QF609
Risk Analysis

VaR parameters

Risk Takers

Time Horizon

1-Day

Reporting Currency

USD

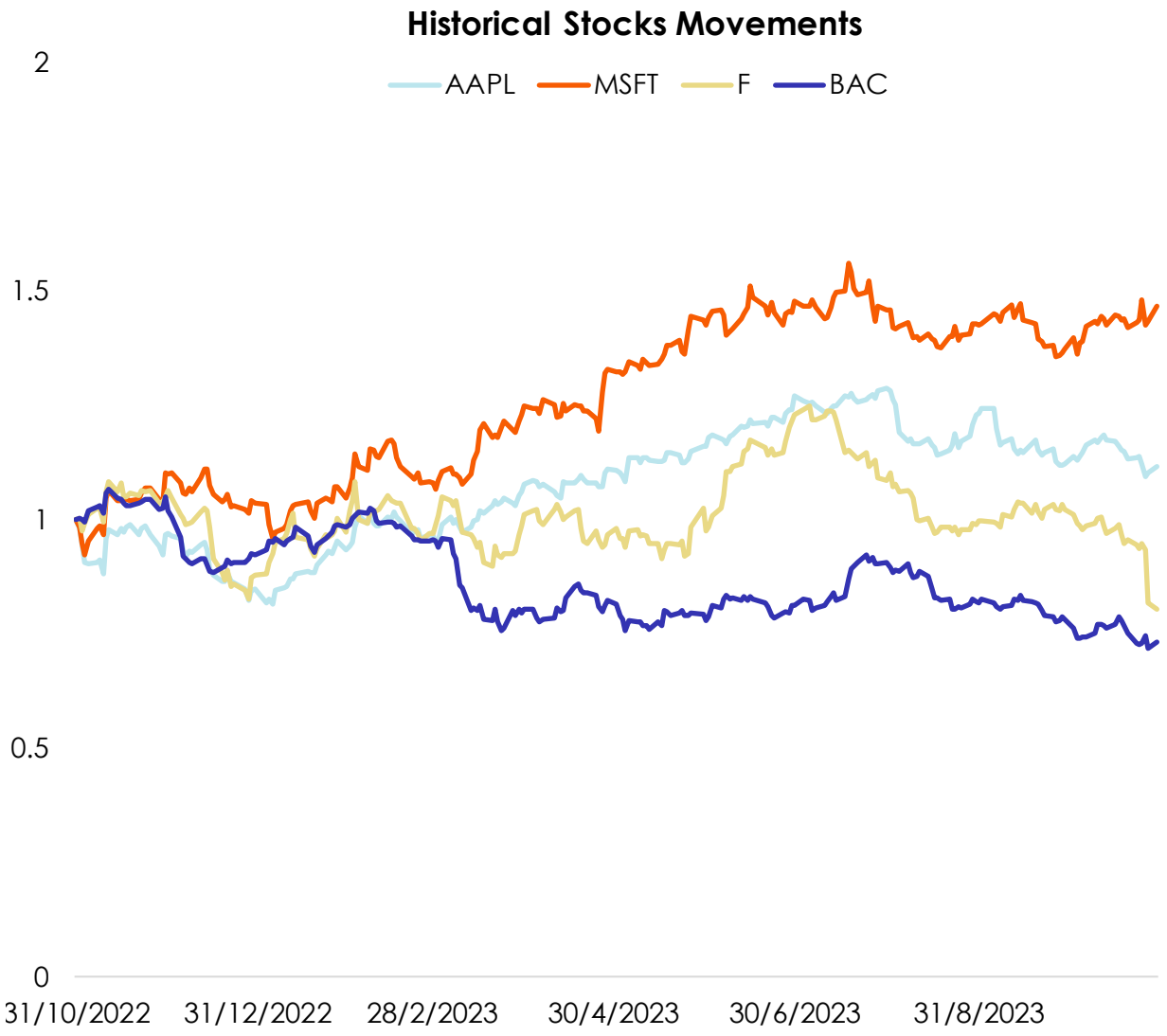
Significance Level

5%

(95% Confidence)

Stock Portfolio Overview

Risk Takers



Holdings @ 30th Oct 2023

Value: $1,000,000
Shares: 5,887 @ $169.85

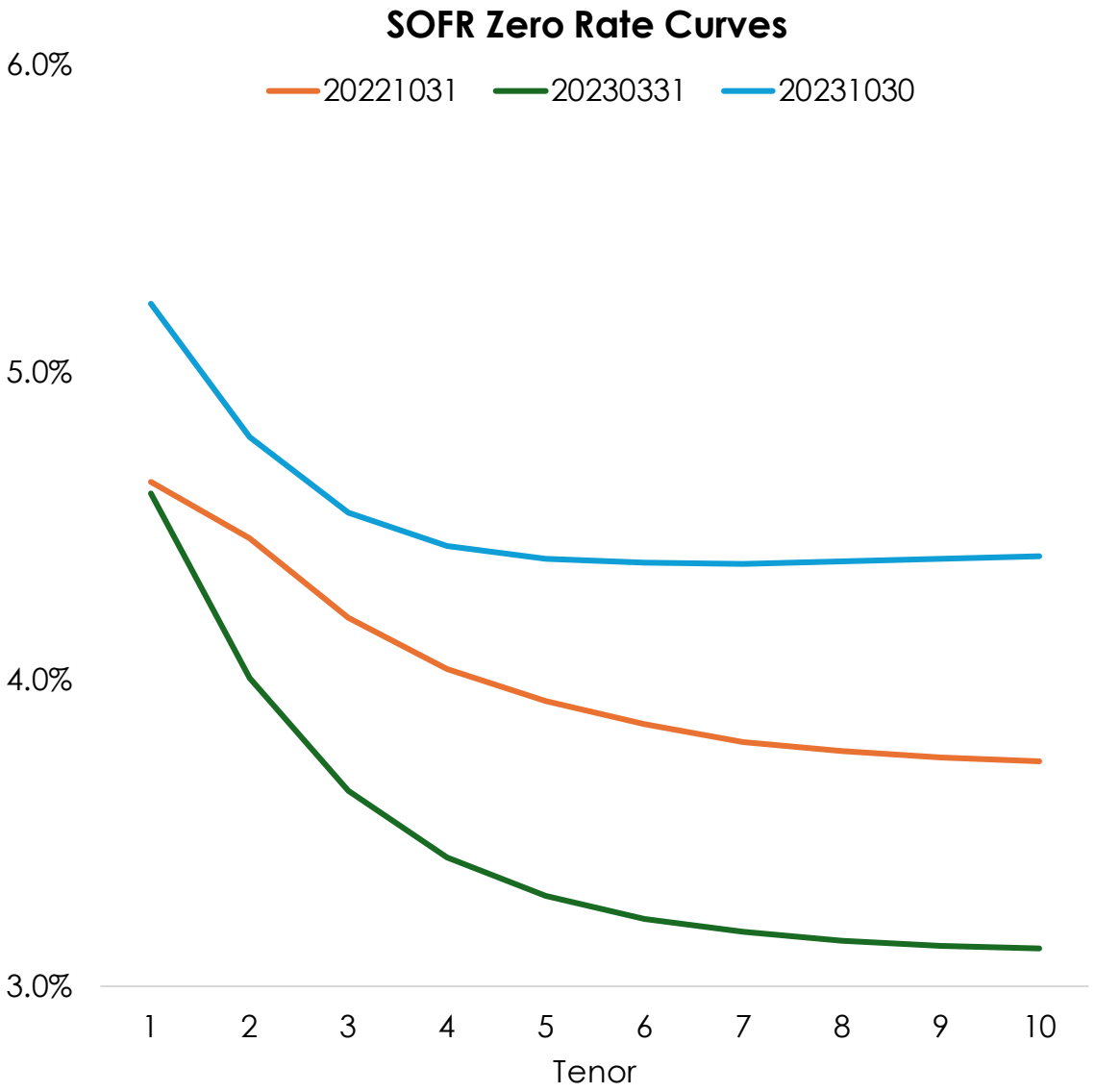
Value: $1,000,000
Shares: 2,970 @ $336.63

Value: $1,000,000
Shares: 103,950 @ $12.74

Value: $1,000,000
Shares: 39,236 @ $25.49

Swap Portfolio Overview

Risk Takers



Payer Swap Valuation @ 30th Oct 2023

Start Date: 30th October 2023

Notional: $100 Million, Fixed Rate: 4.2%

Float Leg Frequency: Annual
Float Leg $= 1 - D(0,10Y)$

Fixed Leg Frequency: Annual
Fixed Leg $= 4.2\% * \sum_{i=1}^{10} D(0,i)$

Swap Value Today

$$Notional * (Float\ Leg\ - Fixed\ Leg)$$

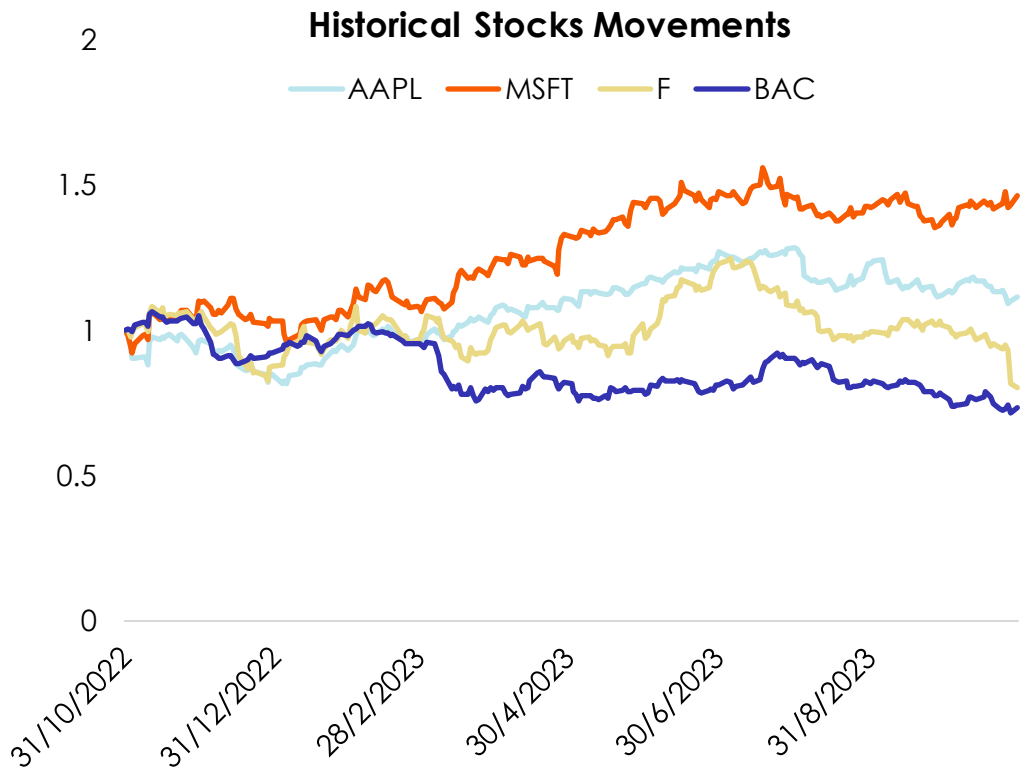
$2,442,902

Risk Factors

Risk Takers

Stock

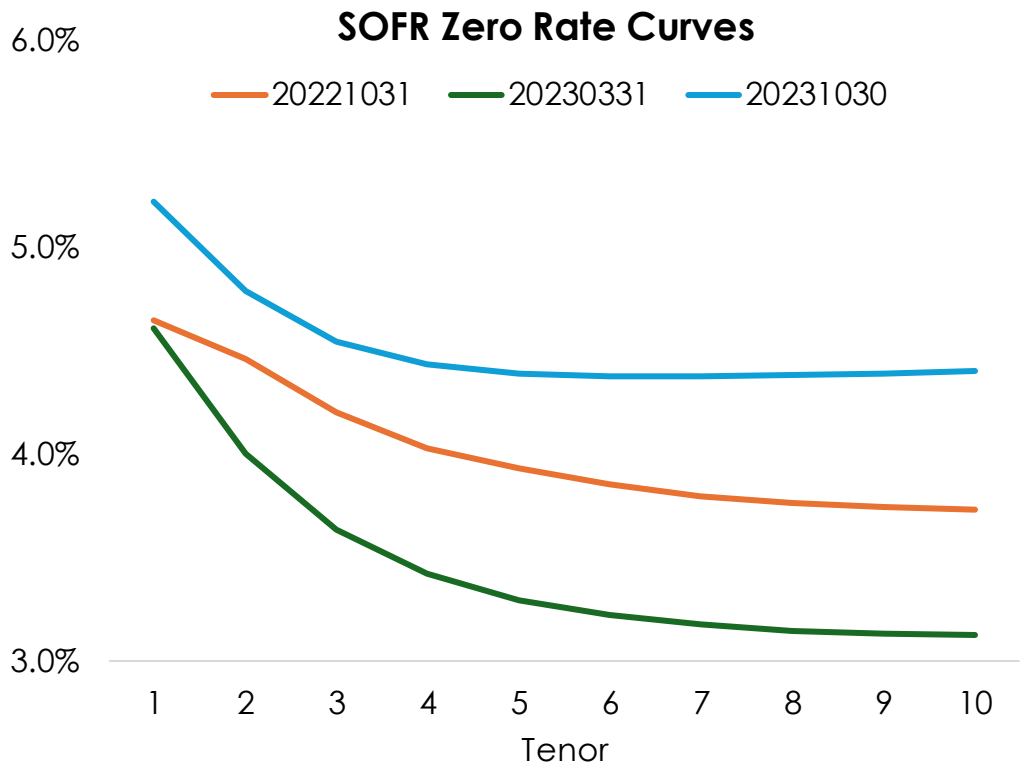
Risk Factor: Price



Daily % Change In Price

SOFR Rates

Risk Factor: 1-10y Zero Rates



Absolute Daily SOFR Rate Change

EDA (Missing Data)

Risk Takers

Stock

Date	Adj Close
14/6/2023	183.2264
15/6/2023	185.2783
16/6/2023	184.1926
19/6/2023	
20/6/2023	184.2823
21/6/2023	183.2364
22/6/2023	186.2644

*AAPL Data

19th June 2023

7th April 2023

SOFR Rates

Tenor	20231005	20231006	20231009	20231010	20231011
1D	0.052979	0.052939		0.052965	0.053028
1M	0.053099	0.053121		0.05309	0.053101
2M	0.053335	0.053462		0.053304	0.053256
3M	0.053532	0.053725		0.053488	0.053444
6M	0.053658	0.053969		0.053555	0.053602
9M	0.053223	0.053644		0.053028	0.053118
1Y	0.052471	0.05294		0.052191	0.052308
2Y	0.04805	0.048569		0.047529	0.047714
3Y	0.045262	0.045783		0.044683	0.044764

9th October 2023

11th November 2023

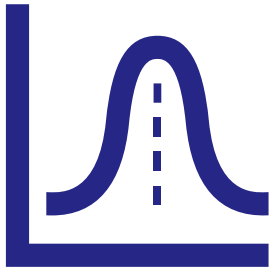
Solution

Linear Interpolation

251 > 253 rows

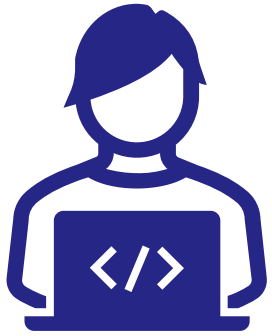
Parametric VaR

Risk Takers



Assumes a certain distribution (typically normal)

μ & σ to estimate VaR



Simple & Fast Calculations

Parametric VaR (Stock)

Risk Takers

Mean Returns

```
Stock_rets = Stocks.pct_change()
```

	Mean Return
AAPL	0.000568
MSFT	0.001692
F	-0.000619
BAC	-0.001105

Covariance of Returns

```
Stock_rets = Stock_rets.cov()
```

	AAPL	MSFT	F	BAC
AAPL	0.000252	0.000183	0.000135	0.000075
MSFT	0.000183	0.000320	0.000134	0.000070
F	0.000135	0.000134	0.000508	0.000168
BAC	0.000075	0.000070	0.000168	0.000268

$$Parametric\ mean$$
$$w * \mu = 536.21$$

$$Parametric\ Standard\ Deviation$$
$$\sqrt{w * \sum * w^T} = 53{,}662.35$$

Parametric VaR (Swap)

Risk Takers

Computing KR01 (For Every Tenor)

Tenor	20231030
1Y	0.052245
2Y	0.047904
3Y	0.045429
4Y	0.044345
5Y	0.043928
6Y	0.043794
7Y	0.043779
8Y	0.043828
9Y	0.043915
10Y	0.044023

+1bp

20231030
0.052345
0.047904
0.045429
0.044345
0.043928
0.043794
0.043779
0.043828
0.043915
0.044023

Compute Discount Factor

Continuous compounding e^{-rt}

Discount
0.949001
0.908638
0.872592
0.837461
0.802808
0.768923
0.736053
0.704249
0.673522
0.643888

Change In Swap Value

Tenor	KR01 / %
1Y	3,986,007
2Y	7,631,797
3Y	10,993,001
4Y	14,066,554
5Y	16,854,751
6Y	19,371,056
7Y	21,632,336
8Y	23,653,228
9Y	25,447,769
10Y	670,595,263

Mean Absolute Change In Rates

Tenor	Mean
1Y	0.000023
2Y	0.000013
3Y	0.000014
4Y	0.000016
5Y	0.000019
6Y	0.000021
7Y	0.000023
8Y	0.000025
9Y	0.000026
10Y	0.000027

$$w * \mu = 20{,}927.25$$

Covariance Of Absolute Change In Rates

	1Y	2Y	…	9Y	10Y
1Y	0.0000006434	0.0000006910	…	0.0000003809	0.0000003612
2Y	0.0000006910	0.0000008367	…	0.0000005335	0.0000005078
…	…	…	…	…	…
9Y	0.0000003809	0.0000005335	…	0.0000005202	0.0000005079
10Y	0.0000003612	0.0000005078	…	0.0000005079	0.0000004972

$$\sqrt{w * \sum * w^T} = 53{,}662.35$$

Parametric VaR (Portfolio)

Risk Takers

Mean Change In Risk Factors

	W
1Y	3,986,007
2Y	7,631,797
3Y	10,993,001
4Y	14,066,554
5Y	16,854,751
6Y	19,371,056
7Y	21,632,336
8Y	23,653,228
9Y	25,447,769
10Y	670,595,263
AAPL	1,000,000
MSFT	1,000,000
F	1,000,000
BAC	1,000,000

	μ
1Y	0.000023
2Y	0.000013
3Y	0.000014
4Y	0.000016
5Y	0.000019
6Y	0.000021
7Y	0.000023
8Y	0.000025
9Y	0.000026
10Y	0.000027
AAPL	0.000568
MSFT	0.001692
F	-0.000619
BAC	-0.001105

Parametric Mean: $w * \mu = 21{,}291.78$

Covariance Risk Factors

	1y	2y	...	9y	10y	AAPL	MSFT	F	BAC
1y	6.4e-07	6.8e-07	...	3.8e-07	3.6e-07	-2.7e-07	-1e-06	1.8e-06	4.5e-06
2y	6.8e-07	8.3e-07	...	5.3e-07	5e-07	-8.4e-07	-1.5e-06	1.4e-06	4.5e-06
...	...	...	...	...	...	...	...	...	...
9y	3.8e-07	5.3e-07	...	5.1e-07	5e-07	-1.4e-06	-1.2e-06	-7.7e-07	1.7e-06
10y	3.6e-07	5e-07	...	5e-07	4.9e-07	-1.4e-06	-1.1e-06	-9.2e-07	1.6e-06
AAPL	-2.7e-07	-8.4e-07	...	-1.4e-06	-1.4e-06	0.00025	0.00018	0.00013	7.5e-05
MSFT	-1e-06	-1.5e-06	...	-1.2e-06	-1.1e-06	0.00018	0.00032	0.00013	6.9e-05
F	1.8e-06	1.4e-06	...	-7.7e-07	-9.2e-07	0.00013	0.00013	0.0005	0.00017
BAC	4.5e-06	4.5e-06	...	1.7e-06	1.6e-06	7.5e-05	6.9e-05	0.00017	0.00027

Parametric SD: $\sqrt{w * \sum * w^T} = \mathbf{576{,}604.50}$

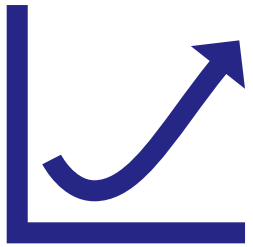
95% Parametric VaR

$$|\mu + z * \sigma| = |21{,}291.78 + (-1.644) * 576{,}604.50| = \$\mathbf{927{,}138.23}$$

Monte Carlo VaR Simulation

Risk Takers

Simulate possible portfolio outcomes using μ, σ^2, & ρ



Does not require normal distribution

Ability to model complex, non-linear relationships & path-dependent options accurately

Monte Carlo VaR Simulation

Risk Takers

Sample Generation

```
np.random.default_rng().multivariate_normal(portfolio_mean,
  portfolio_cov, size = 2 ** 14 - 1,  method = 'cholesky')
```

	1y	2y	3y	4y	5y	6y	7y	8y	9y	10y	AAPL	MSFT	F	BAC
0	-0.000122	0.000058	0.000197	0.000307	0.000348	0.000393	0.000442	0.000463	0.000465	0.000466	0.002634	0.023410	0.007983	0.029459
1	0.000453	0.000345	0.000203	0.000072	0.000058	0.000049	0.000015	-0.000004	-0.000009	-0.000012	0.011222	0.005479	0.001908	0.004710
2	-0.000696	-0.000712	-0.000590	-0.000531	-0.000554	-0.000581	-0.000586	-0.000574	-0.000559	-0.000561	0.004697	0.011977	0.006328	0.018192
3	0.002262	0.002291	0.002028	0.001801	0.001675	0.001612	0.001574	0.001554	0.001544	0.001535	0.012298	0.011177	-0.013796	-0.003296
4	-0.001105	-0.001044	-0.000730	-0.000555	-0.000374	-0.000294	-0.000293	-0.000301	-0.000300	-0.000290	0.041934	0.016775	0.034330	-0.011176
...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
16378	0.000674	0.000660	0.000488	0.000427	0.000372	0.000315	0.000260	0.000204	0.000153	0.000114	-0.000379	-0.001437	0.011634	0.004586
16379	0.000783	0.000265	-0.000119	-0.000191	-0.000314	-0.000387	-0.000411	-0.000435	-0.000461	-0.000484	0.050214	0.040602	0.018115	0.013311
16380	0.000743	0.000816	0.000483	0.000206	-0.000041	-0.000197	-0.000283	-0.000354	-0.000414	-0.000453	0.029472	0.011619	0.004441	0.011247
16381	0.000489	0.000928	0.000804	0.000702	0.000579	0.000477	0.000410	0.000334	0.000263	0.000219	0.012512	-0.021983	0.000260	-0.016049
16382	0.000148	0.000434	0.000600	0.000557	0.000551	0.000597	0.000635	0.000631	0.000599	0.000565	-0.017340	-0.005116	0.013078	0.010021

Monte Carlo VaR Methodology (Stock)

Risk Takers

Full Revaluation

Stock

Applying Risk Factor

$$Current\ Stock\ Price\ *(1+\textbf{\textit{Sim}}\ Return)$$

Computing New Stock Value

$$Share\ Holding\ *New\ Stock\ Price$$

Computing Stock PnL Change

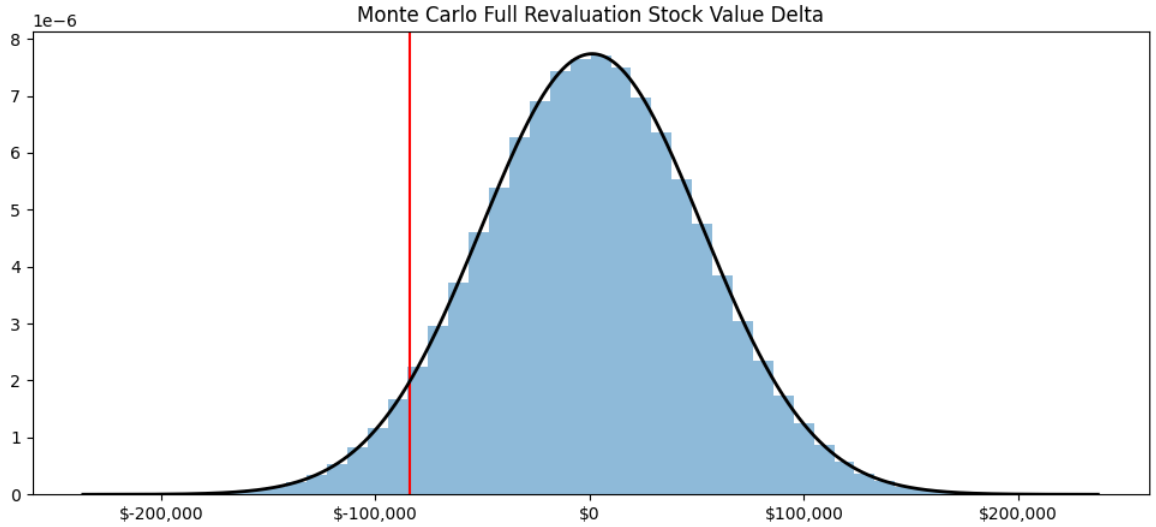
$$New\ Portfolio\ Value-\$1{,}000{,}000$$

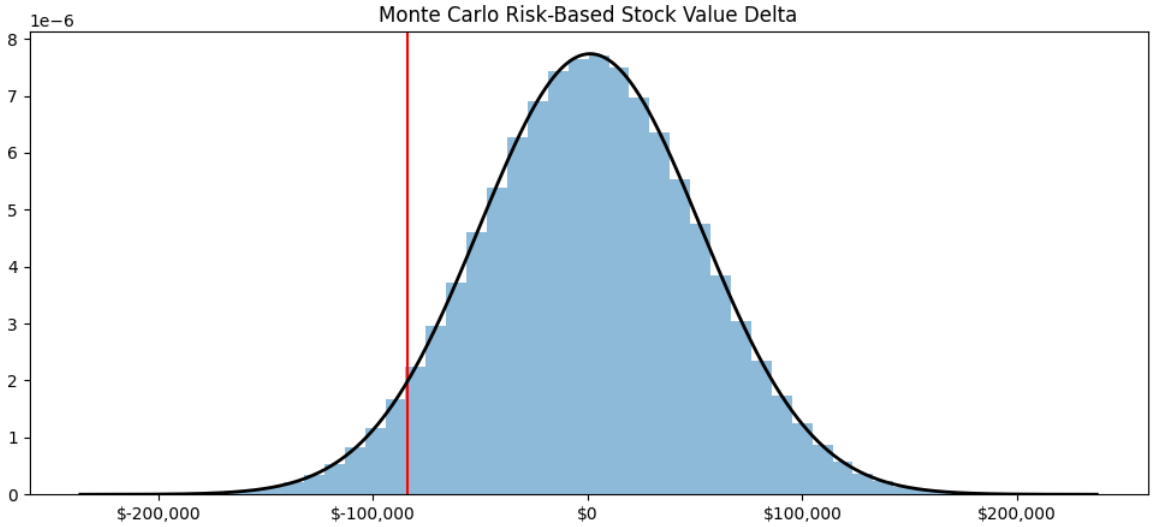
Risk-Based

Stock

Computing Stock PnL Change

$$\$1{,}000{,}000 * \textbf{\textit{Sim}}\ Return$$





Monte Carlo VaR Methodology (Swap)

Risk Takers

Full Revaluation

Swap

Applying Risk Factor

$$Current\ Rates + \textbf{\textit{Sim}}\ Rate\ Change$$

Computing New Discount Factor

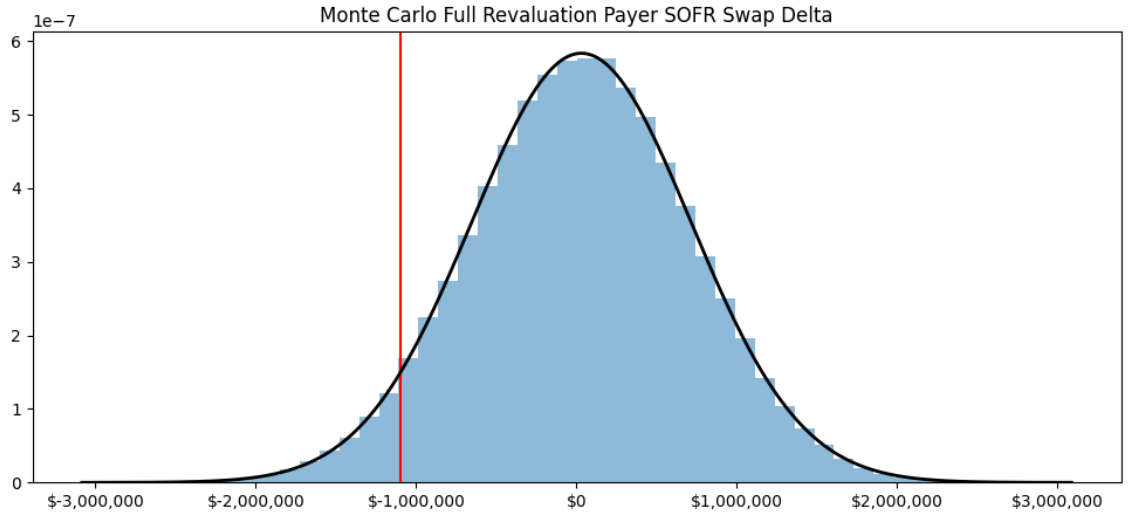
$$\text{Exp}(-New\ Rate * Tenor)$$

Computing New Swap Value

$$Notional * (Float\ Leg\ - Fixed\ Leg\ )$$

Computing Swap PnL Change

$$New\ Swap\ Value - \$2{,}442{,}902$$

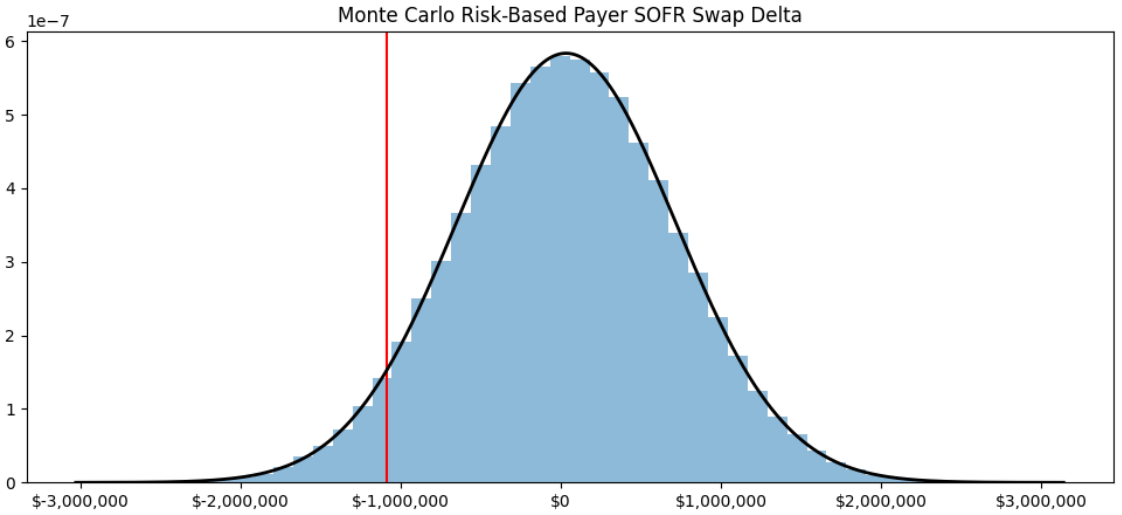


Risk-Based

Swap

Computing Swap PnL Change

$$KR01 * \textbf{\textit{Sim}}\ Rate\ Change$$

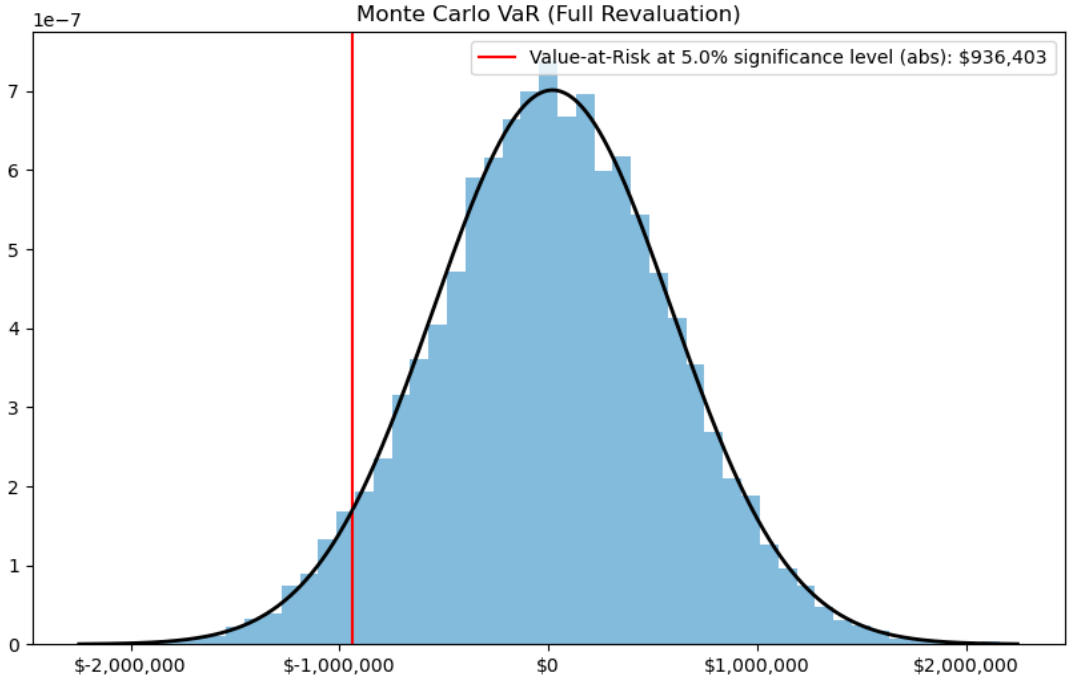


Monte Carlo VaR (Portfolio)

Risk Takers

Full Revaluation

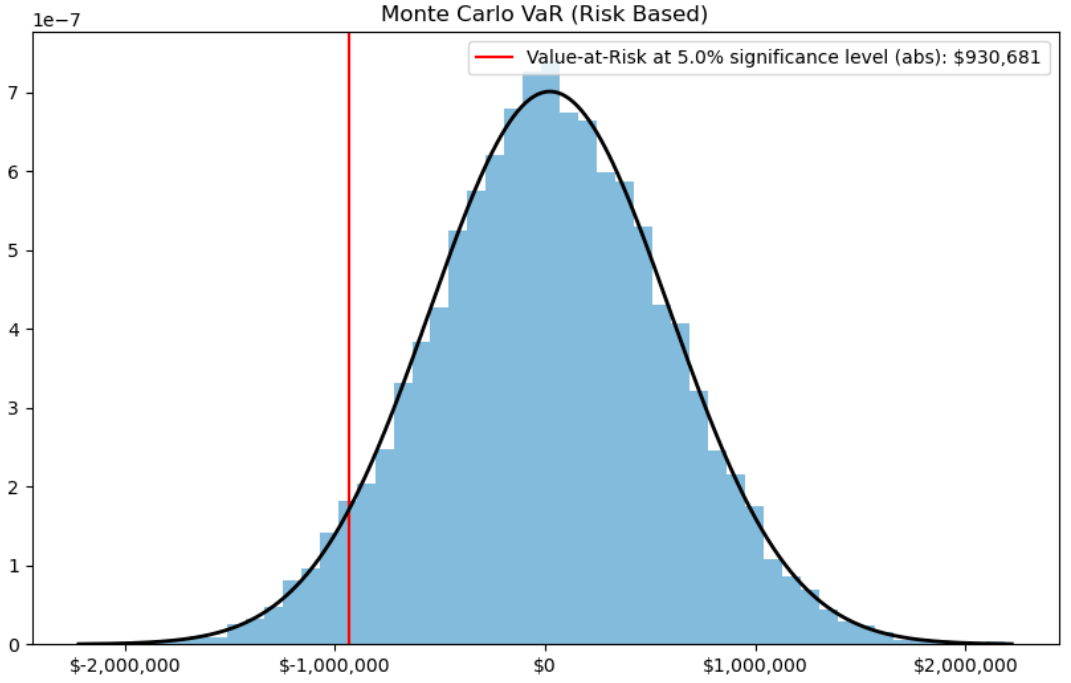
```
abs(np.percentile(MC_full_reval_portfolio, 5))
```



Full Revaluation 95% VaR
$936,402.89

Risk-Based

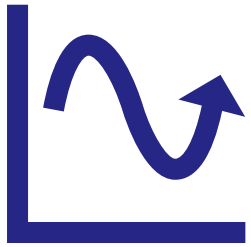
```
abs(np.percentile(MC_Risk_Based_portfolio, 5))
```

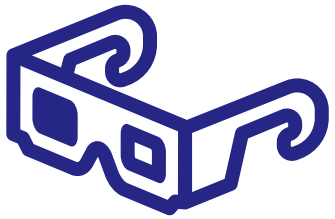


Risk-Based 95% VaR
$930,680.76

Historical VaR

Risk Takers





Calculates VaR directly from **historical returns**, assuming historical movements are a good indicator of future risks

No assumptions about return distribution

straightforward & easy to understand

Backwards-looking, may not be as effective in capturing the risk of new or unprecedented market events

Less responsive to recent market conditions if the historical window is too long.

Historical VaR Methodology (Stock)

Risk Takers

Full Revaluation

Stock

Applying Risk Factor

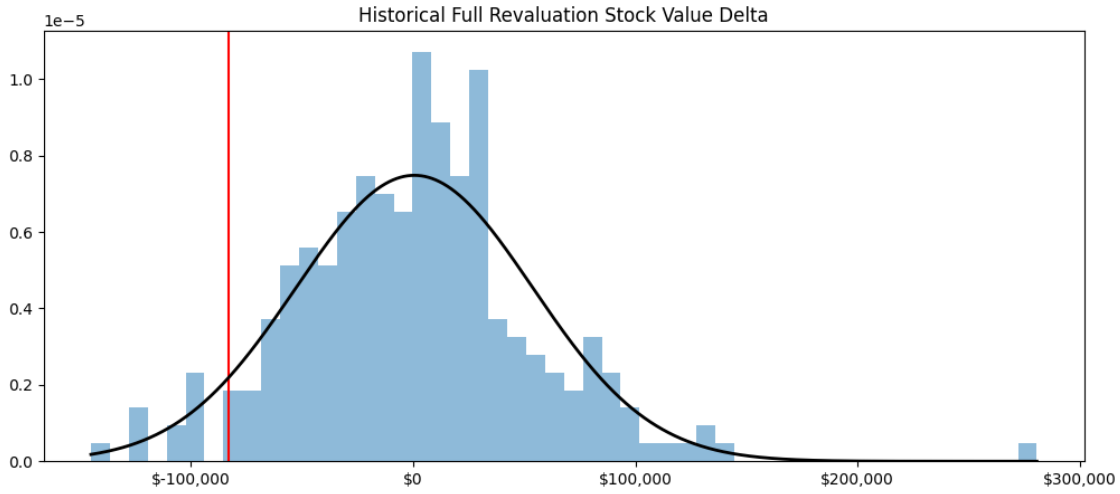
$$Current\ Stock\ Price\ *(1+\boldsymbol{Hist}\ Return)$$

Computing New Portfolio Value

$$Share\ Holding\ *New\ Stock\ Price$$

Computing Stock PnL Change

$$New\ Portfolio\ Value-\$1{,}000{,}000$$

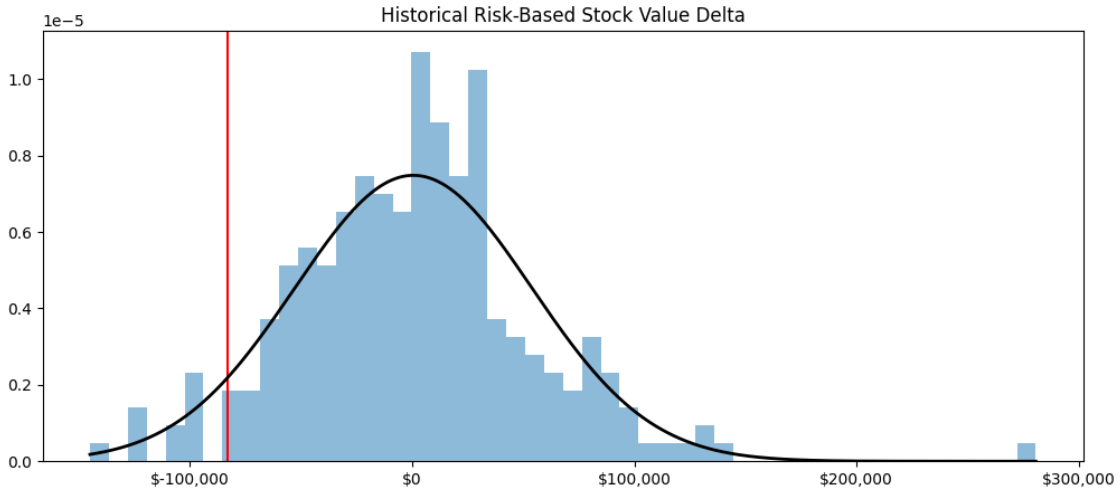


Risk-Based

Stock

Computing Stock PnL Change

$$\$1{,}000{,}000 * \boldsymbol{Hist}\ Return$$



Historical VaR Methodology (Swap)

Risk Takers

Full Revaluation

Swap

Applying Risk Factor

$$Current\ Rates + \textbf{\textit{Hist}}\ Rate\ Change$$

Computing New Discount Factor

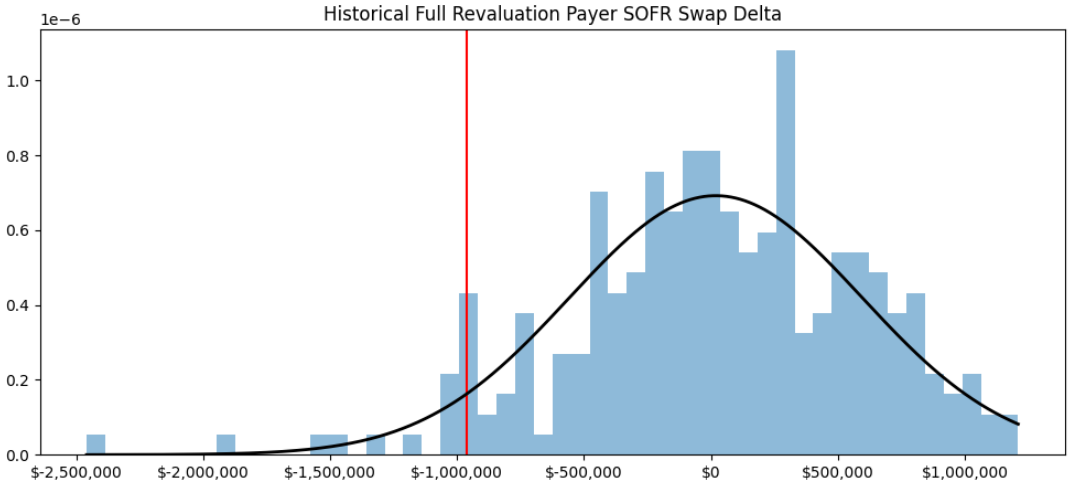
$$\text{Exp}(-New\ Rate * Tenor)$$

Computing New Swap Value

$$Notional * (Float\ Leg\ - Fixed\ Leg)$$

Computing Swap PnL Change

$$New\ Swap\ Value - \$2{,}442{,}902$$

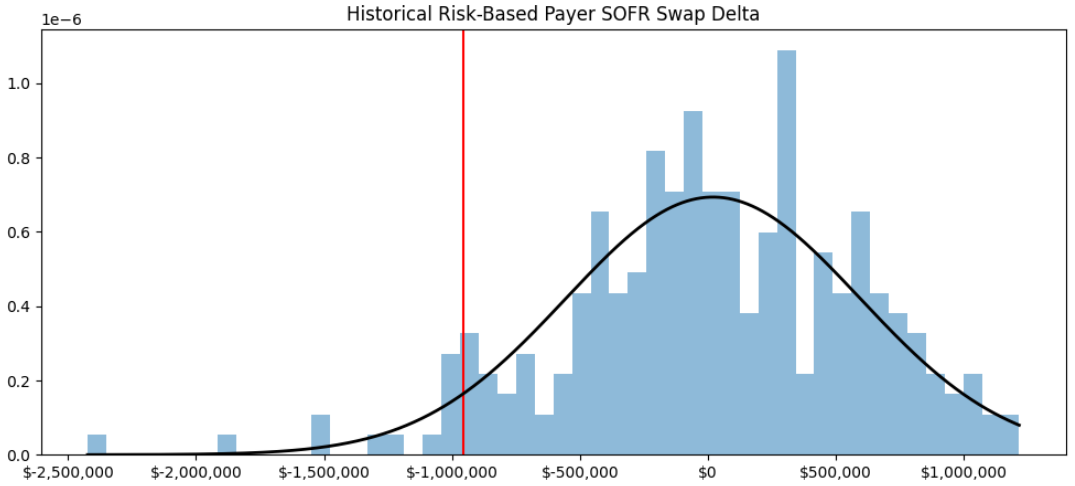


Risk-Based

Swap

Computing Swap PnL Change

$$KR01 * \textbf{\textit{Hist}}\ Rate\ Change$$

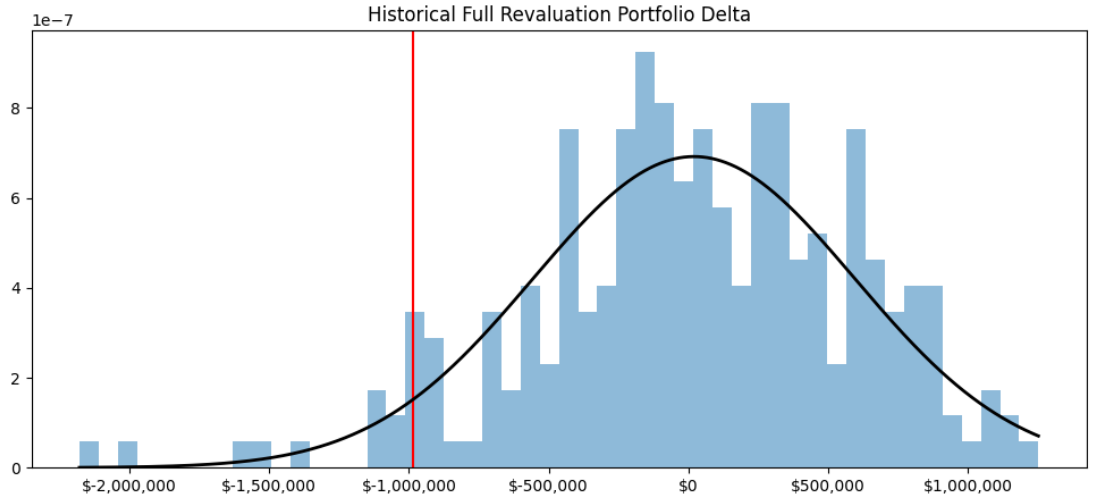


Historical VaR (Portfolio)

Risk Takers

Full Revaluation

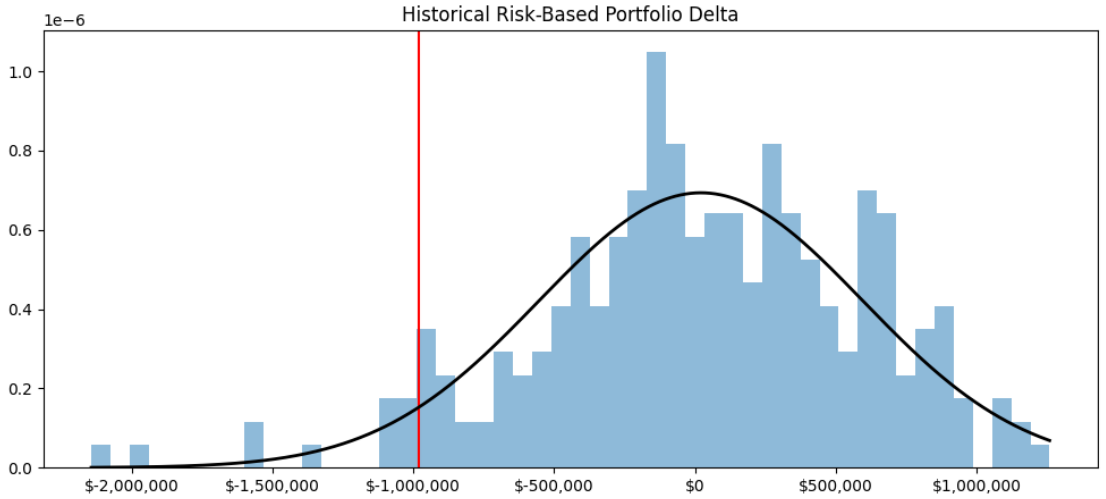
```
abs(np.percentile(Hist_full_reval_portfolio, 5))
```



Full Revaluation 95% VaR
$984,781.39

Risk-Based

```
abs(np.percentile(Hist_Risk_Based_portfolio, 5))
```



Risk-Based 95% VaR
$978,693.27

VaR Model Evaluation

Risk Takers

Parametric

- Ease of application

- Assumes risk factors normally distributed, market tend to have fatter tails (leptokurtosis)
- Poor at handling non-linear exposures.

Monte Carlo

- Accounts for both linear & non-linear risks.
- Incorporate various scenarios, including extreme events not seen in the historical data.

- Many assumptions on distribution, most computational effort
- Results can vary significantly based on model assumptions
- Requires detailed data on all portfolio components

Historical

- No need to assume any particular distribution (Nonparametric)

- Backwards-looking: Assumes past performance is representative of the future
- Less responsive to recent market conditions.

Need to Weigh

Available computational resources

Acceptable level of approximation vs detail in risk estimation

Summary

Risk Takers

	1 Day 95% Value-at-Risk	
Parametric	$927,138.23	
	Full Revaluation	**Risk-Based**
Monte Carlo	$936,402.89	$930,680.76
Historical	$984,781.39	$978,693.27