

proof of inverse transform sampling

Let F be a CDF and let F^{-1} be its (pseudo) inverse. Let $U \sim \text{Uniform}(0,1)$.

$$F^{-1}(u) = \inf\{x \mid F(x) \geq u\}$$

$$\text{WTS } P(F^{-1}(U) \leq x) = F(x)$$

$$P(F^{-1}(U) \leq x) = P(U \leq F(x)) \quad \text{by applying the inverse}$$

$$= F(x) \quad \text{because } P(U \leq k) = k \text{ for } \text{Uniform}(0,1)$$