HF Sourcing:
FFTs on Sourcing Data

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Review of continuous Fourier transforms:

- Fourier transform (FT) definition
  - $F(u) = \int_{-\infty}^{+\infty} f(t)e^{-2\pi iut}dt$
- FTs have real and imaginary components
  - Real: $\mathcal{R}(F)$
  - Imaginary: $\mathcal{I}(F)$
- FTs have magnitude and phase in complex space:
  - Magnitude: $|F| = |\mathcal{R}(F)^2 + \mathcal{I}(F)^2|^{1/2}$
  - Phase: $\phi(F) = \tan^{-1} \frac{\mathcal{R}(F)}{\mathcal{I}(F)}$

Review of discrete, fast Fourier transforms:

- ROOT can run fast Fourier transforms (FFTs) with `fftw`
- Notes from `fftw` online documentation:
  - We want to do a 1D discrete Fourier transform (DFT)
  - We want a transform from real to complex space (R2C)
  - For a 1D R2C DFT on an input with n bins, `fftw` calculates the following:

$$Y_k = \sum_{j=0}^{n-1} X_j e^{-2\pi jk\sqrt{-1}/n}$$

Where:

- n is the number of bins in the input histogram
- Y_k is the FFT output (content of output bin k)
- X_j is the original input (content of input bin j)

Review of discrete, fast Fourier transforms:

- For an input histogram with n bins, `fftw` outputs a vector of $\frac{n}{2}+1$ complex elements ($\frac{n}{2}$ rounds down):
  - 0th element is the "DC": purely real (omitted from plots)
  - $\frac{n}{2}$th element is the "Nyquist" frequency: purely real
  - All elements have Hermitian symmetry: $Y_k = Y^*_{n-k}$
- Performing the transform introduces a factor of $\sqrt{n}$, which must be divided out by the user

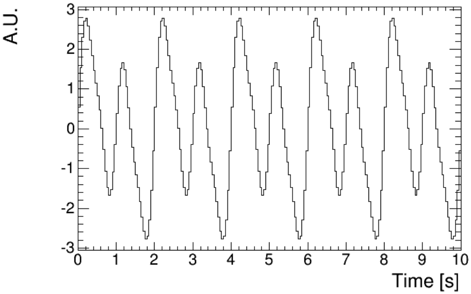
Review of discrete, fast Fourier transforms:

- Just like continuous FTs, we're interested in the real and imaginary components of `fftw` output
- Consider real and imaginary output from `fftw` (Y_k)
  - Real: $\mathcal{R}(Y_k)$
  - Imaginary: $\mathcal{I}(Y_k)$
- Consider magnitude and phase in complex space:
  - Magnitude: $|Y_k| = |\mathcal{R}(Y_k)^2 + \mathcal{I}(Y_k)^2|^{1/2}$
  - Phase: $\phi(Y_k) = \tan^{-1} \frac{\mathcal{R}(Y_k)}{\mathcal{I}(Y_k)}$

Method

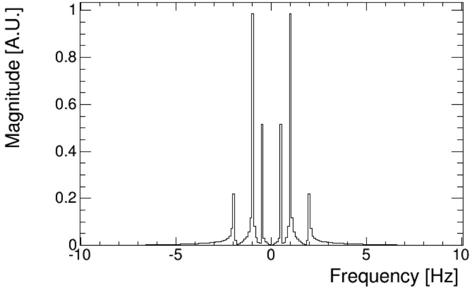
- Two very simple steps
  1. Use sine functions to test ROOT FFT software
  2. Use ROOT FFT software to analyze sourcing data
- All of this code is on git:
  1. Link to code for testing FFTs
  2. Link to code for running FFTs on sourcing data

FFT test on sine functions



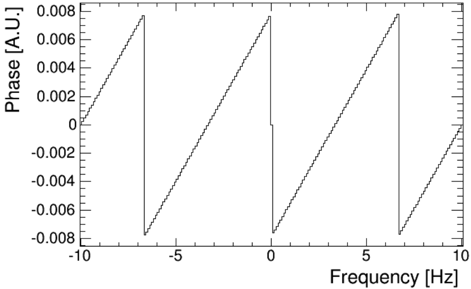
- Fill a histogram using linear combo of sine functions:
  - $f(t) = \sum_{i=0}^{3} A_i \cdot \sin(2\pi \cdot f_i \cdot t)$
  - $A_i = \{1.0, 2.0, 0.5\}$ [A.U.]
  - $f_i = \{0.5, 1.0, 2.0\}$ [Hz]

Look at magnitude of FFT



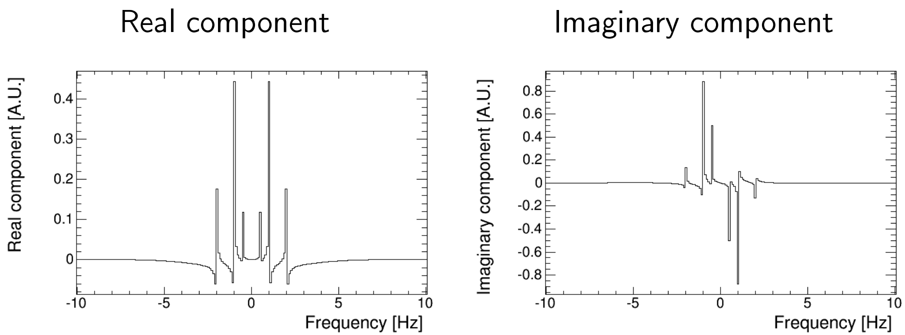
- Magnitude (even function) returns A_i and f_i
  - $f(t) = \sum_{i=0}^{3} A_i \cdot \sin(2\pi \cdot f_i \cdot t)$
  - $A_i = \{1.0, 2.0, 0.5\}$ [A.U.]
  - $f_i = \{0.5, 1.0, 2.0\}$ [Hz]

Look at phase of FFT



- Phase information not useful for our purposes...
  - $f(t) = \sum_{i=0}^{3} A_i \cdot \sin(2\pi \cdot f_i \cdot t)$
  - $A_i = \{1.0, 2.0, 0.5\}$ [A.U.]
  - $f_i = \{0.5, 1.0, 2.0\}$ [Hz]

Look at real and imaginary components of FFT:



- Real/imaginary components similar to magnitude:
  - $f(t) = \sum_{i=0}^{3} A_i \cdot \sin(2\pi \cdot f_i \cdot t)$
  - $A_i = \{1.0, 2.0, 0.5\}$ [A.U.]
  - $f_i = \{0.5, 1.0, 2.0\}$ [Hz]

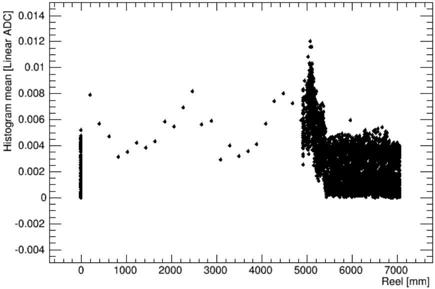
Conclusion of test:

- Can use ROOT FFT software
- ROOT FFT software can reconstruct parameters of sines
  - FT magnitude contains useful information for analysis
  - FT phase not useful for this analysis (?)
  - Can use FT phase to reconstruct original function (inverse FFT)
- Ready to try sourcing data

Look at sourcing data

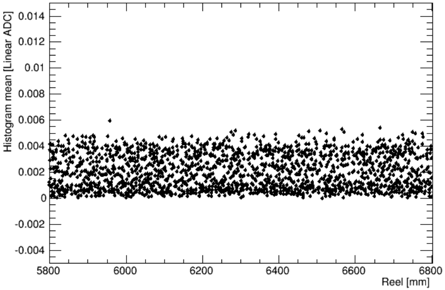
- histogram name:
  “HFP13_ETA38_PHI25_T10_SRCTUBE_
  Ieta38_Iphi25_Depth2
  Run 221509reelPosition”
- x-axis: Reel [mm]
- y-axis: Histogram mean [linear ADC]

Look at sourcing data: full range of reel vals



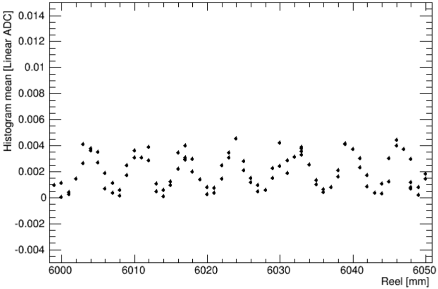
- Next: focus on reel ϵ $[5800, 6800]$ [mm], where amplitude is stable

Look at sourcing data: zoomed reel vals (TGraph)



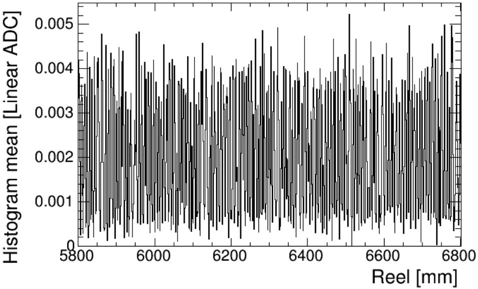
- This looks stable. We can do our FFT on this data.
- Next: zoom in even further ($[6000, 6050]$) to see what frequency we suspect naively

Look at sourcing data: zoomed reel vals (TGraph)



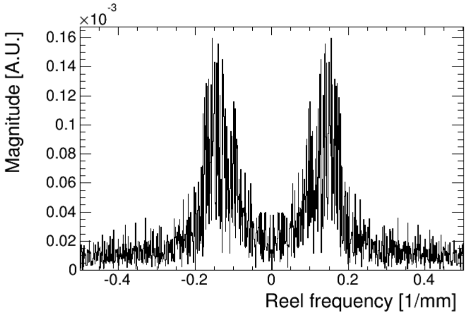
- 7 periods in 50mm:
  - $T = 50\ [\text{mm}]/7 = 7.14\ [\text{mm}]$
  - $f = 1/T = 0.14\ [1/\text{mm}]$
- Only a naive guess for the frequency!

Look at sourcing data: zoomed reel vals (TH1F)



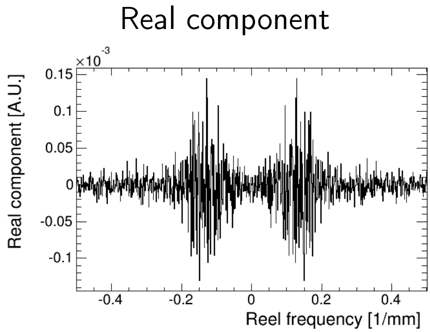
- Now make a histogram from the TGraph (result above)
  - If multiple points in TGraph have the same x-value, use their mean y-value on y-axis for histogram
- Next: do FFT on this histogram

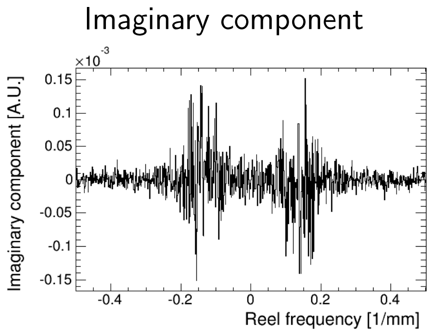
Look at sourcing data: FFT magnitude



- FFT magnitude peaks between “reel frequency” (in [1/mm]) of $|0.10|$ and $|0.20|$
- Roughly matches our naive guess (0.14)

Look at sourcing data: FFT real & imaginary





- Real/imaginary components similar to magnitude

Conclusion

- We can use ROOT software to do FFTs
  - Tests done on sine waves in time / frequency space
  - Prelim. results on data in reel / "reel frequency" space
- Prelim. results show peak in "reel frequency"
  - Around 0.12 - 0.13 [1/mm]
- Would be nice to repeat the study on sourcing data in time (OrN)
  - Need plots for this