Lecture 1

- What happens if there are no financial institutions
  - Funds flow from households directly to corporations but with low level
  - High monitoring cost
  - Low liquidity
  - Price risk
- Functions of Fis
  - Brokerage
    - Agent providing information and transaction service
    - Cost reduction with economies of scale
  - Asset transformer
    - Purchase primary securities to be sold to households
    - Financial products sold to investors allow for transformation of financial risk
  - Achieve profitable financial risk transformation
    - Lower information cost
    - More timely information
    - Reduce information asymmetry
    - Secondary claims sold to investors have low price risk and high liquidity
- Risks
  - Interest rate risk – net interest income losses due to mismatch of assets and liabilities and changing interest rate environment
    - Refinancing risk, reinvestment risk, balancing hedging or asset and liabilities matching is inconsistent with FI's role as asset transformer
  - Credit risk – promised cash flows on financial claims will not be paid in full
    - Systematic risk – default associated with general economy affecting all borrowers
    - Firm-specific risk
  - FX risk – movement in FX affect the value assets and liabilities denominated in foreign currencies
  - Market risk – losses from securities and derivatives positions in trading books
    - Depending on instrument, positions can be exposed to interest / FX / credit / commodity / equity and other asset price risk
    - Daily PnL fluctuations due to market movements and marked to market
  - Liquidity risk – FI unable to meet financial obligations on time without sustaining unacceptable losses
    - Asset / market liquidity risk – market participants may not be able to execute a trade or to liquidate a position immediately
    - Funding liquidity risk – sudden surge in cash outflow may leave FI seeking funding to meet requirements
    - Liquidity risk may generate bank runs which can turn into solvency problem
- Trading book – short term trading
- Banking book – positions held to maturity (customer loans, deposits from retail / corporate, derivatives used to hedge exposure arising from banking activities)
- Accounting methods – accrual accounting / mark to market accounting (PV of future cash flows on every time period excluding current period cash flow because net realized cash add separately)
- Repricing model framework – banking book position data, repricing gaps, IRRBB Metrics
- IRRBB Metrics – net income interest, economic value at risk, price sensitivities
- Repricing model (current and prospective risk of bank earnings and capital due to adverse interest rate movements) – nearest future interest rate update event associated with a financial clam (contractual interest rate fixing aka IR swap, rollover / renewal of expiring instrument)
- Repricing gap – notional / book value of contract subjected to repricing
- Repricing date – date which the interest rate gets repriced

Lecture 2

- Note that deposit is +ve interest paid from deposit is -ve
- If instrument gets repriced periodically, only the nearest next repricing date is considered
- Floating rate instrument
  - Interest income sensitivity – at repricing date 3 month L1, consider the payment at 6 month
  - Economic value sensitivity – less sensitive to interest rate changes because of the offsetting of interest and discounting
- Forward starting fixed rate deposit and loan – give loan / deposit some time in the future
  - Need to reprice 2 times, when issue the loan and during maturity (assume that interest is decided at time 0)
- 1 Month Forward starting deposit paying 3M Libor
  - First fixing is when the deposit start
  - Another fixing on the same time the deposit start because of LIBOR fixed in advance paid in arrears
- For IRS, if no exchange of notional then only have 1 repricing
- NII sensitivity (short term change in earnings) = change in interest rate * repricing gap * (time horizon – mid point of bucket of time to repricing)
  - When distributing NII across period, use weighted average, full period not mid point
  - Usually focus on short term, time horizon = 1
- EVE sensitivity = scenario EV by changing interest rate – base EV
  - Look at impact over longer period
- Repricing gap limitations
  - Doesn't account for repricing risk of interest payments
  - Standardized repricing buckets over (if payments are close but fall in different buckets) / under (if payments are far but fall in different buckets) state actual repricing risk
  - No indication of basis risk, offsetting items may not reprice by same amount
  - Behavioural assumptions need to be embedded
  - NII assumes that asset and liabilities have like for like replacement as they run off – can rollover indefinitely

Lecture 3

- Converting nominal rate to different compounding frequency: $\left(1+\frac{y^{(m)}}{m}\right)^{-mt}=\left(1+\frac{y^{(n)}}{n}\right)^{-nt} \Rightarrow y^{(n)}=n\left[\left(1+\frac{y^{(m)}}{m}\right)^{\frac{m}{n}}-1\right]$, higher compounding frequency, smaller the nominal rate
- Coupon bond price and durations
  - Price increasing when coupon > yield
  - Yield increase, price decrease at decreasing rate
  - Maturity increase, duration increase at decreasing rate
  - Yield increase, duration increase, rate depends (there is an inflexion point)
- Duration of IRS = duration of fixed leg because the floating rate value is at par, no interest rate risk (when rate go up, coupon go up but discount rate will also offset), wont change
- Modified duration can estimate bond price volatility: VOL(% change in bond price) = $-D * VOL(\Delta y)$
- Duration for asset / liability = weighted average of the duration of each asset / liability
- Immunizing equity change in balance sheet: $\Delta E = \Delta A - \Delta L$
  - $\Delta E=-[D_A*A-D_L L]*\frac{\Delta y}{1+y}=-\left[D_A-\frac{L}{A}D_L\right]*A*\frac{\Delta y}{1+y}$, (**<u>Macaulay Duration</u>**)
  - leverage ratio $\frac{L}{A}$
  - $D_A - D_L\frac{L}{A}$ is the leverage adjusted duration gap (D-gap)
    - If D-gap is +ve, more duration on asset side so can try to reduce asset duration or increase liability duration
  - Since A and $\frac{\Delta y}{1+y}$!= 0, can only make $D_A - D_L\frac{L}{A}$ = 0 to immunize
  - Limitations:
    - Time
    - Transaction cost
    - Large interest rate movements (1st order, does not include convexity)
    - Assumes parallel shift / flat yield curve

Lecture 4

- Geometric – failures until first success
- Negative binomial – trials until nth success (generalization of geometric)
  - Before nth success we need n-1 success from k-1 trials so formula is
  - $C(k-1,n-1)*p*p^{n-1}*(1-p)^{k-n}$
- Poisson – number of events within a time frame
- Exponential – time until first event occurs
  - Set x = 0 in poisson distribution $\frac{(\lambda T)^0 e^{-\lambda T}}{0!}$
- Given CDF of X, if do inverse of uniform distribution, we get a random variable that follows the distribution of X (making X the subject)
- Monte Carlo – boils down to simulating correlated standard normal random variables
  - To simulated correlated sample
    - Compute decomposition of correlation matrix
    - Simulate standard normal random variables
    - Apply inverse CDF to the simulated variables
    - Apply the decomposition matrix
  - Simplified case, to simulate 2 correlated standard normal random variable with correlation ρ
    - $X_1 = Z_1, X_2 = aZ_1 + bZ_2$,
      - Z_2 doesn't affect correlation with Z_1 so a = ρ
      - since standard normal, mean is 0 variance is 1 means variance of X_2 =1, $\rho^2 + b^2 = 1, b = \sqrt{1-\rho}$
      - $X_2 = \rho Z_1 + \sqrt{1-\rho}Z_2$
  - Estimating π using monte carlo,
    - use area of circle in a unit square
    - simulate values, if value falls in circle (where distance of point to middle of the circle with coordinates 0.5,0.5 < radius), tag as 1 else 0
    - estimated probability = $\frac{\frac{\pi}{4}}{1}$
    - then *4

Lecture 5

- library to generate uniform variables: scipy.stats.qmc
  - `from scipy.stats import qmc`
  - `sampler = qmc.Sobol(d=1, scramble=False)`
  - `sample = sampler.random_base2(m=5)`
- Cholesky decomposition
  - Variance = 1, upper triangle of decomposition matrix = 0
  - Finding the decomposition
    - Correlation between standard normal
      - If Independent standard normal, correlation = 0
      - Correlation between 2 standard normal is the product of each coefficient
- Eigen decomposition
- To check if decomposition is valid,
  - $decomposition\ matrix\ X\ (decomposition\ matrix)^T = original\ correlation\ matrix$
- VaR (define params, identify portfolio, identify risk factors, model joint distribution of risk factor over risk horizon, build distribution, calculate VaR)
  - Returns
    - Absolute return
    - Simple return
    - Log return (additive, for short horizon, log return ~ simple return)
    - Simple and log are more appropriate for positive risk factors
  - Building distribution of portfolio PnL
    - Base scenario (current portfolio value)
    - Sample from join distribution of risk factor changes
    - Calculate portfolio PnL as a result of the risk factor (PnL = Pi – Po)
      - Full revaluation (usually or exotic products, non-linear) – take the change, add/minus to the current risk factors, feed into the scenario and call re-pricer to give new PnL
      - Risk-based approach – generate sensitivity, using change * eg PV01
  - Parametric VaR (toy model, only useful if portfolio is simple)
    - Assumes daily risk factor change are multivariate normally distributed, means and covariance matrix estimated from historical data
    - Daily change are IID and additive
    - Portfolio PV over horizon is linear function of VaR risk factor change
  - Monte Carlo VaR
    - For n sample of PnL, if there is no 1 percentile value, interpolate
    - If risk horizon > 1 day, can just use $\sqrt{h} * VaR$ approximation
  - Historical VaR
    - Look at historical daily change (usually 1 year) as scenario

Lecture 7

- Principal component analysis
  - First PC = eigenvector associated with largest eigenvalue of correlation matrix of V
  - If V is symmetric and positive semi definite, Trace(V) which is the sum of the diagonals of a covariance matrix which is the variance of variables preserves the total variance of the original data
  - Running PCA
    - Standardize raw data
    - Calculate correlation/covariance (same because data is standardized) matrix of the dataset
    - Compute eigenvector of V and associated eigenvalues and order according to eigen value
    - Each principal component = individual value of the eigenvalue
    - Determine number of PC to keep by using % of explained vairnace eg. 95%
- Full rank reduction
  - Correlation matrix is full rank if there is a decomposition A = $\gamma * \gamma^T$ such that factor loading matrix, γ, is invertible
  - If A is invertible, none of x can be expressed as a linear combination of other x
  - To truncate correlation matrix A with minimal distortion, use eigen decomposition, keeping the largest eigen value from the diagonal matrix
    - Full rank mean eigen value will be non-zero
    - Identify how much to keep, truncate the eigen vector matrix and diagonal matrix
    - Compute C which is the scaling factor to rescale H to get the correlation matrix and factor loading matrix
      - Condition is that the length of each row of the truncated factor loading matrix = 1
      - So to compute the scaling divide each element in each row of the factor loading matrix by the sqrt of sum of squares of all the elements in each row
  - Can be used to fix an invalid correlation matrix (non positive semi definite) just rank the diagonal matrix and remove the negative eigen values and their corresponding eigen vectors