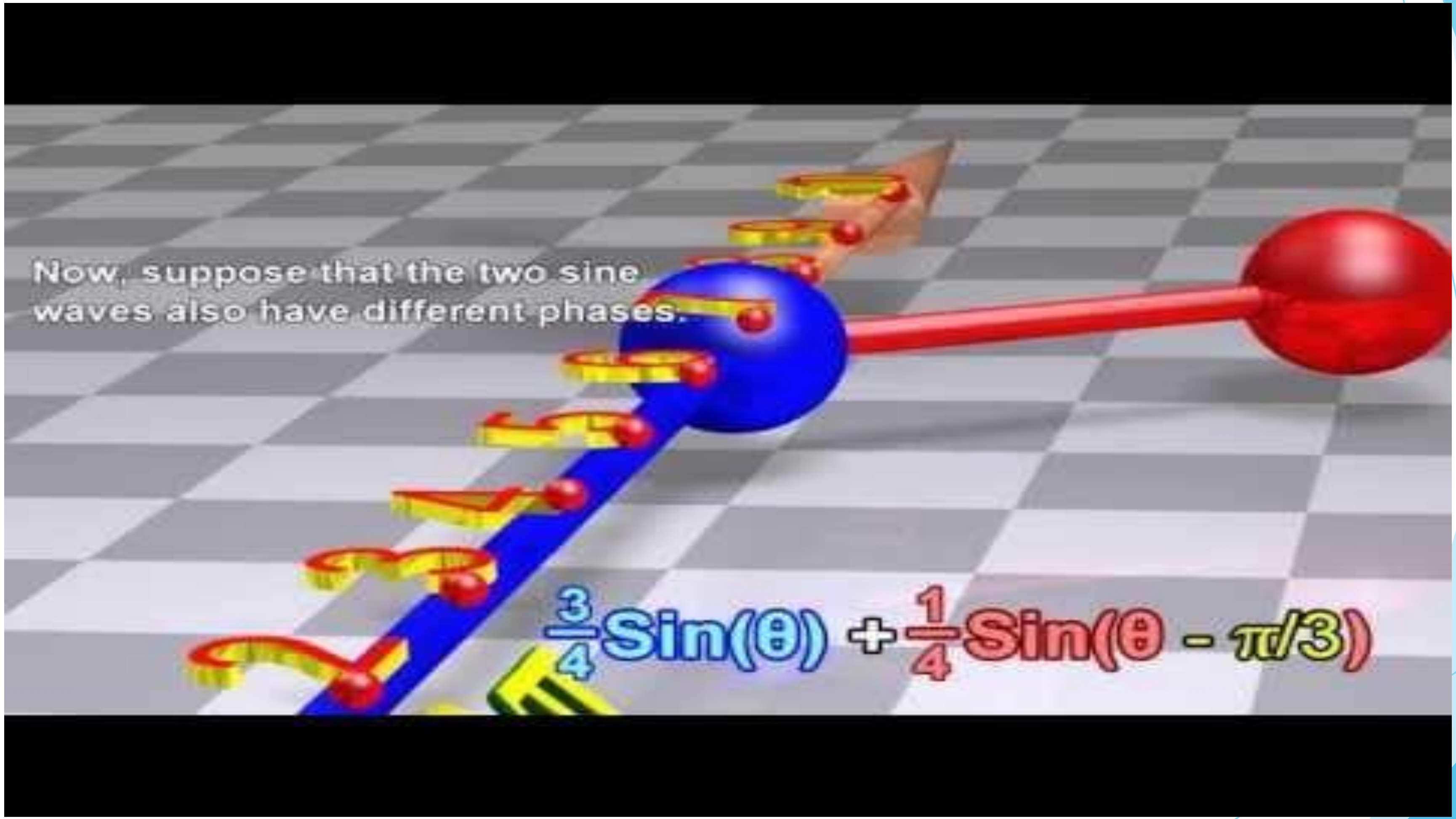
SERIES DE FOURIER

Comunicación datos I

DEFINICIÓN


Now, suppose that the two sine waves also have different phases.
$\frac{3}{4}\mathrm{Sin}(\theta) + \frac{1}{4}\mathrm{Sin}(\theta - \pi/3)$

Serie de una Función

La **serie de Fourier** de una función f definida en el intervalo $(-p, p)$ está dada por

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left(a_n \cos\frac{n\pi}{p}x + b_n \operatorname{sen}\frac{n\pi x}{p}\right), \tag{8}$$

donde

$$a_0 = \frac{1}{p}\int_{-p}^{p} f(x)\,dx \tag{9}$$

$$a_n = \frac{1}{p}\int_{-p}^{p} f(x)\cos\frac{n\pi x}{p}\,dx \tag{10}$$

$$b_n = \frac{1}{p}\int_{-p}^{p} f(x)\operatorname{sen}\frac{n\pi x}{p}\,dx. \tag{11}$$

Ejemplo1

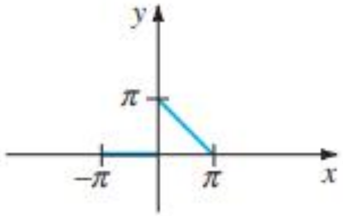


FIGURA 11.2.1 Función definida por tramos del ejemplo 1.

Desarrolle $$f(x) = \begin{cases} 0, & -\pi < x < 0 \\ \pi - x, & 0 \le x < \pi \end{cases} \qquad (12)$$

en una serie de Fourier.

SOLUCIÓN En la figura 11.2.1 se presenta la gráfica de f. Con $p = \pi$ tenemos de las ecuaciones (9) y (10) que

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\,dx = \frac{1}{\pi}\left[\int_{-\pi}^{0} 0\,dx + \int_{0}^{\pi}(\pi - x)\,dx\right] = \frac{1}{\pi}\left[\pi x - \frac{x^2}{2}\right]_0^{\pi} = \frac{\pi}{2}$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\,dx = \frac{1}{\pi}\left[\int_{-\pi}^{0} 0\,dx + \int_{0}^{\pi}(\pi - x)\cos nx\,dx\right]$$

$$= \frac{1}{\pi}\left[(\pi - x)\frac{\operatorname{sen} nx}{n}\Big|_0^{\pi} + \frac{1}{n}\int_0^{\pi}\operatorname{sen} nx\,dx\right] \quad \text{integración por partes}$$

$$= -\frac{1}{n\pi}\frac{\cos nx}{n}\Big|_0^{\pi} = \frac{1 - (-1)^n}{n^2\pi},$$

donde hemos usado $\cos n\pi = (-1)^n$. En forma similar encontramos de (11) que

$$b_n = \frac{1}{\pi}\int_0^{\pi}(\pi - x)\operatorname{sen} nx\,dx = \frac{1}{n}.$$

Por tanto $$f(x) = \frac{\pi}{4} + \sum_{n=1}^{\infty}\left\{\frac{1 - (-1)^n}{n^2\pi}\cos nx + \frac{1}{n}\operatorname{sen} nx\right\}. \qquad (13)$$ ■

integrales

$$\int u\,dv = uv - \int v\,du$$

5. $\int \operatorname{sen} u\,du = -\cos u + C$

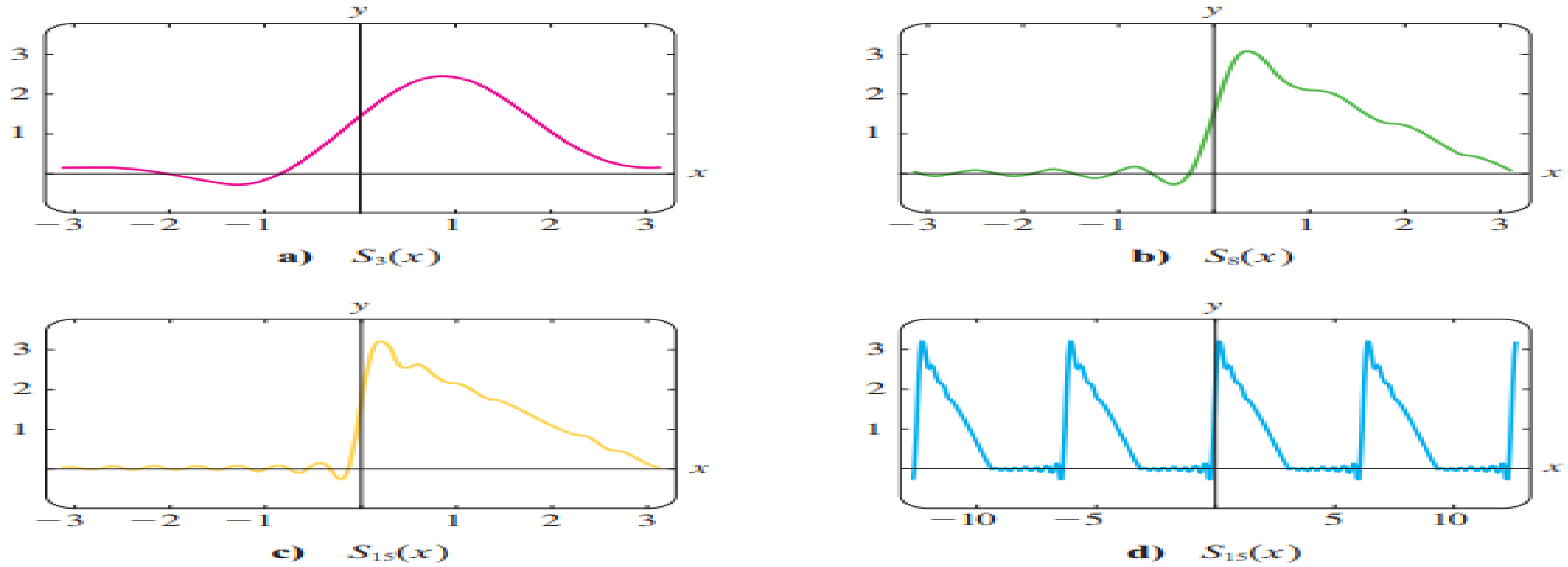
6. $\int \cos u\,du = \operatorname{sen} u + C$

15. $\int u \operatorname{sen} u\,du = \operatorname{sen} u - u\cos u + C$

16. $\int u \cos u\,du = \cos u + u \operatorname{sen} u + C$

SUCESIÓN DE SUMAS PARCIALES

$$S_1(x) = \frac{\pi}{4}, \quad S_2(x) = \frac{\pi}{4} + \frac{2}{\pi}\cos x + \operatorname{sen} x, \quad \text{y} \quad S_3(x) = \frac{\pi}{4} + \frac{2}{\pi}\cos x + \operatorname{sen} x + \frac{1}{2}\operatorname{sen} 2x.$$



Ejemplo 2

Calcular la serie de Fourier de

$$f(x) = \begin{cases} 0 , & -\pi < x < 0 , \\ x , & 0 < x < \pi . \end{cases}$$

En este caso, $T = \pi$. Usamos las fórmulas (9) y (10) para tener

$$\begin{aligned} a_n &= \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx\, dx = \frac{1}{\pi} \int_0^{\pi} x \cos nx\, dx \\ &= \frac{1}{\pi n^2} \int_0^{\pi n} u \cos u\, du = \frac{1}{\pi n^2} \left[\cos u + u \operatorname{sen} u \right] \Big|_0^{\pi n} \\ &= \frac{1}{\pi n^2} (\cos n\pi - 1) = \frac{1}{\pi n^2} \left[(-1)^n - 1 \right] , \qquad n = 1, 2, 3, \ldots , \\ a_0 &= \frac{1}{\pi} \int_{-\pi}^{\pi} f(x)\, dx = \frac{1}{\pi} \int_0^{\pi} x\, dx = \frac{x^2}{2\pi} \Big|_0^{\pi} = \frac{\pi}{2} . \end{aligned}$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\operatorname{sen} nx\,dx = \frac{1}{\pi}\int_0^{\pi} x\operatorname{sen} nx\,dx$$

$$= \frac{1}{\pi n^2}\int_0^{\pi n} u\operatorname{sen} u\,du = \frac{1}{\pi n^2}\left[\operatorname{sen} u - u\cos u\right]\Big|_0^{\pi n}$$

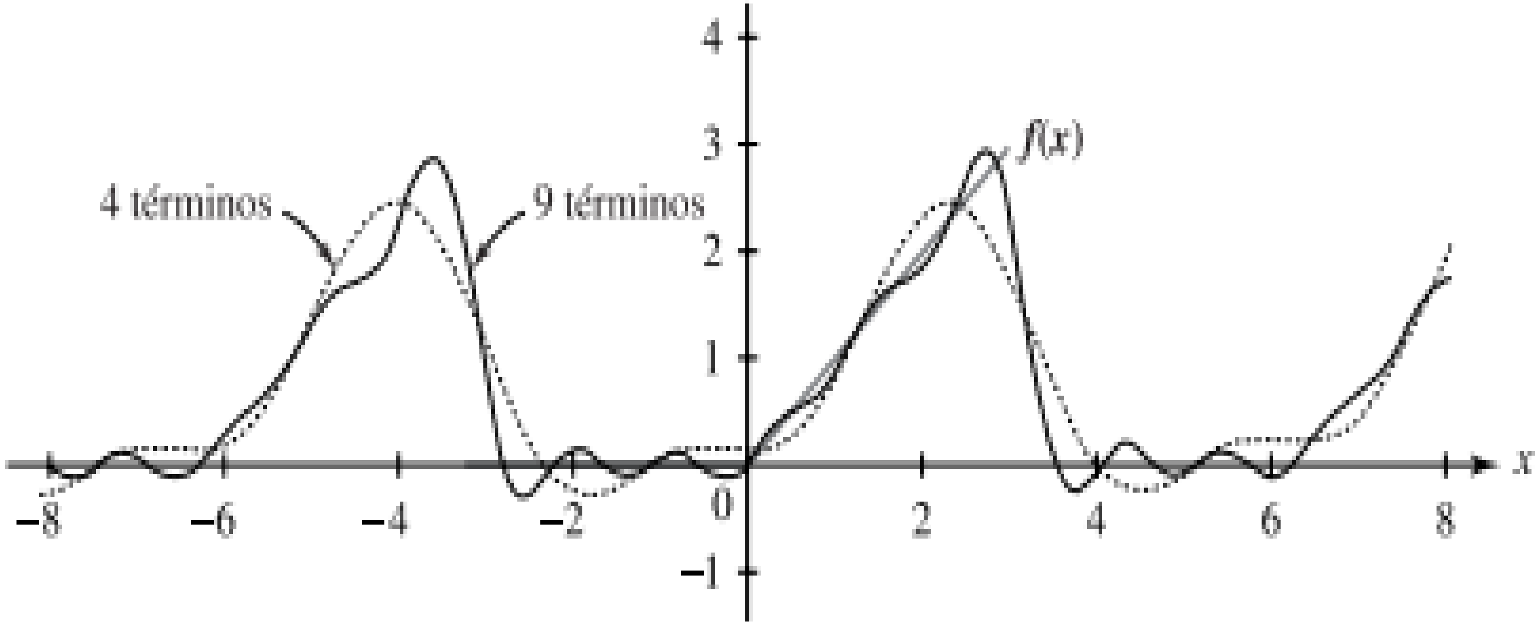
$$= \frac{-\cos n\pi}{n} = \frac{(-1)^{n+1}}{n}, \qquad n = 1, 2, 3, \ldots .$$

Por lo tanto,

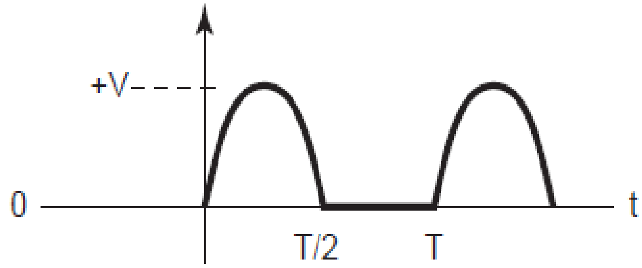
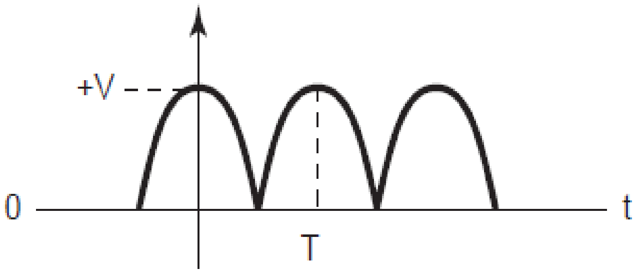
$$\textbf{(11)} \qquad f(x) \sim \frac{\pi}{4} + \sum_{n=1}^{\infty}\left\{\frac{1}{\pi n^2}\left[(-1)^n - 1\right]\cos nx + \frac{(-1)^{n+1}}{n}\operatorname{sen} nx\right\}$$

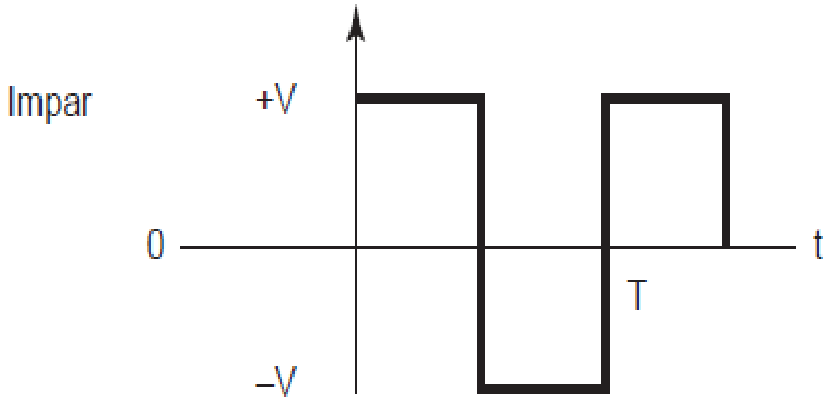
$$= \frac{\pi}{4} - \frac{2}{\pi}\left\{\cos x + \frac{1}{9}\cos 3x + \frac{1}{25}\cos 5x + \cdots\right\}$$

$$+ \left\{\operatorname{sen} x - \frac{1}{2}\operatorname{sen} 2x + \frac{1}{3}\operatorname{sen} 3x + \cdots\right\} .$$


4 términos
9 términos
f(x)
x
4
3
2
1
0
−1
−8
−6
−4
−2
2
4
6
8

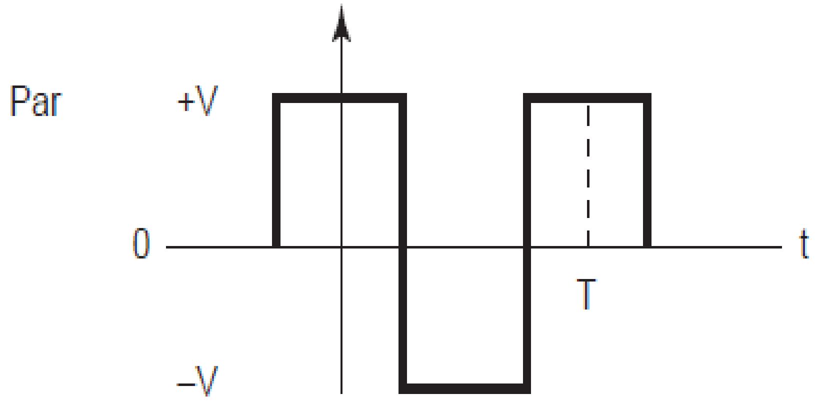
Taller # 5 Graficar las siguientes series

	Forma de onda	Serie de Fourier
Impar		$v(t) = \frac{V}{\pi} + \frac{V}{2}\text{sen } \omega t - \frac{2V}{3\pi}\cos 2\omega t - \frac{2V}{15\pi}\cos 4\omega t + \cdots$ $v(t) = \frac{V}{\pi} + \frac{V}{2}\text{sen } \omega t + \sum_{N=2}^{\infty} \frac{V[1 + (-1)^N]}{\pi(1 - N^2)}\cos N\omega t$
Par		$v(t) = \frac{2V}{\pi} + \frac{4V}{3\pi}\cos \omega t - \frac{4V}{15\pi}\cos 2\omega t + \cdots$ $v(t) = \frac{2V}{\pi} + \sum_{N=1}^{\infty} \frac{4V(-1)^N}{\pi[1 - (2N)^2]}\cos N\omega t$



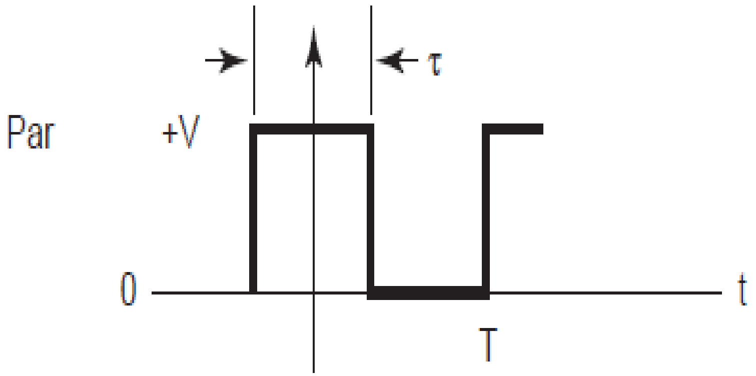
$$v(t) = \frac{4V}{\pi}\,\text{sen}\,\omega t + \frac{4V}{3\pi}\,\text{sen}\,3\omega t + \cdots$$

$$v(t) = \sum_{N=\text{impar}}^{\infty} \frac{4V}{N\pi}\,\text{sen}\,N\omega t$$

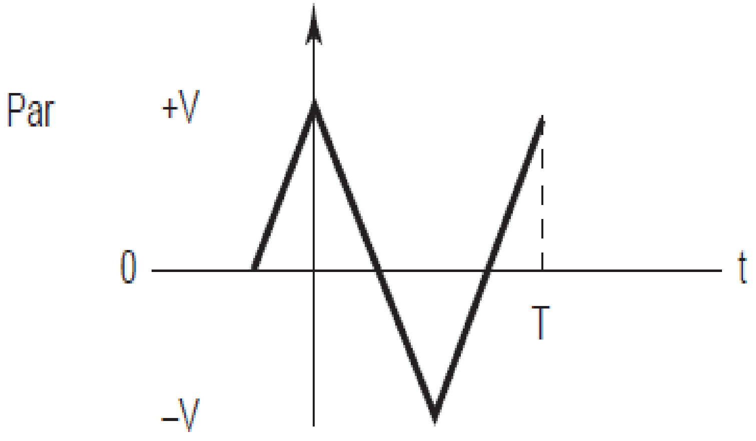


$$v(t) = \frac{4V}{\pi}\cos\omega t - \frac{4V}{3\pi}\cos 3\omega t + \frac{4V}{5\pi}\cos 5\omega t + \cdots$$

$$v(t) = \sum_{N=\text{impar}}^{\infty} \frac{V\,\text{sen}\,N\pi/2}{N\pi/2}\cos N\omega t$$



$$v(t) = \frac{V\tau}{T} + \sum_{N=1}^{\infty} \left(\frac{2V\tau}{T} \frac{\text{sen } N\omega t/T}{N\pi t/T} \right) \cos N\pi t$$



$$v(t) = \frac{8V}{\pi^2} \cos \omega t + \frac{8V}{(3\pi)^2} \cos 3\omega t + \frac{8V}{(5\pi)^2} \cos 5\omega t + \cdots$$

$$v(t) = \sum_{N=\text{impar}}^{\infty} \frac{8V}{(N\pi)^2} \cos N\omega t$$

TALLER REALIZAR LA SERIE DE FOURIER

$$f(x) = \begin{cases} 0, & -\pi < x < 0 \\ 1, & 0 \leq x < \pi \end{cases}$$

$$f(x) = \begin{cases} -1, & -\pi < x < 0 \\ 2, & 0 \leq x < \pi \end{cases}$$

$$f(x) = \begin{cases} 1, & -1 < x < 0 \\ x, & 0 \leq x < 1 \end{cases}$$

$$f(x) = \begin{cases} 0, & -1 < x < 0 \\ x, & 0 \leq x < 1 \end{cases}$$

$$f(x) = \begin{cases} 0, & -\pi < x < 0 \\ x^2, & 0 \leq x < \pi \end{cases}$$

$$f(x) = \begin{cases} \pi^2, & -\pi < x < 0 \\ \pi^2 - x^2, & 0 \leq x < \pi \end{cases}$$

$$f(x) = x + \pi, \quad -\pi < x < \pi$$

$$f(x) = 3 - 2x, \quad -\pi < x < \pi$$

$$f(x) = \begin{cases} 0, & -\pi < x < 0 \\ \operatorname{sen} x, & 0 \leq x < \pi \end{cases}$$

$$f(x) = \begin{cases} 0, & -\pi/2 < x < 0 \\ \cos x, & 0 \leq x < \pi/2 \end{cases}$$

$$f(x) = \begin{cases} 0, & -2 < x < -1 \\ -2, & -1 \leq x < 0 \\ 1, & 0 \leq x < 1 \\ 0, & 1 \leq x < 2 \end{cases}$$

$$f(x) = \begin{cases} 0, & -2 < x < 0 \\ x, & 0 \leq x < 1 \\ 1, & 1 \leq x < 2 \end{cases}$$

$$f(x) = \begin{cases} 1, & -5 < x < 0 \\ 1 + x, & 0 \leq x < 5 \end{cases}$$

$$f(x) = \begin{cases} 2 + x, & -2 < x < 0 \\ 2, & 0 \leq x < 2 \end{cases}$$

$$f(x) = e^x, \quad -\pi < x < \pi$$

$$f(x) = \begin{cases} 0, & -\pi < x < 0 \\ e^x - 1, & 0 \leq x < \pi \end{cases}$$

TALLER # 6

Realice una aplicación para trazar unas cuantas sumas parciales de la serie de Fourier. n=variable, para comprobar los resultados