Consumption Inequality and Partial Insurance: A Correction for Time Aggregation

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Abstract

The extent to which households are able to insure themselves against shocks to their income plays a key role in business cycle dynamics. In an influential paper making use of the covariance structure in PSID data Blundell, Pistaferri, and Preston (2008) (BPP) find almost full insurance against transitory shocks to income. This result diverges from the majority of the literature aimed at measuring the marginal propensity to consume that finds consumption responds sharply to transitory income shocks. In this paper I show that the discretization of time in the BPP model is far from a benign assumption. Allowing shocks to arrive in continuous time throughout each year changes the covariance structure of the model. I repeat the minimum distance estimation exercise matching moments from the equivalent continuous time model and find the pass through from income to consumption for transitory shocks increases from around 5% to 24%.

Keywords Consumption, Insurance

JEL codes D12, D31, D91, E21

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Thanks to...

1 Introduction

Blundell, Pistaferri, and Preston (2008) Working (1960) Kaplan and Violante (2010) "we argue that the BPP insurance coefficients should become central in quantitative macroeconomics"

1.1 Estimates of the Marginal Propensity to Consume

Show the BPP estimate is far away from consensus. Jappelli and Pistaferri (2010)

1.2 Applications of the BPP methodology

Violante, Kaplan, and Weidner (2014) Auclert (2015)

2 The Time Aggregation Problem

The results of this paper derive from the insight of Working (1960). He was the first to note that the use of averages in time series data can results in correlations that are not present in the original series. Intuition on this result can be obtained by thinking about a random walk in continuous time (such as a Weiner process W_t) where we are only able to observe a discrete series $\bar{W}_T$ for $T \in \{1, 2, 3...\}$ corresponding to the mean value of the series between $T-1$ and T. If a shock occurs halfway through a period, $\bar{W}_T$ will increase by half the value of the shock and $\bar{W}_{T+1}$ will be expected to be larger than $\bar{W}_T$ by half the value of the shock again. In this way an autocorrelation is induced in the time-aggregated series, even though the underlying series is a pure random walk. In continous time this correlation takes a value of $\frac{1}{6}$.

Once the problem is recognized it is immediately apparent that it may have important implications for the BPP methodology that relies on the covariance structure of income and consumption to identify the insurance coefficients. In this section I first describe the BPP methodology in discrete time as originally proposed. I then illustrate the time-aggregation problem by dividing time up into two sub-periods. Finally I write down an equivalent continuous time model and use it to derive a time-aggregate corrected set of moments to use in the minimum distance estimation.

2.1 BPP Moments in Discrete Time

Here I briefly describe the method followed by Blundell, Pistaferri, and Preston (2008). For a more detail please refer to their original paper. The core of the model are their assumptions on the income and consumption processes. They assume that (unexplained) income growth for household i follows the process:

$$\Delta y_{i,t} = \zeta_{i,t} + \Delta \nu_{i,t}$$

where time is discrete, the permanent shock component $\zeta_{i,t}$ is serially uncorrelated and the transitory shock component $\nu_{i,t}$ follows an MA(q) process. The (unexplained) change in log consumption is assumed to be:

$$\Delta c_{i,t} = \phi_{i,t}\zeta_{i,t} + \psi_{i,t}\nu_{i,t} + \xi_{i,t}$$

where $\phi_{i,t}$ and $\psi_{i,t}$ are the *partial insurance* parameters for permanent and transitory shocks respectively. $\xi_{i,t}$ represents unobserved taste shocks.

Identification of the parameters is achieved by matching the covariance structure of the model with that in the data. In the simple case in which $\nu_{i,t}$ is a random walk (q=0), and assuming stationarity, the two insurance parameters are identified by:

$$\phi = \frac{cov(\Delta c_t, \Delta y_{t-1} + \Delta y_t + \Delta y_{t+1})}{cov(\Delta y_t, \Delta y_{t-1} + \Delta y_t + \Delta y_{t+1})} \tag{1}$$

$$\psi = \frac{cov(\Delta c_t, \Delta y_{t+1})}{cov(\Delta y_t, \Delta y_{t+1})} \tag{2}$$

2.2 A Two Sub-Period Example

This section aims to give intuition to the problem that arises from the discrete time assumption made by BPP. I will calculate the moments 1 and 2 but under the assumption that the true process is semi-annual while we only observe the sum of income and consumption every year.

As in BPP the underlying processes for income and consumption growth are:

$$\Delta y_t = \zeta_t + \varepsilon_t - \varepsilon_{t-1}$$
$$\Delta c_t = \phi\zeta_t + \psi\varepsilon_t$$

where now t denotes 6-month periods. Summing up (assuming $y_0 = c_0 = 0$) gives:

$$y_t = \sum_{i=1}^{t} \zeta_i + \varepsilon_t$$
$$c_t = \phi\sum_{i=1}^{t} \zeta_i + \psi\sum_{i=1}^{t} \varepsilon_i$$

We observe y_T^{obs} and c_T^{obs} for $T \in \{2, 4, 6...\}$ (annually) where:

$$y_T^{obs} = y_T + y_{T-1}$$
$$c_T^{obs} = c_T + c_{T-1}$$

Our observed (annual) changes in income and consumption are therefore:

$$\Delta^2 y_T^{obs} \equiv y_T^{obs} - y_{T-2}^{obs} = y_T + y_{T-1} - y_{T-2} - y_{T-3}$$
$$\Delta^2 c_T^{obs} \equiv c_T^{obs} - c_{T-2}^{obs} = c_T + c_{T-1} - c_{T-2} - c_{T-3}$$

which implies:

$$\Delta^2 y_T^{obs} = \zeta_T + 2\zeta_{T-1} + \zeta_{T-2} + \varepsilon_T + \varepsilon_{T-1} - \varepsilon_{T-2} - \varepsilon_{T-3}$$
$$\Delta^2 c_T^{obs} = \phi(\zeta_T + 2\zeta_{T-1} + \zeta_{T-2}) + \psi(\varepsilon_T + 2\varepsilon_{T-1} + \varepsilon_{T-2})$$

I can use these to calculate the estimates retrieved by using the identifying moments 1 and 2 for the permanent and transitory insurance parameters:

$$\begin{aligned}
\mathbb{E}\hat{\phi} &= \frac{cov(\Delta^2 c_T^{obs}, \Delta^2 y_{T+2}^{obs} + \Delta^2 y_T^{obs} + \Delta^2 y_{T-2}^{obs})}{cov(\Delta^2 y_T^{obs}, \Delta^2 y_{T+2}^{obs} + \Delta^2 y_T^{obs} + \Delta^2 y_{T-2}^{obs})} \\
&= \frac{\phi(2var(\zeta_T) + 4var(\zeta_{T-1}) + 2var(\zeta_{T-2}))}{2var(\zeta_T) + 4var(\zeta_{T-1}) + 2var(\zeta_{T-2})} \\
&= \phi
\end{aligned} \tag{3}$$

$$\begin{aligned}
\mathbb{E}\hat{\psi} &= \frac{cov(\Delta^2 c_T^{obs}, \Delta^2 y_{T+2}^{obs})}{cov(\Delta^2 y_T^{obs}, \Delta^2 y_{T+2}^{obs})} \\
&= \frac{\phi var(\zeta_T) - \psi(var(\varepsilon_T) + 2var(\varepsilon_{T-1}))}{var(\zeta_T) - var(\varepsilon_T) - var(\varepsilon_{T-1})} \\
&= \frac{\phi var(\zeta_T) - 3\psi var(\varepsilon_T)}{var(\zeta_T) - 2var(\varepsilon_T)}
\end{aligned} \tag{4}$$

From equation 3 we can see the estimator for the permanent shock insurance parameter is still unbiased, but equation 4 shows this is not the case for the transitory shock insurance parameter. To take a simple example consider the permanent income hypothesis with $\phi = 1$ and $\psi = 0$. Using the annual BPP methodology with a semi-annual shock process would yield a correct estimate of 1 for ϕ, but the estimate for ψ would be $\frac{var(\zeta_T)}{var(\zeta_T) - 2var(\varepsilon_T)}$, a number that could take on almost any value depending on the relative variances of permanent and transitory shocks.

2.3 Continuous Time Moments in PSID Data

It should now be clear that assuming a discrete annual time period for the model income and consumption processes will not suffice. In order for the BPP method to give us a reasonable estimate of the transitory shock insurance parameter the model will need to account for the fact that shocks can arrive at any point during the year. In this section I derive the covariance structure of a model that is as close to the original BPP model but derived in continuous time.

In the stationary continuous time model we have two underlying martingale processes (possibly with jumps), P_t and Q_t such that for all $s_1 > s_2 > s_3 > s_4 > 0$:

$$var(P_{s_1} - P_{s_2}) = (s_1 - s_2)\sigma_P^2$$
$$cov(P_{s_1} - P_{s_2}, P_{s_3} - P_{s_4}) = 0$$

$$P_s = 0 \qquad \text{if } s < 0$$

and similarly for Q_t. Instantaneous income in a period dt is given by:

$$y_t dt = \Big(\int_0^t dP_s \Big) dt + dQ_t \tag{5}$$

so that P_t and Q_t are exactly analogous to the permanent and transitory shocks in the discrete time model (with $q = 0$ in the MA(q) transitory component - see appendix ***** to relax this assumption). Keeping with the assumption that consumption is a random walk with insurance parameters ϕ and ψ, instantaneous consumption is given by

$$c_t dt = \phi \Big(\int_0^t dP_s \Big) dt + \psi \Big(\int_0^t dQ_s \Big) dt + \Big(\int_0^t d\xi_s \Big) dt \tag{6}$$

where ξ_t is also a martingale process similar to P_t and Q_t and represents innovations in consumption (taste shocks) that are independent of those in income (c.f. ξ_t in BPP).

Equations 5 and 6 give the instantaneous income and consumption process in continuous time. To bring this model to the covariance matrix in the PSID data it is necessary to calculate the model implied covariance matrix, which requires paying attention to exactly what is being measured in the PSID data. For income, the survey asks about total income in the previous calendar year. In the model this is equivalent to the quantity $\bar{y}_T$ where

$$\bar{y}_T = \int_{T-1}^{T} y_t dt$$

for $T \in \{1, 2, 3...\}$. BPP use questions about food consumption to impute the level of total consumption. The questionnaire asks about food consumption in a typical week, but unfortunately the timing of this 'typical week' is less clear. The questionnaire is usually given at the end of March in the following year. See Altonji and Siow (1987) and Hall and Mishkin (1982) for differing views. Here I will assume the 'typical week' occurs exactly at the end of the calendar year, so it measures the snapshot of consumption c_T for $T \in \{1, 2, 3...\}$. In appendix ***** I show that the data does not fit an alternative assumption that the 'typical week' is an average for the previous calendar year.

The covariance structure is based on observable annual changes in income and consumption:

$$\begin{aligned}
\Delta \bar{y}_T &= \int_{T-1}^{T} y_t dt - \int_{T-2}^{T-1} y_t dt \\
&= \int_{T-1}^{T} \int_0^t dP_s dt - \int_{T-2}^{T-1} \int_0^t dP_s dt + \int_{T-1}^{T} dQ_t - \int_{T-2}^{T-1} dQ_t \\
&= \int_{T-1}^{T} \int_{t-1}^{t} dP_s dt + \int_{T-1}^{T} dQ_t - \int_{T-2}^{T-1} dQ_t \\
&= \Big(\int_{T-2}^{T-1} (s - (T-2)) dP_s + \int_{T-1}^{T} (T - s) dP_s \Big)
\end{aligned}$$

$$+\Big(\int_{T-1}^{T} dQ_t - \int_{T-2}^{T-1} dQ_t\Big) \tag{7}$$

$$\begin{aligned}\Delta c_T &= c_T - c_{T-1}\\ &= \phi\int_{T-1}^{T} dP_s + \psi\int_{T-1}^{T} dQ_s + \int_{T-1}^{T} d\xi_s \end{aligned}\tag{8}$$

The key moments of interest are the covariances of consumption change with differing lags of income change:

$$\begin{aligned} cov(\Delta c_T^*, \Delta\bar{y}_T) &= \mathbb{E}\Big(\phi\int_{T-1}^{T}(T-s)dP_s dP_s + \psi\int_{T-1}^{T} dQ_s dQ_s\Big)\\ &= \frac{1}{2}\phi\sigma_P^2 + \psi\sigma_Q^2 \end{aligned}\tag{9}$$

$$\begin{aligned} cov(\Delta c_T^*, \Delta\bar{y}_{T+1}) &= \mathbb{E}\Big(\phi\int_{T-1}^{T}(s-(T-1))dP_s dP_s - \psi\int_{T-1}^{T} dQ_s dQ_s\Big)\\ &= \frac{1}{2}\phi\sigma_P^2 - \psi\sigma_Q^2 \end{aligned}\tag{10}$$

$$cov(\Delta c_T^*, \Delta\bar{y}_{T-1}) = 0 \tag{11}$$

$$cov(\Delta c_T^*, \Delta\bar{y}_S) = 0 \qquad \forall S,T \text{ such that } |S-T|>1 \tag{12}$$

For the baseline model I relax the assumption of stationarity and also add measurement error in consumption making exactly analogous assumptions to those in the original BPP paper. The full set of moments are calculated in appendix *********.

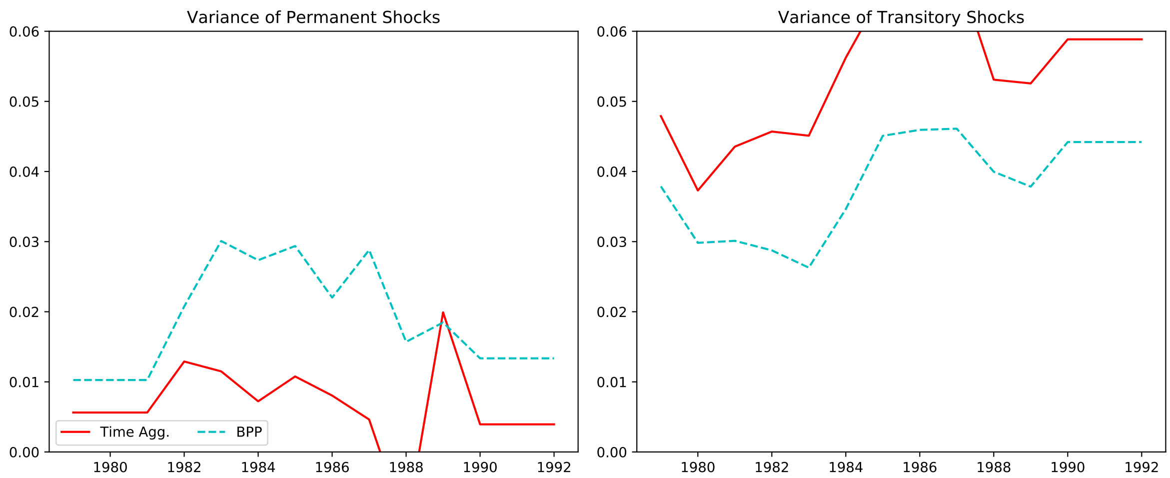
3 The Evidence

Table 1 replicates the first 3 columns from Table 6 in the original BPP paper, comparing the BPP numbers with those I calculated using the time aggregated model covariance moments.

Notes on Table 1:

- ψ takes on much higher values in the time aggregated case
- ϕ takes much lower values. Why is this? The theory says that time aggregation shouldn't make a difference for the permanent insurance estimate. Note that when I restrict to the stationary model the BPP estimate is much smaller (how could this be?)
- College educated have significantly lower ψ and *phi* suggesting much more insurance
- Note - the BPP estimates for the $\sigma_{Q,T}^2$ for the whole sample are transcribed wrong in the BPP paper (at least that is the only thing I can think to explain why my numbers don't match, everything else matches perfectly apart from σ_ξ^2 which is also off, but I match the standard errors...)

Figure 1 Shock Variances in the 1980's



Notes: BPP plots the variances from Table 6 of the original BPP paper. Time Agg. plots the equivalent variances corrected for the time aggregation problem.

- The fit of my model is significantly better than BPP - the objective function returns a number about 20% lower using my model

Figure 1 plots the implied shock variances through the 1980's using the original BPP method and the time aggregated model.

Notes on Figure 1:

- The BPP permanent numbers should match up with Figure 4 in their paper but don't. It would be great to understand where BPP get the numbers for their Figure 4 from. They say they are taking them from their Table 6, but that is not the case. (Similarly their Figure 6 should match the transitory shocks but doesn't)

- The pattern of increasing permanent shocks is not really visible now. What has changed?

- The transitory shocks look more or less the same with the two methods

- The time aggregated method suffers from larger standard errors. The identification of each year's variance is less accurate, I think because each year merges with its neighbor in the time aggregated case. The years 1988 and 1989 are an extreme case of this where clearly the 'truth' is closer to the average of the two. One possible solution is to limit the amount of variation in shock variance so that it can only change every two years.

Table 2 replicates Table 7 from the original BPP paper.

Table 3 replicates Table 8 from the original BPP paper.

Table 1 Minimum-Distance Partial Insurance and Variance Estimates

		Whole Sample		No College		College	
		BPP	Time Agg.	BPP	Time Agg.	BPP	Time Agg.
$\sigma^2_{P,T}$	1979-1981	0.0103	0.0056	0.0067	0.0018	0.0100	0.0065
(Variance perm. shock)		(0.0035)	(0.0033)	(0.0037)	(0.0019)	(0.0053)	(0.0050)
	1982	0.0208	0.0129	0.0154	0.0051	0.0253	0.0253
		(0.0041)	(0.0066)	(0.0052)	(0.0041)	(0.0060)	(0.0112)
	1983	0.0301	0.0115	0.0317	0.0118	0.0234	0.0006
		(0.0057)	(0.0066)	(0.0074)	(0.0080)	(0.0090)	(0.0093)
	1984	0.0274	0.0072	0.0332	0.0050	0.0176	0.0129
		(0.0049)	(0.0068)	(0.0073)	(0.0058)	(0.0060)	(0.0118)
	1985	0.0294	0.0108	0.0286	0.0118	0.0207	-0.0039
		(0.0096)	(0.0074)	(0.0073)	(0.0083)	(0.0152)	(0.0118)
	1986	0.0220	0.0080	0.0171	0.0007	0.0310	0.0290
		(0.0060)	(0.0088)	(0.0068)	(0.0073)	(0.0102)	(0.0185)
	1987	0.0288	0.0046	0.0201	0.0026	0.0354	-0.0071
		(0.0063)	(0.0137)	(0.0073)	(0.0167)	(0.0098)	(0.0200)
	1988	0.0157	-0.0108	0.0117	-0.0300	0.0183	0.0122
		(0.0069)	(0.0119)	(0.0079)	(0.0164)	(0.0110)	(0.0148)
	1989	0.0185	0.0199	0.0107	0.0163	0.0274	0.0207
		(0.0059)	(0.0111)	(0.0101)	(0.0114)	(0.0061)	(0.0138)
	1990-92	0.0134	0.0039	0.0091	0.0015	0.0216	0.0085
		(0.0042)	(0.0035)	(0.0045)	(0.0025)	(0.0066)	(0.0066)
$\sigma^2_{Q,T}$	1979	0.0379	0.0479	0.0465	0.0590	0.0301	0.0378
(Variance trans. shock)		(0.0059)	(0.0071)	(0.0096)	(0.0115)	(0.0057)	(0.0063)
	1980	0.0298	0.0373	0.0330	0.0371	0.0283	0.0380
		(0.0039)	(0.0062)	(0.0053)	(0.0091)	(0.0059)	(0.0085)
	1981	0.0301	0.0435	0.0364	0.0521	0.0253	0.0354
		(0.0035)	(0.0061)	(0.0053)	(0.0083)	(0.0046)	(0.0079)
	1982	0.0288	0.0457	0.0376	0.0582	0.0213	0.0363
		(0.0039)	(0.0066)	(0.0063)	(0.0103)	(0.0042)	(0.0064)
	1983	0.0263	0.0451	0.0372	0.0652	0.0186	0.0269
		(0.0037)	(0.0063)	(0.0063)	(0.0100)	(0.0037)	(0.0067)
	1984	0.0346	0.0562	0.0405	0.0680	0.0304	0.0477
		(0.0039)	(0.0072)	(0.0059)	(0.0105)	(0.0051)	(0.0083)
	1985	0.0451	0.0657	0.0356	0.0531	0.0496	0.0687
		(0.0075)	(0.0117)	(0.0056)	(0.0093)	(0.0130)	(0.0190)
	1986	0.0459	0.0677	0.0475	0.0688	0.0452	0.0729
		(0.0058)	(0.0101)	(0.0076)	(0.0131)	(0.0085)	(0.0140)
	1987	0.0461	0.0690	0.0520	0.0815	0.0421	0.0569
		(0.0054)	(0.0094)	(0.0082)	(0.0144)	(0.0071)	(0.0117)
	1988	0.0400	0.0531	0.0472	0.0615	0.0343	0.0491
		(0.0047)	(0.0083)	(0.0074)	(0.0132)	(0.0060)	(0.0096)
	1989	0.0378	0.0526	0.0539	0.0759	0.0219	0.0311
		(0.0067)	(0.0101)	(0.0126)	(0.0189)	(0.0051)	(0.0075)
	1990-92	0.0442	0.0589	0.0537	0.0680	0.0345	0.0514
		(0.0040)	(0.0055)	(0.0062)	(0.0080)	(0.0049)	(0.0068)
θ		0.1132	N/A	0.1269	N/A	0.1084	N/A
(Serial correl. trans. shock)		(0.0247)		(0.0317)		(0.0342)	
σ^2_ξ		0.0098	0.0079	0.0066	0.0030	0.0133	0.0126
(Variance unobs. slope heterog.)		(0.0041)	(0.0049)	(0.0080)	(0.0102)	(0.0040)	(0.0042)
ϕ		0.6427	0.9284	0.9471	1.7679	0.4159	0.5324
(Partial insurance perm. shock)		(0.0946)	(0.4118)	(0.1789)	(0.8784)	(0.0919)	(0.2318)
ψ		0.0534	0.2292	0.0767	0.2909	0.0279	0.1426
(Partial insurance trans. shock)		(0.0435)	(0.0387)	(0.0601)	(0.0550)	(0.0551)	(0.0455)

Table 2 Minimum-Distance Partial Insurance and Variance Estimates

Consumption:	Nondurable		Nondurable		Nondurable	
Income:	net income		earnings only		male earnings	
Sample:	baseline		baseline		baseline	
	BPP	Time Agg.	BPP	Time Agg.	BPP	Time Agg.
ϕ	0.6427	0.9284	0.3096	0.2921	0.2240	0.1341
(Partial insurance perm. shock)	(0.0946)	(0.4118)	(0.0574)	(0.1059)	(0.0493)	(0.0455)
ψ	0.0534	0.2292	0.0634	0.1513	0.0502	0.1157
(Partial insurance trans. shock)	(0.0435)	(0.0387)	(0.0309)	(0.0260)	(0.0294)	(0.0242)

Table 3 Minimum-Distance Partial Insurance and Variance Estimates

Consumption: Income: Sample:	Nondurable net income baseline		Nondurable excluding help baseline		Nondurable net income low wealth	
	BPP	Time Agg.	BPP	Time Agg.	BPP	Time Agg.
ϕ	0.6427	0.9284	0.6217	0.9791	0.8481	1.6267
(Partial insurance perm. shock)	(0.0946)	(0.4118)	(0.0896)	(0.4485)	(0.2846)	(0.6449)
ψ	0.0534	0.2292	0.0500	0.2257	0.2879	0.5145
(Partial insurance trans. shock)	(0.0435)	(0.0387)	(0.0434)	(0.0381)	(0.1143)	(0.0982)

Consumption: Income: Sample:	Nondurable net income high wealth		Total net income low wealth		Nondurable net income baseline+SEO	
	BPP	Time Agg.	BPP	Time Agg.	BPP	Time Agg.
ϕ	0.6261	1.5612	1.0338	2.0620	0.7658	2.3632
(Partial insurance perm. shock)	(0.1002)	(0.8444)	(0.3516)	(0.8079)	(0.1031)	(1.3201)
ψ	0.0109	0.1726	0.3684	0.6547	0.1211	0.3080
(Partial insurance trans. shock)	(0.0414)	(0.0367)	(0.1465)	(0.1254)	(0.0354)	(0.0334)

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A Full Model Implied Moments

In this appendix I calculate the full set of moments for the non-stationary model with measurement error in consumption. With classical measurement error on consumption the observables are now $\bar{y}_T$ and c_T^* where

$$\bar{y}_T = \int_{T-1}^{T} y_t dt$$
$$c_T^* = c_T + u_T$$

I am interested in the full set of observable covariances:

$$cov(\Delta\bar{y}_T, \Delta\bar{y}_S)$$
$$cov(\Delta c_T^*, \Delta c_S^*)$$
$$cov(\Delta c_T^*, \Delta\bar{y}_S)$$

for all T and S in $\{1, 2, ...\}$. I further make the assumption that while the variance of the permanent and transitory shocks and insurance coefficients can change from year to year, within each year these are constant. The variance the permanent shock in year T is $\sigma_{P,T}^2$ and the transitory shock $\sigma_{Q,T}^2$. Using equations 7 and 8 we get:

$$\begin{aligned}
var(\Delta\bar{y}_T) &= \mathbb{E}\Big(\int_{T-2}^{T-1}(s-(T-2))^2 dP_s dP_s + \int_{T-1}^{T}(T-s)^2 dP_s dP_s\Big) \\
&\quad + \mathbb{E}\Big(\int_{T-1}^{T} dQ_t dQ_t + \int_{T-2}^{T-1} dQ_t dQ_t\Big) \\
&= \frac{1}{3}\sigma_{P,T}^2 + \frac{1}{3}\sigma_{P,T-1}^2 + \sigma_{Q,T}^2 + \sigma_{Q,T-1}^2 && (13) \\
cov(\Delta\bar{y}_T, \Delta\bar{y}_{T+1}) &= \mathbb{E}\Big(\int_{T-1}^{T}(T-s)(s-(T-1))dP_s dP_s\Big) - \mathbb{E}\Big(\int_{T-1}^{T} dQ_t dQ_t\Big) \\
&= \frac{1}{6}\sigma_{P,T}^2 - \sigma_{Q,T}^2 && (14) \\
cov(\Delta\bar{y}_T, \Delta\bar{y}_{T-1}) &= \frac{1}{6}\sigma_{P,T-1}^2 - \sigma_{Q,T-1}^2 && (15) \\
cov(\Delta\bar{y}_T, \Delta\bar{y}_S) &= 0 \qquad \forall S, T \text{ such that } |S-T| > 1 && (16)
\end{aligned}$$

$$\begin{aligned}
var\Delta c_T^* &= \phi^2\mathbb{E}\Big(\int_{T-1}^{T} dP_s dP_s\Big) + \psi^2\mathbb{E}\Big(\int_{T-1}^{T} dQ_s dQ_s\Big) + \mathbb{E}\Big(\int_{T-1}^{T} d\xi_s d\xi_s\Big) + \sigma_{u,T}^2 + \sigma_{u,T-1}^2 \\
&= \phi^2\sigma_{P,T}^2 + \psi^2\sigma_{Q,T}^2 + \sigma_{\xi,T}^2 + \sigma_{u,T}^2 + \sigma_{u,T-1}^2 && (17) \\
cov(\Delta c_T^*, \Delta c_{T+1}^*) &= -\sigma_{u,T}^2 && (18) \\
cov(\Delta c_T^*, \Delta c_{T-1}^*) &= -\sigma_{u,T-1}^2 && (19) \\
cov(\Delta c_T^*, \Delta c_S^*) &= 0 \qquad \forall S, T \text{ such that } |S-T| > 1 && (20)
\end{aligned}$$

$$cov(\Delta c_T^*, \Delta\bar{y}_T) = \mathbb{E}\Big(\phi_T\int_{T-1}^{T}(T-s)dP_s dP_s + \psi_T\int_{T-1}^{T} dQ_s dQ_s\Big)$$

$$
\begin{aligned}
&= \frac{1}{2}\phi_T\sigma^2_{P,T} + \psi_T\sigma^2_{Q,T} && (21)\\
cov(\Delta c^*_T, \Delta\bar{y}_{T+1}) &= \mathbb{E}\Big(\phi_T \int_{T-1}^{T} (s-(T-1))dP_s dP_s - \psi_T \int_{T-1}^{T} dQ_s dQ_s\Big)\\
&= \frac{1}{2}\phi_T\sigma^2_{P,T} - \psi_T\sigma^2_{Q,T} && (22)\\
cov(\Delta c^*_T, \Delta\bar{y}_{T-1}) &= 0 && (23)\\
cov(\Delta c^*_T, \Delta\bar{y}_{S}) &= 0 \qquad \forall S,T \text{ such that } |S-T|>1 && (24)
\end{aligned}
$$

B Persistence in Transitory Shock

To introduce persistence in the transitory shock

$$
y_t dt = \Big(\int_0^t dP_s\Big)dt + \Big(\int_{t-\tau}^{t} \frac{1}{\tau}dQ_s\Big)dt
$$

So that the observable change in income is given by:

$$
\begin{aligned}
\Delta\bar{y}_T &= \int_{T-1}^{T} y_t dt - \int_{T-2}^{T-1} y_t dt\\
&= \int_{T-1}^{T}\int_0^t dP_s dt - \int_{T-2}^{T-1}\int_0^t dP_s dt\\
&\quad + \int_{T-1}^{T}\int_{t-\tau}^{t} \frac{1}{\tau} dQ_s dt - \int_{T-2}^{T-1}\int_{t-\tau}^{t}\frac{1}{\tau}dQ_s dt\\
&= \Big(\int_{T-2}^{T-1}(s-(T-2))dP_s + \int_{T-1}^{T}(T-s)dP_s\Big)\\
&\quad + \frac{1}{\tau}\Big(\int_{T-\tau}^{T}(T-s)dQ_s \quad + \int_{T-1}^{T-\tau}\tau dQ_s \quad + \int_{T-1-\tau}^{T-1}(s-(T-1-\tau))dQ_s\Big) && (25)\\
&\quad - \frac{1}{\tau}\Big(\int_{T-1-\tau}^{T-1}(T-1-s)dQ_s + \int_{T-2}^{T-1-\tau}\tau dQ_s + \int_{T-2-\tau}^{T-2}(s-(T-2-\tau))dQ_s\Big)\\
&= \Big(\int_{T-2}^{T-1}(s-(T-2))dP_s + \int_{T-1}^{T}(T-s)dP_s\Big)\\
&\quad + \frac{1}{\tau}\Big(\int_{T-\tau}^{T}(T-s)dQ_s + \int_{T-1}^{T-\tau}\tau dQ_s\Big)\\
&\qquad + \frac{1}{\tau}\int_{T-1-\tau}^{T-1}(\tau - 2(s-(T-1-\tau))dQ_s\\
&\quad - \frac{1}{\tau}\Big(\int_{T-2}^{T-1-\tau}\tau dQ_s + \int_{T-2-\tau}^{T-2}(s-(T-2-\tau))dQ_s\Big) && (26)
\end{aligned}
$$

The full set of moments for this model are:

$$\begin{aligned}
var(\Delta\bar{y}_T) = \mathbb{E}\Big(&\int_{T-2}^{T-1}(s-(T-2))^2 dP_s dP_s + \int_{T-1}^{T}(T-s)^2 dP_s dP_s\Big)\\
&+ \frac{1}{\tau^2}\mathbb{E}\Big(\int_{T-\tau}^{T}(T-s)^2 dQ_s dQ_s + \int_{T-1}^{T-\tau}\tau^2 dQ_s dQ_s\Big)\\
&\quad + \frac{1}{\tau^2}\int_{T-1-\tau}^{T-1}(\tau-2(s-(T-1-\tau))^2 dQ_s dQ_s\\
&+ \frac{1}{\tau^2}\mathbb{E}\Big(\int_{T-2}^{T-1-\tau}\tau^2 dQ_s dQ_s + \int_{T-2-\tau}^{T-2}(s-(T-2-\tau))^2 dQ_s dQ_s\Big)\\
= &\frac{1}{3}\sigma^2_{P,T} + \frac{1}{3}\sigma^2_{P,T-1}\\
&+ \frac{1}{3}\tau\sigma^2_{Q,T} + (1-\tau)\sigma^2_{Q,T}\\
&\quad + \frac{1}{3}\tau\sigma^2_{Q,T-1}\\
&+ (1-\tau)\sigma^2_{Q,T-1} + \frac{1}{3}\tau\sigma^2_{Q,T-2}
\end{aligned} \tag{27}$$

$$\begin{aligned}
cov(\Delta\bar{y}_T, \Delta\bar{y}_{T+1}) = \mathbb{E}\Big(&\int_{T-1}^{T}(T-s)(s-(T-1))dP_s dP_s\Big)\\
&+ \mathbb{E}\Big(\frac{1}{\tau^2}\int_{T-\tau}^{T}(\tau-2(s-(T-\tau))(T-s)dQ_s dQ_s\Big)\\
&- \mathbb{E}\Big(\frac{1}{\tau^2}\int_{T-2}^{T-1-\tau}\tau^2 dQ_s dQ_s\Big)\\
&- \mathbb{E}\Big(\frac{1}{\tau^2}\int_{T-1-\tau}^{T-1}(\tau-2(s-(T-1-\tau)))(s-(T-1-\tau))dQ_s dQ_s\Big)\\
= \frac{1}{6}&\sigma^2_{P,T} + \frac{1}{6}\tau\sigma^2_{Q,T} - (1-\tau)\sigma^2_{Q,T-1} - \frac{1}{6}\tau\sigma^2_{Q,T-1}
\end{aligned} \tag{28}$$

$$\begin{aligned}
cov(\Delta\bar{y}_T, \Delta\bar{y}_{T+2}) &= -\mathbb{E}\Big(\frac{1}{\tau^2}\int_{T-\tau}^{T}(T-s)(s-(T-\tau))dQ_s dQ_s\Big)\\
&= -\frac{1}{6}\tau\sigma^2_{Q,T}
\end{aligned} \tag{29}$$

The above equations also work for $cov(\Delta\bar{y}_T, \Delta\bar{y}_{T-1})$ and $cov(\Delta\bar{y}_T, \Delta\bar{y}_{T-2})$ due to symmetry.

$$cov(\Delta\bar{y}_T, \Delta\bar{y}_S) = 0 \qquad \forall S, T \text{ such that } |S-T| > 2 \tag{30}$$

The covariance matrix $cov(\Delta c^*_T, \Delta c^*_S)$ is the same as before.

$$cov(\Delta c^*_T, \Delta\bar{y}_T) = \phi_T\mathbb{E}\Big(\int_{T-1}^{T}(T-s)dP_s dP_s\Big)$$

$$
\begin{aligned}
&+ \psi_T \mathbb{E}\Big(\frac{1}{\tau}\int_{T-\tau}^{T}(T-s)dQ_s dQ_s + \frac{1}{\tau}\int_{T-1}^{T-\tau}\tau dQ_s dQ_s\Big) \\
&= \frac{1}{2}\phi_T \sigma_{P,T}^2 + \psi_T \sigma_{Q,T}^2 (1-\frac{1}{2}\tau) \qquad (31)
\end{aligned}
$$

$$
\begin{aligned}
cov(\Delta c_T^*, \Delta \bar{y}_{T+1}) &= \phi_T \mathbb{E}\Big(\int_{T-1}^{T}(s-(T-1))dP_s dP_s\Big) \\
&\psi_T \mathbb{E}\Big(\frac{1}{\tau}\int_{T-\tau}^{T}(\tau - 2(s-(T-\tau)))dQ_s dQ_s - \frac{1}{\tau}\int_{T-1}^{T-\tau}\tau dQ_s dQ_s\Big) \\
&= \frac{1}{2}\phi_T \sigma_{P,T}^2 - (1-\tau)\psi_T \sigma_{Q,T}^2 \qquad (32)
\end{aligned}
$$

$$
\begin{aligned}
cov(\Delta c_T^*, \Delta \bar{y}_{T+2}) &= -\psi_T \mathbb{E}\Big(\frac{1}{\tau}\int_{T-\tau}^{T}(s-(T-\tau))dQ_s dQ_s\Big) \\
&= -\psi_T \frac{1}{2}\tau \sigma_{Q,T}^2 \qquad (33)
\end{aligned}
$$