

$$egin{aligned} e^{ix} &= 1 + ix + rac{(ix)^2}{2!} + rac{(ix)^3}{3!} + rac{(ix)^4}{4!} + rac{(ix)^5}{5!} + rac{(ix)^6}{6!} + rac{(ix)^7}{7!} + rac{(ix)^8}{8!} + \cdots \ &= 1 + ix - rac{x^2}{2!} - rac{ix^3}{3!} + rac{x^4}{4!} + rac{ix^5}{5!} - rac{x^6}{6!} - rac{ix^7}{7!} + rac{x^8}{8!} + \cdots \ &= \left(1 - rac{x^2}{2!} + rac{x^4}{4!} - rac{x^6}{6!} + rac{x^8}{8!} - \cdots 
ight) + i\left(x - rac{x^3}{3!} + rac{x^5}{5!} - rac{x^7}{7!} + \cdots 
ight) \end{aligned}$$

 $=\cos x + i\sin x.$ 

$$\cos \theta = \frac{a}{r}$$

$$\sin \theta = \frac{b}{r}$$

$$\theta = a \cos \theta$$

$$\theta = b \sin \theta$$

$$z = a + bi$$

$$z = r \cos \theta + i r \sin \theta$$

$$z = r(\cos \theta + i \sin \theta)$$

$$ln(z) = ln(r) + ln(\cos \theta + i \sin \theta)$$

$$ln(z) = ln(r) + ln(e^{i\theta})$$

$$ln(z) = ln(r) + i\theta$$

$$(x + ix_i)^{y+iy_i}$$

$$= e^{ln(x+ix_i)^{y+iy_i}}$$

$$= e^{(y+iy_i)ln(x+ix_i)}$$

$$= e^{(y+iy_i)ln(x+ix_i)}$$

$$= e^{yt+iyt_i+iy_it-y_it_i}$$

$$= e^{yt-y_it_i+i(yt_i+y_it)}$$

$$= e^{yt-y_it_i} \cdot e^{i(yt_i+y_it)}$$

$$= e^{yt-y_it_i} (\cos(yt_i + y_it) + i \sin(yt_i + y_it))$$

$$= e^{yt-y_it_i} \cos(yt_i + y_it)$$

$$+ ie^{yt-y_it_i} \sin(yt_i + y_it)$$

$$ln(x + ix_i) = ln \sqrt{x^2 + x_i^2} + i \cos^{-1} \frac{x}{\sqrt{x^2 + x_i^2}}$$

$$let t = ln \sqrt{x^2 + x_i^2}$$

$$let t_i = \cos^{-1} \frac{x}{\sqrt{x^2 + x_i^2}}$$

$$ln \sqrt{x^2 + x_i^2} + i \cos^{-1} \frac{x}{\sqrt{x^2 + x_i^2}}$$

$$= t + it_i$$