DPIR Experimental Methods Lecture 6

Adaptive Design

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Road Map

- Treatment adaptive design
- Thompson Sampling
- Augmented Controls
- Exploration Sampling

Treatment-adaptive designs

Types of Sequential Randomised Experiments

- Non-adaptive - assignment probabilities fixed
- Treatment-adapative - change based on number of subjects in treatment
- Covariate-adaptive - change based on covariate profiles of new and previous subjects
- Responsive-adaptive - change as function of previous units' outcomes

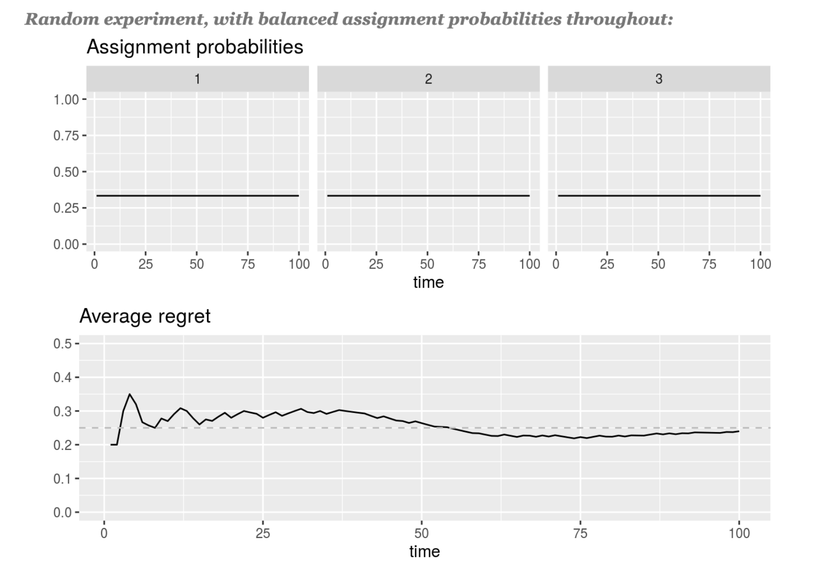
Treatment-adaptive designs

- *ATE* is not always quantity of interest
- Particularly online firms such as Google, Tiktok, Meta, etc.
    - Randomly assign sampled users to different arms and dynamically re-orient sample based on which is more successful/more informative
    - Identify which of many will get the most clicks
- But also of interest to political scientists: Ballot initiatives and malfeasance information
- How do adaptive multi-arm trials work?

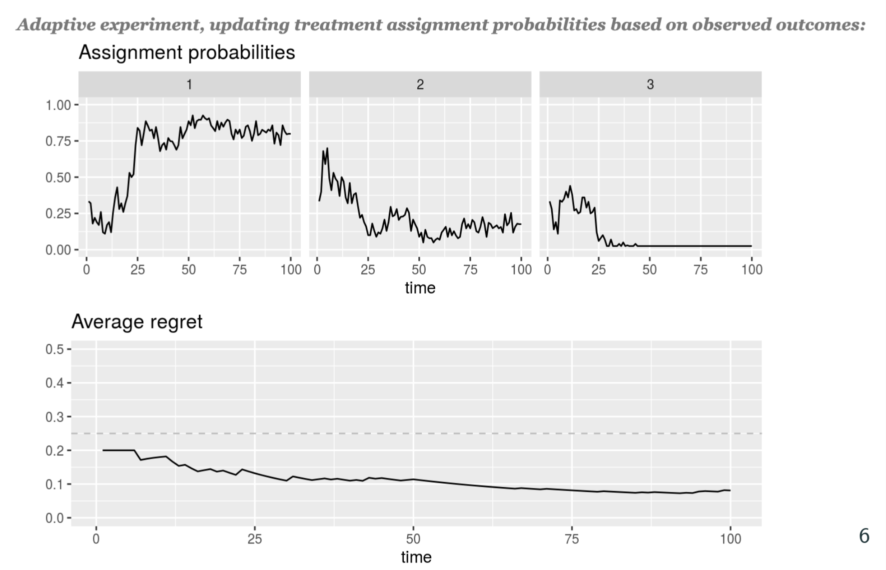
Regret

- The difference between the average outcomes we would have observed under optimal assignment and the average outcomes we actually observe under a given assignment algorithm
- Example
    - if the best prototype gives us a 90% click-through rate on average
    - a different prototype gives us a 40% rate on average
    - the regret from assigning the sub-optimal arm is 0.5

Regret: True arms 1 (.8) 2 (.6) 3 (.3)



Regret: True arms 1 (.8) 2 (.6) 3 (.3)



Treatment-adaptive design: Thompson Sampling

- When researchers are initially agnostic about the relative performance of the K arms, priors are distributed uniformly over parameter space, i.e. $\beta_{1,1}$
- In each t, treatment is randomly assigned according to probability of arms being best (= highest success rate)

$$P\left[\Theta_k = \max_k\{\Theta_1, ..., \Theta_K\} | (X_1^{n_{1,t}}, ..., X_K^{n_{K,t}})\right]$$

for K arms, vector of responses under treatment arm k observed up until and including t $X_k^{\{n_{k,t}\}}$ and Θ_k distributions of success rates

best_binomial_bandit

```
c(10,20,30,50)
n=c(100,102,120,130)
arm_probabilities = best_binomial_bandit(x,n)
print(arm_probabilities)
[1] 1.611266e-07 8.048293e-04 1.142867e-02 9.877663e-01
sum(arm_probabilities)
[1] 1
```

Treatment-adaptive design: Thompson Sampling

- Simulations to illustrate design and estimation
- Sample 100 observations for each of 10 periods, updating posterior probability of being best after each period, and assign treatment probabilities in the subsequent period accordingly
- In the first case, one arm has a true 0.20 probability of success, and the remaining 8 arms have a 0.10 probability of success

Treatment-adaptive designs

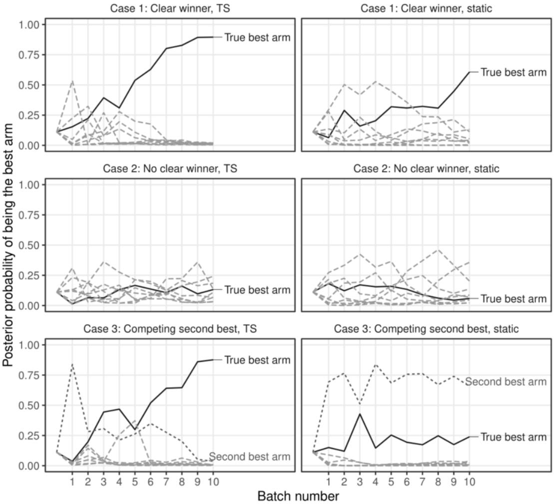
TABLE 1 Iterated Simulation Statistics

Design			RMSE		Coverage	
Assignment algorithm	**Case**	**Best arm selected**	**Best arm**	**ATE**	**Best arm**	**ATE**
TS	1: Clear winner	0.968	0.021	–	0.958	–
	2: No clear winner	0.193	0.033	–	0.880	–
	3: Competing second best	0.715	0.025	–	0.956	–
Static	1: Clear winner	0.909	0.031	0.038	0.941	0.949
	2: No clear winner	0.180	0.024	0.033	0.935	0.947
	3: Competing second best	0.635	0.031	0.038	0.940	0.945
TS,	1: Clear winner	0.956	0.023	0.029	0.957	0.952
Control-Augmented	2: No clear winner	0.174	0.034	0.041	0.879	0.886
	3: Competing second best	0.683	0.029	0.035	0.946	0.937

Note: Assignment algorithms are Thompson sampling (TS), balanced static design (Static), and control-augmented Thompson sampling (TS, Control-Augmented). "Best arm selected" column presents the portion of simulations under which the true best arm was selected. RMSE is average root mean squared error of the estimate of the mean of the true best arm, and the average treatment effect of the true best arm relative to the control. Coverage is with respect to 95% confidence intervals around the estimate. In all cases one of the inferior arms with a true success rate of 0.10 is selected as the control comparison.

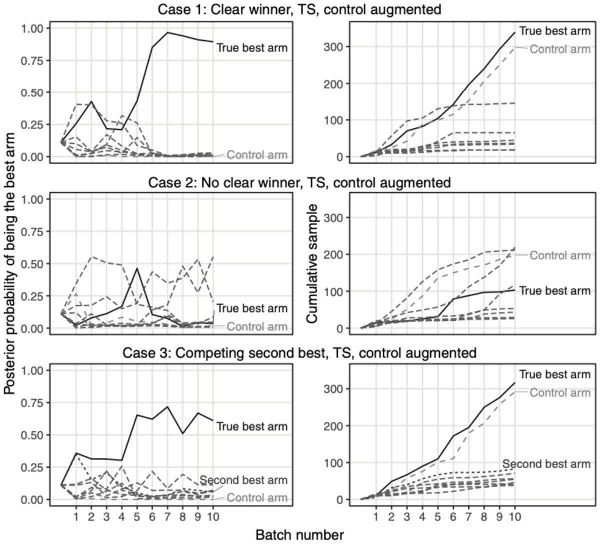
Treatment-adaptive designs

Figure 1: Simulated Posterior Probabilities Over Time, Thompson Sampling and Static Designs



Treatment-adaptive designs

FIGURE 2 Simulated Posterior Probabilities over Time and Cumulative Sample, Control-Augmented Adaptive Design



Molly Offer-Westort et al 2020

Table 2: Study One, Treatments and Outcome Measures

	Minimum Wage	Right to Work
Question Text	Imagine that the following ballot measure were up for a vote in your state. [**ballot measure text**]. If this measure were on the ballot in your state, would you vote in favor or against? [I would vote in favor of this measure; I would vote against this measure]	Imagine that the following ballot measure were up for a vote in your state. [**ballot measure text**]. If this measure were on the ballot in your state, would you vote in favor or against? [I would vote in favor of this measure; I would vote against this measure]
Proposal 1	The measure would: increase the minimum wage [**from {current}**] to {current + 1} per hour, adjusted annually for inflation, and provide that no more than $3.02 per hour in tip income may be used to offset the minimum wage of employees who regularly receive tips.	The measure would [**amend the State Constitution to**]: prohibit, as a condition of employment, forced membership in a labor organization (union) or forced payments of dues or fees, in full or pro-rata ("fair-share"), to a union. The measure will also make any activity which violates employees' rights provided by the bill illegal and ineffective and allow legal remedies for anyone injured as a result of another person violating or threatening to violate those employees' rights. The measure will not apply to union agreements entered into before the effective date of the measure, unless those agreements are amended or renewed after the effective date of the measure.
Proposal 2	The measure would: raise the minimum wage [**from {current}**] to {current + 1} per hour effective September 30th, 2021. Each September 30th thereafter, minimum wage shall increase by $1.00 per hour until the minimum wage reaches {current + 5} per hour on September 30th, 2026. From that point forward, future minimum wage increases shall revert to being adjusted annually for inflation starting September 30th, 2027.	The measure [**reads / would amend the State Constitution to read**]: The right of persons to work may not be denied or abridged on account of membership or nonmembership in any labor union or labor organization, and all contracts in negation or abrogation of such rights are hereby declared to be invalid, void, and unenforceable.
Proposal 3	The measure reads: Shall the minimum wage for adults over the age of 18 be raised [**from {current}**] to {current + 1} per hour by January 1, 2019?	The measure would [**amend the State Constitution to**]: ban any new employment contract that requires employee to resign from or belong to a union, pay union dues, or make other payment to a union. Required contributions to charity or other third party instead of payments to union are also banned. Employees must authorize payroll deduction to unions. Violations of the section is a misdemeanor.
Proposal 4	The measure would: raise the minimum wage [**from {current}**] to {current + 1} per hour worked if the employer provides health benefits, or {current + 2} per hour worked if the employer does not provide health benefits.	The measure [**reads / would amend the State Constitution to read**]: No person shall be deprived of life, liberty or property without due process of law. The right of persons to work shall not be denied or abridged on account of membership or nonmembership in any labor union, or labor organization.
Proposal 5	The measure would: raise the State minimum wage rate [**from {current}**] to at least {current + 1} per hour, and require annual increases in that rate if there are annual increases in the cost of living.	

Boldface text indicates randomly varied elements.

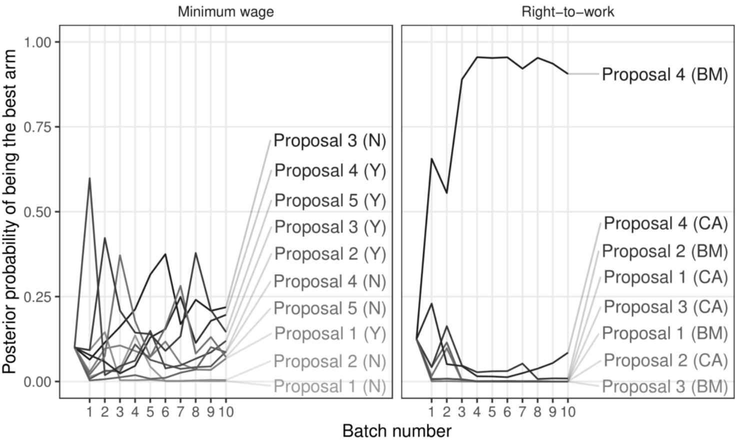
Ballot Initiative Ex. from Offer-Westort et al 2020

	id	Y_mw	Y_rtw	Z_mw	Z_rtw	weights_rtw	prob_rtw	weights_mw	prob_mw	batch
1	1	1	1	Proposal 4 (N)	Proposal 2 (CA)	8.000000	0.12500	10.000000	0.10000	1
2	2	1	1	Proposal 4 (N)	Proposal 4 (BM)	8.000000	0.12500	10.000000	0.10000	1
3	3	1	0	Proposal 3 (N)	Proposal 1 (CA)	8.000000	0.12500	10.000000	0.10000	1
4	4	1	0	Proposal 2 (N)	Proposal 3 (BM)	8.000000	0.12500	10.000000	0.10000	1
5	5	1	1	Proposal 2 (N)	Proposal 2 (BM)	8.000000	0.12500	10.000000	0.10000	1
6	6	1	1	Proposal 3 (N)	Proposal 4 (BM)	8.000000	0.12500	10.000000	0.10000	1
7	7	1	1	Proposal 1 (N)	Proposal 3 (BM)	8.000000	0.12500	10.000000	0.10000	1
8	8	1	1	Proposal 2 (Y)	Proposal 3 (BM)	8.000000	0.12500	10.000000	0.10000	1
9	9	1	1	Proposal 4 (Y)	Proposal 3 (BM)	8.000000	0.12500	10.000000	0.10000	1
10	10	1	1	Proposal 3 (N)	Proposal 1 (CA)	8.000000	0.12500	10.000000	0.10000	1
11	11	1	1	Proposal 3 (Y)	Proposal 1 (CA)	8.000000	0.12500	10.000000	0.10000	1
12	12	1	1	Proposal 5 (Y)	Proposal 4 (CA)	8.000000	0.12500	10.000000	0.10000	1
13	13	0	1	Proposal 5 (Y)	Proposal 2 (BM)	8.000000	0.12500	10.000000	0.10000	1
14	14	1	1	Proposal 5 (Y)	Proposal 3 (CA)	8.000000	0.12500	10.000000	0.10000	1
15	15	1	1	Proposal 1 (Y)	Proposal 1 (BM)	8.000000	0.12500	10.000000	0.10000	1
16	16	1	1	Proposal 3 (Y)	Proposal 4 (CA)	8.000000	0.12500	10.000000	0.10000	1
17	17	1	0	Proposal 1 (Y)	Proposal 2 (CA)	8.000000	0.12500	10.000000	0.10000	1
18	18	1	1	Proposal 5 (Y)	Proposal 1 (BM)	8.000000	0.12500	10.000000	0.10000	1

Showing 1 to 18 of 1,000 entries, 10 total columns

Treatment-adaptive designs

Figure 3: Study One, Overtime Posterior Probabilities



Variations on Thompson Sampling

Augmented Control (Offer-Westort et al 2021)

- two treatment arms, with success rates $T1$ and $T2$; control arm, with success rate C
- The true, but unknown values of $T1 = 0.75$ and $T2 = 0.5$ and $C = 0.25$;
- set prior for all arms as uniform over the parameter space, i.e., Beta(1; 1)
- Batch design:
  - 9 observations per period
  - period 1: 3 observations per arm
  - then control augmented assignment in next 2 periods

Augmented Control Example: Period 1

- Results:
  - T1: 3 successes
  - T2: 1 success/2 failures
  - C: 3 failures

- Posteriors:
  - θ_{T1}: Beta(4,1)
  - θ_{T2}: Beta(2,3)
  - θ_C: Beta(1,4)

Augmented Control Example

Probability that $T1$ is the best control arm:

$$
\begin{aligned}
&P\left(\Theta_{T1} \geq \Theta_{T2} \middle| X_{T1}^{\{3\}} = x_{T1}^{\{3\}}, X_{T2}^{\{3\}} = x_{T2}^{\{3\}}\right) \\
&\qquad = \frac{\Gamma(5)}{\Gamma(4)\Gamma(1)} \frac{\Gamma(5)}{\Gamma(2)\Gamma(3)} \int_{\theta_{T1}=0}^{1} \theta_{T1}^{3} \int_{\theta_{T2}=0}^{\theta_{T1}} \theta_{T2}(1-\theta_{T2})^2 d\theta_{T2} d\theta_{T1} \\
&\qquad = 0.929.
\end{aligned}
$$

Probability that $T2$ is best treatment arm: 0.071

Augmented Control Example: Period 2

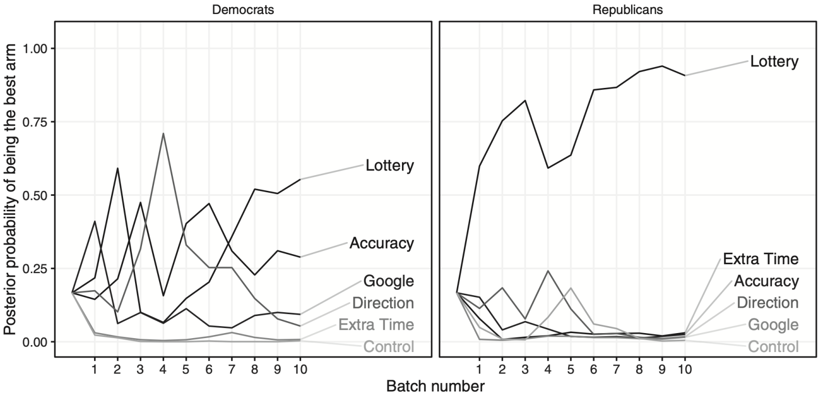
- Probabilities:
  - $p_{t1,2}$: $0.929 * 2/3 = 0.619$
  - $p_{t2,2}$: $0.071 * 2/3 = 0.048$
  - $p_{c,2}$: $1/3$
- Assignments:
  - T1: 6
  - T2: 1
  - C: 2
- Results:
  - T1: 3 successes/3 failures
  - T2: 1 success
  - C: 1 success/1 failure

Augmented Control Example: Period 3

- Posteriors (cumulative):
  - T1: 0.713 (sample 9)
  - T2: 0.287 (sample 4)
  - C: (sample 5)
- Probabilities:
  - $p_{t1,3}$: $0.713 * 2/3 * 5/9 = 0.264$
  - $p_{t2,3}$: $0.287 * 2/3 * 5/9 = 0.106$
  - $p_{c,3}$: $4/9 + 1/3 * 5/9 = 0.630$
- Assignments:
  - T1: 2
  - T2: 1
  - C: 6

Control Augmented Example: Partisan Bias

FIGURE 5 Study 2, Overtime Posterior Probabilities



Note: Respondents are coded as Democrat if their response is 1, 2, or 3 on the 7-point partisanship scale, and Republican if their response is 5, 6, or 7. Posterior probabilities are updated after each day's data collection according to the control-augmented Thompson sampling algorithm.

Exploration Sampling (Kasy et al 2021): Definition

Consider an usual scenario which adaptive designs are suitable for:

- A policymaker wants to maximize the expected value of a binary outcome variable (i.e. success rate), $Y \in \{0, 1\}$
- She can choose one of k different treatments (i.e. policies), with $k >= 3$
- Treatments are assessed via an experiment carried out in multiple waves $t = 1, ..., T$

Exploration Sampling (Kasy et al 2021): Definition

Replace Thompson assignment p_t^1 ... p_t^k with:

$$q_t^d = S_t \cdot p_t^d \cdot (1 - p_t^d), \qquad S_t = \frac{1}{\sum_d p_t^d \cdot (1 - p_t^d)}.$$

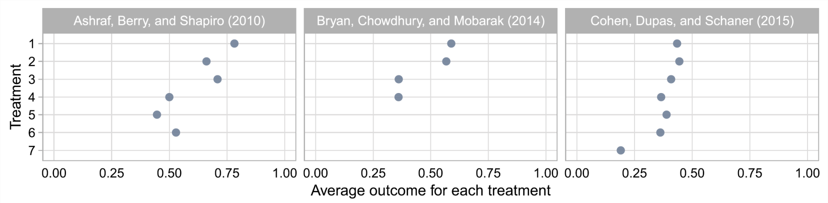
Notes:

- Mapping from p_t^k to q_t^d is monotonically increasing and concave
- Weight shifted in favor of the closest competitors of the best arm
- Equivalent to Thompson sampling if same treatment assignment never assigned twice in a row

Exploration Sampling (Kasy et al 2021): Evidence

- Simulation evidence on performance of alternative treatment assignment algorithms: non-adaptive, Thompson sampling, exploration sampling
- Parameter vectors and sample size calibrated to experimental data from published experiments in development economics
- Assume the goal is to maximize the average measured outcome

Exploration Sampling (Kasy et al 2021): Evidence



- 100,000 simulation draws
- Vector θ equals average outcomes in the original experiment in all draws
- Total sample size as in the original experiment

Exploration Sampling (Kasy et al 2021): Evidence

TABLE I

SIMULATION RESULTS[a]

Statistic	2 waves	4 waves	10 waves
Ashraf, Berry, and Shapiro (2010)			
Average policy regret			
exploration sampling	0.0017	0.0010	0.0008
expected Thompson	0.0022	0.0014	0.0013
non-adaptive	0.0051	0.0050	0.0051
Share optimal			
exploration sampling	0.978	0.987	0.989
expected Thompson	0.971	0.981	0.982
non-adaptive	0.933	0.935	0.933
Average in-sample regret			
exploration sampling	0.1126	0.0828	0.0701
expected Thompson	0.1007	0.0617	0.0416
non-adaptive	0.1776	0.1776	0.1776
Units per wave	502	251	100

Exploration Sampling (Kasy et al 2021): Evidence

- Evidence from field experiment in India
- Precision Agriculture for Development (PAD) provides phone-based personalized agricultural extension service to farmers
- Choose optimal call method to enroll rice farmers among 6 alternatives
- Outcome $Y \in \{0, 1\}$, where 1 means the call recipient answered five questions enabling enrollment

Exploration Sampling (Kasy et al 2021): Evidence

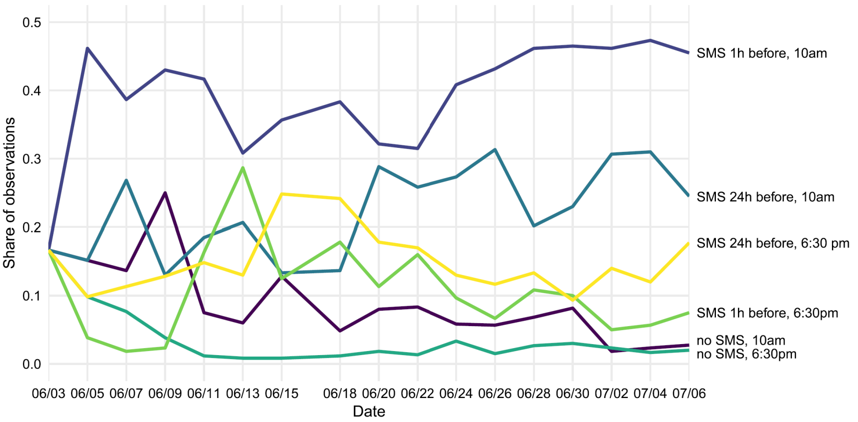


FIGURE 2.—Assignment frequencies over time.

Exploration Sampling (Kasy et al 2021): Evidence

TABLE II

OUTCOMES OF THE ADAPTIVE EXPERIMENT FOR PAD[a]

Treatment		Outcomes			Posterior		
Call time	SMS alert	m_T^d	r_T^d	r_T^d/m_T^d	Mean	SD	p_T^d
10 am	–	903	145	0.161	0.161	0.012	0.009
10 am	1 h ahead	3931	757	0.193	0.193	0.006	0.754
10 am	24 h ahead	2234	400	0.179	0.179	0.008	0.073
6:30 pm	–	366	53	0.145	0.147	0.018	0.011
6:30 pm	1 h ahead	1081	182	0.168	0.169	0.011	0.027
6:30 pm	24 h ahead	1485	267	0.180	0.180	0.010	0.126

[a]For each treatment arm: total observations, total successes, share of successes, posterior mean, and standard deviation of the success rate, and probability that the arm is optimal. 10,000 units and 17 waves.

Exploration Sampling (Kasy et al 2021): Summary

- Modification of Thompson sampling
- Avoids assigning more than 50% of sample to the highest performing treatment
- In large samples equalizes power for rejecting each sub-optimal treatment
- Maximize convergence rate of social welfare (choosing the optimal treatment after the experiment)

Text Nudge India (Bahety et al 2021)

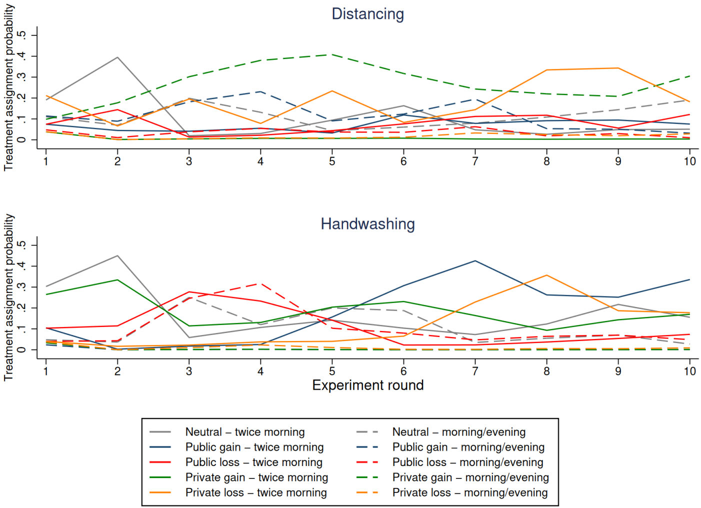
- Two adaptive randomized controlled trials
- Evaluation of the impact of a SMS-based information campaign on preventive health behaviour
  - Handwashing
  - Social distancing
- Treatment variables:
  - Message framing - highlighting public gain or loss, private gain or loss, or neutral (i.e. 5 levels)
  - Delivery time - twice in the morning / once in the morning and once in the evening
- To minimise the risk of experimenter demand, health behaviour elicited with open-ended question: "What are you doing to protect against the virus?"

Text Nudge India (Bahety et al 2021)

- 5 (Message framing) x 2 (Delivery time) = 10 treatment arms + 1 control (no SMS)
- Particularly suitable for an adaptive trial strategy given the large number of treatments
- Implementation of the exploration sampling algorithm over 10 waves

Text Nudge India (Bahety et al 2021)

Figure A.2: Treatment Shares by Round



Note: Figure A.2 shows the posterior probabilities for each of the 10 treatment arms for Round 1 to 10 for distancing in Panel A and handwashing in Panel B. Round 1 comprises of first five rounds that are used as priors.

Text Nudge India (Bahety et al 2021)

Table 2
First-Stage Results on Self-Reported Receipt of COVID-Related SMS.

	Any SMS	# SMS	SD SMS | Any SMS	SD SMS | Any SMS	HW SMS | Any SMS	HW SMS | Any SMS
Pooled treatment	0.286**	1.045**	0.038		0.122*	
	(0.026)	(0.137)	(0.061)		(0.050)	
Treatment - SD				0.136		
				(0.093)		
Treatment - HW						0.209**
						(0.059)
R^2	0.21	0.22	0.23	0.23	0.16	0.16
N	1,988	1,949	791	791	773	773
Control Mean	0.16	0.68	0.27	0.19	0.09	0.13
F-statistic	118.43	58.10	0.38	2.17	5.86	12.41

Table 2 shows the first stage results for four self-reported measures of receipt of any COVID-related SMS: any SMS, number of SMS received in Column 1 and 2, recall of social distancing in Column 3 and 4 and handwashing messages in Column 5 and 6, respectively. The last four measures are conditional on receiving any COVID-related SMS. The regressions include fixed effects for gender, occupation, education, age, target behavior, block, day of the week, round of the experiment, enumerator, and (random) order of the knowledge and action question for the key outcomes. Robust standard errors in parentheses. Asymptotic p-values are denoted by: * $p<0.1$; ** $p<0.05$; *** $p<0.001$.

Text Nudge India (Bahety et al 2021)

Table 3
ITT Results by Pooled Treatment.

	Distancing		Handwashing	
	Know	Act	Know	Act
Treatment - SD	−0.002	−0.003	−0.035	−0.051*
	(0.030)	(0.029)	(0.029)	(0.029)
	[0.957]	[0.928]	[0.206]	[0.056]
Treatment - HW	−0.001	0.018	0.034	0.002
	(0.028)	(0.027)	(0.027)	(0.028)
	[0.954]	[0.448]	[0.158]	[0.943]
Adjusted R^2	0.09	0.05	0.05	0.05
N	3,563	3,563	3,563	3,563
Control Mean	0.49	0.36	0.32	0.35

Table 3 shows the ITT results by pooled treatment for the four main outcomes. The regressions include fixed effects for gender, occupation, education, age, target behavior, block, day of the week, round of the experiment, enumerator, and (random) order of the knowledge and action question for the key outcomes. Robust standard errors are in parentheses and Fisher exact *p*-values are in square brackets. Asymptotic *p*-values are denoted by: * p<0.1; ** p<0.05; *** p<0.001.

Adaptive experimentation tutorial

- Molly Offer-Westort, Vitor Hadad, Susan Athey
- `https://mollyow.shinyapps.io/adaptive/`