The foundations of the rigorous study of *analysis* were laid in the nineteenth century, notably by the mathematicians Cauchy and Weierstrass. Central to the study of this subject are the formal definitions of *limits* and continuity.

Let D be a subset of  $\mathbf{R}$  and let  $f:D\to\mathbf{R}$  be a real-valued function on D. The function f is said to be *continuous* on D if, for all  $\epsilon>0$  and for all  $x\in D$ , there exists some  $\delta>0$  (which may depend on x) such that if  $y\in D$  satisfies

$$|y-x|<\delta$$

then

$$|f(y) - f(x)| < \epsilon.$$

One may readily verify that if f and g are continuous functions on D then the functions f+g, f-g and f.g are continuous. If in addition g is everywhere non-zero then f/g is continuous.