{+3.20\times10^{-19}\text{ C}}$

mass $=40.078\left(1.66\times10^{-27}\text{ kg}\right)-2\left(9.11\times10^{-31}\text{ kg}\right)=\boxed{6.65\times10^{-26}\text{ kg}}$

(e) charge of $N^{3-}=3\left(-1.60\times10^{-19}\text{ C}\right)=\boxed{-4.80\times10^{-19}\text{ C}}$

mass $=14.007\left(1.66\times10^{-27}\text{ kg}\right)+3\left(9.11\times10^{-31}\text{ kg}\right)=\boxed{2.33\times10^{-26}\text{ kg}}$

(f) charge of $N^{4+}=4\left(1.60\times10^{-19}\text{ C}\right)=\boxed{+6.40\times10^{-19}\text{ C}}$

mass $=14.007\left(1.66\times10^{-27}\text{ kg}\right)-4\left(9.11\times10^{-31}\text{ kg}\right)=\boxed{2.32\times10^{-26}\text{ kg}}$

(g) We think of a nitrogen nucleus as a seven-times ionized nitrogen atom.

charge $=7\left(1.60\times10^{-19}\text{ C}\right)=\boxed{1.12\times10^{-18}\text{ C}}$

mass $=14.007\left(1.66\times10^{-27}\text{ kg}\right)-7\left(9.11\times10^{-31}\text{ kg}\right)=\boxed{2.32\times10^{-26}\text{ kg}}$

(h) charge $=\boxed{-1.60\times10^{-19}\text{ C}}$

mass $=\left[2(1.007\,9)+15.999\right]1.66\times10^{-27}\text{ kg}+9.11\times10^{-31}\text{ kg}=\boxed{2.99\times10^{-26}\text{ kg}}$

P23.2 (a) $$N = \left(\frac{10.0 \text{ grams}}{107.87 \text{ grams/mol}}\right)\left(6.02 \times 10^{23} \frac{\text{atoms}}{\text{mol}}\right)\left(47 \frac{\text{electrons}}{\text{atom}}\right) = \boxed{2.62 \times 10^{24}}$$

(b) $$\# \text{ electrons added} = \frac{Q}{e} = \frac{1.00 \times 10^{-3} \text{ C}}{1.60 \times 10^{-19} \text{ C/electron}} = 6.25 \times 10^{15}$$

or $\boxed{2.38 \text{ electrons for every } 10^9 \text{ already present}}$.

Section 23.2 **Charging Objects by Induction**

Section 23.3 **Coulomb's Law**

P23.3 If each person has a mass of ≈ 70 kg and is (almost) composed of water, then each person contains

$$N \cong \left(\frac{70\,000 \text{ grams}}{18 \text{ grams/mol}}\right)\left(6.02 \times 10^{23} \frac{\text{molecules}}{\text{mol}}\right)\left(10 \frac{\text{protons}}{\text{molecule}}\right) \cong 2.3 \times 10^{28} \text{ protons}.$$

With an excess of 1% electrons over protons, each person has a charge

$$q = 0.01\left(1.6 \times 10^{-19} \text{ C}\right)\left(2.3 \times 10^{28}\right) = 3.7 \times 10^{7} \text{ C}.$$

So $$F = k_e \frac{q_1 q_2}{r^2} = \left(9 \times 10^9\right)\frac{\left(3.7 \times 10^7\right)^2}{0.6^2} \text{ N} = 4 \times 10^{25} \text{ N} \boxed{\sim 10^{26} \text{ N}}.$$

This force is almost enough to lift a weight equal to that of the Earth:

$$Mg = 6 \times 10^{24} \text{ kg}\left(9.8 \text{ m/s}^2\right) = 6 \times 10^{25} \text{ N} \sim 10^{26} \text{ N}.$$

***P23.4** The force on one proton is $\mathbf{F} = \frac{k_e q_1 q_2}{r^2}$ away from the other proton. Its magnitude is

$$\left(8.99 \times 10^9 \text{ N}\cdot\text{m/C}^2\right)\left(\frac{1.6 \times 10^{-19} \text{ C}}{2 \times 10^{-15} \text{ m}}\right)^2 = \boxed{57.5 \text{ N}}.$$

P23.5 (a) $$F_e = \frac{k_e q_1 q_2}{r^2} = \frac{\left(8.99 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2\right)\left(1.60 \times 10^{-19} \text{ C}\right)^2}{\left(3.80 \times 10^{-10} \text{ m}\right)^2} = \boxed{1.59 \times 10^{-9} \text{ N}} \text{ (repulsion)}$$

(b) $$F_g = \frac{G m_1 m_2}{r^2} = \frac{\left(6.67 \times 10^{-11} \text{ N}\cdot\text{m}^2/\text{C}^2\right)\left(1.67 \times 10^{-27} \text{ kg}\right)^2}{\left(3.80 \times 10^{-10} \text{ m}\right)^2} = \boxed{1.29 \times 10^{-45} \text{ N}}$$

The electric force is $\boxed{\text{larger by } 1.24 \times 10^{36} \text{ times}}$.

(c) If $k_e \frac{q_1 q_2}{r^2} = G \frac{m_1 m_2}{r^2}$ with $q_1 = q_2 = q$ and $m_1 = m_2 = m$, then

$$\frac{q}{m} = \sqrt{\frac{G}{k_e}} = \sqrt{\frac{6.67 \times 10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2}{8.99 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2}} = \boxed{8.61 \times 10^{-11} \text{ C/kg}}.$$

P23.6 We find the equal-magnitude charges on both spheres:

$$F = k_e \frac{q_1 q_2}{r^2} = k_e \frac{q^2}{r^2} \quad \text{so} \quad q = r\sqrt{\frac{F}{k_e}} = (1.00 \text{ m})\sqrt{\frac{1.00 \times 10^4 \text{ N}}{8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2}} = 1.05 \times 10^{-3} \text{ C}.$$

The number of electron transferred is then

$$N_{\text{xfer}} = \frac{1.05 \times 10^{-3} \text{ C}}{1.60 \times 10^{-19} \text{ C}/e^-} = 6.59 \times 10^{15} \text{ electrons}.$$

The whole number of electrons in each sphere is

$$N_{\text{tot}} = \left(\frac{10.0 \text{ g}}{107.87 \text{ g/mol}}\right)\left(6.02 \times 10^{23} \text{ atoms/mol}\right)\left(47 \ e^-/\text{atom}\right) = 2.62 \times 10^{24} \ e^-.$$

The fraction transferred is then

$$f = \frac{N_{\text{xfer}}}{N_{\text{tot}}} = \left(\frac{6.59 \times 10^{15}}{2.62 \times 10^{24}}\right) = \boxed{2.51 \times 10^{-9}} = 2.51 \text{ charges in every billion.}$$

P23.7

$$F_1 = k_e \frac{q_1 q_2}{r^2} = \frac{\left(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2\right)\left(7.00 \times 10^{-6} \text{ C}\right)\left(2.00 \times 10^{-6} \text{ C}\right)}{(0.500 \text{ m})^2} = 0.503 \text{ N}$$

$$F_2 = k_e \frac{q_1 q_2}{r^2} = \frac{\left(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2\right)\left(7.00 \times 10^{-6} \text{ C}\right)\left(4.00 \times 10^{-6} \text{ C}\right)}{(0.500 \text{ m})^2} = 1.01 \text{ N}$$

$$F_x = 0.503 \cos 60.0° + 1.01 \cos 60.0° = 0.755 \text{ N}$$
$$F_y = 0.503 \sin 60.0° - 1.01 \sin 60.0° = -0.436 \text{ N}$$
$$\mathbf{F} = (0.755 \text{ N})\hat{\mathbf{i}} - (0.436 \text{ N})\hat{\mathbf{j}} = \boxed{0.872 \text{ N at an angle of } 330°}$$

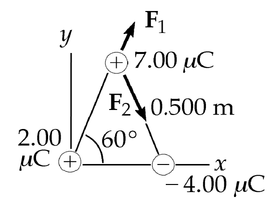


FIG. P23.7

P23.8

$$F = k_e \frac{q_1 q_2}{r^2} = \frac{\left(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2\right)\left(1.60 \times 10^{-19} \text{ C}\right)^2\left(6.02 \times 10^{23}\right)^2}{\left[2\left(6.37 \times 10^6 \text{ m}\right)\right]^2} = \boxed{514 \text{ kN}}$$

P23.9 (a) The force is one of $\boxed{\text{attraction}}$. The distance r in Coulomb's law is the distance between centers. The magnitude of the force is

$$F = \frac{k_e q_1 q_2}{r^2} = \left(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2\right)\frac{\left(12.0 \times 10^{-9} \text{ C}\right)\left(18.0 \times 10^{-9} \text{ C}\right)}{(0.300 \text{ m})^2} = \boxed{2.16 \times 10^{-5} \text{ N}}.$$

(b) The net charge of -6.00×10^{-9} C will be equally split between the two spheres, or -3.00×10^{-9} C on each. The force is one of $\boxed{\text{repulsion}}$, and its magnitude is

$$F = \frac{k_e q_1 q_2}{r^2} = \left(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2\right)\frac{\left(3.00 \times 10^{-9} \text{ C}\right)\left(3.00 \times 10^{-9} \text{ C}\right)}{(0.300 \text{ m})^2} = \boxed{8.99 \times 10^{-7} \text{ N}}.$$

P23.10 Let the third bead have charge Q and be located distance x from the left end of the rod. This bead will experience a net force given by

$$\mathbf{F} = \frac{k_e(3q)Q}{x^2}\hat{\mathbf{i}} + \frac{k_e(q)Q}{(d-x)^2}\left(-\hat{\mathbf{i}}\right).$$

The net force will be zero if $\frac{3}{x^2} = \frac{1}{(d-x)^2}$, or $d - x = \frac{x}{\sqrt{3}}$.

This gives an equilibrium position of the third bead of $x = \boxed{0.634d}$.

The equilibrium is $\boxed{\text{stable if the third bead has positive charge}}$.

P23.11 (a) $$F = \frac{k_e e^2}{r^2} = \left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\frac{\left(1.60\times10^{-19}\ \text{C}\right)^2}{\left(0.529\times10^{-10}\ \text{m}\right)^2} = \boxed{8.22\times10^{-8}\ \text{N}}$$

(b) We have $F = \frac{mv^2}{r}$ from which

$$v = \sqrt{\frac{Fr}{m}} = \sqrt{\frac{8.22\times10^{-8}\ \text{N}\left(0.529\times10^{-10}\ \text{m}\right)}{9.11\times10^{-31}\ \text{kg}}} = \boxed{2.19\times10^6\ \text{m/s}}.$$

P23.12 The top charge exerts a force on the negative charge $\frac{k_e qQ}{\left(\frac{d}{2}\right)^2 + x^2}$ which is directed upward and to the left, at an angle of $\tan^{-1}\left(\frac{d}{2x}\right)$ to the x-axis. The two positive charges together exert force

$$\left(\frac{2k_e qQ}{\left(\frac{d^2}{4}+x^2\right)}\right)\left(\frac{(-x)\hat{\mathbf{i}}}{\left(\frac{d^2}{4}+x^2\right)^{1/2}}\right) = m\mathbf{a} \text{ or for } x \ll \frac{d}{2},\ \mathbf{a} \approx \frac{-2k_e qQ}{md^3/8}\mathbf{x}.$$

(a) The acceleration is equal to a negative constant times the excursion from equilibrium, as in $\mathbf{a} = -\omega^2\mathbf{x}$, so we have Simple Harmonic Motion with $\omega^2 = \frac{16k_e qQ}{md^3}$.

$T = \frac{2\pi}{\omega} = \boxed{\frac{\pi}{2}\sqrt{\frac{md^3}{k_e qQ}}}$, where m is the mass of the object with charge $-Q$.

(b) $$v_{\max} = \omega A = \boxed{4a\sqrt{\frac{k_e qQ}{md^3}}}$$

Section 23.4 **The Electric Field**

P23.13 For equilibrium, $\mathbf{F}_e = -\mathbf{F}_g$

or $q\mathbf{E} = -mg\left(-\hat{\mathbf{j}}\right)$.

Thus, $\mathbf{E} = \dfrac{mg}{q}\hat{\mathbf{j}}$.

(a) $$\mathbf{E} = \frac{mg}{q}\hat{\mathbf{j}} = \frac{\left(9.11\times10^{-31}\text{ kg}\right)\left(9.80\text{ m/s}^2\right)}{\left(-1.60\times10^{-19}\text{ C}\right)}\hat{\mathbf{j}} = \boxed{-\left(5.58\times10^{-11}\text{ N/C}\right)\hat{\mathbf{j}}}$$

(b) $$\mathbf{E} = \frac{mg}{q}\hat{\mathbf{j}} = \frac{\left(1.67\times10^{-27}\text{ kg}\right)\left(9.80\text{ m/s}^2\right)}{\left(1.60\times10^{-19}\text{ C}\right)}\hat{\mathbf{j}} = \boxed{\left(1.02\times10^{-7}\text{ N/C}\right)\hat{\mathbf{j}}}$$

P23.14 $\sum F_y = 0:\ QE\hat{\mathbf{j}} + mg\left(-\hat{\mathbf{j}}\right) = 0$

$$\therefore m = \frac{QE}{g} = \frac{\left(24.0\times10^{-6}\text{ C}\right)\left(610\text{ N/C}\right)}{9.80\text{ m/s}^2} = \boxed{1.49\text{ grams}}$$

P23.15 The point is designated in the sketch. The magnitudes of the electric fields, E_1, (due to the -2.50×10^{-6} C charge) and E_2 (due to the 6.00×10^{-6} C charge) are

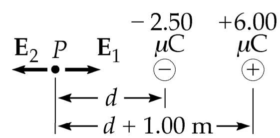


FIG. P23.15

$$E_1 = \frac{k_e q}{r^2} = \frac{\left(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2\right)\left(2.50\times10^{-6}\text{ C}\right)}{d^2} \qquad (1)$$

$$E_2 = \frac{k_e q}{r^2} = \frac{\left(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2\right)\left(6.00\times10^{-6}\text{ C}\right)}{(d+1.00\text{ m})^2} \qquad (2)$$

Equate the right sides of (1) and (2)

to get $(d+1.00\text{ m})^2 = 2.40d^2$

or $d + 1.00\text{ m} = \pm1.55d$

which yields $d = 1.82$ m

or $d = -0.392$ m.

The negative value for d is unsatisfactory because that locates a point between the charges where both fields are in the same direction.

Thus, $d = \boxed{1.82\text{ m to the left of the }-2.50\ \mu\text{C charge}}$.

P23.16 If we treat the concentrations as point charges,

$$\mathbf{E}_+ = k_e \frac{q}{r^2} = \left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\frac{(40.0\ \text{C})}{(1\,000\ \text{m})^2}\left(-\hat{\mathbf{j}}\right) = 3.60\times10^5\ \text{N/C}\left(-\hat{\mathbf{j}}\right)(\text{downward})$$

$$\mathbf{E}_- = k_e \frac{q}{r^2} = \left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\frac{(40.0\ \text{C})}{(1\,000\ \text{m})^2}\left(-\hat{\mathbf{j}}\right) = 3.60\times10^5\ \text{N/C}\left(-\hat{\mathbf{j}}\right)(\text{downward})$$

$$\mathbf{E} = \mathbf{E}_+ + \mathbf{E}_- = \boxed{7.20\times10^5\ \text{N/C downward}}$$

***P23.17** The first charge creates at the origin field $\frac{k_eQ}{a^2}$ to the right.

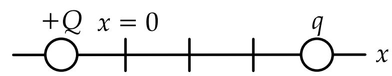


FIG. P23.17

Suppose the total field at the origin is to the right. Then q must be negative:

$$\frac{k_eQ}{a^2}\hat{\mathbf{i}} + \frac{k_eq}{(3a)^2}\left(-\hat{\mathbf{i}}\right) = \frac{2k_eQ}{a^2}\hat{\mathbf{i}} \qquad \boxed{q=-9Q}.$$

In the alternative, the total field at the origin is to the left:

$$\frac{k_eQ}{a^2}\hat{\mathbf{i}} + \frac{k_eq}{9a^2}\left(-\hat{\mathbf{i}}\right) = \frac{2k_eQ}{a^2}\left(-\hat{\mathbf{i}}\right) \qquad \boxed{q=+27Q}.$$

P23.18 (a)

$$E_1 = \frac{k_eq}{r^2} = \frac{\left(8.99\times10^9\right)\left(7.00\times10^{-6}\right)}{(0.500)^2} = 2.52\times10^5\ \text{N/C}$$

$$E_2 = \frac{k_eq}{r^2} = \frac{\left(8.99\times10^9\right)\left(4.00\times10^{-6}\right)}{(0.500)^2} = 1.44\times10^5\ \text{N/C}$$

$$E_x = E_2 - E_1\cos60° = 1.44\times10^5 - 2.52\times10^5\cos60.0° = 18.0\times10^3\ \text{N/C}$$

$$E_y = -E_1\sin60.0° = -2.52\times10^5\sin60.0° = -218\times10^3\ \text{N/C}$$

$$\mathbf{E} = \left[18.0\hat{\mathbf{i}} - 218\hat{\mathbf{j}}\right]\times10^3\ \text{N/C} = \boxed{\left[18.0\hat{\mathbf{i}} - 218\hat{\mathbf{j}}\right]\ \text{kN/C}}$$

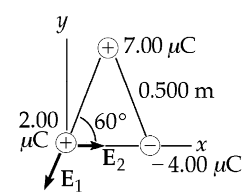


FIG. P23.18

(b)

$$\mathbf{F} = q\mathbf{E} = \left(2.00\times10^{-6}\ \text{C}\right)\left(18.0\hat{\mathbf{i}} - 218\hat{\mathbf{j}}\right)\times10^3\ \text{N/C} = \left(36.0\hat{\mathbf{i}} - 436\hat{\mathbf{j}}\right)\times10^{-3}\ \text{N} = \boxed{\left(36.0\hat{\mathbf{i}} - 436\hat{\mathbf{j}}\right)\ \text{mN}}$$

P23.19 (a)

$$\mathbf{E}_1 = \frac{k_e|q_1|}{r_1^2}\left(-\hat{\mathbf{j}}\right) = \frac{\left(8.99\times10^9\right)\left(3.00\times10^{-9}\right)}{(0.100)^2}\left(-\hat{\mathbf{j}}\right) = -\left(2.70\times10^3\ \text{N/C}\right)\hat{\mathbf{j}}$$

$$\mathbf{E}_2 = \frac{k_e|q_2|}{r_2^2}\left(-\hat{\mathbf{i}}\right) = \frac{\left(8.99\times10^9\right)\left(6.00\times10^{-9}\right)}{(0.300)^2}\left(-\hat{\mathbf{i}}\right) = -\left(5.99\times10^2\ \text{N/C}\right)\hat{\mathbf{i}}$$

$$\mathbf{E} = \mathbf{E}_2 + \mathbf{E}_1 = \boxed{-\left(5.99\times10^2\ \text{N/C}\right)\hat{\mathbf{i}} - \left(2.70\times10^3\ \text{N/C}\right)\hat{\mathbf{j}}}$$

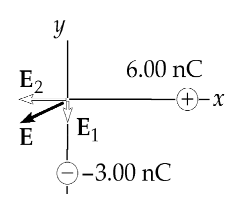


FIG. P23.19

(b)

$$\mathbf{F} = q\mathbf{E} = \left(5.00\times10^{-9}\ \text{C}\right)\left(-599\hat{\mathbf{i}} - 2\,700\hat{\mathbf{j}}\right)\ \text{N/C}$$

$$\mathbf{F} = \left(-3.00\times10^{-6}\hat{\mathbf{i}} - 13.5\times10^{-6}\hat{\mathbf{j}}\right)\ \text{N} = \boxed{\left(-3.00\hat{\mathbf{i}} - 13.5\hat{\mathbf{j}}\right)\ \mu\text{N}}$$

P23.20 (a) $E = \frac{k_e q}{r^2} = \frac{(8.99\times10^9)(2.00\times10^{-6})}{(1.12)^2} = 14\,400\ \text{N/C}$

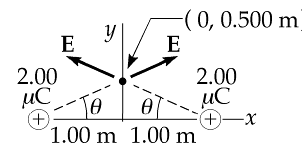


FIG. P23.20

$E_x = 0$ and $E_y = 2(14\,400)\sin 26.6° = 1.29\times10^4\ \text{N/C}$

so $\boxed{\mathbf{E} = 1.29\times10^4\,\hat{\mathbf{j}}\ \text{N/C}}$.

(b) $\mathbf{F} = q\mathbf{E} = (-3.00\times10^{-6})(1.29\times10^4\,\hat{\mathbf{j}}) = \boxed{-3.86\times10^{-2}\,\hat{\mathbf{j}}\ \text{N}}$

P23.21 (a) $\mathbf{E} = \frac{k_e q_1}{r_1^2}\hat{\mathbf{r}}_1 + \frac{k_e q_2}{r_2^2}\hat{\mathbf{r}}_2 + \frac{k_e q_3}{r_3^2}\hat{\mathbf{r}}_3 = \frac{k_e(2q)}{a^2}\hat{\mathbf{i}} + \frac{k_e(3q)}{2a^2}(\hat{\mathbf{i}}\cos 45.0° + \hat{\mathbf{j}}\sin 45.0°) + \frac{k_e(4q)}{a^2}\hat{\mathbf{j}}$

$\mathbf{E} = 3.06\frac{k_e q}{a^2}\hat{\mathbf{i}} + 5.06\frac{k_e q}{a^2}\hat{\mathbf{j}} = \boxed{5.91\frac{k_e q}{a^2}\ \text{at}\ 58.8°}$

(b) $\mathbf{F} = q\mathbf{E} = \boxed{5.91\frac{k_e q^2}{a^2}\ \text{at}\ 58.8°}$

P23.22 The electric field at any point x is

$$E = \frac{k_e q}{(x-a)^2} - \frac{k_e q}{(x-(-a))^2} = \frac{k_e q(4ax)}{(x^2-a^2)^2}.$$

When x is much, much greater than a, we find $E \cong \boxed{\frac{4a(k_e q)}{x^3}}$.

P23.23 (a) One of the charges creates at P a field $\mathbf{E} = \frac{k_e Q/n}{R^2+x^2}$ at an angle θ to the x-axis as shown.

When all the charges produce field, for $n > 1$, the components perpendicular to the x-axis add to zero.

The total field is $\frac{nk_e(Q/n)\hat{\mathbf{i}}}{R^2+x^2}\cos\theta = \boxed{\frac{k_e Qx\hat{\mathbf{i}}}{(R^2+x^2)^{3/2}}}$.

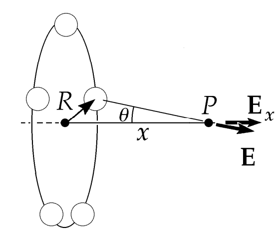


FIG. P23.23

(b) A circle of charge corresponds to letting n grow beyond all bounds, but the result does not depend on n. Smearing the charge around the circle does not change its amount or its distance from the field point, so it $\boxed{\text{does not change the field}}$.

P23.24 $\mathbf{E} = \sum\frac{k_e q}{r^2}\hat{\mathbf{r}} = \frac{k_e q}{a^2}(-\hat{\mathbf{i}}) + \frac{k_e q}{(2a)^2}(-\hat{\mathbf{i}}) + \frac{k_e q}{(3a)^2}(-\hat{\mathbf{i}}) + \ldots = \frac{-k_e q\hat{\mathbf{i}}}{a^2}\left(1 + \frac{1}{2^2} + \frac{1}{3^2} + \ldots\right) = \boxed{-\frac{\pi^2 k_e q}{6a^2}\hat{\mathbf{i}}}$

Section 23.5 **Electric Field of a Continuous Charge Distribution**

P23.25 $$E=\frac{k_e\lambda\ell}{d(\ell+d)}=\frac{k_e(Q/\ell)\ell}{d(\ell+d)}=\frac{k_eQ}{d(\ell+d)}=\frac{\left(8.99\times10^9\right)\left(22.0\times10^{-6}\right)}{(0.290)(0.140+0.290)}$$

$$\mathbf{E}=\boxed{1.59\times10^6\ \text{N/C, directed toward the rod.}}$$

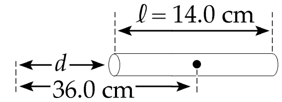


FIG. P23.25

P23.26 $E=\int\frac{k_e dq}{x^2}$, where $dq=\lambda_0 dx$

$$E=k_e\lambda_0\int_{x_0}^{\infty}\frac{dx}{x^2}=k_e\lambda_0\left(-\frac{1}{x}\right)\Bigg|_{x_0}^{\infty}=\boxed{\frac{k_e\lambda_0}{x_0}}$$

$\boxed{\text{The direction is } -\hat{\mathbf{i}} \text{ or left for } \lambda_0>0}$

P23.27 $$E=\frac{k_exQ}{\left(x^2+a^2\right)^{3/2}}=\frac{\left(8.99\times10^9\right)\left(75.0\times10^{-6}\right)x}{\left(x^2+0.100^2\right)^{3/2}}=\frac{6.74\times10^5\,x}{\left(x^2+0.010\,0\right)^{3/2}}$$

(a) At $x=0.010\,0$ m, $\mathbf{E}=6.64\times10^6\,\hat{\mathbf{i}}$ N/C $=\boxed{6.64\hat{\mathbf{i}}\ \text{MN/C}}$

(b) At $x=0.050\,0$ m, $\mathbf{E}=2.41\times10^7\,\hat{\mathbf{i}}$ N/C $=\boxed{24.1\hat{\mathbf{i}}\ \text{MN/C}}$

(c) At $x=0.300$ m, $\mathbf{E}=6.40\times10^6\,\hat{\mathbf{i}}$ N/C $=\boxed{6.40\hat{\mathbf{i}}\ \text{MN/C}}$

(d) At $x=1.00$ m, $\mathbf{E}=6.64\times10^5\,\hat{\mathbf{i}}$ N/C $=\boxed{0.664\hat{\mathbf{i}}\ \text{MN/C}}$

P23.28 $$\mathbf{E}=\int d\mathbf{E}=\int_{x_0}^{\infty}\left[\frac{k_e\lambda_0x_0dx\left(-\hat{\mathbf{i}}\right)}{x^3}\right]=-k_e\lambda_0x_0\hat{\mathbf{i}}\int_{x_0}^{\infty}x^{-3}dx=-k_e\lambda_0x_0\hat{\mathbf{i}}\left(-\frac{1}{2x^2}\Bigg|_{x_0}^{\infty}\right)=\boxed{\frac{k_e\lambda_0}{2x_0}\left(-\hat{\mathbf{i}}\right)}$$

P23.29 $$E=\frac{k_eQx}{\left(x^2+a^2\right)^{3/2}}$$

For a maximum, $$\frac{dE}{dx}=Qk_e\left[\frac{1}{\left(x^2+a^2\right)^{3/2}}-\frac{3x^2}{\left(x^2+a^2\right)^{5/2}}\right]=0$$

$x^2+a^2-3x^2=0$ or $x=\frac{a}{\sqrt{2}}$.

Substituting into the expression for E gives

$$E=\frac{k_eQa}{\sqrt{2}\left(\frac{3}{2}a^2\right)^{3/2}}=\frac{k_eQ}{3\frac{\sqrt{3}}{2}a^2}=\boxed{\frac{2k_eQ}{3\sqrt{3}a^2}}=\boxed{\frac{Q}{6\sqrt{3}\pi\in_0a^2}}.$$

P23.30 $E = 2\pi k_e \sigma\left(1 - \frac{x}{\sqrt{x^2 + R^2}}\right)$

$$E = 2\pi\left(8.99\times10^{9}\right)\left(7.90\times10^{-3}\right)\left(1 - \frac{x}{\sqrt{x^2 + (0.350)^2}}\right) = 4.46\times10^{8}\left(1 - \frac{x}{\sqrt{x^2 + 0.123}}\right)$$

(a) At $x = 0.050\,0\text{ m}$, $E = 3.83\times10^{8}\text{ N/C} = \boxed{383\text{ MN/C}}$

(b) At $x = 0.100\text{ m}$, $E = 3.24\times10^{8}\text{ N/C} = \boxed{324\text{ MN/C}}$

(c) At $x = 0.500\text{ m}$, $E = 8.07\times10^{7}\text{ N/C} = \boxed{80.7\text{ MN/C}}$

(d) At $x = 2.00\text{ m}$, $E = 6.68\times10^{8}\text{ N/C} = \boxed{6.68\text{ MN/C}}$

P23.31 (a) From Example 23.9: $E = 2\pi k_e \sigma\left(1 - \frac{x}{\sqrt{x^2 + R^2}}\right)$

$\sigma = \frac{Q}{\pi R^2} = 1.84\times10^{-3}\text{ C/m}^2$

$E = \left(1.04\times10^{8}\text{ N/C}\right)(0.900) = 9.36\times10^{7}\text{ N/C} = \boxed{93.6\text{ MN/C}}$

appx: $E = 2\pi k_e \sigma = \boxed{104\text{ MN/C (about 11\% high)}}$

(b) $E = \left(1.04\times10^{8}\text{ N/C}\right)\left(1 - \frac{30.0\text{ cm}}{\sqrt{30.0^2 + 3.00^2}\text{ cm}}\right) = \left(1.04\times10^{8}\text{ N/C}\right)(0.004\,96) = \boxed{0.516\text{ MN/C}}$

appx: $E = k_e \frac{Q}{r^2} = \left(8.99\times10^{9}\right)\frac{5.20\times10^{-6}}{(0.30)^2} = \boxed{0.519\text{ MN/C (about 0.6\% high)}}$

P23.32 The electric field at a distance x is $E_x = 2\pi k_e \sigma\left[1 - \frac{x}{\sqrt{x^2 + R^2}}\right]$

This is equivalent to $E_x = 2\pi k_e \sigma\left[1 - \frac{1}{\sqrt{1 + R^2/x^2}}\right]$

For large x, $\frac{R^2}{x^2} << 1$ and $\sqrt{1 + \frac{R^2}{x^2}} \approx 1 + \frac{R^2}{2x^2}$

so $E_x = 2\pi k_e \sigma\left(1 - \frac{1}{\left[1 + R^2/\left(2x^2\right)\right]}\right) = 2\pi k_e \sigma \frac{\left(1 + R^2/\left(2x^2\right) - 1\right)}{\left[1 + R^2/\left(2x^2\right)\right]}$

Substitute $\sigma = \frac{Q}{\pi R^2}$, $E_x = \frac{k_e Q\left(1/x^2\right)}{\left[1 + R^2/\left(2x^2\right)\right]} = k_e Q\left(x^2 + \frac{R^2}{2}\right)$

But for $x >> R$, $\frac{1}{x^2 + R^2/2} \approx \frac{1}{x^2}$, so $\boxed{E_x \approx \frac{k_e Q}{x^2}\text{ for a disk at large distances}}$

P23.33 Due to symmetry $E_y = \int dE_y = 0$, and $E_x = \int dE \sin\theta = k_e \int \frac{dq \sin\theta}{r^2}$

where $dq = \lambda ds = \lambda r d\theta$,

so that, $E_x = \frac{k_e \lambda}{r}\int_0^\pi \sin\theta d\theta = \frac{k_e\lambda}{r}(-\cos\theta)\Big|_0^\pi = \frac{2k_e\lambda}{r}$

where $\lambda = \frac{q}{L}$ and $r = \frac{L}{\pi}$.

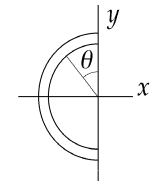


FIG. P23.33

Thus, $E_x = \frac{2k_e q\pi}{L^2} = \frac{2(8.99\times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(7.50\times 10^{-6}\ \text{C})\pi}{(0.140\ \text{m})^2}$.

Solving, $E_x = 2.16\times 10^7\ \text{N/C}$.

Since the rod has a negative charge, $\mathbf{E} = \left(-2.16\times 10^7\,\hat{\mathbf{i}}\right)\ \text{N/C} = \boxed{-21.6\,\hat{\mathbf{i}}\ \text{MN/C}}$.

P23.34 (a) We define $x = 0$ at the point where we are to find the field. One ring, with thickness dx, has charge $\frac{Qdx}{h}$ and produces, at the chosen point, a field

$$d\mathbf{E} = \frac{k_e x}{\left(x^2+R^2\right)^{3/2}}\frac{Qdx}{h}\hat{\mathbf{i}}.$$

The total field is

$$\mathbf{E} = \int_{\text{all charge}} d\mathbf{E} = \int_d^{d+h}\frac{k_e Qxdx}{h\left(x^2+R^2\right)^{3/2}}\hat{\mathbf{i}} = \frac{k_e Q\hat{\mathbf{i}}}{2h}\int_{x=d}^{d+h}\left(x^2+R^2\right)^{-3/2}2xdx$$

$$\mathbf{E} = \frac{k_e Q\hat{\mathbf{i}}}{2h}\frac{\left(x^2+R^2\right)^{-1/2}}{(-1/2)}\Bigg|_{x=d}^{d+h} = \boxed{\frac{k_e Q\hat{\mathbf{i}}}{h}\left[\frac{1}{\left(d^2+R^2\right)^{1/2}} - \frac{1}{\left((d+h)^2+R^2\right)^{1/2}}\right]}$$

(b) Think of the cylinder as a stack of disks, each with thickness dx, charge $\frac{Qdx}{h}$, and charge-per-area $\sigma = \frac{Qdx}{\pi R^2 h}$. One disk produces a field

$$d\mathbf{E} = \frac{2\pi k_e Qdx}{\pi R^2 h}\left(1 - \frac{x}{\left(x^2+R^2\right)^{1/2}}\right)\hat{\mathbf{i}}.$$

So, $$\mathbf{E} = \int_{\text{all charge}} d\mathbf{E} = \int_{x=d}^{d+h}\frac{2k_e Qdx}{R^2 h}\left(1 - \frac{x}{\left(x^2+R^2\right)^{1/2}}\right)\hat{\mathbf{i}}$$

$$\mathbf{E} = \frac{2k_e Q\hat{\mathbf{i}}}{R^2 h}\left[\int_d^{d+h}dx - \frac{1}{2}\int_{x=d}^{d+h}\left(x^2+R^2\right)^{-1/2}2xdx\right] = \frac{2k_e Q\hat{\mathbf{i}}}{R^2 h}\left[x\Big|_d^{d+h} - \frac{1}{2}\frac{\left(x^2+R^2\right)^{1/2}}{1/2}\Bigg|_d^{d+h}\right]$$

$$\mathbf{E} = \frac{2k_e Q\hat{\mathbf{i}}}{R^2 h}\left[d+h-d-\left((d+h)^2+R^2\right)^{1/2}+\left(d^2+R^2\right)^{1/2}\right]$$

$$\mathbf{E} = \boxed{\frac{2k_e Q\hat{\mathbf{i}}}{R^2 h}\left[h+\left(d^2+R^2\right)^{1/2}-\left((d+h)^2+R^2\right)^{1/2}\right]}$$

P23.35 (a) The electric field at point P due to each element of length dx, is $dE = \dfrac{k_e dq}{x^2+y^2}$ and is directed along the line joining the element to point P. By symmetry,

$E_x = \int dE_x = 0$ and since $dq = \lambda dx$,

$E = E_y = \int dE_y = \int dE\cos\theta$ where $\cos\theta = \dfrac{y}{\sqrt{x^2+y^2}}$.

Therefore, $E = 2k_e\lambda y \displaystyle\int_0^{\ell/2} \frac{dx}{\left(x^2+y^2\right)^{3/2}} = \boxed{\frac{2k_e\lambda\sin\theta_0}{y}}$.

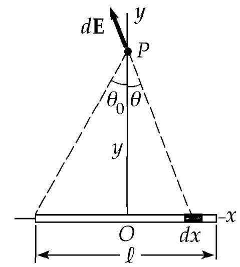


FIG. P23.35

(b) For a bar of infinite length, $\theta_0 = 90°$ and $E_y = \boxed{\dfrac{2k_e\lambda}{y}}$.

P23.36 (a) The whole surface area of the cylinder is $A = 2\pi r^2 + 2\pi rL = 2\pi r(r+L)$.

$Q = \sigma A = \left(15.0\times10^{-9}\ \text{C/m}^2\right)2\pi(0.025\,0\ \text{m})\left[0.025\,0\ \text{m} + 0.060\,0\ \text{m}\right] = \boxed{2.00\times10^{-10}\ \text{C}}$

(b) For the curved lateral surface only, $A = 2\pi rL$.

$Q = \sigma A = \left(15.0\times10^{-9}\ \text{C/m}^2\right)\left[2\pi(0.025\,0\ \text{m})(0.060\,0\ \text{m})\right] = \boxed{1.41\times10^{-10}\ \text{C}}$

(c) $Q = \rho V = \rho\pi r^2 L = \left(500\times10^{-9}\ \text{C/m}^3\right)\left[\pi(0.025\,0\ \text{m})^2(0.060\,0\ \text{m})\right] = \boxed{5.89\times10^{-11}\ \text{C}}$

P23.37 (a) Every object has the same volume, $V = 8(0.030\,0\ \text{m})^3 = 2.16\times10^{-4}\ \text{m}^3$.

For each, $Q = \rho V = \left(400\times10^{-9}\ \text{C/m}^3\right)\left(2.16\times10^{-4}\ \text{m}^3\right) = \boxed{8.64\times10^{-11}\ \text{C}}$

(b) We must count the $9.00\ \text{cm}^2$ squares painted with charge:

(i) $6\times4 = 24$ squares

$Q = \sigma A = \left(15.0\times10^{-9}\ \text{C/m}^2\right)24.0\left(9.00\times10^{-4}\ \text{m}^2\right) = \boxed{3.24\times10^{-10}\ \text{C}}$

(ii) 34 squares exposed

$Q = \sigma A = \left(15.0\times10^{-9}\ \text{C/m}^2\right)34.0\left(9.00\times10^{-4}\ \text{m}^2\right) = \boxed{4.59\times10^{-10}\ \text{C}}$

(iii) 34 squares

$Q = \sigma A = \left(15.0\times10^{-9}\ \text{C/m}^2\right)34.0\left(9.00\times10^{-4}\ \text{m}^2\right) = \boxed{4.59\times10^{-10}\ \text{C}}$

(iv) 32 squares

$Q = \sigma A = \left(15.0\times10^{-9}\ \text{C/m}^2\right)32.0\left(9.00\times10^{-4}\ \text{m}^2\right) = \boxed{4.32\times10^{-10}\ \text{C}}$

(c) (i) total edge length: $\ell = 24\times(0.030\,0\ \text{m})$

$Q = \lambda\ell = \left(80.0\times10^{-12}\ \text{C/m}\right)24\times(0.030\,0\ \text{m}) = \boxed{5.76\times10^{-11}\ \text{C}}$

(ii) $Q = \lambda\ell = \left(80.0\times10^{-12}\ \text{C/m}\right)44\times(0.030\,0\ \text{m}) = \boxed{1.06\times10^{-10}\ \text{C}}$

continued on next page

(iii) $Q = \lambda\ell = \left(80.0\times10^{-12}\ \text{C/m}\right)64\times\left(0.030\,0\ \text{m}\right) = \boxed{1.54\times10^{-10}\ \text{C}}$

(iv) $Q = \lambda\ell = \left(80.0\times10^{-12}\ \text{C/m}\right)40\times\left(0.030\,0\ \text{m}\right) = \boxed{0.960\times10^{-10}\ \text{C}}$

Section 23.6 **Electric Field Lines**

P23.38

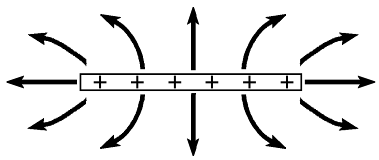


FIG. P23.38

P23.39

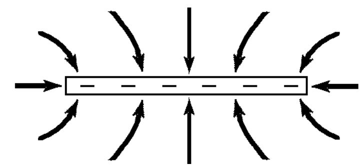


FIG. P23.39

P23.40 (a) $\dfrac{q_1}{q_2} = \dfrac{-6}{18} = \boxed{-\dfrac{1}{3}}$

(b) $\boxed{q_1 \text{ is negative, } q_2 \text{ is positive}}$

P23.41 (a) The electric field has the general appearance shown. It is zero $\boxed{\text{at the center}}$, where (by symmetry) one can see that the three charges individually produce fields that cancel out.

In addition to the center of the triangle, the electric field lines in the second figure to the right indicate three other points near the middle of each leg of the triangle where $E = 0$, but they are more difficult to find mathematically.

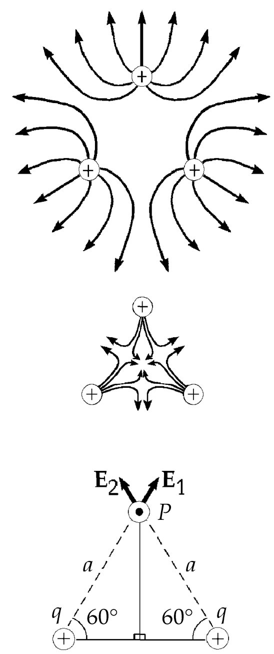


FIG. P23.41

(b) You may need to review vector addition in Chapter Three. The electric field at point P can be found by adding the electric field vectors due to each of the two lower point charges: $\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2$.

The electric field from a point charge is $\mathbf{E} = k_e \dfrac{q}{r^2}\hat{\mathbf{r}}$.

As shown in the solution figure at right,

$\mathbf{E}_1 = k_e \dfrac{q}{a^2}$ to the right and upward at 60°

$\mathbf{E}_2 = k_e \dfrac{q}{a^2}$ to the left and upward at 60°

$$\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2 = k_e \frac{q}{a^2}\left[\left(\cos 60°\,\hat{\mathbf{i}} + \sin 60°\,\hat{\mathbf{j}}\right) + \left(-\cos 60°\,\hat{\mathbf{i}} + \sin 60°\,\hat{\mathbf{j}}\right)\right] = k_e \frac{q}{a^2}\left[2\left(\sin 60°\,\hat{\mathbf{j}}\right)\right]$$

$$= \boxed{1.73 k_e \frac{q}{a^2}\hat{\mathbf{j}}}$$

Section 23.7 **Motion of Charged Particles in a Uniform Electric Field**

P23.42 $F = qE = ma \qquad a = \dfrac{qE}{m}$

$v_f = v_i + at \qquad v_f = \dfrac{qEt}{m}$

electron: $v_e = \dfrac{(1.602\times10^{-19})(520)(48.0\times10^{-9})}{9.11\times10^{-31}} = \boxed{4.39\times10^{6}\ \text{m/s}}$

in a direction opposite to the field

proton: $v_p = \dfrac{(1.602\times10^{-19})(520)(48.0\times10^{-9})}{1.67\times10^{-27}} = \boxed{2.39\times10^{3}\ \text{m/s}}$

in the same direction as the field

P23.43 (a) $a = \dfrac{qE}{m} = \dfrac{1.602\times10^{-19}(640)}{1.67\times10^{-27}} = \boxed{6.14\times10^{10}\ \text{m/s}^2}$

(b) $v_f = v_i + at \qquad 1.20\times10^{6} = (6.14\times10^{10})t \qquad t = \boxed{1.95\times10^{-5}\ \text{s}}$

(c) $x_f - x_i = \dfrac{1}{2}(v_i + v_f)t \qquad x_f = \dfrac{1}{2}(1.20\times10^{6})(1.95\times10^{-5}) = \boxed{11.7\ \text{m}}$

(d) $K = \dfrac{1}{2}mv^2 = \dfrac{1}{2}(1.67\times10^{-27}\ \text{kg})(1.20\times10^{6}\ \text{m/s})^2 = \boxed{1.20\times10^{-15}\ \text{J}}$

P23.44 (a) $|a| = \dfrac{qE}{m} = \dfrac{(1.602\times10^{-19})(6.00\times10^{5})}{(1.67\times10^{-27})} = 5.76\times10^{13}\ \text{m/s so } \mathbf{a} = \boxed{-5.76\times10^{13}\,\hat{\mathbf{i}}\ \text{m/s}^2}$

(b) $v_f = v_i + 2a(x_f - x_i)$

$0 = v_i^2 + 2(-5.76\times10^{13})(0.070\,0) \qquad \boxed{\mathbf{v}_i = 2.84\times10^{6}\,\hat{\mathbf{i}}\ \text{m/s}}$

(c) $v_f = v_i + at$

$0 = 2.84\times10^{6} + (-5.76\times10^{13})t \qquad t = \boxed{4.93\times10^{-8}\ \text{s}}$

P23.45 The required electric field will be $\boxed{\text{in the direction of motion}}$.

Work done $= \Delta K$

so, $-Fd = -\dfrac{1}{2}mv_i^2$ (since the final velocity $= 0$)

which becomes $eEd = K$

and $E = \boxed{\dfrac{K}{ed}}$.

P23.46 The acceleration is given by

$v_f^2 = v_i^2 + 2a(x_f - x_i)$ or $v_f^2 = 0 + 2a(-h)$.

Solving $a = -\dfrac{v_f^2}{2h}$.

Now $\sum \mathbf{F} = m\mathbf{a}: \quad -mg\hat{\mathbf{j}} + q\mathbf{E} = -\dfrac{mv_f^2\hat{\mathbf{j}}}{2h}$.

Therefore $q\mathbf{E} = \left(-\dfrac{mv_f^2}{2h} + mg\right)\hat{\mathbf{j}}$.

(a) Gravity alone would give the bead downward impact velocity

$$\sqrt{2(9.80\ \text{m/s}^2)(5.00\ \text{m})} = 9.90\ \text{m/s}.$$

To change this to 21.0 m/s down, a $\boxed{\text{downward}}$ electric field must exert a downward electric force.

(b) $q = \dfrac{m}{E}\left(\dfrac{v_f^2}{2h} - g\right) = \dfrac{1.00\times10^{-3}\ \text{kg}}{1.00\times10^{4}\ \text{N/C}}\left(\dfrac{\text{N}\cdot\text{s}^2}{\text{kg}\cdot\text{m}}\right)\left[\dfrac{(21.0\ \text{m/s})^2}{2(5.00\ \text{m})} - 9.80\ \text{m/s}^2\right] = \boxed{3.43\ \mu\text{C}}$

P23.47 (a) $t = \dfrac{x}{v_x} = \dfrac{0.050\,0}{4.50\times10^5} = 1.11\times10^{-7}\ \text{s} = \boxed{111\ \text{ns}}$

(b) $a_y = \dfrac{qE}{m} = \dfrac{(1.602\times10^{-19})(9.60\times10^3)}{(1.67\times10^{-27})} = 9.21\times10^{11}\ \text{m/s}^2$

$y_f - y_i = v_{yi}t + \dfrac{1}{2}a_y t^2$: $\quad y_f = \dfrac{1}{2}(9.21\times10^{11})(1.11\times10^{-7})^2 = 5.68\times10^{-3}\ \text{m} = \boxed{5.68\ \text{mm}}$

(c) $v_x = \boxed{4.50\times10^5\ \text{m/s}}$ $\quad v_{yf} = v_{yi} + a_y t = (9.21\times10^{11})(1.11\times10^{-7}) = \boxed{1.02\times10^5\ \text{m/s}}$

***P23.48** The particle feels a constant force: $\mathbf{F} = q\mathbf{E} = (1\times10^{-6}\ \text{C})(2\,000\ \text{N/C})(-\hat{\mathbf{j}}) = 2\times10^{-3}\ \text{N}(-\hat{\mathbf{j}})$

and moves with acceleration: $\mathbf{a} = \dfrac{\sum\mathbf{F}}{m} = \dfrac{(2\times10^{-3}\ \text{kg}\cdot\text{m/s}^2)(-\hat{\mathbf{j}})}{2\times10^{-16}\ \text{kg}} = (1\times10^{13}\ \text{m/s}^2)(-\hat{\mathbf{j}})$.

Its x-component of velocity is constant at $(1.00\times10^5\ \text{m/s})\cos 37° = 7.99\times10^4\ \text{m/s}$. Thus it moves in a parabola opening downward. The maximum height it attains above the bottom plate is described by

$v_{yf}^2 = v_{yi}^2 + 2a_y(y_f - y_i)$: $\quad 0 = (6.02\times10^4\ \text{m/s})^2 - (2\times10^{13}\ \text{m/s}^2)(y_f - 0)$

$y_f = 1.81\times10^{-4}\ \text{m}$.

continued on next page

Since this is less than 10 mm, the particle does not strike the top plate, but moves in a symmetric parabola and strikes the bottom plate after a time given by

$$y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2 \qquad 0 = 0 + \left(6.02\times10^4 \text{ m/s}\right)t + \frac{1}{2}\left(-1\times10^{13} \text{ m/s}^2\right)t^2$$

since $t > 0$, $\quad t = 1.20\times10^{-8}$ s.

The particle's range is $\quad x_f = x_i + v_x t = 0 + \left(7.99\times10^4 \text{ m/s}\right)\left(1.20\times10^{-8} \text{ s}\right) = 9.61\times10^{-4}$ m.

In sum,

$\boxed{\text{The particle strikes the negative plate after moving in a parabola with a height of 0.181 mm and a width of 0.961 mm.}}$

P23.49 $v_i = 9.55\times10^3$ m/s

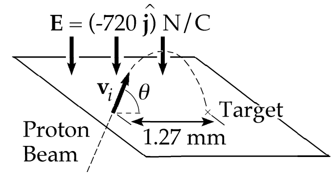


FIG. P23.49

(a) $$a_y = \frac{eE}{m} = \frac{\left(1.60\times10^{-19}\right)(720)}{\left(1.67\times10^{-27}\right)} = 6.90\times10^{10} \text{ m/s}^2$$

$$R = \frac{v_i^2 \sin 2\theta}{a_y} = 1.27\times10^{-3} \text{ m so that}$$

$$\frac{\left(9.55\times10^3\right)^2 \sin 2\theta}{6.90\times10^{10}} = 1.27\times10^{-3}$$

$\sin 2\theta = 0.961 \qquad \theta = \boxed{36.9°} \qquad 90.0° - \theta = \boxed{53.1°}$

(b) $t = \dfrac{R}{v_{ix}} = \dfrac{R}{v_i \cos\theta}$ $\qquad$ If $\theta = 36.9°$, $t = \boxed{167 \text{ ns}}$. $\qquad$ If $\theta = 53.1°$, $t = \boxed{221 \text{ ns}}$.

Additional Problems

***P23.50** The two given charges exert equal-size forces of attraction on each other. If a third charge, positive or negative, were placed between them they could not be in equilibrium. If the third charge were at a point $x > 15$ cm, it would exert a stronger force on the 45 μC than on the -12 μC, and could not produce equilibrium for both. Thus the third charge must be at $x = -d < 0$. Its equilibrium requires

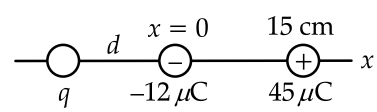


FIG. P23.50

$$\frac{k_e q(12\ \mu\text{C})}{d^2} = \frac{k_e q(45\ \mu\text{C})}{(15 \text{ cm} + d)^2} \qquad \left(\frac{15 \text{ cm} + d}{d}\right)^2 = \frac{45}{12} = 3.75$$

$15 \text{ cm} + d = 1.94d \qquad d = 16.0$ cm.

The third charge is at $\boxed{x = -16.0 \text{ cm}}$. The equilibrium of the -12 μC requires

$$\frac{k_e q(12\ \mu\text{C})}{(16.0 \text{ cm})^2} = \frac{k_e(45\ \mu\text{C})12\ \mu\text{C}}{(15 \text{ cm})^2} \qquad \boxed{q = 51.3\ \mu\text{C}}.$$

All six individual forces are now equal in magnitude, so we have equilibrium as required, and this is the only solution.

P23.51 The proton moves with acceleration $\left|a_p\right| = \dfrac{qE}{m} = \dfrac{\left(1.60 \times 10^{-19}\ \text{C}\right)(640\ \text{N/C})}{1.673 \times 10^{-27}\ \text{kg}} = 6.13 \times 10^{10}\ \text{m/s}^2$

while the e^- has acceleration $\left|a_e\right| = \dfrac{\left(1.60 \times 10^{-19}\ \text{C}\right)(640\ \text{N/C})}{9.110 \times 10^{-31}\ \text{kg}} = 1.12 \times 10^{14}\ \text{m/s}^2 = 1\,836 a_p$.

(a) We want to find the distance traveled by the proton (i.e., $d = \frac{1}{2}a_p t^2$), knowing:

$$4.00\ \text{cm} = \frac{1}{2}a_p t^2 + \frac{1}{2}a_e t^2 = 1\,837\left(\frac{1}{2}a_p t^2\right).$$

Thus, $d = \dfrac{1}{2}a_p t^2 = \dfrac{4.00\ \text{cm}}{1\,837} = \boxed{21.8\ \mu\text{m}}$.

(b) The distance from the positive plate to where the meeting occurs equals the distance the sodium ion travels (i.e., $d_{\text{Na}} = \frac{1}{2}a_{\text{Na}}t^2$). This is found from:

$4.00\ \text{cm} = \dfrac{1}{2}a_{\text{Na}}t^2 + \dfrac{1}{2}a_{\text{Cl}}t^2$: $\quad 4.00\ \text{cm} = \dfrac{1}{2}\left(\dfrac{eE}{22.99\ \text{u}}\right)t^2 + \dfrac{1}{2}\left(\dfrac{eE}{35.45\ \text{u}}\right)t^2$.

This may be written as $\quad 4.00\ \text{cm} = \dfrac{1}{2}a_{\text{Na}}t^2 + \dfrac{1}{2}(0.649a_{\text{Na}})t^2 = 1.65\left(\dfrac{1}{2}a_{\text{Na}}t^2\right)$

so $\quad d_{\text{Na}} = \dfrac{1}{2}a_{\text{Na}}t^2 = \dfrac{4.00\ \text{cm}}{1.65} = \boxed{2.43\ \text{cm}}$.

P23.52 (a) The field, E_1, due to the 4.00×10^{-9} C charge is in the $-x$ direction.

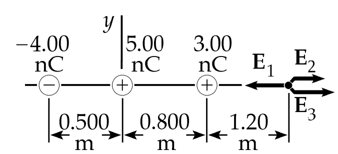


FIG. P23.52(a)

$$\mathbf{E}_1 = \frac{k_e q}{r^2}\hat{\mathbf{r}} = \frac{\left(8.99 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(-4.00 \times 10^{-9}\ \text{C}\right)}{(2.50\ \text{m})^2}\hat{\mathbf{i}}$$

$$= -5.75\hat{\mathbf{i}}\ \text{N/C}$$

Likewise, E_2 and E_3, due to the 5.00×10^{-9} C charge and the 3.00×10^{-9} C charge are

$$\mathbf{E}_2 = \frac{k_e q}{r^2}\hat{\mathbf{r}} = \frac{\left(8.99 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(5.00 \times 10^{-9}\ \text{C}\right)}{(2.00\ \text{m})^2}\hat{\mathbf{i}} = 11.2\ \text{N/C}\ \hat{\mathbf{i}}$$

$$\mathbf{E}_3 = \frac{\left(8.99 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(3.00 \times 10^{-9}\ \text{C}\right)}{(1.20\ \text{m})^2}\hat{\mathbf{i}} = 18.7\ \text{N/C}\ \hat{\mathbf{i}}$$

$\mathbf{E}_R = \mathbf{E}_1 + \mathbf{E}_2 + \mathbf{E}_3 = \boxed{24.2\ \text{N/C}}$ in $+x$ direction.

(b)

$$\mathbf{E}_1 = \frac{k_e q}{r^2}\hat{\mathbf{r}} = (-8.46\ \text{N/C})\left(0.243\hat{\mathbf{i}} + 0.970\hat{\mathbf{j}}\right)$$

$$\mathbf{E}_2 = \frac{k_e q}{r^2}\hat{\mathbf{r}} = (11.2\ \text{N/C})\left(+\hat{\mathbf{j}}\right)$$

$$\mathbf{E}_3 = \frac{k_e q}{r^2}\hat{\mathbf{r}} = (5.81\ \text{N/C})\left(-0.371\hat{\mathbf{i}} + 0.928\hat{\mathbf{j}}\right)$$

$E_x = E_{1x} + E_{3x} = -4.21\hat{\mathbf{i}}$ N/C $\qquad E_y = E_{1y} + E_{2y} + E_{3y} = 8.43\hat{\mathbf{j}}$ N/C

$E_R = \boxed{9.42\ \text{N/C}} \qquad \theta = \boxed{63.4°\ \text{above}\ -x\ \text{axis}}$

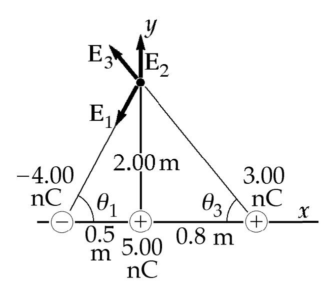


FIG. P23.52(b)

***P23.53** (a) Each ion moves in a quarter circle. The electric force causes the centripetal acceleration.

$$\sum F = ma \qquad qE = \frac{mv^2}{R} \qquad \boxed{E = \frac{mv^2}{qR}}$$

(b) For the x-motion, $v_{xf}^2 = v_{xi}^2 + 2a_x\left(x_f - x_i\right)$

$$0 = v^2 + 2a_x R \qquad a_x = -\frac{v^2}{2R} = \frac{F_x}{m} = \frac{qE_x}{m}$$

$E_x = -\dfrac{mv^2}{2qR}$. Similarly for the y-motion,

$$v^2 = 0 + 2a_y R \qquad a_y = +\frac{v^2}{2R} = \frac{qE_y}{m} \qquad E_y = \frac{mv^2}{2qR}$$

The magnitude of the field is

$$\sqrt{E_x^2 + E_y^2} = \boxed{\frac{mv^2}{\sqrt{2}qR} \text{ at } 135° \text{ counterclockwise from the } x\text{-axis}}.$$

P23.54 From the free-body diagram shown,

$$\sum F_y = 0: \qquad T\cos 15.0° = 1.96 \times 10^{-2} \text{ N}.$$

So $\qquad T = 2.03 \times 10^{-2}$ N.

From $\sum F_x = 0$, we have $\qquad qE = T\sin 15.0°$

$$\text{or} \qquad q = \frac{T\sin 15.0°}{E} = \frac{\left(2.03 \times 10^{-2} \text{ N}\right)\sin 15.0°}{1.00 \times 10^3 \text{ N/C}} = 5.25 \times 10^{-6} \text{ C} = \boxed{5.25\ \mu\text{C}}.$$

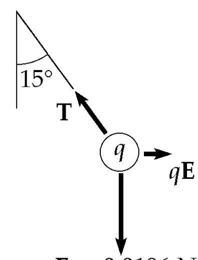


FIG. P23.54

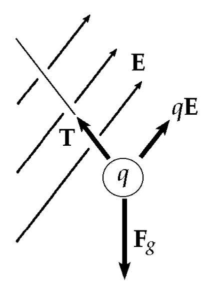
P23.55 (a) Let us sum force components to find

$$\sum F_x = qE_x - T\sin\theta = 0, \text{ and } \sum F_y = qE_y + T\cos\theta - mg = 0.$$

Combining these two equations, we get

$$q = \frac{mg}{\left(E_x \cot\theta + E_y\right)} = \frac{\left(1.00 \times 10^{-3}\right)(9.80)}{(3.00\cot 37.0° + 5.00) \times 10^5} = 1.09 \times 10^{-8} \text{ C}$$

$$= \boxed{10.9 \text{ nC}}$$



Free Body Diagram

FIG. P23.55

(b) From the two equations for $\sum F_x$ and $\sum F_y$ we also find

$$T = \frac{qEx}{\sin 37.0°} = 5.44 \times 10^{-3} \text{ N} = \boxed{5.44 \text{ mN}}.$$

P23.56 This is the general version of the preceding problem. The known quantities are A, B, m, g, and θ. The unknowns are q and T.

The approach to this problem should be the same as for the last problem, but without numbers to substitute for the variables. Likewise, we can use the free body diagram given in the solution to problem 55.

Again, Newton's second law: $\sum F_x = -T\sin\theta + qA = 0$ (1)

and $\sum F_y = +T\cos\theta + qB - mg = 0$ (2)

(a) Substituting $T = \dfrac{qA}{\sin\theta}$, into Eq. (2), $\dfrac{qA\cos\theta}{\sin\theta} + qB = mg$.

Isolating q on the left, $\boxed{q = \dfrac{mg}{(A\cot\theta + B)}}$.

(b) Substituting this value into Eq. (1), $\boxed{T = \dfrac{mgA}{(A\cos\theta + B\sin\theta)}}$.

If we had solved this general problem first, we would only need to substitute the appropriate values in the equations for q and T to find the numerical results needed for problem 55. If you find this problem more difficult than problem 55, the little list at the first step is useful. It shows what symbols to think of as known data, and what to consider unknown. The list is a guide for deciding what to solve for in the analysis step, and for recognizing when we have an answer.

P23.57 $F = \dfrac{k_e q_1 q_2}{r^2}$: $\tan\theta = \dfrac{15.0}{60.0}$

$\theta = 14.0°$

$$F_1 = \frac{(8.99\times10^9)(10.0\times10^{-6})^2}{(0.150)^2} = 40.0\text{ N}$$

$$F_3 = \frac{(8.99\times10^9)(10.0\times10^{-6})^2}{(0.600)^2} = 2.50\text{ N}$$

$$F_2 = \frac{(8.99\times10^9)(10.0\times10^{-6})^2}{(0.619)^2} = 2.35\text{ N}$$

$$F_x = -F_3 - F_2\cos 14.0° = -2.50 - 2.35\cos 14.0° = -4.78\text{ N}$$

$$F_y = -F_1 - F_2\sin 14.0° = -40.0 - 2.35\sin 14.0° = -40.6\text{ N}$$

$$F_{\text{net}} = \sqrt{F_x^2 + F_y^2} = \sqrt{(4.78)^2 + (40.6)^2} = \boxed{40.9\text{ N}}$$

$$\tan\phi = \frac{F_y}{F_x} = \frac{-40.6}{-4.78}$$

$$\phi = \boxed{263°}$$

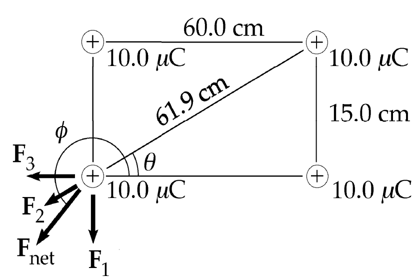


FIG. P23.57

P23.58 From Figure A: $d\cos 30.0° = 15.0\text{ cm}$,

or $d = \dfrac{15.0\text{ cm}}{\cos 30.0°}$

From Figure B: $\theta = \sin^{-1}\left(\dfrac{d}{50.0\text{ cm}}\right)$

$$\theta = \sin^{-1}\left(\frac{15.0\text{ cm}}{50.0\text{ cm}(\cos 30.0°)}\right) = 20.3°$$

$$\frac{F_q}{mg} = \tan\theta$$

or $F_q = mg\tan 20.3°$ (1)

From Figure C: $F_q = 2F\cos 30.0°$

$$F_q = 2\left[\frac{k_e q^2}{(0.300\text{ m})^2}\right]\cos 30.0° \qquad (2)$$

Combining equations (1) and (2),

$$2\left[\frac{k_e q^2}{(0.300\text{ m})^2}\right]\cos 30.0° = mg\tan 20.3°$$

$$q^2 = \frac{mg(0.300\text{ m})^2\tan 20.3°}{2k_e\cos 30.0°}$$

$$q^2 = \frac{(2.00\times 10^{-3}\text{ kg})(9.80\text{ m/s}^2)(0.300\text{ m})^2\tan 20.3°}{2(8.99\times 10^9\text{ N}\cdot\text{m}^2/\text{C}^2)\cos 30.0°}$$

$$q = \sqrt{4.20\times 10^{-14}\text{ C}^2} = 2.05\times 10^{-7}\text{ C} = \boxed{0.205\ \mu\text{C}}$$

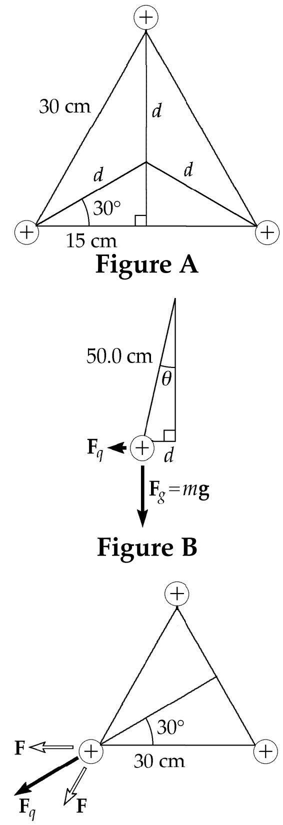


Figure A

Figure B

Figure C

FIG. P23.58

P23.59 Charge $\dfrac{Q}{2}$ resides on each block, which repel as point charges: $F = \dfrac{k_e(Q/2)(Q/2)}{L^2} = k(L - L_i)$.

Solving for Q, $Q = \boxed{2L\sqrt{\dfrac{k(L-L_i)}{k_e}}}$.

***P23.60** If we place one more charge q at the 29th vertex, the total force on the central charge will add up to zero: $\mathbf{F}_{28\text{ charges}} + \dfrac{k_e qQ}{a^2}$ away from vertex $29 = 0$ $\qquad \mathbf{F}_{28\text{ charges}} = \boxed{\dfrac{k_e qQ}{a^2}\text{ toward vertex 29}}$.

P23.61 According to the result of Example 23.7, the left-hand rod creates this field at a distance d from its right-hand end:

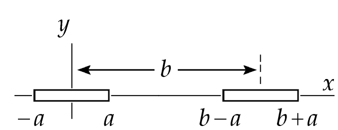


FIG. P23.61

$$E = \frac{k_e Q}{d(2a+d)}$$

$$dF = \frac{k_e QQ}{2a}\frac{dx}{d(d+2a)}$$

$$F = \frac{k_e Q^2}{2a}\int_{x=b-2a}^{b}\frac{dx}{x(x+2a)} = \frac{k_e Q^2}{2a}\left(-\frac{1}{2a}\ln\frac{2a+x}{x}\right)_{b-2a}^{b}$$

$$F = \frac{+k_e Q^2}{4a^2}\left(-\ln\frac{2a+b}{b} + \ln\frac{b}{b-2a}\right) = \frac{k_e Q^2}{4a^2}\ln\frac{b^2}{(b-2a)(b+2a)} = \boxed{\left(\frac{k_e Q^2}{4a^2}\right)\ln\left(\frac{b^2}{b^2-4a^2}\right)}$$

P23.62 At equilibrium, the distance between the charges is $r = 2(0.100\text{ m})\sin 10.0° = 3.47\times 10^{-2}\text{ m}$

Now consider the forces on the sphere with charge $+q$, and use $\sum F_y = 0$:

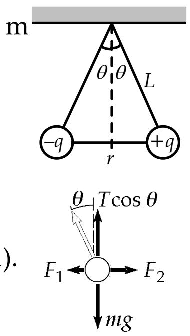


FIG. P23.62

$$\sum F_y = 0: \qquad T\cos 10.0° = mg\text{, or } T = \frac{mg}{\cos 10.0°} \qquad (1)$$

$$\sum F_x = 0: \qquad F_{\text{net}} = F_2 - F_1 = T\sin 10.0° \qquad (2)$$

F_{net} is the net electrical force on the charged sphere. Eliminate T from (2) by use of (1).

$$F_{\text{net}} = \frac{mg\sin 10.0°}{\cos 10.0°} = mg\tan 10.0° = \left(2.00\times 10^{-3}\text{ kg}\right)\left(9.80\text{ m/s}^2\right)\tan 10.0° = 3.46\times 10^{-3}\text{ N}$$

F_{net} is the resultant of two forces, F_1 and F_2. F_1 is the attractive force on $+q$ exerted by $-q$, and F_2 is the force exerted on $+q$ by the external electric field.

$$F_{\text{net}} = F_2 - F_1 \text{ or } F_2 = F_{\text{net}} + F_1$$

$$F_1 = \left(8.99\times 10^9\text{ N}\cdot\text{m}^2/\text{C}^2\right)\frac{\left(5.00\times 10^{-8}\text{ C}\right)\left(5.00\times 10^{-8}\text{ C}\right)}{\left(3.47\times 10^{-3}\text{ m}\right)^2} = 1.87\times 10^{-2}\text{ N}$$

Thus, $F_2 = F_{\text{net}} + F_1$ yields $F_2 = 3.46\times 10^{-3}\text{ N} + 1.87\times 10^{-2}\text{ N} = 2.21\times 10^{-2}\text{ N}$

and $F_2 = qE$, or $E = \dfrac{F_2}{q} = \dfrac{2.21\times 10^{-2}\text{ N}}{5.00\times 10^{-8}\text{ C}} = 4.43\times 10^5\text{ N/C} = \boxed{443\text{ kN/C}}$.

P23.63 $Q = \int \lambda\, d\ell = \int_{-90.0°}^{90.0°} \lambda_0 \cos\theta R\, d\theta = \lambda_0 R \sin\theta\Big|_{-90.0°}^{90.0°} = \lambda_0 R[1-(-1)] = 2\lambda_0 R$

$Q = 12.0\ \mu\text{C} = (2\lambda_0)(0.600)\text{ m} = 12.0\ \mu\text{C} \quad$ so $\quad \lambda_0 = 10.0\ \mu\text{C/m}$

$$dF_y = \frac{1}{4\pi\epsilon_0}\left(\frac{(3.00\ \mu\text{C})(\lambda\, d\ell)}{R^2}\right)\cos\theta = \frac{1}{4\pi\epsilon_0}\left(\frac{(3.00\ \mu\text{C})\left(\lambda_0\cos^2\theta R\, d\theta\right)}{R^2}\right)$$

$$F_y = \int_{-90.0°}^{90.0°}\left(8.99\times 10^9\text{ N}\cdot\text{m}^2/\text{C}^2\right)\frac{\left(3.00\times 10^{-6}\text{ C}\right)\left(10.0\times 10^{-6}\text{ C/m}\right)}{(0.600\text{ m})}\cos^2\theta\, d\theta$$

$$F_y = \frac{8.99(30.0)}{0.600}\left(10^{-3}\text{ N}\right)\int_{-\pi/2}^{\pi/2}\left(\frac{1}{2}+\frac{1}{2}\cos 2\theta\right)d\theta$$

$$F_y = (0.450\text{ N})\left(\frac{1}{2}\theta + \frac{1}{4}\sin 2\theta\Big|_{-\pi/2}^{\pi/2}\right) = \boxed{0.707\text{ N}}\text{ Downward.}$$

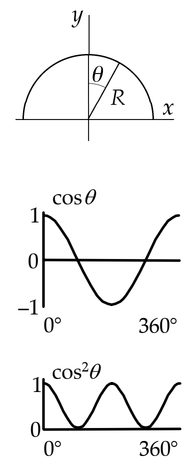


FIG. P23.63

Since the leftward and rightward forces due to the two halves of the semicircle cancel out, $F_x = 0$.

P23.64 At an equilibrium position, the net force on the charge Q is zero. The equilibrium position can be located by determining the angle θ corresponding to equilibrium.

In terms of lengths s, $\frac{1}{2}a\sqrt{3}$, and r, shown in Figure P23.64, the charge at the origin exerts an attractive force

$$\frac{k_e Qq}{\left(s+\frac{1}{2}a\sqrt{3}\right)^2}$$

continued on next page

The other two charges exert equal repulsive forces of magnitude $\dfrac{k_e Qq}{r^2}$. The horizontal components of the two repulsive forces add, balancing the attractive force,

$$F_{\text{net}} = k_e Qq\left[\frac{2\cos\theta}{r^2} - \frac{1}{\left(s+\frac{1}{2}a\sqrt{3}\right)^2}\right] = 0$$

From Figure P23.64 $\qquad r = \dfrac{\frac{1}{2}a}{\sin\theta} \qquad s = \dfrac{1}{2}a\cot\theta$

The equilibrium condition, in terms of θ, is $\qquad F_{\text{net}} = \left(\dfrac{4}{a^2}\right)k_e Qq\left(2\cos\theta\sin^2\theta - \dfrac{1}{\left(\sqrt{3}+\cot\theta\right)^2}\right) = 0$.

Thus the equilibrium value of θ satisfies $\qquad 2\cos\theta\sin^2\theta\left(\sqrt{3}+\cot\theta\right)^2 = 1$.

One method for solving for θ is to tabulate the left side. To three significant figures a value of θ corresponding to equilibrium is 81.7°.

The distance from the vertical side of the triangle to the equilibrium position is

$$s = \frac{1}{2}a\cot 81.7° = \boxed{0.072\,9a}\,.$$

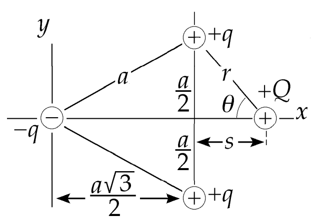


FIG. P23.64

θ	$2\cos\theta\sin^2\theta\left(\sqrt{3}+\cot\theta\right)^2$
60°	4
70°	2.654
80°	1.226
90°	0
81°	1.091
81.5°	1.024
81.7°	0.997

A second zero-field point is on the negative side of the x-axis, where $\theta = -9.16°$ and $s = -3.10a$.

P23.65 (a) From the $2Q$ charge we have $\quad F_e - T_2\sin\theta_2 = 0$ and $mg - T_2\cos\theta_2 = 0$.

Combining these we find $\quad \dfrac{F_e}{mg} = \dfrac{T_2\sin\theta_2}{T_2\cos\theta_2} = \tan\theta_2$.

From the Q charge we have $\quad F_e = T_1\sin\theta_1 = 0$ and $mg - T_1\cos\theta_1 = 0$.

Combining these we find $\quad \dfrac{F_e}{mg} = \dfrac{T_1\sin\theta_1}{T_1\cos\theta_1} = \tan\theta_1$ or $\boxed{\theta_2 = \theta_1}$.

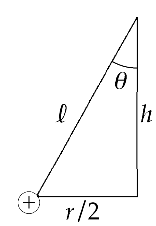


FIG. P23.65

(b) $\quad F_e = \dfrac{k_e 2QQ}{r^2} = \dfrac{2k_e Q^2}{r^2}$

If we assume θ is small then $\quad \tan\theta \approx \dfrac{r/2}{\ell}$.

Substitute expressions for F_e and $\tan\theta$ into either equation found in part (a) and solve for r.

$\dfrac{F_e}{mg} = \tan\theta$ then $\dfrac{2k_e Q^2}{r^2}\left(\dfrac{1}{mg}\right) \approx \dfrac{r}{2\ell}$ and solving for r we find $r \approx \left(\dfrac{4k_e Q^2\ell}{mg}\right)^{1/3}$.

P23.66 (a) The distance from each corner to the center of the square is

$$\sqrt{\left(\frac{L}{2}\right)^2+\left(\frac{L}{2}\right)^2}=\frac{L}{\sqrt{2}}.$$

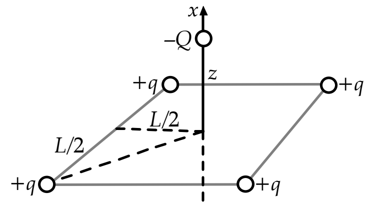


FIG. P23.66

The distance from each positive charge to $-Q$ is then $\sqrt{z^2+\frac{L^2}{2}}$. Each positive charge exerts a force directed along the line joining q and $-Q$, of magnitude

$$\frac{k_eQq}{z^2+L^2/2}.$$

The line of force makes an angle with the z-axis whose cosine is

$$\frac{z}{\sqrt{z^2+L^2/2}}$$

The four charges together exert forces whose x and y components add to zero, while the z-components add to

$$\mathbf{F}=\boxed{-\frac{4k_eQqz}{\left(z^2+L^2/2\right)^{3/2}}\hat{\mathbf{k}}}$$

(b) For $z>>L$, the magnitude of this force is $F_z=-\frac{4k_eQqz}{\left(L^2/2\right)^{3/2}}=-\left(\frac{4(2)^{3/2}k_eQq}{L^3}\right)z=ma_z$

Therefore, the object's vertical acceleration is of the form $\boxed{a_z=-\omega^2z}$

with $\omega^2=\frac{4(2)^{3/2}k_eQq}{mL^3}=\frac{k_eQq\sqrt{128}}{mL^3}$.

Since the acceleration of the object is always oppositely directed to its excursion from equilibrium and in magnitude proportional to it, the object will execute simple harmonic motion with a period given by

$$T=\frac{2\pi}{\omega}=\frac{2\pi}{(128)^{1/4}}\sqrt{\frac{mL^3}{k_eQq}}=\boxed{\frac{\pi}{(8)^{1/4}}\sqrt{\frac{mL^3}{k_eQq}}}.$$

P23.67 (a) The total non-contact force on the cork ball is: $F=qE+mg=m\left(g+\frac{qE}{m}\right)$,

which is constant and directed downward. Therefore, it behaves like a simple pendulum in the presence of a modified uniform gravitational field with a period given by:

$$T=2\pi\sqrt{\frac{L}{g+qE/m}}=2\pi\sqrt{\frac{0.500\text{ m}}{9.80\text{ m/s}^2+\left[\left(2.00\times10^{-6}\text{ C}\right)\left(1.00\times10^5\text{ N/C}\right)/1.00\times10^{-3}\text{ kg}\right]}}$$

$$=\boxed{0.307\text{ s}}$$

(b) $\boxed{\text{Yes}}$. Without gravity in part (a), we get $T=2\pi\sqrt{\frac{L}{qE/m}}$

$$T=2\pi\sqrt{\frac{0.500\text{ m}}{\left(2.00\times10^{-6}\text{ C}\right)\left(1.00\times10^5\text{ N/C}\right)/1.00\times10^{-3}\text{ kg}}}=0.314\text{ s}\ \text{(a 2.28\% difference)}.$$

P23.68 The bowl exerts a normal force on each bead, directed along the radius line or at 60.0° above the horizontal. Consider the free-body diagram of the bead on the left:

$$\sum F_y = n\sin 60.0° - mg = 0\,,$$

or $$n = \frac{mg}{\sin 60.0°}\,.$$

Also, $$\sum F_x = -F_e + n\cos 60.0° = 0\,,$$

or $$\frac{k_e q^2}{R^2} = n\cos 60.0° = \frac{mg}{\tan 60.0°} = \frac{mg}{\sqrt{3}}\,.$$

Thus, $$q = \boxed{R\left(\frac{mg}{k_e\sqrt{3}}\right)^{1/2}}\,.$$

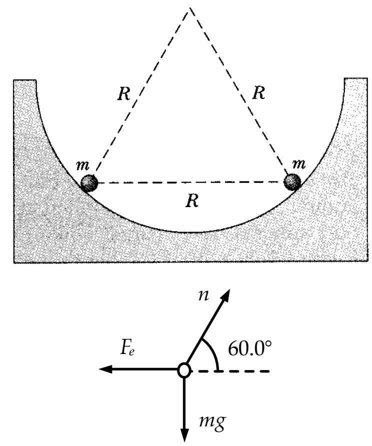


FIG. P23.68

P23.69 (a) There are 7 terms which contribute:

3 are s away (along sides)

3 are $\sqrt{2}s$ away (face diagonals) and $\sin\theta = \frac{1}{\sqrt{2}} = \cos\theta$

1 is $\sqrt{3}s$ away (body diagonal) and $\sin\phi = \frac{1}{\sqrt{3}}$.

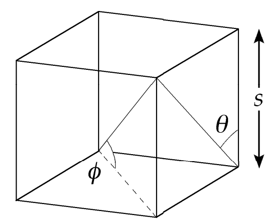


FIG. P23.69

The component in each direction is the same by symmetry.

$$\mathbf{F} = \frac{k_e q^2}{s^2}\left[1 + \frac{2}{2\sqrt{2}} + \frac{1}{3\sqrt{3}}\right]\left(\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}\right) = \boxed{\frac{k_e q^2}{s^2}(1.90)\left(\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}\right)}$$

(b) $$F = \sqrt{F_x^2 + F_y^2 + F_z^2} = \boxed{3.29\frac{k_e q^2}{s^2} \text{ away from the origin}}$$

P23.70 (a) Zero contribution from the same face due to symmetry, opposite face contributes

$$4\left(\frac{k_e q}{r^2}\sin\phi\right) \quad \text{where} \quad r = \sqrt{\left(\frac{s}{2}\right)^2 + \left(\frac{s}{2}\right)^2 + s^2} = \sqrt{1.5}s = 1.22s$$

$$\sin\phi = \frac{s}{r} \qquad E = 4\frac{k_e q s}{r^3} = \frac{4}{(1.22)^3}\frac{k_e q}{s^2} = \boxed{2.18\frac{k_e q}{s^2}}$$

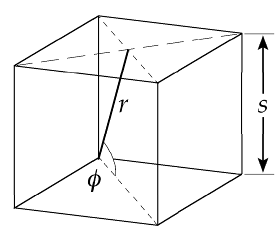


FIG. P23.70

(b) $\boxed{\text{The direction is the } \hat{\mathbf{k}} \text{ direction.}}$

P23.71 The field on the axis of the ring is calculated in Example 23.8, $E = E_x = \dfrac{k_e x Q}{\left(x^2 + a^2\right)^{3/2}}$

The force experienced by a charge $-q$ placed along the axis of the ring is $F = -k_e Qq \left[\dfrac{x}{\left(x^2+a^2\right)^{3/2}}\right]$

and when $x << a$, this becomes $F = -\left(\dfrac{k_e Qq}{a^3}\right)x$

This expression for the force is in the form of Hooke's law, with an effective spring constant of $k = \dfrac{k_e Qq}{a^3}$

Since $\omega = 2\pi f = \sqrt{\dfrac{k}{m}}$, we have $f = \boxed{\dfrac{1}{2\pi}\sqrt{\dfrac{k_e Qq}{ma^3}}}$.

P23.72

$$d\mathbf{E} = \frac{k_e dq}{x^2 + (0.150\text{ m})^2}\left(\frac{-x\hat{\mathbf{i}} + 0.150\text{ m}\hat{\mathbf{j}}}{\sqrt{x^2+(0.150\text{ m})^2}}\right) = \frac{k_e \lambda\left(-x\hat{\mathbf{i}} + 0.150\text{ m}\hat{\mathbf{j}}\right)dx}{\left[x^2+(0.150\text{ m})^2\right]^{3/2}}$$

$$\mathbf{E} = \int_{\text{all charge}} d\mathbf{E} = k_e\lambda \int_{x=0}^{0.400\text{ m}} \frac{\left(-x\hat{\mathbf{i}} + 0.150\text{ m}\hat{\mathbf{j}}\right)dx}{\left[x^2+(0.150\text{ m})^2\right]^{3/2}}$$

$$\mathbf{E} = k_e\lambda\left[\left.\frac{+\hat{\mathbf{i}}}{\sqrt{x^2+(0.150\text{ m})^2}}\right|_0^{0.400\text{ m}} + \left.\frac{(0.150\text{ m})\hat{\mathbf{j}}x}{(0.150\text{ m})^2\sqrt{x^2+(0.150\text{ m})^2}}\right|_0^{0.400\text{ m}}\right]$$

$$\mathbf{E} = \left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(35.0\times10^{-9}\ \text{C/m}\right)\left[\hat{\mathbf{i}}(2.34-6.67)\text{ m}^{-1} + \hat{\mathbf{j}}(6.24-0)\text{ m}^{-1}\right]$$

$$\mathbf{E} = \left(-1.36\hat{\mathbf{i}} + 1.96\hat{\mathbf{j}}\right)\times10^3\ \text{N/C} = \boxed{\left(-1.36\hat{\mathbf{i}} + 1.96\hat{\mathbf{j}}\right)\ \text{kN/C}}$$

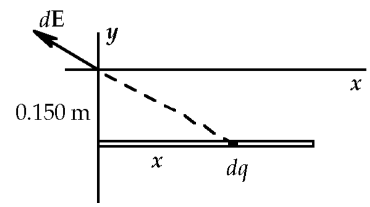


FIG. P23.72

P23.73 The electrostatic forces exerted on the two charges result in a net torque $\tau = -2Fa\sin\theta = -2Eqa\sin\theta$.

For small θ, $\sin\theta \approx \theta$ and using $p = 2qa$, we have $\tau = -Ep\theta$.

The torque produces an angular acceleration given by $\tau = I\alpha = I\dfrac{d^2\theta}{dt^2}$.

Combining these two expressions for torque, we have $\dfrac{d^2\theta}{dt^2} + \left(\dfrac{Ep}{I}\right)\theta = 0$.

This equation can be written in the form $\dfrac{d^2\theta}{dt^2} = -\omega^2\theta$ where $\omega^2 = \dfrac{Ep}{I}$.

This is the same form as Equation 15.5 and the frequency of oscillation is found by comparison with Equation 15.11, or $f = \dfrac{1}{2\pi}\sqrt{\dfrac{pE}{I}} = \boxed{\dfrac{1}{2\pi}\sqrt{\dfrac{2qaE}{I}}}$.

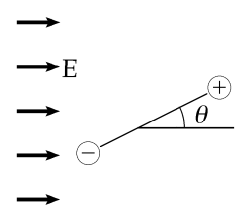


FIG. P23.73

ANSWERS TO EVEN PROBLEMS

P23.2 (a) 2.62×10^{24}; (b) 2.38 electrons for every 10^9 present

P23.4 57.5 N

P23.6 2.51×10^{-9}

P23.8 514 kN

P23.10 $x = 0.634d$. The equilibrium is stable if the third bead has positive charge.

P23.12 (a) period $= \frac{\pi}{2}\sqrt{\frac{md^3}{k_e qQ}}$ where m is the mass of the object with charge $-Q$; (b) $4a\sqrt{\frac{k_e qQ}{md^3}}$

P23.14 1.49 g

P23.16 720 kN/C down

P23.18 (a) $\left[18.0\hat{\mathbf{i}} - 218\hat{\mathbf{j}}\right]$ kN/C; (b) $\left(36.0\hat{\mathbf{i}} - 436\hat{\mathbf{j}}\right)$ mN

P23.20 (a) $12.9\hat{\mathbf{j}}$ kN/C; (b) $-38.6\hat{\mathbf{j}}$ mN

P23.22 see the solution

P23.24 $-\frac{\pi^2 k_e q}{6a^2}\hat{\mathbf{i}}$

P23.26 $\frac{k_e\lambda_0}{x_0}\left(-\hat{\mathbf{i}}\right)$

P23.28 $\frac{k_e\lambda_0}{2x_0}\left(-\hat{\mathbf{i}}\right)$

P23.30 (a) 383 MN/C away; (b) 324 MN/C away; (c) 80.7 MN/C away; (d) 6.68 MN/C away

P23.32 see the solution

P23.34 (a) $\frac{k_e Q\hat{\mathbf{i}}}{h}\left[\left(d^2+R^2\right)^{-1/2} - \left((d+h)^2+R^2\right)^{-1/2}\right]$; (b) $\frac{2k_e Q\hat{\mathbf{i}}}{R^2 h}\left[h + \left(d^2+R^2\right)^{1/2} - \left((d+h)^2+R^2\right)^{1/2}\right]$

P23.36 (a) 200 pC; (b) 141 pC; (c) 58.9 pC

P23.38 see the solution

P23.40 (a) $-\frac{1}{3}$; (b) q_1 is negative and q_2 is positive

P23.42 electron: 4.39 Mm/s; proton: 2.39 km/s

P23.44 (a) $-57.6\hat{\mathbf{i}}$ Tm/s^2; (b) $2.84\hat{\mathbf{i}}$ Mm/s; (c) 49.3 ns

P23.46 (a) down; (b) 3.43 μC

P23.48 The particle strikes the negative plate after moving in a parabola 0.181 mm high and 0.961 mm.

P23.50 Possible only with +51.3 μC at $x = -16.0$ cm

P23.52 (a) 24.2 N/C at 0°; (b) 9.42 N/C at 117°

P23.54 5.25 μC

P23.56 (a) $\frac{mg}{A\cot\theta + B}$; (b) $\frac{mgA}{A\cos\theta + B\sin\theta}$

P23.58 0.205 μC

P23.60 $\frac{k_e qQ}{a^2}$ toward the 29th vertex

P23.62 $443\,\hat{\mathbf{i}}$ kN/C

P23.64 $0.072\,9a$

P23.66 see the solution; the period is $\frac{\pi}{8^{1/4}}\sqrt{\frac{mL^3}{k_e Qq}}$

P23.68 $R\left(\frac{mg}{k_e\sqrt{3}}\right)^{1/2}$

P23.70 (a) see the solution; (b) $\hat{\mathbf{k}}$

P23.72 $\left(-1.36\hat{\mathbf{i}} + 1.96\hat{\mathbf{j}}\right)$ kN/C