Take-home Exam Assignment
02619 Model Predictive Control
Modified Four Tank System
Autumn 2021

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You must hand in a report (Building 303B Office 110) no later than 13:00 January 3, 2022. You must also upload an electronic version (pdf) to CampusNet. This submission should also include a zip-file with all Matlab files that you have used as well as original Word / Latex files. You are allowed to work in groups, but you must hand in an individual report that you write yourself. Your answer to the problems should be written as a professional report providing necessary theory and figures. We have uploaded a latex template in Learn that you are welcome to use.

General Problem Description

This problem consider Model Predictive Control of the Modified Four Tank System. The Modified Four Tank System is illustrated in Figure 1. Use $\gamma_1 = 0.6$ and $\gamma_2 = 0.75$.

The purpose of the assignment and problem is that you demonstrate that you master all aspects involved in the design of a Model Predictive Control system. The assignment is open-ended and you do not necessarily have to answer all questions, but you should demonstrate in the report that you master design and evaluation of a Model Predictive Control system.

Problem 1
Control Structure

Consider the Modified Four Tank System. Assume that we measure the levels in tank 1, tank 2. The flows F_1 and F_2 can be controlled by manipulating the two pumps. Assume that the flows F_3 and F_4 are unmeasured flow rates that we cannot manipulate. F_3 and F_4 are stochastic variables (normally distributed). We want to control the levels in tank 1 and tank 2 to desired set points $\bar{z}_1$ and $\bar{z}_2$.

1. What are the states, x, the measurement, y, the manipulated variables (MVs), u, the measured disturbance variables (DVs), and the controlled variables (CVs) for this system?
2. Draw a block-diagram of the Modified Four Tank System with an MPC system. The MPC block should illustrate both the state-estimator and regulator of the MPC.

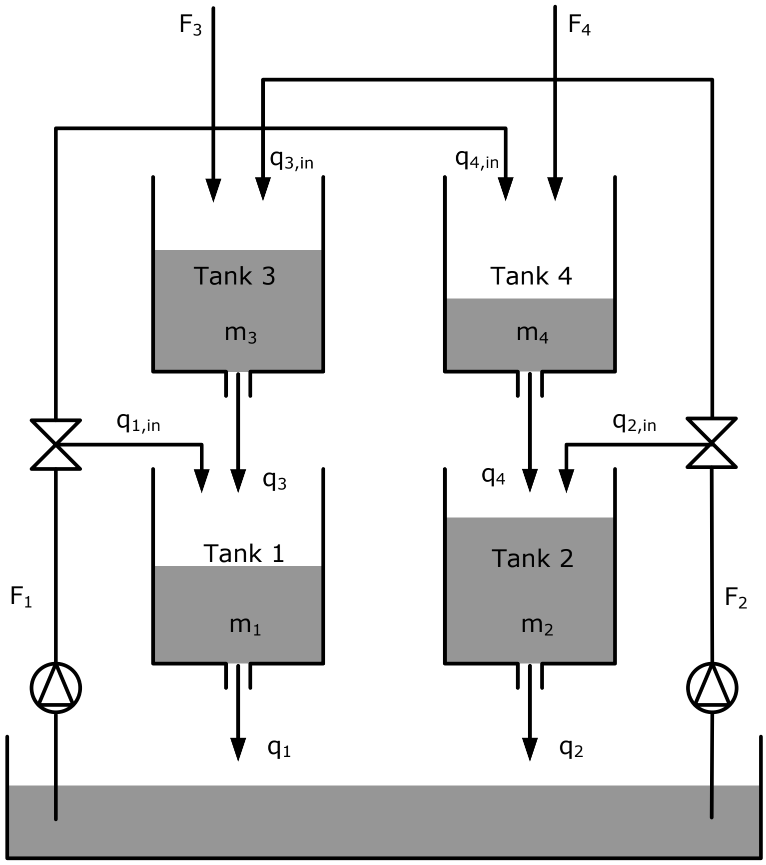

Figure 1: The Modified Four Tank System.

Problem 2
Deterministic and Stochastic Nonlinear Modeling

In the following you must select one or several set of parameters for the Modified Four Tank System. You should explain and discuss the properties of the model for the parameters that you select. E.g. select a parameter set that make the system a minimum phase system and a parameter set that makes the system a non-minimum phase system. You should select an operating point.

2.1 Deterministic Nonlinear Model

1. Develop a deterministic mathematical model for the dynamics of the system. It should be in the form $\dot{x}(t) = f(x(t), u(t), d(t), p)$.

2. Develop a mathematical model for the sensors (measurements). It should be in the form $y(t) = g(x(t), p)$.

3. Develop a mathematical model for the outputs, $z(t) = h(x(t), p)$.

2.2 Stochastic Nonlinear Model

Assume that the disturbances F_3 and F_4 are stochastic variables but piecewise constant.

1. Develop a deterministic mathematical model for the dynamics of the system. It should be in the form $\dot{x}(t) = f(x(t), u(t), d(t), p)$ with $d(t) = d_k$ for $t_k \leq t < t_{k+1}$.
2. Develop a mathematical model for the sensors (measurements). It should be in the form $y(t) = g(x(t), p) + v(t)$ with $v(t) \sim N(0, R_{vv}(p))$.
3. Develop a mathematical model for the outputs, $z(t) = h(x(t), p)$.

2.3 Stochastic Nonlinear Model (SDE)

Assume that the disturbances F_3 and F_4 can be modeled as Brownian motion with some variance.

1. Develop a mathematical model (stochastic differential equation system) for the dynamics of the system. It should be in the form

$$d\mathbf{x}(t) = f(\mathbf{x}(t), u(t), d(t), p)dt + \sigma(\mathbf{x}(t), u(t), d(t), p)d\omega(t) \tag{0.1}$$

   where ω models the unmeasured disturbances F_3 and F_4.
2. Develop a mathematical model for the sensors (measurements). It should be in the form $\mathbf{y}(t) = g(\mathbf{x}(t), p) + \mathbf{v}(t)$ with $\mathbf{v}(t) \sim N(0, R_{vv}(p))$.
3. Develop a mathematical model for the outputs, $\mathbf{z}(t) = h(\mathbf{x}(t), p)$.

Problem 3
Nonlinear Simulation - Step Responses

1. Simulate the step responses for 10%, 25% and 50% steps in the manipulated variables. Do this for the deterministic model.
2. Simulate the step responses for 10%, 25% and 50% steps in the manipulated variables. In this case you should include process and measurement noise. Try 3 different noise levels (low noise, medium noise, and high noise).
3. In all cases compute and plot (in appropriate plots) the normalized steps
4. From the normalized steps, identify a transfer function for the four tank system (transfer function from u to y).
5. Report the identified linear model estimate from the step responses. Discuss the accuracy of the model and the requirements of a step experiment.
6. Compute the corresponding impulse response coefficients (Markov paramaters, discrete-time, you choose a sampling time) and plot them in appropriate plots.

Problem 4
Linearization and Discretization

1. Compute continuous-time linearized models for the 3 models developed in Problem 2.
2. Compute the gains, poles and zeros of these models.
3. Compute the continuous-time transfer functions for the continuous-time linearized models.
4. Compare the gains and time constants to the gains and time constants obtained from the step response experiments in Problem 3.
5. Compute discrete-time state space models using a sampling time of your choice.
6. Compute the Markov parameters for these discrete-time state space models and compare them to the the Markov parameters you obtained from the step response experiments
7. Discuss and comment on the linearization approach for obtaining discrete-time linear state space models.

Problem 5
State Estimation for the Discrete-Time Linear System

1. Show how the models in Problem 3 and Problem 4 can be represented as linear state space models in discrete time.
2. Design and evaluate static and dynamic Kalman filters for the linear models identified in Problem 3 and Problem 4. You should simulate the case where the unknown disturbances are stochastic variables but do not contains step changes.
3. Design and evaluate static and dynamic Kalman filters for the linear models identified in Problem 3 and Problem 4. You should simulate the case where the unknown disturbances are stochastic variables but DO CONTAIN step changes. Design Kalman Filters that do not have steady state offsets.
4. Discuss and evaluate the Kalman filters by simulation on the linear and nonlinear models.

Problem 6
QP solver interface

Implement a QP solver interface

```
function [x,info] = qpsolver(H,g,l,u,A,bl,bu,xinit)
```

for solution of the convex quadratic program

$$
\begin{aligned}
\min_{x \in \mathbb{R}^n} \quad & \phi = \frac{1}{2} x' H x + g' x \\
s.t. \quad & l \leq x \leq u \\
& b_l \leq Ax \leq b_u
\end{aligned}
$$

using the QP solver in Matlab, i.e. `quadprog`.

Problem 7
Unconstrained MPC

1. Implement a function for design of an Unconstrained MPC based on discrete-time state space models. You should explain in the report how your Matlab function work and its theoretical background.
2. Design unconstrained MPC for the models identified in Problem 3 and Problem 4.
3. Implement and discuss a compute and prediction function for this MPC.

Problem 8
Input Constrained MPC

1. Implement a function for design of an input constrained MPC based on discrete-time state space models. You should explain in the report how your Matlab function work and its theoretical background.
2. Design input constrained MPC for the models identified in Problem 3 and Problem 4.
3. Implement and discuss a compute and prediction function for this MPC.

Problem 9
MPC with Input Constraints and Soft Output Constraints

1. Implement a function for design of an MPC based on discrete-time state space models. The MPC should have input constraints and soft constraints. You should explain in the report how your Matlab function work and its theoretical background.
2. Design the input- and soft-constrained MPC for the models identified in Problem 3 and Problem 4.
3. Implement and discuss a compute and prediction function for this MPC.

Problem 10
Closed-Loop Simulations

Do closed-simulations of your MPCs. You should do the simulation for both linear and nonlinear models. Discuss the results. Present movies and plots that illustrate the performance of your MPCs.

Problem 11 - Nonlinear MPC

In this problem you design a Nonlinear MPC based on stochastic differential equations.

1. Provide a continuous-discrete mathematical model for the modified four tank system.
2. Provide and implement a continuous-discrete extended Kalman filter for the modified four tank system and test it by simulation.
3. Implement a prediction-error method for the modified four tank system. Do a stochastic simulation and use the data from this simulation to identify parameters using the prediction-error method. Compare the identified parameters to the true parameters. Compare the predictions of the identified model to the predictions with the true model.
4. Implement a bound-constrained NMPC for the modified tank system.
5. Test the NMPC by closed-loop simulations.
6. Compare the closed-loop simulations of the NMPC to the closed-loop simulations of the linear MPC (Problem 9).

Problem 12 - Economic Linear MPC and Nonlinear MPC

In this problem we consider a controller that must minimize the pumping cost. Assume that the pumping cost is proportianl to the water pumped and that the unit pumping cost may depend on the time. Denote this unit pumping cost $c(t)$ (in discrete time c_k). In this problem we assume that the heights in lower tanks should be above a minimum level.

Linear Economic MPC:

1. Formulate the optimal control problem for the linear economic MPC.
2. Implement the optimal control problem for the linear economic MPC.
3. Do closed-loop simulations with a Kalman filter for the linear economic MPC
4. Compare and discuss the linear economic MPC to other controllers

Nonlinear Economic MPC:

1. Formulate the optimal control problem for the nonlinear economic MPC.
2. Implement the optimal control problem for the nonlinear economic MPC.
3. Do closed-loop simulations with an Extendede Kalman filter for the linear economic MPC
4. Compare and discuss the nonlinear economic MPC to other controllers

Problem 13 - PID control

1. Discuss the pairing of inputs and outputs for the four tank system.
2. Implement a P-, a PI- and a PID-controller for the four tank system.
3. Test the controllers by closed-loop simulation
4. Compare the PID type controllers to other controllers e.g. MPC.

Problem 14
Discussion and Conclusion

Discuss and comment on your results. Provide a discussion on the pros and conss of the different controllers.