uflvector_excercise

April 6, 2022

1 Problem

We want to use quadratic finite elements over

$$\Omega = [-\frac{1}{2}, \frac{1}{2}]^2$$

to solve the vector valued PDE:

$$\begin{aligned} -\Delta u_1 + u_1 &= f_1 \ , \qquad \nabla u_1 \cdot n = 0 \ , \\ -\Delta u_2 + u_2 &= f_2 \ , \qquad \nabla u_2 \cdot n = g \ . \end{aligned}$$

where f_1, f_2, g are chosen so that

$$u_1 = \sin(\pi x) + \sin(\pi y) \ , \qquad u_2 = \sin(4\pi x \cdot x) \ .$$

1.1 Necessary imports

1.2 Setup of grid and space

We want to setup a quadratic finite element space V_h over $\mathcal{T}_h$.

```
Created parallel ALUGrid<2,2,simplex,conforming> from input stream.
```

```
GridParameterBlock: Parameter 'refinementedge' not specified, defaulting to
'ARBITRARY'.
WARNING (ignored): Could not open file 'alugrid.cfg', using default values 0 <
[balance] < 1.2, partitioning method 'ALUGRID_SpaceFillingCurve(9)'.
```

```
You are using DUNE-ALUGrid, please don't forget to cite the paper:
Alkaemper, Dedner, Kloefkorn, Nolte. The DUNE-ALUGrid Module, 2016.
```

The exact solution is given by $u = (u_1, u_2)^T$.

The weak form is given by

$$\begin{aligned} a(u, v) &= \int_\Omega \nabla u \nabla v + uv \, dx \\ b(v) &= \int_\Omega (\Delta u_1 + u_1) v_1 \, dx + \int_\Omega (\Delta u_2 + u_2) v_2 \, dx + \int_\Omega (\nabla u_2 \cdot n) v_2 \, ds \end{aligned}$$

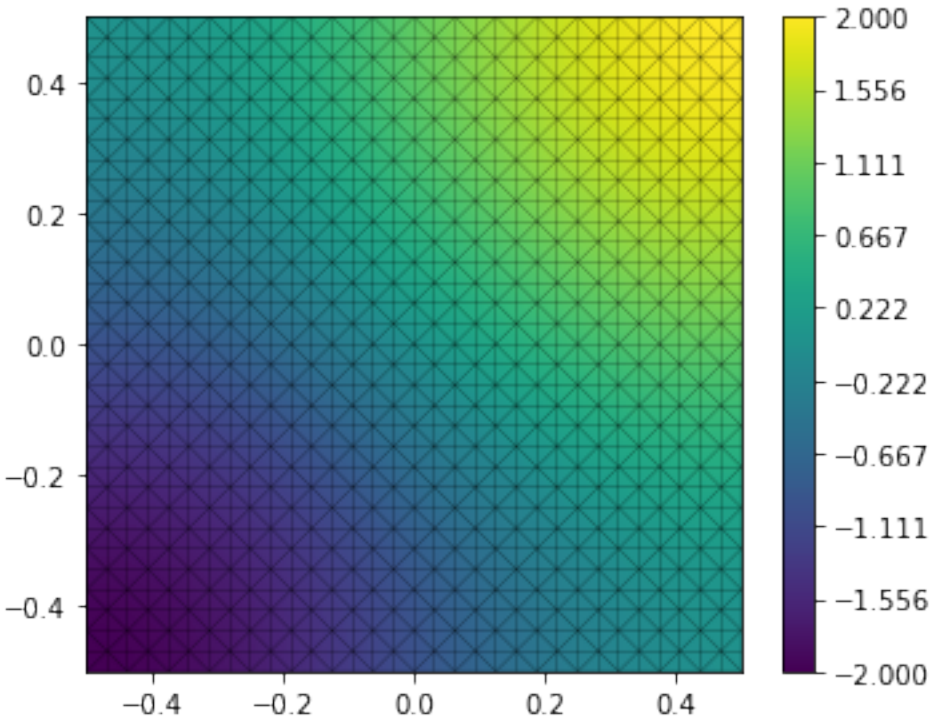
By default the scheme uses a *gmres* solver but the given bilinear form is symmetric so we can use *cg* instead:

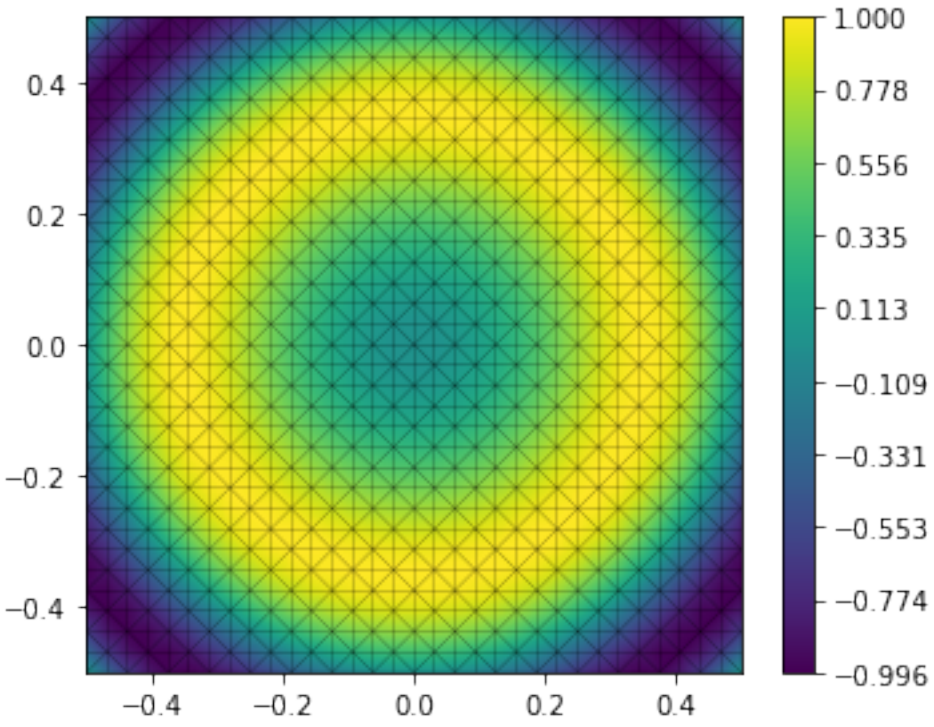
Create $u_h \in V_h$.

Refine the grid a few times and on each level compute the solution and compute the errors

$$\|e_h\|_0^2 := \int_\Omega |e_h|^2 \ , \qquad \|e_h\|_1^2 := \int_\Omega |\nabla e_h|^2 \ .$$

```
128 578 [0.01040168 0.60037702]
512 2178 [0.00181493 0.21224711]
2048 8450 [0.00024122 0.05405574]
```




0.4
0.2
0.0
−0.2
−0.4
−0.4
−0.2
0.0
0.2
0.4
1.000
0.778
0.556
0.335
0.113
−0.109
−0.331
−0.553
−0.774
−0.996