



$$\begin{aligned}
 e^{ix} &= 1 + ix + \frac{(ix)^2}{2!} + \frac{(ix)^3}{3!} + \frac{(ix)^4}{4!} + \frac{(ix)^5}{5!} + \frac{(ix)^6}{6!} + \frac{(ix)^7}{7!} + \frac{(ix)^8}{8!} + \dots \\
 &= 1 + ix - \frac{x^2}{2!} - \frac{ix^3}{3!} + \frac{x^4}{4!} + \frac{ix^5}{5!} - \frac{x^6}{6!} - \frac{ix^7}{7!} + \frac{x^8}{8!} + \dots \\
 &= \left(1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} - \dots\right) + i \left(x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots\right) \\
 &= \cos x + i \sin x.
 \end{aligned}$$

$$\begin{aligned}
 \cos \theta &= \frac{a}{r} \\
 \sin \theta &= \frac{b}{r}
 \end{aligned}$$

$$\theta = a \cos \theta$$

$$\theta = b \sin \theta$$

$$z = a + bi$$

$$z = r \cos \theta + i r \sin \theta$$

$$z = r(\cos \theta + i \sin \theta)$$

$$\ln(z) = \ln(r) + \ln(\cos \theta + i \sin \theta)$$

$$\ln(z) = \ln(r) + \ln(e^{i\theta})$$

$$\ln(z) = \ln(r) + i\theta$$

$$(x + ix_i)^{y+iy_i}$$

$$= e^{\ln(x+ix_i)^{y+iy_i}}$$

$$= e^{(y+iy_i) \ln(x+ix_i)}$$

$$= e^{(y+iy_i)(t+it_i)}$$

$$= e^{yt+iyt_i+iy_it-y_it_i}$$

$$= e^{yt-y_it_i+i(yt_i+y_it)}$$

$$= e^{yt-y_it_i} \cdot e^{i(yt_i+y_it)}$$

$$= e^{yt-y_it_i} (\cos(yt_i + y_it) + i \sin(yt_i + y_it))$$

$$= e^{yt-y_it_i} \cos(yt_i + y_it)$$

$$+ i e^{yt-y_it_i} \sin(yt_i + y_it)$$

$$\ln(x + ix_i)$$

$$= \ln \sqrt{x^2 + x_i^2} + i \cos^{-1} \frac{x}{\sqrt{x^2 + x_i^2}}$$

$$\text{let } t = \ln \sqrt{x^2 + x_i^2}$$

$$\text{let } t_i = \cos^{-1} \frac{x}{\sqrt{x^2 + x_i^2}}$$

$$\ln \sqrt{x^2 + x_i^2} + i \cos^{-1} \frac{x}{\sqrt{x^2 + x_i^2}}$$

$$= t + it_i$$