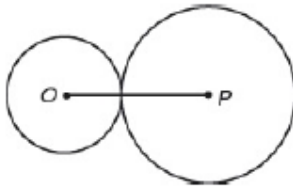## LESSON 6-3 CIRCLES AND THEIR EQUATIONS

Multiple-Choice

1. Circles O and P intersect at exactly one point, as shown in the figure above. If the radius of circle O is 2 and the radius of circle P is 6, what is the circumference of any circle that has OP as a diameter?

   (A) 4π
   (B) 8π
   (C) 12π
   (D) 16π

2. What is the area of a circle with a circumference of 10π?

   (A) $\sqrt{10\pi}$
   (B) 5π
   (C) 25π
   (D) 100π

3. Every time the pedals go through a 360° rotation on a certain bicycle, the tires rotate three times. If the tires are 24 inches in diameter, what is the minimum number of complete rotations of the pedals needed for the bicycle to travel at least 1 mile? [1 mile = 5,280 feet]

   (A) 24
   (B) 281
   (C) 561
   (D) 5,280
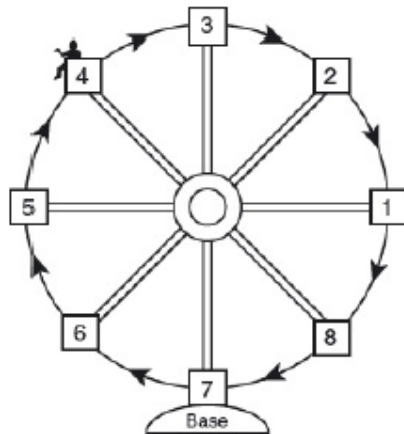
4. Kristine is riding in car 4 of the ferris wheel represented in the diagram above, which is $\frac{84}{\pi}$ meters from car 8. The ferris wheel is rotating in the direction indicated by the arrows. If each of the cars are equally spaced around the circular wheel, what is the best estimate of the number of meters in the distance through which Kristine's car will travel to reach the bottom of the ferris wheel before her car returns to the same position?

(A) 42.0
(B) 52.50
(C) 64.75
(D) 105.0
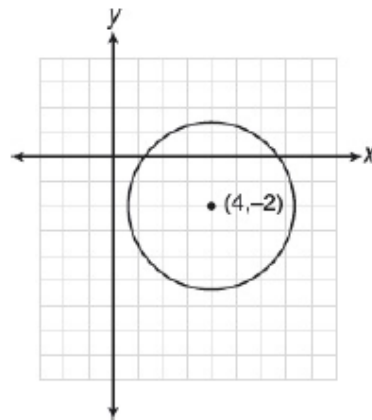
5. Which of the following could be an equation of the circle above?

(A) $(x - 4)^2 + (y + 2)^2 = 17$
(B) $(x + 4)^2 + (y - 2)^2 = 17$
(C) $(x - 4)^2 + (y + 2)^2 = 13$
(D) $(x + 4)^2 + (y - 2)^2 = 13$

6. If the equation of a circle is $x^2 - 10x + y^2 + 6y = -9$, which of the following lines contains a diameter of the circle?

(A) $y = 2x - 7$
(B) $y = -2x + 7$
(C) $y = 2x + 13$
(D) $y = -2x + 13$
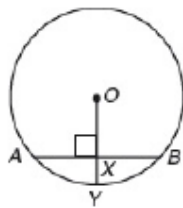
7. In the figure above, if the radius length of circle O is 10, $\overline{OY} \perp \overline{AB}$, and $AB = 16$, what is the length of segment XY?
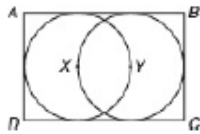
(A) 2

(B) 3
(C) 4
(D) 6

8. If a bicycle wheel has traveled $\frac{f}{\pi}$ feet after n complete revolutions, what is the length in feet of the diameter of the bicycle wheel?

(A) $\frac{f}{n\pi^2}$

(B) $\frac{\pi^2}{fn}$

(C) $\frac{nf}{\pi^2}$

(D) nf

$$x^2 + y^2 - 6x + 8y = 56$$

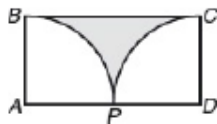9. What is the area of a circle whose equation is given above?

(A) 25π
(B) 81π
(C) 162π
(D) $6{,}561\pi$

10. In the figure above, X and Y are the centers of two overlapping circles. If the area of each circle is 7, what is the area of rectangle $ABCD$?

(A) $14 - \frac{17}{\pi}$

(B) $7 + \frac{14}{\pi}$

(C) $\frac{28}{\pi}$

(D) $\frac{42}{\pi}$

11. In rectangle $ABCD$ above, arcs BP and CP are quarter circles with centers at points A and D, respectively. If the area of each quarter circle is π, what is the area of the shaded region?

(A) $4 - \frac{\pi}{2}$

(B) $4 - \pi$

(C) $8 - \pi$

(D) $8 - 2\pi$
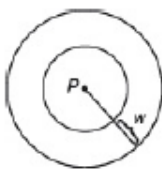
12. In the figure above, point P is the center of each circle. The circumference of the larger circle exceeds the circumference of the smaller circle by 12π. What is the width, w, of the region between the two circles?

(A) 4
(B) 6
(C) 8
(D) 9
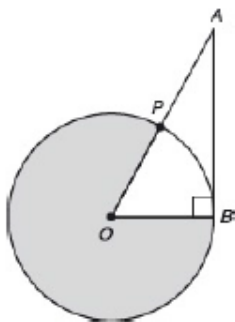
13. The circle shown above has center O and a radius length of 12. If P is the midpoint of $\overline{OA}$ and $\overline{AB}$ is tangent to circle O at B, what is the area of the shaded region?
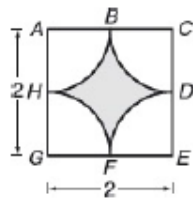
(A) 81π
(B) 96π
(C) 120π
(D) 128π
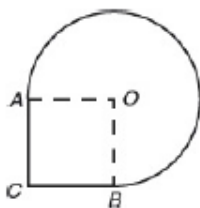
14. In the figure above, $OACB$ is a square with area $4x^2$. If OA and OB are radii of a sector of a circle O, what is the perimeter, in terms of x, of the unbroken figure?

(A) $x(4 + 3\pi)$
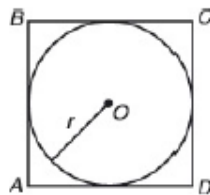(B) $x(3 + 4\pi)$
(C) $x(6 + 4\pi)$
(D) $4(x + 2\pi)$
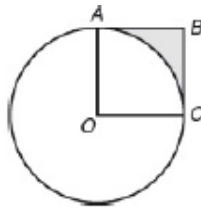
15. In the figure above, *OABC* is a square. If the area of circle *O* is 2π, what is the area of the shaded region?

(A) $\dfrac{\pi}{2} - 1$

(B) $2 - \dfrac{\pi}{2}$

(C) $\pi - 2$

(D) $\dfrac{\pi - 1}{2}$

16. In the figure above, the vertices of square *ACEG* are the centers of four quarter circles of equal area. What is the best approximation for the area of the shaded region? (Use $\pi = 3.14$.)
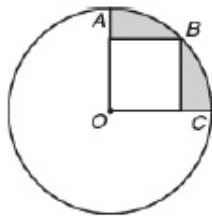
(A) 0.64
(B) 0.79
(C) 0.86
(D) 1.57

17. In the figure above, if circle *O* is inscribed in square *ABCD* in such a way that each side of the square is tangent to the circle, which of the following statements must be true?

   I. $AB \times CD < \pi \times r \times r$

   II. Area $ABCD = 4r^2$

   III. $r < \dfrac{2(CD)}{\pi}$

(A) I and II
(B) I and III
(C) II and III
(D) II only

18. In the figure above, *OABC* is a square and *B* is a point on the circle with center *O*. If *AB* = 6, what is the area of the shaded region?

   (A) $9(\pi - 2)$
   (B) $9(\pi - 1)$
   (C) $12(\pi - 2)$
   (D) $18(\pi - 2)$
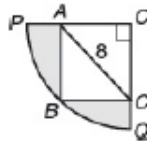
19. In the figure above, arc *PBQ* is one-quarter of a circle with center at *O*, and *OABC* is a rectangle. If *AOC* is an isosceles right triangle with *AC* = 8, what is the perimeter of the figure that encloses the shaded region?

   (A) $24 - 4\pi$
   (B) $24 - 4\sqrt{2} + 4\pi$
   (C) $16 - 4\sqrt{2} + 4\pi$
   (D) $16 + 4\pi$

20. The center of circle *Q* has coordinates $(3, -2)$ in the *xy*-plane. If an endpoint of a radius of circle *Q* has coordinates $R(7, 1)$, what is an equation of circle *Q*?

   (A) $(x - 3)^2 + (y + 2)^2 = 5$
   (B) $(x + 3)^2 + (y - 2)^2 = 25$
   (C) $(x - 3)^2 + (y + 2)^2 = 25$
   (D) $(x + 3)^2 + (y - 2)^2 = 5$
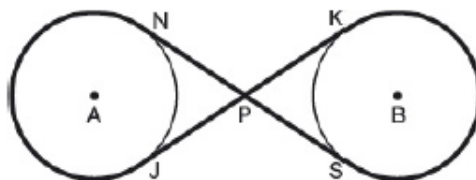
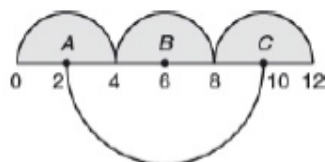21. In the pully system illustrated in the figure above, a belt with negligible thickness is stretched tightly around two identical wheels represented by circles *A* and *B*. If the radius of each wheel is $\frac{12}{\pi}$ inches and the measure of $\angle NPJ$ is 60°, what is the length of the belt?

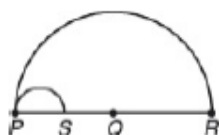(A) $32 + \dfrac{48\sqrt{3}}{\pi}$

(B) $48 + \dfrac{32\sqrt{3}}{\pi}$

(C) $\left(32 + 48\sqrt{3}\right)\pi$

(D) $80\pi\sqrt{3}$

Grid-In

1.

In the figure above, the sum of the areas of the three shaded semicircles with centers at A, B, and C is X, and the area of the larger semicircle below the line is Y. If $Y - X = k\pi$, what is the value of k?

2. In the figure above, each arc is a semicircle. If S is the midpoint of PQ and Q is the midpoint of PR, what is the ratio of the area of semicircle PS to the area of semicircle PR?

3. What is the distance in the xy-plane from the point $(3, -6)$ to the center of the circle whose equation is $x(x + 4) + y(y - 12) = 9$?
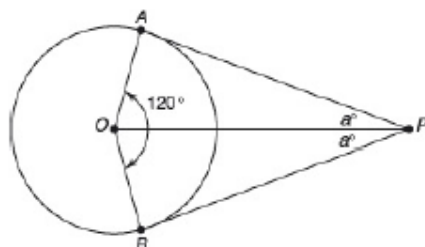
4. In the figure above, $\overline{PA}$ is tangent to circle O at point A, $\overline{PB}$ is tangent to circle O at point B. Angle AOB measures $120°$ and $\overline{OP} = \dfrac{24}{\pi}$. What is the length of minor arc AB?
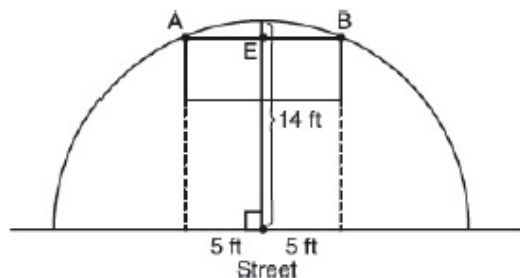
5. The diagram above shows a semicircular arch over a street that has a radius of 14 feet. A banner is attached to the arch at points A and B, such that $AE = EB = 5$ feet. How many feet above the ground are these points of attachment for the banner, correct to the *nearest tenth* of a foot?