1 Sufficient Statistics

I match the expressions in your paper with the sufficient statistics in Auclert (2017) as below.

$$
\begin{aligned}
M &= E_I\Big[\frac{Y_i}{Y}MPC_i\Big]\frac{Y}{C} \\
\gamma\mathcal{E}_Y &= E_I\Big[MPC_i\Big(\frac{dY_i}{dY}-\frac{Y_i}{Y}\Big)\Big] = E_I\Big[MPC_i\Big(\frac{dY_i}{Y_i}-\frac{dY}{Y}\Big)\frac{Y_i}{Y}\frac{Y}{dY}\Big]\frac{Y}{C} \\
\mathcal{E}_p &= COV_I(MPC_i, NNP_i)/C \\
\mathcal{E}_R &= COV_I(MPC_i, URE_i)/C \\
\sigma\mathcal{S} &= E_I[\sigma_i(1-MPC_i)c_i]/C
\end{aligned}
$$

where $Y = C+G$ and $MPC_i = \widehat{MPC}_i$ since $MPN_i = \frac{\partial n}{\partial y} = 0$ with GHH preference[1] in the one asset IOU model. I recognize $\frac{dYi}{Yi}$ and $\frac{dY}{Y}$ as impulse responses of individual households' income and aggregate income, respectively. Individual household income is sum of labor income and profit as in Auclert (2017).

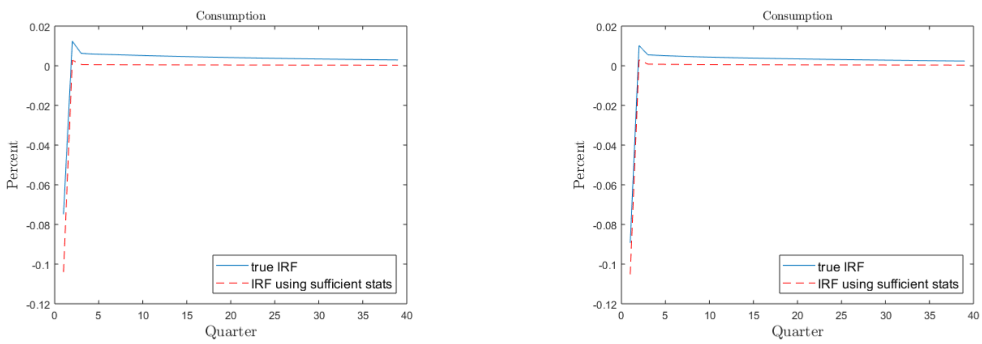
The values of the sufficient statistics in the model are as the following.[2] The signs of all statistics in the model are same with those in your data except earning heterogeneity channel.

Table 1: Sufficient statistics in One asset IOU model

M	$\gamma\mathcal{E}_Y$	$\mathcal{E}_p$	$\mathcal{E}_R$	$\sigma\mathcal{S}$
0.21	0.16	-0.51	-0.48	0.43

The figure shows two impulse responses of aggregate consumption. The true IRF denotes impulse responses that are derived directly from the model and the other one is indirectly derived by using above sufficient statistics and other impulse responses of relevant endogeneous variables. Figure 1 compares two impulse responses with respect to 0.1% monetary policy shock.

Figure 1: Comparison of consumption Impulse Responses



(a) $\frac{B}{Y} = 2$ (b) $\frac{B}{Y} = 1.5$

I find that the impulse responses highly depend on the ratio of government debt to output in the steady state. I set the ratio 2.0 (1.5), which results in the saving

[1] No wealth effect in labor supply with GHH preference

[2] This result is based on $\frac{B}{Y} = 2$

rate as 0.46 (0.15)% (quarterly). The difference between true IRFs and IRFs by sufficient statistics goes down as the ratio of debt to output decreases. However, too low debt to output ratio can lead to excessively low saving rate.

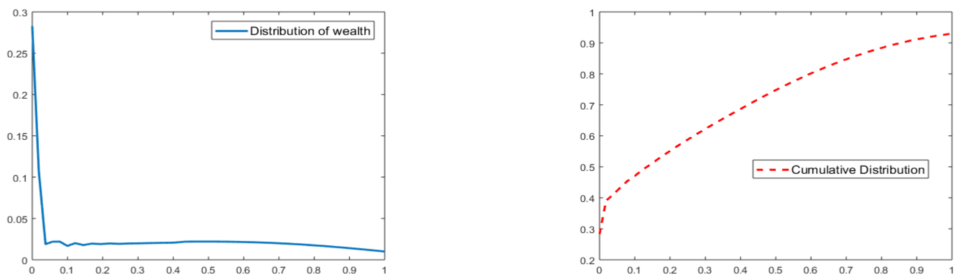
Table 2 shows values of calibrated parameters. I set the persistence of the Taylor rule 0 since it causes persistence of the impulse responses. However, I use high persistence parameter for the fiscal policy, which is nearly irrelevant to the shape of impulse responses. I set CRRA parameter as 1 in order to make mass at the constraint larger.

Table 2: Calibrated Parameters

Calibrated Parameters	Description	Value
Preferences		
β	Discount factor	0.985
σ_c	Relative risk aversion	1
γ	Inverse of Frisch elasticity	1
Income process		
ρ_h	Persistence of productivity shock	0.979
σ_h	Variance of productivity shock	0.059
$P(E \text{ } or \text{ } B)$	Prob. to become entrepreneurs or bankers	0.0005
$P(W)$	Probability to become workers again	0.0625
Final Goods		
κ	Price stickiness	0.0883
μ	5% Markup	0.05
Government		
τ_t	Labor tax	5%
$\frac{B}{Y}$	Government debt/output	200%
Monetary policy		
ϕ_π^M	reaction to inflation	1.25
ρ_R	Persistence	0.0
Fiscal policy		
ϕ_π^F	reaction to inflation	1.5
ρ_R	Reaction to tax	0.5
ρ_B	Persistence	0.99

Figure 2b shows wealth distribution around the borrowing constraint. Almost 30 % of households are at the constraint, which leads to high aggregate income and earning heterogeneity channels.

Figure 2: Wealth distributions around borrowing constraint



(a) Distribution

(b) Cumulative distribution

Appendix

A Details in each channel

A.1 Earning heterogeneity channel

$$E_I[MPC_i(\frac{dY_i}{Y_i} - \frac{dY}{Y})\frac{Y_i}{Y}\frac{Y}{dY}] = E_I[MPC_i(IRF_{Y_i} - IRF_Y)\frac{Y_i}{Y}\frac{1}{IRF_Y}]$$

, where IRF_{Y_i} and IRF_Y are impulse responses of individual income (labor or profit) and aggregate output, respectively.

A.2 Substitution channel

$$\sigma_i = -\frac{u^{'}}{u^{''}c_{it}} = \frac{x_{it}}{\xi c_{it}}$$

, where $u = \frac{(c_{it} - \frac{h_{it}}{1+\gamma}n_{it}^{1+\gamma})^{1-\xi}}{1-\xi}$ and $x_{it} = c_{it} - \frac{h_i t}{1+\gamma}n_{it}^{1+\gamma}$