parabolic_excercise

April 6, 2022

1 Problem

$$\partial_t u - \nabla \cdot K(\nabla u)\nabla u = f \tag{1}$$

$f = f(x,t)$ is a time dependent forcing term and the diffusion tensor is

$$K(\nabla u) = \frac{2}{1+\sqrt{1+4|\nabla u|}} \tag{2}$$

On the boundary we prescribe Neumann boundary conditions and require initial conditions $u(\cdot,0) = u_0(\cdot)$.

We solve this problem in variational form using Implicit Euler in time

$$\int_\Omega \frac{u^{n+1}-u^n}{\Delta t}\varphi + K(\nabla u^{n+1})\nabla u^{n+1}\cdot\nabla\varphi\, dx = \int_\Omega f(x,t^n+\Delta t)\varphi\, dx + \int_{\partial\Omega} g(x,t^n+\Delta t)\varphi\, ds\ .$$

on a domain $\Omega = [0,1]^2$. We choose f, g so that the exact solution is

$$u(x,t) = e^{-2t}\left(\frac{1}{2}(x^2+y^2) - \frac{1}{3}(x^3-y^3)\right) + 1$$

Setup grid and space

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GridParameterBlock: Parameter 'refinementedge' not specified, defaulting to
'ARBITRARY'.
WARNING (ignored): Could not open file 'alugrid.cfg', using default values 0 <
[balance] < 1.2, partitioning method 'ALUGRID_SpaceFillingCurve(9)'.

You are using DUNE-ALUGrid, please don't forget to cite the paper:
Alkaemper, Dedner, Kloefkorn, Nolte. The DUNE-ALUGrid Module, 2016.


Created parallel ALUGrid<2,2,simplex,conforming> from input stream.
```

Import UFL variables and define spatial coordinate, test/trial function

Define initial condition and exact solution

Define discrete functions, one for u^{n+1} and one for u^n. We also define two constants which can be used as floats in ufl expressions but can be changed at a later stage without requiring any recompilation.

Now setup the model and the scheme:

$$\int_\Omega \frac{u^{n+1}-u^n}{\Delta t}\varphi + K(\nabla u^{n+1})\nabla u^{n+1}\cdot\nabla\varphi\, dx = \int_\Omega f(x,t^n+\Delta t)\varphi\, dx + \int_{\partial\Omega} g(x,t^n+\Delta t)\varphi\, ds\ .$$

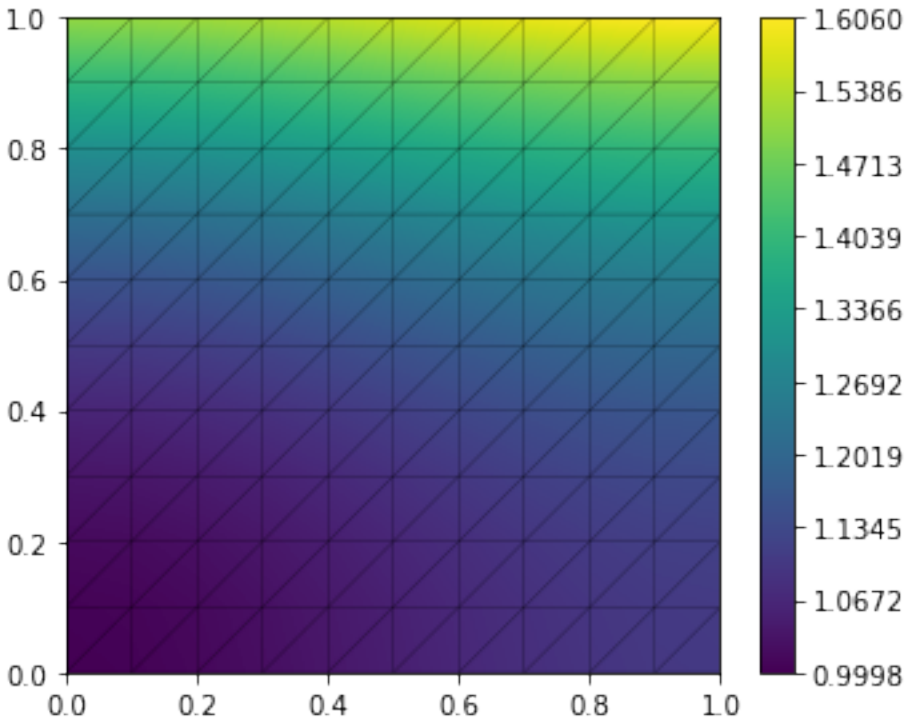
1.1 Task

Implement the form **a** and right hand side **b** as described above, using terms K, f, and g.

A Galerkin scheme is created that allows us to solve the equation $a = b$.

Time loop: first choose a time step and then iterate from $t^0 = 0$ to $t^N = N\Delta t = 0.25$. We write vtk files every after every 0.05 time interval.

Plot result and compute error

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error at final time= 0.16418668639785838
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