

Proof : The period of Trigonometric functions in complex world is 2π

$$\theta = \frac{\ln(i \sin \theta \pm \sqrt{1 - \sin^2 \theta})}{i} = \frac{\ln|i \sin \theta \pm \sqrt{1 - \sin^2 \theta}| + i \operatorname{Arg}(i \sin \theta \pm \sqrt{1 - \sin^2 \theta})}{i}$$

$$= \frac{\ln|i \sin \theta \pm \sqrt{1 - \sin^2 \theta}| + i \arg(i \sin \theta \pm \sqrt{1 - \sin^2 \theta}) + 2k\pi i}{i} \quad (k \in \mathbb{Z})$$

$$= \frac{\ln|i \sin \theta \pm \sqrt{1 - \sin^2 \theta}| + i \arg(i \sin \theta \pm \sqrt{1 - \sin^2 \theta})}{i} + 2k\pi$$

$$\theta = \frac{\ln(\cos \theta \pm \sqrt{\cos^2 \theta - 1})}{i} = \frac{\ln|\cos \theta \pm \sqrt{\cos^2 \theta - 1}| + i \operatorname{Arg}(\cos \theta \pm \sqrt{\cos^2 \theta - 1})}{i}$$

$$= \frac{\ln|\cos \theta \pm \sqrt{\cos^2 \theta - 1}| + i \arg(\cos \theta \pm \sqrt{\cos^2 \theta - 1}) + 2k\pi i}{i} \quad (k \in \mathbb{Z})$$

$$= \frac{\ln|\cos \theta \pm \sqrt{\cos^2 \theta - 1}| + i \arg(\cos \theta \pm \sqrt{\cos^2 \theta - 1})}{i} + 2k\pi$$

$$\theta = \frac{\ln\left(\frac{\pm i \sqrt{\tan^2 \theta + 1}}{\tan \theta + 1}\right)}{i} = \frac{\ln\left|\frac{\pm i \sqrt{\tan^2 \theta + 1}}{\tan \theta + 1}\right| + i \operatorname{Arg}\left(\frac{\pm i \sqrt{\tan^2 \theta + 1}}{\tan \theta + 1}\right)}{i}$$

$$= \frac{\ln\left|\frac{\pm i \sqrt{\tan^2 \theta + 1}}{\tan \theta + 1}\right| + i \arg\left(\frac{\pm i \sqrt{\tan^2 \theta + 1}}{\tan \theta + 1}\right) + 2k\pi i}{i} \quad (k \in \mathbb{Z})$$

$$= \frac{\ln\left|\frac{\pm i \sqrt{\tan^2 \theta + 1}}{\tan \theta + 1}\right| + i \arg\left(\frac{\pm i \sqrt{\tan^2 \theta + 1}}{\tan \theta + 1}\right)}{i} + 2k\pi$$

Find the relationship between the plus minus sign and Trigonometric Identities

$$\pi - \theta = \pi - \frac{\ln(i \sin \theta \pm \sqrt{1 - \sin^2 \theta})}{i} = \frac{\pi i - \ln(i \sin \theta \pm \sqrt{1 - \sin^2 \theta})}{i}$$

$$= \frac{\ln(-1) - \ln(i \sin \theta \pm \sqrt{1 - \sin^2 \theta})}{i} = \frac{\ln\left(\frac{-1}{i \sin \theta \pm \sqrt{1 - \sin^2 \theta}}\right)}{i}$$

$$= \frac{\ln\left(\frac{-(i \sin \theta \mp \sqrt{1 - \sin^2 \theta})}{(i \sin \theta \pm \sqrt{1 - \sin^2 \theta})(i \sin \theta \mp \sqrt{1 - \sin^2 \theta})}\right)}{i}$$

$$= \frac{\ln\left(\frac{-(i \sin \theta \mp \sqrt{1 - \sin^2 \theta})}{(i \sin \theta)^2 - (\sqrt{1 - \sin^2 \theta})^2}\right)}{i} = \frac{\ln\left(\frac{-(i \sin \theta \mp \sqrt{1 - \sin^2 \theta})}{-\sin^2 \theta - (1 - \sin^2 \theta)}\right)}{i}$$

$$= \frac{\ln\left(\frac{-(i \sin \theta \mp \sqrt{1 - \sin^2 \theta})}{-1}\right)}{i} = \frac{\ln(i \sin \theta \mp \sqrt{1 - \sin^2 \theta})}{i}$$

$$2\pi - \theta = 2\pi - \frac{\ln(\cos \theta \pm \sqrt{\cos^2 \theta - 1})}{i} = \frac{2\pi i - \ln(\cos \theta \pm \sqrt{\cos^2 \theta - 1})}{i}$$

$$= \frac{\ln(1) - \ln(\cos \theta \pm \sqrt{\cos^2 \theta - 1})}{i} = \frac{\ln\left(\frac{1}{\cos \theta \pm \sqrt{\cos^2 \theta - 1}}\right)}{i}$$

$$= \frac{\ln\left(\frac{\cos \theta \mp \sqrt{\cos^2 \theta - 1}}{(\cos \theta \pm \sqrt{\cos^2 \theta - 1})(\cos \theta \mp \sqrt{\cos^2 \theta - 1})}\right)}{i}$$

$$= \frac{\ln\left(\frac{\cos \theta \mp \sqrt{\cos^2 \theta - 1}}{(\cos \theta)^2 - (\sqrt{\cos^2 \theta - 1})^2}\right)}{i} = \frac{\ln\left(\frac{\cos \theta \mp \sqrt{\cos^2 \theta - 1}}{\cos^2 \theta - (\cos^2 \theta - 1)}\right)}{i}$$

$$= \frac{\ln\left(\frac{\cos \theta \mp \sqrt{\cos^2 \theta - 1}}{1}\right)}{i} = \frac{\ln(\cos \theta \mp \sqrt{\cos^2 \theta - 1})}{i}$$

$$\pi + \theta = \pi + \frac{\ln\left(\frac{\pm i \sqrt{\tan^2 \theta + 1}}{\tan \theta + 1}\right)}{i} = \frac{\pi i + \ln\left(\frac{\pm i \sqrt{\tan^2 \theta + 1}}{\tan \theta + 1}\right)}{i}$$

$$= \frac{\ln(-1) + \ln\left(\frac{\pm i \sqrt{\tan^2 \theta + 1}}{\tan \theta + 1}\right)}{i} = \frac{\ln\left((-1)\left(\frac{\pm i \sqrt{\tan^2 \theta + 1}}{\tan \theta + 1}\right)\right)}{i} = \frac{\ln\left(\frac{\mp i \sqrt{\tan^2 \theta + 1}}{\tan \theta + 1}\right)}{i}$$