

FOR ONLINE PUBLICATION

B1. Continuous Time Model as Limit of Discrete Model with m Sub-periods

The identifying equations in the paper are calculated using a ‘log’ income process that does not directly align with any real-world concept of income. In the data we take logs on the sum of income over the entire year, but the process we use in the model informally aligns with log income over an instantaneous period dt . This is a problem as transitory income arrive as a point mass, making it difficult to interpret what the ‘log’ income process really represents. Here I show how the identifying equations can be derived as the limit of discrete time model with m sub-periods. I show that in the limit the variance of observed log income growth is the same as derived in the informal model (to a first order approximation). The rest of the identifying equations can be shown in the same way.

Let p_t for $t \in \mathbb{R}^+$ be a martingale process (possibly with jumps) with independent stationary increments and ν be such that $\mathbb{E}(e^{p_t - p_{t-1}}) = e^\nu$. Define permanent income as:

$$P_t = e^{p_t - t\nu}$$

Note that $\mathbb{E}\left(\frac{P_{t+s}}{P_t}\right) = 1$ for all $s \geq 0$. Define the variance of log permanent shocks to be:

$$\sigma_P^2 = \text{Var}\left(\log\left(\frac{P_{t+1}}{P_t}\right)\right) = \text{Var}(p_{t+1} - p_t)$$

We will assume changes in permanent income over a one year period are small

enough such that:

$$\begin{aligned}\text{Var}\left(\frac{P_{t+1}}{P_t}\right) &= \text{Var}\left(\frac{P_{t+1} - P_t}{P_t}\right) \\ &\approx \text{Var}\left(\log\left(1 + \frac{P_{t+1} - P_t}{P_t}\right)\right) \\ &= \text{Var}\left(\log\left(\frac{P_{t+1}}{P_t}\right)\right) = \sigma_P^2\end{aligned}$$

For transitory shocks, we define an increasing stochastic process, Θ_t , which also has independent stationary increments. The increments in this process will define the transitory shocks. We set the expectation of increments, and the variance of the log of an increment of length 1 as:

$$\begin{aligned}\mathbb{E}(\Theta_{t+s} - \Theta_t) &= s \\ \text{Var}\left(\log(\Theta_{t+1} - \Theta_t)\right) &= \sigma_\Theta^2\end{aligned}$$

Note that for this to be well defined, Θ_t must not only be increasing but also its increments are almost surely strictly positive (so that log of the increment is defined almost everywhere). Examples of such a stochastic process would be a gamma process, or a process that increases linearly with time (non-stochastically) but is also subject to positive shocks that arrive as a Poisson process. The stochastic part of this process has no Brownian motion component as this would necessarily lead to non-zero probability of a decreasing increment.

We will use these two processes to define an income process in discrete time with m intervals per period, and then look at the limit as $m \rightarrow \infty$. Define $\theta_{t,m}$ for $t \in \{\frac{1}{m}, \frac{2}{m}, \frac{3}{m}, \dots\}$ to be the increment of Θ_t from $t - \frac{1}{m}$ to t :

$$\theta_{t,m} = \Theta_t - \Theta_{t-\frac{1}{m}}$$

Income is defined for each period $t \in \{\frac{1}{m}, \frac{2}{m}, \frac{3}{m} \dots\}$ as:

$$Y_{t,m} = P_t \theta_{t,m}$$

Therefore the underlying income process has a pure division into permanent and transitory shocks. Income is observed for $T \in \{1, 2, 3 \dots\}$ as the sum of income in each of the subperiods:

$$\bar{Y}_{T,m} = \sum_{i=0}^{m-1} P_{T-\frac{i}{m}} \theta_{T-\frac{i}{m},m}$$

Note that for $m = 1$ this is the same as the underlying income process, with permanent and transitory variance as defined above. We are interested in the log of observable income growth:

$$\begin{aligned} \Delta \bar{y}_{T,m} &= \log \bar{Y}_{T,m} - \log \bar{Y}_{T-1,m} \\ &= \log \left(\sum_{i=0}^{m-1} P_{T-\frac{i}{m}} \theta_{T-\frac{i}{m},m} \right) - \log \left(\sum_{i=0}^{m-1} P_{T-1-\frac{i}{m}} \theta_{T-1-\frac{i}{m},m} \right) \\ &= \log \left(\sum_{i=0}^{m-1} \frac{P_{T-\frac{i}{m}}}{P_{T-1}} \theta_{T-\frac{i}{m},m} \right) - \log \left(\sum_{i=0}^{m-1} \frac{P_{T-1-\frac{i}{m}}}{P_{T-1}} \theta_{T-1-\frac{i}{m},m} \right) \end{aligned}$$

As P_t and Θ_t have independent increments, the covariance between each of the two parts of the sum above is 0. Therefore:

$$\text{Var}(\Delta \bar{y}_{T,m}) = \text{Var} \left(\log \left(\sum_{i=0}^{m-1} \frac{P_{T-\frac{i}{m}}}{P_{T-1}} \theta_{T-\frac{i}{m},m} \right) \right) + \text{Var} \left(\log \left(\sum_{i=0}^{m-1} \frac{P_{T-1-\frac{i}{m}}}{P_{T-1}} \theta_{T-1-\frac{i}{m},m} \right) \right)$$

We will treat each of these two variances individually. We begin by looking at

the variable:

$$\begin{aligned}
\log \left(\sum_{i=0}^{m-1} \frac{P_{T-\frac{i}{m}}}{P_{T-1}} \theta_{T-\frac{i}{m},m} \right) &= \log \left(\sum_{i=0}^{m-1} \theta_{T-\frac{i}{m},m} + \sum_{i=0}^{m-1} \left(\frac{P_{T-\frac{i}{m}}}{P_{T-1}} - 1 \right) \theta_{T-\frac{i}{m},m} \right) \\
&= \log \left(\Theta_T - \Theta_{T-1} \right) + \log \left(1 + \sum_{i=0}^{m-1} \left(\frac{P_{T-\frac{i}{m}}}{P_{T-1}} - 1 \right) \frac{\theta_{T-\frac{i}{m},m}}{\sum_{l=0}^{m-1} \theta_{T-\frac{l}{m},m}} \right) \\
&\approx \log \left(\Theta_T - \Theta_{T-1} \right) + \sum_{i=0}^{m-1} \left(\frac{P_{T-\frac{i}{m}}}{P_{T-1}} - 1 \right) \frac{\theta_{T-\frac{i}{m},m}}{\sum_{l=0}^{m-1} \theta_{T-\frac{l}{m},m}}
\end{aligned}$$

Where the approximation comes from the fact that the shocks to permanent income in a one year period are small. Defining

$$\zeta_{t,m} = \frac{P_t}{P_{t-\frac{1}{m}}}$$

we have that

$$\begin{aligned}
\text{Var}\left(\log\left(\sum_{i=0}^{m-1} \frac{P_{T-\frac{i}{m}}}{P_{T-1}} \theta_{T-\frac{i}{m},m}\right)\right) &\approx \sigma_{\Theta}^2 + \text{Var}\left(\sum_{i=0}^{m-1} \left(\prod_{j=i}^{m-1} \zeta_{T-\frac{j}{m}} - 1\right) \frac{\theta_{T-\frac{i}{m},m}}{\sum_{l=0}^{m-1} \theta_{T-\frac{l}{m},m}}\right) \\
&= \sigma_{\Theta}^2 + \mathbb{E}\left[\sum_{i=0}^{m-1} \left(\prod_{j=i}^{m-1} \zeta_{T-\frac{j}{m}} - 1\right) \frac{\theta_{T-\frac{i}{m},m}}{\sum_{l=0}^{m-1} \theta_{T-\frac{l}{m},m}}\right]^2 \\
&= \sigma_{\Theta}^2 + \mathbb{E}\left[\sum_{i=0}^{m-1} \left(\left(\prod_{j=i}^{m-1} \zeta_{T-\frac{j}{m}} - 1\right)^2 \left(\frac{\theta_{T-\frac{i}{m},m}}{\sum_{l=0}^{m-1} \theta_{T-\frac{l}{m},m}}\right)^2\right.\right. \\
&\quad \left.\left.+ 2 \sum_{k< i} \left(\prod_{j=k}^{m-1} \zeta_{T-\frac{j}{m}} - 1\right) \left(\prod_{j=i}^{m-1} \zeta_{T-\frac{j}{m}} - 1\right) \frac{\theta_{T-\frac{k}{m},m} \theta_{T-\frac{i}{m},m}}{\left(\sum_{l=0}^{m-1} \theta_{T-\frac{l}{m},m}\right)^2}\right)\right] \\
&= \sigma_{\Theta}^2 + \frac{\sigma_P^2}{m} \sum_{i=0}^{m-1} \left(i \mathbb{E}\left(\frac{\theta_{T-\frac{i}{m},m}}{\sum_{l=0}^{m-1} \theta_{T-\frac{l}{m},m}}\right)^2 + 2 \sum_{k< i} (m-1-i) \mathbb{E}\left(\frac{\theta_{T-\frac{k}{m},m} \theta_{T-\frac{i}{m},m}}{\left(\sum_{l=0}^{m-1} \theta_{T-\frac{l}{m},m}\right)^2}\right)\right) \\
&= \sigma_{\Theta}^2 + \frac{\sigma_P^2}{m} \frac{m(m-1)}{2} \mathbb{E}\left(\frac{\theta_{T-\frac{i}{m},m}}{\sum_{l=0}^{m-1} \theta_{T-\frac{l}{m},m}}\right)^2 \\
&\quad + 2 \frac{\sigma_P^2}{m} \sum_{i=1}^{m-1} i(m-1-i) \mathbb{E}\left(\frac{\theta_{T-\frac{k}{m},m} \theta_{T-\frac{i}{m},m}}{\left(\sum_{l=0}^{m-1} \theta_{T-\frac{l}{m},m}\right)^2}\right) \\
&= \sigma_{\Theta}^2 + \sigma_P^2 \frac{m-1}{2} \mathbb{E}\left(\frac{\theta_{T-\frac{i}{m},m}}{\sum_{l=0}^{m-1} \theta_{T-\frac{l}{m},m}}\right)^2 \\
&\quad + \sigma_P^2 \left[(m-1)^2 - \frac{(m-1)(2m-1)}{3}\right] \mathbb{E}\left(\frac{\theta_{T-\frac{k}{m},m} \theta_{T-\frac{i}{m},m}}{\left(\sum_{l=0}^{m-1} \theta_{T-\frac{l}{m},m}\right)^2}\right)
\end{aligned}$$

Note that:

$$\begin{aligned}
1 &= \mathbb{E}\left(\sum_{i=0}^{m-1} \frac{\theta_{T-\frac{i}{m},m}}{\sum_{l=0}^{m-1} \theta_{T-\frac{l}{m},m}}\right)^2 \\
&= \sum_{i=0}^{m-1} \mathbb{E}\left(\frac{\theta_{T-\frac{i}{m},m}}{\sum_{l=0}^{m-1} \theta_{T-\frac{l}{m},m}}\right)^2 + 2 \sum_{k< i} \mathbb{E}\left(\frac{\theta_{T-\frac{k}{m},m} \theta_{T-\frac{i}{m},m}}{\left(\sum_{l=0}^{m-1} \theta_{T-\frac{l}{m},m}\right)^2}\right)
\end{aligned}$$

So that

$$\mathbb{E}\left(\frac{\theta_{T-\frac{k}{m},m}\theta_{T-\frac{i}{m},m}}{\left(\sum_{l=0}^{m-1}\theta_{T-\frac{l}{m},m}\right)^2}\right) = \frac{1}{m(m-1)} - \frac{1}{m-1}\mathbb{E}\left(\frac{\theta_{T-\frac{i}{m},m}}{\sum_{l=0}^{m-1}\theta_{T-\frac{l}{m},m}}\right)^2$$

This gives:

$$\begin{aligned} \text{Var}\left(\log\left(\sum_{i=0}^{m-1}\frac{P_{T-\frac{i}{m}}}{P_{T-1}}\theta_{T-\frac{i}{m},m}\right)\right) &\approx \sigma_{\Theta}^2 + \text{Var}\left(\sum_{i=0}^{m-1}\left(\prod_{j=i}^{m-1}\zeta_{T-\frac{j}{m}} - 1\right)\frac{\theta_{T-\frac{i}{m},m}}{\sum_{l=0}^{m-1}\theta_{T-\frac{l}{m},m}}\right) \\ &\approx \sigma_{\Theta}^2 + \frac{m-2}{3m}\sigma_P^2 + \frac{m+1}{6}\mathbb{E}\left(\frac{\theta_{T-\frac{i}{m},m}}{\sum_{l=0}^{m-1}\theta_{T-\frac{l}{m},m}}\right)^2\sigma_P^2 \\ &\rightarrow \sigma_{\Theta}^2 + \frac{1}{3}\sigma_P^2 \quad \text{as } m \rightarrow \infty \end{aligned}$$

A very similar calculation shows that:

$$\text{Var}\left(\log\left(\sum_{i=0}^{m-1}\frac{P_{T-1-\frac{i}{m}}}{P_{T-1}}\theta_{T-1-\frac{i}{m},m}\right)\right) \rightarrow \sigma_{\Theta}^2 + \frac{1}{3}\sigma_P^2 \quad \text{as } m \rightarrow \infty$$

Putting these together gives:

$$\text{Var}(\Delta\bar{y}_{T,m}) \rightarrow \frac{2}{3}\sigma_P^2 + 2\sigma_{\Theta}^2 \quad \text{as } m \rightarrow \infty$$

This is the same as the identifying equation for $\text{Var}(\Delta y_T^{obs})$ (equation A1 from appendix A.A1, assuming shock variances are constant over time), and the rest of the identifying equations can be shown as the limit of the discrete time model in a similar way.

B2. Persistence in Transitory Shock

This appendix shows how to extend the time aggregated model to include persistence in the transitory shock.

LINEAR DECAY MODEL

I will walk though the derivation of the moments for the linear decay model in detail and then just list the moments for the uniform model. In the linear decay model, a shock of size 1 will arrive with a flow intensity of $\frac{2}{\tau}$ and over the subsequent time τ the total flow of transitory income will sum to 1. Instantaneous income can be written as:

$$dy_t = \left(\int_0^t dP_s \right) dt + \left(\int_{t-\tau}^t \frac{2}{\tau} (s - (t - \tau)) dQ_s \right) dt$$

So that the observable change in income is given by:

$$\begin{aligned} \Delta y_T^{obs} &= \int_{T-1}^T y_t dt - \int_{T-2}^{T-1} y_t dt \\ &= \int_{T-1}^T \int_0^t dP_s dt - \int_{T-2}^{T-1} \int_0^t dP_s dt \\ &\quad + \int_{T-1}^T \int_{t-\tau}^t \frac{2}{\tau} (s - (t - \tau)) dQ_s dt - \int_{T-2}^{T-1} \int_{t-\tau}^t \frac{2}{\tau} (s - (t - \tau)) dQ_s dt \\ &= \left(\int_{T-2}^{T-1} (s - (T - 2)) dP_s + \int_{T-1}^T (T - s) dP_s \right) \\ &\quad + \frac{2}{\tau} \left(\int_{T-\tau}^T \frac{1}{2} \left(\tau - \frac{(s - (T - \tau))^2}{\tau} \right) dQ_s + \int_{T-1}^{T-\tau} \frac{1}{2} \tau dQ_s + \int_{T-1-\tau}^{T-1} \frac{1}{2} \frac{(s - (T - 1 - \tau))^2}{\tau} dQ_s \right) \\ &\quad - \frac{2}{\tau} \left(\int_{T-1-\tau}^{T-1} \frac{1}{2} \left(\tau - \frac{(s - (T - 1 - \tau))^2}{\tau} \right) dQ_s + \int_{T-2}^{T-1-\tau} \frac{1}{2} \tau dQ_s \right. \\ &\quad \left. + \int_{T-2-\tau}^{T-2} \frac{1}{2} \frac{(s - (T - 2 - \tau))^2}{\tau} dQ_s \right) \\ &= \int_{T-2}^{T-1} (s - (T - 2)) dP_s + \int_{T-1}^T (T - s) dP_s \\ &\quad + \int_{T-\tau}^T 1 - \left(\frac{s - (T - \tau)}{\tau} \right)^2 dQ_s + \int_{T-1}^{T-\tau} dQ_s \\ &\quad - \int_{T-1-\tau}^{T-1} 1 - 2 \left(\frac{s - (T - 1 - \tau)}{\tau} \right)^2 dQ_s \\ &\quad - \int_{T-2}^{T-1-\tau} dQ_s - \int_{T-2-\tau}^{T-2} \left(\frac{s - (T - 2 - \tau)}{\tau} \right)^2 dQ_s \end{aligned}$$

The full set of identification equations used in this model are:

$$\begin{aligned}
\text{Var}(\Delta y_T^{obs}) &= \mathbb{E} \left(\int_{T-2}^{T-1} (s - (T-2))^2 dP_s dP_s + \int_{T-1}^T (T-s)^2 dP_s dP_s \right) \\
&\quad + \mathbb{E} \left(\int_{T-\tau}^T \left(1 - \left(\frac{s - (T-\tau)}{\tau} \right)^2 \right)^2 dQ_s dQ_s + \int_{T-1}^{T-\tau} dQ_s dQ_s \right) \\
&\quad + \mathbb{E} \left(\int_{T-1-\tau}^{T-1} \left(1 - 2 \left(\frac{s - (T-1-\tau)}{\tau} \right)^2 \right)^2 dQ_s dQ_s \right) \\
&\quad + \mathbb{E} \left(\int_{T-2}^{T-1-\tau} dQ_s dQ_s + \int_{T-2-\tau}^{T-2} \left(\frac{s - (T-2-\tau)}{\tau} \right)^4 dQ_s dQ_s \right) \\
&= \frac{1}{3} \sigma_{P,T}^2 + \frac{1}{3} \sigma_{P,T-1}^2 \\
&\quad + \frac{8}{15} \tau \sigma_{Q,T}^2 + (1-\tau) \sigma_{Q,T}^2 \\
&\quad + \frac{7}{15} \tau \sigma_{Q,T-1}^2 \\
&\quad + (1-\tau) \sigma_{Q,T-1}^2 + \frac{1}{5} \tau \sigma_{Q,T-2}^2 \\
&= \frac{1}{3} \sigma_{P,T}^2 + \frac{1}{3} \sigma_{P,T-1}^2 + \left(1 - \frac{7}{15} \tau \right) \sigma_{Q,T}^2 + \left(1 - \frac{8}{15} \tau \right) \sigma_{Q,T-1}^2 + \frac{1}{5} \tau \sigma_{Q,T-2}^2
\end{aligned}$$

$$\begin{aligned}
\text{Cov}(\Delta y_T^{obs}, \Delta y_{T+1}^{obs}) &= \mathbb{E} \left(\int_{T-1}^T (T-s)(s - (T-1)) dP_s dP_s \right) \\
&\quad - \mathbb{E} \left(\int_{T-\tau}^T \left(1 - \left(\frac{s - (T-\tau)}{\tau} \right)^2 \right) \left(1 - 2 \left(\frac{s - (T-\tau)}{\tau} \right)^2 \right) dQ_s dQ_s \right) \\
&\quad - \mathbb{E} \left(\int_{T-1}^{T-\tau} dQ_s dQ_s \right) \\
&\quad + \mathbb{E} \left(\int_{T-1-\tau}^{T-1} \left(1 - 2 \left(\frac{s - (T-1-\tau)}{\tau} \right)^2 \right) \left(\frac{s - (T-1-\tau)}{\tau} \right)^2 dQ_s dQ_s \right) \\
&= \frac{1}{6} \sigma_{P,T}^2 - \frac{2}{5} \tau \sigma_{Q,T}^2 - (1-\tau) \sigma_{Q,T}^2 - \frac{1}{15} \sigma_{Q,T-1}^2
\end{aligned}$$

$$\begin{aligned}
\text{Cov}(\Delta y_T^{obs}, \Delta y_{T+2}^{obs}) &= -\mathbb{E} \left(\int_{T-\tau}^T \left(1 - \left(\frac{s - (T-\tau)}{\tau} \right)^2 \right) \left(\frac{s - (T-\tau)}{\tau} \right)^2 dQ_s dQ_s \right) \\
&= -\frac{2}{15} \tau \sigma_{Q,T}^2
\end{aligned}$$

The above equations also work for $\text{Cov}(\Delta y_T^{obs}, \Delta y_{T-1}^{obs})$ and $\text{Cov}(\Delta y_T^{obs}, \Delta y_{T-2}^{obs})$ due to symmetry.

$$\text{Cov}(\Delta y_T^{obs}, \Delta y_S^{obs}) = 0 \quad \forall S, T \text{ such that } |S - T| > 2$$

The covariance matrix $\text{Cov}(\Delta c_T^{obs}, \Delta c_S^{obs})$ is the same as in appendix A.A1.

$$\begin{aligned} \text{Cov}(\Delta c_T^{obs}, \Delta y_T^{obs}) &= \phi_T \mathbb{E} \left(\int_{T-1}^T (T-s) dP_s dP_s \right) \\ &\quad + \psi_T \mathbb{E} \left(\int_{T-\tau}^T \left(1 - \left(\frac{s-(T-\tau)}{\tau} \right)^2 \right) dQ_s dQ_s + \int_{T-1}^{T-\tau} dQ_s dQ_s \right) \\ &= \frac{1}{2} \phi_T \sigma_{P,T}^2 + \psi_T \left(1 - \frac{1}{3} \tau \right) \sigma_{Q,T}^2 \end{aligned}$$

$$\begin{aligned} \text{Cov}(\Delta c_T^{obs}, \Delta y_{T+1}^{obs}) &= \phi_T \mathbb{E} \left(\int_{T-1}^T (s-(T-1)) dP_s dP_s \right) \\ &\quad - \psi_T \mathbb{E} \left(\int_{T-\tau}^T \left(1 - 2 \left(\frac{s-(T-\tau)}{\tau} \right)^2 \right) dQ_s dQ_s + \int_{T-1}^{T-\tau} dQ_s dQ_s \right) \\ &= \frac{1}{2} \phi_T \sigma_{P,T}^2 - \left(1 - \frac{2}{3} \tau \right) \psi_T \sigma_{Q,T}^2 \end{aligned}$$

$$\begin{aligned} \text{Cov}(\Delta c_T^{obs}, \Delta y_{T+2}^{obs}) &= -\psi_T \mathbb{E} \left(\int_{T-\tau}^T \left(\frac{s-(T-\tau)}{\tau} \right)^2 dQ_s dQ_s \right) \\ &= -\frac{1}{5} \psi_T \tau \sigma_{Q,T}^2 \end{aligned}$$

THE UNIFORM MODEL

In the uniform model, transitory shocks consist of a constant flow of income that lasts for a time period τ . The full set of moments for this model are:

$$\text{Var}(\Delta y_T^{obs}) = \frac{1}{3} \sigma_{P,T}^2 + \frac{1}{3} \sigma_{P,T-1}^2 + \left(1 - \frac{2}{3} \tau \right) \sigma_{Q,T}^2 + \left(1 - \frac{2}{3} \tau \right) \sigma_{Q,T-1}^2 + \frac{1}{3} \tau \sigma_{Q,T-2}^2$$

$$\text{Cov}(\Delta y_T^{obs}, \Delta y_{T+1}^{obs}) = \frac{1}{6}\sigma_{P,T}^2 - \frac{1}{6}\tau\sigma_{Q,T}^2 - (1-\tau)\sigma_{Q,T}^2 - \frac{1}{15}\sigma_{Q,T-1}^2$$

$$\text{Cov}(\Delta y_T^{obs}, \Delta y_{T+2}^{obs}) = -\frac{1}{6}\tau\sigma_{Q,T}^2$$

The above equations also work for $\text{Cov}(\Delta y_T^{obs}, \Delta y_{T-1}^{obs})$ and $\text{Cov}(\Delta y_T^{obs}, \Delta y_{T-2}^{obs})$ due to symmetry.

$$\text{Cov}(\Delta y_T^{obs}, \Delta y_S^{obs}) = 0 \quad \forall S, T \text{ such that } |S - T| > 2$$

The covariance matrix $\text{Cov}(\Delta c_T^{obs}, \Delta c_S^{obs})$ is the same as in appendix A.A1.

$$\text{Cov}(\Delta c_T^{obs}, \Delta y_T^{obs}) = \frac{1}{2}\phi_T\sigma_{P,T}^2 + \psi_T(1 - \frac{1}{2}\tau)\sigma_{Q,T}^2$$

$$\text{Cov}(\Delta c_T^{obs}, \Delta y_{T+1}^{obs}) = \frac{1}{2}\phi_T\sigma_{P,T}^2 - (1-\tau)\psi_T\sigma_{Q,T}^2$$

$$\text{Cov}(\Delta c_T^{obs}, \Delta y_{T+2}^{obs}) = -\frac{1}{2}\psi_T\tau\sigma_{Q,T}^2$$

B3. Effect of Timing of Consumption in the PSID

BPP impute annual consumption from the question in the PSID asking about food consumption in a ‘typical’ week. Unfortunately it is not clear if this relates to an average of the previous calendar year, or some more recent week closer to when the interview was conducted (normally in March of the following year). In the paper I have assumed the answer gives a snapshot of consumption at the end of the calendar year. Here I show that assuming the ‘typical’ week is an average of consumption over the previous calendar year, the identifying equation from BPP for transitory insurance coefficient is different again, and still significantly biased. Under this new assumption the equation for the permanent insurance coefficient

is unbiased as before:

$$\frac{\text{Cov}(\Delta c_T^{obs}, \Delta y_{T-1}^{obs} + \Delta y_T^{obs} + \Delta y_{T+1}^{obs})}{\text{Cov}(\Delta y_T^{obs}, \Delta y_{T-1}^{obs} + \Delta y_T^{obs} + \Delta y_{T+1}^{obs})} = \phi$$

While the identifying equation for the transitory insurance coefficient is:

$$\frac{\text{Cov}(\Delta c_T^{obs}, \Delta y_{T+1}^{obs})}{\text{Cov}(\Delta y_T^{obs}, \Delta y_{T+1}^{obs})} = \frac{-\phi \frac{1}{6} \sigma_P^2 + \frac{1}{2} \psi \sigma_Q^2}{-\frac{1}{6} \sigma_P^2 + \sigma_Q^2} \neq \psi$$

Under the permanent income hypothesis with $\phi = 1$, $\psi = 0$ and permanent and transitory variances approximately equal, the BPP estimate of ψ would be -0.2.

B4. Other Tables from the BPP paper

Table B1 replicates Table 6 from the original BPP paper.

Table B2 replicates Table 7 from the original BPP paper.

Table B3 replicates Table 8 from the original BPP paper.

TABLE B1—MINIMUM-DISTANCE PARTIAL INSURANCE AND VARIANCE ESTIMATES

		Whole Sample		No College		College	
		BPP	Time Agg.	BPP	Time Agg.	BPP	Time Agg.
$\sigma_{P,T}^2$ (Variance perm. shock)	1979-1981	0.0103	0.0247	0.0068	0.0234	0.0101	0.0189
		(0.0034)	(0.0043)	(0.0037)	(0.0063)	(0.0053)	(0.0050)
	1982	0.0208	0.0358	0.0156	0.0290	0.0253	0.0455
		(0.0041)	(0.0071)	(0.0052)	(0.0099)	(0.0060)	(0.0099)
	1983	0.0301	0.0333	0.0318	0.0553	0.0234	0.0086
		(0.0057)	(0.0100)	(0.0074)	(0.0128)	(0.0090)	(0.0148)
	1984	0.0274	0.0292	0.0334	0.0232	0.0177	0.0361
		(0.0049)	(0.0114)	(0.0073)	(0.0131)	(0.0060)	(0.0161)
	1985	0.0295	0.0363	0.0287	0.0504	0.0208	0.0025
		(0.0096)	(0.0124)	(0.0073)	(0.0145)	(0.0152)	(0.0205)
	1986	0.0221	0.0327	0.0173	0.0247	0.0311	0.0597
		(0.0060)	(0.0136)	(0.0067)	(0.0172)	(0.0101)	(0.0202)
	1987	0.0289	0.0420	0.0202	0.0478	0.0354	0.0229
		(0.0063)	(0.0143)	(0.0073)	(0.0182)	(0.0098)	(0.0211)
	1988	0.0158	0.0082	0.0117	-0.0069	0.0183	0.0302
		(0.0069)	(0.0137)	(0.0079)	(0.0209)	(0.0110)	(0.0149)
	1989	0.0185	0.0531	0.0107	0.0639	0.0274	0.0414
		(0.0059)	(0.0129)	(0.0101)	(0.0214)	(0.0061)	(0.0149)
	1990-92	0.0135	0.0291	0.0093	0.0265	0.0217	0.0291
		(0.0042)	(0.0042)	(0.0045)	(0.0063)	(0.0065)	(0.0057)
$\sigma_{Q,T}^2$ (Variance trans. shock)	1979	0.0379	0.0310	0.0465	0.0364	0.0301	0.0261
		(0.0059)	(0.0049)	(0.0096)	(0.0080)	(0.0056)	(0.0043)
	1980	0.0298	0.0240	0.0330	0.0247	0.0283	0.0238
		(0.0039)	(0.0033)	(0.0053)	(0.0046)	(0.0059)	(0.0047)
	1981	0.0300	0.0265	0.0363	0.0305	0.0253	0.0222
		(0.0035)	(0.0032)	(0.0053)	(0.0048)	(0.0046)	(0.0040)
	1982	0.0287	0.0280	0.0375	0.0332	0.0213	0.0237
		(0.0039)	(0.0034)	(0.0063)	(0.0057)	(0.0042)	(0.0036)
	1983	0.0262	0.0276	0.0371	0.0378	0.0185	0.0169
		(0.0037)	(0.0034)	(0.0063)	(0.0056)	(0.0037)	(0.0040)
	1984	0.0346	0.0350	0.0404	0.0388	0.0304	0.0315
		(0.0039)	(0.0038)	(0.0059)	(0.0058)	(0.0051)	(0.0046)
	1985	0.0450	0.0427	0.0355	0.0338	0.0496	0.0465
		(0.0075)	(0.0071)	(0.0056)	(0.0053)	(0.0130)	(0.0122)
	1986	0.0458	0.0404	0.0474	0.0373	0.0452	0.0464
		(0.0058)	(0.0055)	(0.0076)	(0.0068)	(0.0085)	(0.0084)
	1987	0.0461	0.0445	0.0520	0.0486	0.0421	0.0385
		(0.0054)	(0.0053)	(0.0082)	(0.0078)	(0.0071)	(0.0069)
	1988	0.0399	0.0327	0.0471	0.0360	0.0343	0.0313
		(0.0047)	(0.0044)	(0.0074)	(0.0072)	(0.0060)	(0.0055)
	1989	0.0378	0.0343	0.0539	0.0475	0.0219	0.0215
		(0.0067)	(0.0061)	(0.0126)	(0.0117)	(0.0051)	(0.0044)
	1990-92	0.0441	0.0359	0.0535	0.0408	0.0345	0.0322
		(0.0040)	(0.0027)	(0.0062)	(0.0047)	(0.0049)	(0.0032)
θ (Serial correl. trans. shock)		0.1126	N/A	0.1260	N/A	0.1082	N/A
		(0.0248)		(0.0319)		(0.0342)	
σ_{ξ}^2 (Variance unobs. slope heterog.)		0.0097	0.0122	0.0065	0.0114	0.0132	0.0146
		(0.0041)	(0.0039)	(0.0079)	(0.0070)	(0.0040)	(0.0039)
ϕ (Partial insurance perm. shock)		0.6456	0.3384	0.9484	0.4365	0.4180	0.2729
		(0.0941)	(0.0471)	(0.1773)	(0.0738)	(0.0913)	(0.0603)
ψ (Partial insurance trans. shock)		0.0501	0.2421	0.0724	0.2870	0.0260	0.1590
		(0.0430)	(0.0431)	(0.0593)	(0.0616)	(0.0546)	(0.0504)

TABLE B2—MINIMUM-DISTANCE PARTIAL INSURANCE AND VARIANCE ESTIMATES

Consumption: Income: Sample:	Nondurable net income baseline		Nondurable earnings only baseline		Nondurable male earnings baseline	
	BPP	Time Agg.	BPP	Time Agg.	BPP	Time Agg.
ϕ	0.6456	0.3384	0.3101	0.1761	0.2240	0.1232
(Partial insurance perm. shock)	(0.0941)	(0.0471)	(0.0572)	(0.0339)	(0.0492)	(0.0316)
ψ	0.0501	0.2421	0.0630	0.1625	0.0502	0.1181
(Partial insurance trans. shock)	(0.0430)	(0.0431)	(0.0306)	(0.0280)	(0.0293)	(0.0244)

TABLE B3—MINIMUM-DISTANCE PARTIAL INSURANCE AND VARIANCE ESTIMATES

Consumption: Income: Sample:	Nondurable net income baseline		Nondurable excluding help baseline		Nondurable net income low wealth	
	BPP	Time Agg.	BPP	Time Agg.	BPP	Time Agg.
ϕ	0.6456	0.3384	0.6244	0.3422	0.8339	0.8584
(Partial insurance perm. shock)	(0.0941)	(0.0471)	(0.0891)	(0.0466)	(0.2762)	(0.2498)
ψ	0.0501	0.2421	0.0469	0.2404	0.2853	0.4926
(Partial insurance trans. shock)	(0.0430)	(0.0431)	(0.0429)	(0.0427)	(0.1154)	(0.1050)

Consumption: Income: Sample:	Nondurable net income high wealth		Total net income low wealth		Nondurable net income baseline+SEO	
	BPP	Time Agg.	BPP	Time Agg.	BPP	Time Agg.
ϕ	0.6278	0.2691	1.0207	1.0580	0.7663	0.4630
(Partial insurance perm. shock)	(0.0998)	(0.0420)	(0.3426)	(0.3099)	(0.1028)	(0.0499)
ψ	0.0088	0.1838	0.3647	0.6185	0.1201	0.3232
(Partial insurance trans. shock)	(0.0409)	(0.0409)	(0.1477)	(0.1344)	(0.0352)	(0.0367)