Python Code for Monte Carlo Simulation of Selected Processes with Applications to Finance

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1 Geometric Brownian Motion

$$dS_t = uS_t dt + \sigma S_t dW_t$$

code used to generate process:

```
 \begin{array}{lll} & \text{for i in range} \, (2\,,100) \colon \\ & \text{sp.iloc} \, [\,i\,,1\,] \, = \, \text{sp.iloc} \, [\,i\,-1\,,1\,] \, + \, \text{mu*delta} \, + \, \text{sigma*math.sqrt} \, (\,\text{delta}\,) * \text{np.random.normal} \, (0\,,1) \\ & \text{print} \, (\,\text{jdx}\,) \end{array}
```

2 Ornstein - Uhlenbeck Process

$$dS_t = \theta(u - xt) + \sigma S_t dW_t$$

code used to generate process:

```
 \begin{array}{lll} \text{for x in range(2,100):} \\ & \text{spou.iloc[x,1] = spou.iloc[x-1,1]*exp(-lambda1*delta1) + mul*(1-math.exp(-lambda1*delta1)) + (sig1*sqrt((1-math.exp(-lambda1*delta1))) + (sig1*sqrt((1-math.exp(-lambda1*delta1)))) + (sig1*sqrt((1-math.exp(-lambda1*delta1)))) + (sig1*sqrt((1-math.exp(-lambda1*delta1)))) } \end{array}
```

3 Process with Synthetic Trends

$$dS_t = u\Delta_t + \sigma\sqrt{\Delta_t} + \Delta_t\lambda\sum_{i=N}^{N} \frac{1}{N}r_{t-i}$$

code used to generate process:

```
 \begin{array}{l} \text{for i in } \operatorname{range}(21,100) \colon \\ \text{for k in } \operatorname{range}(i-n,i) \colon \\ \text{sum1} = \operatorname{sum1} + \operatorname{sp.iloc}\left[k,1\right] \\ \text{spm.iloc}\left[i,1\right] = \operatorname{spm.iloc}\left[i-1,1\right] + \operatorname{mu*delta} + \operatorname{sigma*math.sqrt}(\operatorname{delta}) * \operatorname{np.random.normal}(0,1) + \operatorname{delta*lamda*}(\operatorname{sum1/normal}(0,1)) \\ \text{spm.iloc}\left[i,1\right] = \operatorname{spm.iloc}\left[i-1,1\right] + \operatorname{mu*delta} + \operatorname{sigma*math.sqrt}(\operatorname{delta}) * \operatorname{np.random.normal}(0,1) \\ \text{spm.iloc}\left[i,1\right] = \operatorname{spm.iloc}\left[i-1,1\right] + \operatorname{mu*delta} + \operatorname{sigma*math.sqrt}(\operatorname{delta}) * \operatorname{np.random.normal}(0,1) \\ \text{spm.iloc}\left[i,1\right] = \operatorname{spm.iloc}\left[i-1,1\right] + \operatorname{mu*delta} + \operatorname{sigma*math.sqrt}(\operatorname{delta}) * \operatorname{np.random.normal}(0,1) \\ \text{spm.iloc}\left[i,1\right] = \operatorname{spm.iloc}\left[i-1,1\right] + \operatorname{mu*delta} + \operatorname{sigma*math.sqrt}(\operatorname{delta}) * \operatorname{np.random.normal}(0,1) \\ \text{spm.iloc}\left[i,1\right] = \operatorname{spm.iloc}\left[i-1,1\right] + \operatorname{mu*delta} + \operatorname{sigma*math.sqrt}(\operatorname{delta}) * \operatorname{np.random.normal}(0,1) \\ \text{spm.iloc}\left[i,1\right] = \operatorname{spm.iloc}\left[i-1,1\right] + \operatorname{mu*delta} + \operatorname{sigma*math.sqrt}(\operatorname{delta}) * \operatorname{np.random.normal}(0,1) \\ \text{spm.iloc}\left[i,1\right] + \operatorname{spm.iloc}\left[i-1,1\right] + \operatorname{mu*delta} + \operatorname{sigma*math.sqrt}(\operatorname{delta}) * \operatorname{np.random.normal}(0,1) \\ \text{spm.iloc}\left[i,1\right] + \operatorname{spm.iloc}\left[i-1,1\right] + \operatorname{mu*delta} + \operatorname{sigma*math.sqrt}(\operatorname{delta}) * \operatorname{np.random.normal}(0,1) \\ \text{spm.iloc}\left[i,1\right] + \operatorname{spm.iloc}\left[i-1,1\right] + \operatorname{mu*delta} + \operatorname{sigma*math.sqrt}(\operatorname{delta}) * \operatorname{np.random.normal}(0,1) \\ \text{spm.iloc}\left[i,1\right] + \operatorname{spm.iloc}\left[i,1\right] + \operatorname{spm.iloc}\left[i,1\right]
```

4 Process with Local Volatility

$$dS_t = \theta(u - xt) + \sigma S_t dW_t$$

code used to generate process:

```
 \begin{array}{lll} \text{for i in } & \text{range} \, (2\,, 10\,0) \colon \\ & & \text{spsk.iloc} \, [\, i\,, 1\,] \, = \, \text{spsk.iloc} \, [\, i\,-1\,, 1\,] \, + \, \text{mu*delta} \, + \, \text{sigma*math.sqrt} \, (\, \text{delta}\,) * \, \text{np.random.normal} \, (0\,, 1) \end{array}
```

5 Jump Diffusion Process

Here, the jump magnitudes J are i.i.d. r.v.'s, i.e. the jump-size J is selected by drawing from a pre-specified probability distribution.

$$dS_t = uS_t dt + \sigma S_t dW_t + J dN$$

6 Short-term momentum plus Long-term mean reversion

This process is specified in the following way

$$dS_t = \theta(u - xt) + \sigma S_t dW_t + \Delta_t \lambda \sum_{i=1}^{N} \frac{1}{N} r_{t-i}$$

code used to generate process: This needs to be changed.

```
 \begin{array}{lll} \text{for x in range(2,100):} \\ & \text{spou.iloc[x,1] = spou.iloc[x-1,1]*exp(-lambda1*delta1) + mul*(1-math.exp(-lambda1*delta1)) + (sig1*sqrt((1-math.exp(-lambda1*delta1))) + (sig1*sqrt((1-math.exp(-lambda1*delta1)))) + (sig1*sqrt((1-math.exp(-lambda1*delta1)))) + (sig1*sqrt((1-math.exp(-lambda1*delta1)))) } \end{array}
```