Consumption Inequality and Partial Insurance: Comment

By Edmund Crawley*

I revisit Blundell, Pistaferri and Preston (2008) paying particular attention to the time aggregated nature of income and consumption data. In 1960 Working noted that time aggregation of a random walk induces serial correlation in the first differences that is not present in the original series. Correcting for this correlation, I find the estimate for partial insurance to transitory shocks, originally estimated to be 0.05, is close to 0.24. This new estimate resolves the dissonance between Blundell, Pistaferri and Preston (2008) and the large literature that uses natural experiments to estimate the marginal propensity to consume out of transitory shocks.
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The extent to which households insure their consumption against permanent and transitory shocks to their income lies at the heart of many micro and macro economic questions. The methodology and estimates of Blundell, Pistaferri and Preston (2008) (henceforth BPP) have had significant influence in this literature. Indeed, the argument in Kaplan and Violante (2010) that "the BPP insurance coeffcients should become central in quantitative macroeconomics" has become widely accepted. However, their finding that households are fully insured against transitory income shocks conflicts with a parallel literature that, using natural experiments, finds large consumption responses to transitory income shocks.[1] This comment resolves the dissonance, showing that the time aggregated nature of income data leads to a large negative bias in the BPP methodology. Correction for this bias, I find the estimate for partial insurance to transitory shocks, originally estimated in BPP to be 0.05, is close to 0.24.

I. What is Time Aggregation?

In a short note in Econometrica, Working (1960) made the simple but important point that time aggregation can induce serial correlation that is not present in the original series. This fact was readily absorbed by the macroeconomic literature, where such time aggregated series are common[2] and a small literature has

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[1] A small sample of this literature includes Parker et al. (2013), Agarwal and Qian (2014), Sahm, Shapiro and Slemrod (2010). Consumers also answer that they have a high marginal propensity to consume when asked, see Fuster, Kaplan and Zafar (2018) and Jappelli and Pistaferri (2014). For an overview of the entire literature on consumption responses to income shocks, see Jappelli and Pistaferri (2010). Note the dissonance between BPP and the natural experiment literature is also addressed by Commault (2017), however this requires structural changes to the underlying model.

[2] For an example see Campbell and Mankiw (1989)

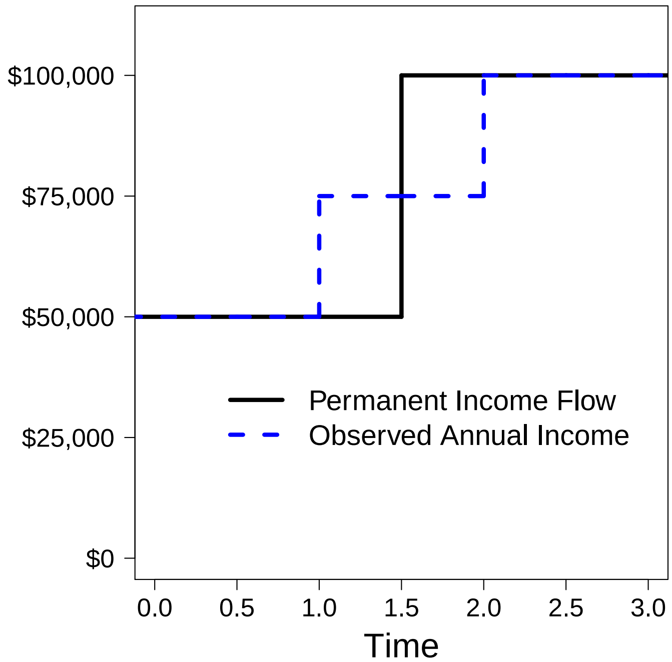


Figure 1. Income Flow and Observed Income

grown around how to account for time aggregation in various settings.[3] Recently, by studying the covariance structure of panel data, much progress has been made in understanding household income and consumption dynamics. However, this literature has not accounted for the serial correlation induced by the time aggregated nature of observed income and consumption data. This oversight can result in significant bias. This paper will focus on the implications of time aggregation for the methodology in Blundell, Pistaferri and Preston (2008), but it applies to a broad swath of the literature.[4]

Time aggregation occurs when a time series is observed at a lower frequency than the underlying data that it is generated by. For example, income is often observed at an annual frequency when it may in fact consist of paychecks arriving at a monthly, biweekly or irregular timetable. To transform income into an annual frequency we sum up all the income that was received by a household

[3] A sample of this literature includes Amemiya and Wu (1972), Weiss (1984) and Drost and Nijman (1993).

[4] Modern approaches that build on BPP, such as Arellano, Blundell and Bonhomme (2017), are likely to show a similar size bias. Other methods, such as Carroll and Samwick (1997), also have bias albeit of smaller size. See section III.C for intuition on why the methodology in BPP is particularly affected by time aggregation.

during the year. The key insight of Working (1960) is that even if there is no correlation between changes in income at the underlying frequency, the resulting time aggregated series will show positive autocorrelation. The intuition behind this can be seen in figure 1 showing the income process of a household that begins with an annual salary of \$50,000 and receives a permanent pay rise to \$100,000 mid-way through the second year. The solid line shows this jump in income flow occurring just once. The dotted line shows the income we actually observe in annual data. During the second year the household receives an annual \$50,000 salary for six months, followed by \$100,000 in the second six month, resulting in a reported income of \$75,000 for the entire year. The single shock to income therefore appears in the time aggregated data as two increases. In this way, an income change in one year is positively correlated with an income change in the following year, even if the underlying income process follows a random walk. Section II lays this out formally and shows that this autocorrelation tends to $\frac{1}{4}$ as the number of time subdivisions increases to infinity.

II. Time Aggregated Random Walk

I this section I formally prove that a time aggregated random walk is autocorrelated and show that this autocorrelation tends to $\frac{1}{4}$ as the number of time subdivisions increases to infinity. I will also introduce continuous time notation that will be used for the underlying model in section III.

A. The two sub-division case

I begin with the two-subdivision case. The underlying income process follows a random walk at discrete time periods $t \in \{0, 1, 2, 3, ...\}$:

$$y_t = \begin{cases} 0 & \text{if } t = 0 \\ y_{t-1} + \varepsilon_t & \text{otherwise} \end{cases}$$

where ε_t is i.i.d. and has variance σ^2. The time aggregated process is observed every two periods at $T \in \{2, 4, 6, ...\}$ and is equal to the sum of income over the two periods leading up to it:

$$y_T^{obs} = y_T + y_{T-1}$$

The observed income change is given by:

$$\begin{aligned} \Delta^2 y_T^{obs} &= y_T^{obs} - y_{T-2}^{obs} \\ &= \varepsilon_T + 2\varepsilon_{T-1} + \varepsilon_{T-2} \end{aligned}$$

This allows for easy calculation of the serial correlation:

$$\begin{aligned}
\mathrm{Cov}(\Delta^2 y_T^{obs}, \Delta^2 y_{T-2}^{obs}) &= \sigma^2 \\
\mathrm{Var}(\Delta^2 y_T^{obs}) &= \sigma^2 + 4\sigma^2 + \sigma^2 \\
&= 6\sigma^2 \\
\mathrm{Corr}(\Delta^2 y_T^{obs}, \Delta^2 y_{T-2}^{obs}) &= \frac{1}{6}
\end{aligned}$$

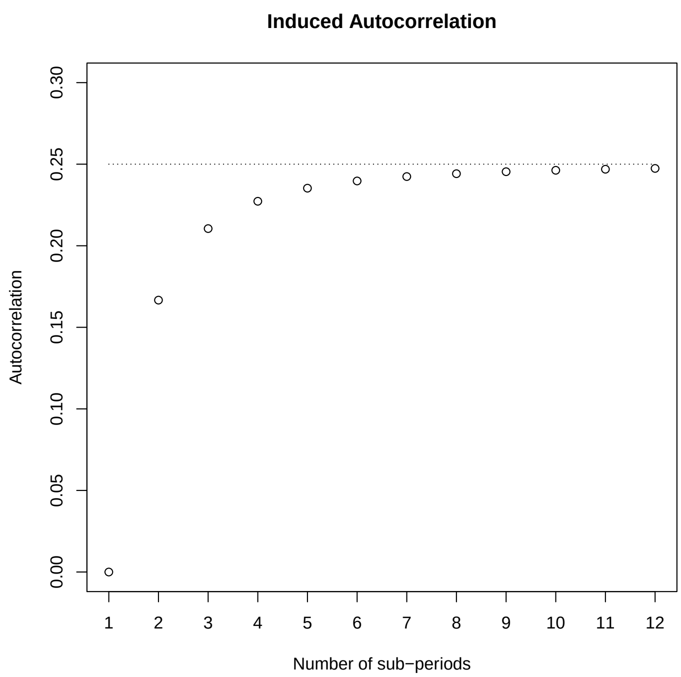


Figure 2. Induced Autocorrelation for different N

B. The N sub-division case

The two sub-division case easily extends to N sub-divisions. Using the same underlying income process, the observable income process is now aggregated over N periods:

$$y_T^{obs} = \sum_{t=T-N+1}^{T} y_t$$

So that the observed change in income is:

$$\begin{aligned}
\Delta^N y_T^{obs} &= \sum_{t=T-N+1}^{T} y_t - y_{t-N} \\
&= \varepsilon_T + \varepsilon_{T-1} + \quad \ldots \quad + \varepsilon_{T-N+2} + \varepsilon_{T-N+1} \\
&\qquad + \varepsilon_{T-1} + \varepsilon_{T-2} + \quad \ldots \quad + \varepsilon_{T-N+1} + \varepsilon_{T-N} \\
&\qquad\qquad + \varepsilon_{T-2} + \varepsilon_{T-3} + \quad \ldots \\
&\qquad\qquad\qquad \ldots \\
&\qquad\qquad\qquad\qquad + \varepsilon_{T-N+1} + \quad \ldots \quad + \varepsilon_{T-2N+2} \\
&= N\varepsilon_{T-N+1} + \sum_{i=1}^{N-1} i\Big(\varepsilon_{T-i+1} + \varepsilon_{T-2N+i+1}\Big)
\end{aligned}$$

We can now calculate the autocorrelation:

$$\begin{aligned}
\mathrm{Cov}(\Delta^N y_T^{obs}, \Delta^N y_{T-N}^{obs}) &= \sum_{i=1}^{N-1} i(N-i)\sigma^2 \\
&= \frac{N(N^2-1)}{6}\sigma^2 \\
\mathrm{Var}(\Delta^N y_T^{obs}) &= N^2\sigma^2 + 2\sum_{i=1}^{N-1} i^2\sigma^2 \\
&= \frac{N(2N^2+1)}{3}\sigma^2 \\
\mathrm{Corr}(\Delta^N y_T^{obs}, \Delta^N y_{T-N}^{obs}) &= \frac{N^2-1}{2(2N^2+1)} \to \frac{1}{4} \text{ as } N \to \infty
\end{aligned}$$

C. *The Continuous Time Case*

It will turn out to be significantly simpler to work with a model in which shocks can occur at any point in continuous time. Here I introduce some notation for such a model, and show that it gives a good approximation even if the actual underlying process is discrete (say quarterly or monthly).

The underlying income process will be modeled as a martingale process in continuous time, y_t, where for all $s_1 > s_2 > s_3 > s_4 > 0$:

$$\begin{aligned}
\mathrm{Var}(y_{s_1} - y_{s_2}) &= (s_1 - s_2)\sigma^2 \\
\mathrm{Cov}(y_{s_1} - y_{s_2}, y_{s_3} - y_{s_4}) &= 0 \\
y_s &= 0 \qquad \text{if } s < 0
\end{aligned}$$

The process has independent increment increments. A Brownian motion would satisfy these criteria, but there is no restriction that it is a continuous process (it may have jumps).[5] The observed income process is the sum of income over a year:

$$\begin{aligned}\bar{y}_T &= \int_{T-1}^{T} y_t dt \\ &= \int_{T-1}^{T}\int_{0}^{t} dy_s dt\end{aligned}$$

So that:

$$\begin{aligned}\Delta\bar{y}_T &= \int_{T-1}^{T}\int_{0}^{t} dy_s dt - \int_{T-2}^{T-1}\int_{0}^{t} dy_s dt \\ &= \int_{T-1}^{T}\int_{t-1}^{t} dy_s dt \\ &= \int_{T-1}^{T}(T-s)dy_s + \int_{T-2}^{T-1}(s-(T-2))dy_s\end{aligned}$$

The autocorrelation can now be calculated:

$$\begin{aligned}\mathrm{Cov}(\Delta\bar{y}_T, \Delta\bar{y}_{T-1}) &= \int_{T-2}^{T-1}(T-1-s)(s-(T-2))\sigma^2 dt \\ &= \frac{1}{6}\sigma^2 \\ \mathrm{Var}(\Delta\bar{y}_T) &= \int_{T-1}^{T}(T-s)^2\sigma^2 dt + \int_{T-2}^{T-1}(s-(T-2))^2\sigma^2 dt \\ &= \frac{2}{3}\sigma^2 \\ \mathrm{Corr}(\Delta\bar{y}_T, \Delta\bar{y}_{T-1}) &= \frac{1}{4}\end{aligned}$$

Which is unsurprisingly the same as the limit of the autocorrelation in the N sub-periods case.

Figure 2 shows how fast the N sub-period case converges towards the continuous time case. When $N = 1$ the time aggregated process is the same as the underlying random walk so the autocorrelation is zero. When income is quarterly ($N = 4$) the autocorrelation is 0.23 and is closely approximated by the continuous time model. With monthly ($N = 12$) or higher frequency for income shocks the discrete and continuous models are almost indistinguishable.

[5]Note that such a process will take both positive and negative values, and therefore may not be a good choice for an income process. In appendix A.A1, by looking at the limit of discrete time models with m sub-periods, I show that under certain assumptions the same results approximately hold when shocks are multiplicative rather than additive.

III. Time Aggregation in Blundell, Pistaferri and Preston (2008)

A. The Model in Discrete Time Without Time Aggregation

Here I briefly describe the method followed by Blundell, Pistaferri and Preston (2008). For more detail please refer to their original paper. The core of the model are their assumptions on the income and consumption processes. The model described here is a simplified version of the original in order to highlight the role played by time aggregation.[6]

Unexplained log income growth for household i follows the process:

$$\Delta y_{i,t} = \zeta_{i,t} + \Delta \nu_{i,t}$$

where $\zeta_{i,t}$ (the change in permanent income) and $\nu_{i,t}$ (transitory income) are each i.i.d. and independent of each other. The variance of permanent shocks ($\sigma^2_\zeta = \mathrm{Var}(\zeta_{i,t})$) and transitory shocks ($\sigma^2_\nu = \mathrm{Var}(\nu_{i,t})$) will be of interest. These variances can be identified from observable data by noting the following identities (where the household identifier i has been removed for clarity):

$$\begin{aligned} \sigma^2_\zeta &= \mathrm{Var}(\zeta_t) \\ &= \mathrm{Cov}(\Delta y_t, \Delta y_{t-1} + \Delta y_t + \Delta y_{t+1}) \quad (1) \\ \sigma^2_\nu &= \mathrm{Var}(\nu_t) \\ &= -\mathrm{Cov}(\Delta y_t, \Delta y_{t+1}) \quad (2) \end{aligned}$$

The unexplained change in log consumption is modeled as a random walk that moves in response to changes in both permanent income and transitory income:

$$\Delta c_{i,t} = \phi \zeta_{i,t} + \psi \nu_{i,t}$$

where ϕ and ψ are the *partial insurance* parameters for permanent and transitory shocks respectively. A value of zero implies full insurance (consumption does not respond at all to the income shock), while a value of one implies no insurance. These insurance parameters can be identified in the data from these identities:

$$\phi = \frac{\mathrm{Cov}(\Delta c_t, \Delta y_{t-1} + \Delta y_t + \Delta y_{t+1})}{\mathrm{Cov}(\Delta y_t, \Delta y_{t-1} + \Delta y_t + \Delta y_{t+1})} \quad (3)$$

$$\psi = \frac{\mathrm{Cov}(\Delta c_t, \Delta y_{t+1})}{\mathrm{Cov}(\Delta y_t, \Delta y_{t+1})} \quad (4)$$

The four equations 1, 2, 3 and 4 are the core of the BPP identification method-

[6] In this simplified model I assume insurance parameters are constant accross both time and households, that the transitory component of income has no persistence, and that there are no taste shocks. These elements are reintroduced in section III.D in which I show the quantitative effect of time aggregation.

ology. In the following section I show how this identification fails when time aggregation is accounted for.

B. The Model in Continuous Time with Time Aggregation

The model in this section will be the exact analog of the discrete time BPP model just described, but embedded in continuous time where shocks are spread uniformly throughout the year.[7] For the income process we will assume two underlying martingale processes (possibly with jumps), P_t and Q_t such that for all $s_1 > s_2 > s_3 > s_4 > 0$:

$$\begin{aligned} \mathrm{Var}(P_{s_1} - P_{s_2}) &= (s_1 - s_2)\sigma_P^2 \\ \mathrm{Cov}(P_{s_1} - P_{s_2}, P_{s_3} - P_{s_4}) &= 0 \\ P_s &= 0 \qquad \text{if } s < 0 \end{aligned}$$

and similarly for Q_t. Instantaneous income in a period dt is given by:[8]

$$dy_t = \Big(\int_0^t dP_s\Big)dt + dQ_t \tag{5}$$

that is they receive their permanent income ($P_t = \int_0^t dP_s$) flow multiplied by time dt in addition to a one-off transitory income dQ_t.

Keeping with the assumption that consumption is a random walk with insurance parameters ϕ and ψ, instantaneous consumption is given by

$$dc_t = \phi\Big(\int_0^t dP_s\Big)dt + \psi\Big(\int_0^t dQ_s\Big)dt \tag{6}$$

that is they consume a proportion ϕ of their permanent income and a proportion ψ of the cumulation of all the transitory income they have received in their lifetime.

The Panel Study of Income Dynamics (PSID) data, we observe the total income received over the previous calendar year:

$$y_T^{obs} = \int_{T-1}^{T} dy_t$$

BPP use data on food consumption to impute total annual consumption. The questionnaire asks about food consumption in a typical week, but unfortunately the timing of this 'typical week' is not clear. The questionnaire is usually given at the end of March in the following year. See Altonji and Siow (1987) and Hall and Mishkin (1982) for differing views. Here I will assume the 'typical week' occurs

[7]There is little formal evidence on the distribution of shocks throughout the year. While this assumption is unlikely to be strictly true, it is more reasonable that the implicit assumption of BPP that shocks all occur 1st January each year.

[8]A more formal treatment of how to relate this to the log income process is given in appendix A.A1

exactly at the end of the calendar year, so it measures a snapshot of consumption at time T

$$c_T^{obs} = \phi\Big(\int_0^t dP_s\Big) + \psi\Big(\int_0^t dQ_s\Big)$$

In appendix A.A4 I show that the timing of the 'typical' week can have a large effect on the results. This is an important drawback to using this method with the PSID data. In a forthcoming paper (Crawley and Kuchler (2018)) we use expenditure data imputed from Danish administrative records in which the timing of expenditure is very clearly defined.

The BPP method makes use of the changes in observable income and consumption:

$$\begin{aligned}\Delta y_T^{obs} &= \Big(\int_{T-2}^{T-1}(s-(T-2))dP_s + \int_{T-1}^{T}(T-s)dP_s\Big) \\ &\quad + \Big(\int_{T-1}^{T} dQ_t - \int_{T-2}^{T-1} dQ_t\Big) \end{aligned} \tag{7}$$

$$\Delta c_T^{obs} = \phi\int_{T-1}^{T} dP_s + \psi\int_{T-1}^{T} dQ_s \tag{8}$$

If we use the identification of permanent and transitory variances in equations 1 and 2 from the discrete time model we get:

$$\begin{aligned}\mathrm{Cov}(\Delta y_T^{obs}, \Delta y_{T-1}^{obs} + \Delta y_T^{obs} + \Delta y_{T+1}^{obs}) &= \sigma_P^2 \\ -\mathrm{Cov}(\Delta y_T^{obs}, \Delta y_{T+1}^{obs}) &= -\frac{1}{6}\sigma_P^2 + \sigma_Q^2 \neq \sigma_Q^2\end{aligned}$$

This shows that identification of the variance of permanent shocks, σ_P^2, is unbiased, while that of transitory shocks is biased down by $\frac{1}{6}\sigma_P^2$. Turning to the identification of ϕ and ψ in equations 3 and 4 we have:

$$\frac{\mathrm{Cov}(\Delta c_T^{obs}, \Delta y_{T-1}^{obs} + \Delta y_T^{obs} + \Delta y_{T+1}^{obs})}{\mathrm{Cov}(\Delta y_T^{obs}, \Delta y_{T-1}^{obs} + \Delta y_T^{obs} + \Delta y_{T+1}^{obs})} = \phi \tag{9}$$

$$\frac{\mathrm{Cov}(\Delta c_T^{obs}, \Delta y_{T+1}^{obs})}{\mathrm{Cov}(\Delta y_T^{obs}, \Delta y_{T+1}^{obs})} = \frac{-\phi\frac{1}{2}\sigma_P^2 + \psi\sigma_Q^2}{-\frac{1}{6}\sigma_P^2 + \sigma_Q^2} \neq \psi \tag{10}$$

Again identification of the permanent insurance coefficient, ϕ, is unbiased, but the transitory insurance coefficient bears little relation to the true value of ψ.

In section III.D I repeat the GMM exercise of BPP, using the same empirical moments, but with identification coming from the continuous time model with time aggregated income. The full set of identification equations, with the model extended to include time varying coefficients, transitory persistence and taste shocks, can be found in appendices A.A2 and A.A3.

C. Intution

While it is clear that time aggregation will introduce some bias, the size of the problem may be surprising. For example, if the household follows the permanent income hypothesis with values $\phi = 1$ and $\psi = 0$, and permanent and transitory income variances are close to equal, the BPP method would estimate ψ to be -0.6.

Some intuition can be gleamed from viewing the estimation of the transitory insurance parameter as an IV regression. The BPP methodology essentially uses the change in income in period $t+1$ as an instrument for the transitory income shock in period t. Figure 3 show why this works in discrete time: transitory shocks are negatively correlated with income changes the following period, while permanent shocks are uncorrelated with income changes in the following period. Thus income changes in period $t+1$ satisfy the exclusion restriction required to be a valid instrument for transitory shocks. However, with time aggregation permanent shocks are positively correlated with income changes in the following period. Therefore BPP method interprets negative permanent shocks as positive transitory shocks. As the negative permanent shocks likely lead to consumption decreasing, and the method associates this with positive transitory shocks, it results in significant downward bias for the estimate of partial transitory insurance.

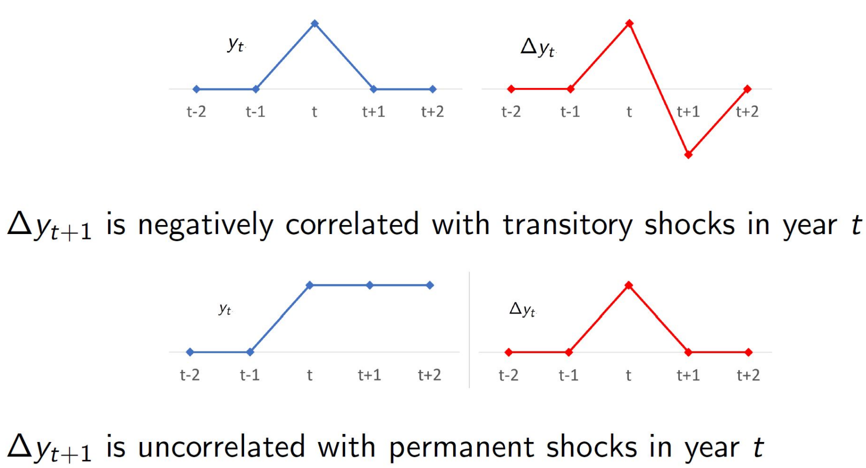


Figure 3. Key to Transitory Shock Identification in BPP

D. The Evidence

The columns labeled 'BPP' in table 1 replicate the columns from table 6 from the original BPP paper. Next to each of these columns I have reported the equivalent estimate from the continuous time model with time aggregation (and no persistence in the transitory shock). The most notable changes are to the partial insurance parameters ϕ and ψ. Given the results from section II.C, it should not be surprising that the coefficient for transitory shocks has changed significantly, from 0.05 to 0.24 in the whole sample. The fact that the coefficient for permanent shock insurance has also changed, from 0.65 to 0.34, is somewhat surprising given the theory suggested it should not change when transitory shocks are not persistent. When there is persistence in transitory shocks, the identification of ϕ in the two models in no longer the same. In section III.E I show how the estimate for ϕ is very sensitive to the degree of persistence in the discrete time model, which can explain why we observe a change in the estimate of ϕ. The estimates for the no college and college sub-samples also move in similar ways, but the qualitative result that college educated households have significantly more insurance against income shocks holds.

The whole sample permanent and transitory variances from table 6 are plotted in figure 4. The transitory shock variances are of similar magnitude and follow the same pattern of increasing in the mid-80's as the original estimates of BPP. The permanent shock variances are now slightly larger (although again this is sensitive to the degree of persistence in transitory shocks). The sharp decrease in 1988, followed by increase in 1989, seems strange. However, the standard errors at these points are relatively large (approx 0.013) such that this pattern may be a result of statistical noise. Note that the standard errors for the permanent variances are approximately twice as large in the time aggregated model compared to the original BPP method.

In appendix B I have reproduced all the estimation tables from the BPP paper, along with the time aggregated estimates. As with the college/no college cohort results, the insurance coefficient accross cohorts move in the same direction as they do in BPP's estimates, but they are quantitatively very different.

E. Persistence in the Transitory Shock

The baseline results for the time aggregated model reported in table 1 had no persistence in the transitory shock. In table 2 I report the insurance coefficients for three different ways of introducing persistence into the continuous time model, along with the estimates for the discrete time model with no persistence. The first method, called 'two-shot', models transitory income as a mass of income arriving at time t, followed by another mass of income, smaller than the first by a factor θ, arriving exactly one year later. This most closely mirrors the MA(1) model of transitory income used in the discrete time model. The second, called 'uniform', models transitory income as a constant flow of income starting at time

Table 1—Minimum-Distance Partial Insurance and Variance Estimates

		Whole Sample		No College		College	
		BPP	Time Agg.	BPP	Time Agg.	BPP	Time Agg.
$\sigma^2_{P,T}$	1979-1981	0.0103	0.0247	0.0068	0.0234	0.0101	0.0189
(Variance perm. shock)		(0.0034)	(0.0043)	(0.0037)	(0.0063)	(0.0053)	(0.0050)
	1982	0.0208	0.0358	0.0156	0.0290	0.0253	0.0455
		(0.0041)	(0.0071)	(0.0052)	(0.0099)	(0.0060)	(0.0099)
	1983	0.0301	0.0333	0.0318	0.0553	0.0234	0.0086
		(0.0057)	(0.0100)	(0.0074)	(0.0128)	(0.0090)	(0.0148)
	1984	0.0274	0.0292	0.0334	0.0232	0.0177	0.0361
		(0.0049)	(0.0114)	(0.0073)	(0.0131)	(0.0060)	(0.0161)
	1985	0.0295	0.0363	0.0287	0.0504	0.0208	0.0025
		(0.0096)	(0.0124)	(0.0073)	(0.0145)	(0.0152)	(0.0205)
	1986	0.0221	0.0327	0.0173	0.0247	0.0311	0.0597
		(0.0060)	(0.0136)	(0.0067)	(0.0172)	(0.0101)	(0.0202)
	1987	0.0289	0.0420	0.0202	0.0478	0.0354	0.0229
		(0.0063)	(0.0143)	(0.0073)	(0.0182)	(0.0098)	(0.0211)
	1988	0.0158	0.0082	0.0117	-0.0069	0.0183	0.0302
		(0.0069)	(0.0137)	(0.0079)	(0.0209)	(0.0110)	(0.0149)
	1989	0.0185	0.0531	0.0107	0.0639	0.0274	0.0414
		(0.0059)	(0.0129)	(0.0101)	(0.0214)	(0.0061)	(0.0149)
	1990-92	0.0135	0.0291	0.0093	0.0265	0.0217	0.0291
		(0.0042)	(0.0042)	(0.0045)	(0.0063)	(0.0065)	(0.0057)
$\sigma^2_{Q,T}$	1979	0.0379	0.0310	0.0465	0.0364	0.0301	0.0261
(Variance trans. shock)		(0.0059)	(0.0049)	(0.0096)	(0.0080)	(0.0056)	(0.0043)
	1980	0.0298	0.0240	0.0330	0.0247	0.0283	0.0238
		(0.0039)	(0.0033)	(0.0053)	(0.0046)	(0.0059)	(0.0047)
	1981	0.0300	0.0265	0.0363	0.0305	0.0253	0.0222
		(0.0035)	(0.0032)	(0.0053)	(0.0048)	(0.0046)	(0.0040)
	1982	0.0287	0.0280	0.0375	0.0332	0.0213	0.0237
		(0.0039)	(0.0034)	(0.0063)	(0.0057)	(0.0042)	(0.0036)
	1983	0.0262	0.0276	0.0371	0.0378	0.0185	0.0169
		(0.0037)	(0.0034)	(0.0063)	(0.0056)	(0.0037)	(0.0040)
	1984	0.0346	0.0350	0.0404	0.0388	0.0304	0.0315
		(0.0039)	(0.0038)	(0.0059)	(0.0058)	(0.0051)	(0.0046)
	1985	0.0450	0.0427	0.0355	0.0338	0.0496	0.0465
		(0.0075)	(0.0071)	(0.0056)	(0.0053)	(0.0130)	(0.0122)
	1986	0.0458	0.0404	0.0474	0.0373	0.0452	0.0464
		(0.0058)	(0.0055)	(0.0076)	(0.0068)	(0.0085)	(0.0084)
	1987	0.0461	0.0445	0.0520	0.0486	0.0421	0.0385
		(0.0054)	(0.0053)	(0.0082)	(0.0078)	(0.0071)	(0.0069)
	1988	0.0399	0.0327	0.0471	0.0360	0.0343	0.0313
		(0.0047)	(0.0044)	(0.0074)	(0.0072)	(0.0060)	(0.0055)
	1989	0.0378	0.0343	0.0539	0.0475	0.0219	0.0215
		(0.0067)	(0.0061)	(0.0126)	(0.0117)	(0.0051)	(0.0044)
	1990-92	0.0441	0.0359	0.0535	0.0408	0.0345	0.0322
		(0.0040)	(0.0027)	(0.0062)	(0.0047)	(0.0049)	(0.0032)
θ		0.1126	N/A	0.1260	N/A	0.1082	N/A
(Serial correl. trans. shock)		(0.0248)		(0.0319)		(0.0342)	
σ^2_{ξ}		0.0097	0.0122	0.0065	0.0114	0.0132	0.0146
(Variance unobs. slope heterog.)		(0.0041)	(0.0039)	(0.0079)	(0.0070)	(0.0040)	(0.0039)
ϕ		0.6456	0.3384	0.9484	0.4365	0.4180	0.2729
(Partial insurance perm. shock)		(0.0941)	(0.0471)	(0.1773)	(0.0738)	(0.0913)	(0.0603)
ψ		0.0501	0.2421	0.0724	0.2870	0.0260	0.1590
(Partial insurance trans. shock)		(0.0430)	(0.0431)	(0.0593)	(0.0616)	(0.0546)	(0.0504)

t and ending at time $t + \tau$ where τ is a measure of persistence. This can be thought of as a member of the household becoming unemployed for a length of time τ. The third, called 'linear decay', models transitory income as a flow of income starting at time t, the size of which decreases linearly until it reaches zero at time $t + \tau$. This tries to capture the fact that some transitory shocks have little persistence, while others are longer lived, so that on average income from a transitory shock will be decreasing over time. The identifying equations for each model can be found in appendix A.A3. The bottom two rows of table 2 report the estimated values of θ and τ in each model. The values of θ in the MA(1) model and the two-shot model are similar, with a fraction 0.1 of the first year's transitory income arriving in the following year. The uniform model estimates transitive periods of high or low income to last for somewhat less than half a year (0.43), while the linear decay model estimates them to last more than half a year (0.61). This makes sense as the 'persistence' associated with a uniform flow of income for a period τ is greater than that of a linearly decaying flow of income over a period τ.

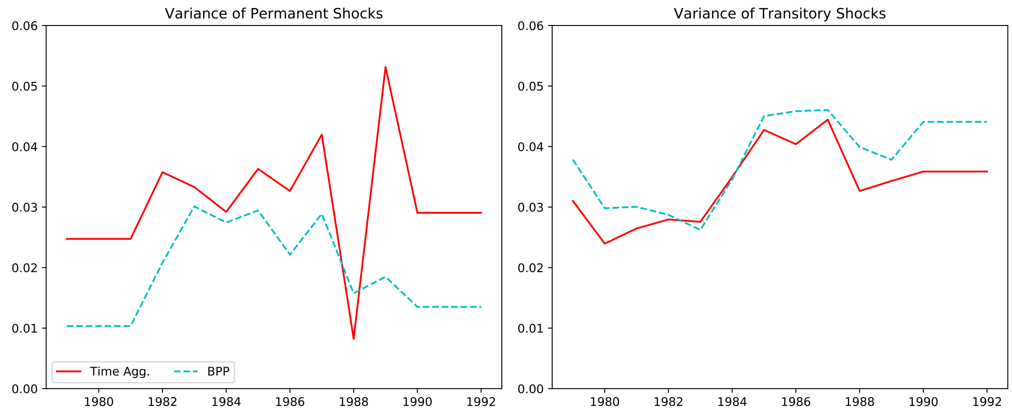
The first two columns of table 2 show that the degree of persistence in the original BPP model makes a big difference to the estimate of ϕ, while all of the time aggregation models show similar estimates for ψ. This suggests the difference we see in the estimates of ϕ between BPP original model and the time aggregated model may be driven, at least in part, by misspecification in the model of transitive income shocks. It is reassuring that, in contrast to the BPP model, the values of both ϕ and ψ are relatively robust to the exact specification of transitive persistence in the time aggregated model.

TABLE 2—MINIMUM-DISTANCE PARTIAL INSURANCE AND VARIANCE ESTIMATES

	BPP		Time Agg.			
Persistence Type:	None	MA(1)	None	Two-shot	Uniform	Linear Decay
ϕ	0.4692	0.6456	0.3384	0.4169	0.3287	0.3516
(Partial insurance perm. shock)	(0.0598)	(0.0941)	(0.0471)	(0.0680)	(0.0580)	(0.0627)
ψ	0.0503	0.0501	0.2421	0.2149	0.2510	0.2403
(Partial insurance tran. shock)	(0.0505)	(0.0430)	(0.0431)	(0.0386)	(0.0428)	(0.0417)
θ or τ	N/A	0.1126	N/A	0.1004	0.4320	0.6140
(Degree of Persistence)	(0.0000)	(0.0248)	(0.0000)	(0.0242)	(0.1008)	(0.1225)

IV. Conclusion

This paper highlights the importance of time aggregation when working with panel data, especially when analyzing the covariance matrix of income and consumption growth. It also resolves the dissonance between BPP's estimates of transitory income insurance and the natural experiment literature on marginal



Notes: BPP plots the variances from Table 6 of the original BPP paper. Time Agg. plots the equivalent variances corrected for the time aggregation problem.

Figure 4. Shock Variances in the 1980's

propensity to consume. Going forward I hope the methods used here to correct for the time aggregation problem can be useful for researchers, especially as more and more high quality panel datasets on income and consumption become available.

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Mathematical Appendix

A1. Continuous Time Model as Limit of Discrete Model with m Sub-periods

The identifying equations in the paper are calculated using a 'log' income process that does not directly align with any real-world concept of income. In the data we take logs on the sum of income over the entire year, but the process we use in the model informally aligns with log income over an instantaneous period dt. This is a problem as transitory income arrive as a point mass, making it difficult to interpret what the 'log' income process really represents. Here I show how the identifying equations can be derived as the limit of discrete time model with m sub-periods. I show that in the limit the variance of observed log income growth is the same as derived in the informal model (to a first order approximation). The rest of the identifying equations can be shown in the same way.

Let p_t for $t \in \mathbb{R}^+$ be a martingale process (possibly with jumps) with independent stationary increments and ν be such that $\mathbb{E}(e^{p_t - p_{t-1}}) = e^{\nu}$. Define permanent income as:

$$P_t = e^{p_t - t\nu}$$

Note that $\mathbb{E}\Big(\frac{P_{t+s}}{P_t}\Big) = 1$ for all $s \geq 0$. Define the variance of log permanent shocks to be:

$$\sigma_P^2 = \mathrm{Var}\Big(\log\big(\frac{P_{t+1}}{P_t}\big)\Big) = \mathrm{Var}(p_{t+1} - p_t)$$

We will assume changes in permanent income over a one year period are small enough such that:

$$\begin{aligned}
\mathrm{Var}\Big(\frac{P_{t+1}}{P_t}\Big) &= \mathrm{Var}\Big(\frac{P_{t+1} - P_t}{P_t}\Big) \\
&\approx \mathrm{Var}\Big(\log\big(1 + \frac{P_{t+1} - P_t}{P_t}\big)\Big) \\
&= \mathrm{Var}\Big(\log\big(\frac{P_{t+1}}{P_t}\big)\Big) = \sigma_P^2
\end{aligned}$$

For transitory shocks, we define an increasing stochastic process, Θ_t, which also has independent stationary increments. The increments in this process will define the transitory shocks. We set the expectation of increments, and the variance of the log of an increment of length 1 as:

$$\begin{aligned}
\mathbb{E}(\Theta_{t+s} - \Theta_t) &= s \\
\mathrm{Var}\Big(\log\big(\Theta_{t+1} - \Theta_t\big)\Big) &= \sigma_\Theta^2
\end{aligned}$$

Note that for this to be well defined, Θ_t must not only be increasing but also its increments are almost surely strictly positive (so that log of the increment is defined almost everywhere). Examples of such a stochastic process would be a gamma process, or a process that increases linearly with time (non-stochastically) but is also subject to positive shocks that arrive as a Poisson process. The stochastic part of this process has no Brownian motion component as this would necessarily lead to non-zero probability of a decreasing increment.

We will use these two processes to define an income process in discrete time with m intervals per period, and then look at the limit as $m \to \infty$. Define $\theta_{t,m}$ for $t \in \{\frac{1}{m}, \frac{2}{m}, \frac{3}{m}...\}$ to be the increment of Θ_t from $t - \frac{1}{m}$ to t:

$$\theta_{t,m} = \Theta_t - \Theta_{1-\frac{1}{m}}$$

Income is defined for each period $t \in \{\frac{1}{m}, \frac{2}{m}, \frac{3}{m}...\}$ as:

$$Y_{t,m} = P_t \theta_{t,m}$$

Therefore the underlying income process has a pure division into permanent and transitory shocks. Income is observed for $T \in \{1, 2, 3...\}$ as the sum of income in each of the subperiods:

$$\bar{Y}_{T,m} = \sum_{i=0}^{m-1} P_{T-\frac{i}{m}} \theta_{T-\frac{i}{m},m}$$

Note that for $m = 1$ this the same as the underlying income process, with permanent and transitory variance as defined above. We are interested in the log of observable income growth:

$$\begin{aligned}
\Delta \bar{y}_{T,m} &= \log \bar{Y}_{T,m} - \log \bar{Y}_{T-1,m} \\
&= \log\left(\sum_{i=0}^{m-1} P_{T-\frac{i}{m}} \theta_{T-\frac{i}{m},m}\right) - \log\left(\sum_{i=0}^{m-1} P_{T-1-\frac{i}{m}} \theta_{T-1-\frac{i}{m},m}\right) \\
&= \log\left(\sum_{i=0}^{m-1} \frac{P_{T-\frac{i}{m}}}{P_{T-1}} \theta_{T-\frac{i}{m},m}\right) - \log\left(\sum_{i=0}^{m-1} \frac{P_{T-1-\frac{i}{m}}}{P_{T-1}} \theta_{T-1-\frac{i}{m},m}\right)
\end{aligned}$$

As P_t and Θ_t have independent increments, the covariance between each of the two parts of the sum above is 0. Therefore:

$$\mathrm{Var}\Big(\Delta^1 \bar{y}_{T,m}\Big) = \mathrm{Var}\Bigg(\log\left(\sum_{i=0}^{m-1} \frac{P_{T-\frac{i}{m}}}{P_{T-1}} \theta_{T-\frac{i}{m},m}\right)\Bigg) + \mathrm{Var}\Bigg(\log\left(\sum_{i=0}^{m-1} \frac{P_{T-1-\frac{i}{m}}}{P_{T-1}} \theta_{T-1-\frac{i}{m},m}\right)\Bigg)$$

We will treat each of these two variances individually. We begin by looking at the variable:

$$\begin{aligned}
\log\left(\sum_{i=0}^{m-1}\frac{P_{T-\frac{i}{m}}}{P_{T-1}}\theta_{T-\frac{i}{m},m}\right) &= \log\left(\sum_{i=0}^{m-1}\theta_{T-\frac{i}{m},m}+\sum_{i=0}^{m-1}\Big(\frac{P_{T-\frac{i}{m}}}{P_{T-1}}-1\Big)\theta_{T-\frac{i}{m},m}\right)\\
&= \log\Big(\Theta_T-\Theta_{T-1}\Big)+\log\left(1+\sum_{i=0}^{m-1}\Big(\frac{P_{T-\frac{i}{m}}}{P_{T-1}}-1\Big)\frac{\theta_{T-\frac{i}{m},m}}{\sum_{l=0}^{m-1}\theta_{T-\frac{l}{m},m}}\right)\\
&\approx \log\Big(\Theta_T-\Theta_{T-1}\Big)+\sum_{i=0}^{m-1}\Big(\frac{P_{T-\frac{i}{m}}}{P_{T-1}}-1\Big)\frac{\theta_{T-\frac{i}{m},m}}{\sum_{l=0}^{m-1}\theta_{T-\frac{l}{m},m}}
\end{aligned}$$

Where the approximation comes from the fact that the shocks to permanent income in a one year period are small. Defining

$$\zeta_{t,m}=\frac{P_t}{P_{t-\frac{1}{m}}}$$

we have that

$$\begin{aligned}
\mathrm{Var}\Bigg(\log\Bigg(\sum_{i=0}^{m-1} \frac{P_{T-\frac{i}{m}}}{P_{T-1}} \theta_{T-\frac{i}{m},m} \Bigg)\Bigg) &\approx \sigma_\Theta^2 + \mathrm{Var}\Bigg(\sum_{i=0}^{m-1} \Big(\prod_{j=i}^{m-1} \zeta_{T-\frac{j}{m}} - 1 \Big) \frac{\theta_{T-\frac{i}{m},m}}{\sum_{l=0}^{m-1} \theta_{T-\frac{l}{m},m}} \Bigg) \\
&= \sigma_\Theta^2 + \mathbb{E}\Bigg[\sum_{i=0}^{m-1} \Big(\prod_{j=i}^{m-1} \zeta_{T-\frac{j}{m}} - 1 \Big) \frac{\theta_{T-\frac{i}{m},m}}{\sum_{l=0}^{m-1} \theta_{T-\frac{l}{m},m}} \Bigg]^2 \\
&= \sigma_\Theta^2 + \mathbb{E}\Bigg[\sum_{i=0}^{m-1} \Bigg(\Big(\prod_{j=i}^{m-1} \zeta_{T-\frac{j}{m}} - 1 \Big)^2 \Bigg(\frac{\theta_{T-\frac{i}{m},m}}{\sum_{l=0}^{m-1} \theta_{T-\frac{l}{m},m}} \Bigg)^2 \\
&\qquad + 2 \sum_{k<i} \Big(\prod_{j=k}^{m-1} \zeta_{T-\frac{j}{m}} - 1 \Big) \Big(\prod_{j=i}^{m-1} \zeta_{T-\frac{j}{m}} - 1 \Big) \frac{\theta_{T-\frac{k}{m},m} \theta_{T-\frac{i}{m},m}}{\Big(\sum_{l=0}^{m-1} \theta_{T-\frac{l}{m},m}\Big)^2} \Bigg) \Bigg] \\
&= \sigma_\Theta^2 + \frac{\sigma_P^2}{m} \sum_{i=0}^{m-1} \Bigg(i\mathbb{E}\Bigg(\frac{\theta_{T-\frac{i}{m},m}}{\sum_{l=0}^{m-1} \theta_{T-\frac{l}{m},m}} \Bigg)^2 + 2\sum_{k<i} (m-1-i) \mathbb{E}\Bigg(\frac{\theta_{T-\frac{k}{m},m} \theta_{T-\frac{i}{m},m}}{\Big(\sum_{l=0}^{m-1} \theta_{T-\frac{l}{m},m}\Big)^2} \Bigg) \Bigg) \\
&= \sigma_\Theta^2 + \frac{\sigma_P^2}{m} \frac{m(m-1)}{2} \mathbb{E}\Bigg(\frac{\theta_{T-\frac{i}{m},m}}{\sum_{l=0}^{m-1} \theta_{T-\frac{l}{m},m}} \Bigg)^2 \\
&\qquad + 2\frac{\sigma_P^2}{m} \sum_{i=1}^{m-1} i(m-1-i) \mathbb{E}\Bigg(\frac{\theta_{T-\frac{k}{m},m} \theta_{T-\frac{i}{m},m}}{\Big(\sum_{l=0}^{m-1} \theta_{T-\frac{l}{m},m}\Big)^2} \Bigg) \\
&= \sigma_\Theta^2 + \sigma_P^2 \frac{m-1}{2} \mathbb{E}\Bigg(\frac{\theta_{T-\frac{i}{m},m}}{\sum_{l=0}^{m-1} \theta_{T-\frac{l}{m},m}} \Bigg)^2 \\
&\qquad + \sigma_P^2 \Bigg[(m-1)^2 - \frac{(m-1)(2m-1)}{3} \Bigg] \mathbb{E}\Bigg(\frac{\theta_{T-\frac{k}{m},m} \theta_{T-\frac{i}{m},m}}{\Big(\sum_{l=0}^{m-1} \theta_{T-\frac{l}{m},m}\Big)^2} \Bigg)
\end{aligned}$$

Note that:

$$\begin{aligned}
1 &= \mathbb{E}\Bigg(\sum_{i=0}^{m-1} \frac{\theta_{T-\frac{i}{m},m}}{\sum_{l=0}^{m-1} \theta_{T-\frac{l}{m},m}} \Bigg)^2 \\
&= \sum_{i=0}^{m-1} \mathbb{E}\Bigg(\frac{\theta_{T-\frac{i}{m},m}}{\sum_{l=0}^{m-1} \theta_{T-\frac{l}{m},m}} \Bigg)^2 + 2\sum_{k<i} \mathbb{E}\Bigg(\frac{\theta_{T-\frac{k}{m},m} \theta_{T-\frac{i}{m},m}}{\Big(\sum_{l=0}^{m-1} \theta_{T-\frac{l}{m},m}\Big)^2} \Bigg)
\end{aligned}$$

So that

$$\mathbb{E}\left(\frac{\theta_{T-\frac{k}{m},m}\theta_{T-\frac{i}{m},m}}{\left(\sum_{l=0}^{m-1}\theta_{T-\frac{l}{m},m}\right)^2}\right) = \frac{1}{m(m-1)} - \frac{1}{m-1}\mathbb{E}\left(\frac{\theta_{T-\frac{i}{m},m}}{\sum_{l=0}^{m-1}\theta_{T-\frac{l}{m},m}}\right)^2$$

This gives:

$$\begin{aligned}
\mathrm{Var}\left(\log\left(\sum_{i=0}^{m-1}\frac{P_{T-\frac{i}{m}}}{P_{T-1}}\theta_{T-\frac{i}{m},m}\right)\right) &\approx \sigma_\Theta^2 + \mathrm{Var}\left(\sum_{i=0}^{m-1}\Big(\prod_{j=i}^{m-1}\zeta_{T-\frac{j}{m}} - 1\Big)\frac{\theta_{T-\frac{i}{m},m}}{\sum_{l=0}^{m-1}\theta_{T-\frac{l}{m},m}}\right)\\
&\approx \sigma_\Theta^2 + \frac{m-2}{3m}\sigma_P^2 + \frac{m+1}{6}\mathbb{E}\left(\frac{\theta_{T-\frac{i}{m},m}}{\sum_{l=0}^{m-1}\theta_{T-\frac{l}{m},m}}\right)^2\sigma_P^2\\
&\to \sigma_\Theta^2 + \frac{1}{3}\sigma_P^2 \qquad \text{as } m\to\infty
\end{aligned}$$

A very similar calculation shows that:

$$\mathrm{Var}\left(\log\left(\sum_{i=0}^{m-1}\frac{P_{T-1-\frac{i}{m}}}{P_{T-1}}\theta_{T-1-\frac{i}{m},m}\right)\right) \to \sigma_\Theta^2 + \frac{1}{3}\sigma_P^2 \qquad \text{as } m\to\infty$$

Putting these together gives:

$$\mathrm{Var}\Big(\Delta\bar{y}_{T,m}\Big) \to \frac{2}{3}\sigma_P^2 + 2\sigma_\Theta^2 \qquad \text{as } m\to\infty$$

This is the same as the identifying equation for $\mathrm{Var}\Big(\Delta y_T^{obs}\Big)$ (equation **??** from appendix A.A2, assuming shock variances are constant over time), and the rest of the identifying equations can be shown as the limit of the discrete time model in a similar way.

A2. *Identification in the Full Model*

In this appendix I calculate the full set of identifying equations for the non-stationary model with measurement error in consumption and taste shocks. Appendix A.A3 extends these to add persistence in the transitory shock. With classical measurement error on consumption the observables are now y_T^{obs} and c_T^{obs} where

$$\begin{aligned}
y_T^{obs} &= \int_{T-1}^{T} dy_t\\
c_T^{obs} &= \int_{T-1}^{T} dc_t + u_T
\end{aligned}$$

I am interested in the full set of observable covariances:

$$\begin{gathered}\mathrm{Cov}(\Delta y_T^{obs}, \Delta y_S^{obs})\\ \mathrm{Cov}(\Delta c_T^{obs}, \Delta c_S^{obs})\\ \mathrm{Cov}(\Delta c_T^{obs}, \Delta y_S^{obs})\end{gathered}$$

for all T and S in $\{1, 2, ...\}$. I further make the assumption that while the variance of the permanent and transitory shocks and insurance coefficients can change from year to year, within each year these are constant. The variance the permanent shock in year T is $\sigma^2_{P,T}$ and the transitory shock $\sigma^2_{Q,T}$. I use equation 7 for the change in observable log income, and extend equation 8 for the change is observable log consumption to include taste shocks (ξ_t) and measurement error:

$$\Delta c_T^{obs} = \phi \int_{T-1}^{T} dP_s + \psi \int_{T-1}^{T} dQ_s + \int_{T-1}^{T} d\xi_s + u_T - u_{T-1}$$

These two equations allow for the calculation of all the required identification equations:

$$\begin{aligned}
\mathrm{Var}(\Delta y_T^{obs}) &= \mathbb{E}\Big(\int_{T-2}^{T-1} (s-(T-2))^2 dP_s dP_s + \int_{T-1}^{T} (T-s)^2 dP_s dP_s\Big)\\
&\quad + \mathbb{E}\Big(\int_{T-1}^{T} dQ_t dQ_t + \int_{T-2}^{T-1} dQ_t dQ_t\Big)\\
&= \frac{1}{3}\sigma^2_{P,T} + \frac{1}{3}\sigma^2_{P,T-1} + \sigma^2_{Q,T} + \sigma^2_{Q,T-1}\\
\mathrm{Cov}(\Delta y_T^{obs}, \Delta y_{T+1}^{obs}) &= \mathbb{E}\Big(\int_{T-1}^{T} (T-s)(s-(T-1)) dP_s dP_s\Big) - \mathbb{E}\Big(\int_{T-1}^{T} dQ_t dQ_t\Big)\\
&= \frac{1}{6}\sigma^2_{P,T} - \sigma^2_{Q,T}\\
\mathrm{Cov}(\Delta y_T^{obs}, \Delta y_{T-1}^{obs}) &= \frac{1}{6}\sigma^2_{P,T-1} - \sigma^2_{Q,T-1}\\
\mathrm{Cov}(\Delta y_T^{obs}, \Delta y_S^{obs}) &= 0 \qquad \forall S, T \text{ such that } |S-T| > 1
\end{aligned}$$

$$\begin{aligned}
\mathrm{Var}\Delta c_T^{obs} &= \phi^2 \mathbb{E}\Big(\int_{T-1}^{T} dP_s dP_s\Big) + \psi^2 \mathbb{E}\Big(\int_{T-1}^{T} dQ_s dQ_s\Big) + \mathbb{E}\Big(\int_{T-1}^{T} d\xi_s d\xi_s\Big) + \sigma^2_{u,T} + \sigma^2_{u,T-1}\\
&= \phi^2 \sigma^2_{P,T} + \psi^2 \sigma^2_{Q,T} + \sigma^2_{\xi,T} + \sigma^2_{u,T} + \sigma^2_{u,T-1}\\
\mathrm{Cov}(\Delta c_T^{obs}, \Delta c_{T+1}^{obs}) &= -\sigma^2_{u,T}\\
\mathrm{Cov}(\Delta c_T^{obs}, \Delta c_{T-1}^{obs}) &= -\sigma^2_{u,T-1}\\
\mathrm{Cov}(\Delta c_T^{obs}, \Delta c_S^{obs}) &= 0 \qquad \forall S, T \text{ such that } |S-T| > 1
\end{aligned}$$

$$
\begin{aligned}
\mathrm{Cov}(\Delta c_T^{obs}, \Delta y_T^{obs}) &= \mathbb{E}\Big(\phi_T \int_{T-1}^{T} (T-s) dP_s dP_s + \psi_T \int_{T-1}^{T} dQ_s dQ_s\Big) \\
&= \frac{1}{2}\phi_T \sigma_{P,T}^2 + \psi_T \sigma_{Q,T}^2 \\
\mathrm{Cov}(\Delta c_T^{obs}, \Delta y_{T+1}^{obs}) &= \mathbb{E}\Big(\phi_T \int_{T-1}^{T} (s-(T-1)) dP_s dP_s - \psi_T \int_{T-1}^{T} dQ_s dQ_s\Big) \\
&= \frac{1}{2}\phi_T \sigma_{P,T}^2 - \psi_T \sigma_{Q,T}^2 \\
\mathrm{Cov}(\Delta c_T^{obs}, \Delta y_{T-1}^{obs}) &= 0 \\
\mathrm{Cov}(\Delta c_T^{obs}, \Delta y_S^{obs}) &= 0 \qquad \forall S, T \text{ such that } |S-T| > 1
\end{aligned}
$$

A3. Persistence in Transitory Shock

This appendix shows how to extend the time aggregated model to include persistence in the transitory shock.

Linear Decay Model

I will walk though the derivation of the moments for the linear decay model in detail and then just list the moments for the two-step and uniform models. In the linear decay model, a shock of size 1 will arrive with a flow intensity of $\frac{2}{\tau}$ and over the subsequent time τ a the total flow of transitory income will sum to 1. Instantaneous income can be written as:

$$
dy_t = \Big(\int_0^t dP_s\Big)dt + \Big(\int_{t-\tau}^{t} \frac{2}{\tau}(s-(t-\tau))dQ_s\Big)dt
$$

So that the observable change in income is given by:

$$
\begin{aligned}
\Delta y_T^{obs} &= \int_{T-1}^{T} y_t dt - \int_{T-2}^{T-1} y_t dt \\
&= \int_{T-1}^{T}\int_0^t dP_s dt - \int_{T-2}^{T-1}\int_0^t dP_s dt \\
&\quad + \int_{T-1}^{T}\int_{t-\tau}^{t} \frac{2}{\tau}(s-(t-\tau))dQ_s dt - \int_{T-2}^{T-1}\int_{t-\tau}^{t} \frac{2}{\tau}(s-(t-\tau))dQ_s dt \\
&= \Big(\int_{T-2}^{T-1}(s-(T-2))dP_s + \int_{T-1}^{T}(T-s)dP_s\Big) \\
&\quad + \frac{2}{\tau}\Big(\int_{T-\tau}^{T}\frac{1}{2}\Big(\tau - \frac{(s-(T-\tau))^2}{\tau}\Big)dQ_s + \int_{T-1}^{T-\tau}\frac{1}{2}\tau dQ_s + \int_{T-1-\tau}^{T-1}\frac{1}{2}\frac{(s-(T-1-\tau))^2}{\tau}dQ_s\Big) \\
&\quad - \frac{2}{\tau}\Big(\int_{T-1-\tau}^{T-1}\frac{1}{2}\Big(\tau - \frac{(s-(T-1-\tau))^2}{\tau}\Big)dQ_s + \int_{T-2}^{T-1-\tau}\frac{1}{2}\tau dQ_s \\
&\qquad\qquad + \int_{T-2-\tau}^{T-2}\frac{1}{2}\frac{(s-(T-2-\tau))^2}{\tau}dQ_s\Big) \\
&= \int_{T-2}^{T-1}(s-(T-2))dP_s + \int_{T-1}^{T}(T-s)dP_s \\
&\quad + \int_{T-\tau}^{T} 1 - \Big(\frac{s-(T-\tau)}{\tau}\Big)^2 dQ_s + \int_{T-1}^{T-\tau} dQ_s \\
&\quad - \int_{T-1-\tau}^{T-1} 1 - 2\Big(\frac{s-(T-1-\tau)}{\tau}\Big)^2 dQ_s \\
&\quad - \int_{T-2}^{T-1-\tau} dQ_s - \int_{T-2-\tau}^{T-2}\Big(\frac{s-(T-2-\tau)}{\tau}\Big)^2 dQ_s
\end{aligned}
$$

The full set of identification equations used in this model are:

$$\begin{aligned}
\mathrm{Var}(\Delta y_T^{obs}) &= \mathbb{E}\Big(\int_{T-2}^{T-1}(s-(T-2))^2 dP_s dP_s + \int_{T-1}^{T}(T-s)^2 dP_s dP_s\Big)\\
&\quad + \mathbb{E}\Big(\int_{T-\tau}^{T}\Big(1-\Big(\frac{s-(T-\tau)}{\tau}\Big)^2\Big)^2 dQ_s dQ_s + \int_{T-1}^{T-\tau} dQ_s Q_s\Big)\\
&\quad + \mathbb{E}\Big(\int_{T-1-\tau}^{T-1}\Big(1-2\Big(\frac{s-(T-1-\tau)}{\tau}\Big)^2\Big)^2 dQ_s dQ_s\Big)\\
&\quad + \mathbb{E}\Big(\int_{T-2}^{T-1-\tau} dQ_s dQ_s + \int_{T-2-\tau}^{T-2}\Big(\frac{s-(T-2-\tau)}{\tau}\Big)^4 dQ_s dQ_s\Big)\\
&= \frac{1}{3}\sigma^2_{P,T} + \frac{1}{3}\sigma^2_{P,T-1}\\
&\quad + \frac{8}{15}\tau\sigma^2_{Q,T} + (1-\tau)\sigma^2_{Q,T}\\
&\quad + \frac{7}{15}\tau\sigma^2_{Q,T-1}\\
&\quad + (1-\tau)\sigma^2_{Q,T-1} + \frac{1}{5}\tau\sigma^2_{Q,T-2}\\
&= \frac{1}{3}\sigma^2_{P,T} + \frac{1}{3}\sigma^2_{P,T-1} + (1-\frac{7}{15}\tau)\sigma^2_{Q,T} + (1-\frac{8}{15}\tau)\sigma^2_{Q,T-1} + \frac{1}{5}\tau\sigma^2_{Q,T-2}
\end{aligned}$$

$$\begin{aligned}
\mathrm{Cov}(\Delta y_T^{obs}, \Delta y_{T+1}^{obs}) &= \mathbb{E}\Big(\int_{T-1}^{T}(T-s)(s-(T-1)) dP_s dP_s\Big)\\
&\quad - \mathbb{E}\Big(\int_{T-\tau}^{T}\Big(1-\Big(\frac{s-(T-\tau)}{\tau}\Big)^2\Big)\Big(1-2\Big(\frac{s-(T-\tau)}{\tau}\Big)^2\Big) dQ_s dQ_s\Big)\\
&\quad - \mathbb{E}\Big(\int_{T-1}^{T-\tau} dQ_s Q_s\Big)\\
&\quad + \mathbb{E}\Big(\int_{T-1-\tau}^{T-1}\Big(1-2\Big(\frac{s-(T-1-\tau)}{\tau}\Big)^2\Big)\Big(\frac{s-(T-1-\tau)}{\tau}\Big)^2 dQ_s dQ_s\Big)\\
&= \frac{1}{6}\sigma^2_{P,T} - \frac{2}{5}\tau\sigma^2_{Q,T} - (1-\tau)\sigma^2_{Q,T} - \frac{1}{15}\sigma^2_{Q,T-1}
\end{aligned}$$

$$\begin{aligned}
\mathrm{Cov}(\Delta y_T^{obs}, \Delta y_{T+2}^{obs}) &= -\mathbb{E}\Big(\int_{T-\tau}^{T}\Big(1-\Big(\frac{s-(T-\tau)}{\tau}\Big)^2\Big)\Big(\frac{s-(T-\tau)}{\tau}\Big)^2 dQ_s dQ_s\Big)\\
&= -\frac{2}{15}\tau\sigma^2_{Q,T}
\end{aligned}$$

The above equations also work for $\mathrm{Cov}(\Delta y_T^{obs}, \Delta y_{T-1}^{obs})$ and $\mathrm{Cov}(\Delta y_T^{obs}, \Delta y_{T-2}^{obs})$ due to symmetry.

$$\mathrm{Cov}(\Delta y_T^{obs}, \Delta y_S^{obs}) = 0 \qquad \forall S, T \text{ such that } |S - T| > 2$$

The covariance matrix $\mathrm{Cov}(\Delta c_T^{obs}, \Delta c_S^{obs})$ is the same as in appendix A.A2.

$$\begin{aligned}\mathrm{Cov}(\Delta c_T^{obs}, \Delta y_T^{obs}) = \phi_T \mathbb{E}\Big(\int_{T-1}^{T} (T-s) dP_s dP_s \Big) \\ + \psi_T \mathbb{E}\Big(\int_{T-\tau}^{T} \Big(1 - \Big(\frac{s-(T-\tau)}{\tau}\Big)^2\Big) dQ_s dQ_s + \int_{T-1}^{T-\tau} dQ_s dQ_s \Big) \\ = \frac{1}{2}\phi_T \sigma_{P,T}^2 + \psi_T (1 - \frac{1}{3}\tau)\sigma_{Q,T}^2\end{aligned}$$

$$\begin{aligned}\mathrm{Cov}(\Delta c_T^{obs}, \Delta y_{T+1}^{obs}) = \phi_T \mathbb{E}\Big(\int_{T-1}^{T} (s-(T-1)) dP_s dP_s \Big) \\ - \psi_T \mathbb{E}\Big(\int_{T-\tau}^{T} \Big(1 - 2\Big(\frac{s-(T-\tau)}{\tau}\Big)^2\Big) dQ_s dQ_s + \int_{T-1}^{T-\tau} dQ_s dQ_s \Big) \\ = \frac{1}{2}\phi_T \sigma_{P,T}^2 - (1 - \frac{2}{3}\tau)\psi_T \sigma_{Q,T}^2\end{aligned}$$

$$\begin{aligned}\mathrm{Cov}(\Delta c_T^{obs}, \Delta y_{T+2}^{obs}) = -\psi_T \mathbb{E}\Big(\int_{T-\tau}^{T} \Big(\frac{s-(T-\tau)}{\tau}\Big)^2 dQ_s dQ_s \Big) \\ = -\frac{1}{5}\psi_T \tau \sigma_{Q,T}^2\end{aligned}$$

The Uniform Model

In the uniform model, transitory shocks consist of a constant flow of income that lasts for a time period τ. The full set of moments for this model are:

$$\mathrm{Var}(\Delta y_T^{obs}) = \frac{1}{3}\sigma_{P,T}^2 + \frac{1}{3}\sigma_{P,T-1}^2 + \big(1 - \frac{2}{3}\tau\big)\sigma_{Q,T}^2 + (1 - \frac{2}{3}\tau)\sigma_{Q,T-1}^2 + \frac{1}{3}\tau\sigma_{Q,T-2}^2$$

$$\mathrm{Cov}(\Delta y_T^{obs}, \Delta y_{T+1}^{obs}) = \frac{1}{6}\sigma_{P,T}^2 - \frac{1}{6}\tau\sigma_{Q,T}^2 - (1-\tau)\sigma_{Q,T}^2 - \frac{1}{15}\sigma_{Q,T-1}^2$$

$$\mathrm{Cov}(\Delta y_T^{obs}, \Delta y_{T+2}^{obs}) = -\frac{1}{6}\tau\sigma_{Q,T}^2$$

The above equations also work for $\text{Cov}(\Delta y_T^{obs}, \Delta y_{T-1}^{obs})$ and $\text{Cov}(\Delta y_T^{obs}, \Delta y_{T-2}^{obs})$ due to symmetry.

$$\text{Cov}(\Delta y_T^{obs}, \Delta y_S^{obs}) = 0 \qquad \forall S, T \text{ such that } |S - T| > 2$$

The covariance matrix $\text{Cov}(\Delta c_T^{obs}, \Delta c_S^{obs})$ is the same as in appendix A.A2.

$$\text{Cov}(\Delta c_T^{obs}, \Delta y_T^{obs}) = \frac{1}{2}\phi_T \sigma_{P,T}^2 + \psi_T(1 - \frac{1}{2}\tau)\sigma_{Q,T}^2$$

$$\text{Cov}(\Delta c_T^{obs}, \Delta y_{T+1}^{obs}) = \frac{1}{2}\phi_T \sigma_{P,T}^2 - (1 - \tau)\psi_T \sigma_{Q,T}^2$$

$$\text{Cov}(\Delta c_T^{obs}, \Delta y_{T+2}^{obs}) = -\frac{1}{2}\psi_T \tau \sigma_{Q,T}^2$$

The Two-shot Model

In the two shot model, transitory shocks consist of a mass of income arriving at time t followed exactly one year later by another mass of size θ of the first. The full set of moments for this model are:

$$\text{Var}(\Delta y_T^{obs}) = \frac{1}{3}\sigma_{P,T}^2 + \frac{1}{3}\sigma_{P,T-1}^2 + \sigma_{Q,T}^2 + (1 - \theta)^2 \sigma_{Q,T-1}^2 + \theta^2 \sigma_{Q,T-2}^2$$

$$\text{Cov}(\Delta y_T^{obs}, \Delta y_{T+1}^{obs}) = \frac{1}{6}\sigma_{P,T}^2 - \theta\sigma_{Q,T}^2 + \theta(1 - \theta)\sigma_{Q,T-1}^2$$

$$\text{Cov}(\Delta y_T^{obs}, \Delta y_{T+2}^{obs}) = -\theta\sigma_{Q,T}^2$$

The above equations also work for $\text{Cov}(\Delta y_T^{obs}, \Delta y_{T-1}^{obs})$ and $\text{Cov}(\Delta y_T^{obs}, \Delta y_{T-2}^{obs})$ due to symmetry.

$$\text{Cov}(\Delta y_T^{obs}, \Delta y_S^{obs}) = 0 \qquad \forall S, T \text{ such that } |S - T| > 2$$

The covariance matrix $\text{Cov}(\Delta c_T^{obs}, \Delta c_S^{obs})$ is the same as in appendix A.A2.

$$\text{Cov}(\Delta c_T^{obs}, \Delta y_T^{obs}) = \frac{1}{2}\phi_T \sigma_{P,T}^2 + \psi_T \sigma_{Q,T}^2$$

$$\text{Cov}(\Delta c_T^{obs}, \Delta y_{T+1}^{obs}) = \frac{1}{2}\phi_T \sigma_{P,T}^2 - (1 - \theta)\psi_T \sigma_{Q,T}^2$$

$$\text{Cov}(\Delta c_T^{obs}, \Delta y_{T+2}^{obs}) = -\psi_T \theta \sigma_{Q,T}^2$$

A4. Effect of Timing of Consumption in the PSID

BPP impute annual consumption from the question in the PSID asking about food consumption in a 'typical' week. Unfortunately it is not clear if this relates to an average of the previous calendar year, or some more recent week closer to when the interview was conducted (normally in March of the following year). In the paper I have assumed the answer gives a snapshot of consumption at the end of the calendar year. Here I show that assuming the 'typical' week is an average of consumption over the previous calendar year, the identifying equation from BPP for transitory insurance coefficient is different again, and still significantly biased. Under this new assumption the equation for the permanent insurance coefficient is unbiased as before:

$$\frac{\text{Cov}(\Delta c_T^{obs}, \Delta y_{T-1}^{obs} + \Delta y_T^{obs} + \Delta y_{T+1}^{obs})}{\text{Cov}(\Delta y_T^{obs}, \Delta y_{T-1}^{obs} + \Delta y_T^{obs} + \Delta y_{T+1}^{obs})} = \phi$$

While the identifying equation for the transitory insurance coefficient is:

$$\frac{\text{Cov}(\Delta c_T^{obs}, \Delta y_{T+1}^{obs})}{\text{Cov}(\Delta y_T^{obs}, \Delta y_{T+1}^{obs})} = \frac{-\phi\frac{1}{6}\sigma_P^2 + \frac{1}{2}\psi\sigma_Q^2}{-\frac{1}{6}\sigma_P^2 + \sigma_Q^2} \neq \psi$$

Under the permanent income hypothesis with $\phi = 1$, $\psi = 0$ and permanent and transitory variances approximately equal, the BPP estimate of ψ would be -0.2.

Other Tables from the BPP paper

Table B1 replicates Table 7 from the original BPP paper.

Table B1—Minimum-Distance Partial Insurance and Variance Estimates

Consumption: Income: Sample:	Nondurable net income baseline		Nondurable earnings only baseline		Nondurable male earnings baseline	
	BPP	Time Agg.	BPP	Time Agg.	BPP	Time Agg.
ϕ	0.6456	0.3384	0.3101	0.1761	0.2240	0.1232
(Partial insurance perm. shock)	(0.0941)	(0.0471)	(0.0572)	(0.0339)	(0.0492)	(0.0316)
ψ	0.0501	0.2421	0.0630	0.1625	0.0502	0.1181
(Partial insurance trans. shock)	(0.0430)	(0.0431)	(0.0306)	(0.0280)	(0.0293)	(0.0244)

Table B2 replicates Table 8 from the original BPP paper.

Table B2—Minimum-Distance Partial Insurance and Variance Estimates

Consumption: Income: Sample:	Nondurable net income baseline		Nondurable excluding help baseline		Nondurable net income low wealth	
	BPP	Time Agg.	BPP	Time Agg.	BPP	Time Agg.
ϕ	0.6456	0.3384	0.6244	0.3422	0.8339	0.8584
(Partial insurance perm. shock)	(0.0941)	(0.0471)	(0.0891)	(0.0466)	(0.2762)	(0.2498)
ψ	0.0501	0.2421	0.0469	0.2404	0.2853	0.4926
(Partial insurance trans. shock)	(0.0430)	(0.0431)	(0.0429)	(0.0427)	(0.1154)	(0.1050)

Consumption: Income: Sample:	Nondurable net income high wealth		Total net income low wealth		Nondurable net income baseline+SEO	
	BPP	Time Agg.	BPP	Time Agg.	BPP	Time Agg.
ϕ	0.6278	0.2691	1.0207	1.0580	0.7663	0.4630
(Partial insurance perm. shock)	(0.0998)	(0.0420)	(0.3426)	(0.3099)	(0.1028)	(0.0499)
ψ	0.0088	0.1838	0.3647	0.6185	0.1201	0.3232
(Partial insurance trans. shock)	(0.0409)	(0.0409)	(0.1477)	(0.1344)	(0.0352)	(0.0367)