CS1133, Spring 2018, Homework 1 (Lect 2, 10 pts each)

Due January 30, 2018 (before 11:00 pm)

Create a script file with a name that properly identify the assignment and yourself. For the first homework, the file name should be `HW01_0123456.m` if your student ID number is `N0123456`.

Make sure that at the top of file you insert your name and student ID number in the form of comments.

Do all the following problems by writing programming statements in this single script file. The programming statements for each problem must be contained within a "cell" (as done in the lab).

Make sure that your program runs with no errors.

Report your results, findings, observations and interpretations as comments at the end of the programming statements for each of the problems.

Submit your script file on NYU Classes; any other forms of submission (e.g. email) will not be accepted. You must first attach the file and then click the submit button. Make sure that you always inspect the file to check to see if it has been properly submitted and that it is the actual file that you really want to submit.

Note that no late homework will be allowed. It is your responsibility to properly submit your assignment. Failure to submit your assignment for whatever reason, is treated the same as failure to complete it.

Your assignments will be graded according to the following criteria (in decreasing order of importance):

- program runs with on errors
- program produces the correct results
- program is written efficiently except when specified
- program uses good programming practices (good choice of names, good use of comments, etc.).

Other comments:

- Each problem must be contained within a separate cell block with the `clear` statement as the first statement so it can be run independently of others.

- Therefore only one programming file needs to be submitted.
- Each problem must be done using scalars only. No vectors are allowed here.
- For this assignment, you should only have one version of the program for each problem. If a program has to be run several times with slightly different input data, you should make the necessary changes in the data and rerun the same program. DO not have multiple versions of the same program having slightly different input data.
- Write down all relevant output results. Collect them and display them inside the program as comment statements at the end of the cell block.

Problem 1 [10 pts]

The first derivative of a function $f(x)$ is defined mathematically as

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}.$$

A method for computing numerically the first derivative of a given function $f(x)$ at a given x is to use the formula

$$\frac{f(x+h) - f(x-h)}{2h} \tag{1}$$

with a "small" step h. This formula approximates the first derivative of $f(x)$. The difference between this approximated expression and the exact derivative has a magnitude that is proportional to h^2 (assuming that h is small). Therefore mathematically speaking, the result is supposed to be more and more accurate as the step size h is made smaller and smaller.

In this problem we consider using this method to compute numerically the first derivative of $f(x) = \ln \cosh(px + q)$ at an arbitrary point x, for given values of p and q.

For this problem we know that the exact solution is given by $p \tanh(px + q)$. This allows one to compute the relative error, r of an approximate result a:

$$r = \left| \frac{a - e}{e} \right|.$$

In this expression e is the exact result.

Find the relative errors for various step sizes. Use only scalar variables and compute the results for one value of h with each run. For both this problem and the next, you can let $x = 1.7$, $p = -0.22$, and $q = 2.1$. Use prope variables, so that your program can easily be modified to use other values.

You need to write down the results for each run to produce a table with a total of two columns. Column 1 is for the value of h used. The second column contain the relative errors for you numerical calculations. Place the table at the end of your program in the form of a comment block.

Do you see a trend? Describe your findings semi-qualitatively. There is no need to explain them.

What values of h should one use? You need to explore and experiment with decreasing values of h that shows this qualitative trend in the best way. What is the largest value of h should one uses? How much to reduce h from one value to the next? When should one stop reducing h further? All of these answers have to come out by exploration and experimentation. To best display the trend in your table of results, arrange h from the largest to the smallest values.

Problem 2 [10 pts]

In the last 10 years or so, if one is using a computer language that can readily handle complex arithmetics as in Matlab, a method employing an imaginary step has been introduced. Here we let $h = i\tilde{h}$ where $\tilde{h}$ is taken to be a real positive parameter, and i is the imaginary unit whose square gives -1. Matlab has a pre-defined variable for this value. The name of that variable is `i`.

With an imaginary step, the first derivative of a function $f(x)$ now has the form

$$f'(x) = \lim_{\tilde{h}\to 0} \frac{f(x+i\tilde{h}) - f(x)}{i\tilde{h}}.$$

For a real function $f(x)$ of a real argument x, the first derivative must also be real. Since the denominator is purely imaginary, this expression is the same as

$$f'(x) = \lim_{\tilde{h}\to 0} \frac{i\,\mathrm{Im}\left(f(x+i\tilde{h}) - f(x)\right)}{i\tilde{h}},$$

where Im(z) refers to the imaginary part of z. Given a variable, such as `complexVariable`, that may contain a complex number, one can use the Matlab built-in function `imag` to obtain its imaginary part by writing: `imag(complexVariable)`.

Since $f(x)$ is real we can simplify this expression as

$$f'(x) = \lim_{\tilde{h}\to 0} \frac{\mathrm{Im}\left(f(x+i\tilde{h})\right)}{\tilde{h}},$$

This gives us a method for computing the first derivative in the form:

$$f'(x) \approx \frac{\mathrm{Im}\left(f(x+i\tilde{h})\right)}{\tilde{h}}. \tag{2}$$

Implement this method (called method 2) for the above problem (the same function $f(x)$ and using the same values of x, p and q) using again the same choice of values for $\tilde{h}$ as h. Comment qualitatively on the results that you obtained. In particular compare with the corresponding results from the previous two methods.

At the end of the program file collect all the results and enter comment lines to display them in a table form. The table should have appropriate headings and a total of three columns. Column 1 should have the step sizes used. The relatively error obtained for the computed results for the first derivative for methods 1 and 2 should be displayed respectively in columns 2 and 3.

Describe and comment, if you can, of the lessons that you have learned in this assignment.