In Search of Lost Time Aggregation

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In 1960, Working noted that time aggregation of a random walk induces serial correlation in the first difference that is not present in the original series. This important contribution has been overlooked in a recent literature analyzing income and consumption in panel data. I examine Blundell, Pistaferri and Preston (2008) as an important example for which time aggregation has quantitatively large effects. Using new techniques to correct for the problem, I find the estimate for the partial insurance to transitory shocks, originally estimated to be 0.05, increases to 0.24. This larger estimate resolves the dissonance between the low partial consumption insurance estimates of Blundell, Pistaferri and Preston (2008) and the high marginal propensities to consume found in the natural experiment literature.

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In a short note in Econometrica, Working (1960) made the simple but important point that time aggregation can induce serial correlation that is not present in the original series. This fact was readily absorbed by the macroeconomic literature, where time aggregated series are common[1] and a small literature has grown around how to account for time aggregation in various settings.[2]

However, the effect of time aggregation has been overlooked in much of the literature studying the covariance structure of household income and consumption dynamics.[3] This oversight can result in significant bias. I examine Blundell, Pistaferri and Preston (2008) (henceforth BPP) not only as a way to demonstrate new techniques to overcome the bias, but also because the consumption responses to transitory and permanent income shocks are of significant economic interest in themselves. Indeed, Kaplan and Violante (2010) argue that "the BPP insurance coefficients should become central in quantitative macroeconomics". Using the same Panel Study of Income Dynmics (PSID) data as in BPP, I update their underlying model to account for time aggregation. I find the estimate for partial insurance to transitory shocks, originally estimated in BPP to be 0.05, to be 0.24 when time aggregation is accounted for. This new estimate resolves the dissonance between BPP's "full insurance of transitory shocks" and a parallel literature that, using natural experiments, finds large consumption responses to transitory income shocks.[4] However, a new puzzle arises from the low estimate for the partial insurance to permanent shocks, now estimated to be around 0.34.

While this paper will focus on the implications of time aggregation for the

[1]For an example see Campbell and Mankiw (1989)

[2]A sample of this literature includes Amemiya and Wu (1972), Weiss (1984) and Drost and Nijman (1993).

[3]The literature goes back to early work such as Hause (1973), Weiss and Lillard (1979) and MaCurdy (1982) that look at the covariance structure of the income process. Following BPP, a number of papers have looked at income and consumption together, for example Arellano, Blundell and Bonhomme (2017) .

[4]A small sample of this literature includes Parker et al. (2013), Agarwal and Qian (2014) and Sahm, Shapiro and Slemrod (2010). Consumers also answer that they have a high marginal propensity to consume when asked, see Fuster, Kaplan and Zafar (2018) and Jappelli and Pistaferri (2014). For an overview of the entire literature on consumption responses to income shocks, see Jappelli and Pistaferri (2010). Note the dissonance between BPP and the natural experiment literature is also addressed by Commault (2017). In contrast to this paper, her approach makes structural changes to the underlying model but does not address time aggregation.

methodology in BPP, the techniques can be applied to a broad swath of the literature.

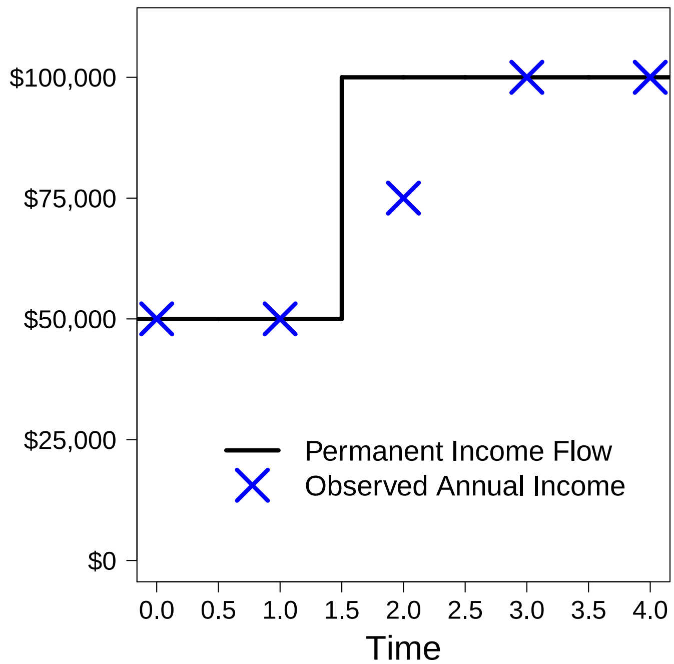

FIGURE 1. INCOME FLOW AND OBSERVED INCOME

I. What is Time Aggregation?

Time aggregation occurs when a time series is observed at a lower frequency than the underlying data that generates it. For example, income is often observed at an annual frequency when it may in fact consist of paychecks arriving at a monthly, biweekly or irregular timetable. To transform income into an annual frequency, we sum up all the income that was received by a household during the year. The key insight of Working (1960) is that even if there is no correlation between changes in income at the underlying frequency, changes in the resulting time aggregated series will show positive autocorrelation. The intuition behind this can be seen in figure 1, showing the income process of a household that begins with an annual salary of \$50,000 and receives a permanent pay rise to \$100,000 mid-way through the second year. The solid line shows this jump in income flow occurring just once. The crosses show the income we actually observe in annual data. During the second year the household receives an annual \$50,000 salary for six months, followed by \$100,000 in the second six months, resulting in a reported income of \$75,000 for the entire year. The single shock to income therefore appears in the time aggregated data as two increases. In this way, an income change in one year is positively correlated with an income change in the following year, even if the underlying income process follows a random walk.

II. Modelling Time Aggregation in Blundell, Pistaferri and Preston (2008)

A. The Model in Discrete Time Without Time Aggregation

Here I briefly describe the method used by Blundell, Pistaferri and Preston (2008) to estimate household consumption responses to permanent and transitory income shocks. The model described here is a simplified version of the original in order to highlight the role played by time aggregation.[5]

The core of the model is the assumptions made on the income and consumption processes. Unexplained log income growth for household i follows the process:

$$\Delta y_{i,t} = \zeta_{i,t} + \Delta\nu_{i,t}$$

where $\zeta_{i,t}$ (the change in permanent income) and $\nu_{i,t}$ (transitory income) are each mean zero, finite variance, i.i.d. and independent of each other.

The unexplained change in log consumption is modeled as a random walk that

[5] In this simplified model I assume insurance parameters are constant across both time and households, that the transitory component of income has no persistence, and that there are no taste shocks. These elements are reintroduced in section III in which I show the quantitative effect of time aggregation.

moves in response to permanent and transitory income shocks:

$$\Delta c_{i,t} = \phi\zeta_{i,t} + \psi\nu_{i,t}$$

where ϕ and ψ are the *partial insurance* parameters. A value of zero implies full insurance (consumption does not respond at all to the income shock), while a value of one implies no insurance. The core of the empirical methodology is to identify these insurance parameters in the data from the following identities:

$$\phi = \frac{\text{Cov}(\Delta c_t, \Delta y_{t-1} + \Delta y_t + \Delta y_{t+1})}{\text{Cov}(\Delta y_t, \Delta y_{t-1} + \Delta y_t + \Delta y_{t+1})} \tag{1}$$

$$\psi = \frac{\text{Cov}(\Delta c_t, \Delta y_{t+1})}{\text{Cov}(\Delta y_t, \Delta y_{t+1})} \tag{2}$$

B. The Model in Continuous Time with Time Aggregation

In this section I show how time aggregation can significantly bias the partial insurance parameter estimates obtained by equations 1 and 2. The model in this section will be the exact analog of the discrete time model just described, but embedded in continuous time where shocks are spread uniformly throughout the year.[6] The main result does not hinge on the use of continuous time, and similar estimates would be obtained by dividing the year into quarters or months.[7]

Time is continuous and one time unit represents one year. For the income process we will assume two underlying martingale processes, P_t and Q_t such that

[6]There is little formal evidence on the distribution of shocks throughout the year. While this assumption is unlikely to be strictly true, it is more reasonable than the implicit assumption of BPP that shocks all occur 1st January each year.

[7]The autocorrelation of a time aggregated random walk is 0.25 in continuous time, compared to 0.23 for a discrete quarterly model and almost indistinguishable from a discrete monthly model. The theoretical moments are however significantly more elegant in continuous time. See online appendix B.B1

for all $s_1 > s_2 > s_3 > s_4 > 0$:

$$\begin{aligned} \mathrm{Var}(P_{s_1} - P_{s_2}) &= (s_1 - s_2)\sigma_P^2 \\ \mathrm{Cov}(P_{s_1} - P_{s_2}, P_{s_3} - P_{s_4}) &= 0 \\ P_s = 0 &\qquad \text{if } s < 0 \end{aligned}$$

and similarly for Q_t. Brownian motion fits these assumptions, but the slightly more general definition allows for jumps in the income process. Allowing for jumps accommodates low-frequency events, such changing job or getting a promotion, that may only occur once every few years, but when they do occur they can be at any point in the year. Instantaneous income in a period dt is given by:[8]

$$dy_t = P_t dt + dQ_t \tag{3}$$

that is they receive their permanent income flow ($P_t = \int_0^t dP_s$) multiplied by time dt in addition to a one-off transitory income dQ_t.

Keeping with the assumption that consumption is a random walk with insurance parameters ϕ and ψ, instantaneous consumption is given by:

$$dc_t = \phi P_t dt + \psi Q_t dt \tag{4}$$

that is, they consume a proportion ϕ of their permanent income and a proportion ψ of the cumulation of all the transitory income they have received in their lifetime ($Q_t = \int_0^t dQ_s$).

In the Panel Study of Income Dynamics (PSID) data, we observe the total income received over the previous calendar year at time T:

$$y_T^{obs} = \int_{T-1}^{T} dy_t$$

[8] A more formal treatment of how to relate this to the log income process is given in online appendix B.B1.

Consumption is measured by a survey at the beginning of the following calendar year, which I map to a snapshot of consumption exactly at the end of the calendar year:[9]

$$c_T^{obs} = \phi P_T + \psi Q_T \tag{5}$$

The BPP method makes use of the changes in observable income and consumption, which in the time aggregated model relate to:

$$\begin{aligned} \Delta y_T^{obs} &= \Big(\int_{T-2}^{T-1} (s-(T-2))dP_s + \int_{T-1}^{T} (T-s)dP_s \Big) \\ &\quad + \Big(\int_{T-1}^{T} dQ_t - \int_{T-2}^{T-1} dQ_t \Big) \end{aligned} \tag{6}$$

$$\Delta c_T^{obs} = \phi \int_{T-1}^{T} dP_s + \psi \int_{T-1}^{T} dQ_s \tag{7}$$

We see that these observable income and consumption changes in equations 1 and 2 recover the permanent, but not the transitory insurance parameter:

$$\frac{\mathrm{Cov}(\Delta c_T^{obs}, \Delta y_{T-1}^{obs} + \Delta y_T^{obs} + \Delta y_{T+1}^{obs})}{\mathrm{Cov}(\Delta y_T^{obs}, \Delta y_{T-1}^{obs} + \Delta y_T^{obs} + \Delta y_{T+1}^{obs})} = \phi \tag{8}$$

$$\frac{\mathrm{Cov}(\Delta c_T^{obs}, \Delta y_{T+1}^{obs})}{\mathrm{Cov}(\Delta y_T^{obs}, \Delta y_{T+1}^{obs})} = \psi - \frac{(3\phi - \psi)\sigma_P^2}{6\sigma_Q^2 - \sigma_P^2} \tag{9}$$

Indeed the transitory insurance coefficient bears little relation to the true value of ψ. For example, if permanent and transitory variances are equal, and households follow the permanent income hypothesis ($\phi = 1$, $\psi = 0$), the estimate for ψ using this method will be *negative* 0.6.

[9]BPP use data on food consumption to impute total annual consumption. The questionnaire asks about food consumption in a typical week, but unfortunately the timing of this 'typical week' is not clear. The questionnaire is usually given at the end of March in the following year. See Altonji and Siow (1987) and Hall and Mishkin (1982) for differing views. In online appendix B.B4 I show that controlling for the interview date barely changes the results. However, in online appendix B.B3 I show that the timing of the 'typical' week can have a large effect on the results. This is an important drawback to using this method with the PSID data. In Crawley and Kuchler (2018) we use expenditure data imputed from Danish administrative records in which the timing of expenditure is very clearly defined.

III. Revised BPP Estimates

In this section I repeat the BPP estimation proceedure, but with the model moments coming from the continuous time model with time aggregated income. While the core identification in BPP is illustrated in equations 1 and 2, the full estimation proceedure minimizes the distance between all the observable covariances ($\text{Cov}(\Delta y_T^{obs}, \Delta y_S^{obs})$, $\text{Cov}(\Delta c_T^{obs}, \Delta c_S^{obs})$ and $\text{Cov}(\Delta c_T^{obs}, \Delta y_S^{obs})$) and their model implied equivalents.[10] The full set of these model implied moments for the continuous time model, extended to include time varying coefficients, transitory persistence and taste shocks, can be found in appendix A.A1 and online appendix B.B2.

Table 1—Minimum-Distance Partial Insurance and Variance Estimates

	BPP		Time Agg.		
Persistence Type:	None	MA(1)	None	Uniform	Linear Decay
ψ	0.0503	0.0501	0.2421	0.2510	0.2403
(Partial insurance tran. shock)	(0.0505)	(0.0430)	(0.0431)	(0.0428)	(0.0417)
ϕ	0.4692	0.6456	0.3384	0.3287	0.3516
(Partial insurance perm. shock)	(0.0598)	(0.0941)	(0.0471)	(0.0580)	(0.0627)

Table 1 shows the estimates for the transitory and permanent insurance parameters, first using BPP's original method and then with time aggregation. As there is no equivalent to an MA(1) process in continuous time, I consider two alternative ways to introduce persistence in the transitory shock, as well as reporting results assuming no persistence. First I assume a transitory shock provides a stream of income uniformly distributed over a short period of time (to be estimated). Second I assume the stream of income decays linearly over a short period.[11] The time aggregated results are not very sensitive to these assumptions.

[10] I follow the exact same diagonally weighted minimum distance proceedure in BPP as described in online appendix D of Blundell, Pistaferri and Preston (2008)

[11] See online appendix B.B2 for details.

The top row of table 1 gives the main result showing the transitory insurance parameter increases from 0.05 in BPP to 0.24 with time aggregation. This new estimate is much more in line with the literature that estimates MPCs using natural experiments. However, the new estimate for the permanent insurance parameter is lower than before, around 0.35. This new puzzle is discussed next.

A. A New Puzzle: Too Much Permanent Insurance?

The low estimate of ϕ coflicts with both consumption theory and other empirical estimates.[12] Furthermore, equation 8 suggests time aggregation should not alter the permanent insurance estimate, at least in the model without transitory persistence. Here I propose a further change to the model that can explain the difference observed in the permant insurance estimate between BPP and the time aggregated model. However, the low estimate for ϕ remains a puzzle which I suggest is a feature of the underlying Consumer Expenditure (CEX) data rather than model misspecification.

I extend the model so that, rather than consumption following a random walk, the consumption response to a transitory shock decays exponentially over time. That this consumption response decays over time is a natural consequence of a large initial response. The details are in appendix ******************. With this change, the half life of the consumption response to transitory shocks is ************, and both ϕ and ψ are estimated to be close to 0.2. The low ϕ estimate is now even more of a puzzle. However, when I simulate a panel of income and consumption data from this model, using the estimated parameters, and then use this simulated panel in the original BPP algorithm, I retrieve close to the estimates in the original BPP paper. This new model specification is therefore able to explain all the differences observed in the estimations.

The low estimate of ϕ appears to be a feature of the data, possibly measurement

[12]Standard buffer stock theory suggests ϕ should be close to 1. The literature on consumption responses to permanent shocks is much smaller than for transitory shocks, but tends to find estimates close to 1. See Gelman et al. (2016) and Crawley and Kuchler (2018) for examples.

error, rather than model misspecification. Indeed the relation between income and consumption in the cross-sectional CEX data is low. Sabelhaus et al. (2014) find "The ratio of spending to income at low-income levels seems implausibly high, and the ratio of spending to income at the top seems implausibly low." This cross-sectional relation in the CEX data is carried over to the imputed PSID consumption data used in BPP, and is reflected in the low estimates for permanent insurance.[13]

IV. Conclusion

This paper highlights the importance of time aggregation when working with panel data, especially when analyzing the covariance matrix of income and consumption growth. It also resolves the dissonance between BPP's estimates of transitory income insurance and the natural experiment literature on marginal propensity to consume. Going forward, I hope the methods used here to correct for the time aggregation problem can be useful for researchers, especially as more and more high quality panel datasets on income and consumption become available.

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[13] See online appendix *************** for more detail on the low ϕ estimate.

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Mathematical Appendix

A1. Identification in the Full Model

In this appendix I calculate the full set of identifying equations for the non-stationary model with measurement error in consumption and taste shocks. Online appendix B.B2 extends these to add persistence in the transitory shock.

I am interested in the full set of observable covariances:

$$\mathrm{Cov}(\Delta y_T^{obs}, \Delta y_S^{obs})$$
$$\mathrm{Cov}(\Delta c_T^{obs}, \Delta c_S^{obs})$$
$$\mathrm{Cov}(\Delta c_T^{obs}, \Delta y_S^{obs})$$

for all T and S in $\{1, 2, ...\}$. I further make the assumption that while the variance of the permanent and transitory shocks and insurance coefficients can change from year to year, within each year these are constant. The variance the permanent shock in year T is $\sigma^2_{P,T}$ and the transitory shock $\sigma^2_{Q,T}$. I use equation 6 for the change in observable log income, and extend equation 7 for the change in observable log consumption to include taste shocks (ξ_t) and measurement error (u_T):

$$\Delta c_T^{obs} = \phi \int_{T-1}^{T} dP_s + \psi \int_{T-1}^{T} dQ_s + \int_{T-1}^{T} d\xi_s + u_T - u_{T-1}$$

These two equations allow for the calculation of all the required identification equations:

$$
\begin{aligned}
\mathrm{Var}(\Delta y_T^{obs}) &= \mathbb{E}\Big(\int_{T-2}^{T-1}(s-(T-2))^2 dP_s dP_s + \int_{T-1}^{T}(T-s)^2 dP_s dP_s\Big) \\
&\quad + \mathbb{E}\Big(\int_{T-1}^{T} dQ_t dQ_t + \int_{T-2}^{T-1} dQ_t dQ_t\Big) \\
&= \frac{1}{3}\sigma^2_{P,T} + \frac{1}{3}\sigma^2_{P,T-1} + \sigma^2_{Q,T} + \sigma^2_{Q,T-1} \qquad \text{(A1)} \\
\mathrm{Cov}(\Delta y_T^{obs}, \Delta y_{T+1}^{obs}) &= \mathbb{E}\Big(\int_{T-1}^{T}(T-s)(s-(T-1)) dP_s dP_s\Big) - \mathbb{E}\Big(\int_{T-1}^{T} dQ_t dQ_t\Big) \\
&= \frac{1}{6}\sigma^2_{P,T} - \sigma^2_{Q,T} \\
\mathrm{Cov}(\Delta y_T^{obs}, \Delta y_{T-1}^{obs}) &= \frac{1}{6}\sigma^2_{P,T-1} - \sigma^2_{Q,T-1} \\
\mathrm{Cov}(\Delta y_T^{obs}, \Delta y_S^{obs}) &= 0 \qquad \forall S, T \text{ such that } |S-T| > 1
\end{aligned}
$$

$$
\begin{aligned}
\mathrm{Var}\Delta c_T^{obs} &= \phi^2 \mathbb{E}\Big(\int_{T-1}^{T} dP_s dP_s\Big) + \psi^2 \mathbb{E}\Big(\int_{T-1}^{T} dQ_s dQ_s\Big) + \mathbb{E}\Big(\int_{T-1}^{T} d\xi_s d\xi_s\Big) + \sigma^2_{u,T} + \sigma^2_{u,T-1} \\
&= \phi^2\sigma^2_{P,T} + \psi^2\sigma^2_{Q,T} + \sigma^2_{\xi,T} + \sigma^2_{u,T} + \sigma^2_{u,T-1} \\
\mathrm{Cov}(\Delta c_T^{obs}, \Delta c_{T+1}^{obs}) &= -\sigma^2_{u,T} \\
\mathrm{Cov}(\Delta c_T^{obs}, \Delta c_{T-1}^{obs}) &= -\sigma^2_{u,T-1} \\
\mathrm{Cov}(\Delta c_T^{obs}, \Delta c_S^{obs}) &= 0 \qquad \forall S, T \text{ such that } |S-T| > 1
\end{aligned}
$$

$$
\begin{aligned}
\mathrm{Cov}(\Delta c_T^{obs}, \Delta y_T^{obs}) &= \mathbb{E}\Big(\phi_T \int_{T-1}^{T} (T-s) dP_s dP_s + \psi_T \int_{T-1}^{T} dQ_s dQ_s\Big) \\
&= \frac{1}{2}\phi_T \sigma_{P,T}^2 + \psi_T \sigma_{Q,T}^2 \\
\mathrm{Cov}(\Delta c_T^{obs}, \Delta y_{T+1}^{obs}) &= \mathbb{E}\Big(\phi_T \int_{T-1}^{T} (s-(T-1)) dP_s dP_s - \psi_T \int_{T-1}^{T} dQ_s dQ_s\Big) \\
&= \frac{1}{2}\phi_T \sigma_{P,T}^2 - \psi_T \sigma_{Q,T}^2 \\
\mathrm{Cov}(\Delta c_T^{obs}, \Delta y_{T-1}^{obs}) &= 0 \\
\mathrm{Cov}(\Delta c_T^{obs}, \Delta y_S^{obs}) &= 0 \qquad \forall S, T \text{ such that } |S-T| > 1
\end{aligned}
$$

For Online Publication

B1. Continuous Time Model as Limit of Discrete Model with m Sub-periods

The identifying equations in the paper are calculated using a 'log' income process that does not directly align with any real-world concept of income. In the data we take logs on the sum of income over the entire year, but the process we use in the model informally aligns with log income over an instantaneous period dt. This is a problem as transitory income arrive as a point mass, making it difficult to interpret what the 'log' income process really represents. Here I show how the identifying equations can be derived as the limit of discrete time model with m sub-periods. I show that in the limit the variance of observed log income growth is the same as derived in the informal model (to a first order approximation). The rest of the identifying equations can be shown in the same way.

Let p_t for $t \in \mathbb{R}^+$ be a martingale process (possibly with jumps) with independent stationary increments and ν be such that $\mathbb{E}(e^{p_t - p_{t-1}}) = e^{\nu}$. Define permanent income as:

$$P_t = e^{p_t - t\nu}$$

Note that $\mathbb{E}\Big(\frac{P_{t+s}}{P_t}\Big) = 1$ for all $s \geq 0$. Define the variance of log permanent shocks to be:

$$\sigma_P^2 = \mathrm{Var}\Big(\log\big(\frac{P_{t+1}}{P_t}\big)\Big) = \mathrm{Var}(p_{t+1} - p_t)$$

We will assume changes in permanent income over a one year period are small enough such that:

$$\begin{aligned}
\mathrm{Var}\Big(\frac{P_{t+1}}{P_t}\Big) &= \mathrm{Var}\Big(\frac{P_{t+1}-P_t}{P_t}\Big) \\
&\approx \mathrm{Var}\Big(\log\big(1+\frac{P_{t+1}-P_t}{P_t}\big)\Big) \\
&= \mathrm{Var}\Big(\log\big(\frac{P_{t+1}}{P_t}\big)\Big) = \sigma_P^2
\end{aligned}$$

For transitory shocks, we define an increasing stochastic process, Θ_t, which also has independent stationary increments. The increments in this process will define the transitory shocks. We set the expectation of increments, and the variance of the log of an increment of length 1 as:

$$\mathbb{E}(\Theta_{t+s}-\Theta_t) = s$$
$$\mathrm{Var}\Big(\log\big(\Theta_{t+1}-\Theta_t\big)\Big) = \sigma_\Theta^2$$

Note that for this to be well defined, Θ_t must not only be increasing but also its increments are almost surely strictly positive (so that log of the increment is defined almost everywhere). Examples of such a stochastic process would be a gamma process, or a process that increases linearly with time (non-stochastically) but is also subject to positive shocks that arrive as a Poisson process. The stochastic part of this process has no Brownian motion component as this would necessarily lead to non-zero probability of a decreasing increment.

We will use these two processes to define an income process in discrete time with m intervals per period, and then look at the limit as $m \to \infty$. Define $\theta_{t,m}$ for $t \in \{\frac{1}{m}, \frac{2}{m}, \frac{3}{m}...\}$ to be the increment of Θ_t from $t-\frac{1}{m}$ to t:

$$\theta_{t,m} = \Theta_t - \Theta_{1-\frac{1}{m}}$$

Income is defined for each period $t \in \{\frac{1}{m}, \frac{2}{m}, \frac{3}{m}...\}$ as:

$$Y_{t,m} = P_t \theta_{t,m}$$

Therefore the underlying income process has a pure division into permanent and transitory shocks. Income is observed for $T \in \{1, 2, 3...\}$ as the sum of income in each of the subperiods:

$$\bar{Y}_{T,m} = \sum_{i=0}^{m-1} P_{T-\frac{i}{m}} \theta_{T-\frac{i}{m},m}$$

Note that for $m = 1$ this the same as the underlying income process, with permanent and transitory variance as defined above. We are interested in the log of observable income growth:

$$\begin{aligned}
\Delta \bar{y}_{T,m} &= \log \bar{Y}_{T,m} - \log \bar{Y}_{T-1,m} \\
&= \log\left(\sum_{i=0}^{m-1} P_{T-\frac{i}{m}} \theta_{T-\frac{i}{m},m}\right) - \log\left(\sum_{i=0}^{m-1} P_{T-1-\frac{i}{m}} \theta_{T-1-\frac{i}{m},m}\right) \\
&= \log\left(\sum_{i=0}^{m-1} \frac{P_{T-\frac{i}{m}}}{P_{T-1}} \theta_{T-\frac{i}{m},m}\right) - \log\left(\sum_{i=0}^{m-1} \frac{P_{T-1-\frac{i}{m}}}{P_{T-1}} \theta_{T-1-\frac{i}{m},m}\right)
\end{aligned}$$

As P_t and Θ_t have independent increments, the covariance between each of the two parts of the sum above is 0. Therefore:

$$\mathrm{Var}\Big(\Delta^1 \bar{y}_{T,m}\Big) = \mathrm{Var}\Bigg(\log\left(\sum_{i=0}^{m-1} \frac{P_{T-\frac{i}{m}}}{P_{T-1}} \theta_{T-\frac{i}{m},m}\right)\Bigg) + \mathrm{Var}\Bigg(\log\left(\sum_{i=0}^{m-1} \frac{P_{T-1-\frac{i}{m}}}{P_{T-1}} \theta_{T-1-\frac{i}{m},m}\right)\Bigg)$$

We will treat each of these two variances individually. We begin by looking at the variable:

$$
\begin{aligned}
\log\left(\sum_{i=0}^{m-1}\frac{P_{T-\frac{i}{m}}}{P_{T-1}}\theta_{T-\frac{i}{m},m}\right) &= \log\left(\sum_{i=0}^{m-1}\theta_{T-\frac{i}{m},m}+\sum_{i=0}^{m-1}\Big(\frac{P_{T-\frac{i}{m}}}{P_{T-1}}-1\Big)\theta_{T-\frac{i}{m},m}\right)\\
&= \log\Big(\Theta_T-\Theta_{T-1}\Big)+\log\left(1+\sum_{i=0}^{m-1}\Big(\frac{P_{T-\frac{i}{m}}}{P_{T-1}}-1\Big)\frac{\theta_{T-\frac{i}{m},m}}{\sum_{l=0}^{m-1}\theta_{T-\frac{l}{m},m}}\right)\\
&\approx \log\Big(\Theta_T-\Theta_{T-1}\Big)+\sum_{i=0}^{m-1}\Big(\frac{P_{T-\frac{i}{m}}}{P_{T-1}}-1\Big)\frac{\theta_{T-\frac{i}{m},m}}{\sum_{l=0}^{m-1}\theta_{T-\frac{l}{m},m}}
\end{aligned}
$$

Where the approximation comes from the fact that the shocks to permanent income in a one year period are small. Defining

$$\zeta_{t,m}=\frac{P_t}{P_{t-\frac{1}{m}}}$$

we have that

$$
\begin{aligned}
\operatorname{Var}\left(\log\left(\sum_{i=0}^{m-1}\frac{P_{T-\frac{i}{m}}}{P_{T-1}}\theta_{T-\frac{i}{m},m}\right)\right) &\approx \sigma_\Theta^2+\operatorname{Var}\left(\sum_{i=0}^{m-1}\Big(\prod_{j=i}^{m-1}\zeta_{T-\frac{j}{m}}-1\Big)\frac{\theta_{T-\frac{i}{m},m}}{\sum_{l=0}^{m-1}\theta_{T-\frac{l}{m},m}}\right)\\
&=\sigma_\Theta^2+\mathbb{E}\left[\sum_{i=0}^{m-1}\Big(\prod_{j=i}^{m-1}\zeta_{T-\frac{j}{m}}-1\Big)\frac{\theta_{T-\frac{i}{m},m}}{\sum_{l=0}^{m-1}\theta_{T-\frac{l}{m},m}}\right]^2\\
&=\sigma_\Theta^2+\mathbb{E}\left[\sum_{i=0}^{m-1}\left(\Big(\prod_{j=i}^{m-1}\zeta_{T-\frac{j}{m}}-1\Big)^2\left(\frac{\theta_{T-\frac{i}{m},m}}{\sum_{l=0}^{m-1}\theta_{T-\frac{l}{m},m}}\right)^2\right.\right.\\
&\qquad\left.\left.+2\sum_{k<i}\Big(\prod_{j=k}^{m-1}\zeta_{T-\frac{j}{m}}-1\Big)\Big(\prod_{j=i}^{m-1}\zeta_{T-\frac{j}{m}}-1\Big)\frac{\theta_{T-\frac{k}{m},m}\theta_{T-\frac{i}{m},m}}{\Big(\sum_{l=0}^{m-1}\theta_{T-\frac{l}{m},m}\Big)^2}\right)\right]\\
&=\sigma_\Theta^2+\frac{\sigma_P^2}{m}\sum_{i=0}^{m-1}\left(i\mathbb{E}\left(\frac{\theta_{T-\frac{i}{m},m}}{\sum_{l=0}^{m-1}\theta_{T-\frac{l}{m},m}}\right)^2+2\sum_{k<i}(m-1-i)\mathbb{E}\left(\frac{\theta_{T-\frac{k}{m},m}\theta_{T-\frac{i}{m},m}}{\Big(\sum_{l=0}^{m-1}\theta_{T-\frac{l}{m},m}\Big)^2}\right)\right)\\
&=\sigma_\Theta^2+\frac{\sigma_P^2}{m}\frac{m(m-1)}{2}\mathbb{E}\left(\frac{\theta_{T-\frac{i}{m},m}}{\sum_{l=0}^{m-1}\theta_{T-\frac{l}{m},m}}\right)^2\\
&\qquad+2\frac{\sigma_P^2}{m}\sum_{i=1}^{m-1}i(m-1-i)\mathbb{E}\left(\frac{\theta_{T-\frac{k}{m},m}\theta_{T-\frac{i}{m},m}}{\Big(\sum_{l=0}^{m-1}\theta_{T-\frac{l}{m},m}\Big)^2}\right)\\
&=\sigma_\Theta^2+\sigma_P^2\frac{m-1}{2}\mathbb{E}\left(\frac{\theta_{T-\frac{i}{m},m}}{\sum_{l=0}^{m-1}\theta_{T-\frac{l}{m},m}}\right)^2\\
&\qquad+\sigma_P^2\left[(m-1)^2-\frac{(m-1)(2m-1)}{3}\right]\mathbb{E}\left(\frac{\theta_{T-\frac{k}{m},m}\theta_{T-\frac{i}{m},m}}{\Big(\sum_{l=0}^{m-1}\theta_{T-\frac{l}{m},m}\Big)^2}\right)
\end{aligned}
$$

Note that:

$$
\begin{aligned}
1&=\mathbb{E}\left(\sum_{i=0}^{m-1}\frac{\theta_{T-\frac{i}{m},m}}{\sum_{l=0}^{m-1}\theta_{T-\frac{l}{m},m}}\right)^2\\
&=\sum_{i=0}^{m-1}\mathbb{E}\left(\frac{\theta_{T-\frac{i}{m},m}}{\sum_{l=0}^{m-1}\theta_{T-\frac{l}{m},m}}\right)^2+2\sum_{k<i}\mathbb{E}\left(\frac{\theta_{T-\frac{k}{m},m}\theta_{T-\frac{i}{m},m}}{\Big(\sum_{l=0}^{m-1}\theta_{T-\frac{l}{m},m}\Big)^2}\right)
\end{aligned}
$$

So that

$$\mathbb{E}\left(\frac{\theta_{T-\frac{k}{m},m}\theta_{T-\frac{i}{m},m}}{\left(\sum_{l=0}^{m-1}\theta_{T-\frac{l}{m},m}\right)^2}\right) = \frac{1}{m(m-1)} - \frac{1}{m-1}\mathbb{E}\left(\frac{\theta_{T-\frac{i}{m},m}}{\sum_{l=0}^{m-1}\theta_{T-\frac{l}{m},m}}\right)^2$$

This gives:

$$\begin{aligned}\mathrm{Var}\left(\log\left(\sum_{i=0}^{m-1}\frac{P_{T-\frac{i}{m}}}{P_{T-1}}\theta_{T-\frac{i}{m},m}\right)\right) &\approx \sigma_\Theta^2 + \mathrm{Var}\left(\sum_{i=0}^{m-1}\Big(\prod_{j=i}^{m-1}\zeta_{T-\frac{j}{m}} - 1\Big)\frac{\theta_{T-\frac{i}{m},m}}{\sum_{l=0}^{m-1}\theta_{T-\frac{l}{m},m}}\right)\\ &\approx \sigma_\Theta^2 + \frac{m-2}{3m}\sigma_P^2 + \frac{m+1}{6}\mathbb{E}\left(\frac{\theta_{T-\frac{i}{m},m}}{\sum_{l=0}^{m-1}\theta_{T-\frac{l}{m},m}}\right)^2\sigma_P^2\\ &\to \sigma_\Theta^2 + \frac{1}{3}\sigma_P^2 \qquad \text{as } m\to\infty\end{aligned}$$

A very similar calculation shows that:

$$\mathrm{Var}\left(\log\left(\sum_{i=0}^{m-1}\frac{P_{T-1-\frac{i}{m}}}{P_{T-1}}\theta_{T-1-\frac{i}{m},m}\right)\right) \to \sigma_\Theta^2 + \frac{1}{3}\sigma_P^2 \qquad \text{as } m\to\infty$$

Putting these together gives:

$$\mathrm{Var}\Big(\Delta\bar{y}_{T,m}\Big) \to \frac{2}{3}\sigma_P^2 + 2\sigma_\Theta^2 \qquad \text{as } m\to\infty$$

This is the same as the identifying equation for $\mathrm{Var}\Big(\Delta y_T^{obs}\Big)$ (equation A1 from appendix A.A1, assuming shock variances are constant over time), and the rest of the identifying equations can be shown as the limit of the discrete time model in a similar way.

B2. Persistence in Transitory Shock

This appendix shows how to extend the time aggregated model to include persistence in the transitory shock.

Linear Decay Model

I will walk though the derivation of the moments for the linear decay model in detail and then just list the moments for the uniform model. In the linear decay model, a shock of size 1 will arrive with a flow intensity of $\frac{2}{\tau}$ and over the subsequent time τ the total flow of transitory income will sum to 1. Instantaneous income can be written as:

$$dy_t = \Big(\int_0^t dP_s\Big)dt + \Big(\int_{t-\tau}^t \frac{2}{\tau}(s-(t-\tau))dQ_s\Big)dt$$

So that the observable change in income is given by:

$$\begin{aligned}
\Delta y_T^{obs} &= \int_{T-1}^T y_t dt - \int_{T-2}^{T-1} y_t dt \\
&= \int_{T-1}^T \int_0^t dP_s dt - \int_{T-2}^{T-1}\int_0^t dP_s dt \\
&\quad + \int_{T-1}^T \int_{t-\tau}^t \frac{2}{\tau}(s-(t-\tau))dQ_s dt - \int_{T-2}^{T-1}\int_{t-\tau}^t \frac{2}{\tau}(s-(t-\tau))dQ_s dt \\
&= \Big(\int_{T-2}^{T-1}(s-(T-2))dP_s + \int_{T-1}^T (T-s)dP_s\Big) \\
&\quad + \frac{2}{\tau}\Big(\int_{T-\tau}^T \frac{1}{2}\Big(\tau - \frac{(s-(T-\tau))^2}{\tau}\Big)dQ_s + \int_{T-1}^{T-\tau}\frac{1}{2}\tau dQ_s + \int_{T-1-\tau}^{T-1}\frac{1}{2}\frac{(s-(T-1-\tau))^2}{\tau}dQ_s\Big) \\
&\quad - \frac{2}{\tau}\Big(\int_{T-1-\tau}^{T-1}\frac{1}{2}\Big(\tau - \frac{(s-(T-1-\tau))^2}{\tau}\Big)dQ_s + \int_{T-2}^{T-1-\tau}\frac{1}{2}\tau dQ_s \\
&\qquad\qquad + \int_{T-2-\tau}^{T-2}\frac{1}{2}\frac{(s-(T-2-\tau))^2}{\tau}dQ_s\Big) \\
&= \int_{T-2}^{T-1}(s-(T-2))dP_s + \int_{T-1}^T (T-s)dP_s \\
&\quad + \int_{T-\tau}^T 1 - \Big(\frac{s-(T-\tau)}{\tau}\Big)^2 dQ_s + \int_{T-1}^{T-\tau} dQ_s \\
&\quad - \int_{T-1-\tau}^{T-1} 1 - 2\Big(\frac{s-(T-1-\tau)}{\tau}\Big)^2 dQ_s \\
&\quad - \int_{T-2}^{T-1-\tau} dQ_s - \int_{T-2-\tau}^{T-2}\Big(\frac{s-(T-2-\tau)}{\tau}\Big)^2 dQ_s
\end{aligned}$$

The full set of identification equations used in this model are:

$$\begin{aligned}
\mathrm{Var}(\Delta y_T^{obs}) &= \mathbb{E}\Big(\int_{T-2}^{T-1}(s-(T-2))^2 dP_s dP_s + \int_{T-1}^{T}(T-s)^2 dP_s dP_s\Big) \\
&\quad + \mathbb{E}\Big(\int_{T-\tau}^{T}\Big(1-\Big(\frac{s-(T-\tau)}{\tau}\Big)^2\Big)^2 dQ_s dQ_s + \int_{T-1}^{T-\tau} dQ_s Q_s\Big) \\
&\quad + \mathbb{E}\Big(\int_{T-1-\tau}^{T-1}\Big(1-2\Big(\frac{s-(T-1-\tau)}{\tau}\Big)^2\Big)^2 dQ_s dQ_s\Big) \\
&\quad + \mathbb{E}\Big(\int_{T-2}^{T-1-\tau} dQ_s dQ_s + \int_{T-2-\tau}^{T-2}\Big(\frac{s-(T-2-\tau)}{\tau}\Big)^4 dQ_s dQ_s\Big) \\
&= \frac{1}{3}\sigma_{P,T}^2 + \frac{1}{3}\sigma_{P,T-1}^2 \\
&\quad + \frac{8}{15}\tau\sigma_{Q,T}^2 + (1-\tau)\sigma_{Q,T}^2 \\
&\quad + \frac{7}{15}\tau\sigma_{Q,T-1}^2 \\
&\quad + (1-\tau)\sigma_{Q,T-1}^2 + \frac{1}{5}\tau\sigma_{Q,T-2}^2 \\
&= \frac{1}{3}\sigma_{P,T}^2 + \frac{1}{3}\sigma_{P,T-1}^2 + (1-\frac{7}{15}\tau)\sigma_{Q,T}^2 + (1-\frac{8}{15}\tau)\sigma_{Q,T-1}^2 + \frac{1}{5}\tau\sigma_{Q,T-2}^2
\end{aligned}$$

$$\begin{aligned}
\mathrm{Cov}(\Delta y_T^{obs}, \Delta y_{T+1}^{obs}) &= \mathbb{E}\Big(\int_{T-1}^{T}(T-s)(s-(T-1)) dP_s dP_s\Big) \\
&\quad - \mathbb{E}\Big(\int_{T-\tau}^{T}\Big(1-\Big(\frac{s-(T-\tau)}{\tau}\Big)^2\Big)\Big(1-2\Big(\frac{s-(T-\tau)}{\tau}\Big)^2\Big) dQ_s dQ_s\Big) \\
&\quad - \mathbb{E}\Big(\int_{T-1}^{T-\tau} dQ_s Q_s\Big) \\
&\quad + \mathbb{E}\Big(\int_{T-1-\tau}^{T-1}\Big(1-2\Big(\frac{s-(T-1-\tau)}{\tau}\Big)^2\Big)\Big(\frac{s-(T-1-\tau)}{\tau}\Big)^2 dQ_s dQ_s\Big) \\
&= \frac{1}{6}\sigma_{P,T}^2 - \frac{2}{5}\tau\sigma_{Q,T}^2 - (1-\tau)\sigma_{Q,T}^2 - \frac{1}{15}\sigma_{Q,T-1}^2
\end{aligned}$$

$$\begin{aligned}
\mathrm{Cov}(\Delta y_T^{obs}, \Delta y_{T+2}^{obs}) &= -\mathbb{E}\Big(\int_{T-\tau}^{T}\Big(1-\Big(\frac{s-(T-\tau)}{\tau}\Big)^2\Big)\Big(\frac{s-(T-\tau)}{\tau}\Big)^2 dQ_s dQ_s\Big) \\
&= -\frac{2}{15}\tau\sigma_{Q,T}^2
\end{aligned}$$

The above equations also work for $\mathrm{Cov}(\Delta y_T^{obs}, \Delta y_{T-1}^{obs})$ and $\mathrm{Cov}(\Delta y_T^{obs}, \Delta y_{T-2}^{obs})$ due to symmetry.

$$\mathrm{Cov}(\Delta y_T^{obs}, \Delta y_S^{obs}) = 0 \qquad \forall S, T \text{ such that } |S-T| > 2$$

The covariance matrix $\mathrm{Cov}(\Delta c_T^{obs}, \Delta c_S^{obs})$ is the same as in appendix A.A1.

$$\begin{aligned}\mathrm{Cov}(\Delta c_T^{obs}, \Delta y_T^{obs}) &= \phi_T \mathbb{E}\Big(\int_{T-1}^{T} (T-s) dP_s dP_s\Big) \\ &\quad + \psi_T \mathbb{E}\Big(\int_{T-\tau}^{T} \Big(1 - \Big(\frac{s-(T-\tau)}{\tau}\Big)^2\Big) dQ_s dQ_s + \int_{T-1}^{T-\tau} dQ_s dQ_s\Big) \\ &= \frac{1}{2}\phi_T \sigma_{P,T}^2 + \psi_T(1 - \frac{1}{3}\tau)\sigma_{Q,T}^2\end{aligned}$$

$$\begin{aligned}\mathrm{Cov}(\Delta c_T^{obs}, \Delta y_{T+1}^{obs}) &= \phi_T \mathbb{E}\Big(\int_{T-1}^{T} (s-(T-1)) dP_s dP_s\Big) \\ &\quad - \psi_T \mathbb{E}\Big(\int_{T-\tau}^{T} \Big(1 - 2\Big(\frac{s-(T-\tau)}{\tau}\Big)^2\Big) dQ_s dQ_s + \int_{T-1}^{T-\tau} dQ_s dQ_s\Big) \\ &= \frac{1}{2}\phi_T \sigma_{P,T}^2 - (1 - \frac{2}{3}\tau)\psi_T\sigma_{Q,T}^2\end{aligned}$$

$$\begin{aligned}\mathrm{Cov}(\Delta c_T^{obs}, \Delta y_{T+2}^{obs}) &= -\psi_T \mathbb{E}\Big(\int_{T-\tau}^{T} \Big(\frac{s-(T-\tau)}{\tau}\Big)^2 dQ_s dQ_s\Big) \\ &= -\frac{1}{5}\psi_T \tau \sigma_{Q,T}^2\end{aligned}$$

The Uniform Model

In the uniform model, transitory shocks consist of a constant flow of income that lasts for a time period τ. The full set of moments for this model are:

$$\mathrm{Var}(\Delta y_T^{obs}) = \frac{1}{3}\sigma_{P,T}^2 + \frac{1}{3}\sigma_{P,T-1}^2 + \big(1 - \frac{2}{3}\tau\big)\sigma_{Q,T}^2 + (1 - \frac{2}{3}\tau)\sigma_{Q,T-1}^2 + \frac{1}{3}\tau\sigma_{Q,T-2}^2$$

$$\text{Cov}(\Delta y_T^{obs}, \Delta y_{T+1}^{obs}) = \frac{1}{6}\sigma_{P,T}^2 - \frac{1}{6}\tau\sigma_{Q,T}^2 - (1-\tau)\sigma_{Q,T}^2 - \frac{1}{15}\sigma_{Q,T-1}^2$$

$$\text{Cov}(\Delta y_T^{obs}, \Delta y_{T+2}^{obs}) = -\frac{1}{6}\tau\sigma_{Q,T}^2$$

The above equations also work for $\text{Cov}(\Delta y_T^{obs}, \Delta y_{T-1}^{obs})$ and $\text{Cov}(\Delta y_T^{obs}, \Delta y_{T-2}^{obs})$ due to symmetry.

$$\text{Cov}(\Delta y_T^{obs}, \Delta y_S^{obs}) = 0 \qquad \forall S, T \text{ such that } |S-T| > 2$$

The covariance matrix $\text{Cov}(\Delta c_T^{obs}, \Delta c_S^{obs})$ is the same as in appendix A.A1.

$$\text{Cov}(\Delta c_T^{obs}, \Delta y_T^{obs}) = \frac{1}{2}\phi_T\sigma_{P,T}^2 + \psi_T(1-\frac{1}{2}\tau)\sigma_{Q,T}^2$$

$$\text{Cov}(\Delta c_T^{obs}, \Delta y_{T+1}^{obs}) = \frac{1}{2}\phi_T\sigma_{P,T}^2 - (1-\tau)\psi_T\sigma_{Q,T}^2$$

$$\text{Cov}(\Delta c_T^{obs}, \Delta y_{T+2}^{obs}) = -\frac{1}{2}\psi_T\tau\sigma_{Q,T}^2$$

B3. Effect of Timing of Consumption in the PSID

BPP impute annual consumption from the question in the PSID asking about food consumption in a ‘typical’ week. Unfortunately it is not clear if this relates to an average of the previous calendar year, or some more recent week closer to when the interview was conducted (normally in March of the following year). In the paper I have assumed the answer gives a snapshot of consumption at the end of the calendar year. Here I show that assuming the ‘typical’ week is an average of consumption over the previous calendar year, the identifying equation from BPP for transitory insurance coefficient is different again, and still significantly biased. Under this new assumption the equation for the permanent insurance coefficient is unbiased as before:

$$\frac{\mathrm{Cov}(\Delta c_T^{obs}, \Delta y_{T-1}^{obs} + \Delta y_T^{obs} + \Delta y_{T+1}^{obs})}{\mathrm{Cov}(\Delta y_T^{obs}, \Delta y_{T-1}^{obs} + \Delta y_T^{obs} + \Delta y_{T+1}^{obs})} = \phi$$

While the identifying equation for the transitory insurance coefficient is:

$$\frac{\mathrm{Cov}(\Delta c_T^{obs}, \Delta y_{T+1}^{obs})}{\mathrm{Cov}(\Delta y_T^{obs}, \Delta y_{T+1}^{obs})} = \frac{-\phi\frac{1}{6}\sigma_P^2 + \frac{1}{2}\psi\sigma_Q^2}{-\frac{1}{6}\sigma_P^2 + \sigma_Q^2} \neq \psi$$

Under the permanent income hypothesis with $\phi = 1$, $\psi = 0$ and permanent and transitory variances approximately equal, the BPP estimate of ψ would be -0.2.

B4. Controlling for Interview Date

To mitigate some of the seasonal effects coming from the timing of consumption in the data, here I control for the interview date at the first stage (before residualizing). Specifically, I include dummies for the time of year (divided into nine bins) that the interview was taken in the first stage regression. The results are little changed from table 1:

Table B1—Replication of Table 1 Controlling for Interview Date

	BPP		Time Agg.		
Persistence Type:	None	MA(1)	None	Uniform	Linear Decay
ψ	0.0503	0.0505	0.2420	0.2512	0.2403
(Partial insurance tran. shock)	(0.0505)	(0.0430)	(0.0431)	(0.0427)	(0.0417)
ϕ	0.4704	0.6452	0.3384	0.3287	0.3515
(Partial insurance perm. shock)	(0.0601)	(0.0941)	(0.0473)	(0.0583)	(0.0629)

B5. Other Tables from the BPP paper

Table B2 replicates Table 6 from the original BPP paper.

Table B3 replicates Table 7 from the original BPP paper.

Table B4 replicates Table 8 from the original BPP paper.

Table B2—Minimum-Distance Partial Insurance and Variance Estimates

		Whole Sample		No College		College	
		BPP	Time Agg.	BPP	Time Agg.	BPP	Time Agg.
$\sigma^2_{P,T}$	1979-1981	0.0103	0.0247	0.0068	0.0234	0.0101	0.0189
(Variance perm. shock)		(0.0034)	(0.0043)	(0.0037)	(0.0063)	(0.0053)	(0.0050)
	1982	0.0208	0.0358	0.0156	0.0290	0.0253	0.0455
		(0.0041)	(0.0071)	(0.0052)	(0.0099)	(0.0060)	(0.0099)
	1983	0.0301	0.0333	0.0318	0.0553	0.0234	0.0086
		(0.0057)	(0.0100)	(0.0074)	(0.0128)	(0.0090)	(0.0148)
	1984	0.0274	0.0292	0.0334	0.0232	0.0177	0.0361
		(0.0049)	(0.0114)	(0.0073)	(0.0131)	(0.0060)	(0.0161)
	1985	0.0295	0.0363	0.0287	0.0504	0.0208	0.0025
		(0.0096)	(0.0124)	(0.0073)	(0.0145)	(0.0152)	(0.0205)
	1986	0.0221	0.0327	0.0173	0.0247	0.0311	0.0597
		(0.0060)	(0.0136)	(0.0067)	(0.0172)	(0.0101)	(0.0202)
	1987	0.0289	0.0420	0.0202	0.0478	0.0354	0.0229
		(0.0063)	(0.0143)	(0.0073)	(0.0182)	(0.0098)	(0.0211)
	1988	0.0158	0.0082	0.0117	-0.0069	0.0183	0.0302
		(0.0069)	(0.0137)	(0.0079)	(0.0209)	(0.0110)	(0.0149)
	1989	0.0185	0.0531	0.0107	0.0639	0.0274	0.0414
		(0.0059)	(0.0129)	(0.0101)	(0.0214)	(0.0061)	(0.0149)
	1990-92	0.0135	0.0291	0.0093	0.0265	0.0217	0.0291
		(0.0042)	(0.0042)	(0.0045)	(0.0063)	(0.0065)	(0.0057)
$\sigma^2_{Q,T}$	1979	0.0379	0.0310	0.0465	0.0364	0.0301	0.0261
(Variance trans. shock)		(0.0059)	(0.0049)	(0.0096)	(0.0080)	(0.0056)	(0.0043)
	1980	0.0298	0.0240	0.0330	0.0247	0.0283	0.0238
		(0.0039)	(0.0033)	(0.0053)	(0.0046)	(0.0059)	(0.0047)
	1981	0.0300	0.0265	0.0363	0.0305	0.0253	0.0222
		(0.0035)	(0.0032)	(0.0053)	(0.0048)	(0.0046)	(0.0040)
	1982	0.0287	0.0280	0.0375	0.0332	0.0213	0.0237
		(0.0039)	(0.0034)	(0.0063)	(0.0057)	(0.0042)	(0.0036)
	1983	0.0262	0.0276	0.0371	0.0378	0.0185	0.0169
		(0.0037)	(0.0034)	(0.0063)	(0.0056)	(0.0037)	(0.0040)
	1984	0.0346	0.0350	0.0404	0.0388	0.0304	0.0315
		(0.0039)	(0.0038)	(0.0059)	(0.0058)	(0.0051)	(0.0046)
	1985	0.0450	0.0427	0.0355	0.0338	0.0496	0.0465
		(0.0075)	(0.0071)	(0.0056)	(0.0053)	(0.0130)	(0.0122)
	1986	0.0458	0.0404	0.0474	0.0373	0.0452	0.0464
		(0.0058)	(0.0055)	(0.0076)	(0.0068)	(0.0085)	(0.0084)
	1987	0.0461	0.0445	0.0520	0.0486	0.0421	0.0385
		(0.0054)	(0.0053)	(0.0082)	(0.0078)	(0.0071)	(0.0069)
	1988	0.0399	0.0327	0.0471	0.0360	0.0343	0.0313
		(0.0047)	(0.0044)	(0.0074)	(0.0072)	(0.0060)	(0.0055)
	1989	0.0378	0.0343	0.0539	0.0475	0.0219	0.0215
		(0.0067)	(0.0061)	(0.0126)	(0.0117)	(0.0051)	(0.0044)
	1990-92	0.0441	0.0359	0.0535	0.0408	0.0345	0.0322
		(0.0040)	(0.0027)	(0.0062)	(0.0047)	(0.0049)	(0.0032)
θ		0.1126	N/A	0.1260	N/A	0.1082	N/A
(Serial correl. trans. shock)		(0.0248)		(0.0319)		(0.0342)	
σ^2_{ξ}		0.0097	0.0122	0.0065	0.0114	0.0132	0.0146
(Variance unobs. slope heterog.)		(0.0041)	(0.0039)	(0.0079)	(0.0070)	(0.0040)	(0.0039)
ϕ		0.6456	0.3384	0.9484	0.4365	0.4180	0.2729
(Partial insurance perm. shock)		(0.0941)	(0.0471)	(0.1773)	(0.0738)	(0.0913)	(0.0603)
ψ		0.0501	0.2421	0.0724	0.2870	0.0260	0.1590
(Partial insurance trans. shock)		(0.0430)	(0.0431)	(0.0593)	(0.0616)	(0.0546)	(0.0504)

Table B3—Minimum-Distance Partial Insurance and Variance Estimates

Consumption:	Nondurable		Nondurable		Nondurable	
Income:	net income		earnings only		male earnings	
Sample:	baseline		baseline		baseline	
	BPP	Time Agg.	BPP	Time Agg.	BPP	Time Agg.
ϕ	0.6456	0.3384	0.3101	0.1761	0.2240	0.1232
(Partial insurance perm. shock)	(0.0941)	(0.0471)	(0.0572)	(0.0339)	(0.0492)	(0.0316)
ψ	0.0501	0.2421	0.0630	0.1625	0.0502	0.1181
(Partial insurance trans. shock)	(0.0430)	(0.0431)	(0.0306)	(0.0280)	(0.0293)	(0.0244)

Table B4—Minimum-Distance Partial Insurance and Variance Estimates

Consumption:	Nondurable		Nondurable		Nondurable	
Income:	net income		excluding help		net income	
Sample:	baseline		baseline		low wealth	
	BPP	Time Agg.	BPP	Time Agg.	BPP	Time Agg.
ϕ	0.6456	0.3384	0.6244	0.3422	0.8339	0.8584
(Partial insurance perm. shock)	(0.0941)	(0.0471)	(0.0891)	(0.0466)	(0.2762)	(0.2498)
ψ	0.0501	0.2421	0.0469	0.2404	0.2853	0.4926
(Partial insurance trans. shock)	(0.0430)	(0.0431)	(0.0429)	(0.0427)	(0.1154)	(0.1050)

Consumption:	Nondurable		Total		Nondurable	
Income:	net income		net income		net income	
Sample:	high wealth		low wealth		baseline+SEO	
	BPP	Time Agg.	BPP	Time Agg.	BPP	Time Agg.
ϕ	0.6278	0.2691	1.0207	1.0580	0.7663	0.4630
(Partial insurance perm. shock)	(0.0998)	(0.0420)	(0.3426)	(0.3099)	(0.1028)	(0.0499)
ψ	0.0088	0.1838	0.3647	0.6185	0.1201	0.3232
(Partial insurance trans. shock)	(0.0409)	(0.0409)	(0.1477)	(0.1344)	(0.0352)	(0.0367)