Can Artificial Intelligence Solve Strategic Decision Problems?

Abstract

Whether, when, and how Artificial Intelligence (AI) will substitute for human labor are all active debates. This paper evaluates the ability of several leading Large Language Models (LLMs) to solve strategic decision-making problems from the social sciences. On average, we find that LLMs perform similarly to highly competent humans, but the type of problem affects solution rates. LLMs perform better when faced with problems that require numerical calculations of expected values. They perform worse when given tasks that are: a) not based on well-known 'textbook' problems, b) involve complex equilibrium reasoning (such commitment problems) or c) require novel economic insights.

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1 Introduction

AI systems are often benchmarked against their substantive mathematical, scientific problem-solving or coding capabilities ([1–10], cf. [11–15]). However, augmenting or replacing some aspects of human labor in a knowledge economy hinges on AI's ability to understand strategic human behavior. We approach this question through the lens of social science research. We know that AI systems might be plausible synthetic research subjects in the social sciences ([16–20], cf. [21]), but they struggle to reproduce social science research without substantial human assistance [22]. This paper tests whether LLMs can analyze human behavior by assessing responses to strategic decision problems from the social sciences.

We test five leading AI systems using social science problems that many humans would be unable solve. We document whether each LLM a) correctly solves the problem and b) provides correct reasoning in its answer. We also analyze variation across five types of task characteristic. First, we assess whether the problem derives from older or recent research. Second, we distinguish between questions that require, or do not require, numerical calculations or use of simple mathematical ideas such as Bayes' Rule. Third, we measure whether the problem involves asymmetric information and thus requires direct commentary about strategic behavior. Fourth, we categorize problems as requiring different types of equilibrium reasoning, for example problems of future commitment, or not. Finally, we assess whether answers to the problem require original economic insight. As outlined in our pre-registered hypotheses, we expect that LLMs will do better when faced with numerical calculations (hypotheses 2a and 2b) and older problems (hypothesis 2c), but do worse when given problems of asymmetric information (hypothesis 3a) or equilibrium reasoning (hypothesis 3b) and when asked for economic insights (hypothesis 4) .[1]

Assessing not just the average ability of AI to solve these problems, but also the specific type of problems that it is able to solve, helps us understand better not just the opportunities, and limitations, of human-AI collaboration [23–28], but the specific use of AI assistance in social science research [17, 18, 29–37]. It also contributes to our understanding of the capability of AI systems to augment or replace human workers in a knowledge economy [28, 38–42].

[1]The pre-registration plan is available at https://osf.io/wdys6/?view_only=da82daf497b144e59c7d16b14caf18a7.

2 Methods

We test whether five LLMs (ChatGPT o3, Claude, Grok 3, LLama, and Gemini) can solve stylized strategic decision-making problems. These problems are from admissions interview passages for undergraduate economics degrees at [institution name removed for review]. They are problems which a top performing secondary school student might be expected to solve, but would not be soluble by many people in the population. Each problem contains a preamble followed by five numbered questions. We used the complete set of 20 problems, and therefore 100 questions, fielded from 2008-2024.

In each problem, the five questions examine human behavior in different contexts. Some problems focus on conflicts between individual incentives and group objectives, others on tensions between commitment and sequential rationality, and others on asymmetric information. Most of these exercises are game-theoretic in nature, but they are all presented without mathematical formalism and in a real, but hypothetical, setting such as countries deciding to go to war, councils deciding how to split a bequest or partners setting up a company. None of the problems are publicly available. The exact wording of the problems is in Appendix D and the solutions are in Appendix E.

Our first concern is whether AI models can competently answer these problems at the level of an academically able human. Our second concern is what characteristics of each problem affect LLM performance. We group these characteristics into five categories. First, there is the age of the problem: is it based on a 'classic' puzzle or more recently published research? Second, are there simple numerical calculations or computations of expected values according to Bayes' rule? Third, does the problem depend on asymmetric information? For example, is there a moral hazard in which there are unobservable costly actions of another party? Fourth, does the problem require equilibrium reasoning? For example, does one need to carry out backward induction based on anticipation of other parties acting rationally in future? Finally, did the answer require economic insight? This typically means comparing answers across the other four questions of the problem.

Some of these types of characteristics, such as economic insight, have a single variable to measure them, but we measure most using multiple variables. In particular, we distinguish between three types of asymmetric information and four types of equilibrium reasoning. The full list of variables is in Table 2. Given the limited number of cases, we also create composite variables for asymmetric information and equilibrium reasoning that simply measure whether the question contains any of the three asymmetric information types or any of the four equilibrium reasoning types. Table 2 shows how frequently the characteristics are found in our data.

Table 1 Independent variable definitions and question counts

Variable	Brief definition	#Q
	H2a and H2b: Computation	
Numerical calculation	Report data (numbers) given in the preamble, carry out simple calculations.	48
Bayes' rule	Calculate or update probabilities using Bayes' rule, compute expected values.	42
	H2c: Salience	
Classic puzzle	Problem based on a classic puzzle or game.	30
More recent puzzle	Problem based on an identifiable publication (academic article) that is more advanced and recent.	55
	H3a: Asymmetric information	
Adverse selection	The problem or question involves private information about the characteristics of agents that they may be tempted to "lie" about.	20
Moral Hazard	The problem or question involves unobservable costly actions of certain agents that they need incentives to carry out.	10
Efficiency reasoning	The problem or question highlights tensions between individual incentives and a group (or social) optimum.	9
	H3b: Equilibrium reasoning	
Dominant strategy	Recognise that in a static setting, rational agents do not use actions that are strictly dominated and that all agents are aware of this fact.	17
Backward induction	Recognise that in a dynamic setting, rational agents anticipate others to act rationally in future and carry out backward induction.	16
Equilibrium reasoning	Recognise that in a predicted equilibrium outcome all agents anticipate each other's behaviour and all act rationally.	44
Commitment	The question concerns repeated interaction or the (im)possibility or value of commitment.	8
	H4: Economic insights	
Economic insight	The question elicits a novel insight, often based on comparing answers to multiple questions on the same problem sheet.	28

To obtain the LLM responses to each question, we freshly installed all five LLMs on a single computer. We then fed each LLM the 20 problem sets in the same randomized order, to account for any potential 'learning' effects during the process. For each problem set, we first inputted the preamble and the first numbered question, and nothing else, into the LLM. We then entered each of the remaining questions one by one. This was repeated for each problem set and each LLM, using the most advanced publicly available LLM version (for example, using 'deep thinking' when available).

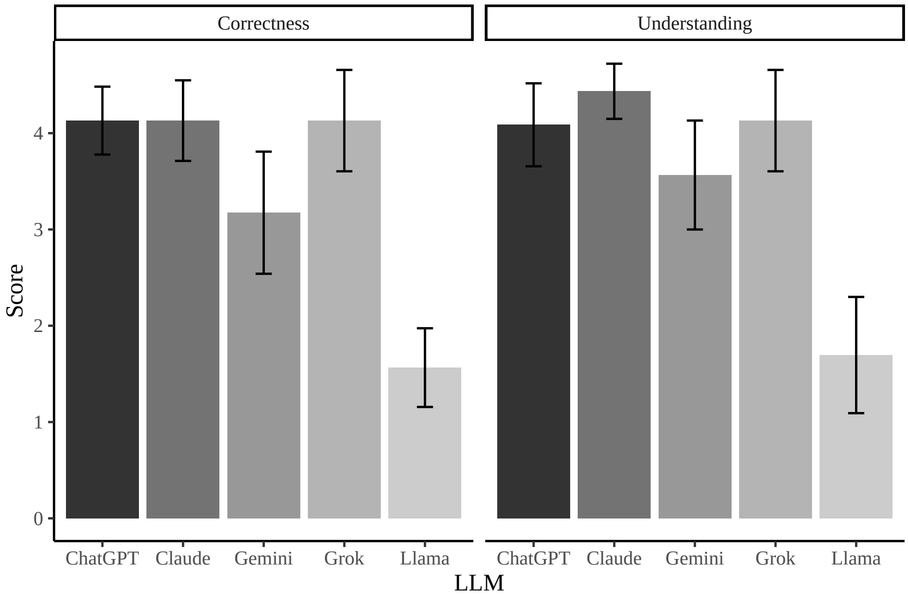
For each question, we judge 'correctness' and 'understanding' [43]. An answer is judged correct if it matches the correct answer in Appendix E. An answer is judged as understanding the question if the explanation given by the LLM shows a logical route to the answer. We triple-marked each question and then took the modal score. The marking was conducted blind to the LLM in question. The LLM responses were collected by a research assistant, who shared with us the anonymized responses. [2] As there are five questions in each problem, to measure correctness and understanding at the problem level, we also summed the relevant question level scores to give two 0-5 scales for each problem.

3 Results

In Figure 1 we show the mean of the 0-5 correctness and understanding scores (with 95 per cent confidence intervals) across all twenty problems for the five LLMs. The mean scores for ChatGPT, Claude and Grok are around four out of five points for both correctness and understanding. Gemini does slightly worse at around three out of five points and Llama performs substantially below this at around 1.5 points out of five. In the standard undergraduate admissions process at the University of Oxford, most admitted students would score 3.5 or more on both scales. These levels therefore represent a benchmark score of a highly competent secondary school student. With the exception of Llama, the LLMs therefore perform at this level and are likely comparable to the top few per cent of the human population with regard to solving strategic decision problems.

[2] Only after marking the responses and agreeing on the final scores did we de-anonymize the LLM.

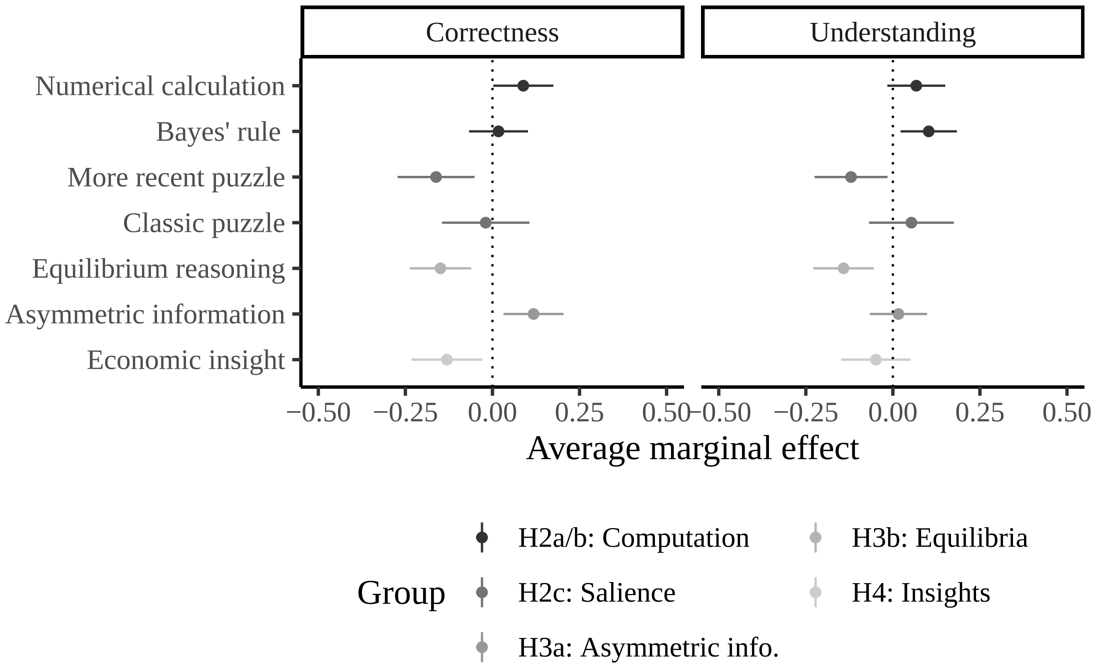
Fig. 1 Large Language Models generally solve strategic decision problems at the level of a competent undergraduate



Note. Average correctness (0-5) and understanding (0-5) scores with their 95 per cent confidence intervals. These results are averages at the problem level.

The LLMs are by no means perfect, however, and there is, in fact, a fair degree of variation in how they perform at different tasks. We can see this when we estimate logit regressions at the question level. The first regression regresses the binary indicator of whether an LLM provided a correct answer on all question characteristics. Here we include equilibrium reasoning and asymmetric information as the composite measures and add LLM fixed effects. The second regression does the same, but for understanding. Figure 2 shows the marginal change in predicted probabilities, with 95 per cent confidence intervals, for each binary characteristic.

Fig. 2 Large Language Models perform consistently worse on problems requiring novel insight, equilibrium reasoning or those based on recently published research

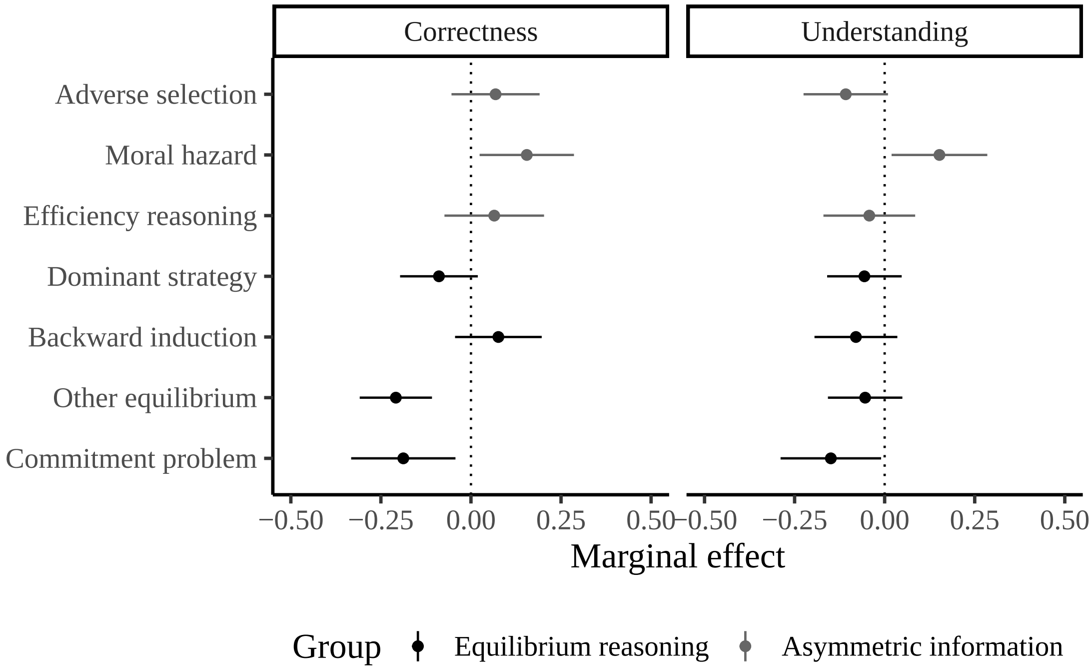


Note. Marginal effects and their 95 per cent confidence intervals. The results are from a logistic regression predicting correctness with question characteristics and LLM fixed effects. See corresponding models in Appendix A. All other covariates are held at their mean, or, for binary variables, at their proportion.

Overall, it appears that LLM performance, while generally high, is systematically affected by some type of question characteristics. As we expected, LLMs do better when presented with numerical calculations and they do worse when given puzzles based on recent research that can not be found in textbooks. Also, as expected, LLMs do worse when the question involves some type of equilibrium reasoning or when the answer requires a broader economic insight. Finally, and unexpectedly, problems of asymmetric information appear to be, ceteris paribus, easier for LLMs to solve, although not understand, than other types of question. Some of these effects are fairly large. For example, questions requiring numerical calculations are around seven percentage points more likely to be marked as correct and understood. Similarly, questions demanding equilibrium reasoning reduce the probability of a correct or understood response by almost fifteen percentage points.

As the effects of the composite categories of asymmetric information and equilibrium reasoning are some of the largest, we break down these categories further in Figure 3. This shows the results of two logit regressions that regress the binary indicator of whether each LLM provided a correct, or understood, answer on all twelve question characteristics shown in Table 1. As Figure 3 shows, including all the sub-categories of equilibrium reasoning and asymmetric information changes the picture slightly. Of the three elements of asymmetric information, it appears that LLMs find it easiest to both solve and understand moral hazard problems. Similarly, only certain elements of equilibrium reasoning appear to affect problem difficulty. While questions that involve backward induction are no more difficult to solve and understand than any other problem, LLMs appear to find commitment problems particularly difficult. Indeed, holding all the other question characteristics constant, LLMs are over 15 percentage points less likely to get correct, and understand, questions that involve commitment.

Fig. 3 Question level marginal effects for the sub-categories of asymmetric information and equilibrium reasoning



Note. Marginal effects and their 95 per cent confidence intervals. The results are from a logistic regression predicting correctness with question characteristics and LLM fixed effects. See corresponding models in Appendix A. All other covariates are held at their mean, or, for binary variables, at their proportion.

All the results in Figures 2 and 3 are robust to the inclusion of problem set fixed effects (see Appendix B). The results at the problem set level are also very similar to those we present here (see Appendix C).

4 Discussion

AI is typically benchmarked against its mathematical and scientific problem-solving capacity, rather than its ability to understand human behavior. In this paper, we tested that ability by asking five leading LLMs to solve social science problems. Overall, we found that all but one of the LLMs performed at a high level and were able to both answer most questions correctly and show good understanding of the underlying problems' logic. We also found that the characteristics of the question systematically affected the quality of the LLM responses. On the one hand, questions that tended to elicit weaker responses (between 10 and 20 percentage points lower) from the LLMs involved more recently published research, necessitated understanding of repeated interactions, or required original economic insights. On the other hand, LLMs tended to perform better on questions that required numerical calculations. In effect, questions requiring computation or regurgitation of textbook knowledge were more easily soluble by the LLMs, but questions requiring the LLM to advance its thinking beyond a textbook level (by providing original insights into the problem or by considering recently published research) saw reductions in performance. Our results suggest that LLMs perform substantially better when tasks allow for training-based memorization, such as recalling or reproducing content seen during pretraining. By contrast, when tasks demand deeper pattern recognition and reasoning beyond surface similarity, performance deteriorates markedly.

There are inevitably limitations to our study. We focus on a specific sample of strategic decision questions and our results may not therefore generalize across the social sciences, especially to problems outside the scope of strategic decision making. We also only gave the LLMs one opportunity to answer each question. While this is equivalent to the way in which these questions were previously asked of humans, we would expect the LLMs to do better if given additional opportunities to correct their responses.

These caveats aside, we show that AI language models are not just capable of solving difficult mathematical problems or answering questions requiring encyclopaedic knowledge. They can also understand human behavior and strategic decision-making. While that understanding is not perfect, their overall performance suggests that LLMs are, or will shortly be, able to replace human workers in many knowledge economy positions as contemporary AI systems are likely to perform similarly to a competent worker. Indeed, advances in AI models such as chain of thought programming might also allow LLMs to perform better when responding to new research or when deriving novel economic insights [44]. We should therefore expect the ability of LLMs to solve strategic decision problems to improve further in the near future.

Competing interests. The authors declare none.

Data and materials availability. The problem sets and answers are available in Appendices D and E. The categorization of questions is available in Appendix F. The AI responses, marks, and the code to reproduce the analyses in the paper and appendices are available at: https://osf.io/tky7x/overview?view_only=2acf46d1b61b48d4a3aec955a2d746b6. We declare no restrictions on sharing or re-use.

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Appendix A: Full models

In this Appendix, in Table 2 and Table 3, we provide the full models for that produced the predicted probabilities shown in Figure 2 and Figure 3. We also provide results for a series of bivariate regressions predicting correctness and understanding with each covariate included separately (along with LLM fixed effects in each case). These are split across Table 4, Table 5, and Table 6.

Table 2 Coefficient estimates for Figure 2

	Correct	Understand
	(1)	(2)
Claude	0.000	0.648
	(0.375)	(0.410)
Gemini	-1.108^{***}	-0.646^{*}
	(0.344)	(0.347)
Grok	0.000	0.141
	(0.375)	(0.375)
Llama	-2.470^{***}	-2.275^{***}
	(0.358)	(0.354)
Numerical calc.	0.533^{**}	0.436
	(0.268)	(0.277)
Bayes' rule	0.105	0.667^{**}
	(0.260)	(0.272)
Classic puzzle	-0.117	0.344
	(0.385)	(0.404)
More recent puzzle	-0.974^{***}	-0.778^{**}
	(0.347)	(0.351)
Asymmetric info.	-0.899^{***}	-0.917^{***}
	(0.280)	(0.295)
Equilibria	0.710^{***}	0.104
	(0.271)	(0.273)
Insight	-0.786^{**}	-0.315
	(0.318)	(0.330)
Constant	2.349^{***}	1.959^{***}
	(0.506)	(0.507)
Observations	500	500
Log Likelihood	-252.499	-237.400
Akaike Inf. Crit.	528.998	498.799
Note:	*p<0.1; **p<0.05; ***p<0.01	

Table 3 Coefficient estimates for Figure 3

	Dependent variable:	
	Score	
	(1)	(2)
Claude	0.000	0.668
	(0.384)	(0.416)
Gemini	-1.173^{***}	-0.674^{*}
	(0.355)	(0.355)
Grok	0.000	0.146
	(0.384)	(0.382)
Llama	-2.619^{***}	-2.389^{***}
	(0.373)	(0.365)
Numerical calc.	0.111	0.123
	(0.296)	(0.314)
Bayes' rule	0.413	0.906^{***}
	(0.288)	(0.302)
Classic puzzle	0.041	0.479
	(0.412)	(0.425)
More recent puzzle	-1.166^{***}	-0.976^{**}
	(0.378)	(0.381)
Dominant strategy	-0.564	-0.379
	(0.352)	(0.357)
Backward induction	0.483	-0.538
	(0.391)	(0.398)
Equilibrium reasoning	-1.323^{***}	-0.365
	(0.341)	(0.356)
Commitment problem	-1.191^{**}	-1.007^{**}
	(0.480)	(0.487)
Adverse selection	0.433	-0.728^{*}
	(0.397)	(0.407)
Moral hazard	0.983^{**}	1.025^{**}
	(0.431)	(0.464)
Efficiency reasoning	0.410	-0.287
	(0.449)	(0.438)
Insight	-0.840^{**}	-0.271
	(0.339)	(0.352)
Constant	2.817^{***}	2.146^{***}
	(0.542)	(0.536)
Observations	500	500
Log Likelihood	-240.989	-228.918
Akaike Inf. Crit.	515.978	491.835
Note:	*p<0.1; **p<0.05; ***p<0.01	

Table 4 Bivariate estimates, part 1

	Score									
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Claude	−0.000	0.617	−0.000	0.612	0.000	0.615	0.000	0.621	−0.000	0.608
	(0.365)	(0.400)	(0.362)	(0.398)	(0.361)	(0.399)	(0.364)	(0.401)	(0.361)	(0.397)
Gemini	−1.037***	−0.602*	−1.011***	−0.595*	−1.010***	−0.599*	−1.027***	−0.608*	−1.003***	−0.588*
	(0.333)	(0.335)	(0.329)	(0.333)	(0.328)	(0.334)	(0.331)	(0.337)	(0.327)	(0.331)
Grok	0.000	0.133	−0.000	0.132	−0.000	0.132	−0.000	0.134	−0.000	0.130
	(0.365)	(0.365)	(0.362)	(0.363)	(0.361)	(0.364)	(0.364)	(0.366)	(0.361)	(0.361)
Llama	−2.290***	−2.073***	−2.225***	−2.043***	−2.223***	−2.067***	−2.265***	−2.100***	−2.204***	−2.013***
	(0.341)	(0.333)	(0.335)	(0.330)	(0.335)	(0.333)	(0.339)	(0.336)	(0.333)	(0.327)
Numerical calc.	0.842***	0.775***								
	(0.218)	(0.225)								
Bayes' rule			0.424*	0.591***						
			(0.217)	(0.228)						
Classic puzzle					0.439*	0.832***				
					(0.236)	(0.259)				
More recent puzzle							−0.716***	−0.927***		
							(0.218)	(0.232)		
Dominant strategy									−0.086	−0.335
									(0.276)	(0.279)
Constant	1.100***	1.059***	1.290***	1.169***	1.331***	1.179***	1.883***	1.959***	1.465***	1.448***
	(0.269)	(0.266)	(0.267)	(0.262)	(0.262)	(0.257)	(0.295)	(0.299)	(0.260)	(0.256)
Observations	500	500	500	500	500	500	500	500	500	500
Note:										*p<0.1; **p<0.05; ***p<0.01

Table 5 Bivariate estimates, part 2

	Dependent variable:									
2-11	Score									
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Claude	0.000	0.620	−0.000	0.616	0.000	0.617	0.000	0.612	0.000	0.608
	(0.363)	(0.401)	(0.366)	(0.399)	(0.370)	(0.400)	(0.361)	(0.398)	(0.361)	(0.397)
Gemini	−1.017***	−0.603*	−1.042***	−0.599*	−1.056***	−0.599*	−1.003***	−0.594*	−1.006***	−0.588*
	(0.330)	(0.335)	(0.334)	(0.334)	(0.336)	(0.334)	(0.327)	(0.333)	(0.328)	(0.331)
Grok	−0.000	0.133	−0.000	0.132	−0.000	0.133	−0.000	0.132	0.000	0.130
	(0.363)	(0.366)	(0.366)	(0.364)	(0.370)	(0.364)	(0.361)	(0.363)	(0.361)	(0.361)
Llama	−2.235***	−2.062***	−2.297***	−2.057***	−2.296***	−2.043***	−2.206***	−2.034***	−2.212***	−2.015***
	(0.336)	(0.332)	(0.342)	(0.331)	(0.342)	(0.330)	(0.333)	(0.329)	(0.334)	(0.327)
Backward induction	−0.677**	−0.941***								
	(0.274)	(0.279)								
Equilibrium reasoning			−0.875***	−0.680***						
			(0.216)	(0.221)						
Commitment problem					−1.607***	−1.046***				
					(0.371)	(0.366)				
Adverse selection							−0.148	−0.614**		
							(0.258)	(0.260)		
Moral hazard									0.435	0.504
									(0.369)	(0.391)
Constant	1.577***	1.572***	1.885***	1.713***	1.632***	1.493***	1.481***	1.527***	1.412***	1.343***
	(0.263)	(0.261)	(0.286)	(0.278)	(0.267)	(0.256)	(0.261)	(0.260)	(0.257)	(0.252)
Observations	500	500	500	500	500	500	500	500	500	500
Note:										*p<0.1; **p<0.05; ***p<0.01

Table 6 Bivariate estimates, part 3

	Score							
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Claude	−0.000	0.606	−0.000	0.607	0.000	0.618	−0.000	0.607
	(0.361)	(0.396)	(0.363)	(0.397)	(0.363)	(0.400)	(0.361)	(0.397)
Gemini	−1.007***	−0.586*	−1.018***	−0.587*	−1.019***	−0.603*	−1.004***	−0.587*
	(0.328)	(0.331)	(0.330)	(0.331)	(0.330)	(0.335)	(0.327)	(0.331)
Grok	0.000	0.130	−0.000	0.130	0.000	0.133	−0.000	0.130
	(0.361)	(0.361)	(0.363)	(0.361)	(0.363)	(0.365)	(0.361)	(0.361)
Llama	−2.215***	−2.005***	−2.239***	−2.008***	−2.245***	−2.078***	−2.207***	−2.011***
	(0.334)	(0.326)	(0.336)	(0.327)	(0.337)	(0.334)	(0.333)	(0.327)
Efficiency reasoning	0.538	0.020						
	(0.395)	(0.379)						
Insight			−0.585**	−0.179				
			(0.228)	(0.238)				
Asymmetric info.					−0.582***	−0.806***		
					(0.214)	(0.227)		
Equilibria							0.184	−0.225
							(0.223)	(0.226)
Constant	1.409***	1.384***	1.635***	1.438***	1.785***	1.862***	1.390***	1.466***
	(0.257)	(0.252)	(0.269)	(0.260)	(0.289)	(0.293)	(0.264)	(0.264)
Observations	500	500	500	500	500	500	500	500
Note:								*p<0.1; **p<0.05; ***p<0.01

Appendix B: Robustness

In this Appendix, we demonstrate the robustness of our analyses to the inclusion of problem set fixed effects. Fixed effects account for systematic differences between the problem sets, for example their year of being written, their length, or the inclusion of graphs and diagrams. The results are generally similar to those in the main text.

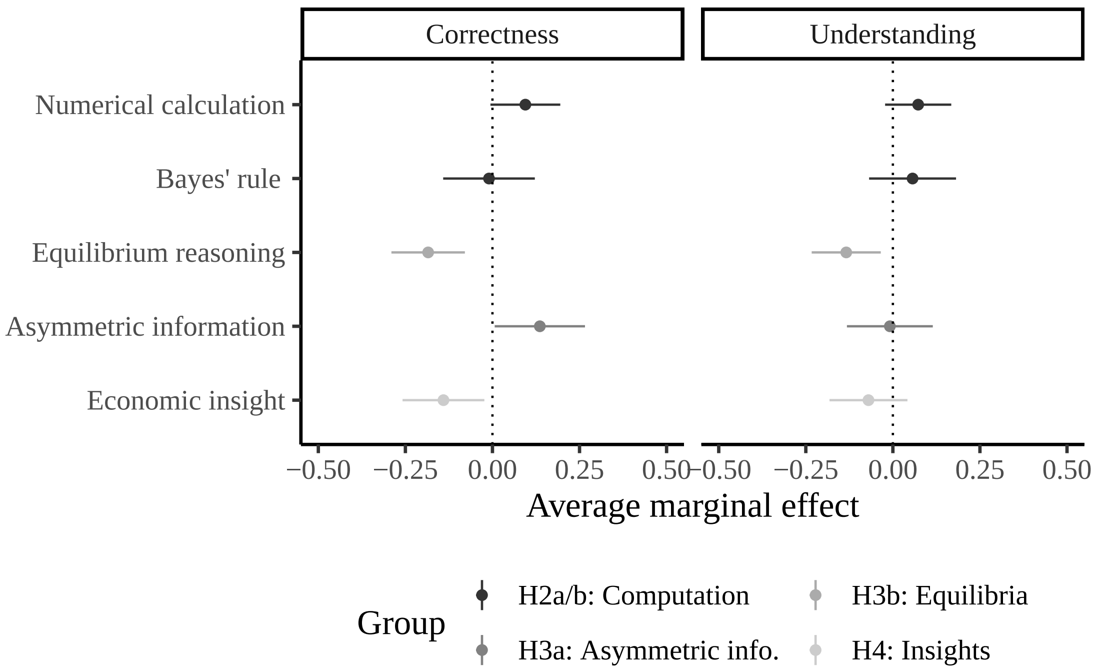


Fig. 4 Question level marginal effects, including LLM and Problem Set fixed effects

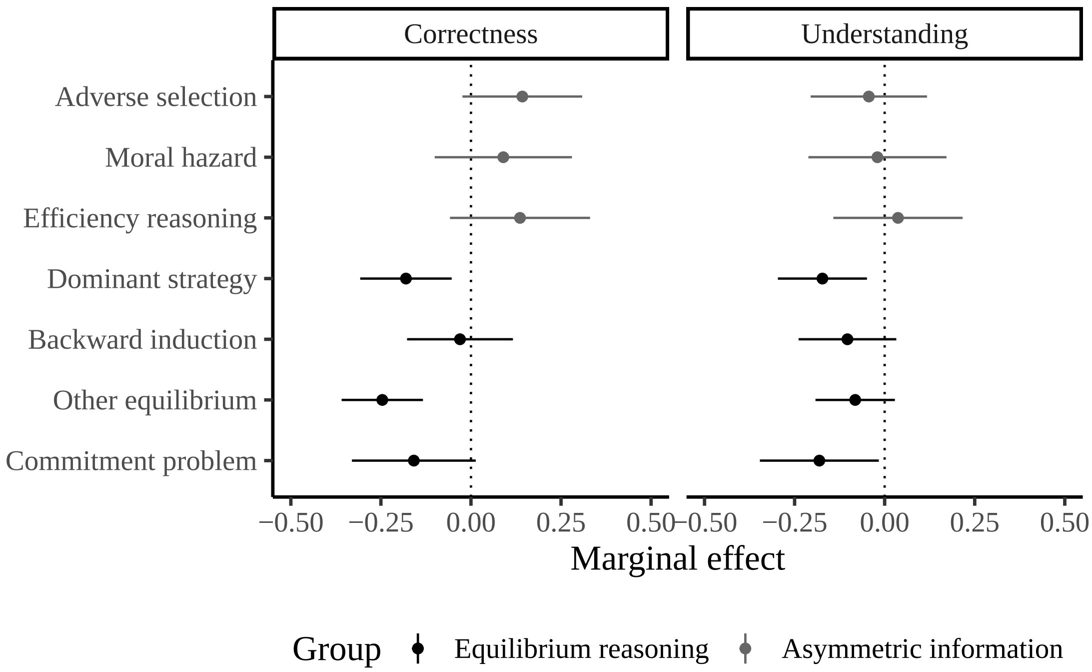


Fig. 5 Question level marginal effects, including more Independent Variables, LLM and Problem Set fixed effects

We also re-ran the models with random effects for LLM instead of fixed effects. The results are again generally similar.

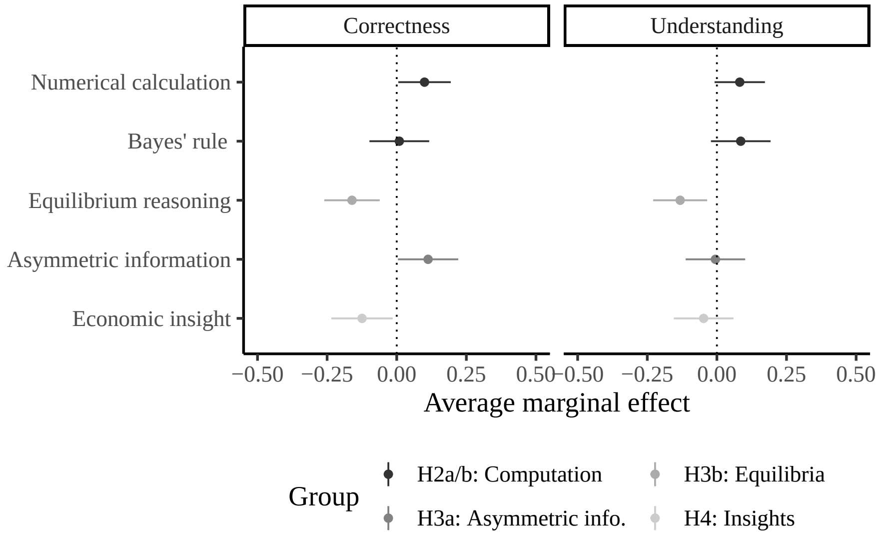


Fig. 6 Question level marginal effects, including LLM and Problem Set fixed effects

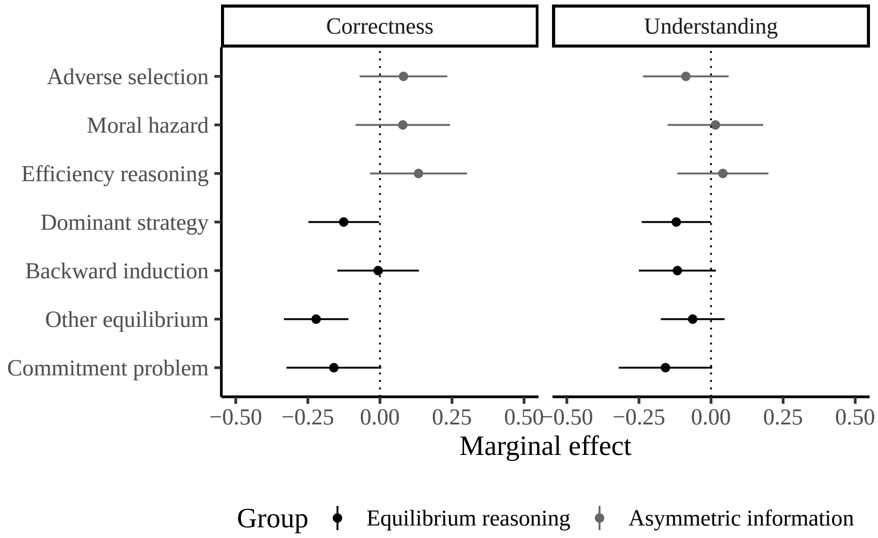


Fig. 7 Question level marginal effects, including more Independent Variables, LLM and Problem Set fixed effects

Finally, we re-ran the question level models with an additional control for the total number of concepts invoked by each question. To avoid multicollinearity, we used the moral hazard category as a baseline. We did not find any significant effects on either correctness or understanding.

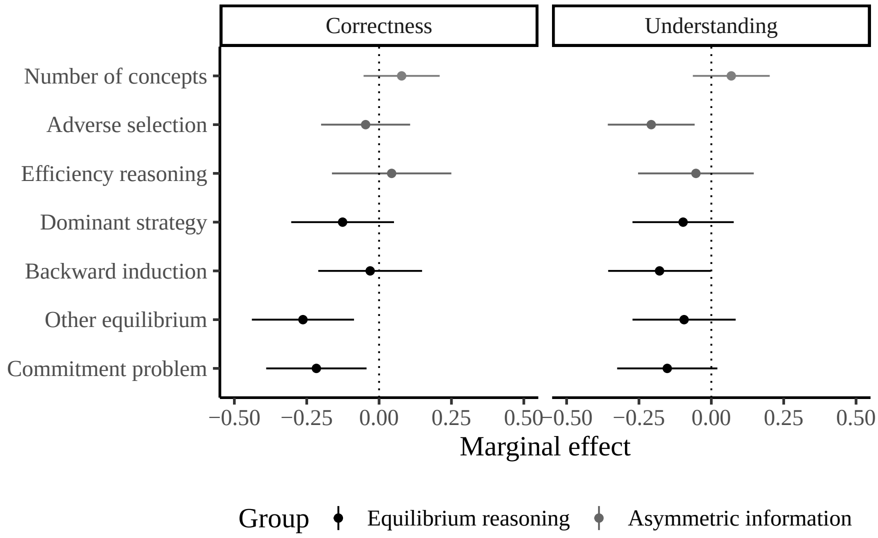


Fig. 8 Question level marginal effects, including LLM fixed effects and controlling for the total number of concepts

Appendix C: Problem set level results

In this Appendix, we replicate our analyses at the level of the problem set. We first report the number of complete problem sets which we identified as belonging to each of the categories of question. There are no complete problem sets in which all questions fit the category of economic insight or commitment problem, so we exclude these categories from the problem set level analysis. We then estimate the marginal effects on correctness and understanding of belonging to each of these categories at the problem set level. In Table 8, we include all relevant problem set variables in the models. In Table 9 and Table 10, we estimate separate models for each problem set level variable. In all cases, we include a fixed effect for LLM.

Table 7 Independent variable definitions and problem set counts

Variable	Brief definition	#PS
	H2a/b: Computations	
Numerical calculation	Report data (numbers) given in the preamble, carry out simple calculations.	1
Bayes' rule	Calculate or update probabilities using Bayes' rule, compute expected values.	9
	H2c: Salience	
Classic puzzle	Problem based on a classic puzzle or game.	6
More recent puzzle	Problem based on an identifiable publication (academic article) that is more advanced and recent.	11
	H3: Equilibria	
	H3a: Asymmetric information	
Asymmetric information	The problem or question involves private information about the characteristics of agents that they may be tempted to "lie" about.	6
Moral Hazard	The problem or question involves unobservable costly actions of certain agents that they need incentives to carry out.	4
Efficiency reasoning	The problem or question highlights tensions between individual incentives and a group (or social) optimum.	2
	H3b: Equilibrium reasoning	
Dominant strategy	Recognise that in a static setting, rational agents do not use actions that are strictly dominated and that all agents are aware of this fact.	3
Backward induction	Recognise that in a dynamic setting, rational agents anticipate others to act rationally in future and carry out backward induction.	3
Equilibrium reasoning	Recognise that in a predicted equilibrium outcome all agents anticipate each other's behaviour and all act rationally.	13

At the problem set level, both understanding and correctness are measured on a 0–5 scale. We estimate linear regressions using these scores as the dependent variables and problem characteristics as independent variables. This analysis is based on 100 observations (20 problem sets across 5 LLMs). At this aggregated level, only the positive effects of numerical calculation and asymmetric information remain statistically significant. The negative effect of more recent puzzles also remains significant when additional control variables are included.

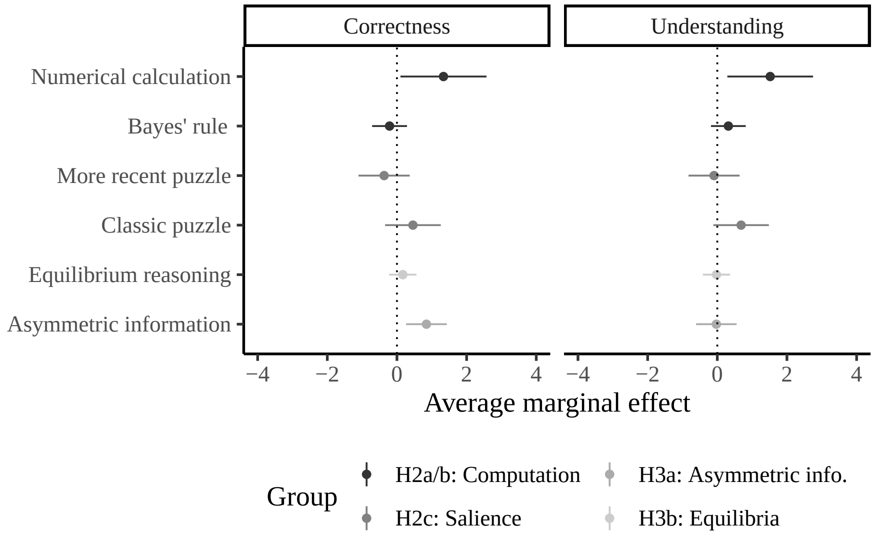


Fig. 9 Problem set level regressions with LLM fixed effects

We also re-ran these models with additional controls for some characteristics of the problem sets which might systematically affect their difficulty. These were: the year the problem set was written, the length (in words) of the problem set, and an indicator for whether the problem set included any graphs or diagrams. The results are generally similar when these additional controls are included.

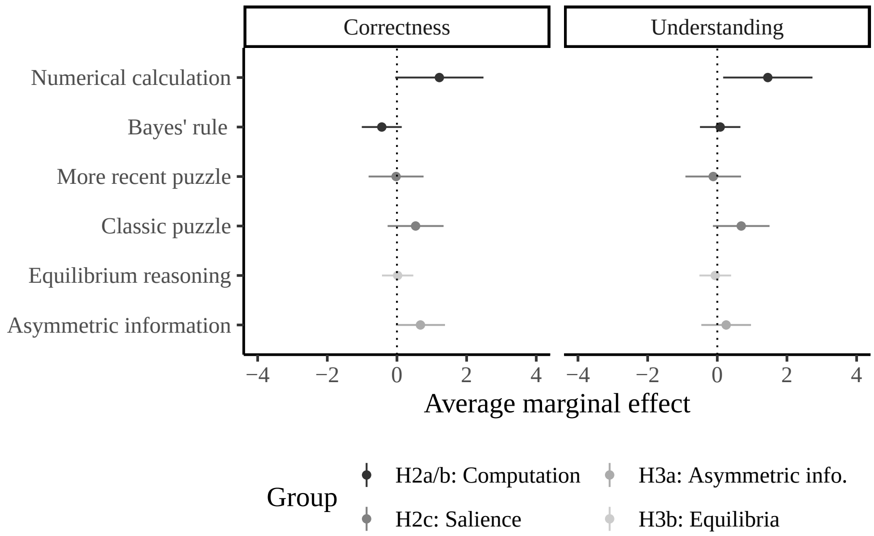


Fig. 10 Problem set level regressions with LLM fixed effects

Finally, we ran these models additionally controlling for the number of concepts invoked in the problem set. To avoid multicollinearity, we use the equilibrium reasoning category as a baseline. We did not find any evidence that the total number of concepts invoked affected either correctness or understanding.

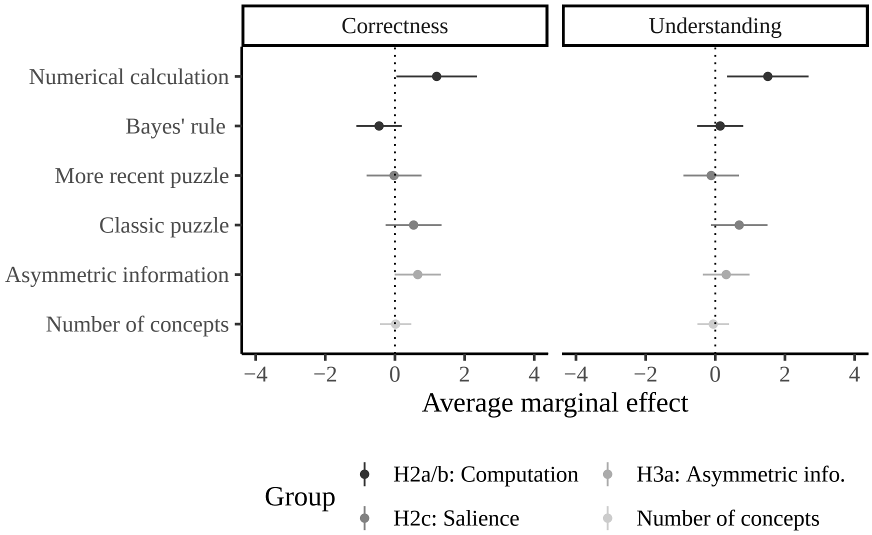


Fig. 11 Problem set level regressions with LLM fixed effects and controlling for the total number of concepts

Table 8 Problem set level coefficient estimates

	Correct	Understand
	(1)	(2)
Claude	−0.000	0.400
	(0.335)	(0.333)
Gemini	−1.000***	−0.550
	(0.335)	(0.333)
Grok	−0.000	0.100
	(0.335)	(0.333)
Llama	−2.450***	−2.250***
	(0.335)	(0.333)
Numerical calc.	1.336**	1.519**
	(0.630)	(0.628)
Bayes' rule	−0.213	0.318
	(0.256)	(0.255)
Classic puzzle	0.459	0.684*
	(0.408)	(0.407)
More recent puzzle	−0.368	−0.094
	(0.376)	(0.374)
Asymmetric info.	0.168	−0.024
	(0.200)	(0.200)
Equilibria	0.846***	−0.028
	(0.298)	(0.297)
Insight	3.521***	3.650***
	(0.570)	(0.568)
Observations	100	100
R^2	0.545	0.539
Adjusted R^2	0.493	0.488
Note:	*p<0.1; **p<0.05; ***p<0.01	

Table 9 Problem set level results: bivariate models, part 1

	Score											
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Claude	−0.000	0.400	−0.000	0.400	−0.000	0.400	−0.000	0.400	−0.000	0.400	−0.000	0.400
	(0.351)	(0.348)	(0.364)	(0.357)	(0.362)	(0.352)	(0.352)	(0.344)	(0.367)	(0.363)	(0.367)	(0.363)
Gemini	−1.000***	−0.550	−1.000***	−0.550	−1.000***	−0.550	−1.000***	−0.550	−1.000***	−0.550	−1.000***	−0.550
	(0.351)	(0.348)	(0.364)	(0.357)	(0.362)	(0.352)	(0.352)	(0.344)	(0.367)	(0.363)	(0.367)	(0.363)
Grok	0.000	0.100	0.000	0.100	0.000	0.100	0.000	0.100	0.000	0.100	0.000	0.100
	(0.351)	(0.348)	(0.364)	(0.357)	(0.362)	(0.352)	(0.352)	(0.344)	(0.367)	(0.363)	(0.367)	(0.363)
LLama	−2.450***	−2.250***	−2.450***	−2.250***	−2.450***	−2.250***	−2.450***	−2.250***	−2.450***	−2.250***	−2.450***	−2.250***
	(0.351)	(0.348)	(0.364)	(0.357)	(0.362)	(0.352)	(0.352)	(0.344)	(0.367)	(0.363)	(0.367)	(0.363)
Numerical calc.	1.516***	1.537***										
	(0.509)	(0.505)										
Bayes' rule			0.280	0.480**								
			(0.230)	(0.226)								
Classic puzzle					0.390	0.657***						
					(0.250)	(0.243)						
More recent puzzle							−0.638***	−0.756***				
							(0.224)	(0.219)				
Dominant strategy									−0.110	0.306		
									(0.325)	(0.322)		
Backward induction											−0.031	−0.322
											(0.325)	(0.322)
Constant	3.974***	3.923***	3.910***	3.760***	3.933***	3.803***	4.401***	4.416***	4.066***	3.954***	4.055***	4.048***
	(0.249)	(0.248)	(0.282)	(0.276)	(0.267)	(0.259)	(0.278)	(0.271)	(0.264)	(0.261)	(0.264)	(0.261)
Observations	100	100	100	100	100	100	100	100	100	100	100	100
Note:	*p<0.1; **p<0.05; ***p<0.01											

Table 10 Problem set level results: bivariate models, part 2

	Score											
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Claude	−0.000	0.400	−0.000	0.400	−0.000	0.400	−0.000	0.400	−0.000	0.400	−0.000	0.400
	(0.360)	(0.354)	(0.367)	(0.356)	(0.356)	(0.353)	(0.362)	(0.358)	(0.363)	(0.360)	(0.357)	(0.365)
Gemini	−1.000***	−0.550	−1.000***	−0.550	−1.000***	−0.550	−1.000***	−0.550	−1.000***	−0.550	−1.000***	−0.550
	(0.360)	(0.354)	(0.367)	(0.356)	(0.356)	(0.353)	(0.362)	(0.358)	(0.363)	(0.360)	(0.357)	(0.365)
Grok	0.000	0.100	0.000	0.100	0.000	0.100	0.000	0.100	0.000	0.100	0.000	0.100
	(0.360)	(0.354)	(0.367)	(0.356)	(0.356)	(0.353)	(0.362)	(0.358)	(0.363)	(0.360)	(0.357)	(0.365)
LLama	−2.450***	−2.250***	−2.450***	−2.250***	−2.450***	−2.250***	−2.450***	−2.250***	−2.450***	−2.250***	−2.450***	−2.250***
	(0.360)	(0.354)	(0.367)	(0.356)	(0.356)	(0.353)	(0.362)	(0.358)	(0.363)	(0.360)	(0.357)	(0.365)
Equilbrium reasoning	−0.486*	−0.610**										
	(0.248)	(0.244)										
Adverse selection			−0.038	−0.533**								
			(0.253)	(0.246)								
Moral hazard					0.675**	0.700**						
					(0.282)	(0.279)						
Efficiency reasoning							0.600	0.733*				
							(0.382)	(0.377)				
Asymmetric info.									−0.240	−0.260		
									(0.162)	(0.161)		
Equilibria											0.533**	0.093
											(0.227)	(0.232)
Constant	4.390***	4.427***	4.061***	4.160***	3.915***	3.860***	3.990***	3.927***	4.290***	4.260***	3.757***	3.949***
	(0.308)	(0.303)	(0.270)	(0.263)	(0.258)	(0.256)	(0.259)	(0.256)	(0.304)	(0.301)	(0.281)	(0.288)
Observations	100	100	100	100	100	100	100	100	100	100	100	100
Note:												*p<0.1; **p<0.05; ***p<0.01

Appendix D: Problem Sets

In this Appendix, we provide the original text of the 20 problem sets.

The problem sheets were used in undergraduate PPE admissions at Jesus College, Oxford, between 2008 and 2024, specifically in the “economics and politics” interview. Human candidates would have had 45 minutes to prepare and about 10-15 minutes to speak.

The year in which the problem sheet was first used appears in its title. (Some have been re-used with different wording or numerical values.) The problem sheets are presented in random order. This is the same order in which LLMs attempted them.

World Cup (2014)

Imagine you are the bureaucrat in charge of organizing the next World Cup. In order for this to be a success you need to build a) a new stadium and b) a new press centre. Unfortunately, you’ve left it until the last minute and so now only have three months left to build both. If either is not built in time, FIFA will withdraw your hosting rights, and they will pay you nothing. If both are built in time, FIFA will pay you (in a brown envelope) £20 million. You are only interested in maximizing the amount of money that you will get from FIFA (multiplied by the probability of success) minus any costs that you will have to pay to get the project finished.

Two companies apply to build the stadium: Acme Ltd and Bob PLC. Acme will definitely build the stadium in three months and will charge you £6 million upfront. Bob can make a choice in month 2 to either “plough ahead” and definitely finish the project which will cost you £7 million, or “cut his losses” which leads to certain failure, but only cost you £1 million. You must hire either Acme or Bob to build the stadium.

Two companies apply to build the press centre: Cowboy Inc and Designs for You. Both companies will charge you upfront for their investment into new broadcasting technologies, and their success is not guaranteed. Cowboy has a 50% chance of completing the press centre in three months and charges you £3 million upfront. Designs also has a 50% chance of completing the press centre in three months but charges you £4 million upfront. Designs, however, will know after the first month whether or not the press centre will be ready before the deadline. You have to hire either Cowboy or Designs to build the press centre.

1. If you hire Acme for building the stadium, would prefer Cowboy or Designs for building the press centre?

2. If you could verify any information possessed by the company you hire (e.g. if you hire Designs, you'll know at the end of the first month whether or not they will eventually succeed), then what would be the best course of action?
3. Suppose that you cannot directly observe any information received nor any action taken by the companies you employ. The building companies are only interested in maximizing the payments they receive from you; they are happy to lie to you and are also able to make (secret) payments to each other. Does this affect your answer to Question 2?
4. As in Question 3, you cannot directly observe any information received / actions taken by the companies you employ; the building companies are self-interested and have no qualms about misrepresenting the truth. You can, however, pay them conditional on the success of both projects, as long as they agree to such an arrangement in advance. Could it be optimal to hire Bob and Designs for the respective tasks?
5. What is the highest upfront cost you would be willing to pay to Designs when employing them to build the press centre?

Fundraising Drive (2021)

Ann and Bob participate in a fundraising drive. If their contributions add up to at least £100 then the drive is successful and a good outcome occurs that they both care about – e.g., children in need get free school lunches, additionally supported by an external charity. The "value" of the success of the fundraising drive, expressed in monetary terms, is £70 for Ann and £80 for Bob. The assumption that the sum of these two valuations exceeds £100 is meant to express that the fundraising drive generates more total satisfaction for them than the total cost of their contributions. Assume that contributions totaling to strictly more than £100 do not generate more value: Ann and Bob still value success at £70 and £80, respectively. In what follows, assume that each person cares about his or her net benefit in terms of "net cash equivalent". For example, if Ann contributes £40 and Bob contributes £60, then the drive is a success; Ann's net benefit is £70 - £40 = £30 and Bob's is £80 - £60 = £20.

Finally, assume that if their contributions add up to less than £100, then Ann gets back 80% of the total amount raised, whereas Bob gets back nothing. The interpretation is that the money raised in an ultimately unsuccessful fundraising drive is spent on a charitable purpose that Ann cares about but Bob does not. So, for example, if each contributes £30 then the fundraising drive is unsuccessful, and Ann ends up with a net benefit of $60 \cdot 80\%–30 = 18$ whereas Bob ends up with –30 (that is, a £30 loss). In this case Bob would be better off not contributing at all.

1. Briefly explain why Ann would never contribute more than £70 and Bob more than £80. Then, explain: If Bob expects Ann to contribute £40, is he better off by "completing the drive" (i.e., donating £60) or by not completing it? If Ann expects Bob to contribute £60 then will she want to add £40 (to complete the drive) or do something else?
2. Suppose they make their contributions simultaneously. Assume that Ann expects Bob to contribute Y pounds. Clearly, Ann can make the fundraising drive successful by contributing X = 100 – Y, or perhaps not make any contribution at all. Explain in words what the following inequality means: $70–(100 - Y) > 0.8 \cdot Y$.
3. For what values of Y does the inequality $70–(100-Y) > 0.8 \cdot Y$ hold? What does that imply for the analysis of the situation Ann and Bob face? Specifically, how much do you think Ann contributes and how much does Bob contribute when they are asked to contribute simultaneously?
4. Now suppose that they are asked to contribute sequentially: first Ann can donate a sum, X, then seeing this, Bob may contribute Y. What is Bob's optimal donation amount, Y, depending on the value of X? Anticipating this, how much will Ann donate in the first place? (Each can donate in 10p increments; it is acceptable not to donate at all.)
5. What insights do we learn from comparing simultaneous and sequential donation protocols?

Used Bikes (2011)

The student council at an Oxford college runs a used bicycle market where graduating students can sell and incoming students buy their means of local transportation. For simplicity, assume that three types of bicycles are owned by graduating students: High, Medium, and Low quality bikes. Low quality bikes tend to break down within a year; Medium quality bikes break down within a year with 50% chance, High quality bikes survive the year without trouble. The proportion of bikes of each quality is the same, 1/3 (i.e., each type of bike is owned by the same number of students). There are many students; graduating and incoming students are in equal numbers.

For simplicity, assume that every owner values a Low quality bike at £70 (i.e., he or she is willing to sell it for £70 or more), a Medium quality bike at £80, and a High quality one at £90. Every potential buyer values a Low quality bike at £60, a Medium quality one at £85, and a High quality one at £110.

Every owner knows the type of his or her bike. In contrast, potential buyers cannot tell what the quality of any given bike is. However, all buyers are aware of the "market conditions" (e.g., that the three types of bikes exist and are owned in equal numbers), and they also know how much each type of bike is worth to its owner. When a buyer is evaluating a bicycle of unknown quality, he or she is willing to pay up to the bike's expected value to him or her. For example, if the buyer knows that the bike is Low or Medium quality with 50-50% chance then the highest price he or she is willing to pay is 0.5*£60+0.5*£85 = £72.50.

1. Suppose that you are a fresher and in this used bike market you come across a seller offering a bicycle for £75. Is it a good idea to buy it? Why or why not?
2. Suppose that you are a fresher and in this used bike market you come across a seller offering a bicycle for £85. Is it a good idea to buy it? Why or why not?
3. Suppose that you are a fresher and in this used bike market you come across a bicycle for sale at price p. At what p would you be willing to buy?
4. Comment on how well the used bicycle market is working.
5. You find a seller who has come up with a simple yet innovative idea. She offers her bicycle for sale at £95 with a rebate of £30 if the bicycle breaks down within a year. Is it a good idea to buy? Why or why not?

Hairy Fairy (2020)

Sophie is six years old, and one of her baby teeth has just fallen out. The agreement with her parents is that she puts the tooth under her pillow when she goes to sleep; at night, the 'tooth fairy' will replace it with £1. The tooth fairy (in reality her father) is busy and forgetful. He forgets to carry out his tooth-fairy duties with 50% chance. Sophie is quite aware of all this, therefore she and her father agree: if the 'tooth fairy' forgets to come at night and take her tooth, then the following night the prize will be doubled: the tooth must be replaced with £2. If the 'tooth fairy' forgets it again (which occurs every night independently with 50% chance) then the prize is doubled again, it will be £4 on the third night, and so on. Once the tooth is exchanged for cash (the first time the tooth fairy does not forget to come), the game is over.

1. When Sophie goes to bed with her baby tooth under her pillow for the first time, how likely is it that the tooth fairy will arrive and replace it on the first night, on the second night, and generally, exactly on the nth night, where n is a positive integer?
2. What is the mathematical expectation of the revenue that the child will get from the tooth fairy, from a single baby tooth?
3. What do you think is the monetary value of this game to a real-life human child, in other words (or as a thought experiment), how much would Sophie be willing to accept up-front for her baby tooth, instead of playing the game with the tooth fairy?
4. If your answers to Questions 2 and 3 (concerning the mathematical expectation of the monetary value of playing the game, and the amount that a child would accept instead of playing the game, respectively) differ, then what do you think is the reason for the difference? Provide one or two possible explanations.

5. Sophie and her dad modify the agreement. If the 'tooth fairy' forgets to replace the tooth with money for 10 days in a row (recall that the 'tooth fairy' comes or forgets to come each night with 50-50% chance, independently across nights), then on the 11th morning Sophie gives up her tooth and gets £10. What is the mathematical expectation of her revenue in this scenario?

Sunny Canarias (2016)

Alice is thinking of hiring Bob to manage her vacation home in the Canary Islands. The rental income from the property is uncertain, but it also depends on Bob's efforts (whether he properly advertises vacancies, maintains the house, and so on). If Bob works hard then the property generates £60K (shorthand for £60,000) or £20K with 50%-50% chance in a year. If he does not work hard ("shirks") then it generates £20K with probability 1. Alice wants to maximize the mathematical expected value of her income, net of whatever she pays to Bob.

For Bob, it costs £5K over the course of the year to work hard; if he shirks then he saves this expenditure. He also cares about the wage he gets from Alice. In what follows assume that he behaves as if he wanted to maximize the mathematical expected value of the square root of his net income (meaning his wage from Alice less his expenses) measured in thousands of pounds. This assumption tries to capture the fact that he cares about a £1,000 increase in his income a lot more when his income is low compared to when his income is high.

For example, if Bob were to receive £30,000 with 50% chance and £6,000 with 50% chance from Alice while working hard, he would find it equivalent to receiving £9,000 for sure and shirking, because $0.5*\sqrt{30-5}+0.5*\sqrt{6-5} = 0.5*5 + 0.5*1 = 3$, whereas the square root of 9, $\sqrt{9}$, is also 3. (Note that we subtract 5 from his income to account for his expenses when he works, but not when he shirks.)

In case Bob agrees to work for Alice, he may work hard (incurring the £5K expense) or shirk (saving the expense) but cannot choose anything in-between. Assume that whenever Bob is indifferent between working hard and shirking (like in the numerical example above) he chooses to work, not shirk. Bob has an outside option, working for the government, which would yield a secure income of £8,999 per year, with no effort or cost to him whatsoever.

1. If Alice could observe whether or not Bob works hard throughout the year, then how much would she have to pay him to entice him to work for her, and how much would Alice earn, on average, from her investment property?
2. From now on assume that Alice cannot observe whether Bob works or shirks when hired. Do you think that the wage found in Question 1 would entice Bob to work hard for Alice? (What could go wrong?)
3. Suppose that Alice offers Bob a wage of £30K when her income from the property is £60K but only £6K when her rental income is £20K. Would this income-sharing arrangement entice Bob to work hard for Alice?
4. Alice thinks of another income-sharing agreement: she would offer Bob £21K in case her rental income is £60K, and £9K in case her rental income is £20K. Would this entice Bob to work hard for Alice?
5. Which wage contract (income sharing agreement) is better from Alice's point of view, the one proposed in Question 3 or the one in Question 4?

Bank Regulation (2012)

In a country far, far away the government is about to reform the regulation of a bank that is "too big to fail". The bank only cares about the profits it expects to make over the next ten years. The government only cares about what it expects the costs to the country will be over the next ten years (excluding the bank's profits).

Excessive risk taking by the bank will cause a financial crisis. The cost of a financial crisis to the country is £50 billion and there is no cost to the bank. The bank can choose to commit itself now to risky (R) or less risky (L) banking over the next ten years. The outcomes for R and L are as below:

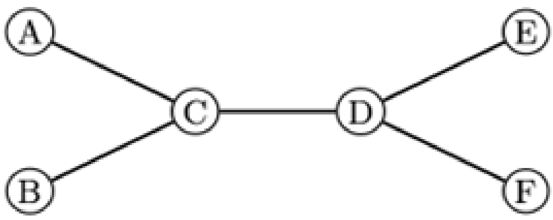
- R: If the bank takes the risky approach, then a financial crisis will definitely occur within the next ten years. The bank will make £3 billion a year in profits.
- L: If the bank takes the less risky approach, then a financial crisis has only a 50% chance of happening within the next ten years, but the country's economic output will decrease by £1 billion per year (i.e., by £10 billion over ten years), for certain. The bank will make £2 billion a year in profits (for certain).

1. Given the assumptions above, will the bank take option L or R?
2. Given the assumptions above, if the government could force the bank to take option L, should it do so?
3. Assume that the government can observe whether or not the bank takes option L or R. How much would the government have to pay to the bank every year in the next ten years if it wanted the bank to take option L? Should the government pay the bank the subsidy?
4. Now assume that the government cannot observe whether or not the bank takes option L or R. Instead, it can promise to pay the bank a subsidy at the end of the ten years based on whether or not a crisis occurred. If there was a crisis, the bank receives no subsidy. How much would the government have to promise to pay to the bank if it wanted the bank to take option L? Should the government promise to pay the bank the subsidy?
5. What assumptions have we made about the government in this account that strike you as implausible? List three such assumptions.

Sharing is Caring (2023)

Six young people share a flat: Ann, Bob, Chabi, Deniz, Ernie, and Fei. They all want to watch Netflix, but each prefers that others buy and share their passwords. Specifically, assume that the monetary benefit of having access to Netflix is b for each person, whereas the cost of paying for a Netflix account is c, with $b > c > 0$.

Assume that each person is happy to share their own Netflix password only with their close friends. Unfortunately, the flatmates are not all close friends with each other. Their close friendship connections are represented by the following graph.

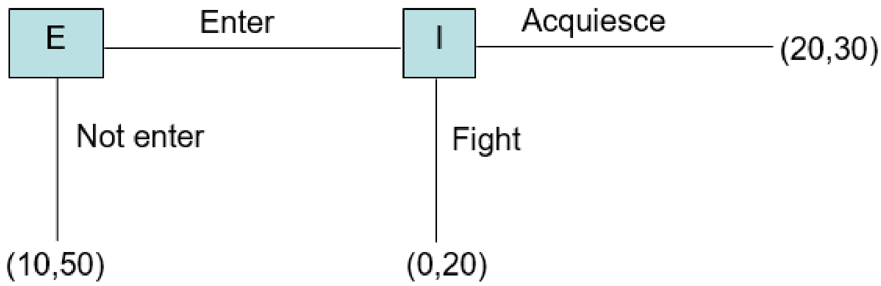


1. Briefly explain who are close friends with whom.
2. Suppose that Netflix does not allow simultaneous streaming (i.e., sharing is not feasible). Then who will buy a subscription and who will not?
3. If there is no limit on simultaneous streaming, then who will buy a subscription and who will not? Explain your reasoning.
4. Try to answer the same question when the number of devices that can simultaneously stream on the same account is two, and also when it is three.
5. Explain what we learn from comparing the answers to Questions 3 and 4.

Entry Deterrence (2008)

Consider the following situation described in the diagram below. There is currently one dominant political party in a country (the incumbent I), and a potential new party that may enter and compete in the system (the entrant E). Depending on the parties' actions they receive certain pay-offs (we could think of these as either direct benefits of being in power or more indirect benefits of achieving policy aims). The entrant E needs to decide whether it should enter the system and compete, or not enter. The incumbent I needs to decide whether to 'put up a fight' when E enters and restrict E's benefits of entry, or to 'acquiesce' and allow E to gain the benefits of entry.

If E does not enter, then the 'game' is over and the payoffs for E and I are, respectively, 10 and 50. If E does enter, then I can decide whether to contest this (to 'fight') or whether to acquiesce and allow E to enter. If I 'fights', the payoffs to E and I are 0 and 20; otherwise the payoffs are 20 and 30.



1. Which outcome is most preferred by E? By I?
2. If you were the incumbent what would you like the entrant to think what action you are going to take?
3. What do you think is the most plausible outcome if the game is played only once?
4. Would your answer for Question 3 change if the game were played repeatedly? If so, how?
5. Aside from issues of game repetition, are there other implicit assumptions that we need to make in order to draw conclusions about the outcome? List three such assumptions.

Partners at Work (2016)

Eric and Roger set up a widget making company in which both are equal partners. In any given month each partner may either work hard at making widgets or shirk.

If they both work hard then there is a 2/3 chance that their partnership earns £12K, but there is also a 1/3 chance that their partnership does not make any money. If they do earn any money, then this money is split equally between them.

If one of them works and the other shirks then the probability of jointly earning £12K goes down to 1/3 and the probability of getting no revenue at all goes up to 2/3. Again, if they do earn any money then this money is split equally between them.

If both shirk, then they earn nothing.

The cost of working (expressed in monetary terms) is £3K per person, for each partner that works hard. Note that this cost is incurred upfront in buying materials and is irrespective of whether the partnership is successful in earning money.

Each partner only cares about his own net expected monetary payoff. For instance, if both work then Eric gets £6K (half of £12K) with 2/3 chance and nothing with 1/3 chance, but also spends £3K on working hard, so his net expected payoff is $2/3*6 + 1/3*0 - 3 =$ £1K.

1. Compute the partners' expected net payoffs in all possible situations: when both work, when one of them works and the other shirks, and when both shirk.
2. Suppose the partners can monitor each other and agree in advance whether or not to work hard. This agreement can be enforced at no cost. What is the best course of action for them?
3. Suppose that each partner can secretly shirk even if they had both agreed to work hard. What do you expect to happen?
4. It is possible to hire a manager who will monitor (and enforce the promise of) the partners' dedication to working hard. Should Eric and Roger hire a manager? What is the highest wage they would be willing to pay a manager?

5. Suppose that instead of engaging a manager as in Question 4 they agree to the following. Each month they flip a coin. If the outcome is 'heads' then Eric works hard and Roger shirks that month; if it is 'tails' then Roger works hard and Eric shirks that month. If the partnership is successful (i.e. it generates £12K in that month) then whoever was supposed to shirk pays the other £3K and takes him out to lunch. What do you expect to happen? Is this a better arrangement than hiring a manager?

Sports Funding (2023)

The university's finance committee will decide how much to spend on student sports next year. They believe the right amount is anywhere between £40k and £60k, with any sum between these extremes being equally likely; that is, from their perspective, the right level of spending is uniformly distributed between £40k and £60k. Assume that the finance committee spends on sport the mathematical expectation of the amount they believe is right for the university, given the information they have. Tom is the president of the students' sports committee and knows exactly how much spending would be optimal for the university. Indeed, the finance committee will ask him for his advice on this matter. The issue is that he would like to secure extra funding for certain sports that he likes and practices the most. Specifically, assume that the ideal spending level from Tom's perspective is £3k more than what is optimal for the university, and he dislikes it equally strongly if the university spends x amount less or (the same) x amount more than his ideal point. The smaller the difference between the eventual spending level and his ideal point the happier he is.

1. How much does the finance committee spend on sports without any input from Tom?
2. Suppose that the finance committee asks Tom what the right level of spending on sports is for the university. Would it be wise for them to take his report at face value? Would it be a good idea for them to ask Tom about the right level of spending and then subtracting £3k from his response?
3. Suppose that Tom manages to convey to the committee whether the right amount of spending on sports for the university is above a certain threshold T, or below it. How much does the finance committee spend on sports if he says it is above T, and how much do they spend on it when he says it is below T?
4. Under what condition is it optimal for Tom to tell the truth regarding whether the optimal level of spending on sports is above or below a given threshold, T? For example, would he tell the truth, if the threshold was $T = 50$ and he knew that the right amount of spending on sports for the university was £49k?
5. Can Tom credibly and truthfully convey to the finance committee whether the right amount of spending on the sports for the university is above or below the threshold £44k? Explain why. Interpret what we have learnt from this exercise, regarding whether communication is possible between a biased expert and a decision maker when communication is entirely costless, i.e., "talk is cheap".

Island Dispute (2017)

Two countries, Freedonia and Sylvania, are in a dispute over who should have the rights to an island. To secure possession, each country may send in its army and occupy the island, but if both armies arrive simultaneously, they will end up in a stalemate.

The advisors of Freedonia's ruler estimate that the up-front monetary cost of sending in their troops to occupy the island is £2 billion, and if their army arrives unopposed, then the value of the island's long-term occupation will be £12 billion. If Sylvania's army also attacks the island then Freedonia's cost of waging the war increases to £7 billion, with no anticipated gain, as most likely neither country will be able to secure the island. If Freedonia decides not to send their army to occupy the island then of course they do not incur any cost, but do not benefit from long-term occupation either. The objective of Freedonia's ruler is to choose the best option as expressed by their country's monetary gains and losses outlined above.

1. Suppose that Sylvania is not expected to attack the island. What are the monetary values of the options available to Freedonia (sending in the troops or not), and which action is best for Freedonia?
2. Suppose that Sylvania is expected to attack the island. Compare the monetary values of the two options available to Freedonia and choose which one is best for them.
3. Suppose that Sylvania's leadership makes the exact same calculations and arrives at the exact same estimates concerning the costs and benefits of attacking the island as the advisors to Freedonia's ruler have done above. Compute each country's monetary payoffs in all possible outcomes: when both attack, when exactly one country attacks, when neither attack. What do you think is the most likely outcome?
4. Freedonia's leadership believes that Sylvania is ruled by an unpredictable dictator who will flip a coin and send its army to occupy the island if the outcome is heads and not send its army if the outcome is tails, that is, each event occurs with 50-50% chance. What is Freedonia's best course of action in light of this assumption?
5. Suppose now that Freedonia has the opportunity to announce in advance that it will invade the island. The cost of this announcement (in terms of military build-up and perhaps also the loss of the country's peaceful reputation) is £1 billion. This cost is incurred no matter whether they end up sending in their troops and does not decrease the cost of military invasion – it is simply added on to any and all other costs. Freedonia's gain from a successful invasion (which occurs if, and only if, Sylvania does not invade) remains the same. These facts are well-known to both Freedonia's and Sylvania's rulers. If Freedonia declares its intention to invade, should Sylvania assume the threat will be carried out? Is it in Freedonia's interest to make such an advance announcement?

Tutors and Colleges (2012)

Three new tutors (Alice, Bob and Carol, referred to as A, B, C) will join three Oxford colleges (Jesus, Keble and Lincoln, or J, K, L), each with one vacancy. The question is which tutor should end up in which college. Each tutor has a preference ranking over the colleges (based on, e.g., the quality of beer). The colleges rank the tutors based on teaching needs.

Assume that tutors A and B both prefer J to K and K to L (denoted by $J > K > L$), whereas tutor C ranks the colleges as $K > J > L$. College J prefers tutor C to tutor A to tutor B, whereas K and L both rank the tutors $B > C > A$. The rankings are summarized below:

A:	$J > K > L$	J:	$C > A > B$
B:	$J > K > L$	K:	$B > C > A$
C:	$K > J > L$	L:	$B > C > A$

A matching (or assignment) of tutors to colleges is called stable if there is no tutor who would rather go to a different college, which would also like him or her better than the tutor originally assigned to it. For example, the assignment "Alice to Lincoln, Bob to Jesus, Carol to Keble" (or "AL, BJ, CK") is not stable because Alice would switch to Jesus from Lincoln, and Jesus would also prefer Alice to Bob.

1. Explain why the matching "Alice to Keble, Bob to Lincoln, Carol to Jesus" (or "AK, BL, CJ") is not stable. Which tutor and college would "break" the assignment? Is the matching "Alice to Lincoln, Bob to Keble, Carol to Jesus" ("AL, BK, CJ") stable?
2. Suppose that tutors pick colleges in alphabetical order (first Alice, then Bob, then Carol), from the colleges not taken by someone else before them. What is the resulting assignment, and is that assignment stable?
3. The matching mechanism is modified so that each tutor picks a college simultaneously, then each college chooses one of the tutors that chose them. Unmatched tutor(s) can pick again from the college(s) that still have a vacancy, then colleges choose one among the tutors that picked them. Assuming that tutors pick according to their true preferences, what outcome do you expect, and is that stable?

4. This is the final proposal for a matching algorithm. First, each tutor simultaneously picks a college; then each college demanded by multiple tutors selects one, provisionally. In each subsequent round, each tutor who has been rejected in the previous round selects a new college – any college (not just one with a vacancy) that he or she had not selected in any previous round. Then, again, colleges that are over-demanded by tutors (including the demand of the provisionally selected tutor) make a new provisional selection. The procedure ends when each college is only demanded by one tutor. Find out how this mechanism works in our example and determine whether the resulting assignment is stable.
5. What insights arise from the comparison of the matching mechanisms studied in Questions 2, 3, and 4?

Money Jars (2024)

Two students, Ana and Barbara, are playing a game with their teacher. The teacher puts either £10 or £30 into Jar A with 50-50% probability. Independently of that, the teacher also puts either £10 or £30 into Jar B with 50-50% probability.

Ana can look into Jar A and see how much money it contains, and Barbara can look into Jar B and see how much money it contains. They do not see the content of the other jar, but they know it is either £10 or £30 with 50-50% probability.

Afterwards, Ana and Barbara compete in an auction. They both write down their bid on separate pieces of paper (the other student does not see their bid) and hand it to their teacher. Whoever bids the higher amount wins. The winner receives all the money in both jars, but they must pay the value of the other student's bid. The losing bidder doesn't pay or receive anything.

If Ana and Barbara both write down the same number, the teacher flips a fair coin to determine which student wins the money in the jar, and the winning student pays the bid. Again, the losing student doesn't pay or receive anything.

1. Let X denote the amount of money in Jar A and Y the amount of money in Jar B. From Ana's point of view, write down the possible values for the total amount of money in both jars, and their probabilities. Repeat for Barbara.
2. Assume that Barbara believes that Ana will bid $X + 20$. Barbara is planning to bid $Y + 20$, but, before submitting her bid, she asks for your advice. Is there a better strategy for Barbara than bidding $Y + 20$?
3. Now assume that Barbara knows that Ana will bid $2X$. Barbara is planning to bid $2Y$. Is there a better strategy for Barbara than bidding $2Y$?
4. Now assume Ana's mother offers to pay Ana an additional £1 if she wins the bidding. So, if Ana wins, her income is $X + Y + 1$. If Barbara wins, her income is still $X + Y$. What do you expect to happen, and what advice would you give to Barbara?
5. Summarize the insights gained from answering Questions 2, 3, and 4.

Shady Deals (2015)

A politician (Ms P) is approached by a shady character called Mr Q. He offers financial support to a foundation run by Ms P's husband in exchange for certain foreign-policy favours. This puts Ms P in a pickle. Accepting the deal would benefit her family but also create a substantial risk for her future as a politician. Rejecting the deal will provoke the ire of Mr Q and make her life very difficult. Overall, she would rather have never met Mr Q.

Ms P assigns numerical "valuations" to the three outcomes mentioned above. Accepting Mr Q's offer is worth 0 (zero) to her. Rejecting the offer is worth -1. The hypothetical situation when Mr Q never made the offer is valued at +2. Mr Q also evaluates the outcomes numerically. If his offer is accepted then his valuation is +1. If he makes the offer and it is rejected, his valuation is -1. If he does not approach Ms P then the outcome is worth 0 to him.

The right way to interpret these valuations is that Mr Q would be indifferent between (i) not approaching Ms P, and (ii) approaching her believing that his offer will be rejected with exactly 50% chance. This is so because $0 = 50\% \times (+1) + 50\% \times (-1)$.

1. Suppose that the situation as it is described above is equally clear to Mr Q and Ms P, including the way each side evaluates the possible outcomes. Will Mr Q approach Ms P, and if so, will Ms P accept the offer?
2. Now assume that the situation is not crystal clear to Mr Q. He cannot be sure whether Ms P is an ordinary politician (thinking the way described above) or that she is an incorruptible politician (which means that she rejects all approaches). Mr Q's assessment of the probability that Ms P is incorruptible is X%; he believes Ms P is an ordinary politician with $(100 - X)$% chance. What should Mr Q do depending on the value of X? Would it benefit Ms P to make Mr Q believe the value of X is high?
3. Imagine that two shady characters are thinking of approaching Ms P: first Mr Q1, and then Mr Q2. The latter observes, before making his own move, whether or not Mr Q1's offer has been rejected by Ms P. Furthermore, suppose that initially both Mr Q1 and Mr Q2 believe Ms P is incorruptible with probability $X = 40\%$. What do you predict will happen? Could an ordinary politician make Mr Q2 believe that she is an incorruptible politician, perhaps by refusing Mr Q1's offer?
4. Continue with the setup of the previous question: both Mr Q1 and Q2 initially believe Ms P is incorruptible with probability $X = 40\%$. Now suppose that Mr Q2 thinks Mr Q1's offer is definitely rejected by an incorruptible Ms P, whereas it is rejected by an ordinary Ms P with 2/3 chance, that is, in 2 out of 3 cases. Try to come up with a percentage value for the chance that Ms P is incorruptible in case Mr Q2 observes she has rejected Mr Q1's offer. (Hint: suppose that in 40 out of 100 cases Ms P is incorruptible and in 60 cases she is an ordinary politician. Then figure out in how many cases you expect to see Mr Q1's offer to be rejected. In how many cases is Ms P incorruptible when Mr Q1's offer is refused?)
5. What, if anything, do we learn from the answers to Questions 3 and 4?

Anna's Bike (2013)

Anna is leaving home to start university, and she wants to sell her bicycle. The only potential buyer is Bob. Anna will make a price offer to Bob, which he can either accept or reject. If he rejects the offer, then neither side gets any surplus as the bike will be given to charity.

The value of the bike for Bob is uncertain to both sides. Initially Anna and Bob agree that there is a 40% chance that the bike will work fine, in which case it is worth £200 to Bob, but there is also a 60% chance that the bike will soon break down, in which case it is worth nothing at all to him. This is a simplification. As Anna prepares her bike for sale, she finds out its true "quality" and hence its value for Bob (either £200 or £0) with 50% chance. Otherwise (also with 50% chance) she learns nothing new. As Bob inspects the bike, he figures out its true value with 25% chance and learns nothing new with 75% chance. This is independent of whether or not Anna has found out the true value of the bike.

Anna and Bob are both good at making rational inferences and calculating the mathematical expectation (or average value) of any random number. This is done by multiplying the probability of each possible outcome by the value of the outcome and then adding up the products. For example, the mathematical expectation of the bike's value for Bob with no additional information is 0.4*£200+0.6*£0 = £80. Assume that Anna and Bob want to maximize the expected value of their own surplus from trade. Each person is keen on carrying out a trade even if his or her expected surplus is zero, but neither agrees to a trade with a strictly negative expected surplus.

1. Suppose that neither Anna nor Bob can convincingly prove to the other side if or when they find out the true value of the bike. Explain what happens (whether or not Bob buys, how much Anna earns on average) if Anna charges £80.
2. Now suppose Anna can prove to Bob the value of the bike if she is lucky enough to have discovered it. (She cannot prove that she does not know the value of the bike.) What is her best strategy for selling the bike in this case? In particular, how much can she charge if she finds out the value of the bike? How much can she charge when she does not discover the value of the bike?

3. Is Anna better off when she is able to prove the value of the bike whenever she discovers it? In other words, is her expected revenue in Question 2 higher than it was under assumptions in Question 1?
4. Suppose that Anna's little brother Seb is an expert on bicycles: everyone knows that he can figure out the true value of the bike for Bob (whether it is £200 or 0), and prove it to him as well. What is Anna's profit if she uses Seb as intermediary?
5. Compare Anna's revenue in the three scenarios studies above: when she cannot prove the bike's value, when she can prove it provided she discovers it, and when she uses Seb's help who can always prove the bike's value to Bob. Are there any general conclusions to draw from your answers?

Committee Voting (2010)

A three-member committee at an Oxford college is to make an investment decision ("invest" or "not invest"). Each committee member initially thinks "invest" is best with 50% chance and "not invest" is best with 50% chance. Each committee member collects information on whether "invest" is the right decision for the college. For simplicity, think of the information that each committee member obtains as a "signal" that can be either GOOD or BAD concerning the desirability of the investment.

A signal is not always correct: sometimes the committee member gets a BAD signal when the right decision is to "invest" and vice versa. Assume that if "invest" is the best decision for the college then a committee member observes GOOD with 60% chance and BAD with 40% chance. This is independent of what any other committee member observes. If the option "not invest" is best for the college, then each committee member independently observes GOOD with 40% chance and BAD with 60% chance.

1. If a committee member observes the GOOD signal, then how likely does he or she think that "invest" is best for the college? Can we put a probability on that, or at least say whether the committee member thinks "invest" is more likely to be better than "not invest" conditional on seeing a GOOD signal?
2. If one committee member observes a GOOD signal, how likely is it that another member of the committee also observes a GOOD signal?
3. Suppose that the committee members cast simultaneous, secret ballots and the proposal ("invest" or "not invest") that gets the majority wins. The committee members do not talk to each other about the signals they have observed before the vote. If each committee member wants to maximize the probability that the best proposal wins, how will they vote, conditional on what they observed?
4. From now on assume that besides wanting the best outcome for the college, each committee member also cares about being on the winning side. Does this alter the way each committee member votes?
5. Change the voting procedure: Suppose that the committee members vote sequentially, that is, they take turns to announce which decision they support. The chair of the committee goes last and the majority wins. If each committee member wants to maximize the probability that the best proposal wins and also cares about being on the winning side, how will they vote? Start the analysis by thinking about how the chair will vote at the end; then consider the voter that goes second, and so on.

Rappers' Battle (2019)

MC Alice (she) and B-Rabbit (he) are two hip-hop artists deciding which mid-winter music festival to play: Hullabalooza or Jinglejam. Each can only play one festival, but each festival can accommodate both. Each rapper has 1,000 die-hard fans who go wherever their favourite artist appears. There are 3,000 additional fans who only attend a festival if both artists play it. A music festival pays the artist(s) a total of £2 per ticket sold, split evenly between the two rappers if both appear at the same festival. There are no other rappers around. Finally, it is well-known that MC Alice lives near Hullabalooza, so it is costless for her to appear there, but she incurs a transportation cost of £2,000 if she plays Jinglejam. Conversely, B-Rabbit incurs a cost of £2,000 if he plays Hullabalooza, but bears no cost if he plays Jinglejam.

Each musician wants to maximize his or her own profit (income minus cost) with no regards to the other's profit or loss. The data (number of fans, payments, costs) are known to everyone.

1. There are four possible outcomes: both rappers play at Hullabalooza; both play at Jinglejam; or they appear at different festivals. Compute each rapper's monetary profit in each of the four outcomes.
2. What should MC Alice do if she anticipates that B-Rabbit will play at Jinglejam? What should she do if she anticipates B-Rabbit going to Hullabalooza? Which outcome or outcomes do you think are likely to happen, and why? Would communication and coordination between the rappers help?
3. MC Alice's agent suggests that instead of playing either festival she could stream a studio concert via social media. This way she would reach a wide audience at a low cost generating a net profit of £4,000. B-Rabbit learns that MC Alice has this option, but unfortunately, he does not and has to play one of the festivals. What should he do? Will MC Alice play at one of the festivals or go for the studio concert? Would communication between the rappers help?
4. Suppose that the studio concert is not feasible for MC Alice after all. However, someone suggests to her that she could publicly burn £1,111 on "international singles day" (11 November), just before deciding on which festival to play at. At first, this seems to be just a costly stunt. But think harder about it: Having burnt this much money, would MC Alice ever sign up to play Jinglejam? If B-Rabbit deduces this, where will he play in case MC Alice publicly burns £1,111 on singles day? Does the costly stunt benefit MC Alice in the end?
5. Having worked through these questions, what do you think is the take-away, are there any insights here that are relevant for politics and economics?

Invade or Wait? (2013)

Two countries, Strongland (S) and Punyland (P), are thinking about going to war with a third country. If the third country is invaded, by anybody, then both Strongland and Punyland will be able to exploit its oil reserves (worth £10 billion) between them. Each country has the choice to invade or to wait. Invading the other country is costly (£2 billion for each invader) and antagonizes locals who do not want to award oil contracts to invaders. Waiting is costless.

All things being equal, Strongland, being stronger, is able to enforce its claims on the oilfields more effectively than Punyland. There are four possible outcomes:

1. What are the relevant net gains for P and S in outcomes 1, 2, 3 and 4?
2. If you were the ruler of Punyland then what would you do?
3. Given your answer to Question 2, which of the four outcomes would you predict would happen?
4. Would your answer to Question 3 change if these same two countries were thinking of invading an indefinite number of other countries in the future, under exactly the same circumstances as above?
5. What implicit assumptions, if any, underpin the conclusions that you drew in Questions 3 and 4? List three such assumptions.

Inheritance (2022)

The city of Oxbridge sits in the county of Oxbridgeshire. The Conservatives run the Oxbridgeshire county council and Labour run the Oxbridge city council. A generous local benefactor has recently died. His will reveals that he has left 50 acres of land and £100,000 to be shared between the city and county councils.

The two councils have different preferences for land and cash. The city council needs land, and so 1 acre of land is worth £20,000 to them. The county council prefers cash, and so 1 acre of land is worth £10,000 to them. Both councils know the other's preferences. As they are controlled by rival parties, each only cares about the part of the bequest they receive and are unwilling to talk to one another other than to comply with the benefactor's requested method of dividing the assets.

1. The local benefactor has stipulated that the procedure to divide the bequest is as follows: "The county will make an offer to the city, which the city either accepts or refuses. If the city refuses, then all assets will be shared equally." What is the offer that the county should make to the city?
2. Now suppose the procedure is that the county divides the inheritance into two parts and the city chooses. The city cannot refuse the split, but it can choose which part to take. What should the county do?
3. Suppose the city divides and the county chooses. What should the city do?
4. After the city council elections both county and city are controlled by the Conservatives. This means that they will talk to one another before the offers are made but they still only care about their own share. Does this change your answers to Questions 1, 2, and 3?
5. Since both are controlled by the same party, imagine that the county council now also cares about the outcome for the city and vice versa. Does that affect your answers to Questions 1, 2, and 3?

Sponsored Links (2015)

An internet search company sells sponsored links to advertisers as follows. When a user initiates a search, the company identifies the key word or expression in the search (called the adword) and compares the bids that advertisers had put in for this adword. For example, the adword could be "xmas lights" and the bids monetary values (e.g., 10p, 4p, 1p). On the search results page the company displays links to websites relevant for the search itself, as well as two sponsored links, which are vertically ordered. The top sponsored link is awarded to the advertiser offering the highest bid for the adword, the bottom link goes to the second highest bid. A bid is valid for one hour and then expires.

The advertiser pays the search company when the user clicks on its sponsored link. However, the payment amount is not the advertiser's own bid, but rather the bid of another advertiser "crowded out" from the same spot. That is, the advertiser at the top sponsored link pays, per click-through, the second-highest bid (which has won the bottom sponsored link), whereas the advertiser winning the bottom sponsored link pays, also per click-through, the third-highest bid (submitted by an advertiser that did not get either link).

Suppose that at a given point in time (today between 10am and 11am) exactly three advertisers are interested in sponsored links for the adword "xmas lights". The three bidders have different valuations for a click-through, for example, because they sell different types of festive decoration. Advertiser A expects to gain 10p from a click-through, B expects to gain 4p, and C expects to gain 1p. Advertisers know that the top sponsored link gets 200 clicks per hour, whereas the bottom sponsored link gets 120 clicks per hour for this adword.

1. Suppose that the three advertisers submit bids equal to their true valuations for a click-through. Who gets which sponsored link, and what are the advertisers' expected profits from buying the sponsored links?
2. Could any of the advertisers achieve a larger expected profit by bidding differently (higher or lower than their true valuation per click-through)?
3. Suppose that one of the parameters of the problem is changed: The bottom sponsored link also gets nearly 200 clicks per hour (instead of 120). Could any of the advertisers achieve a larger expected profit by bidding higher or lower than their true valuation per click through?
4. Suppose that the rules are changed so that each winner has to pay their "impact" on the sum of the other two bidders' expected gains – the total amount by which the valuations (gains before payments are subtracted) of the other two advertisers would have increased had the winner of a given spot never bid at all. What are these payments if all bid their true valuations (re-calculate it for both Questions 2 and 3). Is there now a reason for any of the advertisers to exaggerate or understate their valuation for a click-through?
5. Summarize the insights from answering Questions 2, 3, and 4.

Appendix E: Correct Answers

In this Appendix, we provide the marking criteria for each of the 20 problem sets.

The passages and the corresponding questions are paraphrased in brief as well. For the original, complete passages see Appendix D.

The answers are often given using more advanced technical terms than what is expected from an applicant. This is for brevity and clarity.

World Cup (2014)

Must build a) a new stadium and b) a new press centre within 6 months to get £20m. Stadium: must hire either Acme or Bob. Acme charges £6m upfront and will finish. Bob can make a choice in month 2 to either "plough ahead" and finish at cost £7m, or cancel at cost £1m. Press centre: hire Cowboy or Designs. Cowboy charges £3m upfront and has a 50% chance of completing the build in 3 months. Designs charges £4m upfront, will know after month 1 if it will finish, which has a 50% chance as well.

Question 1. If you hire Acme for the stadium, would prefer Cowboy or Designs for building the press centre?

Answer: Cowboy. Same prob of completing at lower upfront cost. No gain from learning if Design is successful or not after a month as Acme cannot be cancelled.

Question 2. If you could verify any information possessed by the company you hire (e.g. if you hire Designs, you'll know at the end of the first month whether or not they will eventually succeed), then what would be the best course of action?

Answer: Acme+Cowboy costs £9m and yields a payout of £20m with 50% chance. Bob+Design with the option to cancel Bob's contract if Design cannot finish yields £20m minus £7m+£4m with 50% chance, or £0 minus £1m+£4m with 50% chance. The expected profit is 0.5*(£9m)+0.5*(-£5m)=£2m, hence choose the latter.

Question 3. You cannot directly observe any information received nor any action taken by the companies you employ. The building companies are only interested in maximizing the payments they receive from you; they are happy to lie to you and are also able to make (secret) payments to each other. Does this affect your answer?

Answer: Yes. Design will not disclose if they cannot complete after month 1, hence we do not benefit from the option of cancelling Bob if Design is unsuccessful.

Question 4. Still cannot directly observe any information received / actions taken by the companies you employ; the building companies are self-interested and have no qualms about misrepresenting the truth. You can, however, pay them conditional on the success of both projects, as long as they agree to such an arrangement in advance. Could it be optimal to hire Bob and Designs for the respective tasks?

Answer: Yes. Pay Design £4m upfront and ask them to report to Bob after one month whether the press centre will succeed (implying that Bob should also plough ahead) or not (implying that Bob should cancel). Promise to pay Bob £7m if both projects succeed and £1m if either project fails. Design has no reason to lie to Bob, and Bob has no reason to ask (bribe) Design to misrepresent the truth either.

Question 5. What is the highest upfront cost you would be willing to pay to Designs when employing them to build the press centre?

Answer: Not more than £5m; at that price Acme+Cowboy becomes cheaper.

Fundraising drive (2021)

If Ann and Bob jointly contribute at least £100 then the drive is a success, which is worth £70 for Ann and £80 for Bob. Otherwise the drive fails; Ann gets back 80% of the total amount raised and Bob gets back nothing. Each person cares about his or her net benefit. For example, if Ann contributes £40 and Bob contributes £60, then the drive is a success; Ann's net benefit is £70-£40=£30 and Bob's is £80-£60=£20. For another example, if each contributes £30 then the drive is unsuccessful; Ann ends up with a net benefit of £60*80% – £30 = £18 whereas Bob ends up with –£30 (that is, a £30 loss). In this case Bob would be better off not contributing at all.

Question 1. Explain why Ann would never contribute more than £70 and Bob more than £80. If Bob expects Ann to contribute £40, is he better off by "completing the drive" (i.e., donating £60) or by not completing it? If Ann expects Bob to contribute £60 then will she want to add £40 (to complete the drive) or do something else?

Answer: Neither donates more than his or her valuation: £70 for Ann, £80 for Bob.

If Bob expects Ann to give £40 then Bob gets £80-£60=£20 by completing and nothing from not completing, hence he completes. If Ann expects Bob to give £60 then she ends up with £70-£40=£30 by completing and 0.8*£60=£48 by not completing ("stealing the pot"). She is better off with the latter.

Question 2. Simultaneous contributions. Ann expects Bob to contribute Y; she induces success by giving $X = 100–Y$. Explain what $70–(100 - Y) > 0.8 * Y$ means.

Answer: This is the condition on Bob's donation Y that makes Ann want to complete.

Question 3. When does $70–(100 - Y) > 0.8 * Y$ hold and what does it imply?

Answer: The inequality can be rearranged to $Y > 150$, which never holds (Bob never sets $Y > 80$), hence Bob cannot make Ann want to complete, hence he won't contribute, therefore Ann won't contribute either. The drive fails.

Question 4. Sequential contributions: Ann donates X, then seeing this, Bob gives Y. What is Bob's optimal Y given X? Anticipating this, how much will Ann donate in the first place? (Each can donate in 10p increments; it is acceptable not to donate at all.)

Answer: If $X < 20$ then Bob is better off not giving, $Y = 0$, but if $X > 20$ then Bob is strictly better off setting $Y = 100 - X$. In words: Bob will complete Ann's donation to £100 provided her initial donation is at least £20. Anticipating this, Ann donates £20. The fundraising drive is successful!

Question 5. Insights from comparing simultaneous vs sequential protocols.

Answer: By providing "seed money" Ann can induce Bob to contribute and complete the fundraising drive. This is impossible when they donate simultaneously. Empirical fact (candidate need not be aware): sequential donations (or someone committing to donate in case others do) raise more money in fundraising drives. It is interesting to note that the sequential protocol doesn't work if Bob goes first. Second insight: the person who has the most incentive to "steal the pot" should go first.

Used bikes (2011)

There are equal numbers of High, Medium, and Low quality bikes. (High: never breaks down, Medium: breaks down with 50%, Low: for sure.) Only the owner knows the bike's type. Every owner values a Low quality bike at £70, a Medium quality bike at £80, and a High quality one at £90. Every potential buyer values a Low quality bike at £60, a Medium quality one at £85, and a High quality one at £110.

Question 1. Is it a good idea for a potential buyer to buy a bike selling for £75?

Answer: No. Only low-quality bikes are sold at that price and buyers value those £60.

Question 2. Is it a good idea for a potential buyer to buy a bike selling for £85?

Answer: No. Only low- and medium-quality bikes are sold at that price; the expected value for a potential buyer of a bike sold at £85 is therefore (£60+£85)/2=£72.5 (this calculation is helpfully given in the text), which is less than the price charged.

Question 3. Suppose that you are a fresher and in this used bike market you come across a bicycle for sale at price p. At what p would you be willing to buy?

Answer: In Question 1 we argued it is not worth buying at p=£70 (in fact, at any p below £80) because only low-quality bikes are offered at that price and the buyer values such bikes at £60 $< p$. In Question 2 we considered p between £80 and £90: only low- and medium-quality bikes are offered, buyers value them at £72.5 $< p$, not worth it either. All bikes are offered at p=£90 and above; the buyer's willingness to pay for an average bike is

$$(60 + 85 + 110)/3 = 85 < p$$

, hence no trade either. There is no p at which it is worth buying a bike that is put on sale by the owner.

Question 4. Comment on how well the used bicycle market is working.

Answer: It does not work at all. No bikes are traded even though buyers value medium- and high-quality bikes more than current owners do. The market unravels; this is an example of adverse selection, Akerlof's market for lemons.

Question 5. You find a seller who has come up with a simple yet innovative idea. She offers her bicycle for sale at £95 with a rebate of £30 if the bicycle breaks down within a year. Is it a good idea to buy? Why or why not?

Answer: Such contract yields £65 for a Low owner (the bike breaks down for sure), less than the owner's valuation of £70, hence not rational for him to offer.

In contrast, Medium and High bike owners are happy to offer this contract: in expectation they get £95-£15=£80 and £95 respectively, which are weakly more than their own valuations (£80 and £90) for their respective bikes. The expected net payments made by potential buyers (£80 if the bike is Medium, £90 if it is High) are strictly lower than their willingness to pay for such bikes, hence they buy. This "warranty contract" is a market solution to the market failure noted in Question 4.

Hairy fairy (2020)

Based on a true story. Sophie gets £1 for her tooth if the tooth fairy (her father) remembers to take it on the first night, £2 if he forgets it on the first night but remembers it on the second night, and so on: £2^{n-1} if the first time he does not forget to visit and take the tooth is night number n=1, 2, 3, ... Conditional on not having come earlier, the tooth fairy remembers to come with an iid 50% chance. The game is over as soon as the tooth fairy remembers to visit.

Question 1. When Sophie goes to bed for the first time, how likely is it that the tooth fairy will arrive and replace it on the first night, on the second night, and generally, exactly on the nth night, where n is a positive integer?

Answer: First night: 50% (given in the text), second night: 25% (0.5*0.5, the question is clear that we compute this ex ante, not conditional on forgetting to visit on the first night!), generally, the ex-ante probability that the tooth fairy first visits for the first time on night n is $\frac{1}{2}^n$, because each "draw" (come or not if haven't) is iid with 50%.

Question 2. What is the mathematical expectation of the revenue that the child will get from the tooth fairy for a single baby tooth?

Answer: The probability of the fairy first visiting on night n is $(1/2)^n$; the payoff conditional on that is £2^{n-1}. Each event "Sophie gets paid on night n" is mutually exclusive for n=1,2, ... The expected value of the payoff is $\sum_{n=1}^{\infty}(1/2)^n * 2^{n-1} = \sum_{n=1}^{\infty}(1/2) = \infty$, that is, infinitely many times (1/2), which is infinite!

Question 3. What do you think, how much would Sophie be willing to accept up-front for her baby tooth, instead of playing the game with the tooth fairy?

Answer: Most children would accept a small sum, maybe £4-£6. Any answer except "infinity" is acceptable, any integer under £10-£15 is reasonable.

Question 4. If your answers to Questions 2 and 3 differ, then what do you think is the reason for the difference?

Answer: The expected value is infinite, a human's valuations for this gamble is typically finite and indeed quite small. This is known as the St Petersburg paradox, by Nicolas and Daniel Bernoulli in the early 18th century. (The mathematical "surprise" is that the payoff is finite almost surely, yet its expected value is infinite.) The usual explanation for a finite willingness to pay for such a gamble is risk aversion, i.e., a strictly concave (perhaps logarithmic) utility for monetary payoffs, which may yield a finite and potentially smallish certainty equivalent for the gamble. It could be that Sophie doesn't believe her father would be able to pay an arbitrarily large amount or that she discounts payoffs that occur far in the future.

Question 5. Modified agreement. If the 'tooth fairy' fails to come for 10 days in a row then on the 11th morning Sophie gives up her tooth and gets £10. What is the mathematical expectation of her revenue in this scenario?

Answer: She gets £$\frac{1}{2}$*10 + £10*$\frac{1}{1024}$ = £5.1ish. (£$\frac{1}{2}$ in expectation for each of the first 10 nights, and £10 with prob $(\frac{1}{2})^{10} = \frac{1}{1024}$, which is about £0.1).

Sunny Canarias (2016)

Alice may hire Bob. If she does and Bob works hard then she earns £60K or £20K with 50%-50% chance. If he is hired but does not work hard ("shirks") then Alice gets £20K with probability 1. Alice maximizes her expected income net of Bob's wage.

For Bob the upfront cost of working is £5K, shirking is free. He maximizes the expected value of the square root of his net income (wage minus effort cost) measured in £K. E.g., if Bob receives £30K with 50% chance and £6K with 50% chance from Alice while working hard, he finds it equivalent to receiving £9K for sure and shirking, because $0.5 \times \sqrt{30-5} + 0.5 \times \sqrt{6-5} = 0.5 \times 5 + 0.5 \times 1 = 3$, whereas the square root of 9, $\sqrt{9}$, is also 3. Assume that if Bob is indifferent between working and shirking then he works. Bob has an outside option, £8.999K for sure, no effort.

Question 1. If Alice could observe if Bob works or shirks then how much would she have to pay him to entice him to work for her, and how much would Alice earn?

Answer: Need to pay him £9K+£5K = £14K (outside option plus cost of effort). Alice then earns 0.5*(£60K+£20K)=£40K less what she pays Bob, i.e., net profit £26K.

Question 2. Assume that Alice cannot observe whether Bob works or shirks. Does the wage in Question 1 entice Bob to work hard?

Answer: No. He gets £14K if he works or shirks; he'd rather shirk and save £5K.

Question 3. Suppose that Alice offers Bob a wage of £30K if her income is £60K but only £6K when her income is £20K. Does this entice Bob to work hard for Alice?

Answer: Yes. If he works hard his expected utility is $0.5 \times \sqrt{30-5} + 0.5 \times \sqrt{6-5} = 3$, given as an example in the setup. If he shirks his expected utility is $\sqrt{6} < 3$.

Bob is happy to take up Alice's contract as it is better than his outside option, too.

Alice's expected profit is $0.5 \times (60-30) + 0.5 \times (20-6) = 22$ (in £K).

Question 4. Alice offers Bob £21K when her income is £60K and £9K when her income is £20K. Would this entice Bob to work hard for Alice?

Answer: Yes. His expected utility from working is $0.5 \times \sqrt{21-5} + 0.5 \times \sqrt{9-5} = 3$, whereas his expected utility from shirking is $\sqrt{9} = 3$ as well. By assumption he works when indifferent. He is also happy to take Alice's contract as it is better than his outside option. Alice's net profit is now $0.5 \times (60-21) + 0.5 \times (20-9) = 25$ (in £K).

Question 5. Which wage contract (income sharing agreement) is better from Alice's point of view, the one proposed in Question 3 or the one in Question 4?

Answer: Alice's net profit from the wage contract in Question 3 is £22K, whereas her net profit from the wage contract in Question 4 is £25K. The latter is better.

Indeed, this is the best performance wage contract from her perspective that she can offer to Bob that he takes voluntarily and induces him to work hard. This is so because both his incentive compatibility constraint and his participation constraint bind in this contract.

Bank regulation (2012)

Bank can be prudent (L) or a risk-taker (R) over the next ten years. The latter will cause a financial crisis whose cost to the country is £50 billion and 0 to the bank. Action R yields £3 billion per year for ten years to the bank. If the bank chooses L then the financial crisis occurs with 50% chance, but the country's economic output reduces for sure by £1 billion per year. The bank will make £2 billion a year in profits.

The bank cares about its own profit. The government cares about the country's economic output and does not take the bank's profits in account.

Question 1. Will the bank take option L or R?

Answer: R. It yields £30bn over 10 years whereas L yields only £20bn.

Question 2. If the government could force the bank to take option L, should it do so?

Answer: Yes. R yields a certain loss of £50bn due to the financial crisis. L yields a certain loss of £10bn (lower economic output) plus a loss of £50bn (financial crisis) with 50% chance. The expected loss is $35bn < 50bn$, i.e., a £15bn saving.

Assume that the government cannot force the bank to choose option R or L, but it can provide incentives in terms of subsidies (cash transfers).

Question 3. The government can observe whether the bank takes option L or R. How much would the government have to pay to the bank every year in the next ten years if it wanted the bank to take option L? Is it worth it for the government?

Answer: The government can induce the bank to take L instead of R by paying it £1bn (plus a penny) per year for 10 years, i.e., £10bn. It's worth saving £15bn.

Question 4. The government cannot observe whether the bank takes option L or R. Instead, it can promise to pay the bank a subsidy at the end of the ten years conditional on no financial crisis. (If there is a crisis, the bank receives no subsidy.) How much would the government have to promise to pay to the bank if it wanted the bank to take option L? Should the government promise to pay the bank the subsidy?

Answer: If the subsidy conditional on avoiding a financial crisis is X (in £bn) then the bank's payoff from L becomes $0.5X + 20$ (in billions of £) whereas its payoff from R remains £30bn. The bank picks L as long as the subsidy is more than £20bn.

The government pays out, in expectation, £10bn. This is the same amount as in the answer to Question 3, so the subsidy scheme is worth undertaking.

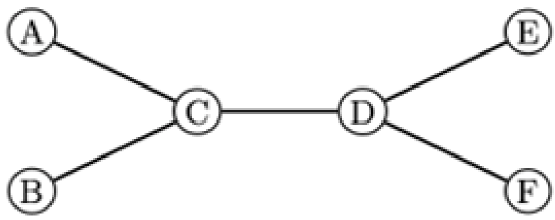
Question 5. What assumptions have we made about the government in this account that strike you as implausible? List three.

Answer: (1) We assumed that the government knows the probability and social cost of the financial crisis given actions L and R, and that it knows the profits of the bank from L and R as well. (2) We assumed that the government's objective is social welfare, that they plan over a 10-year horizon and that they will honour their promises. (3) The situation is played out in isolation.

Sharing is caring (2023)

Ann, Bob, Chabi, Deniz, Ernie, Fei share a flat. Each gets benefit b from watching Netflix. The cost of a Netflix account is c. Assume $b > c > 0$. Each prefers that others buy and share their passwords. Each person is happy to share their own Netflix password only with their close friends, with connections represented below:

Everyone with a subscription shares their password with as many close friends as possible, as long as they can all stream simultaneously. Cannot re-share password. No "binding contracts". All prefer to use someone else's subscription rather than buying their own.



Question 1. Explain who are close friends with whom.

Answer: A with C, B with C, but C's friends are A, B, and D. Similarly, E with D, also F with D, but D's friends are E, F, as well as C.

Question 2. No simultaneous streaming/sharing. Who buys a subscription?

Answer: Each buys a subscription as $b > c$.

Question 3. No limit on sharing: who buys a subscription? Explain.

Answer: A minimal set of buyers S must be such that nobody in S has a friend in S (because then he or she would prefer to drop out of S and free-ride), but everybody is either in S or has a friend in S. Hence $S = \{A, B, E, F\}$, or $S = \{A, B, D\}$, or, $S = \{C, E, F\}$. The latter two are most cost-effective.

Question 4. Redo assuming the number of devices that can simultaneously stream on the same account is two, and also when it is three.

Answer: If exactly two people can stream simultaneously, then the only candidate is $S = \{A, B, E, F\}$. All on the periphery buy, but central agents do not. This outcome is worse for them (but better for Netflix) than that under unlimited sharing.

When three can stream simultaneously then the most efficient buyers' set is $S = \{C, D\}$. Each central person shares with their two peripheral friends.

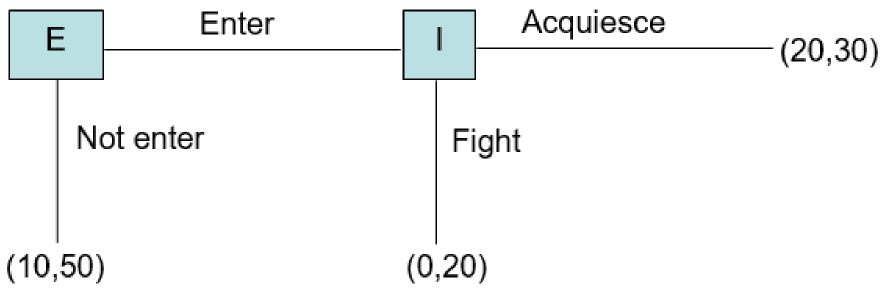
If more than three can share then we are back to the case of unlimited sharing.

Question 5. Explain what we learn from comparing answers to Questions 3 and 4.

Answer: By restricting sharing Netflix may actually make the flatmates better off. Reason: their incentives to free-ride on each other and ultimately underprovide a public good (Netflix subscription) goes up when the public good can be shared more widely. The example is based on Gerke, Gutin, Hwang and Neary "Public goods in networks with constraints on sharing" (WP from June 2023).

Entry deterrence (2008)

Textbook entry deterrence game as given below; E's payoff written first.



Question 1. Which outcome is most preferred by E? By I?

Answer: E prefers (Enter, Acquiesce) to (Not enter), to (Enter, Fight). The incumbent prefers (Not enter) to (Enter, Acquiesce) to (Enter, Fight).

Question 2. If you were the incumbent what would you like the entrant to think what action you are going to take?

Answer: The incumbent wants E to think I will fight if E enters in order to keep E out.

Question 3. What do you think is the most plausible outcome if the game is played only once?

Answer: If E enters then I's best reply is to acquiesce (i.e., rationally, I does not fight once E is in). Anticipating this E is better off by entering. I's threat of fighting upon entry is not credible or time-consistent.

Question 4. Would your answer for Question 3 change if the game were played repeatedly? If so, how?

Answer: If the game is played a fixed number of times then the analysis above applies when the game is played for the final time: E enters, I acquiesces, no matter what has happened before. Therefore, E enters in the second-to-last game as well, and so on, in every game, and player I always acquiesces. This is known as Selten's chain store paradox: "deterrence" (e.g., fighting the first few entrants to keep the rest out) would be ideal for the incumbent but it is not a credible strategy.

Question 5. Aside from issues of game repetition, are there other implicit assumptions that we need to make in order to draw conclusions about the outcome? List three such assumptions.

Answer: (1) We are assuming that all players know each other's available actions and payoffs. (2) We assume both are completely rational and fully capable of analysing the game as we have done above. (3) The game takes place in isolation, there are no norms, customs, communication, side-payments. When the game is repeated, a subsequent entrant may draw inferences concerning the incumbent's type (intentions, payoffs, perhaps norms or customs) from earlier play. See more at `https://en.wikipedia.org/wiki/Chainstore_paradox`.

Partners at work (2016)

Eric and Roger (reference to E. Maskin and R. Myerson whose paper with R. Radner the question is based on) either work or shirk each period. If they both work, then their partnership earns £12K with $\frac{2}{3}$ and nothing with $\frac{1}{3}$ chance. If one works and the other shirks then they earn £12K with $\frac{1}{3}$ and nothing with $\frac{2}{3}$ chance. All earnings are split equally. If both shirk, then they earn nothing. The cost of working is £3K for each person that works. This is incurred before the revenue is realized. Each partner maximizes his own payoff and is risk neutral.

Question 1. Compute the partners' expected net payoffs in all possible situations: when both of them work, when one of them works and the other one shirks, and when both shirk.

Answer: If both work then each gets £1K. If one works the other shirks then the worker gets -£1K and the shirker £2K. If both shirk then each gets 0. Classic Prisoner's Dilemma.

Question 2. Perfect monitoring. What is the best course of action for them?

Answer: Both should work, then their total payoff is highest, £2K.

Question 3. Suppose that each partner can secretly shirk even if they had both agreed to work hard. What do you expect to happen?

Answer: Both will shirk. It is a strictly dominant strategy, i.e., best for the person no matter what the other does.

Question 4. They can hire a manager who will monitor and enforce the promise of working. What is the highest wage they are willing to pay a manager?

Answer: Their expected surplus goes from 0 to £2K, so that's the most they would pay for the manager's services of monitoring and enforcing effort.

Question 5. Instead of hiring a manager they may agree on the following. Each month they flip a coin. If the outcome is 'heads' then Eric works and Roger shirks. If it is 'tails' then Roger works and Eric shirks. If the partnership is successful (i.e. it generates £12K in that month) then whoever was supposed to shirk pays the other £3K plus a lunch. What do you expect to happen? Is it better than hiring a manager?

Answer: The arrangement will work. A player whose turn it is to work gets -£1K (per original payoffs) for sure plus £3K and a lunch with $\frac{1}{3}$ chance; in expectation this is slightly more than zero payoff. If he shirks then he gets zero for sure, hence he is better off working. The player whose turn it is to shirk gets £2K (per original payoffs) for sure, less £3K and a lunch with $\frac{1}{3}$ chance. In expectation his payoff is slightly less than £1K, happy to shirk. This arrangement secures a total joint payoff of £1K for Eric and Roger. It is better than no arrangement (shirking, 0 payoffs). It is better than hiring a manager if the manager costs more than £1K per month. If the manager costs less than £1K then they should hire one.

Comment: the main result of Radner, Myerson Maskin, that full cooperation is impossible in this version of a repeated PD with imperfect public monitoring is not tackled by the question.

Sports funding (2023)

The right amount of spending on sports is uniform on [£40k, £60k]. The finance committee wants to set the level right but does not know the realization. Tom knows but his ideal point is £3k more. He has symmetric loss around his ideal point.

Question 1. How much does the finance committee spend on sports without any input from Tom?

Answer: They would set it equal to the unconditional expected value of a uniform draw on [£40k, £60k], that is, £50k.

Question 2. Suppose that the finance committee asks Tom what the right level of spending on sports is for the university. Would it be wise for them to take his report at face value? Would it be a good idea for them to ask Tom about the right level of spending and then subtracting £3k from his response?

Answer: No, it would not be wise. If they believed him then, anticipating this, Tom would report £3k more. It is not a good idea to subtract £3k from his report: if this is anticipated by Tom he will report £6k more.

Question 3. Suppose that Tom manages to convey to the committee whether the right amount of spending on sports for the university is above a certain threshold T, or below it. How much does the finance committee spend on sports if he says it is above T, and how much do they spend on it when he says it is below T?

Answer: If he conveys that the right amount is above T then they pick the expected value conditional on the value exceeding T, that is, $\frac{60+T}{2}$. If he conveys that the true value is below T then they set the spending level at $\frac{40+T}{2}$.

Question 4. When is it optimal for Tom to tell the truth whether the optimal level of spending on sports is above or below T? Would he tell the truth if $T = 50$ and he knows that the right amount of spending on sports for the university is £49k?

Answer: When the spending level is X, Tom will claim it is above T if, and only if, $X + 3$ is closer to $\frac{60+T}{2}$ than it is to $\frac{40+T}{2}$, i.e.,

$$\frac{60+T}{2} - (X+3) < (X+3) - \frac{40+T}{2}$$

which simplifies to

$$44 + T < 2X.$$

For Tom to be always honest this should hold iff $X > T$. By continuity, we must have $44 + T = 2T$ at $X = T$, that is, $T = 44$. In the numerical example, if $T = 50$ and $X = 49$, he is better off saying X exceeds 50, because his ideal point is 52, which is closer to 55 than it is to 45.

Question 5. Can Tom credibly and truthfully convey to the finance committee whether the right amount of spending on the sports for the university is above or below the threshold £44k? Explain. Interpret what we have learnt about communication between biased expert and decision maker when "talk is cheap".

Answer: The threshold $T = 44$ that is consistent with truth-telling is derived in the answer to Question 4; some candidates may verify it here "manually". Take-away: a biased expert cannot convey all the information he has, but communication in "imprecise terms" can still arise in equilibrium and benefit both parties. The example is based on Crawford and Sobel's (1982) paper on cheap-talk communication.

Island dispute (2017)

If F sends its army and S does not then F's net gain is £12bn - £2bn = £10bn. If S also attacks then F gets a net -£7bn. If F does not attack then it gets 0. Risk neutrality.

Question 1. Suppose S is not expected to attack. Which action is best for F?

Answer: Clearly, to attack, for a net gain of £10bn.

Question 2. Suppose S is expected to attack. Which action is best for F?

Answer: Clearly, not to attack as zero exceeds -£7bn.

Question 3. Suppose that S has the same costs and benefits as F. Compute payoffs in all four outcomes.

Answer: Payoffs for F and S: (0,0) if neither attacks; (10,0) if F attacks and S does not; (0,10) if F doesn't attack and S does; (-7,-7) if both attack. Hawk-Dove game; two pure Nash equilibria: F attacks and S does not, and S attacks and F does not. Both are "self-fulfilling prophecies" and hard to predict which one occurs. Maybe they will fail to coordinate...

Question 4. F's leadership believes that S is ruled by an unpredictable dictator who does and does not attack with 50-50% chance. What is F's best course of action?

Answer: F gets $0.5 \times (10 - 7) = 1.5$ if it attacks and 0 if it does not; it is best to attack.

Question 5. Suppose that F can announce in advance that it will attack. The cost of this announcement is £1 billion, incurred no matter whether they end up attacking, and it is not deducted from the cost of attack. Other costs and gains remain the same and all parties are aware of this. If F makes the announcement, should S assume the threat will be carried out? Is it in F's interest to make such an announcement?

Answer: The baseline game has two pure Nash equilibria: either F attacks and S does not, or vice versa. (There is a third, symmetric mixed equilibrium as well.)

F strictly prefers the equilibrium in which it attacks and S does not.

The act of "burning" £1bn indicates that F will attack, because for F, "not attack" (which yields a payoff of zero no matter what S does) strictly dominates "burn £1bn and not attack" (yielding a payoff of -1, no matter what S does).

S knows this, therefore having seen F burn £1bn, it will not attack. Trusting S's ability to deduce all this it seems to be in F's interest to burn £1bn to ensure his preferred equilibrium at a relatively low cost.

Finally, following Ben-Porath and Dekel (2002, "On Forward Induction and Money Burning Games," Econ Theory 19: 637-648) one can argue that F's mere ability to publicly burn £1bn is sufficient to induce F's preferred equilibrium, without F actually incurring the cost of money burning. As we have established, if F burns £1bn then it deters S from attacking and secures £9bn for F. If F does not burn £1bn then S must infer that F anticipates playing an equilibrium in which F's payoff exceeds £9bn. The only such continuation is where F attacks and S doesn't, with payoffs 10 and 0. Hence even if F does not burn money S understands F will attack and S will not.

Tutors and colleges (2012)

Match tutors A, B, C to colleges J, K, L. Preferences $J > K > L$ for A, B and $K > J > L$ for C.

$C > A > B$ for J and $B > C > A$ for K, L. Stability defined in text; "AL, BJ, CK" is shown not to be stable as A and J break it.

Question 1. Explain why "AK, BL, CJ" is not stable. Is "AL, BK, CJ" stable?

Answer: "AK, BL, CJ" is broken by BK or CK. "AL, BK, CJ" is however stable: colleges J and K get their top picks, L would prefer either B or C to its allocated tutor A, but B and C prefer their allocated colleges (K and J respectively) to L.

Question 2. Tutors pick in alphabetical order. Is the resulting assignment stable?

Answer: Tutors will choose their favourite among the remaining colleges, so AJ, BK, CL will be the outcome. Not stable: CJ breaks it.

Question 3. Simultaneous choice by tutors; each college picks a tutor that has chosen them. Unmatched tutors & colleges repeat. Tutors choose honestly. What's the outcome, stable?

Answer: A and B both pick J and C picks K, the latter pairing is done. J keeps A and rejects B who has to go to L. The resulting "AJ, BL, CK" is not stable, BK breaks it. Note that B has an incentive to lie in the first round (pick K not J to avoid L); anticipating this C might want to lie and pick J not K.

Question 4. Each tutor simultaneously picks a college; over-demanded colleges provisionally selects a tutor. In subsequent rounds, each tutor w/o provisional assignment selects any college that he or she had not picked before, and over-demanded colleges update their provisional tutor choice. The procedure ends when each college is only demanded by one tutor. Find the resulting outcome and check if it is stable.

Answer: First, A and B pick J, C picks K. J selects A, hence B must pick again.

Second, B picks K, which is now over-demanded; K rejects C who must pick again.

Third, C picks J, which rejects provisionally selected A, who must pick again.

Fourth, A picks K, but K is happy with B, and A must pick again: ends up with L.

The resulting assignment "AL, BK, CJ" is stable as seen in Question 1.

Comments: This algorithm is the Gale-Shapley deferred acceptance protocol; economics Nobel prize in 2012. Under honest behaviour it results in a stable matching. If there are more than one stable allocations, then it selects the tutor-optimal one. Tutors cannot manipulate it. Colleges can manipulate it by revealing their preferences dishonestly.

Question 5. Any insights from comparing answers to Q2, Q3, Q4?

Answer: Sequential or simultaneous "greedy" matching algorithms do not always yield a stable outcome even if agents are honest, and they may have an incentive to lie. "Deferred acceptance" algorithm (Gale-Shapley) ensures stability by flexibility.

Money jars (2024)

Teacher puts either £10 or £30 into Jar A with 50-50% probability. Independently of that, he puts either £10 or £30 into Jar B with 50-50% probability. Ana sees what's in Jar A, Barb sees what's in Jar B; neither sees the other's jar. Both know the rules. Ana and Barb submit sealed bids. The highest bidder wins the money in both jars and pays the bid of the loser (50-50 tie break). The loser gets and pays nothing.

Question 1. X = money in Jar A, Y = money in Jar B. Write down the possible values of $X + Y$ and their probs from Ana's then Barb's perspective.

Answer: From Ana's perspective, the total is either $X+10$ or $X+30$ with 50-50% chance. From Barb's it is either $Y + 10$ or $Y + 30$ with 50-50% chance.

Question 2. Assume that Barb believes that Ana will bid $X + 20$. Barb is planning to bid $Y + 20$. Is there a better strategy for Barbara than bidding $Y + 20$?

Answer: There is. By assumption, Ana bids either 30 or 50 (whether $X = 10$ or $X = 30$). If Barb sees $Y = 10$, it's best not to bid at all, because Ana's bid is more than the money in the two jars combined, hence if Barb wins, she will have to pay too much. Barb's strategy of "trying not to win" when $Y = 10$ strictly improves on bidding $Y + 20$ (i.e., 30 with $Y = 10$), because a bid of 30 occasionally wins. Barb can do better than bidding $Y + 20 = 50$ when $Y = 30$ as well. With that bid, against Ana's bid of $X + 20$, Barb gains 10 when she wins, which occurs with 75% chance. If Barb bids $50 + \varepsilon$ instead of 50 with $Y = 30$ then she always wins and makes a profit of nearly 10.

Question 3. Now assume that Barbara knows that Ana will bid $2X$. Barbara is planning to bid $2Y$. Is there a better strategy for Barbara than bidding $2Y$?

Answer: This is an ex-post equilibrium, that is, it is optimal for Barb to bid $2Y$ even knowing whether $X = 10$ or $X = 30$. If $X = 10$ and hence Ana bids 20, then Barb with $Y = 10$ doesn't want to bid more than 20 anyway; it does not matter what she does, she gets 0 profit. So bidding $2Y = 20$ optimal. If $Y = 30$ then Barb wants to win for sure (because she will pay $2X = 20$, less than $X+Y = 40$), so bidding $2X = 60 > 20$ is optimal. If $X = 30$ and hence Ana bids 60, then Barb cannot and will not make any profit no matter how much she bids with either $Y = 10$ or $Y = 30$.

Question 4. Ana's mother pays Ana an additional £1 if she wins the bidding. What do you expect to happen, and what advice would you give to Barbara?

Answer: Ana is willing to bid up to 61 if she sees 30, but Barb is not. So if both are 30, she knows she cannot win. If Barb sees 10, she cannot win either. The only case she can win is if $Y = 30$ and $X = 10$. Ana is always willing to bid slightly more aggressively. But Ana knows this and anticipating this, if Barb is not willing to participate, she can make large profits because she stays alone in the game.

Question 5. Summarize the insights from answering Q2, Q3, and Q4.

Answer: The winner's curse raises its ugly head in Q2, it's not good for Barb to bid $Y + E[X]$. Indeed as we saw in Q3, it is (ex-post) weakly optimal to bid $2Y$, as if Barb won in a tie. A toe-hold gives a bidder great advantage is the take-away in Q4. The last point is from Bulow, Huang, and Klemperer (1999) "Toeholds and takeovers".

Shady deals (2015)

Mr Q may approach Ms P offering a shady deal that Ms P can either accept or reject. If she accepts the deal, then she gets 0 and he gets 1. If she rejects then both get -1. If Mr Q does not make the offer then she gets 2 and he gets 0. Both are risk neutral. The setup is equivalent to that in "entry deterrence" but the questions will be different.

Question 1. Suppose all is clear to both. What happens?

Answer: By backward induction, if Mr Q offers Ms P accepts, hence Mr Q offers.

Question 2. Mr Q believes Ms P is "ordinary" (as in the setup) with (100 - X)% chance and "incorruptible" (committed to reject) with X% chance. What should Mr Q do, depending on the value of X? Does Ms P want Mr Q believe the value of X is high?

Answer: Mr Q's offer is rejected with X% and accepted with (100 - X)% chance, hence his payoff, multiplied by 100, is $(-1) \times X + 100 - X = 100 - 2X$. He prefers to make an offer whenever this exceeds 0 (his payoff from not offering), equivalently $X < 50$. Ms P would prefer not to get an offer, therefore she wants Mr Q to believe $X > 50$.

Question 3. Mr Q1 then Mr Q2 may approach Ms P, the latter observing whether Mr Q1's offer (if made) has been rejected. Initially all believe $X = 40\%$. Could an ordinary Ms P make Mr Q2 believe that she is incorruptible by refusing Mr Q1's offer?

Answer: Not with 100% success. If Ms P refusing Mr Q1's offer makes Mr Q2 believe that she is incorruptible then Mr Q2 does not make an offer; Ms P gets a total payoff of $-1 + 2 = 1$. If she accepts Mr Q1's offer she gets $0 + 0$. Hence, if refusing the first offer deters the second offer then an ordinary Ms P would "overuse" it to gain 1.

Question 4. Continue to assume the initial probability of Ms P being incorruptible is $X = 40\%$. Suppose that Mr Q2 thinks Mr Q1's offer is always rejected by an incorruptible Ms P, whereas it is rejected by an ordinary Ms P with $\frac{2}{3}$ chance. How likely is that Ms P is incorruptible if she rejects Mr Q1's offer?

Answer: Apply Bayes' rule. The total probability of Mr Q1's offer being rejected is $40\% + \frac{2}{3} \times 60\% = 80\%$. Out of this, Ms P is truly incorruptible half the time (the first 40%). Hence

$$\Pr(\text{Ms P incorruptible} \mid \text{Mr Q1's offer rejected}) = 50\%.$$

Question 5. What if anything do we learn from answers to Q3 and Q4?

Answer: By refusing Mr Q1's offer, an "ordinary" Ms P may build a reputation for being incorruptible and therefore keep Mr Q2 at bay. However, if she always refuses Mr Q1's approach (i.e., ordinary and incorruptible Ms P are expected to refuse Mr Q1 just the same) then Mr Q2 won't infer anything from this action. So, when she is an ordinary politician, she should not refuse Mr Q1's offer with 100% chance, otherwise "reputation building" won't work. In Q4 we assumed Ms P mixes between accepting and refusing Mr Q1's offer (2/3 to 1/3) so that Mr Q2 ends up believing that Ms P having refused Mr Q1 she is incorruptible with exactly 50% chance. This makes Mr Q2 indifferent between offering and not.

Anna's bike (2013)

Anna wants to sell her bike to Bob via a take-it-or-leave-it offer. The bike is worth nothing to Anna. It is worth to Bob £200 with 40% chance and 0 with 60% chance. (Initially neither knows.) Anna finds out which it is (£200 or £0) with 50% chance. Independently, Bob finds out the bike's value with 25% chance. Both are risk neutral and happy to trade when their own net surplus is zero.

Question 1. Neither Anna nor Bob can prove when they find out the bike's value. Explain what happens if Anna charges £80.

Answer: Uninformed Bob buys at £80 because the expected value of the bike for him is $200 \times 0.4 = 80$. Bob informed of $v = 200$ also buys at £80. The probability that Bob is uninformed is 75%; the probability that he knows the bike is worth £200 is $0.4 \times 0.25 = 10\%$. Anna's expected profit is therefore $(0.75 + 0.1) \times 80 = 68$.

Question 2. Now Anna can prove to Bob the value of the bike when she discovers it. (She cannot prove that she does not know it.) What is her best strategy for selling the bike in this case? Price when she finds out the bike's value, price when she does not.

Answer: If Anna learns $v = 200$ then she proves it, charges Bob £200, and he buys. This happens with $0.4 \times 0.5 = 20\%$ chance. If Bob finds out that the bike is worthless then there is no sale; this happens with $0.6 \times 0.25 = 15\%$ chance. In all other cases Anna will sell the bike to Bob at the highest price that uninformed Bob is willing to pay. Uninformed Bob reckons either $v = 0$ (which has an a-priori 60% chance), or $v = 200$ but Anna hasn't found it out (this happens with 20% chance). Hence the expected value of the bike conditional on this event is

$$\frac{0.6 \times 0 + 0.2 \times 200}{0.8} = 50.$$

This is the most that Anna can (and will) charge if she cannot prove $v = 200$.

Question 3. Is Anna better off (in terms of expected revenue) when she is able to prove the value of the bike whenever she discovers it?

Answer: We have found that her expected revenue was £68 in Question 1 (she cannot prove the bike's value and charges £80, which is indeed the best she can do). Under the conditions of Question 2, using her best strategy, she gets £200 with 20% chance, nothing with 15% chance, and £50 with 65% chance. This is £72.50, greater.

Question 4. Anna's brother Seb can find out the value of the bike and prove it to Bob. What is Anna's profit if she uses Seb as intermediary?

Answer: If Seb finds out that the bike is worth £200 then he proves it to Bob who buys. Otherwise (if Seb discovers that the value is zero, proves to Bob or not), Bob will buy at price 0. Anna's expected revenue is $0.4 \times 200 = 80$.

Question 5. Compare Anna's profit in three scenarios (Q1, Q2, Q4). Interpret.

Answer: It's best to use Seb, second best if she has evidence, third best if no info:

$$80 > 72.50 > 68$$

. Key point to make: being able to prove the bike's value is a "burden" because Bob becomes suspicious when it's not proven to him. E.g., in the absence of evidence he buys at £50 in Q2, at 0 in Q4. But, overall, Anna is better off if she (or Seb) is more likely to be able to prove the bike's value because when it's £200 she can charge a high price.

Committee voting (2010)

Ideal decision is "invest" or "not invest" with 50-50% prior probabilities. Each of three committee members independently gets a GOOD signal realization with 60% chance or a BAD one with 40% chance conditional on "invest". The probabilities of these signal realizations are reversed conditional on "not invest".

Question 1. If a committee member observes the GOOD signal, then how likely does he or she think the "invest" is best? [...]

Answer: The total probability of getting a GOOD signal is $0.5 \times 60\% + 0.5 \times 40\% = 50\%$. The GOOD signal is received when the state is INVEST with $0.5 \times 60\% = 30\%$ chance. Hence the probability of INVEST being true conditional on GOOD is

$$\frac{0.3}{0.5} = 60\%.$$

Question 2. If a committee member observes a GOOD signal, then how likely is it that another member of the committee also observes a GOOD signal?

Answer: Having observed GOOD, we update INVEST has probability 0.6 and NOT INVEST 0.4. Hence another committee member sees GOOD with probability

$$0.6 \times 60\% + 0.4 \times 40\% = 52\%.$$

Question 3. Simultaneous majority voting. If each committee member wants to maximize the probability that the correct decision wins, how do they vote?

Answer: Each should vote INVEST if they see GOOD and NOT INVEST if they see BAD.

Question 4. Each committee member also cares about being on the winning side. Does this alter the way each committee member votes?

Answer: No. If I see GOOD it's more likely that others have seen GOOD too and that INVEST will win, therefore they have even more reason to vote for it.

Question 5. Sequential voting with the chair going last, majority wins. All want to maximize the probability of the correct decision and all else equal prefer to be in the majority as well. How do they vote? Start by analysing the chair's behaviour.

Answer: If the chair sees a split vote (one vote for INVEST and one for NOT) then, assuming this is so because the other two members have seen one GOOD and one BAD signal, she votes honestly, i.e., INVEST when she sees GOOD and NOT when she sees a BAD signal. (This is optimal as the other two signals cancel each other and the chair's signal is informative.) If both other committee members have voted for the same outcome, then the chair votes for it as well, no matter her signal: she cannot change the outcome anyway, and she wants to be in the majority.

However, given the chair's anticipated behaviour the second voter will not vote honestly when his signal differs from that revealed by the first vote. If the second voter's signal differs from what is revealed by the first vote then he believes INVEST and NOT have 50-50% probabilities (the two signals cancel each other), so there is no "informational" reason to vote according to his own signal. By voting for the same outcome as the first voter he ensures that the chair will ignore her own signal and there will be total unanimity, and he (the second voter) will be in the majority as well.

Conclusion: the first voter casts an honest vote, the rest will herd, which is inefficient.

Rappers' battle (2019 PPE)

Rap artistes A and B decide whether to perform at festival H or J. Each can only do one, but one festival can host both. Each rapper has 1k fans who go wherever s/he goes. Additional 3k fans turn up for both. A festival pays the artist(s) £2 per ticket, split evenly if both play.

The cost of performing at H is 0 for A, £2k for B. The cost of performing at J is £2k for A and 0 for B. Each rapper maximizes his or her own expected payoff.

Question 1. Compute each rapper's monetary profit in each of the four outcomes.

Answer: A picks row, B picks column, A's payoff is written first.

	H	J
H	$(5,3)$	$(2,2)$
J	$(0,0)$	$(3,5)$

Question 2. What should A do if she anticipates that B plays J, etc? What is likely to happen? Would communication and coordination between the rappers help?

Answer: This is a coordination game. Each rapper would prefer to match the action of the other. There are two pure-strategy Nash equilibria: (H,H) and (J,J). Rapper A prefers the former, rapper B the latter. Mixed Nash: A picks H with probability $\frac{5}{6}$ and J with $\frac{1}{6}$, and B mixes conversely. They could easily mis-coordinate. Maybe communication could help avoid that though it is unclear whose preferred Nash will prevail.

Question 3. Rapper A can opt out and get £4k. What to expect?

Answer: A opts out unless she is reasonably sure they will play the (H,H) equilibrium. B should perhaps anticipate this and play H. A's attractive outside option may help them coordinate on (H,H), especially if communication is allowed.

Question 4. No studio option, but A can publicly burn £1,111 just before deciding on which festival to play at. Having burnt this much money would A ever play J? What should B deduce from A burning money and then do?

Answer: For A, playing H strictly dominates burning £1.111k and playing J, hence the latter is unlikely to occur (she won't play a strictly dominated strategy). Therefore B must infer that A will play H when she burns money. Hence B plays H as well if A burns money. This much analysis is expected from a good answer. Even more clever: if A does not burn money B must infer that A anticipates an outcome even better than that of the equilibrium where A burns money and they play (H,H). Hence B ought to play H even if A does not burn money. This is a bit adventurous: it combines a forward-induction argument with the previous backward-induction one.

Question 5. Any take-aways from the preceding analysis for politics and economics?

Answer: In the original setup (Q2) we saw a coordination game with two pure equilibria that the two players rank differently; prediction is hard, miscoordination may occur. Communication could help. In Questions 3 and 4 one of the players had an "outside option" (beating one of the equilibria) and a "costly signal" (money burning) that could also help coordinate them to play that player's favourite equilibrium.

Invade or wait? (2013)

Strongland (S) and Punyland (P) simultaneously decide whether to invade a third country or not (i.e., wait). Invasion costs £2bn to each invader. The gain of invasion is £10bn in total. There are four possible outcomes:

- Outcome 1. If both countries wait then S and P get zero payoff.
- Outcome 2. If S and P both invade then each spends £2bn on the invasion. However, S gets £7bn and P gets £3bn from the gains of invasion.
- Outcome 3. If S invades and P waits, then S spends £2bn and P nothing. Now S and P split the gains of invasion equally, i.e., each gets £5bn gross payoff.
- Outcome 4. If S waits and P invades, then P spends £2bn and S nothing. Here S gets £9bn and P gets £1bn from the gains of invasion.

Question 1. What are the relevant net gains for P and S in outcomes 1, 2, 3 and 4?

Answer: The net payoffs are (0,0) if both wait; 5 for S and 1 for P if both invade; 3 for S and 5 for P if S invades and P waits; 7 for S and -1 for P if P invades and S waits.

Question 2. If you were the ruler of Punyland what would you do?

Answer: If P invades then it gets 1 or -1 depending on whether S invades or waits; if P waits then it gets 5 or 0 depending on whether S invades or waits. Strategy "wait" strictly dominates "invade" for P as $5 > 1$ and $0 > -1$, hence P should wait.

Question 3. Which of the four outcomes would you predict would happen?

Answer: S knows P will wait. If S also waits then it gets 0, if it invades then it gets 3, hence S invades. S invades and P waits, so S gets £3bn and P gets £5bn.

Question 4. Would your answer to Question 3 change if these same two countries were thinking of invading an indefinite number of other countries in the future, under the same circumstances as above?

Answer: If the game is played finitely many times, then no: there is unique Nash in the final period, by backward induction we expect the same in every period. Infinite repetitions: (perfect) Folk Theorem applies. E.g., both invading every period is sustained by the threat that if either S or P ever deviates (waits) then both will wait for five periods before returning to both invading. This is subgame perfect equilibrium for sufficiently patient players.

Question 5. Discuss implicit assumptions. List three.

Answer: (1) All payoffs and the structure of the game are commonly known, including discounting (if the situation is repeated). (2) The players are rational and capable of the same analysis that we have carried out. (3) We assumed S and P move simultaneously, there is no commitment, no interaction outside the game. In "threat-based" repeated-game equilibria there is no off-path renegotiation.

Inheritance (2022)

50 acres of land and £100k to be shared between city and county. The city values 1 acre at £20k. The country values 1 acre at £10k. Labour rule the city, conservatives the county. They know each other's preferences but only care about their own share.

Question 1. Rule: "The county will make an offer to the city, which the city either accepts or refuses. If the city refuses, then all assets will be shared equally." What is the offer that the county should make to the city?

Answer: By refusing the county's offer the city gets 25 acres and £50,000. This is equivalent to £550,000 or 27.5 acres for the city. For the county it is better if she offers the city land: it is "cheaper" for the county as it values 27.5 acres at £275,000. So, the county should offer 27.5 acres (+ maybe £1 to make sure the city takes it), and the city will take this offer.

Question 2. New rule: the county divides the lot and the city chooses. The city cannot refuse the split, but it can choose which part to take. What should the county do?

Answer: The county makes two parts that the city values roughly equally and makes sure that the city picks what the county likes less. County splits the inheritance into "27.501 acres" (27.5 acres plus a little bit of land) and "22.499 acres + £100,000" (22.5 acres minus a little bit of land + £100,000) so the city picks the former and the county gets to keep the latter.

Question 3. Suppose the city divides and the county chooses. What should the city do, and what happens?

Answer: City divides into "£100,000 + 20.001 acres" (£100,000 + 20 acres and a little bit of land) and "29,999 acres" (30 acres minus a little bit of land). County picks the former.

Question 4. After the city council elections both county and city are controlled by the Conservatives. This means that they will talk to one another before the offers are made but they still only care about their own share. Does this change your answers to Questions 1, 2, and 3?

Answer: No, communication should not matter, only preferences do.

Question 5. Since both are controlled by the same party, imagine that the county council now also cares about the outcome for the city and vice versa. Does that affect your answers to Questions 1, 2, and 3?

Answer: As long as the city gets all the land the outcome is efficient. None of the three outcomes above are efficient. If they care about their joint outcome, all land should go to the city and the division of cash is irrelevant for the sum of the city's and council's outcome.

Sponsored links (2015)

Advertisers A, B, C want to buy two sponsored links. The top link yields 200 clicks, the bottom one 120. The profit per click is 10p for A, 4p for B, and 1p for C. In the auction each buyer submits a bid for a click. The highest bidder gets the top link, the second highest the bottom link. A winner pays per click the bid that they crowded out.

Question 1. Suppose that the advertisers submit bids equal to their true valuations. Who wins which link, and what are their expected profits?

Answer: A gets the top link and B the bottom link. A pays 4p per click, B pays 2p per click. Hence A's profit is $200 \times (10p - 4p) = 1,200p$ and B's profit is $120 \times (4p - 1p) = 360p$.

Question 2. Could any of the advertisers achieve a larger expected profit by bidding differently (higher or lower than their true valuation per click-through)?

Answer: No. Obviously, A is better off winning one of the spots than none. As long as A bids above 4p there is no change in the outcome. If A bids a bit less than B then it gets the bottom link, 120 clicks at 2p per click, for a profit of $120 \times (10p - 1p) = 1080p < 1,200p$. So A cannot increase its profit by not being honest. B could either bid more than A but then overpays (has to pay more than 10p per click) or less than C but then loses completely. Hence B has no strategy better than honest bidding either.

Question 3. Change of parameters: The bottom sponsored link also gets nearly 200 clicks per hour (instead of 120). Could any of the advertisers achieve a larger expected profit by bidding higher or lower than their true valuation for a click through?

Answer: Yes. Now it is worth for A to underbid B and get the bottom link at 2p per click. This deviation yields nearly $200 \times (10p - 1p) = 1,800p$ for A.

Question 4. Change in payment rule: each winner pays their "impact" on the sum of the other two bidders' expected gains – the total amount by which the valuations (gains before payments are subtracted) of the other two advertisers would have increased had the winner of a given spot never bid at all. Compute the payments (for Q2 and Q3 respectively) if all bid their true valuations. Is there a reason for any of the advertisers to exaggerate or understate their valuation for a click-through?

Answer: Vickrey's auction, truth-telling is dominant. Under the original assumptions (bottom link yields 120 clicks), if A is truthful then it pays $4p \times (200 - 120) + 1p \times 120 = 440p$ and gets a profit of $10p \times 200 - 440p = 1,560p$. If A bids just below 4p then it gets $(10p - 1p) \times 120 = 1080p$ instead, which is less. Under the modified assumptions (bottom link: 200 clicks), if A is truthful then it pays $1p \times 200 = 200p$ (as A has no impact on B who gets a link either way), and its profit is $10p \times 200 - 200p = 1,800p$. The profit is the same with a bid between 2p and 4p; and it drops to 0 if A bids below 2p, which is therefore not an improvement either.

Question 5. Summarize the insights from answering Q2, Q3, Q4.

Answer: In Q2 it is optimal for all to bid honestly, in Q3 it is not. In Q4, under the modified rules (Vickrey payments) it is weakly dominant to bid truthfully. However, the rules for computing payments are arguably more complicated.

Appendix F: Question Characterisation

PSName	Question	NC	PB	ClPz	AdPz	DS	BI	Eq	RC	AS	MH	Eff	Ins	Dg	Yr	W
Entry deterrence	all			1			1	1						1	‘08	292
Entry deterrence	1	1		1												
Entry deterrence	2			1				1								
Entry deterrence	3			1			1	1								
Entry deterrence	4			1			1	1	1							
Entry deterrence	5			1									1			
Committee voting	all		1		1			1		1					‘10	425
Committee voting	1	1	1		1											
Committee voting	2		1		1			1								
Committee voting	3		1		1			1		1						
Committee voting	4		1		1			1		1						
Committee voting	5		1		1		1	1		1		1				
Used bikes	all		1	1				1		1					‘11	458
Used bikes	1	1	1	1				1		1						
Used bikes	2	1	1	1				1		1						
Used bikes	3		1	1		1		1		1						
Used bikes	4			1						1			1			
Used bikes	5		1	1				1		1			1			
Tutors and colleges	all				1			1			1			1	‘12	528
Tutors and colleges	1	1			1											
Tutors and colleges	2	1			1											
Tutors and colleges	3	1			1			1			1					
Tutors and colleges	4				1		1	1			1					
Tutors and colleges	5				1								1			
Bank regulation	all	1	1								1				‘12	453
Bank regulation	1	1														
Bank regulation	2	1	1													
Bank regulation	3	1	1			1										
Bank regulation	4		1			1		1			1					
Bank regulation	5												1			
Invade or wait?	all					1		1							‘13	395
Invade or wait?	1	1														

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PSName	Question	NC	PB	ClPz	AdPz	DS	BI	Eq	RC	AS	MH	Eff	Ins	Dg	Yr	W
Invade or wait?	2	1				1										
Invade or wait?	3					1		1								
Invade or wait?	4					1	1	1	1							
Invade or wait?	5												1			
World Cup	all		1					1		1					‘14	525
World Cup	1	1	1													
World Cup	2	1	1													
World Cup	3							1		1		1				
World Cup	4		1					1		1		1				
World Cup	5	1											1			
Shady deals	all				1		1	1							‘15	602
Shady deals	1	1			1		1	1								
Shady deals	2	1	1		1		1	1								
Shady deals	3		1		1		1	1								
Shady deals	4		1		1		1	1								
Shady deals	5				1								1			
Partners at work	all		1	1		1						1			‘16	448
Partners at work	1	1	1	1												
Partners at work	2	1	1	1								1				
Partners at work	3	1	1	1		1						1				
Partners at work	4	1		1								1	1			
Partners at work	5		1	1				1	1			1	1			
Sponsored links	all				1			1		1					‘15	509
Sponsored links	1	1			1											
Sponsored links	2				1	1				1						
Sponsored links	3				1	1				1						
Sponsored links	4				1	1				1						
Sponsored links	5				1								1			
Island dispute	all			1				1							‘17	495
Island dispute	1	1		1		1										
Island dispute	2	1		1		1										
Island dispute	3	1		1				1					1			

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PSName	Question	NC	PB	ClPz	AdPz	DS	BI	Eq	RC	AS	MH	Eff	Ins	Dg	Yr	W
Island dispute	4	1	1	1												
Island dispute	5			1			1	1	1				1			
Sunny Canarias	all		1		1						1				‘16	544
Sunny Canarias	1	1	1		1											
Sunny Canarias	2	1			1						1		1			
Sunny Canarias	3		1		1						1					
Sunny Canarias	4		1		1						1					
Sunny Canarias	5	1			1								1			
Anna bike	all		1		1					1					‘13	575
Anna bike	1	1	1		1											
Anna bike	2		1		1			1		1						
Anna bike	3	1			1											
Anna bike	4		1		1			1		1						
Anna bike	5	1			1								1			
Fundraising drive	all				1						1	1			‘21	520
Fundraising drive	1	1			1											
Fundraising drive	2	1			1						1		1			
Fundraising drive	3	1			1	1		1			1	1				
Fundraising drive	4				1		1	1			1	1				
Fundraising drive	5				1						1		1			
Inheritance	all				1		1	1							‘22	338
Inheritance	1	1			1		1	1								
Inheritance	2	1			1		1	1								
Inheritance	3	1			1		1	1								
Inheritance	4				1								1			
Inheritance	5				1								1			
Sharing is caring	all				1			1						1	‘23	324
Sharing is caring	1	1			1											
Sharing is caring	2	1			1	1										
Sharing is caring	3				1			1								
Sharing is caring	4				1			1								
Sharing is caring	5				1								1			

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PSName	Question	NC	PB	ClPz	AdPz	DS	BI	Eq	RC	AS	MH	Eff	Ins	Dg	Yr	W
Rappers battle	all			1				1							‘19	465
Rappers battle	1	1		1												
Rappers battle	2	1		1				1								
Rappers battle	3			1			1	1	1				1			
Rappers battle	4			1			1	1	1				1			
Rappers battle	5			1									1			
Sports funding	all		1		1			1		1					‘23	451
Sports funding	1	1	1		1											
Sports funding	2		1		1	1		1		1						
Sports funding	3	1	1		1											
Sports funding	4		1		1			1		1						
Sports funding	5		1		1			1		1			1			
Hairy fairy	all		1	1											‘20	408
Hairy fairy	1	1	1	1												
Hairy fairy	2	1	1	1												
Hairy fairy	3			1									1			
Hairy fairy	4			1									1			
Hairy fairy	5	1	1	1												
Money jars	all		1		1	1		1							‘24	372
Money jars	1	1	1		1											
Money jars	2		1		1	1		1		1						
Money jars	3		1		1	1		1		1						
Money jars	4		1		1			1	1							
Money jars	5				1				1				1			

Dg = Contains diagram, Yr = Year when PS was used for admissions, W = word count