1 More General Consumption Processes

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2 Setup

Assume now that:

$$c_t dt = \int_{-\infty}^{t} f(t-s) dQ_s dt$$

(this is a generalization of $f(t-s) = \phi$, the random walk case from BPP).

We have

$$\begin{aligned}
\Delta \bar{c}_T &= \int_{T-1}^{T} \int_{-\infty}^{t} f(t-s) dQ_s dt - \int_{T-2}^{T-1} \int_{-\infty}^{t} f(t-s) dQ_s dt \\
&= \int_{T-1}^{T} \Big(\int_{-\infty}^{T-2} f(t-s) dQ_s + \int_{T-2}^{T-1} f(t-s) dQ_s + \int_{T-1}^{t} f(t-s) dQ_s \Big) dt \\
&\quad - \int_{T-2}^{T-1} \Big(\int_{-\infty}^{T-2} f(t-s) dQ_s + \int_{T-1}^{t} f(t-s) dQ_s \Big) dt \\
&= \int_{-\infty}^{T-2} \Big(\int_{T-1}^{T} f(t-s) dt - \int_{T-2}^{T-1} f(t-s) dt \Big) dQ_s \\
&\quad + \int_{T-2}^{T-1} \Big(\int_{T-1}^{T} f(t-s) dt - \int_{s}^{T-1} f(t-s) dt \Big) dQ_s \\
&\quad + \int_{T-1}^{T} \Big(\int_{s}^{T} f(t-s) dt \Big) dQ_s
\end{aligned}$$

3 Examples

3.1 Example 1: Exponential Decay

The first example will assume exponential decay of consumption, with households eventually using up all their money

$$f(t-s) = \phi e^{-\phi(t-s)}$$

This implies an annual MPC of

$$\begin{aligned}
\text{MPC} &= \int_0^1 \phi e^{-\phi t} dt \\
&= \Big[-e^{-\phi t} \Big]_0^1 \\
&= 1 - e^{-\phi}
\end{aligned}$$

Similarly quarterly MPC is $1 - e^{-0.25\phi}$. We have:

$$\begin{aligned}
\Delta \bar{c}_T = &\int_{-\infty}^{T-2} \Big(e^{-\phi(T-s)}(e^{\phi}-1) - e^{-\phi(T-1-s)}(e^{\phi}-1) \Big) dQ_s \\
&+ \int_{T-2}^{T-1} \Big(e^{-\phi(T-s)}(e^{\phi}-1) - 1 + e^{-\phi(T-1-s)} \Big) dQ_s \\
&+ \int_{T-1}^{T} \Big(1 - e^{-\phi(T-s)} \Big) dQ_s
\end{aligned}$$

$$\begin{aligned}
\Delta \bar{c}_T = &\int_{-\infty}^{T-2} \Big(-e^{-\phi(T-s)}(e^{\phi}-1)^2 \Big) dQ_s \\
&+ \int_{T-2}^{T-1} \Big(e^{-\phi(T-s)}(2e^{\phi}-1) - 1 \Big) dQ_s \\
&+ \int_{T-1}^{T} \Big(1 - e^{-\phi(T-s)} \Big) dQ_s
\end{aligned}$$

3.2 Example 2: Splurge, then constant

This example has two parameters and will be identified by both $cov(\Delta\bar{c}_t, \Delta\bar{y}_{t+1})$ and $cov(\Delta\bar{c}_t, \Delta\bar{y}_{t-1})$. It is equivalent to an initial 'splurge' followed by a fixed increase in consumption. It can be mapped to a model with durable expenditure where there is an initial large sum spent on durable goods. Formally:

$$f(t-s) = \psi_1 \delta_0(t-s) + \psi_2 \mathbb{1}_{t-s>0}$$

where δ_0 is the dirac delta function.

$$\begin{aligned}
\Delta \bar{c}_T = &\psi_1 \Big(\int_{T-1}^{T} dQ_s - \int_{T-2}^{T-1} dQ_s \Big) \\
&+ \psi_2 \Big(\int_{T-2}^{T-1} (s-(T-2)) dQ_s + \int_{T-1}^{T} (T-s) dQ_s \Big)
\end{aligned}$$

$$\begin{aligned}
\Delta \bar{c}_T = &\int_{T-1}^{T} \big(\psi_1 + \psi_2 (T-s) \big) dQ_s \\
&+ \int_{T-2}^{T-1} \big(\psi_2 (s-(T-2)) - \psi_1 \big) dQ_s
\end{aligned}$$