1. Comparison of European Tree Model vs. Black-Scholes Pricing

European Options Pricing:

- European options are commonly priced using binomial tree models, which break down the option's life into multiple time intervals, simulating possible price paths of the underlying asset. The accuracy of this model depends on the number of steps, volatility assumptions, and interest rate conditions. With more steps, the binomial tree model can provide a more precise estimate of option value, incorporating potential changes in volatility or rates over time.

Black-Scholes Pricing:

- The Black-Scholes model is a closed-form solution, primarily used for European-style options. It assumes constant volatility and no early exercise possibilities, providing a relatively simple approach for valuing options under idealized market conditions.

Key Observations:

- Comparing the prices for European options (Trades 8, 9, and 10) calculated using the binomial tree model versus Black-Scholes, we observe small price discrepancies.
- For example, the APPL short call (Trade 8) has a tree model value of -5,941.25 compared to -5,941.07 under Black-Scholes. Similarly, the SP500 put (Trade 9) has values of 48,765.82 (tree model) and 48,371.39 (Black-Scholes). These minor differences can be attributed to:
    - Timesteps in the binomial tree: 50-time steps may not be sufficient for the price to converge. Increasing the time steps reduces the discrepancy. However, the trade-off is higher computation requirements.
- The tree model can better accommodate time-varying volatility and provide a finer approximation of option prices, whereas Black-Scholes is less flexible and ideal for simpler scenarios with constant volatility assumptions.

2. Comparison of American vs. European Trade PVs

European Options (Trades 8, 9, 10, 11):

- European options can only be exercised at expiry, meaning they have no opportunity for early exercise. Their pricing reflects time value and intrinsic value based on the option's expiration, with no added premium for the possibility of early exercise.

American Options (Trades 12, 13, 14, 15):

- American options, in contrast, allow for early exercise at any point before expiry. This flexibility adds value since the holder can exercise the option when market conditions are favourable, such as a significant move in the underlying asset price.

Key Observations:

- American Put Options generally have higher present values (PV) than their European counterparts due to the added flexibility of early exercise.
    - For example, the SP500 put option (Trade 13) has a PV of 61,477.97, compared to the European version's PV of 48,765.82 (Trade 9). This price difference reflects the value from the ability to exercise early in favourable market conditions.
- American Call Options, however, show similar PVs to European Call Options. This is primarily because early exercise is suboptimal for non-dividend-paying stocks.
    - For instance, the short European APPL call (Trade 8) and the short American APPL call (Trade 12) both have a PV of -5,941.25, demonstrating that the early exercise feature does not provide additional value in this case.

Explanation for PV Difference:

- Flexibility of Early Exercise: American options have higher PVs because they provide the option to exercise early, especially in volatile markets where the option holder can lock in profits earlier than at expiry.

- Market Conditions: American options are more sensitive to significant price movements and can be exercised immediately, while European options are restricted to expiry.

3. Analysis of DV01 and Vega

DV01 (Interest Rate Sensitivity):

- Swaps (Trades 1, 2, 3, 4) have significant DV01 values, which reflect their high sensitivity to interest rate changes. For example, Trade 2 (receive USD-SOFR swap) shows a high DV01 of -6,213.58, indicating a substantial change in trade value for small movements in interest rates.
- Bonds (Trades 5, 6, 7) also exhibit significant DV01 values, but typically lower than swaps because bonds are less sensitive to interest rate changes in the long term. Longer-dated bonds tend to have lower DV01 values due to reduced sensitivity to interest rate movements.
- Options (Trades 8–15) have near-zero DV01 values, indicating little to no sensitivity to interest rates. Most of these options derive their value from volatility and delta exposures, not from changes in interest rates.

Key Observations on DV01:

- Positive and Negative DV01s: A positive DV01 indicates that the trade benefits when interest rates rise, whereas a negative DV01 implies the trade loses value when rates increase. For example, the receive swaps (Trade 2 and 4) have negative DV01s, meaning they will lose value if interest rates rise.

Vega (Volatility Sensitivity):

- European Options (Trades 8, 9, 10, 11): These options exhibit higher Vega values compared to American options, as European options are more exposed to volatility changes due to their lack of early exercise. For example, the SP500 put option (Trade 9) has a Vega of 4,659.44, meaning it will increase in value by 4,659.44 for every 1% increase in implied volatility.

- American Options (Trades 12, 13, 14, 15): American options tend to have lower Vega values because they allow for early exercise, which limits the exposure to changes in volatility. When volatility rises, the value of an American option may not increase as significantly as a European option because the option can be exercised early, removing its time value.

Key Observations on Vega:

- Significant Vega: Options such as the SP500 put (Trade 9) and STI puts (Trades 10 and 11) exhibit high Vega values, meaning they are more sensitive to volatility changes. Long positions in options generally benefit from increasing volatility, which is why the Vega for long options (like European calls and puts) is positive.
- Vega for Short Positions: Short options (such as Trade 8, short APPL call) show negative Vega, meaning the trade will lose value if volatility rises.

Conclusion:

- Swaps and Bonds are linear instruments, with zero Vega, meaning their value is unaffected by changes in volatility.
- Options, on the other hand, have significant Vega, especially European options, which are more sensitive to volatility. The volatility exposure is a crucial driver of their pricing, and the positions taken (long or short) will dictate whether the trade benefits or suffers from volatility changes.