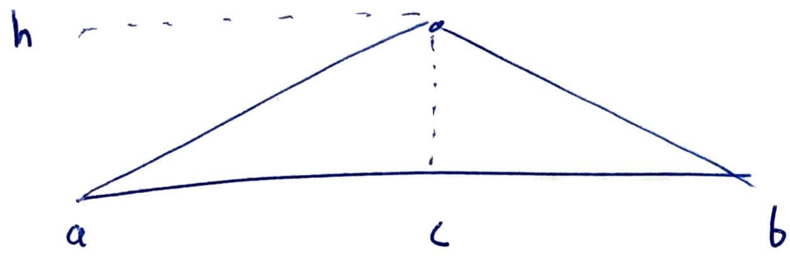
Triangle Distribution



FIND THE HEIGHT

First, lets solve for the height

left right triangle's area + right right triangle's area $= 1$

$$\Rightarrow \frac{(c-a)h}{2} + \frac{(b-c)h}{2} = 1$$

$$\Rightarrow (c-a)h + (b-c)h = 2$$

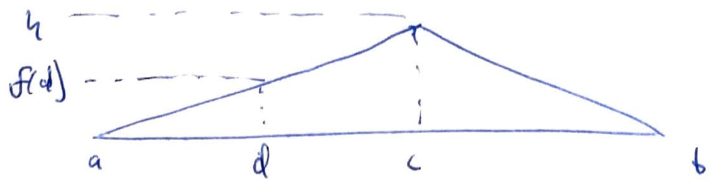
$$\Rightarrow [(c-a) + (b-c)]h = 2$$

$$\Rightarrow [c-a+b-c]h = 2$$

$$\Rightarrow [b-a]h = 2$$

$$\Rightarrow h = \frac{2}{b-a}$$

FIND THE PDF



Now, lets find the pdf

if $x < a$, then the pdf is 0

also if $x > b$, then the pdf is 0

we care about the intervals (a, c) and (c, b)

lets start w/ (a, c)

pick $d \in (a, c)$

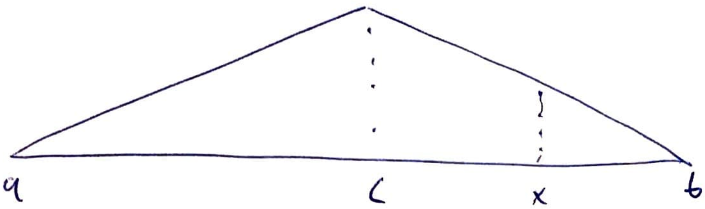
note the triangle w/ legs (a, d) and $(d, f(d))$

is similar to the triangle (a, c), h

so, $$\frac{f(d)}{(d-a)} = \frac{h}{(c-a)}$$

by subs $$\frac{f(d)}{d-a} = \frac{2}{(b-a)}\left(\frac{1}{c-a}\right) = \frac{2}{(b-a)(c-a)}$$

$$\Rightarrow f(d) = \frac{2(d-a)}{(b-a)(c-a)}$$



now pick $x \in (c, b)$

$$\frac{f(x)}{(b-x)} = \frac{h}{(b-c)}$$

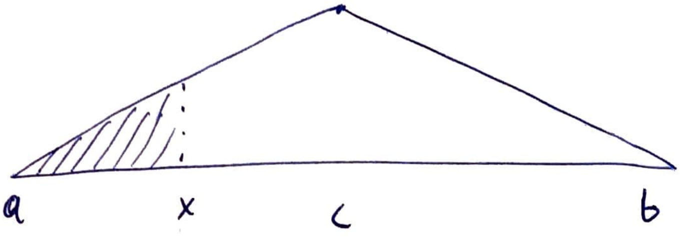
by subs

$$\frac{f(x)}{b-x} = \frac{2}{(b-a)}\frac{1}{(b-c)} = \frac{2}{(b-a)(b-c)}$$

$$f(x) = \frac{2(b-x)}{(b-a)(b-c)}$$

Finally the pdf is

$$f(x) = \begin{cases} 0 & \text{if } x < a \\ \dfrac{2(x-a)}{(b-a)(c-a)} & \text{if } x \in [a, c) \\ \dfrac{2(b-x)}{(b-a)(b-c)} & \text{if } x \in [c, b) \\ 0 & \text{if } x \geq b \end{cases}$$



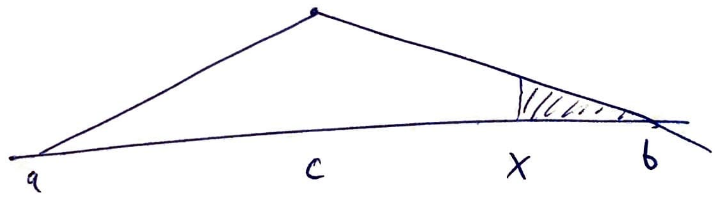
FIND THE CDF

It is easy to see $F(x)=0$ for $x<a$ and $F(x)=1$ for $x\ge b$.

Lets look at $x\in[a,c)$, the area of the triangle is

$$\frac{(x-a)\,f(a)}{2} = \frac{(x-a)}{2}\,\frac{2(x-a)}{(b-a)(c-a)} \quad \text{by subs}$$

$$= \frac{(x-a)^2}{(b-a)(c-a)}$$



Lets look at $x\in[c,b)$. To compute $F(x)$, lets compute its complement given by the shaded triangle the area of the triangle is $\frac{(b-x)f(x)}{2}$

$$= \frac{b-x}{2}\,\frac{2(b-x)}{(b-a)(b-c)}$$

$$= \frac{(b-x)^2}{(b-a)(b-c)}$$

So, $F(x) = 1 - \frac{(b-x)^2}{(b-a)(b-c)}$

Finally, the CDF is

$$F(x) = \begin{cases} 0 & \text{if } x<a \\ \frac{(x-a)^2}{(b-a)(c-a)} & \text{if } x\in[a,c) \\ 1-\frac{(b-x)^2}{(b-a)(b-c)} & \text{if } x\in[c,b) \\ 1 & \text{if } x\ge b \end{cases}$$