ESSAYS ON CONSUMPTION

by

Edmund Crawley

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Abstract

These essays study individual consumption behavior and its implications for macroeconomics. Starting with microeconomic measurement, chapter one improves on methods to estimate how households respond to income shocks. Chapter two builds on these methods, applying them to registry data from Denmark, and uses the resulting estimates to calculate the size of redistribution channels of monetary policy. Chapter three models the redistribution channels in a New Keynesian framework.

Chapter one builds on Working's observation that time aggregation of a random walk induces serial correlation in the first differences that is not present in the original series. This important contribution has been overlooked in a large recent literature analyzing income and consumption in panel data. This chapter takes Blundell, Pistaferri, and Preston (2008) as an example and shows how to correct for this problem. I find the estimate for the partial insurance to transitory shocks, originally estimated to be 5%, is equal to 24% when corrected for time aggregation. This estimate is much closer to estimates from the literature that uses natural experiments to estimate the marginal propensity to consume out of transitory shocks.

ABSTRACT

Chapter two aims to test the microfoundations of consumption models and quantify the macro implications of consumption heterogeneity. We propose a new empirical method to estimate the sensitivity of consumption to permanent and transitory income shocks for different groups of households. We then apply this method to administrative data from Denmark. The large sample size, along with detailed household balance sheet information, allows us to finely divide the population along relevant dimensions. For example, we find that households who stand to lose from an interest rate hike are significantly more sensitive to income shocks than those who stand to gain. Following a one percentage point rate increase, we estimate consumption will decrease by 26 basis points through this interest rate exposure channel alone, making this channel substantially larger than the intertemporal substitution channel that is the key mechanism in representative agent New Keynesian models.

In chapter three we analyze the transmission mechanism of monetary policy to consumption in New Keynesian models with heterogeneous agents. We show that in these models the countercyclical nature of profits, empirically false, plays a large role in amplifying the intertemporal substitution channel. On the other hand the interest rate exposure channel, empirically large, plays a small role. Our analysis makes use of the partial equilibrium decomposition of Auclert (2017) which we show to perform well even in models where the assumptions do not hold. We suggest expanding the role of the interest rate exposure channel, while dampening the amplification effect of countercyclical profits, is of primary quantitative importance in future work.

ABSTRACT

Keywords Consumption, Marginal Propensity to Consume, Heterogeneity, Monetary Policy, Redistribution

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Dedication

This thesis is dedicated to Christine.

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Chapter 1

Time Aggregation in Panel Data on Income and Consumption

1.1 Abstract

In 1960 Working noted that time aggregation of a random walk induces serial correlation in the first differences that is not present in the original series. This important contribution has been overlooked in a large recent literature analyzing income and consumption in panel data. This paper takes Blundell, Pistaferri, and Preston (2008) as an example and shows how to correct for this problem. I find the estimate for the partial insurance to transitory shocks, originally estimated to be 5%, is equal to 24% when corrected for time aggregation. This estimate is much closer to estimates from the literature that uses natural experiments to estimate the marginal

propensity to consume out of transitory shocks.

1.2 Introduction

In a short note in Econometrica, Working (1960) made the simple but important point that time aggregation can induce serial correlation that is not present in the original series. This fact was readily absorbed by the macroeconomic literature, where such time aggregated series are common (for an example see Campbell and Mankiw (1989)). Recently, by studying the covariance structure of panel data, much progress has been made in understanding household income and consumption dynamics. However, this literature has not accounted for the serial correlation induced by the time aggregated nature of observed income and consumption data. This oversight can result in significant bias. This paper will focus on the implications of time aggregation for the methodology in Blundell, Pistaferri, and Preston (2008) (henceforth BPP), but it applies to a broad swath of the literature. I show that the pass through from transitory income to consumption, originally estimated by BPP to be 5%, is close to 25% when the serial correlation in the data induced by time aggregation is accounted for.

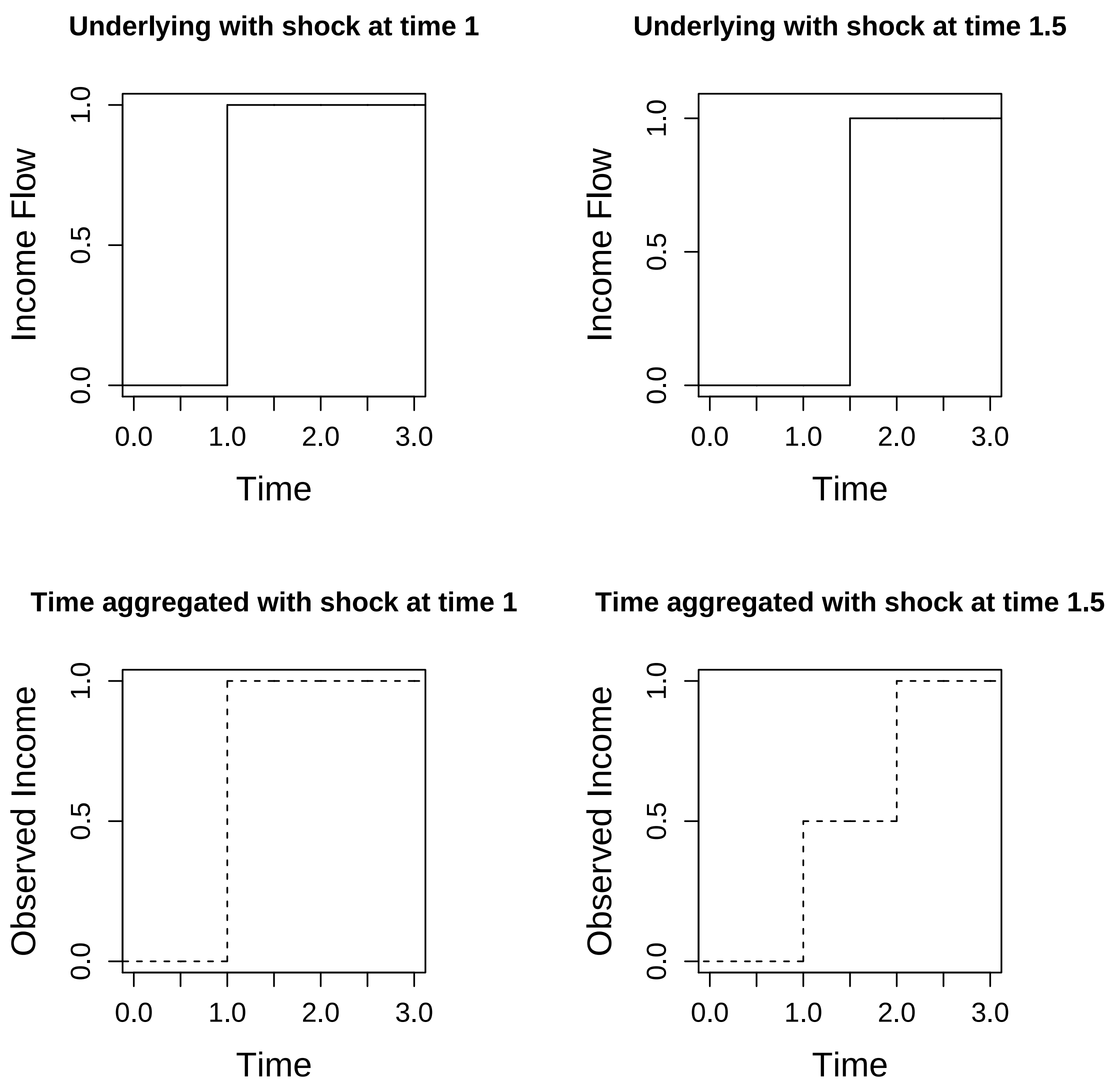
1.2.1 What is Time Aggregation?

Many observed time series in economics are given at a lower frequency than the underlying data that generates them. For example, income is often observed at an annual frequency when it may in fact consist of paychecks arriving at a monthly, biweekly or irregular timetable. To transform income into an annual frequency we sum up all the income that was received by a household during the year, a process known as time aggregation. The key insight of Working (1960) is that even if there is no correlation between changes in income at the underlying frequency, the resulting time aggregated series will show positive autocorrelation. The intuition behind this can be seen in figure 1.1 showing an income process that begins at zero and increases to one in the second year. The top left graph shows the path of income if the shock occurs exactly at the start of the second year, and the bottom left graph shows the time aggregated process exactly mirrors that. There is no income in the first year and one unit of income in each of the second and third years. The top right shows an alternative income process in which the shock occurs half way through the second year. Now the resulting time aggregated process (bottom right) does not mirror the underlying. As before there is no income in the first year, but in the second year the individual receives an income of one for half the year, resulting in a time aggregated income of 0.5. In the third year the individual receives an income of one for the entire year, and hence a time aggregated income of one. If we can only see the time aggregated process, when we observe income increasing from year one to year two we do not know if the shock occurred at the beginning of the year or half way through. If it occurred at the beginning of the year, as in the left hand graphs of figure 1.1, then we would not expect to see any further increase in the time aggregated process associated with it. However, if it occurred half way through, as in the right hand graphs of figure 1.1, we would only have observed half the total increase in income and would expect the time aggregated process to continue to increasing in the following period. Therefore, assuming there is some positive probability that the shock occurred half way through the second period, we would expect to see further increases in the observed process. This is how time aggregation induces serial correlation in the first difference of an observed process even when the underlying process is a random walk. Section 1.3 lays this out formally and shows that this autocorrelation tends to $\frac{1}{4}$ as the number of time subdivisions increases to infinity.

1.3 Time Aggregated Random Walk

I this section I formally prove that a time aggregated random walk is autocorrelated and show that this autocorrelation tends to $\frac{1}{4}$ as the number of time subdivisions increases to infinity. I will also introduce continuous time notation that will be used for the underlying model in section 1.4.

Figure 1.1: Time Aggregation Induces Serial Correlation



1.3.1 The two sub-division case

I begin with the two-subdivision case. The underlying income process follows a random walk at discrete time periods $t \in \{0, 1, 2, 3, ...\}$:

$$y_t = \begin{cases} 0 & \text{if } t = 0 \\ y_{t-1} + \varepsilon_t & \text{otherwise} \end{cases}$$

where ε_t is i.i.d. and has variance σ^2. The time aggregated process is observed every two periods at $T \in \{2, 4, 6, ...\}$ and is equal to the sum of income over the two periods leading up to it:

$$y_T^{obs} = y_T + y_{T-1}$$

The observed income change is given by:

$$\begin{aligned} \Delta^2 y_T^{obs} &= y_T^{obs} - y_{T-2}^{obs} \\ &= \varepsilon_T + 2\varepsilon_{T-1} + \varepsilon_{T-2} \end{aligned}$$

This allows for easy calculation of the serial correlation:

$$\begin{aligned} \text{Cov}(\Delta^2 y_T^{obs}, \Delta^2 y_{T-2}^{obs}) &= \sigma^2 \\ \text{Var}(\Delta^2 y_T^{obs}) &= \sigma^2 + 4\sigma^2 + \sigma^2 \\ &= 6\sigma^2 \\ \text{Corr}(\Delta^2 y_T^{obs}, \Delta^2 y_{T-2}^{obs}) &= \frac{1}{6} \end{aligned}$$

1.3.2 The N sub-division case

The two sub-division case easily extends to N sub-divisions. Using the same underlying income process, the observable income process is now aggregated over N periods:

$$y_T^{obs} = \sum_{t=T-N+1}^{T} y_t$$

So that the observed change in income is:

$$\begin{aligned}
\Delta^N y_T^{obs} &= \sum_{t=T-N+1}^{T} y_t - y_{t-N} \\
&= \varepsilon_T + \varepsilon_{T-1} + \quad \dots \quad + \varepsilon_{T-N+2} + \varepsilon_{T-N+1} \\
&\qquad + \varepsilon_{T-1} + \varepsilon_{T-2} + \quad \dots \quad + \varepsilon_{T-N+1} + \varepsilon_{T-N} \\
&\qquad\qquad + \varepsilon_{T-2} + \varepsilon_{T-3} + \quad \dots \\
&\qquad\qquad\qquad \dots \\
&\qquad\qquad\qquad\qquad + \varepsilon_{T-N+1} + \quad \dots \quad + \varepsilon_{T-2N+2} \\
&= N\varepsilon_{T-N+1} + \sum_{i=1}^{N-1} i\Big(\varepsilon_{T-i+1} + \varepsilon_{T-2N+i+1}\Big)
\end{aligned}$$

We can now calculate the autocorrelation:

$$\begin{aligned}
\mathrm{Cov}(\Delta^N y_T^{obs}, \Delta^N y_{T-N}^{obs}) &= \sum_{i=1}^{N-1} i(N-i)\sigma^2 \\
&= \frac{N(N^2-1)}{6}\sigma^2 \\
\mathrm{Var}(\Delta^N y_T^{obs}) &= N^2\sigma^2 + 2\sum_{i=1}^{N-1} i^2\sigma^2 \\
&= \frac{N(2N^2+1)}{3}\sigma^2 \\
\mathrm{Corr}(\Delta^N y_T^{obs}, \Delta^N y_{T-N}^{obs}) &= \frac{N^2-1}{2(2N^2+1)} \to \frac{1}{4} \text{ as } N \to \infty
\end{aligned}$$

1.3.3 The Continuous Time Case

It will turn out to be significantly simpler to work with a model in which shocks can occur at any point in continuous time. Here I introduce some notation for such a model, and show that it gives a good approximation even if the actual underlying process is discrete (say quarterly or monthly).

The underlying income process will be modeled as a martingale process in continuous time, y_t, where for all $s_1 > s_2 > s_3 > s_4 > 0$:

$$\mathrm{Var}(y_{s_1} - y_{s_2}) = (s_1 - s_2)\sigma^2$$

$$\mathrm{Cov}(y_{s_1} - y_{s_2}, y_{s_3} - y_{s_4}) = 0$$

$$y_s = 0 \qquad \text{if } s < 0$$

The process has independent increment increments. A Brownian motion would satisfy these criteria, but although in continuous time there is no restriction that it is a continuous process (it may have jumps).[1] The observed income process is the sum of income over a year:

$$\begin{aligned}\bar{y}_T &= \int_{T-1}^{T} y_t dt \\ &= \int_{T-1}^{T}\int_{0}^{t} dy_s dt\end{aligned}$$

So that:

$$\begin{aligned}\Delta\bar{y}_T &= \int_{T-1}^{T}\int_{0}^{t} dy_s dt - \int_{T-2}^{T-1}\int_{0}^{t} dy_s dt \\ &= \int_{T-1}^{T}\int_{t-1}^{t} dy_s dt \\ &= \int_{T-1}^{T}(T-s)dy_s + \int_{T-2}^{T-1}(s-(T-2))dy_s\end{aligned}$$

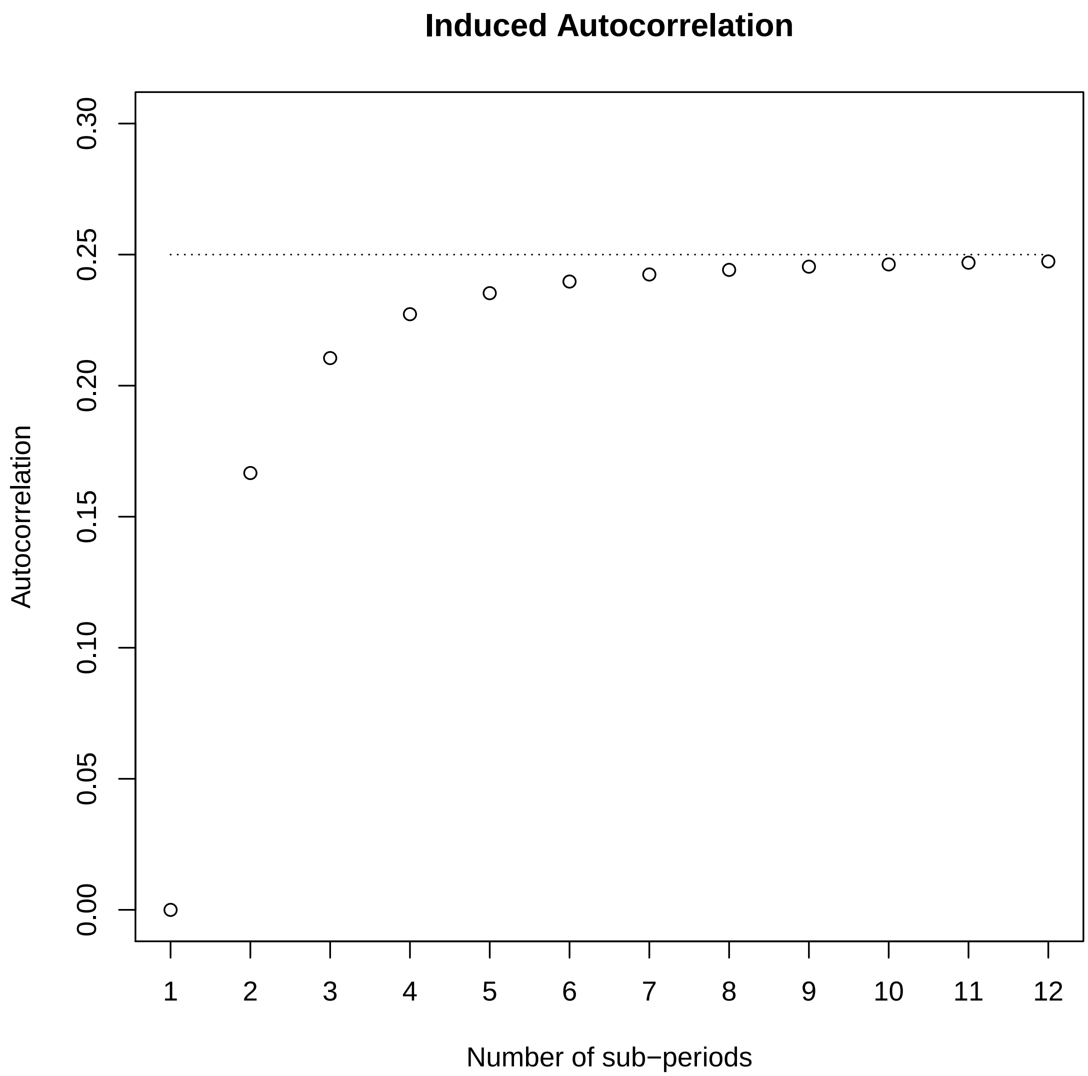
The autocorrelation can now be calculated:

$$\begin{aligned}\text{Cov}(\Delta\bar{y}_T, \Delta\bar{y}_{T-1}) &= \int_{T-2}^{T-1}(T-1-s)(s-(T-2))\sigma^2 dt \\ &= \frac{1}{6}\sigma^2 \\ \text{Var}(\Delta\bar{y}_T) &= \int_{T-1}^{T}(T-s)^2\sigma^2 dt + \int_{T-2}^{T-1}(s-(T-2))^2\sigma^2 dt \\ &= \frac{2}{3}\sigma^2 \\ \text{Corr}(\Delta\bar{y}_T, \Delta\bar{y}_{T-1}) &= \frac{1}{4}\end{aligned}$$

Which is unsurprisingly the same as the limit of the autocorrelation in the N sub-periods case. Figure 1.2 shows how fast the N sub-period case converges towards the

[1] Note that such a process will take both positive and negative values, and therefore may not be a good choice for an income process. In appendix 1.6.1, by looking at the limit of discrete time models with m sub-periods, I show that under certain assumptions the same results approximately hold when shocks are multiplicative rather than additive.

Figure 1.2: Induced Autocorrelation for different N



continuous time case. When $N = 1$ the time aggregated process is the same as the underlying random walk so the autocorrelation is zero. When income is quarterly ($N = 4$) the autocorrelation is 0.23 and is closely approximated by the continuous time model. With monthly ($N = 12$) or higher frequency for income shocks the discrete and continuous models are almost indistinguishable.

1.4 Time Aggregation in Blundell, Pistaferri, and Preston (2008)

I will focus on the methodology for estimating partial "insurance" coefficients to transitory and permanent shocks introduced by Blundell, Pistaferri, and Preston (2008). I choose this paper both because it provides a clear example where time aggregation biases the results quantitatively, and also because the methodology has become common place in the literature and could now be considered a workhorse model. Indeed Kaplan and Violante (2010) state in their paper that applies the method to simulated data that "we argue that the BPP insurance coeffcients should become central in quantitative macroeconomics"

1.4.1 The Model in Discrete Time Without Time Aggregation

Here I briefly describe the method followed by Blundell, Pistaferri, and Preston (2008). For more detail please refer to their original paper. The core of the model are their assumptions on the income and consumption processes. The model described here is a simplified version of the original in order to highlight the role played by time aggregation.[2]

Unexplained log income growth for household i follows the process:

$$\Delta y_{i,t} = \zeta_{i,t} + \Delta \nu_{i,t}$$

where $\zeta_{i,t}$ (the change in permanent income) and $\nu_{i,t}$ (transitory income) are each i.i.d. and independent of each other. The variance of permanent shocks ($\sigma_\zeta^2 = \mathrm{Var}(\zeta_{i,t})$) and transitory shocks ($\sigma_\nu^2 = \mathrm{Var}(\nu_{i,t})$) will be of interest. These variances can be identified from observable data by noting the following identities (where the household identifier

[2] In this simplified model I assume insurance parameters are constant accross both time and households, that the transitory component of income has no persistence, and that there are no taste shocks. These elements are reintroduced in section 1.4.3 in which I show the quantitative effect of time aggregation.

i has been removed for clarity):

$$
\begin{aligned}
\sigma_\zeta^2 &= \mathrm{Var}(\zeta_t) \\
&= \mathrm{Cov}(\Delta y_t, \Delta y_{t-1} + \Delta y_t + \Delta y_{t+1}) && (1.1) \\
\sigma_\nu^2 &= \mathrm{Var}(\nu_t) \\
&= -\mathrm{Cov}(\Delta y_t, \Delta y_{t+1}) && (1.2)
\end{aligned}
$$

The unexplained change in log consumption is modeled as a random walk that moves in response to changes in both permanent income and transitory income:

$$\Delta c_{i,t} = \phi \zeta_{i,t} + \psi \nu_{i,t}$$

where ϕ and ψ are the *partial insurance* parameters for permanent and transitory shocks respectively. A value of zero implies full insurance (consumption does not respond at all to the income shock), while a value of one implies no insurance. These insurance parameters can be identified in the data from these identities:

$$\phi = \frac{\mathrm{Cov}(\Delta c_t, \Delta y_{t-1} + \Delta y_t + \Delta y_{t+1})}{\mathrm{Cov}(\Delta y_t, \Delta y_{t-1} + \Delta y_t + \Delta y_{t+1})} \tag{1.3}$$

$$\psi = \frac{\mathrm{Cov}(\Delta c_t, \Delta y_{t+1})}{\mathrm{Cov}(\Delta y_t, \Delta y_{t+1})} \tag{1.4}$$

It is useful to think of equations 1.3 and 1.4 as IV regressions of consumption growth on income growth where (1.3) uses income growth over 3 periods as an instrument to identify permanent shocks, while (1.4) uses income growth in the following period to identify transitory shocks (a transitory shock to income today predicts that income will go down by the same amount in the following period).

The four equations 1.1, 1.2, 1.3 and 1.4 are the core of the BPP identification methodology. In the following section I will show how this identification fails when time aggregation is accounted for.

1.4.2 The Model in Continuous Time with Time Aggregation

The model in this section will be the exact analog of the discrete time BPP model just described, but embedded in continuous time where shocks are spread uniformly throughout the year.[3] For the income process we will assume two underlying martingale processes (possibly with jumps), P_t and Q_t such that for all $s_1 > s_2 > s_3 > s_4 > 0$:

$$\begin{aligned}\mathrm{Var}(P_{s_1} - P_{s_2}) &= (s_1 - s_2)\sigma_P^2 \\ \mathrm{Cov}(P_{s_1} - P_{s_2}, P_{s_3} - P_{s_4}) &= 0 \\ P_s = 0 \qquad &\text{if } s < 0\end{aligned}$$

and similarly for Q_t. Instantaneous income in a period dt is given by:[4]

$$dy_t = \Big(\int_0^t dP_s\Big)dt + dQ_t \tag{1.5}$$

that is they receive their permanent income ($P_t = \int_0^t dP_s$) flow multiplied by time dt in addition to a one-off transitory income dQ_t.

[3] There is little formal evidence on the distribution of shocks throughout the year. While this assumption is unlikely to be strictly true, it is more reasonable that the implicit assumption of BPP that shocks all occur 1st January each year.

[4] A more formal treatment of how to relate this to the log income process is given in appendix 1.6.1

Keeping with the assumption that consumption is a random walk with insurance parameters ϕ and ψ, instantaneous consumption is given by

$$dc_t = \phi\Big(\int_0^t dP_s\Big)dt + \psi\Big(\int_0^t dQ_s\Big)dt \tag{1.6}$$

that is they consume a proportion ϕ of their permanent income and a proportion ψ of the cumulation of all the transitory income they have received in their lifetime.

The Panel Study of Income Dynamics (PSID) data, we observe the total income received over the previous calendar year:

$$y_T^{obs} = \int_{T-1}^{T} dy_t$$

BPP use data on food consumption to impute total annual consumption. The questionnaire asks about food consumption in a typical week, but unfortunately the timing of this 'typical week' is not clear. The questionnaire is usually given at the end of March in the following year. See Altonji and Siow (1987) and Hall and Mishkin (1982) for differing views. Here I will assume the 'typical week' occurs exactly at the end of the calendar year, so it measures a snapshot of consumption at time T

$$c_T^{obs} = \phi\Big(\int_0^t dP_s\Big) + \psi\Big(\int_0^t dQ_s\Big)$$

In appendix 1.6.4 I show that the timing of the 'typical' week can have a big effect on the results. This is a big drawback to using this method with the PSID data. In a chapter 2 of these essays we use expenditure data imputed from Danish administrative records in which the timing of expenditure is very clearly defined.

The BPP method makes use of the changes in observable income and consumption:

$$\begin{aligned}\Delta y_T^{obs} &= \Big(\int_{T-2}^{T-1}(s-(T-2))dP_s + \int_{T-1}^{T}(T-s)dP_s\Big) \\ &\quad + \Big(\int_{T-1}^{T} dQ_t - \int_{T-2}^{T-1} dQ_t\Big) \qquad (1.7) \\ \Delta c_T^{obs} &= \phi\int_{T-1}^{T} dP_s + \psi\int_{T-1}^{T} dQ_s \qquad (1.8)\end{aligned}$$

If we use the identification of permanent and transitory variances in equations 1.1 and 1.2 from the discrete time model we get:

$$\begin{aligned}\mathrm{Cov}(\Delta y_T^{obs}, \Delta y_{T-1}^{obs} + \Delta y_T^{obs} + \Delta y_{T+1}^{obs}) &= \sigma_P^2 \\ -\mathrm{Cov}(\Delta y_T^{obs}, \Delta y_{T+1}^{obs}) &= -\frac{1}{6}\sigma_P^2 + \sigma_Q^2 \neq \sigma_Q^2\end{aligned}$$

This shows that identification of the variance of permanent shocks, σ_P^2, is unbiased, while that of transitory shocks is biased down by $\frac{1}{6}\sigma_P^2$. Turning to the identification of ϕ and ψ in equations 1.3 and 1.4 we have:

$$\begin{aligned}\frac{\mathrm{Cov}(\Delta c_T^{obs}, \Delta y_{T-1}^{obs} + \Delta y_T^{obs} + \Delta y_{T+1}^{obs})}{\mathrm{Cov}(\Delta y_T^{obs}, \Delta y_{T-1}^{obs} + \Delta y_T^{obs} + \Delta y_{T+1}^{obs})} &= \phi \qquad (1.9) \\ \frac{\mathrm{Cov}(\Delta c_T^{obs}, \Delta y_{T+1}^{obs})}{\mathrm{Cov}(\Delta y_T^{obs}, \Delta y_{T+1}^{obs})} &= \frac{-\phi\frac{1}{2}\sigma_P^2 + \psi\sigma_Q^2}{-\frac{1}{6}\sigma_P^2 + \sigma_Q^2} \neq \psi \qquad (1.10)\end{aligned}$$

Again identification of the permanent insurance coefficient, ϕ, is unbiased, but the transitory insurance coefficient bears little relation to the true value of ψ. For example, if the household follows the permanent income hypothesis with values $\phi = 1$ and $\psi = 0$, and permanent and transitory income variances are close to equal, the BPP method would estimate ψ to be -0.6. The fact that BPP estimate ψ to be close to zero suggests the numerator in equation 1.10 is close to zero, that is in fact $\psi \approx \frac{1}{2}\phi\frac{\sigma_P^2}{\sigma_Q^2}$. With approximately equal permanent and transitory variances this suggest the transitory insurance coefficient, far from being close to zero, is in fact about half the value of the permanent insurance coefficient. In section 1.4.3 I repeat the GMM exercise of BPP, using the same empirical moments, but with identification coming from the continuous time model with time aggregated income. The full set of identification equations, with the model extended to include time varying coefficients, transitory persistence and taste shocks, can be found in appendices 1.6.2 and 1.6.3.

1.4.3 The Evidence

The columns labeled 'BPP' in table 1.1 replicate the columns from table 6 from the original BPP paper. Next to each of these columns I have reported the equivalent estimate from the continuous time model with time aggregation (and no persistence in the transitory shock). The most notable changes are to the partial insurance parameters ϕ and ψ. Given the results from section 1.3.3, it should not be surprising that the coefficient for transitory shocks has changed significantly, from 5% to 24% in the whole sample. The fact that the coefficient for permanent shock insurance has also changed, from 65% to 34%, is somewhat surprising given the theory suggested it should not change when transitory shocks are not persistent. When there is persistence in transitory shocks, the identification of ϕ in the two models in no longer the same. In section 1.4.4 I show how the estimate for ϕ is very sensitive to the degree of persistence in the discrete time model, which can explain why we observe a change in the estimate of ϕ. The estimates for the no college and college sub-samples also move in similar ways, but the qualitative result that college educated households have significantly more insurance against income shocks holds.

The whole sample permanent and transitory variances from table 6 are plotted in figure 1.3. The transitory shock variances are of similar magnitude and follow the same pattern of increasing in the mid-80's as the original estimates of BPP. The permanent shock variances are now slightly larger (although again this is sensitive to the degree of persistence in transitory shocks). The sharp decrease in 1988, followed by increase in 1989, seems strange. However, the standard errors at these points are relatively large (approx 0.013) such that this pattern may be a result of statistical noise. Note that the standard errors for the permanent variances are approximately twice as large in the time aggregated model compared to the original BPP method.

In appendix 1.6.5 I have reproduced all the estimation tables from the BPP paper, along with the time aggregated estimates. As with the college/no college cohort results, the insurance coefficient accross cohorts move in the same direction as they do in BPP's estimates, but they are quantitatively very different.

1.4.4 Persistence in the Transitory Shock

The baseline results for the time aggregated model reported in table 1.1 had no persistence in the transitory shock. In table 1.2 I report the insurance coefficients

Table 1.1: Partial Insurance Estimates Replicating Table 6 from BPP

		Whole Sample		No College		College	
		BPP	Time Agg.	BPP	Time Agg.	BPP	Time Agg.
$\sigma^2_{P,T}$	1979-1981	0.0103	0.0247	0.0068	0.0234	0.0101	0.0189
(Variance perm. shock)		(0.0034)	(0.0043)	(0.0037)	(0.0063)	(0.0053)	(0.0050)
	1982	0.0208	0.0358	0.0156	0.0290	0.0253	0.0455
		(0.0041)	(0.0071)	(0.0052)	(0.0099)	(0.0060)	(0.0099)
	1983	0.0301	0.0333	0.0318	0.0553	0.0234	0.0086
		(0.0057)	(0.0100)	(0.0074)	(0.0128)	(0.0090)	(0.0148)
	1984	0.0274	0.0292	0.0334	0.0232	0.0177	0.0361
		(0.0049)	(0.0114)	(0.0073)	(0.0131)	(0.0060)	(0.0161)
	1985	0.0295	0.0363	0.0287	0.0504	0.0208	0.0025
		(0.0096)	(0.0124)	(0.0073)	(0.0145)	(0.0152)	(0.0205)
	1986	0.0221	0.0327	0.0173	0.0247	0.0311	0.0597
		(0.0060)	(0.0136)	(0.0067)	(0.0172)	(0.0101)	(0.0202)
	1987	0.0289	0.0420	0.0202	0.0478	0.0354	0.0229
		(0.0063)	(0.0143)	(0.0073)	(0.0182)	(0.0098)	(0.0211)
	1988	0.0158	0.0082	0.0117	-0.0069	0.0183	0.0302
		(0.0069)	(0.0137)	(0.0079)	(0.0209)	(0.0110)	(0.0149)
	1989	0.0185	0.0531	0.0107	0.0639	0.0274	0.0414
		(0.0059)	(0.0129)	(0.0101)	(0.0214)	(0.0061)	(0.0149)
	1990-92	0.0135	0.0291	0.0093	0.0265	0.0217	0.0291
		(0.0042)	(0.0042)	(0.0045)	(0.0063)	(0.0065)	(0.0057)
$\sigma^2_{Q,T}$	1979	0.0379	0.0310	0.0465	0.0364	0.0301	0.0261
(Variance trans. shock)		(0.0059)	(0.0049)	(0.0096)	(0.0080)	(0.0056)	(0.0043)
	1980	0.0298	0.0240	0.0330	0.0247	0.0283	0.0238
		(0.0039)	(0.0033)	(0.0053)	(0.0046)	(0.0059)	(0.0047)
	1981	0.0300	0.0265	0.0363	0.0305	0.0253	0.0222
		(0.0035)	(0.0032)	(0.0053)	(0.0048)	(0.0046)	(0.0040)
	1982	0.0287	0.0280	0.0375	0.0332	0.0213	0.0237
		(0.0039)	(0.0034)	(0.0063)	(0.0057)	(0.0042)	(0.0036)
	1983	0.0262	0.0276	0.0371	0.0378	0.0185	0.0169
		(0.0037)	(0.0034)	(0.0063)	(0.0056)	(0.0037)	(0.0040)
	1984	0.0346	0.0350	0.0404	0.0388	0.0304	0.0315
		(0.0039)	(0.0038)	(0.0059)	(0.0058)	(0.0051)	(0.0046)
	1985	0.0450	0.0427	0.0355	0.0338	0.0496	0.0465
		(0.0075)	(0.0071)	(0.0056)	(0.0053)	(0.0130)	(0.0122)
	1986	0.0458	0.0404	0.0474	0.0373	0.0452	0.0464
		(0.0058)	(0.0055)	(0.0076)	(0.0068)	(0.0085)	(0.0084)
	1987	0.0461	0.0445	0.0520	0.0486	0.0421	0.0385
		(0.0054)	(0.0053)	(0.0082)	(0.0078)	(0.0071)	(0.0069)
	1988	0.0399	0.0327	0.0471	0.0360	0.0343	0.0313
		(0.0047)	(0.0044)	(0.0074)	(0.0072)	(0.0060)	(0.0055)
	1989	0.0378	0.0343	0.0539	0.0475	0.0219	0.0215
		(0.0067)	(0.0061)	(0.0126)	(0.0117)	(0.0051)	(0.0044)
	1990-92	0.0441	0.0359	0.0535	0.0408	0.0345	0.0322
		(0.0040)	(0.0027)	(0.0062)	(0.0047)	(0.0049)	(0.0032)
θ		0.1126	N/A	0.1260	N/A	0.1082	N/A
(Serial correl. trans. shock)		(0.0248)		(0.0319)		(0.0342)	
σ^2_{ξ}		0.0097	0.0122	0.0065	0.0114	0.0132	0.0146
(Variance unobs. slope heterog.)		(0.0041)	(0.0039)	(0.0079)	(0.0070)	(0.0040)	(0.0039)
ϕ		0.6456	0.3384	0.9484	0.4365	0.4180	0.2729
(Partial insurance perm. shock)		(0.0941)	(0.0471)	(0.1773)	(0.0738)	(0.0913)	(0.0603)
ψ		0.0501	0.2421	0.0724	0.2870	0.0260	0.1590
(Partial insurance trans. shock)		(0.0430)	(0.0431)	(0.0593)	(0.0616)	(0.0546)	(0.0504)

for three different ways of introducing persistence into the continuous time model, along with the estimates for the discrete time model with no persistence. The first method, called 'two-shot', models transitory income as a mass of income arriving at time t, followed by another mass of income, smaller than the first by a factor θ, arriving exactly one year later. This most closely mirrors the MA(1) model of transitory income used in the discrete time model. The second, called 'uniform', models transitory income as a constant flow of income starting at time t and ending at time $t+\tau$ where τ is a measure of persistence. This can be thought of as a member of the household becoming unemployed for a length of time τ. The third, called 'linear decay', models transitory income as a flow of income starting at time t, the size of which decreases linearly until it reaches zero at time $t+\tau$. This tries to capture the fact that some transitory shocks have little persistence, while others are longer lived, so that on average income from a transitory shock will be decreasing over time. The identifying equations for each model can be found in appendix 1.6.3. The bottom two rows of table 1.2 report the estimated values of θ and τ in each model. The values of θ in the MA(1) model and the two-shot model are similar, with about 10% of the first year's transitory income arriving in the following year. The uniform model estimates transitive periods of high or low income to last for somewhat less than half a year (0.43), while the linear decay model estimates them to last more than half a year (0.61). This makes sense as the 'persistence' associated with a uniform flow of income for a period τ is greater than that of a linearly decaying flow of income over

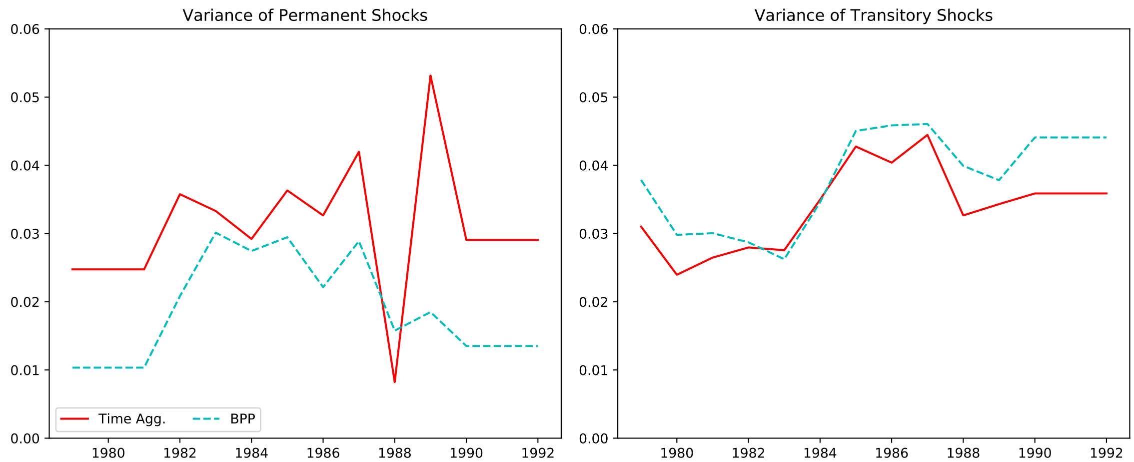
Table 1.2: Partial Insurance Estimates by Persistence Type

	BPP		Time Agg.			
Persistence Type:	None	MA(1)	None	Two-shot	Uniform	Linear Decay
ϕ	0.4692	0.6456	0.3384	0.4169	0.3287	0.3516
(Partial insurance perm. shock)	(0.0598)	(0.0941)	(0.0471)	(0.0680)	(0.0580)	(0.0627)
ψ	0.0503	0.0501	0.2421	0.2149	0.2510	0.2403
(Partial insurance tran. shock)	(0.0505)	(0.0430)	(0.0431)	(0.0386)	(0.0428)	(0.0417)
θ or τ	N/A	0.1126	N/A	0.1004	0.4320	0.6140
(Degree of Persistence)	(0.0000)	(0.0248)	(0.0000)	(0.0242)	(0.1008)	(0.1225)

a period τ.

The first two columns of table 1.2 show that the degree of persistence in the original BPP model makes a big difference to the estimate of ϕ, while all of the time aggregation models show similar estimates for ψ. This suggests the difference we see in the estimates of ϕ between BPP original model and the time aggregated model may be driven, at least in part, by misspecification in the model of transitive income shocks. It is reassuring that, in contrast to the BPP model, the values of both ϕ and ψ are relatively robust to the exact specification of transitive persistence in the time aggregated model.

Figure 1.3: Shock Variances in the 1980's



Notes: BPP plots the variances from Table 6 of the original BPP paper. Time Agg. plots the equivalent variances corrected for the time aggregation problem.

1.5 Conclusion

This paper highlights the importance of time aggregation when working with panel data, especially when analyzing the covariance matrix of income and consumption growth. It also resolves the dissonance between BPP's estimates of transitory income insurance and the natural experiment literature on marginal propensity to consume. Going forward I hope the methods used here to correct for the time aggregation problem can be useful for researchers, especially as more and more high quality panel datasets on income and consumption become available.

1.6 Appendicies

1.6.1 Continuous Time Model as Limit of Discrete Model with m Sub-periods

The identifying equations in the paper are calculated using a 'log' income process that does not directly align with any real-world concept of income. In the data we take logs on the sum of income over the entire year, but the process we use in the model informally aligns with log income over an instantaneous period dt. This is a problem as transitory income arrive as a point mass, making it difficult to interpret what the 'log' income process really represents. Here I show how the identifying equations can be derived as the limit of discrete time model with m sub-periods. I show that in the limit the variance of observed log income growth is the same as derived in the informal model (to a first order approximation). The rest of the identifying equations can be shown in the same way.

Let p_t for $t \in \mathbb{R}^+$ be a martingale process (possibly with jumps) with independent stationary increments and ν be such that $\mathbb{E}(e^{p_t - p_{t-1}}) = e^{\nu}$. Define permanent income as:

$$P_t = e^{p_t - t\nu}$$

Note that $\mathbb{E}\left(\frac{P_{t+s}}{P_t}\right) = 1$ for all $s \geq 0$. Define the variance of log permanent shocks to be:

$$\sigma_P^2 = \mathrm{Var}\Big(\log\big(\frac{P_{t+1}}{P_t}\big)\Big) = \mathrm{Var}(p_{t+1} - p_t)$$

We will assume changes in permanent income over a one year period are small enough such that:

$$\begin{aligned}
\mathrm{Var}\Big(\frac{P_{t+1}}{P_t}\Big) &= \mathrm{Var}\Big(\frac{P_{t+1} - P_t}{P_t}\Big) \\
&\approx \mathrm{Var}\Big(\log\big(1 + \frac{P_{t+1} - P_t}{P_t}\big)\Big) \\
&= \mathrm{Var}\Big(\log\big(\frac{P_{t+1}}{P_t}\big)\Big) = \sigma_P^2
\end{aligned} \tag{1.11}$$

For transitory shocks, we define an increasing stochastic process, Θ_t, which also has independent stationary increments. The increments in this process will define the transitory shocks. We set the expectation of increments, and the variance of the log of an increment of length 1 as:

$$\begin{aligned}
\mathbb{E}(\Theta_{t+s} - \Theta_t) &= s \\
\mathrm{Var}\Big(\log\big(\Theta_{t+1} - \Theta_t\big)\Big) &= \sigma_\Theta^2
\end{aligned}$$

Note that for this to be well defined, Θ_t must not only be increasing but also its increments are almost surely strictly positive (so that log of the increment is defined almost everywhere). Examples of such a stochastic process would be a gamma process, or a process that increases linearly with time (non-stochastically) but is also subject to positive shocks that arrive as a Poisson process. The stochastic part of this process has no Brownian motion component as this would necessarily lead to non-zero probability of a decreasing increment.

We will use these two processes to define an income process in discrete time with m intervals per period, and then look at the limit as $m \to \infty$. Define $\theta_{t,m}$ for $t \in \{\frac{1}{m}, \frac{2}{m}, \frac{3}{m}...\}$ to be the increment of Θ_t from $t - \frac{1}{m}$ to t:

$$\theta_{t,m} = \Theta_t - \Theta_{1-\frac{1}{m}}$$

Income is defined for each period $t \in \{\frac{1}{m}, \frac{2}{m}, \frac{3}{m}...\}$ as:

$$Y_{t,m} = P_t \theta_{t,m}$$

Therefore the underlying income process has a pure division into permanent and transitory shocks. Income is observed for $T \in \{1, 2, 3...\}$ as the sum of income in each of the subperiods:

$$\bar{Y}_{T,m} = \sum_{i=0}^{m-1} P_{T-\frac{i}{m}} \theta_{T-\frac{i}{m},m}$$

Note that for $m = 1$ this the same as the underlying income process, with permanent and transitory variance as defined above. We are interested in the log of observable income growth:

$$\begin{aligned}
\Delta \bar{y}_{T,m} &= \log \bar{Y}_{T,m} - \log \bar{Y}_{T-1,m} \\
&= \log \left(\sum_{i=0}^{m-1} P_{T-\frac{i}{m}} \theta_{T-\frac{i}{m},m} \right) - \log \left(\sum_{i=0}^{m-1} P_{T-1-\frac{i}{m}} \theta_{T-1-\frac{i}{m},m} \right) \\
&= \log \left(\sum_{i=0}^{m-1} \frac{P_{T-\frac{i}{m}}}{P_{T-1}} \theta_{T-\frac{i}{m},m} \right) - \log \left(\sum_{i=0}^{m-1} \frac{P_{T-1-\frac{i}{m}}}{P_{T-1}} \theta_{T-1-\frac{i}{m},m} \right)
\end{aligned}$$

As P_t and Θ_t have independent increments, the covariance between each of the two parts of the sum above is 0. Therefore:

$$\mathrm{Var}\Big(\Delta^1 \bar{y}_{T,m}\Big) = \mathrm{Var}\Bigg(\log\left(\sum_{i=0}^{m-1} \frac{P_{T-\frac{i}{m}}}{P_{T-1}} \theta_{T-\frac{i}{m},m} \right)\Bigg) + \mathrm{Var}\Bigg(\log\left(\sum_{i=0}^{m-1} \frac{P_{T-1-\frac{i}{m}}}{P_{T-1}} \theta_{T-1-\frac{i}{m},m} \right)\Bigg)$$

We will treat each of these two variances individually. We begin by looking at the variable:

$$\begin{aligned}
\log\left(\sum_{i=0}^{m-1} \frac{P_{T-\frac{i}{m}}}{P_{T-1}} \theta_{T-\frac{i}{m},m} \right) &= \log\left(\sum_{i=0}^{m-1} \theta_{T-\frac{i}{m},m} + \sum_{i=0}^{m-1} \Big(\frac{P_{T-\frac{i}{m}}}{P_{T-1}} - 1 \Big) \theta_{T-\frac{i}{m},m} \right) \\
&= \log\Big(\Theta_T - \Theta_{T-1}\Big) + \log\left(1 + \sum_{i=0}^{m-1} \Big(\frac{P_{T-\frac{i}{m}}}{P_{T-1}} - 1 \Big) \frac{\theta_{T-\frac{i}{m},m}}{\sum_{l=0}^{m-1} \theta_{T-\frac{l}{m},m}} \right) \\
&\approx \log\Big(\Theta_T - \Theta_{T-1}\Big) + \sum_{i=0}^{m-1} \Big(\frac{P_{T-\frac{i}{m}}}{P_{T-1}} - 1 \Big) \frac{\theta_{T-\frac{i}{m},m}}{\sum_{l=0}^{m-1} \theta_{T-\frac{l}{m},m}}
\end{aligned}$$

Where the approximation comes from the fact that the shocks to permanent income in a one year period are small. Defining

$$\zeta_{t,m} = \frac{P_t}{P_{t-\frac{1}{m}}}$$

we have that

$$\begin{aligned}
\mathrm{Var}\Bigg(\log\Bigg(\sum_{i=0}^{m-1}\frac{P_{T-\frac{i}{m}}}{P_{T-1}}\theta_{T-\frac{i}{m},m}\Bigg)\Bigg) &\approx \sigma_\Theta^2+\mathrm{Var}\Bigg(\sum_{i=0}^{m-1}\Big(\prod_{j=i}^{m-1}\zeta_{T-\frac{j}{m}}-1\Big)\frac{\theta_{T-\frac{i}{m},m}}{\sum_{l=0}^{m-1}\theta_{T-\frac{l}{m},m}}\Bigg)\\
&=\sigma_\Theta^2+\mathbb{E}\Bigg[\sum_{i=0}^{m-1}\Big(\prod_{j=i}^{m-1}\zeta_{T-\frac{j}{m}}-1\Big)\frac{\theta_{T-\frac{i}{m},m}}{\sum_{l=0}^{m-1}\theta_{T-\frac{l}{m},m}}\Bigg]^2\\
&=\sigma_\Theta^2+\mathbb{E}\Bigg[\sum_{i=0}^{m-1}\Bigg(\Big(\prod_{j=i}^{m-1}\zeta_{T-\frac{j}{m}}-1\Big)^2\Bigg(\frac{\theta_{T-\frac{i}{m},m}}{\sum_{l=0}^{m-1}\theta_{T-\frac{l}{m},m}}\Bigg)^2\\
&\qquad+2\sum_{k<i}\Big(\prod_{j=k}^{m-1}\zeta_{T-\frac{j}{m}}-1\Big)\Big(\prod_{j=i}^{m-1}\zeta_{T-\frac{j}{m}}-1\Big)\frac{\theta_{T-\frac{k}{m},m}\theta_{T-\frac{i}{m},m}}{\Big(\sum_{l=0}^{m-1}\theta_{T-\frac{l}{m},m}\Big)^2}\Bigg)\Bigg]\\
&=\sigma_\Theta^2+\frac{\sigma_P^2}{m}\sum_{i=0}^{m-1}\Bigg(i\mathbb{E}\Bigg(\frac{\theta_{T-\frac{i}{m},m}}{\sum_{l=0}^{m-1}\theta_{T-\frac{l}{m},m}}\Bigg)^2+2\sum_{k<i}(m-1-i)\mathbb{E}\Bigg(\frac{\theta_{T-\frac{k}{m},m}\theta_{T-\frac{i}{m},m}}{\Big(\sum_{l=0}^{m-1}\theta_{T-\frac{l}{m},m}\Big)^2}\Bigg)\Bigg)\\
&=\sigma_\Theta^2+\frac{\sigma_P^2}{m}\frac{m(m-1)}{2}\mathbb{E}\Bigg(\frac{\theta_{T-\frac{i}{m},m}}{\sum_{l=0}^{m-1}\theta_{T-\frac{l}{m},m}}\Bigg)^2\\
&\qquad+2\frac{\sigma_P^2}{m}\sum_{i=1}^{m-1}i(m-1-i)\mathbb{E}\Bigg(\frac{\theta_{T-\frac{k}{m},m}\theta_{T-\frac{i}{m},m}}{\Big(\sum_{l=0}^{m-1}\theta_{T-\frac{l}{m},m}\Big)^2}\Bigg)\\
&=\sigma_\Theta^2+\sigma_P^2\frac{m-1}{2}\mathbb{E}\Bigg(\frac{\theta_{T-\frac{i}{m},m}}{\sum_{l=0}^{m-1}\theta_{T-\frac{l}{m},m}}\Bigg)^2\\
&\qquad+\sigma_P^2\Bigg[(m-1)^2-\frac{(m-1)(2m-1)}{3}\Bigg]\mathbb{E}\Bigg(\frac{\theta_{T-\frac{k}{m},m}\theta_{T-\frac{i}{m},m}}{\Big(\sum_{l=0}^{m-1}\theta_{T-\frac{l}{m},m}\Big)^2}\Bigg)
\end{aligned}$$

Note that:

$$\begin{aligned}
1&=\mathbb{E}\Bigg(\sum_{i=0}^{m-1}\frac{\theta_{T-\frac{i}{m},m}}{\sum_{l=0}^{m-1}\theta_{T-\frac{l}{m},m}}\Bigg)^2\\
&=\sum_{i=0}^{m-1}\mathbb{E}\Bigg(\frac{\theta_{T-\frac{i}{m},m}}{\sum_{l=0}^{m-1}\theta_{T-\frac{l}{m},m}}\Bigg)^2+2\sum_{k<i}\mathbb{E}\Bigg(\frac{\theta_{T-\frac{k}{m},m}\theta_{T-\frac{i}{m},m}}{\Big(\sum_{l=0}^{m-1}\theta_{T-\frac{l}{m},m}\Big)^2}\Bigg)
\end{aligned}$$

So that

$$\mathbb{E}\left(\frac{\theta_{T-\frac{k}{m},m}\theta_{T-\frac{i}{m},m}}{\left(\sum_{l=0}^{m-1}\theta_{T-\frac{l}{m},m}\right)^2}\right)=\frac{1}{m(m-1)}-\frac{1}{m-1}\mathbb{E}\left(\frac{\theta_{T-\frac{i}{m},m}}{\sum_{l=0}^{m-1}\theta_{T-\frac{l}{m},m}}\right)^2$$

This gives:

$$\begin{aligned}\mathrm{Var}\left(\log\left(\sum_{i=0}^{m-1}\frac{P_{T-\frac{i}{m}}}{P_{T-1}}\theta_{T-\frac{i}{m},m}\right)\right)&\approx\sigma_\Theta^2+\mathrm{Var}\left(\sum_{i=0}^{m-1}\Big(\prod_{j=i}^{m-1}\zeta_{T-\frac{j}{m}}-1\Big)\frac{\theta_{T-\frac{i}{m},m}}{\sum_{l=0}^{m-1}\theta_{T-\frac{l}{m},m}}\right)\\&\approx\sigma_\Theta^2+\frac{m-2}{3m}\sigma_P^2+\frac{m+1}{6}\mathbb{E}\left(\frac{\theta_{T-\frac{i}{m},m}}{\sum_{l=0}^{m-1}\theta_{T-\frac{l}{m},m}}\right)^2\sigma_P^2\\&\to\sigma_\Theta^2+\frac{1}{3}\sigma_P^2\qquad\text{as } m\to\infty\end{aligned}$$

A very similar calculation shows that:

$$\mathrm{Var}\left(\log\left(\sum_{i=0}^{m-1}\frac{P_{T-1-\frac{i}{m}}}{P_{T-1}}\theta_{T-1-\frac{i}{m},m}\right)\right)\to\sigma_\Theta^2+\frac{1}{3}\sigma_P^2\qquad\text{as } m\to\infty$$

Putting these together gives:

$$\mathrm{Var}\Big(\Delta\bar{y}_{T,m}\Big)\to\frac{2}{3}\sigma_P^2+2\sigma_\Theta^2\qquad\text{as } m\to\infty$$

This is the same as the identifying equation for $\mathrm{Var}\Big(\Delta y_T^{obs}\Big)$ (equation 1.12 from appendix 1.6.2, assuming shock variances are constant over time), and the rest of the identifying equations can be shown as the limit of the discrete time model in a similar way.

1.6.2 Identification in the Full Model

In this appendix I calculate the full set of identifying equations for the non-stationary model with measurement error in consumption and taste shocks. Appendix 1.6.3 extends these to add persistence in the transitory shock. With classical measurement error on consumption the observables are now y_T^{obs} and c_T^{obs} where

$$y_T^{obs} = \int_{T-1}^{T} dy_t$$
$$c_T^{obs} = \int_{T-1}^{T} dc_t + u_T$$

I am interested in the full set of observable covariances:

$$\mathrm{Cov}(\Delta y_T^{obs}, \Delta y_S^{obs})$$

$$\mathrm{Cov}(\Delta c_T^{obs}, \Delta c_S^{obs})$$

$$\mathrm{Cov}(\Delta c_T^{obs}, \Delta y_S^{obs})$$

for all T and S in $\{1, 2, ...\}$. I further make the assumption that while the variance of the permanent and transitory shocks and insurance coefficients can change from year to year, within each year these are constant. The variance the permanent shock in year T is $\sigma^2_{P,T}$ and the transitory shock $\sigma^2_{Q,T}$. I use equation 1.7 for the change in observable log income, and extend equation 1.8 for the change is observable log consumption to include taste shocks (ξ_t) and measurement error:

$$\Delta c_T^{obs} = \phi \int_{T-1}^{T} dP_s + \psi \int_{T-1}^{T} dQ_s + \int_{T-1}^{T} d\xi_s + u_T - u_{T-1}$$

These two equations allow for the calculation of all the required identification equations:

$$
\begin{aligned}
\mathrm{Var}(\Delta y_T^{obs}) &= \mathbb{E}\Big(\int_{T-2}^{T-1}(s-(T-2))^2 dP_s dP_s + \int_{T-1}^{T}(T-s)^2 dP_s dP_s\Big) \\
&\quad + \mathbb{E}\Big(\int_{T-1}^{T} dQ_t dQ_t + \int_{T-2}^{T-1} dQ_t dQ_t\Big) \\
&= \frac{1}{3}\sigma^2_{P,T} + \frac{1}{3}\sigma^2_{P,T-1} + \sigma^2_{Q,T} + \sigma^2_{Q,T-1} && (1.12) \\
\mathrm{Cov}(\Delta y_T^{obs}, \Delta y_{T+1}^{obs}) &= \mathbb{E}\Big(\int_{T-1}^{T}(T-s)(s-(T-1)) dP_s dP_s\Big) - \mathbb{E}\Big(\int_{T-1}^{T} dQ_t dQ_t\Big) \\
&= \frac{1}{6}\sigma^2_{P,T} - \sigma^2_{Q,T} && (1.13) \\
\mathrm{Cov}(\Delta y_T^{obs}, \Delta y_{T-1}^{obs}) &= \frac{1}{6}\sigma^2_{P,T-1} - \sigma^2_{Q,T-1} && (1.14) \\
\mathrm{Cov}(\Delta y_T^{obs}, \Delta y_S^{obs}) &= 0 \qquad \forall S,T \text{ such that } |S-T|>1 && (1.15)
\end{aligned}
$$

$$
\begin{aligned}
\mathrm{Var}\Delta c_T^{obs} &= \phi^2\mathbb{E}\Big(\int_{T-1}^{T} dP_s dP_s\Big) + \psi^2\mathbb{E}\Big(\int_{T-1}^{T} dQ_s dQ_s\Big) + \mathbb{E}\Big(\int_{T-1}^{T} d\xi_s d\xi_s\Big) + \sigma^2_{u,T} + \sigma^2_{u,T-1} \\
&= \phi^2\sigma^2_{P,T} + \psi^2\sigma^2_{Q,T} + \sigma^2_{\xi,T} + \sigma^2_{u,T} + \sigma^2_{u,T-1} && (1.16) \\
\mathrm{Cov}(\Delta c_T^{obs}, \Delta c_{T+1}^{obs}) &= -\sigma^2_{u,T} && (1.17) \\
\mathrm{Cov}(\Delta c_T^{obs}, \Delta c_{T-1}^{obs}) &= -\sigma^2_{u,T-1} && (1.18) \\
\mathrm{Cov}(\Delta c_T^{obs}, \Delta c_S^{obs}) &= 0 \qquad \forall S,T \text{ such that } |S-T|>1 && (1.19)
\end{aligned}
$$

$$\begin{aligned}
\mathrm{Cov}(\Delta c_T^{obs}, \Delta y_T^{obs}) &= \mathbb{E}\Big(\phi_T \int_{T-1}^{T} (T-s) dP_s dP_s + \psi_T \int_{T-1}^{T} dQ_s dQ_s\Big) \\
&= \frac{1}{2}\phi_T \sigma_{P,T}^2 + \psi_T \sigma_{Q,T}^2 \qquad (1.20) \\
\mathrm{Cov}(\Delta c_T^{obs}, \Delta y_{T+1}^{obs}) &= \mathbb{E}\Big(\phi_T \int_{T-1}^{T} (s-(T-1)) dP_s dP_s - \psi_T \int_{T-1}^{T} dQ_s dQ_s\Big) \\
&= \frac{1}{2}\phi_T \sigma_{P,T}^2 - \psi_T \sigma_{Q,T}^2 \qquad (1.21) \\
\mathrm{Cov}(\Delta c_T^{obs}, \Delta y_{T-1}^{obs}) &= 0 \qquad (1.22) \\
\mathrm{Cov}(\Delta c_T^{obs}, \Delta y_S^{obs}) &= 0 \qquad \forall S, T \text{ such that } |S-T| > 1 \qquad (1.23)
\end{aligned}$$

1.6.3 Persistence in Transitory Shock

This appendix shows how to extend the time aggregated model to include persistence in the transitory shock.

1.6.3.1 Linear Decay Model

I will walk though the derivation of the moments for the linear decay model in detail and then just list the moments for the two-step and uniform models. In the linear decay model, a shock of size 1 will arrive with a flow intensity of $\frac{2}{\tau}$ and over the subsequent time τ a the total flow of transitory income will sum to 1. Instantaneous income can be written as:

$$dy_t = \Big(\int_0^t dP_s\Big)dt + \Big(\int_{t-\tau}^{t} \frac{2}{\tau}(s-(t-\tau))dQ_s\Big)dt$$

So that the observable change in income is given by:

$$
\begin{aligned}
\Delta y_T^{obs} &= \int_{T-1}^{T} y_t dt - \int_{T-2}^{T-1} y_t dt \\
&= \int_{T-1}^{T}\int_{0}^{t} dP_s dt - \int_{T-2}^{T-1}\int_{0}^{t} dP_s dt \\
&\quad + \int_{T-1}^{T}\int_{t-\tau}^{t} \frac{2}{\tau}(s-(t-\tau))dQ_s dt - \int_{T-2}^{T-1}\int_{t-\tau}^{t} \frac{2}{\tau}(s-(t-\tau))dQ_s dt \\
&= \Big(\int_{T-2}^{T-1}(s-(T-2))dP_s + \int_{T-1}^{T}(T-s)dP_s\Big) \\
&\quad + \frac{2}{\tau}\Big(\int_{T-\tau}^{T}\frac{1}{2}\Big(\tau - \frac{(s-(T-\tau))^2}{\tau}\Big)dQ_s + \int_{T-1}^{T-\tau}\frac{1}{2}\tau dQ_s + \int_{T-1-\tau}^{T-1}\frac{1}{2}\frac{(s-(T-1-\tau))^2}{\tau}dQ_s \\
&\quad - \frac{2}{\tau}\Big(\int_{T-1-\tau}^{T-1}\frac{1}{2}\Big(\tau - \frac{(s-(T-1-\tau))^2}{\tau}\Big)dQ_s + \int_{T-2}^{T-1-\tau}\frac{1}{2}\tau dQ_s \\
&\qquad\qquad + \int_{T-2-\tau}^{T-2}\frac{1}{2}\frac{(s-(T-2-\tau))^2}{\tau}dQ_s\Big) \\
&= \int_{T-2}^{T-1}(s-(T-2))dP_s + \int_{T-1}^{T}(T-s)dP_s \\
&\quad + \int_{T-\tau}^{T} 1 - \Big(\frac{s-(T-\tau)}{\tau}\Big)^2 dQ_s + \int_{T-1}^{T-\tau} dQ_s \\
&\quad - \int_{T-1-\tau}^{T-1} 1 - 2\Big(\frac{s-(T-1-\tau)}{\tau}\Big)^2 dQ_s \\
&\quad - \int_{T-2}^{T-1-\tau} dQ_s - \int_{T-2-\tau}^{T-2}\Big(\frac{s-(T-2-\tau)}{\tau}\Big)^2 dQ_s
\end{aligned}
\tag{1.24}
$$

The full set of identification equations used in this model are:

$$
\begin{aligned}
\mathrm{Var}(\Delta y_T^{obs}) = \mathbb{E}\Big(&\int_{T-2}^{T-1}(s-(T-2))^2 dP_s dP_s + \int_{T-1}^{T}(T-s)^2 dP_s dP_s\Big)\\
&+\mathbb{E}\Big(\int_{T-\tau}^{T}\Big(1-\Big(\frac{s-(T-\tau)}{\tau}\Big)^2\Big)^2 dQ_s dQ_s + \int_{T-1}^{T-\tau} dQ_s Q_s\Big)\\
&+\mathbb{E}\Big(\int_{T-1-\tau}^{T-1}\Big(1-2\Big(\frac{s-(T-1-\tau)}{\tau}\Big)^2\Big)^2 dQ_s dQ_s\Big)\\
&+\mathbb{E}\Big(\int_{T-2}^{T-1-\tau} dQ_s dQ_s + \int_{T-2-\tau}^{T-2}\Big(\frac{s-(T-2-\tau)}{\tau}\Big)^4 dQ_s dQ_s\Big)\\
=\ &\frac{1}{3}\sigma^2_{P,T} + \frac{1}{3}\sigma^2_{P,T-1}\\
&+\frac{8}{15}\tau\sigma^2_{Q,T} + (1-\tau)\sigma^2_{Q,T}\\
&+\frac{7}{15}\tau\sigma^2_{Q,T-1}\\
&+(1-\tau)\sigma^2_{Q,T-1} + \frac{1}{5}\tau\sigma^2_{Q,T-2}\\
=\ &\frac{1}{3}\sigma^2_{P,T} + \frac{1}{3}\sigma^2_{P,T-1} + \big(1-\frac{7}{15}\tau\big)\sigma^2_{Q,T} + (1-\frac{8}{15}\tau)\sigma^2_{Q,T-1} + \frac{1}{5}\tau\sigma^2_{Q,T-2}
\end{aligned}
\tag{1.25}
$$

$$
\begin{aligned}
\mathrm{Cov}(\Delta y_T^{obs}, \Delta y_{T+1}^{obs}) = \mathbb{E}\Big(&\int_{T-1}^{T}(T-s)(s-(T-1)) dP_s dP_s\Big)\\
&-\mathbb{E}\Big(\int_{T-\tau}^{T}\Big(1-\Big(\frac{s-(T-\tau)}{\tau}\Big)^2\Big)\Big(1-2\Big(\frac{s-(T-\tau)}{\tau}\Big)^2\Big) dQ_s dQ_s\Big)\\
&-\mathbb{E}\Big(\int_{T-1}^{T-\tau} dQ_s Q_s\Big)\\
&+\mathbb{E}\Big(\int_{T-1-\tau}^{T-1}\Big(1-2\Big(\frac{s-(T-1-\tau)}{\tau}\Big)^2\Big)\Big(\frac{s-(T-1-\tau)}{\tau}\Big)^2 dQ_s dQ_s\Big)\\
=\ &\frac{1}{6}\sigma^2_{P,T} - \frac{2}{5}\tau\sigma^2_{Q,T} - (1-\tau)\sigma^2_{Q,T} - \frac{1}{15}\sigma^2_{Q,T-1}
\end{aligned}
$$

$$\begin{aligned}
\mathrm{Cov}(\Delta y_T^{obs}, \Delta y_{T+2}^{obs}) &= -\mathbb{E}\Big(\int_{T-\tau}^{T}\Big(1-\Big(\frac{s-(T-\tau)}{\tau}\Big)^2\Big)\Big(\frac{s-(T-\tau)}{\tau}\Big)^2 dQ_s dQ_s\Big) \\
&= -\frac{2}{15}\tau\sigma_{Q,T}^2 \qquad (1.26)
\end{aligned}$$

The above equations also work for $\mathrm{Cov}(\Delta y_T^{obs}, \Delta y_{T-1}^{obs})$ and $\mathrm{Cov}(\Delta y_T^{obs}, \Delta y_{T-2}^{obs})$ due to symmetry.

$$\mathrm{Cov}(\Delta y_T^{obs}, \Delta y_S^{obs}) = 0 \qquad \forall S, T \text{ such that } |S-T| > 2 \qquad (1.27)$$

The covariance matrix $\mathrm{Cov}(\Delta c_T^{obs}, \Delta c_S^{obs})$ is the same as in appendix 1.6.2.

$$\begin{aligned}
\mathrm{Cov}(\Delta c_T^{obs}, \Delta y_T^{obs}) &= \phi_T\mathbb{E}\Big(\int_{T-1}^{T}(T-s)dP_s dP_s\Big) \\
&\quad + \psi_T\mathbb{E}\Big(\int_{T-\tau}^{T}\Big(1-\Big(\frac{s-(T-\tau)}{\tau}\Big)^2\Big)dQ_s dQ_s + \int_{T-1}^{T-\tau} dQ_s dQ_s\Big) \\
&= \frac{1}{2}\phi_T\sigma_{P,T}^2 + \psi_T(1-\frac{1}{3}\tau)\sigma_{Q,T}^2 \qquad (1.28)
\end{aligned}$$

$$\begin{aligned}
\mathrm{Cov}(\Delta c_T^{obs}, \Delta y_{T+1}^{obs}) &= \phi_T\mathbb{E}\Big(\int_{T-1}^{T}(s-(T-1))dP_s dP_s\Big) \\
&\quad - \psi_T\mathbb{E}\Big(\int_{T-\tau}^{T}\Big(1-2\Big(\frac{s-(T-\tau)}{\tau}\Big)^2\Big)dQ_s dQ_s + \int_{T-1}^{T-\tau} dQ_s dQ_s\Big) \\
&= \frac{1}{2}\phi_T\sigma_{P,T}^2 - (1-\frac{2}{3}\tau)\psi_T\sigma_{Q,T}^2 \qquad (1.29)
\end{aligned}$$

$$\begin{aligned}
\mathrm{Cov}(\Delta c_T^{obs}, \Delta y_{T+2}^{obs}) &= -\psi_T\mathbb{E}\Big(\int_{T-\tau}^{T}\Big(\frac{s-(T-\tau)}{\tau}\Big)^2 dQ_s dQ_s\Big) \\
&= -\frac{1}{5}\psi_T\tau\sigma_{Q,T}^2 \qquad (1.30)
\end{aligned}$$

1.6.3.2 The Uniform Model

In the uniform model, transitory shocks consist of a constant flow of income that lasts for a time period τ. The full set of moments for this model are:

$$\mathrm{Var}(\Delta y_T^{obs}) = \frac{1}{3}\sigma_{P,T}^2 + \frac{1}{3}\sigma_{P,T-1}^2 + \big(1 - \frac{2}{3}\tau\big)\sigma_{Q,T}^2 + (1 - \frac{2}{3}\tau)\sigma_{Q,T-1}^2 + \frac{1}{3}\tau\sigma_{Q,T-2}^2 \tag{1.31}$$

$$\mathrm{Cov}(\Delta y_T^{obs}, \Delta y_{T+1}^{obs}) = \frac{1}{6}\sigma_{P,T}^2 - \frac{1}{6}\tau\sigma_{Q,T}^2 - (1-\tau)\sigma_{Q,T}^2 - \frac{1}{15}\sigma_{Q,T-1}^2 \tag{1.32}$$

$$\mathrm{Cov}(\Delta y_T^{obs}, \Delta y_{T+2}^{obs}) = -\frac{1}{6}\tau\sigma_{Q,T}^2 \tag{1.33}$$

The above equations also work for $\mathrm{Cov}(\Delta y_T^{obs}, \Delta y_{T-1}^{obs})$ and $\mathrm{Cov}(\Delta y_T^{obs}, \Delta y_{T-2}^{obs})$ due to symmetry.

$$\mathrm{Cov}(\Delta y_T^{obs}, \Delta y_S^{obs}) = 0 \qquad \forall S, T \text{ such that } |S - T| > 2 \tag{1.34}$$

The covariance matrix $\mathrm{Cov}(\Delta c_T^{obs}, \Delta c_S^{obs})$ is the same as in appendix 1.6.2.

$$\mathrm{Cov}(\Delta c_T^{obs}, \Delta y_T^{obs}) = \frac{1}{2}\phi_T\sigma_{P,T}^2 + \psi_T(1 - \frac{1}{2}\tau)\sigma_{Q,T}^2 \tag{1.35}$$

$$\mathrm{Cov}(\Delta c_T^{obs}, \Delta y_{T+1}^{obs}) = \frac{1}{2}\phi_T\sigma_{P,T}^2 - (1-\tau)\psi_T\sigma_{Q,T}^2 \tag{1.36}$$

$$\mathrm{Cov}(\Delta c_T^{obs}, \Delta y_{T+2}^{obs}) = -\frac{1}{2}\psi_T\tau\sigma_{Q,T}^2 \tag{1.37}$$

1.6.3.3 The Two-shot Model

In the two shot model, transitory shocks consist of a mass of income arriving at time t followed exactly one year later by another mass of size θ of the first. The full set of moments for this model are:

$$\mathrm{Var}(\Delta y_T^{obs}) = \frac{1}{3}\sigma_{P,T}^2 + \frac{1}{3}\sigma_{P,T-1}^2 + \sigma_{Q,T}^2 + (1-\theta)^2\sigma_{Q,T-1}^2 + \theta^2\sigma_{Q,T-2}^2 \tag{1.38}$$

$$\mathrm{Cov}(\Delta y_T^{obs}, \Delta y_{T+1}^{obs}) = \frac{1}{6}\sigma_{P,T}^2 - \theta\sigma_{Q,T}^2 + \theta(1-\theta)\sigma_{Q,T-1}^2 \tag{1.39}$$

$$\mathrm{Cov}(\Delta y_T^{obs}, \Delta y_{T+2}^{obs}) = -\theta\sigma_{Q,T}^2 \tag{1.40}$$

The above equations also work for $\mathrm{Cov}(\Delta y_T^{obs}, \Delta y_{T-1}^{obs})$ and $\mathrm{Cov}(\Delta y_T^{obs}, \Delta y_{T-2}^{obs})$ due to symmetry.

$$\mathrm{Cov}(\Delta y_T^{obs}, \Delta y_S^{obs}) = 0 \qquad \forall S, T \text{ such that } |S-T| > 2 \tag{1.41}$$

The covariance matrix $\mathrm{Cov}(\Delta c_T^{obs}, \Delta c_S^{obs})$ is the same as in appendix 1.6.2.

$$\mathrm{Cov}(\Delta c_T^{obs}, \Delta y_T^{obs}) = \frac{1}{2}\phi_T\sigma_{P,T}^2 + \psi_T\sigma_{Q,T}^2 \tag{1.42}$$

$$\mathrm{Cov}(\Delta c_T^{obs}, \Delta y_{T+1}^{obs}) = \frac{1}{2}\phi_T\sigma_{P,T}^2 - (1-\theta)\psi_T\sigma_{Q,T}^2 \tag{1.43}$$

$$\mathrm{Cov}(\Delta c_T^{obs}, \Delta y_{T+2}^{obs}) = -\psi_T\theta\sigma_{Q,T}^2 \tag{1.44}$$

1.6.4 Effect of Timing of Consumption in the PSID

BPP impute annual consumption from the question in the PSID asking about food consumption in a ‘typical’ week. Unfortunately it is not clear if this relates to an average of the previous calendar year, or some more recent week closer to when the interview was conducted (normally in March of the following year). In the paper I have assumed the answer gives a snapshot of consumption at the end of the calendar year. Here I show that assuming the ‘typical’ week is an average of consumption over the previous calendar year, the identifying equation from BPP for transitory insurance coefficient is different again, and still significantly biased. Under this new assumption the equation for the permanent insurance coefficient is unbiased as before:

$$\frac{\mathrm{Cov}(\Delta c_T^{obs}, \Delta y_{T-1}^{obs} + \Delta y_T^{obs} + \Delta y_{T+1}^{obs})}{\mathrm{Cov}(\Delta y_T^{obs}, \Delta y_{T-1}^{obs} + \Delta y_T^{obs} + \Delta y_{T+1}^{obs})} = \phi$$

While the identifying equation for the transitory insurance coefficient is:

$$\frac{\mathrm{Cov}(\Delta c_T^{obs}, \Delta y_{T+1}^{obs})}{\mathrm{Cov}(\Delta y_T^{obs}, \Delta y_{T+1}^{obs})} = \frac{-\phi\frac{1}{6}\sigma_P^2 + \frac{1}{2}\psi\sigma_Q^2}{-\frac{1}{6}\sigma_P^2 + \sigma_Q^2} \neq \psi$$

Under the permanent income hypothesis with $\phi = 1$, $\psi = 0$ and permanent and transitory variances approximately equal, the BPP estimate of ψ would be -0.2.

1.6.5 Other Tables from the BPP paper

Table 1.3 replicates Table 7 from the original BPP paper.

Table 1.4 replicates Table 8 from the original BPP paper.

Table 1.3: Partial Insurance Estimates Replicating Table 7 from BPP

Consumption:	Nondurable		Nondurable		Nondurable	
Income:	net income		earnings only		male earnings	
Sample:	baseline		baseline		baseline	
	BPP	Time Agg.	BPP	Time Agg.	BPP	Time Agg.
ϕ	0.6456	0.3384	0.3101	0.1761	0.2240	0.1232
(Partial insurance perm. shock)	(0.0941)	(0.0471)	(0.0572)	(0.0339)	(0.0492)	(0.0316)
ψ	0.0501	0.2421	0.0630	0.1625	0.0502	0.1181
(Partial insurance trans. shock)	(0.0430)	(0.0431)	(0.0306)	(0.0280)	(0.0293)	(0.0244)

Table 1.4: Partial Insurance Estimates Replicating Table 8 from BPP

Consumption:	Nondurable		Nondurable		Nondurable	
Income:	net income		excluding help		net income	
Sample:	baseline		baseline		low wealth	
	BPP	Time Agg.	BPP	Time Agg.	BPP	Time Agg.
ϕ	0.6456	0.3384	0.6244	0.3422	0.8339	0.8584
(Partial insurance perm. shock)	(0.0941)	(0.0471)	(0.0891)	(0.0466)	(0.2762)	(0.2498)
ψ	0.0501	0.2421	0.0469	0.2404	0.2853	0.4926
(Partial insurance trans. shock)	(0.0430)	(0.0431)	(0.0429)	(0.0427)	(0.1154)	(0.1050)

Consumption:	Nondurable		Total		Nondurable	
Income:	net income		net income		net income	
Sample:	high wealth		low wealth		baseline+SEO	
	BPP	Time Agg.	BPP	Time Agg.	BPP	Time Agg.
ϕ	0.6278	0.2691	1.0207	1.0580	0.7663	0.4630
(Partial insurance perm. shock)	(0.0998)	(0.0420)	(0.3426)	(0.3099)	(0.1028)	(0.0499)
ψ	0.0088	0.1838	0.3647	0.6185	0.1201	0.3232
(Partial insurance trans. shock)	(0.0409)	(0.0409)	(0.1477)	(0.1344)	(0.0352)	(0.0367)

Chapter 2

Consumption Heterogeneity: Micro Drivers and Macro Implications

2.1 Abstract

This paper aims to test the microfoundations of consumption models and quantify the macro implications of consumption heterogeneity. We propose a new empirical method to estimate the sensitivity of consumption to permanent and transitory income shocks for different groups of households. We then apply this method to administrative data from Denmark. The large sample size, along with detailed household balance sheet information, allows us to finely divide the population along relevant

dimensions. For example, we find that households who stand to lose from an interest rate hike are significantly more sensitive to income shocks than those who stand to gain. Following a one percentage point rate increase, we estimate consumption will decrease by 26 basis points through this interest rate exposure channel alone, making this channel substantially larger than the intertemporal substitution channel that is the key mechanism in representative agent New Keynesian models.

2.2 Introduction

How do differences in household consumption behavior affect the business cycle? Recent heterogeneous agent models suggest that wealth redistribution between households with high and low marginal propensity to consume (MPC) may play a dominant role in propagating macroeconomic shocks, particularly for monetary policy. Testing the microfoundations of these models empirically, and quantifying the macroeconomic importance of redistribution, often boils down to measuring how MPCs vary systematically over dimensions such as wealth and exposure to interest rates. However, shortcomings in both the empirical methods used to measure MPCs and in the available data have limited the literature's ability to do this.

This paper overcomes some of these empirical shortcomings. We present a new method to measure MPCs from income and consumption panel data. We then apply it to different groups of households in administrative data from Denmark.

Our method builds upon that of Blundell, Pistaferri, and Preston (2008) (henceforth BPP), following their lead by imposing identifying restrictions on household income and consumption dynamics.[1] However, Crawley (2018) shows that the time aggregated nature of observed income data results in significant bias in BPP's estimates. In contrast to BPP, our identifying restrictions both account for time aggregation and allow for short-lived consumption responses.[2] Where BPP find the economy's overall MPC is close to zero, our estimate of 0.50 is encouragingly close to estimates obtained from natural experiments.

Our data consist of a panel of income and expenditure for the entire Danish population, along with details of the interest rate sensitivity of households' financial assets and liabilities that we require to estimate the redistribution effects of monetary policy. Income and wealth data are largely third party reported to the tax authority and correspondingly accurate. We use the intertemporal budget constraint to back out expenditure, an increasingly common approach with Scandinavian data. As our sample covers the whole economy, we can use the national accounts to reconcile with aggregates.

Speaking to the microfoundations of consumption behavior, we uncover a clear negative monotonic relation between MPC and liquid wealth. We show that the sign of this relationship is in line with standard buffer-stock models, although the

[1] The BPP method, and those closely related to it, have become a standard tool in the literature. See for example Violante, Kaplan, and Weidner (2014), Auclert (2017) and Manovskii and Hryshko (2017)

[2] We assume the consumption response to a transitory income shock lasts no more than two years. In BPP consumption follows a random walk. See section 2.4.3.2 for a discussion.

magnitude of MPCs, especially for the most liquid households, is difficult to reconcile with theory. The monotonic relationship fails when we include illiquid wealth, such as housing, which is consistent with the wealthy hand-to-mouth model of Violante, Kaplan, and Weidner (2014).

The strength of our method and data over previous studies can be clearly seen when we move to quantifying the size of monetary policy redistribution channels. We follow the decomposition of Auclert (2017) who identifies the relevant dimensions of redistribution, but being limited by the econometric methods he has to hand (including BPP), and to publicly available data sources, he finds it is a challenge to get a clear picture of how MPCs vary over these dimensions.

In our data we can clearly see three groups with distinct MPCs and exposure to interest rates: the 'wealthy hand-to-mouth', with MPCs around 0.5, who typically own houses and have mortgages and other debts; the 'poor hand-to-mouth', with MPCs around 0.8, who own few assets, liquid or otherwise; and the 'wealthy', with MPCs around 0.25, who typically own houses and also have large liquid bank balances.[3].

We estimate that a 1% rise in the real interest rate, which redistributes wealth from the 'wealthy hand-to-mouth' who pay the higher rate to the 'wealthy' who receive it, reduces aggregate consumption by 26 basis points through this redistribution channel alone.

[3] These groups loosely line up with those of the same name in Violante, Kaplan, and Weidner (2014), who define 'wealthy hand-to-mouth' as households with significant illiquid assets but little or no liquid assets. We observe that these three groups are naturally separated along the dimension of unhedged interest rate exposure. See figure 2.8.

We believe the kind of detail we are able to provide on the relationships between MPC, home ownership, liquidity and interest rate exposure could be used to discipline microfounded macroeconomic models going forward. As a small step towards this goal, we propose extending the standard buffer-stock model to include large transitory preference shocks, which can be thought of as unexpected costs, such as roof repair. This helps to replicate the high MPCs we observe in the data.

A growing number of large, high quality panel datasets on income and consumption are becoming available to economists.[4] With this, the value of robust econometric methods that can uncover household behavior will increase. Beyond the applications in this paper, the method we present here has a wide variety of potential applications in the consumption, household finance and labor literature.

2.3 Background

The need for better methods and data to measure consumption behavior at the household level has grown with the increasing recognition that household heterogeneity may play a key role in macroeconomic dynamics. Kaplan and Violante (2018) provide a nice overview of the theoretical literature incorporating household heterogeneity into models of economic fluctuations. Computational and methodological

[4] While access is very restricted, panel data from financial aggregation platforms has been highly informative about consumption behavior. The US examples include Gelman, Kariv, Shapiro, Silverman, and Tadelis (2014), Ganong and Noel (2017) and Baker (2015), while Vardardottir and Pagel (2016) use data from Iceland.

limitations, along with early work by Krusell and Smith (1998) showing that the aggregate dynamics of a TFP shock were not much changed in a heterogeneous agent model, have resulted in a slow start for this literature. However, recent advances have allowed for a new generation of Heterogeneous Agent New Keynesian (HANK) models that, as their name suggests, combine elements from both the heterogeneous agent and New Keynesian literature. These models not only match the growing evidence on micro-level consumption behavior, but also imply very different aggregate dynamics and/or propagation mechanisms following macroeconomic shocks, compared to their representative agent equivalents. In particular the transmission mechanism of monetary policy can look very different in a HANK model.[5]

While these HANK models make clear the potential importance of heterogeneity in economic fluctuations, particularly for monetary and fiscal policy, their quantitative results hinge on assumptions, such as the tenure of debt instruments and the government's fiscal rule, that were unimportant in representative agent models. Thus far the ability of these models to help us distinguish transmission channels *empirically* has been limited. Auclert (2017), in contrast to the fully structural HANK models, takes a simplified approach to aggregate dynamics, and one that we will follow in this paper.[6] He derives a set of sufficient statistics, directly measurable from a suitable dataset, that is highly informative about the relative size of different monetary policy

[5]For example, in the model of Kaplan, Moll, and Violante (2016) the intertemporal substitution channel is dwarfed by indirect general equilibrium effects, in stark contrast to a representative agent model.

[6]Wong (2016) also takes an empirical approach by identifying how the consumption response to monetary policy shocks varies with age.

transmission channels. His methodology benefits from being transparent and closely tied to the data, reducing the problem to that of measuring the distribution of MPCs across relevant dimensions of redistribution. However, as we will see in the following section, evidence on how MPCs vary across the population has been hard to come by.[7]

2.3.1 Existing Empirical Evidence on Heterogeneity in Consumption Behavior

Most micro-empirical evidence on consumption behavior comes in the form of an estimate of the marginal propensity to consume out of a one time source of income over the following three months to one year. Table 2.1 shows a selection of the population average estimates from the literature. Most of these studies do not have the power to say much if anything about heterogeneity within the population.

Three methods are used to empirically determine the marginal propensity to consume. The first is to identify a natural experiment and measure the consumption response to it. Often this is done using the Consumer Expenditure Survey in the US. For example Johnson, Parker, and Souleles (2006) use randomly assigned timing of 2001 tax rebates and questions in the Consumer Expenditure Survey to identify

[7] Fagereng, Holm, and Natvik (2016) also estimate Aulert's sufficient statistics, imputing MPCs from lottery winnings in Norway, but they are limited by sample size. Ampudia, Georgarakos, Slacalek, Tristani, Vermeulen, and Violante (2018) look at differences in Auclert's statistics between European countries, but do not attempt to estimate MPCs.

a three month aggregate marginal propensity to consume of 0.2-0.4. Of the three methods, natural experiments likely have the strongest identification, but estimates vary, and there is no strong consensus. Identification issues arise as to when exactly households learn about the payment versus when it is received, and the extent to which external validity extends from these natural experiments to the kinds of transitory shocks found in heterogeneous agent models is unclear.[8] As most of these studies rely on consumer survey data they tend to lack power due to high measurement error and low sample sizes. As a result, they have produced very little evidence of how the MPC varies among different groups in the economy. A recent paper by Fagereng, Holm, and Natvik (2016) overcomes some of these problems. By using lottery data, the shock to income is truly random.[9] They use registry data from Norway similar to the data we use from Denmark and have a sample of over 30,000 lottery winners over 10 years. As a result, they can identify the MPC for households with differing liquid wealth, as well as by the size of the lottery win. They find that households in the lowest quartile of liquid wealth have an MPC of approximately 0.61 over a 6 month period, as opposed to 0.45 for households in the highest quartile of liquid wealth. In another study using data from a financial aggregator, Gelman (2016) has enough power to identify large differences in the impulse response to a tax rebate at

[8]Many studies find a smaller MPC for positive shocks than negative shocks, for example Bunn, Le Roux, Reinold, and Surico (2018). In this paper we implicitly assume that the response is symmetric. In reality our estimates represent an average of positive and negative shock reactions.

[9]We should note that even lottery winnings have some problems. First the results hold for winners of the lottery who may not be representative of the wider population. Second the consumption response to a lottery win may be very different to other income shocks. For example you may spend a significant portion of a small lottery win just celebrating the fact.

a monthly frequency between household quintiles of cash-on-hand.

The second method is simply to ask individuals how much of a transitory income change they would consume, as was done in the Italian Survey of Household Income and Wealth in 2010 and the NY Fed's Survey of Consumer Expectations in 2016-2017. Jappelli and Pistaferri (2014) find an aggregate MPC of 0.48 using this Italian data and are able to identify clear differences across levels of liquid wealth. Fuster, Kaplan, and Zafar (2018) find a lower aggregate MPC in the NY Fed's survey, but find heterogeneity by both size and sign of the shock. While this method holds great promise, it is clearly limited by the accuracy of households' own response to the question.

The third method, which we will follow, is to impose covariance restrictions on panel data of income and consumption and use these to identify the consumption response to income shocks of differing persistence. This method has the advantage that it can be used in a panel dataset with no natural experiment, such as the Danish administrative data we use or the PSID. The most well known paper to use this method is by Blundell, Pistaferri, and Preston (2008), who use imputed non-durable consumption data based on food expenditure reported in PSID data. They estimate a consumption elasticity (closely related to an MPC if households' consumption level is close to their income) and find very little consumption response to transitory shocks; however, as we will discuss in section 2.4.2, this estimate is strongly downward biased.

This paper also adds to the limited literature on consumption responses to perma-

Table 2.1: Estimates of the Marginal Propensity to Consume from Income Shocks

	Consumption Measure				
Permanent Shocks	Nondurables	Total PCE	Horizon	Method	Event/Sample
Blundell, Pistaferri, and Preston (2008)*	0.65		~	3	Estimation Sample: 1980–92
Gelman, Gorodnichenko, Kariv, Koustas, Shapiro, Silverman, and Tadelis (2016)		1.0	~	1	Gasoline Price Shock
Transitory Shocks					
Agarwal and Qian (2014)		0.90	10m	1	Singapore Growth Dividend 2011
Blundell, Pistaferri, and Preston (2008)*	0.05			3	Estimation Sample: 1980–92
Browning and Collado (2001)		~ 0		1	Spanish ECPF Data, 1985–95
Coronado, Lupton, and Sheiner (2005)		0.36	1y	1	2003 Tax Cut
Fuster, Kaplan, and Zafar (2018)		0.08–0.31	3m	2	NY Fed Survey Cons. Expectations
Hausman (2012)		0.6–0.75	1y	1	1936 Veterans' Bonus
Hsieh (2003)*	~ 0	0.6–0.75		1	CEX, 1980–2001
Jappelli and Pistaferri (2014)	0.48			2	Italy, 2010
Johnson, Parker, and Souleles (2009)	~ 0.25		3m	1	2003 Child Tax Credit
Lusardi (1996)*	0.2–0.5			3	Estimation Sample: 1980–87
Parker (1999)	0.2		3m	1	Estimation Sample: 1980–93
Parker, Souleles, Johnson, and McClelland (2013)	0.12–0.30	0.50–0.90	3m	1	2008 Economic Stimulus
Sahm, Shapiro, and Slemrod (2010)		$\sim 1/3$	1y	1	2008 Economic Stimulus
Shapiro and Slemrod (2009)		$\sim 1/3$	1y	1	2008 Economic Stimulus
Souleles (1999)	0.045–0.09	0.34–0.64	3m	1	Estimation Sample: 1980–91
Souleles (2002)	0.6–0.9		1y	1	The Reagan Tax Cuts of the Early 1980s

* Elasticity. Methods: 1) Natural experiment 2) Survey question 3) Covariance restrictions

This table is adapted from Carroll, Slacalek, Tokuoka, and White (2017).

nent shocks to income. Natural experiments for permanent shocks are very hard to come by. Gelman, Gorodnichenko, Kariv, Koustas, Shapiro, Silverman, and Tadelis (2016) use shocks to gasoline prices as a proxy for a permanent shock to income and find an MPC close to 1 across the population. BPP find a consumption elasticity to permanent shocks to income around 0.65 (the permanent shock elasticity is less affected by the time aggregation problem). For a more complete overview of the literature on consumption responses to income changes, see Jappelli and Pistaferri (2010).

2.4 Empirical Strategy

We will take a reduced form approach to estimate four parameters: the variance of permanent and transitory income shocks and the marginal propensity to consume out of permanent and transitory income shocks. To do this we will make identifying restrictions on income and consumption dynamics. Specifically, we will assume that income is made up of a permanent component that moves as a random walk and a transitory component with persistence of less than two years. For consumption, we assume it responds permanently to a permanent income shock, but has a short-lived response of no more than two years to a transitory income shock. Our model will be in continuous time in order to correctly account for the time aggregated nature of our data. These restrictions allow use to calculate a set of observable moments with which we can estimate the four parameters of interest using GMM.

While this strategy allows us to precisely estimate these quantities, in some ways it obscures from the key features of the data that are driving the results. Therefore, in the next section we build some intuition on where identification is coming from by running some simple regressions.

2.4.1 Methodology Intuition

In this section we present some very simple regressions of expenditure growth on income growth and compare them with what we would expect in some very well understood baseline models.

We will look at the estimate of β^N in the model

$$\Delta^N c_{it} = \alpha^N + \beta^N \Delta^N y_{it} + \varepsilon_{it}$$

where N, the number of years over which growth is measured, varies from 1 to 10. Our full identification will come from the fact that transitory income shocks make up a relatively large proportion of the variance of income growth over a short period, while permanent income shocks dominate the variance of income growth over a long period. Figure 2.1 shows what we would expect to see under three well known models, as well as what we actually observe in the data. In a complete markets model in which all idiosyncratic shocks to income are insured against there is no relation between income growth and consumption growth, as represented by the blue horizontal line at zero. In the Solow growth model, and also in some old Keynesian models, households' expenditure is a constant proportion of income in that period, regardless of transitory shocks to income. The green horizontal line around 0.75 shows what we would see in a model of this type where households spend 75% of their income each period. The red line shows the results for a typical buffer-stock saving model.[10] In this model the regression of consumption growth on income growth over one year yields a relatively small number as households are able to self-insure against the transitory shocks that dominate at this frequency. As the time period over which income growth is measured increases the observed income growth is proportionally more permanent and self-insurance is not possible. The red line asymptotes towards 1.0 as N gets large.

The gray line, along with 95% confidence intervals, shows the results of these regressions using all households in the Danish sample. It is striking that the data appears to be closest to the Solow model, with only a small decrease in the regression coefficient over short periods. However, aggregating all households in this way hides a large degree of heterogeneity, particularly across households with different levels of liquid wealth. The two black lines show the regression coefficients where the sample is restricted to households in the lowest and highest quintiles of liquid wealth (averaged over the observed period) respectively. For households in the lowest quintile there is no evidence of consumption smoothing. As the regression coefficient is relatively

[10]These regression results come from the benchmark model presented in section 2.8.1, calibrated to the distribution of liquid wealth in Denmark.

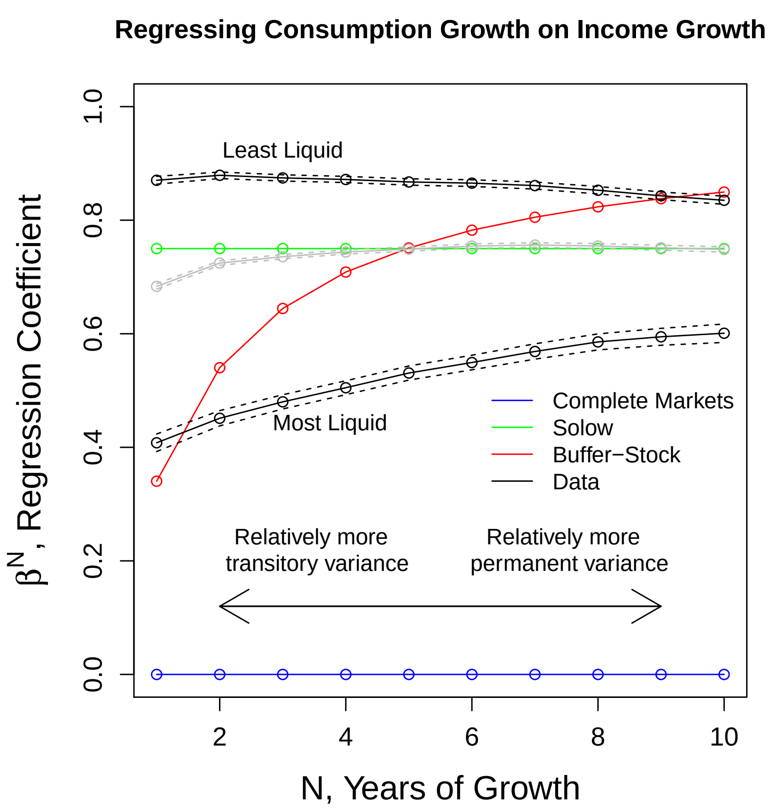


Figure 2.1: Regression Coefficients of Consumption Growth on Income Growth

stable for this group over N, the result, that the marginal propensity to consume out of both transitory and permanent shocks are similar and very high, is robust to a large degree of model misspecification, as discussed in the next section. Households in the top quintile of liquid wealth show a clear upward slope in figure 2.1, indicating a substantial degree of consumption smoothing. The fact that the regression coefficient for this group appears to asymptote well below 1 also suggests, in contrast to standard buffer-stock models, that the MPC out of permanent shocks for liquid households is significantly lower than 1.

2.4.2 Aside: Why Not BPP? A Brief Introduction to the Time Aggregation Problem

As explained above, our identification is going to come from the shape of income and consumption covariance over increasing periods of time. An obvious question is why we have chosen not to use the well known methodology of Blundell, Pistaferri, and Preston (2008) who achieve identification of transitory shocks from the fact that a transitory shock in period t will mean-revert in period $t+1$.[11] Unfortunately the method is not robust to the time aggregation problem of Working (1960). While macroeconomists have long been aware of the importance of time aggregation in time

[11] Kaplan and Violante (2010) show in discrete time simulations that the methodology works reasonably well for standard calibrations of buffer-stock models and end up concluding “The BPP insurance coefficients should become central in quantitative macroeconomics”. However, some recent papers such as Commault (2017) and Hryshko and Manovskii (2018) have pointed to other potential problems of the methodology.

series regressions (see Campbell and Mankiw (1989) for a well known example), the problem has been overlooked by the household finance and labor economics literature. We will therefore briefly describe the problem here. For a more detailed account with particular attention to BPP, see Crawley (2018).

Figure 2.2 shows how the problem arises. The solid 'Income flow' line shows the true income flow of a household who receives zero income throughout year 1, zero income for the first half of year 2, and then a constant income flow of 1.0 per year during the second half of year 2 and in year 3. The dashed line shows the observed *total* income of the household in years 1, 2 and 3: zero in year 1, 0.5 in year 2 and 1.0 in year 3. The important thing to note is that despite there only being one 'shock' to the income flow over the three year period, the naïve observer would assume there had been two shocks, one between years 1 and 2 and another between years 2 and 3. This effect is of particular importance to econometric techniques that make use of the auto-covariance structure of data processes. For example the first difference of a random walk in discrete time has no autocorrelation, but the first difference of a time-aggregated random walk in continuous time has an autocorrelation equal to 1/4. BPP use time aggregated income data and achieve identification of transitory variance precisely through the auto-covariance structure. This is why the problem is particularly pervasive for this methodology: in a simulated dataset where households follow the permanent income hypothesis, that is they respond one-for-one to shocks to permanent income but not at all to transitory income shocks, the estimate for

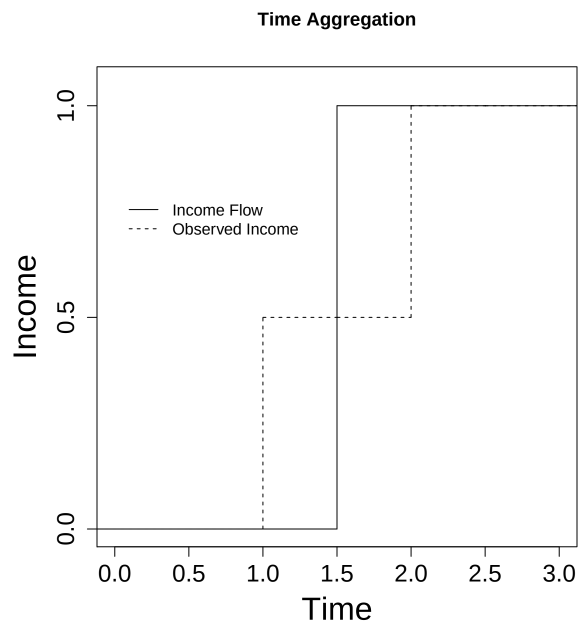


Figure 2.2: The Time Aggregation Problem

the consumption response to transitory income shocks using the BPP methodology is *negative* 0.6.[12]

While it would be possible to stick very closely to the original BPP model and adjust the covariance restrictions to take account of the time aggregation problem,[13] we have found that in practice the underlying assumptions made by BPP (in particular that consumption follows a random walk) do not fit with the data.[14] Therefore

[12]This is for a simulation in which permanent and transitory shock variances are equal and shocks are uniformly distributed over the year.

[13]Crawley (2018) takes this more straightforward approach using the same PSID data as used in BPP.

[14]Kaplan and Violante (2010) show that without time aggregation, the BPP method correctly identifies the transitory consumption response in the period of the income shock regardless of the consumption dynamics going forward. This fact is again not robust to the time aggregation problem. With time aggregation taken into account the estimates are highly sensitive to assumptions about short-term consumption dynamics.

we have chosen to attain identification in a manner similar to Carroll and Samwick (1997) which allows us to be agnostic about the exact short-term dynamics of income and consumption.

2.4.3 Covariance Restrictions

2.4.3.1 Income Dynamics: Carroll and Samwick (1997)

Our identification of permanent and transitory income variance will follow the methodology of Carroll and Samwick (1997) closely. Our method will correctly account for time aggregation, but due to identification coming from income growth over 3, 4 and 5 years, rather than the covariance of income growth at time $t+1$ with time t as in BPP, time aggregation only introduces a small bias in the estimates of Carroll and Samwick (1997). We will first describe the method without time aggregation and then show how the estimates need to be adjusted.

Carroll and Samwick (1997) assume that income is composed of a permanent component that follows a random walk and a transitory MA(2) component. That is:

$$y_t = p_t + \varepsilon_t + \theta_1 \varepsilon_{t-1} + \theta_2 \varepsilon_{t-2}$$

$$p_t = p_{t-1} + \zeta_t$$

where ζ_t and ε_t are mean zero random variables, independent of each other and of themselves over time. Each has constant variance, σ_ζ and σ_ε respectively. For $N \geq 3$

we have:

$$\Delta^N y_t = \zeta_t + \zeta_{t-1} + ... + \zeta_{t-N+1}$$
$$+ \varepsilon_t + \theta_1\varepsilon_{t-1} + \theta_2\varepsilon_{t-2} - (\varepsilon_{t-N} + \theta_1\varepsilon_{t-1-N} + \theta_2\varepsilon_{t-2-N})$$
$$\Rightarrow \mathrm{Var}(\Delta^N y_t) = N\sigma_\zeta + 2\underbrace{(1+\theta_1^2+\theta_2^2)\sigma_\varepsilon}_{\text{`Total' transitory variance}} \qquad \text{for } N \geq 3 \tag{2.1}$$

Equation 2.1 shows that the variance of income growth grows linearly with the number of years of growth beyond 3. The transitory component adds variance at the beginning and end of the growth period, but any transitory shock to income that occurs in the middle of the period does not affect income growth as it will have died out by the end. Carroll and Samwick (1997) use this to identify the variance of permanent shocks, σ_ζ, and the 'total' transitory variance, $(1+\theta_1^2+\theta_2^2)\sigma_\varepsilon$. While similar to BPP, it is important to note that BPP attempts to identify the variance of initial impact of the transitory shock, $\mathrm{Var}(\varepsilon)$, rather than the 'total' transitory variance. While the notion of 'total' transitory variance will carry over naturally into the time aggregated case, the variance of the initial impact does not have a natural interpretation.

The administrative data we use in this paper is at an annual frequency and measures the sum of income over the observed year. If shocks to income occurred only on 1st January every year then we could use equation 2.1 to identify permanent and transitory variance. It is important to distinguish between a model in which shocks happen about once a year (for example) but can occur at any point in the year, versus a model in which shocks to income happen on a timetable. The former can be modeled in continuous time with jumps occurring as a Poisson process approximately once a year. The latter is best modeled as a discrete time model. In this paper we will take the former approach. While some types of jobs may have a regular schedule on which pay appraisals take place, many of the larger permanent shocks to income occur when a worker changes job which can occur at any point in the year. Low, Meghir, and Pistaferri (2010) show that a significant portion of permanent income variance is explained by job mobility. We (along with the literature) lack a clear understanding of what makes up the bulk of the transitory shocks to income and the Danish data is a potentially rich source for further research in this area.[15] Furthermore, even if each individual household experienced shocks on a pre-set timetable, if the timetable itself varies across the year for different households, our approach would yield unbiased results. While there is a big change in going from an underlying annual process to a quarterly process, the further change from quarterly to continuous time is much smaller.[16] As the exposition is much simpler in continuous time, we will therefore choose to present our own method in continuous time.

To write the equivalent model in continuous time we define two underlying martingale processes (possibly with jumps), P_t and Q_t, where P_t will represent the *flow* of permanent income at time t and the change in Q_t provides the transitory *impulses*

[15] While we use annual income data in this paper in order to match with our expenditure data, monthly labor income data is collected, along with employer-employee matching.

[16] See Crawley (2018).

that generate the transitory income. We assume that for all $s_1 > s_2 > s_3 > s_4 > 0$:

$$\begin{aligned}
\mathrm{Var}(P_{s_1} - P_{s_2}) &= (s_1 - s_2)\sigma_P^2 \\
\mathrm{Cov}(P_{s_1} - P_{s_2}, P_{s_3} - P_{s_4}) &= 0 \\
P_s = 0 \qquad &\text{if } s < 0
\end{aligned}$$

and similarly for Q_t. That is, these martingales have independent increments. As a useful benchmark, two Brownian motions satisfy these criteria.

The natural generalization of the MA(2) transitory income process from Carroll and Samwick (1997) is to allow for a generically shaped transitory income shock that decays to zero in under 2 years.[17] Figure 2.3 shows an example of such a transitory income shape $f(t)$, but the model also allows for completely transitory shocks in which case $f(t)$ would be a delta function with all the income from the transitory shocks arriving as a mass at the time of the shock. In this model the *flow* of income arriving at time t is given by the flow of permanent income and the sum of income arising from any transitory shocks to income that have occurred in the previous 2 years:

$$y_t = P_t + \int_{t-2}^{t} f(t-s)dQ_s$$

We do not observe y_t directly but instead $\bar{y}_T$, the time aggregated income over each

[17] Previous studies have found little evidence of transitory dynamics lasting more that one year, but to be conservative and in line with BPP we allow transitory income to persist for up to 2 years.

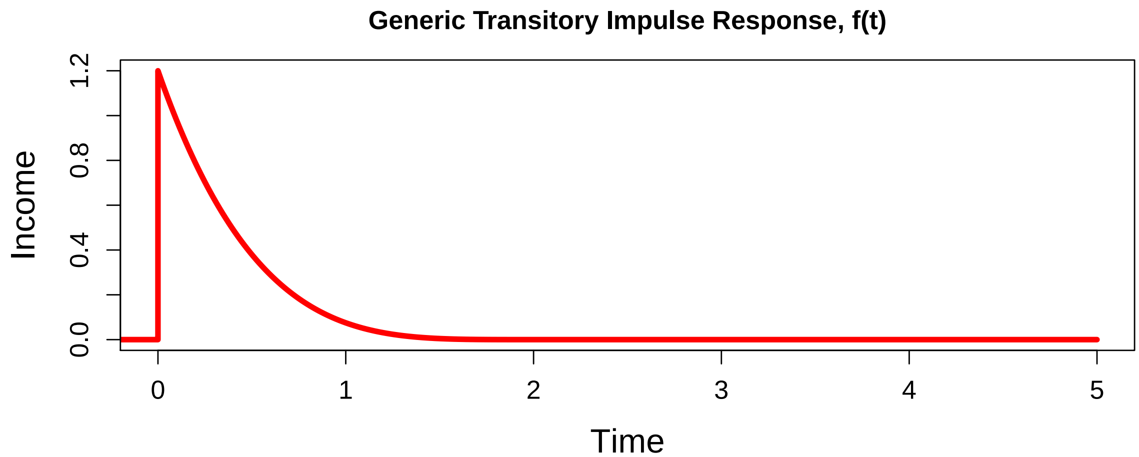


Figure 2.3: Generic Transitory Shock Impulse Response

one year period.

$$\bar{y}_T = \int_{T-1}^{T} y_t dt \text{ for } T \in \{1, 2, 3...\} \tag{2.2}$$

Taking the N^{th} difference for $N \geq 3$ we get:

$$\begin{aligned} \Delta^N \bar{y}_T &= \int_{T-1}^{T} y_t dt - \int_{T-N-1}^{T-N} y_t dt \\ &= \int_{T-1}^{T} (T-s) dP_s + (P_{T-1} - P_{T-N}) + \int_{T-N-1}^{T-N} (s - (T-2)) dP_s \\ &\quad + \left(\int_{T-1}^{T} \int_{t-2}^{t} f(t-s) dQ_t dt - \int_{T-N-1}^{T-N} \int_{t-2}^{t} f(t-s) dQ_t dt \right) \end{aligned} \tag{2.3}$$

The variance of time aggregated income of an N year period is therefore:[18]

$$\text{Var}(\Delta^N \bar{y}_T) = (N - \frac{1}{3})\sigma_P^2 + 2\text{Var}(\tilde{y}) \text{ for } n \geq 3 \tag{2.4}$$

This is similar to the non-time aggregated case (equation 2.1) except that the coefficient on permanent variance is $N - \frac{1}{3}$. This error, though having less serious

[18] See appendix 2.11.1 for full details of this derivation, including how we can approximate a log income process with levels.

consequences than for BPP, has nevertheless been overlooked by the large literature that studies income dynamics using panel data.[19] As with the MA(2) case the transitory variance identified is the variance of 'total' transitory income received in the year, $\tilde{y}$, where this is defined as:

$$\tilde{y_T} = \int_{T-1}^{T} \int_{t-2}^{t} f(t-s) dQ_s dt \tag{2.5}$$

Equation 2.4 with $N \in \{3, 4, 5\}$ provides us with the observable moments for income dynamics that we will use in our GMM estimation.

2.4.3.2 Consumption Dynamics

Our approach will be to extend the identification of income variance by using growth over 3, 4 and 5 years to also identify the covariance of income and consumption. In contrast to BPP, which assumes that consumption follows a random walk, we will instead assume that the impulse response to a transitory shock follows a generic path, $g(t)$, that, like the transitory income shock, has fallen to zero 2 years after the news of the shock. Figure 2.4 shows possible paths for both income and consumption, along with the alternative random walk impulse response of BPP. The best evidence for the speed at which the consumption response decays comes from Gelman (2016) and Fagereng, Holm, and Natvik (2016), both of which show that the response has entirely or almost entirely decayed 2 years after the shock. In section 2.9.3 we will show how

[19] For examples, see Moffitt and Gottschalk (2012), Meghir and Pistaferri (2004), Nielsen and Vissing-jorgensen (2004), Heathcote, Perri, and Violante (2010) and more recent quantile regression approaches such as Arellano, Blundell, and Bonhomme (2017).

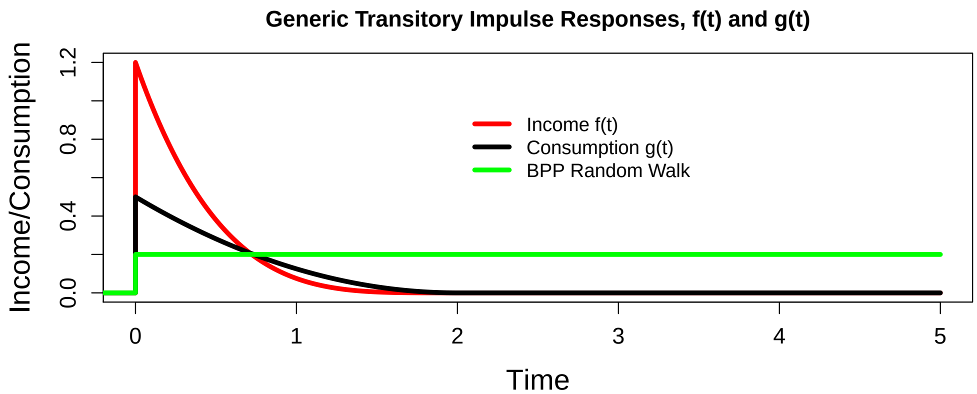


Figure 2.4: Generic Transitory Shock Impulse Response

this assumption may potentially bias the transitory consumption response down, but that this bias is small, especially for all but the most liquid households. We will maintain the assumption from BPP that the consumption response to a permanent shock to income follows a random walk proportional to the permanent shock. Under these assumptions the instantaneous flow of consumption is given by:

$$c_t = \phi P_s + \int_{t-2}^{t} g(t-s)dQ_s$$

and the covariance of time aggregated income and consumption growth over $N \geq 3$ years is given by

$$\text{Cov}(\Delta^N \bar{c_T}, \Delta^N \bar{y_T}) = \phi(N - \frac{1}{3})\sigma_p^2 + 2\text{Cov}(\tilde{c}, \tilde{y}) \text{ for } N \geq 3 \tag{2.6}$$

where total transitory income, $\tilde{y}$, is given by equation 2.5 and total transitory consumption, $\tilde{c}$, is defined by:

$$\tilde{c_T} = \int_{T-1}^{T} \int_{t-2}^{t} g(t-s) dQ_s dt \tag{2.7}$$

Using the equations for variance (2.4) and covariance (2.6) of observed income and consumption growth over N years for at least 2 different values of N, we are able to estimate the 4 unknowns we are interested in:[20]

1. σ_p^2 Variance of permanent shocks
2. $\sigma_{\tilde{q}}^2 = \mathrm{Var}(\tilde{y})$ Variance of transitory income received in a year
3. ϕ Marginal Propensity to eXpend (MPX) w.r.t. permanent income
4. $\psi = \frac{\mathrm{Cov}(\tilde{c},\tilde{y})}{\mathrm{Var}(\tilde{y})}$ Regression coefficient of transitory consumption w.r.t. transitory income over a year (MPX out of transitory income).

Our panel data covers 13 years and we choose to use growth over 3, 4 and 5 years to balance greater identification (longer growth periods give more power) with three identification problems that grow with N. The first is the fact that many households drop out of the sample if we demand they have reliable data for too many consecutive years. The second is that if the permanent shock in fact decays slowly over time (e.g.

[20] We have a total of 96 moments (We have 8 consecutive five year periods, each of which has three 3 year growth periods, two 4 year growth periods and one 5 year growth period. $8 \times (3+2+1) = 48$. Each of these growth periods has both a variance and a covariance moment, $48 \times 2 = 96$). With only four parameters to estimate the system is over identified. We strongly reject the null of the Sargen-Hansen J-test when run on our data, but this is not surprising given the sample size of our data.

is in fact AR(1)), the bias this introduces will be larger for large N. Finally, the validity of running the regressions in levels (rather than logs) is reduced over large N when the potential for the variance of income to change significantly from start to end of the sample is high. In section 2.9 and appendix 2.11.9 we test the importance of these issues.

We follow BPP and use diagonally weighted minimum distance estimation, although our results are not significantly changed by using other popular weighting methods.[21]

As the main part of our analysis will focus on the parameter ψ is it worth describing exactly what this is and why we have labeled it the marginal propensity to expend out of transitory income. If we were able to exactly observe transitory income and consumption resulting from transitory income then ψ would be the regression coefficient of this transitory consumption on transitory income. If transitory income shocks have no persistence this is approximately a six month MPX (on average the shock will happen six months into the year so that the regression will pick up the change in consumption in the following six months). If transitory income shocks have a little persistence (appendix 2.11.2 shows evidence of a small amount of transitory income persistence) ψ can only loosely be interpreted as the MPX to an income shock, and the reader should bear in mind that the true interpretation is, 'if income is higher

[21] As our sample size is large, the motivation for using diagonally weighted minimum distance (DWMD) over optimal minimum distance (OMD) is small, see Altonji and Segal (1996). We get very similar results using OMD. In general our results may be subject to misspecification problems, but the sample size of our data means that standard errors are small.

by one unit this year due to transitory factors, then consumption this year will be expected to be higher by ψ units'.

2.5 Data

Our panel data on income and expenditure comes from Danish registry data from 2003-2015. This data has a number of advantages over survey based measures. First, the sample contains millions of households rather than thousands. Second, households are required by law to report their data so there is much less risk of selection bias through drop outs. Third, measurement error in income data is largely eradicated, as employees' income data is third party reported by their employer, compared to survey data where self reported income has been shown to be particularly unreliable for irregular income.[22]

2.5.1 Income

We are interested in income and consumption decisions at the household level. We define a household as having either one or two adult members. Two adults are considered to be in the same household if they are living together and a) are married to each other or have entered into a registered partnership, b) have at least one common child registered in the Civil Registration System or, c) are of opposite sex and have

[22] See David, Marquis, Moore, Stinson, and Welniak (1997) for a survey of income measurement error issues in survey data.

an age difference of 15 years or less, are not closely related and live in a household with no other adults.[23] In the panel data, an individual's household will change if he or she gets married or divorced. This leads to some selection bias given that we require households to survive for at least 5 years. Following the literature our baseline results will be reported using the labor income of the head of household.[24] We will use after tax and transfer income as we are interested in the consumption response to these changes in income, although the method could be used to measure the extent of consumption insurance provided by the tax and transfer system. Our data comes from the administrative records from the tax authority. The tax reporting system in Denmark is highly automated and individuals bear little of the reporting burden. For employees income is reported by their employer and is thought to be highly accurate. The underground economy in Denmark is small. We remove business owners from the sample as their income may be less accurately reported, but more importantly, because the expenditure imputation method does not work well for them (see section 2.5.2).

We work with the residual of income after controlling for observable characteristics of households that may affect their income and consumption. To start with we

[23] Adults living at the same address but not meeting one of the three criteria are regarded as separate families. Children living with their parents are regarded as members of their parents' family if they are under 25 years old, have never been married or entered into a registered partnership and do not themselves have children. A family meeting these criteria can consist of only two generations. If three or more generations live at the same address, the two younger generations are considered one family, while the members of the eldest generation constitute a separate family.

[24] See Moffitt and Zhang (2018) for an overview of the literature on income volatility in the PSID. In contrast to the PSID literature we define the head of household as the highest earner over the 13 year period in our sample. We believe this definition better fits the social structure in Denmark.

remove households in the top and bottom 1% of the income distribution. We then normalize by average household income over the observed period, and regress income on dummies for age, year, highest level of education, marital status, homeowner status and number of children along with interaction of age with education, marital status and homeowner status. We take the change in the residuals of this regression to be the unexpected income change for a household from one year to the next and remove households in the top and bottom 1% of the unexpected income *change* distribution.

2.5.2 Imputed Expenditure

Our expenditure data comes from imputing expenditure from income and wealth. Along with other Scandinavian countries, Denmark is unusual in that tax reporting includes information about wealth along with income, a legacy from the wealth tax that was phased out between 1989 and 1997. Following the methodology from Browning and Leth-Petersen (2003) and Fagereng and Halvorsen (2015) we impute expenditure using the identity:

$$\bar{C}_t \equiv \bar{Y}_t - \bar{S}_t = \bar{Y}_t - P_t - \Delta NW_t$$

where $\bar{C}_t$, $\bar{Y}_t$ and $\bar{S}_t$ are the sum of expenditure, income and savings over the year t respectively. P_t is contributions to privately administered pension schemes, for which we have very accurate data due to tax deductibility, ΔNW_t is the change in (non-pension, non-housing) net worth measured at the end of years t and $t-1$. Banks and brokers are required to report the value of their clients' accounts on 31st December each year, and the tax reporting year runs from 1st January to 31st December, so the data for income and wealth reported in the tax returns matches with that required to use this identity to impute consumption.

The method works well for households with simple financial lives. One of the biggest problems with the method is its inability to handle capital gains well. The income used in the imputation includes all labor income and capital income, however it excludes capital gains. The value of assets will vary both due to savings from reported income but also due to capital gains and losses. We handle this in a number of ways. First, we completely exclude housing wealth from our measures of net worth and saving, treating housing as an off balance sheet asset. The problem with treating housing in this way is that we must exclude households in years in which they are involved in a housing transaction. For the self-employed, it is also difficult to distinguish between expenditure and investment in their business, so we exclude all households who receive more than a trivial amount of their income from business ventures. Finally, households that hold significant equity investments are likely to see sizable capital gains and losses. We make a naive adjustment by making the assumption that they hold a diversified index of stocks. While this will likely lead to significant measurement error for these individuals, the concern is mitigated first by the fact that stock holding is much more unusual in Denmark than in the US for example. Only around 10% of households hold any stocks, and for many of those stocks make up only a small proportion of their total wealth. Furthermore, as we will explain in section 2.9.4, measurement error in consumption is not a concern unless it is correlated with changes in income. This seems unlikely to be the case, except for households that hold significant equity in the firms in which they work. Another concern with the imputation method is transfers of wealth, say between family members or friends. Indeed imputed expenditure is negative for approximately 3% of households and this may explain a proportion of that. We discard both income and expenditure data for households in years in which their expenditure is negative. In appendix 2.11.9 we test the robustness of our results to sample selection bias problems that these issues may give rise to.

As with income, we work with the residual of expenditure after normalizing by mean household income and controlling for the same observable features as income. We follow exactly the same steps as described in section 2.5.1.

In evaluating how much we can learn from such a measure, it should be compared to the best alternatives available to economists. In the original BPP paper the authors only have access to food expenditures from the PSID data and impute total non-durable consumption by comparison with the Consumer Expenditure Survey. Self reported consumption is also of notoriously poor quality even in comparison to self reported income. Furthermore, in the PSID data the questions regarding food expenditure are ambiguous as to which period exactly the question is referring to. A recent paper by Abildgren, Kuchler, Rasmussen, and Sorensen (2018) shows that

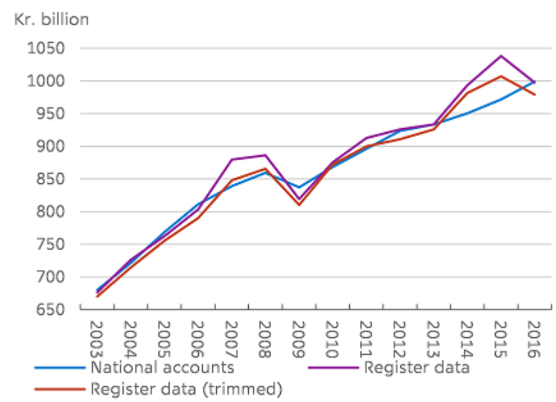


Figure 2.5: Imputed Register Measure and National Account Measure of Expenditure (from Abildgren, Kuchler, Rasmussen, and Sorensen (2018))

the mean levels of expenditure from this imputation method are close to those from the national accounts (see figure 2.5). They find relatively large differences at the household level between the consumer survey and imputed expenditure although it is not clear that this is a problem with the imputation method as opposed to the survey measure. Indeed for car purchases, for which highly accurate register data is available, the consumer survey shows significant underreporting, consistent with Koijen, Nieuwerburgh, and Vestman (2014) who find 30% underreporting of car purchases in the Swedish consumer survey. We believe that, with the exception of transaction level data reported by financial aggregation applications, the imputation method we use results in some of the highest quality expenditure data available to researches for the types of questions we are addressing.

2.5.3 Sample Selection

As our methodology requires income uncertainty to be relatively constant through the observed period and the young and old are likely to have predictable income trends unobservable to the econometrician, we limit the sample to households headed by an individual between the age of 30 and 55 in 2008.[25] Our final sample contains 7.7 million observations over 2004-2015 from an age group population totaling 18.1 million. The selection criterion that reduces the sample size the most is the requirement that a household does not make a housing transaction for a period of 5 years. Table 2.5.3 shows summary statistics for all Danish households whose head fits into this age group as a whole as well as the sample we use in estimation. It is reassuring that both the mean and median values for after tax income and consumption are similar in the estimation sample and the population. Our estimation sample has much lower standard deviations as a mechanical result of excluding the top and bottom 1% of the income and consumption distributions which contain extreme values. Sample selection shows up in homeownership and car ownership as we exclude those households that buy a house at the end of a 5 year period but who otherwise would be counted as renters. This also results in our sample being on average 1 year older than the population. Unhedged Interest Rate Exposure (URE) and Net Nominal Position (NNP) will be discussed in section 2.7, but again the significant differences here are due to the housing transaction criteria.

[25] Appendix 2.11.2 shows the assumption holds for this age group.

	Estimation Sample			Population (Age 30-55)		
	Mean	Median	Std Dev	Mean	Median	Std Dev
After Tax Income	59,261	57,804	28,819	58,312	53,304	68,799
Consumption	52,680	48,344	28,581	54,022	46,373	38,126
Liquid Assets	18,438	6,856	33,016	23,331	6,578	81,473
Net Worth	74,937	19,115	157,295	85,799	12,952	564,404
Homeowner	0.57	1.00	0.50	0.50	1.00	0.50
Car Owner	0.66	1.00	0.47	0.55	1.00	0.50
Higher Education	0.31	0.00	0.46	0.33	0.00	0.47
Age	43.5	44.0	7.1	42.5	42.0	7.3
URE	-28,052	-12,627	108,382	-47,589	-19,374	243,604
NNP	-109,685	-65,810	156,523	-158,321	-85,207	542,498
No. Household-year obs		7,664,360			18,050,340	

Notes: Values are 2015 USD. Age refers to the age in 2008 of the main income earner in the household. For the purposes of calculation of consumption in the population, top and bottom 1% in terms of consumption have been excluded. URE and NNP can only be calculated in the period 2009-2015 due to mortgage information being insufficiently detailed in the previous years.

Table 2.2: Summary Statistics

2.6 Income and Consumption Characteristics by Household Wealth

Liquidity constraints are the key microfoundation for the lack of consumption smoothing in heterogeneous agent models. In this section we look at the empirical relation between liquid wealth and the MPX out of both permanent and transitory shocks to income. We find a strong monotonic negative relation. We also look at net wealth and find such a monotonic relation no longer holds. In section 2.8 we show how these empirical results compare to a standard buffer-stock savings model.

Using our entire estimation sample we find a mean MPX out of transitory shocks of 0.50 and a mean MPX out of permanent shocks of 0.72. However, these averaged results hide a significant amount of heterogeneity. From the standpoint of consumption theory it is the ability of households to self-insure with their own wealth that most determines how much they smooth their consumption over shocks. We divide our estimation sample into quintiles according to both liquid wealth (which we define as bank deposits[26]) and net wealth. In each case wealth is measured as the mean household wealth holdings over the entire sample period.

Figure 2.6 shows the estimated income variances and MPX's for households in each quintile of liquid wealth.[27] Looking at the left hand variance panel first, it is noticeable

[26]The results are little changed using any other definition of liquid wealth as long as housing and debts are excluded. See appendix 2.11.9.

[27]For these graphs, and all similar ones in this paper, the 95% confidence intervals are shown above and below each quantile estimate.

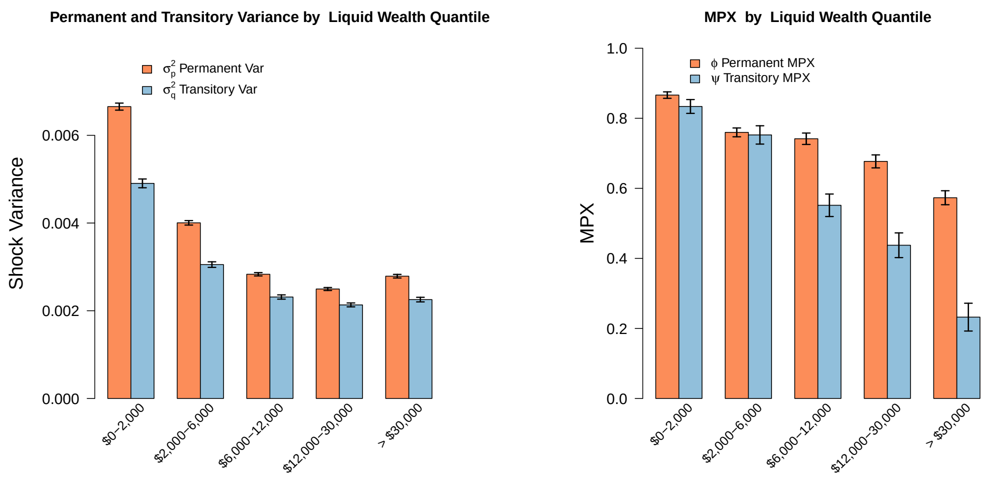


Figure 2.6: Variance and MPX by Liquid Wealth Quintile

that income uncertainty, and particularly permanent income uncertainty, is highest for households in the lowest quintile of liquid wealth. This provides some evidence towards the idea that heterogeneous tastes (e.g. discount factors of risk aversion) may be more important than income risk in determining wealth held for precautionary saving. For households in the top three quintiles of liquid wealth it is remarkable how similar their level of income risk is. Note that in contrast to standard estimates of the US income process, permanent income variance in Denmark is slightly higher than transitory variance, likely due to the high levels of social insurance available in Denmark. The variance level, at just over 0.002 for these top three quintiles, represents a standard deviation of just below 5% of permanent income per year.

Note that the estimates of income variance we obtain are highly sensitive to our treatment of outliers, but our MPX estimates do not change.[28]

The right hand panel of figure 2.6 shows our estimates for the MPX out of permanent and transitory shocks by liquid wealth quintile. The lowest wealth quintile, who hold less than $2,000 in bank deposits on average over the sample period, look somewhat like hand-to-mouth consumers. They respond almost equally to permanent and transitory shocks, spending over 80% of income shocks in the year that it arrives. However, the fact that both permanent and transitory MPXs are very similar and significantly less than 1 suggests that these households may be more accurately modeled as saving in an illiquid asset such as housing or a pension following a rule of thumb (say 20% of income) and then living hand to mouth on the remainder. As the quintile of liquid wealth increases, the MPX out of both transitory and permanent income decreases. In the top quintile, formed of households that maintained a mean bank balance above $30,000, the MPX out of permanent shocks is 0.57 and out of transitory shocks 0.23. From the point of view of theory the responsiveness of spending out of permanent shocks in this quintile is low, while that of transitory shocks is high. A more thorough discussion of how these results compare to a standard model calibrated to Danish characteristics will wait until section 2.8.

Figure 2.7 shows the estimates for households grouped by quintiles of net wealth. Here the pattern is slightly different. The quintile with the highest MPX out of both transitory and permanent income is the second lowest, the quintile that contains zero

[28]See appendix 2.11.9 for evidence of this as well as a discussion of why this is the case.

Figure 2.7: Variance and MPX by Net Wealth Quintile

net worth. Households in the lowest quintile, those with over $20,000 in net debt, do not seem to distinguish between permanent and transitory income shocks in their consumption responses, but their MPX for both is about 10 percentage points lower than the quintile with close to zero net wealth. The pattern for quintiles 3 to 5 looks similar to that for liquid wealth: the MPX out of transitory shocks falls sharply to around 0.28, while that out of permanent shocks also falls but more slowly to 0.62.

These results are broadly in line with the literature. The population mean of 0.5 for transitory MPX is a little higher than most estimates from table 2.1, but bearing in mind that our estimate includes durables and is best compared to a six month MPC, it is certainly not an outlier. The MPX out of permanent shocks of 0.72 is also between the BPP estimate of 0.65[29] and the estimate of 1.0 from

[29]The permanent 'insurance' coefficient estimated by BPP does not suffer as much from the time aggregation problem as the transitory coefficient.

Gelman, Gorodnichenko, Kariv, Koustas, Shapiro, Silverman, and Tadelis (2016). The strength of the relationship between liquid wealth and MPC is similar to that found in Gelman (2016) and stronger than in Fagereng, Holm, and Natvik (2016).

2.7 Monetary Policy and the Redistribution Channel

Auclert (2017) lays out a clear and intuitive theory as to how heterogeneity in the MPC out of transitory shocks affects the transmission mechanism of monetary policy. He identifies five channels through which monetary policy can act, three of which are absent without heterogeneity.[30] He then uses this theory to identify a small set of sufficient statistics that help distinguish which of these channels are of quantitative importance.

While these statistics in theory are highly informative about the transmission mechanism of monetary policy, in his paper he has neither the data nor the methods to be able to estimate them convincingly. He states, "As administrative quality household surveys become available and more sophisticated identification methods

[30] The key assumption made to link MPC with monetary policy redistribution is that households respond to redistribution in the same way as a transitory shock to income. We believe this is a reasonable assumption for the interest rate exposure channel, where households will have to pay a higher or lower rate out of liquid assets, but perhaps not for the Fisher channel. If the price level goes up 1%, a $100,000 debt is made smaller by $1,000 in real terms. However, the liquidity position of the household with this debt is not changed. As as result we have reservations about the reliability of our estimate of the size of the Fisher channel, which we estimate to be very large.

for MPCs arise, a priority for future work is to refine the estimates I provide here". Given we have administrative data, along with a new method to estimate MPCs, a natural application of our work is to estimate Auclert's sufficient statistics. Our data has two significant advantages over previous efforts.[31] First, our sample size is very large, containing a large percentage of all households in Denmark. Second, we have detailed balance sheet information for not only households within our sample, but also for those excluded from our sample. Furthermore, we are able to identify interest rate risk and nominal positions held by firms, foreigners and government so that the aggregate position is zero, as required in equilibrium. This allows us to avoid some of the more problematic assumptions used in aggregating household data.

2.7.1 Distribution of MPX Across NNP, URE and Income

The redistribution effects of monetary policy depend crucially on two household characteristics, their Net Nominal Position and Unhedged Interest Rate Exposure.

- **Net Nominal Position (NNP)** is the net value of a household's nominal assets and liabilities. It's relevance for analyzing the redistributive effects of monetary policy comes from the fact that an unexpected rise in the price level will decrease the wealth of households with positive nominal assets, redistributing

[31] As well as Auclert (2017), a new version of Fagereng, Holm, and Natvik (2016) also attempts to estimate these statistics.

it to those with negative NNP (who now have less real debt). In administrative data we are able to observe directly held nominal positions at the household level, including bank deposits and loans, bond holdings and mortgages. In aggregate the directly held NNP position of the household sector is negative, which from the national accounts we will see is balanced by the financial sector as well as foreigners.

- **Unhedged Interest Rate Exposure (URE)** measures the total amount that a household plans to save at the going interest rate during that period. It is the difference between all maturing[32] assets (including income) and liabilities (including planned consumption). For example, households with a large variable rate mortgage will likely have very negative URE. For them the entire value of their mortgage will be adjusted to the new rate. When the interest rate rises for one period they will see their disposable income (after mortgage payments) go down, and hence if they have a high MPX their spending will also decrease. To calculate URE we assume all bank deposits and bank debt to have a variable rate that changes instantaneously. For mortgage debt we directly observe the amount resetting over the following year and assume that the new rate will only apply for half of the year.[33] For all other assets and liabilities we assume

[32]We define 'maturing' assets and liabilites as those which are due to having their interest rates reset, also if they contractually exist for a longer period. For example, a 30 year variable rate mortgage with annual interest rate fixation periods is 'maturing' each year in our definition.

[33]See appendix 2.11.3 for more details on the Danish mortgage market. Note that prevalence of fixed rate mortgages will strongly influence the distribution of URE. To the extent that the US has more fixed rate mortgages than Denmark, the interest rate exposure channel is likely to be smaller

a maturity of 5 years. As with NNP we find households on aggregate have a negative URE position in our data and this is counterbalanced by the interest rate position of the financial sector. See appendix 2.11.4 for more details on how we calculate NNP and URE positions.

Figure 2.8 shows how the MPX varies across household values for URE, NNP and income. In each case the value on the x-axis has been divided by the mean level of expenditure of the entire sample. The top chart shows the estimated MPX for each decile of unhedged interest rate exposure. The deciles on the left contain households most negatively exposed to a rise in interest rates, those in the middle deciles have little exposure, while the two top deciles on the right have the most to gain from an interest rate rise. We have included in this figure data on both rates of homeownership and median liquid assets for each decile. A clear pattern emerges in which we can roughly categorize the deciles into three groups following Violante, Kaplan, and Weidner (2014):

- **Wealthy Hand-to-Mouth:** The first five deciles contain households with high levels of homeownership but relatively few liquid assets. These households have relatively high MPXs and it is likely that their wealth is locked up in illiquid assets (mostly housing) and that they have large mortgages.
- **Poor Hand-to-Mouth:** The next three deciles tend to be renters with very

in the US. A detailed examination of the Survey of Consumer Finance would be valuable exercise in determining the likely differences.

little in the way of liquid assets either. These households have very high MPXs and are close to being truly hand-to-mouth. As they have few assets they have very little exposure to interest rates and cannot easily substitute consumption between periods, therefore their consumption behavior is likely not affected by changes in interest rates directly.[34]

- **Wealthy:** The top two deciles contain households who are both likely to be homeowners and hold very large liquid asset balances. These are likely to be households who own their house outright without a mortgage and have been able to build up a large stock of liquid assets. Relative to the other deciles they have low MPX and are likely able to use their assets to effectively consumption smooth.

The distribution of MPX with net nominal position follows a similar pattern. As mortgages in Denmark are a mixture of fixed and variable rates (see appendix 2.11.3 for details on the Danish mortgage market), we can think of a typical household with negative URE or NNP as having a large mortgage, while those with positive URE or NNP are wealthy households with lots of liquid wealth. This pattern has not been evident in previous attempts to measure the distribution of MPX across these dimensions. Most importantly for the theory, the average MPX for those with negative URE and NNP positions is significantly greater than for those with positive

[34]Neither the interest rate exposure channel nor the intertemporal substitution channel will have much impact on their consumption. Monetary policy will impact their expenditure strongly through income effects.

URE or NNP. This confirms the intuition that households who owe a lot of floating rate debt have higher MPXs than those who own this debt, and leads to an interest rate exposure channel in which lowering interest rates increases expenditure. Note, the mean levels of both URE and NNP are negative for the households in our estimation sample, so even a constant (positive) MPX would result in interest rate hikes reducing their expenditure if not balanced by indirectly held exposures.[35]

The final chart in figure 2.8 shows the distribution of MPX with total household income. There is a clear downward trend. If the income of lower income households decreases more than that of high income households during a monetary policy contraction, then expenditure will go down by more than the mean income weighted MPX that would be the result of a representative agent model.

For comparison the distribution of MPX out of permanent income shocks across these three dimensions can be found in appendix 2.11.10.

2.7.2 Theoretical Setup and Sufficient Statistics

Auclert's method is to consider individual households' consumption response to a monetary policy shock in which i) the real rate of interest changes for one period by dR, ii) the price level makes a one time change of dP and then remains at the new level, and iii) aggregate income makes a transitory change of dY. While the dynamics

[35]In contrast, Auclert (2017) finds a mean positive URE across households. We believe the difference is partly due to the prevalence of fixed rate mortgages in the US, but also due to underreporting of expenditures, especially in the PSID data.

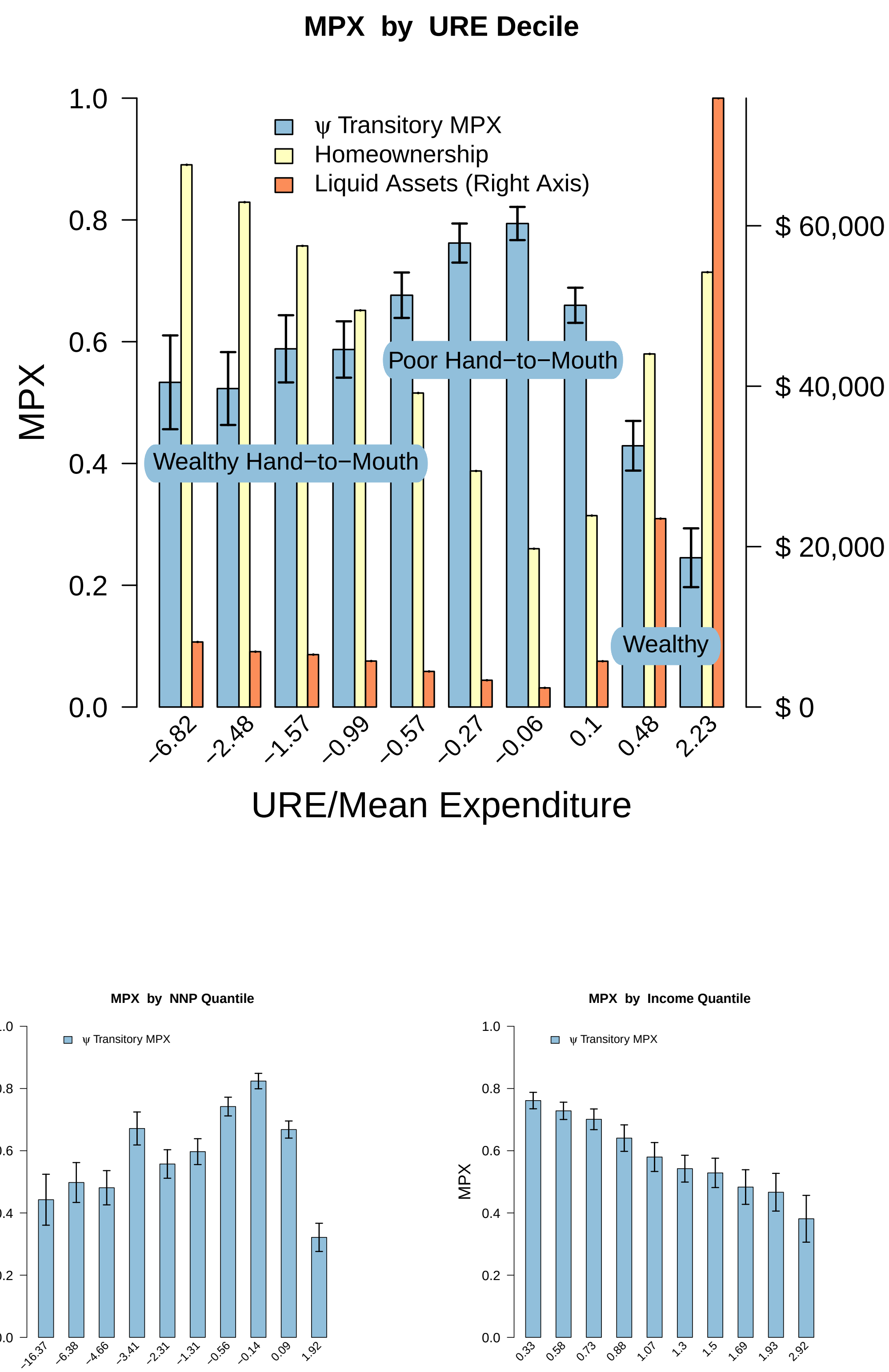


Figure 2.8: MPX Distribution by URE, NNP and Income

here are clearly stylized, and in particular lack any lag in the economy's response, we believe such a simple experiment to be highly informative as to the relative sizes of each transmission channel.

Auclert (2017) divides the effect of monetary policy on aggregate consumption into five distinct channels:

$$\frac{dC}{C} = \overbrace{\mathcal{M}\frac{dY}{Y}}^{\text{Aggregate Income Channel}} \quad \overbrace{+\gamma\mathcal{E}_Y\frac{dY}{Y}}^{\text{Earnings Heterogeneity Channel}} \quad \overbrace{-\mathcal{E}_P\frac{dP}{P}}^{\text{Fisher Channel}} \quad \underbrace{+\mathcal{E}_R\frac{dR}{R}}_{\text{Interest Rate Exposure Channel}} \quad \underbrace{-\sigma\mathcal{S}\frac{dR}{R}}_{\text{Intertemporal Substitution Channel}} \tag{2.8}$$

where σ is the elasticity of intertemporal substitution, γ is the elasticity of relative income to aggregate income[36] and the five sufficient statistics, $\mathcal{M}$, $\mathcal{E}_Y$, $\mathcal{E}_P$, $\mathcal{E}_R$ and $\mathcal{S}$, are measurable in the data and defined in table 2.3. We choose to define these statistics to include the consumption effects coming from exposures not directly held by households. We allocate the aggregate URE and NNP exposure from our estimation sample into seven bins so that the total exposure across the economy is zero. These bins include households with (i) young (¡30) and (ii) old (¿55) heads, and exposures held by households indirectly through (iii) pensions funds, (iv) government, (v) non-financial corporates, (vi) financials and (vii) exposures held by the rest of the world. Within each of these bins we assume no heterogeneity so that the MPX with respect to these exposures is constant. This is a conservative assumption, likely to underestimate the size of the heterogeneous agent channels. Our assumptions on the

[36] Here we are making the simplifying assumptions that these quantities are common for all households, see Auclert (2017) for a discussion.

level of these MPXs can be seen in table 2.7.3.

We define $\mathcal{E}_R$ as:

$$\mathcal{E}_R = \frac{1}{C}\left[\sum_{i \in \text{URE deciles}} \text{MPX}_i \text{URE}_i + \sum_{j \in \text{bins}} \text{MPX}_j \text{URE}_j\right] \tag{2.9}$$

where i sums over the ten deciles of URE, j over the seven bins defined above and C is aggregate household expenditure in the economy. This method of dealing with the fact that aggregate exposure does not equal zero in the estimation sample is different to the approach taken by Auclert. He assumes the residual exposure is distributed equally across households in the sample. By making use of the national accounts we believe we are able to get a better handle on the likely MPXs to attach to this residual exposure. Table 2.3 shows the definitions we use for each of the five measurable statistics in equation 2.8.

Table 2.3: Sufficient Statistics Definitions

Statistic	**Definition**	**Description**
$\mathcal{M}$	$\frac{1}{C}\left[\sum_{i \in \text{Income deciles}} \text{MPX}_i Y_i + \sum_{j \in \{\text{young,old}\}} \text{MPX}_j Y_j\right]$	Income-weighted MPX
$\mathcal{E}_Y$	$\mathcal{M} - \overline{\text{MPX}}\frac{Y}{C}$	Redistribution elasticity for Y
$\mathcal{E}_P$	$\frac{1}{C}\left[\sum_{i \in \text{NNP deciles}} \text{MPX}_i \text{NNP}_i + \sum_{j \in \text{bins}} \text{MPX}_j \text{NNP}_j\right]$	Redistribution elasticity for P
$\mathcal{E}_R$	$\frac{1}{C}\left[\sum_{i \in \text{URE deciles}} \text{MPX}_i \text{URE}_i + \sum_{j \in \text{bins}} \text{MPX}_j \text{URE}_j\right]$	Redistribution elasticity for R
$\mathcal{S}$	$1 - \frac{1}{C}\left[\sum_{i \in \text{Consumption deciles}} \text{MPX}_i C_i + \sum_{j \in \{\text{young,old}\}} \text{MPX}_j C_j\right]$	Hicksian scaling factor

Note: $\overline{\text{MPX}}$ is the mean MPX over all households in the economy. Y and C are aggregate household income and consumption respectively. Bins refers to the seven categories for which we have allocated URE and NNP exposures outside our estimation sample. {young,old} are the two bins that contain young and old households (the other five bins are only relevant for URE and NNP exposures as Y and C measure *household* income and consumption).

2.7.3 Out of Sample MPX

The assumptions we make about the MPX of the young and the old, as well as out of indirectly held URE and NNP exposures are shown in table 2.7.3. In each case we believe we have made conservative choices that will underestimate the size of the interest rate exposure channel of monetary policy. For the young we choose an MPX of 0.5, in line with the rest of the population. As the young have aggregate negative exposures, choosing an MPX on the low side is conservative. Similarly for the old we choose an MPX of 0.5, on the high side for this age group. The assumption that there is no heterogeneity in MPX within these groups is also a very conservative assumption.

Much of the URE and NNP exposure is not held directly on the balance sheet of households, but instead indirectly through pension funds, corporates and the government. There is significant evidence that the MPX out of shocks to the value of pension wealth, stocks or the government balance sheet is substantially lower than the MPX from income. We choose to use the estimate from Maggio, Kermani, and Majlesi (2018) that households' MPX from changes in stock market wealth is about 10%. This choice is the most quantitatively important as the bin containing the most exposure is the financial sector, which is positively exposed to interest rate increases. This may seem surprising as banks are typically thought to have long-term assets and short-term debt that would result in negative URE exposure. However, our findings are in line with Landier, Sraer, and Thesmar (2013) who find that the aggregate financial sector benefits from interest rate hikes, although there is a large amount of heterogeneity between different banks. An important caveat is due here: we focus on the MPX out of changes in the assets indirectly held by households through the financial sector and do not assume any spending or lending response at the bank level. While this may be a reasonable assumption in good times when banks are not credit constrained, it is especially not the case during a banking crisis. This could possibly result in monetary policy being much less effective during a banking crisis as the interest rate exposure channel to household spending is counterbalanced by a channel from bank balance sheet interest rate exposure to lending.[37]

We choose an MPX of zero for government and the rest of the world. There is no evidence that households respond in any significant way to changes in the government's balance sheet, and furthermore a low MPX is a conservative assumption for the size of the heterogeneous agent channels. As Denmark is a very small part of the world economy we assume that foreigners spend a negligible proportion of their wealth there.

[37] It should be noted that our analysis is all on the household side. Evidence suggests that firms are also sensitive to changes in cash flow, for example see Blanchard, Lopez-de Silanes, and Shleifer (1994).

	MPX	NNP	URE	$\mathcal{E}_P$ component	$\mathcal{E}_R$ component
Estimation Sample	**See Distribution**	**-204**	**-61**	**-0.78**	**-0.29**
Young	0.5	-32	-15	-0.12	-0.06
Old	0.5	-23	6	-0.09	0.02
Pension Funds	0.1	137	37	0.10	0.03
Government	0.0	-85	-23	0.00	0.00
Non-financial Corp.	0.1	-49	-13	-0.04	-0.01
Financial Sector	0.1	223	61	0.17	0.05
Rest of World	0.0	33	9	0.00	0.00
Total		**0**	**-0**	**-0.75**	**-0.26**

Notes: NNP and URE numbers are in billions of 2015 USD. Pension Funds includes special saving such as children's savings accounts. See appendix 2.11.4 for detail.

Table 2.4: Aggregating Redistribution Elasticities

2.7.4 Results

Our estimates of the five sufficient statistics are shown in table 2.5. The aggregate income channel is summarized by $\mathcal{M}$ that we estimate to be 0.52. This means that if income for all households in the economy increased by 1%, aggregate consumption would increase by 52 basis points. This is broadly in line with calibrations of saver-spender models designed to fit evidence from Campbell and Mankiw (1989). We find little role for the redistribution effect of income, $\mathcal{E}_Y$, despite the clear negative correlation between income and MPX seen in figure 2.8. $\mathcal{S}$, the Hicksian scaling factor, is 0.49, which reduces the size of the intertemporal substitution channel by close to a half.

The two most interesting statistics are $\mathcal{E}_P$ and $\mathcal{E}_R$, both of which act through redistribution from households with low MPX to those with high MPX. $\mathcal{E}_P$ is estimated to be -0.75 suggesting that a one time increase in the price level of 1% increases aggregate consumption by 75 basis points due to redistribution from those with large nominal assets to those with large nominal debts. This Fisher channel of monetary policy is emphasized in Doepke and Schneider (2006). The interest rate exposure channel is also large. We estimate $\mathcal{E}_R$ to be -0.26, suggesting that a 1% increase in the interest rate decreases expenditure by 26 basis points.[38]

For both of these channels, but particularly the interest rate exposure channel, it is informative to compare to the size of the intertemporal substitution channel. An increase in the real interest rate reduces aggregate consumption today by $\sigma\mathcal{S}$ multiplied by the percent change in the rate where σ is the intertemporal elasticity of substitution. Reliable estimates of σ have been elusive to the economics profession, but there is very little evidence of a large positive number. Havranek (2015) provides a meta-study of the elasticity of intertemporal substitution and finds a mean of zero from studies using macrodata, and 0.3-0.4 for those using microdata. Many of

[38]These estimates are significantly different to the results found in Auclert (2017). Our estimate of the Fisher channel is an order of magnitude larger, while our estimate of the interest rate exposure channel is over twice as large.

these micro-level studies suffer from identification problems.[39] A recent paper of Best, Cloyne, Ilzetzki, and Kleven (2018) makes use of mortgage notches in the UK to overcome some of these problems. They estimate the average elasticity of intertemporal substitution to be 0.1, which would result in a size of the intertemporal substitution channel of monetary policy being 0.05, over five times smaller than our estimate of the interest rate exposure channel.[40]

Table 2.5: Sufficient Statistics

$\mathcal{M}$	$\mathcal{E}_Y$	$\mathcal{E}_P$	$\mathcal{E}_R$	$\mathcal{S}$
0.52	-0.03	-0.75	-0.26	0.49

A long outstanding question in monetary economics is why monetary policy acts with a lag. Two competing theories are habits models such as Fuhrer (2000) and sticky information models such as Mankiw and Reis (2002). A recent paper by Carroll, Crawley, Slacalek, Tokuoka, and White (2018) finds evidence towards the idea that households react fast to their own idiosyncratic income shocks but news about macroeconomic shocks takes time to be absorbed. A possible third alternative to both of these is that households respond strongly to their realized income today, but not to income anticipated in the future. As it takes time for variable rate mortgages

[39]See Carroll (2001) for a critique of many older studies of the elasticity of intertemporal substitution.

[40]Our decomposition does not allow easy comparison of the interest rate exposure channel with the aggregate income channel, as we do not make assumptions about how much aggregate income changes. Cloyne, Ferreira, and Surico (2016) compare mortgagors with outright homeowners and find the aggregate income channel is larger than the direct effect of higher mortgage payments.

to reset (typically from 6 months up to 5 years in Denmark), this would result in the interest rate exposure channel acting with a delay. Indeed the literature on consumption responses to transitory income shocks has generally found little difference between anticipated and unanticipated responses. Many of the estimates in table 2.1 use anticipated shocks (such as tax rebates) as an instrument and find large MPCs, suggesting households do not necessarily pay attention to anticipated cash flows until they arrive. A recent paper by Ganong and Noel (2017) shows this very clearly: there is a sharp consumption drop in the month that unemployment benefits expire, an entirely anticipated event. A model which takes these results seriously, along with a large role for the interest rate exposure channel of monetary policy, could be a fruitful area of future research.

2.8 Benchmark Model and Taste Shock Extension

In this section we calibrate a standard incomplete markets model to Danish characteristics, including the liquid wealth distribution in Denmark, and use it to see if we can match the consumption responses we measure in the data.[41]

Motivated by the fact that the standard model results in lower transitory MPX

[41] By calibrating to the liquid wealth distribution, we are implicitly assuming that households cannot access any of their illiquid wealth. A model such as Violante, Kaplan, and Weidner (2014) or Gorea and Midrigan (2017) which allows households to buy and sell an illiquid asset at a cost would result in lower overall MPCs.

numbers than we find in the data, we make a simple extension to the model to account for potentially large preference shocks. We propose that such shocks, which have generally played a much smaller role in the literature than income shocks, are perhaps quantitatively more important for precautionary savings behavior.

2.8.1 Benchmark Model Calibrated to Danish Data

Our baseline model is the now very familiar buffer-stock saving model of Carroll (1997). Given market resources ($\mathbf{m}_t$), households in this model maximize expected utility:

$$\mathbb{E}_t \sum_{i=t}^{\infty} \beta^i u(\mathbf{c}_i)$$

subject to the constraints:

$$\begin{aligned}
\mathbf{a}_t &= \mathbf{m}_t - \mathbf{c}_t \\
\mathbf{b}_t &= R\mathbf{a}_t \\
\mathbf{y}_t &= \theta_t \mathbf{p}_t \\
\mathbf{p}_t &= \Psi_t \mathbf{p}_{t-1} \\
\mathbf{m}_t &= \mathbf{b}_t + \mathbf{y}_t
\end{aligned}$$

Where the felicity function, $u(\mathbf{c})$ is CRRA. We calibrate our model to match both the income uncertainty (as measured using our methodology) and the liquid wealth distribution in Denmark. To be able to match the liquid wealth distribution, especially

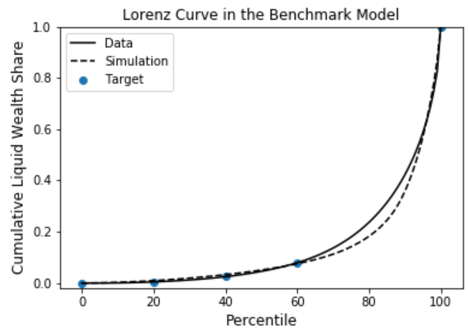


Figure 2.9: Lorenz Curve for Danish Liquid Assets

at the low end, we follow Krusell and Smith (1998) and Carroll, Slacalek, Tokuoka, and White (2017) and allow for ex-ante heterogeneity in the discount factor β. Specifically an agent i has a discount factor β_i where β_i is i.i.d. across agents and follows a uniform distribution between β_{low} and β_{high}. These two parameters allow us to match the fact that while the mean level of liquid assets is high, about half of all households have close to zero liquid assets. Matching the lower part of this distribution is critical to generate transitory consumption elasticities substantially above zero. The Lorenz curve for liquid assets, both in the data and in the model, is shown in figure 2.9.[42]

2.8.2 Model with Preference Shocks

The baseline model exhibits two features in tension with the data. First, the marginal propensity to consume out of transitory income shocks, while exhibiting

[42] We calibrate to the 20th, 40th and 60th percentile of liquid wealth, leaving out the 80th percentile. This is to better match the wealth of the lower half of the distribution, which is necessary to achieve reasonably high MPCs in a model like this. The figure shows that the fit in the upper half of the distribution is less precise.

the right shape relative to the liquid wealth, is too low relative to the data. Second, as would be expected in a consumption smoothing model like this, the path of expenditure is significantly less volatile than income. This is strongly at odds with the data which shows the standard deviation of changes in expenditure to be around 0.37, compared to 0.12 for income. There is very little evidence on the true size of expenditure shocks, partly because of large measurement errors known to be present in consumption survey data. While we believe the 0.37 number from our data also contains measurement error, as well as large expected expenditures such as new cars for which finance may be readily available, it seems likely that the expenditure shocks could be large. Indeed typical financial advice to maintain a buffer stock will mention unexpected costs such as medical bills or a leaky roof before income shocks.[43] A simple tweak to the baseline model can help the model fit the data along both these dimensions. To achieve this we add a preference shock to expected utility:

$$\mathbb{E}_t \sum_{i=t}^{\infty} \beta^i \mathcal{X}_i u(\mathbf{c}_i)$$

where $\mathcal{X}_i$ is i.i.d. and calibrated such that the variance of consumption is large.[44]

[43] For example Forbes Magazine in 2016 suggests "you could find yourself thrown off by a chipped tooth or fender bender. So having an emergency fund padded with nine months of the highest earner's net income may help give you a bit more peace of mind that you could weather a financial storm."

[44] We choose preference shocks with an annual standard deviation of 0.3. While this seems large, the resulting consumption change standard deviation is 0.18, significantly lower than 0.37 that we observe in the data.

2.8.3 MPX by Liquid Wealth

The top panel of figure 2.10 shows how the transitory MPX of the two models compares with the data. While the fact that the MPX decreases with the liquid wealth quintile is robust in both models and in the data, there are two features worth noting.

First, large preference shocks are required to push the transitory MPX close to the levels we see in the data. Many recent papers, such as Krueger, Mitman, and Perri (2016), have attempted to carefully quantify the macroeconomic dynamic consequences of a serious heterogeneous agent model, but thus far have not included significant preference shocks in their calibrations. The evidence here suggests that such shocks may have a quantitatively important role to play, especially in increasing the MPC. To the extent that the precautionary motive is driven by preference shocks as opposed to income shocks, social insurance for unemployment will not reduce precautionary savings as much as these models presently suggest.

Second, neither of the two models is able to explain the high MPX out of transitory shocks that we observe for the top quintile of liquid assets. These households, who hold a mean balance above $30,000, appear very responsive to transitory shocks despite their large buffer stock they could potentially use to smooth income shocks.

The bottom panel of figure 2.10 shows another failure of both these two simple models: neither is able to capture the fact that the consumption response to permanent shocks is substantially below 1, even for middle and low quintiles of liquid

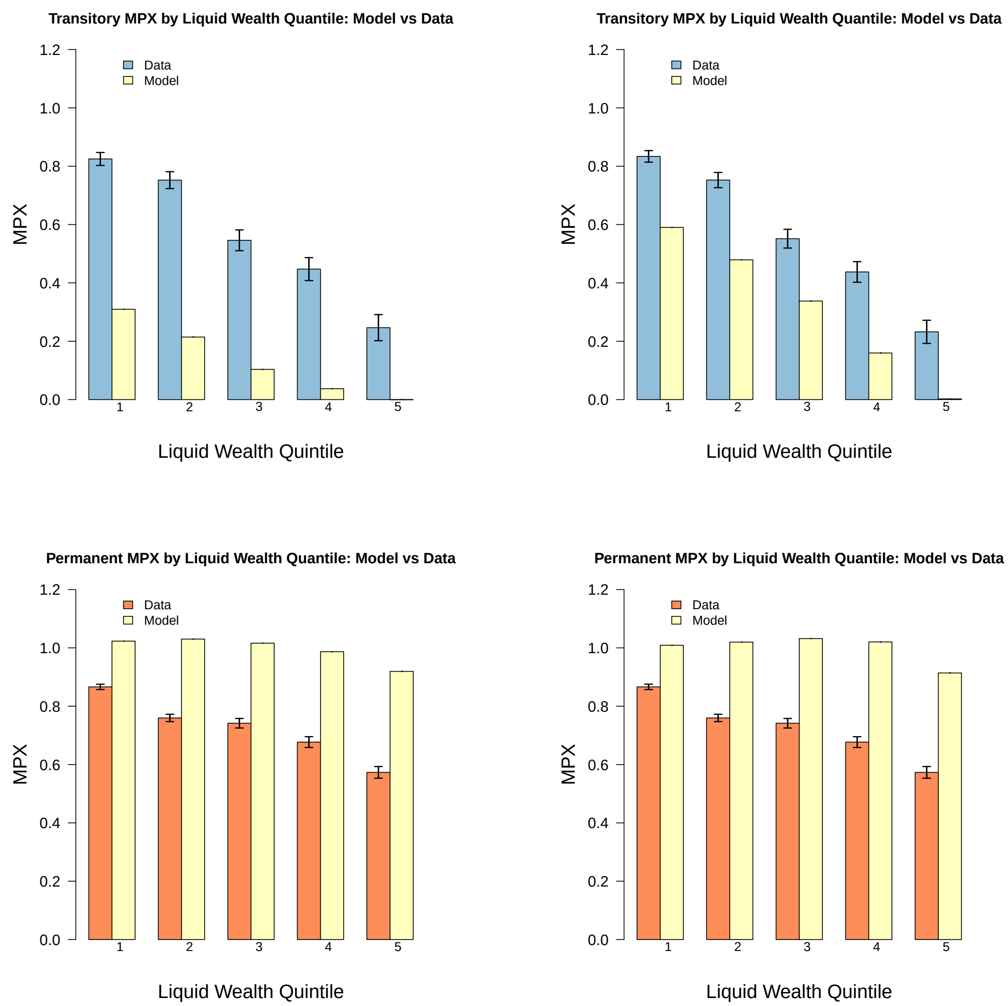


Figure 2.10: Baseline model (LHS) and Preference Shock Model (RHS) with the Data

wealth. Straub (2018) shows that a lifecycle model with non-homothetic preferences may do better along this front.

2.9 Threats to Identification

2.9.1 Durables

A critique of our empirical methodology is that it does not take account of durable goods, while our data includes all spending (except on real estate) and therefore includes large and durable goods such as cars and home improvements. The empirical model assumes that in response to a transitive income shock, expenditure increases temporarily for up to two years. This is entirely consistent with a model that includes durable goods. However, the model assumes that in response to a permanent shock to income, expenditure increases once to a new permanent level. A model that included durable goods would instead imply a large one off expenditure on durable goods to get the households up to their desired stream of durable good services, followed by a decrease back to a permanent level of spending that accounts for replenishing the higher level of depreciating durable goods.

To make this idea more explicit, it will help to write down a simple model. The model will show that our empirical methodology continues to estimate the consumption response to permanent and transitory shocks, but that these need to be interpreted carefully. The model uses the same income process as section 2.4.3. Remembering the income process is made up of two martingale processes, P_t and Q_t, which may have jumps, instantaneous income is given by:

$$dy_t = \Big(\int_0^t dP_s\Big)dt + dQ_t$$

while instantaneous expenditure now has both a durable and a non-durable component:

$$dc_t = \phi_{nd}\Big(\int_0^t dP_s\Big)dt + \phi_d dP_t + \psi dQ_s$$

Here we have assumed that the expenditure response to transitory shocks is instantaneous, but it would not change things to assume as before that the response decays to zero after two years. However, it is important that the durable component of the expenditure response to permanent shocks occurs instantaneously with the shock (or very soon after). Aggregating income and consumption annually gives:

$$\begin{aligned}
\Delta^N \bar{y}_T &= \Big(\int_{T-N-1}^{T-N}(s-(T-N-1))dP_s + \int_{T-N}^{T-1} dP_s + \int_{T-1}^{T}(T-s)dP_s\Big) \\
&\quad + \Big(\int_{T-1}^{T} dQ_t - \int_{T-N-1}^{T-N} dQ_t\Big) \\
\Delta^N \bar{c}_T &= \phi_{nd}\Big(\int_{T-N-1}^{T-N}(s-(T-N-1))dP_s + \int_{T-N}^{T-1} dP_s + \int_{T-1}^{T}(T-s)dP_s\Big) \\
&\quad + \phi_d\Big(\int_{T-1}^{T} dP_t - \int_{T-N-1}^{T-N} dP_t\Big) \\
&\quad + \psi\Big(\int_{T-1}^{T} dQ_t - \int_{T-N-1}^{T-N} dQ_t\Big)
\end{aligned}$$

From this we can calculate the covariance:

$$\begin{aligned}\mathrm{Cov}(\Delta^n \bar{c_T}, \Delta^n \bar{y_T}) = \phi_{nd}\mathrm{Var}(\Delta^n \bar{y_T})& \\ &+ \phi_d\left(\int_{T-1}^{T}(T-s)\sigma_P^2 dt - \int_{T-N-1}^{T-N}(s-(T-N-1))\sigma_P^2 dt\right) \\ &+ \psi\left(\int_{T-1}^{T}\sigma_Q^2 dt + \int_{T-N-1}^{T-N}\sigma_Q^2 dt\right) \\ = \phi_{nd}(n-\frac{1}{3})\sigma_P^2 + 0 + 2\psi\sigma_Q^2&\end{aligned}$$

So the durable component of the covariance cancels out and our identification method correctly identifies ϕ_{nd} and ψ, but is unable to identify ϕ_d.

However, if there is some delay between the household receiving the permanent income shock and purchasing the durable goods, then this introduces an upward bias into the estimate of transitory MPX. The size of the bias grows with the number of months delay between the permanent income shock and the durable goods purchase, plateauing after twelve months at a level of $\frac{\sigma_p^2}{2\sigma_q^2}\phi_d$. Figure 2.11 shows how this bias increases with the delay.

In order to quantify how large this bias may be in practice we make use of the car registry data available in Denmark. Using data on the current value of cars owned by a household, we perform the same residual calculation to find the change in car value that is unpredictable with the household characteristics we are able to observe. We then construct two new expenditure panels, one in which we remove expenditures on cars and a second in which we make a proxy for non-durable consumption by removing expenditures on cars multiplied by $\frac{1}{0.421}$ (car purchases make up 42.1% of

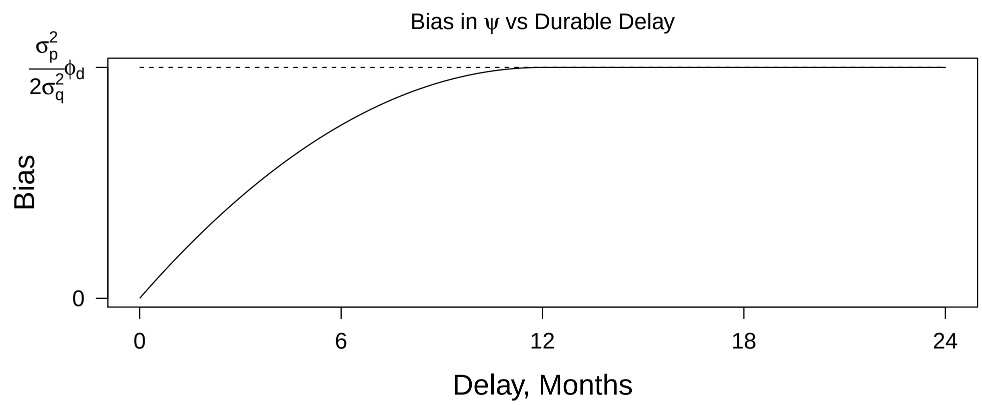


Figure 2.11: Bias in Transitory MPX with Delay in Durable Goods Purchase

durable expenditure in Denmark).

$$C_T^{\text{nocar}} = C_T - \Delta\text{CarValue}$$
$$C_T^{\text{nondurable}} = C_T - \frac{1}{0.421}\Delta\text{CarValue}$$

The second, non-durable proxy consumption panel, can be modeled as the true non-durable consumption panel with classical measurement error added. This classical measurement error does not bias our estimates, so we can use this non-durable proxy panel to estimate an unbiased MPC out of transitory shocks, where the MPC does not include durable expenditures.

The results of this exercise can be seen in figure 2.12. Even without bias, we would expect the non-durable proxy estimates to be lower than those including all expenditures as the definition of transitory MPX changes over the three panels to exclude cars and then all durable goods. For the lower quintiles of liquid wealth it therefore looks as though the bias is likely very small, as non-durable goods make up

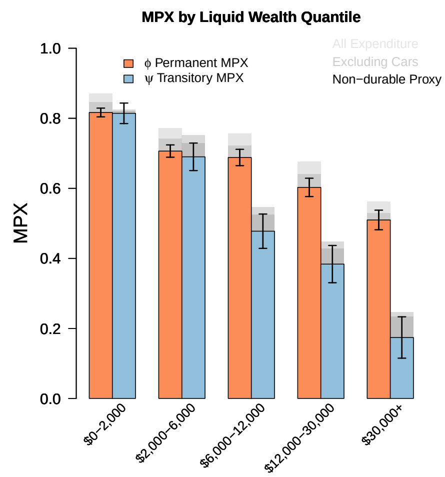


Figure 2.12: MPX Removing Cars and Using the Non-durable Proxy Panel

10% of spending and the MPX estimates are smaller by an amount in this region. For the top quintile of liquid wealth there seems to be some bias, with the estimate of MPX for all expenditures decreasing from 25% to an MPC for non-durable goods of 17%.

While there is some evidence that our results may be biased up for those in the top quintiles of liquid assets, this bias will only have a small effect on our overall conclusions. As the relevant number for the monetary policy exercise is the MPX rather than the MPC, we have chosen not to adjust our baseline results using this method and accept that a small bias may exist in our data. It should be noted that such a bias will cause the heterogeneous channels of monetary policy to appear smaller than they actually are.

2.9.2 Labor Elasticity

The empirical results of this paper estimate MPX to be at the high end of the literature. The results also conflict with standard consumption theory, particularly at the higher end of the liquid wealth distribution. It is possible that these high transitory MPX estimates are being driven by reverse causality: in years when households wish to spend more, they increase their labor supply. In this case our assumption that labor income is exogenous would be false. To get a sense of the quantitative magnitude of the bias such reverse causality could induce, in appendix 2.11.6 we calibrate a model with both preference shocks and labor supply elasticity. We find the effect is quantitatively small, with both the true MPX and the estimates using our method being close to zero for a simulation of households with liquid wealth in the top quintile. However, for extreme values of preference shocks and labor elasticity, we can generate estimates of the transitory MPX to be as high as 0.25, when the true MPX is 0.08.[45]

[45] Estimates of the Frisch elasticity in microdata studies range from 0 to 0.5, while macroeconomic studies generally find a much larger elasticity of between 2 and 4 (see Peterman (2016)). We do not consider estimates of the Frisch elasticity in the macroeconomic range as it seems likely to us that these estimates are high due to labor market frictions over the business cycle, rather than genuine labor supply choices of households. Some of the best evidence comes from Cesarini, Lindqvist, Notowidigdo, and Ostling (2017) who use lottery winnings in Sweden to estimate a Frisch elasticity of 0.14. The extreme values referred to in the text are a Frisch elasticity of 0.5 and an annual preference shock standard deviation of 0.4.

2.9.3 Persistent Consumption Response

Our estimation procedure makes the assumption that the consumption response to a transitory income shock decays to zero in a period of two years or less. A slower decay will lead to a downward bias in our estimates of the transitory MPX. Figure 2.13 shows the results of our estimation procedure on simulated data under two different assumptions about the transitory consumption response.

The exponential decay line assumes that the consumption flow following a transitory shock decays exponentially.[46] We vary the decay rate to match a range of year 1 MPCs and assume that the entire transitory income is eventually consumed. For high MPCs, and especially those over 0.5, there is very little bias. However, for MPCs significantly below 0.5 our method results in downward biased estimates. This bias arises because low MPCs, combined with exponential consumption decay, result in a relatively stable consumption flow over the first few years that has not declined close to 0 after 2 years.

Empirical evidence suggests that in fact the consumption response to a transitory shock decays quickly in the first few months and then more slowly after that.[47] The 'Fagereng et al.' line in figure 2.13 shows the MPC estimate in simulated data in which the consumption response decays according to the estimates made in Fagereng, Holm, and Natvik (2016). In this case the fast decay in the first few months results in a

[46] Standard buffer-stock models give rise to a consumption response that decays very close to exponentially. In appendix 2.11.5 we show how our empirical method performs with data simulated from the model in section 2.8.

[47] Both Fagereng, Holm, and Natvik (2016) and Gelman (2016) provide evidence for this.

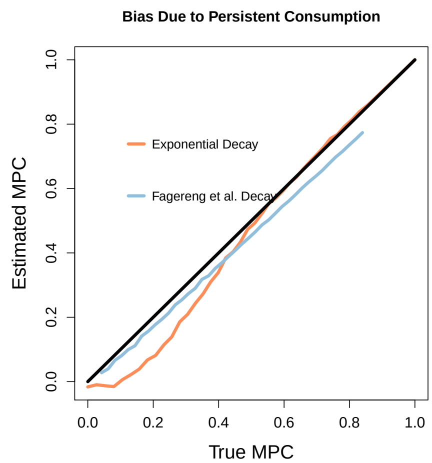


Figure 2.13: Bias from Persistent Consumption

smaller bias than the exponential case for low MPCs, while the fact that the decay is slower following these first months results in a larger bias for high MPCs.[48] Overall it seems likely that our assumption about the persistence of the consumption response leads to a slight downward bias across the range of MPCs.

Appendix 2.11.5 shows that our MPX estimates are not very sensitive to the choice of N (years of growth in our identification equations) between 3 and 6 which lends further support to the fact that assuming a 2 year limit does not bias our results too much.[49]

[48] Details of these simulations can be found in appendix 2.11.5.

[49] Using N equal to 4 and 5 instead of 3, 4 and 5 allows us to extend the consumption response out to 3 years, at the expense of losing data and becoming more sensitive to misspecification of the income process.

2.9.4 Measurement Error

Our identification comes from estimating $\mathrm{Var}(\Delta^N \bar{y})$ and $\mathrm{Cov}(\Delta^N \bar{c}, \Delta^N \bar{y})$ using our observed data. For unbiased estimates of $\mathrm{Var}(\Delta^N \bar{y})$ we require no measurement error in our observed changes in labor income. For unbiased estimation of $\mathrm{Cov}(\Delta^N \bar{c}, \Delta^N \bar{y})$ we only require (further to no measurement error in income growth) that the measurement error in expenditure growth is uncorrelated with labor income growth. As our expenditure is imputed from income and changes in assets, this is potentially more of a concern than would be the case in survey data in which questions about consumption are not directly linked to those on income. Below we examine potential sources of error in labor income and imputed consumption.

2.9.4.1 Labor Income

For most workers labor income is well measured. Third party reporting, along with a high level of trust in government institutions, means that underreporting is likely very low. The black economy in Denmark is small, and to the extent that any growth in unreported income is uncorrelated with growth in reported income this will not bias our estimates.[50] In contrast to survey data, in which measurement error in income is likely to downward bias transitory MPX estimates, this is of very little concern in our data.

[50]Such income may show up as a change in net wealth and hence expenditure, but measurement error in the change in expenditure uncorrelated with the change in labor income will not bias our MPX estimates.

2.9.4.2 Imputed Expenditure

Expenditure is calculated as the residual of total household income (including interest and dividends) after pension contributions and the change in net wealth have been deducted. For households with simple financial lives (which we believe fits most of the Danish population), this should work well. There are a few scenarios which merit further investigation.

- **Stock Market Capital Gains:** Only 10% of Danish households directly own stocks or mutual funds.[51] In appendix 2.11.9 we show that the qualitative patterns we observe are unchanged even when we completely remove these households from the sample. For households that do own stocks, we assume the return they receive is equal to a diversified portfolio of Danish stocks. Given that different households will have their own idiosyncratic portfolios, this methodology will result in significant measurement error. Baker, Kueng, Pagel, and Meyer (2018) show not only that the size of this measurement error is correlated with income and wealth, but also with the business cycle. Furthermore, Fagereng, Guiso, Malacrino, and Pistaferri (2016) show that some groups of investors consistently outperform the market, which would lead us to consistently underestimate their expenditure. Our concern, however, is that the *change* in measurement error of expenditure be correlated with the *change* in labor income. Consistently underestimating expenditure by the same amount is therefore not a problem for us.

[51] In our calculation we directly observe flows in and out of pension accounts, so these can be treated as off balance sheet in which capital gains do not affect our expenditure calculation.

Furthermore, as we have removed all aggregate effects from the labor income residuals that we use in estimation, any measurement error correlated with the business cycle will be uncorrelated with our measure of changes in labor income. We see two potential ways in which mis-measuring stock returns may bias our results. First, if households have significantly invested in the stock of the firm they work for, which is likely only to be the case for high level management. Second, to the extent that households invest their labor income gains halfway through the year, we will underestimate expenditure for those whose income increases, and overestimate it for those whose income decreases. This would lead us to underestimate the MPX. The size of this bias is limited by the size of excess expected returns, so our MPX estimate will be biased by no more than a few percentage points.

- **Family and Friends Transfers:** If a household receives a transfer of money from their parents, for example, imputed expenditure will be lower than true expenditure by this amount. Large transfers typically occur upon death of a parent, which is likely to be uncorrelated with the household head's labor income, or when purchasing a house, years during which we have already excluded in our sample. However, to the extent that friends and family actively insure each other's labor income, our MPX estimates will be upward biased.

- **Off Balance Sheet Assets:** A larger concern is that some forms of saving may be hidden off balance sheet. All Danish banks and brokers are required to report their clients' holdings, so off balance sheet assets are likely to be either off-shore, or non-financial assets. This would be a large concern if we were focused on the expenditure of the super wealthy 0.1%, but is less so when dividing the population into quintiles or deciles as we have done. Our imputation method would interpret this saving as expenditure, so our estimate of the MPX would increase one-to-one for each percentage point of saving out of income shocks performed off balance sheet.

2.9.5 RIP or HIP Income Process?

Our method makes strong assumptions on the income process, namely that there is no persistent idiosyncratic component to income growth and that the process contains a random walk. Guvenen (2009) shows that it is empirically difficult to distinguish between a 'Restricted Income Profile' (RIP) like this and a 'Heterogenous Income Profile' (HIP) income process, in which i) shocks to income are much less persistent (e.g. AR(1) with $\rho \approx 0.8$), and ii) households have a persistent idiosyncratic growth component. The reason the RIP and HIP processes are difficult to tell apart is that the two features (i) and (ii) act in opposite directions on the cross-section variance of income growth. The less persistent income shocks lead the cross-sectional income growth variance not to grow as fast as the HIP model, while the persistent idiosyncratic growth component leads the same variance to grow at a faster rate. The result is that the increase in variance of income growth over 3 to 4 years is approximately the same as the increase from 4 to 5 years.[52] To the extent that the consumption response to these semi-permanent shocks is similar to the response to the idiosyncratic persistent growth component,[53] our methodology will continue to provide reasonable estimates of the 'permanent' MPX and the more familiar transitory MPX. Appendix 2.11.7 has more detail on this point.

2.9.6 Time Varying Risk

We have assumed that idiosyncratic risk remains constant over time. Given that our sample period covers the great recession, this may not be appropriate. In appendix 2.11.8 we show how the variance of income growth has varied over time, peaking just after the crisis in 2010. In order to test how much this time varying risk might bias our results, we simulated data with $\phi = 1$ and $\psi = 0.5$, with permanent variance equal to estimates from the data and transitory variance varying in order to match the time varying income risk pattern observed in the data. When we ran this simulation we found estimates of ϕ and ψ within 1% of their true values.

[52] See appendix 2.11.7 that shows how the variance of income growth over N years grows with N.

[53] See Guvenen (2007) for an example of why this might be the case: if households do not know their own idiosyncratic growth ex-ante, a Bayesian learning process will be very slow, so households (at least initially) will react in similar ways to changes in income due to this persistent growth component as a true income shock.

2.10 Conclusion

In this paper we have presented a new method to measure the sensitivity of consumption to permanent and transitory income shocks for different groups of households. Our focus has been to use this method to test the microfoundations of heterogeneous agent models and quantify the importance of consumption heterogeneity for monetary policy. With administrative data from Denmark we have been able to dig into the distribution of MPC across a variety of dimensions in far more detail than has previously been possible. We find that MPCs vary systematically and in ways that are important for monetary policy transmission, although the current generation of heterogeneous agent models struggle to fit the high sensitivity to income that we observe.

Our hope is that the method we present in this paper, or variants of it, can also be of use to economists in a variety of fields. More and more high quality microdata on consumption is becoming available, such as the administrative data used here, or the even more detailed transaction level data available from financial aggregators. If this continues, as we hope it will, methods such as ours will become even more valuable in bridging the gap between models and data.

2.11 Appendix

2.11.1 Identification with Time Aggregation

In this section we formalize the continuous time model and calculate the relevant variances and covariances. We begin by defining permanent income. Let p_t for $t \in \mathbb{R}^+$ be a martingale process (possibly with jumps) with independent stationary increments and ν_p be such that $\mathbb{E}(e^{p_t - p_{t-1}}) = e^{\nu_p}$. Define the permanent component of income as:

$$P_t = e^{p_t - t\nu_p}$$

Note that $\mathbb{E}\left(\frac{P_{t+s}}{P_t}\right) = 1$ for all $s \geq 0$.

Next we define transitory income. Let q_t on $t \in \mathbb{R}^+$ also be a martingale process, independent of p_t, with independent stationary increments. Let $f : \mathbb{R}^+ \to \mathbb{R}$ be the impulse response of income to changes in q_t. We will assume that the impulse response to a transitory shock to income is over after 2 years, that is $f(s) = 0$ for $s > 2$. The transitory component of income is then defined as:

$$\theta_t = e^{\int_{t-2}^{t} f(t-s)dq_s - \nu_q}$$

where $e^{\nu_q} = \mathbb{E}e^{\int_{t-2}^{t} f(t-s)dq_s}$ so that $\mathbb{E}\theta_t = 1$.

We are now in a position to talk about total income. Total income *flow* at time t

is given by:

$$
\begin{aligned}
Y_t &= P_t\theta_t \\
&= e^{p_t - t\nu_p + \int_{t-2}^{t} f(t-s)dq_s - \nu_q}
\end{aligned}
$$

Observable income is the sum of income *flow* over a 1 year period, that is:

$$
\bar{Y}_T = \int_{T-1}^{T} P_t\theta_t dt
$$

We will be focused on the log of observable income growth over N years:

$$
\begin{aligned}
\Delta^N \log(\bar{y}_T) &= \log\Big(\int_{T-1}^{T} P_t\theta_t dt\Big) - \log\Big(\int_{T-N-1}^{T-N} P_t\theta_t dt\Big) \\
&= \log(\frac{P_{T-1}}{P_{T-N}}) + \log\left(\int_{T-1}^{T} \frac{P_t}{P_{T-1}}\theta_t dt\right) - \log\left(\int_{T-N-1}^{T-N} \frac{P_t}{P_{T-N}}\theta_t dt\right)
\end{aligned}
\tag{2.10}
$$

Note that if $N \geq 3$ each of the three components of equation 2.10 are mutually independent because both p_t and q_t have independent increments, and θ_t is independent of q_s for $s < t-2$ and $s > t$. Defining $\mathcal{P}_{T,N}$, $\mathcal{Q}^1_{T,N}$ and $\mathcal{Q}^2_{T,N}$ to be the three parts of the sum in equation 2.10 respectively, we have:

$$
\begin{aligned}
\mathcal{P}_{T,N} &= \log(\frac{P_{T-1}}{P_{T-N}}) \\
\Rightarrow \mathrm{Var}(\mathcal{P}_{T,N}) &= (N-1)\mathrm{Var}\Big(\log(\frac{P_T}{P_{T-1}})\Big) \\
&= (N-1)\sigma_P^2
\end{aligned}
$$

where σ_P^2 is defined to be $\mathrm{Var}\Big(\log(\frac{P_T}{P_{T-1}})\Big)$, which does not depend on T because p_t has independent increments. Moving on to the components that contain a mix of both permanent and transitory income, and defining $\bar{\theta}_T = \int_{T-1}^{T} \theta_t dt$, we have

$$\begin{aligned}
\mathcal{Q}^1_{T,N} &= \log\left(\int_{T-1}^{T} \frac{P_t}{P_{T-1}}\theta_t dt\right) \\
&= \log\left(\int_{T-1}^{T} \theta_t dt + \int_{T-1}^{T} \Big(\frac{P_t}{P_{T-1}} - 1\Big)\theta_t dt\right) \\
&= \log\left(\bar{\theta}_T\right) + \log\left(1 + \int_{T-1}^{T} \Big(\frac{P_t}{P_{T-1}} - 1\Big)\frac{\theta_t}{\bar{\theta}_T} dt\right) \\
&\approx \log\left(\bar{\theta}_T\right) + \int_{T-1}^{T} \Big(\frac{P_t}{P_{T-1}} - 1\Big)\frac{\theta_t}{\bar{\theta}_T} dt
\end{aligned}$$

Where the approximation holds so long as $\frac{P_t}{P_{T-1}}$ is close to 1 for $T-1 \le t \le T$, that is the permanent shock does not move a lot in the course of 1 year. Define:

$$\sigma^2_\theta = \mathrm{Var}\left(\log\left(\bar{\theta}_T\right)\right)$$

so that

$$\begin{aligned}
\mathrm{Var}\big(\mathcal{Q}^1_{T,N}\big) &\approx \sigma^2_\theta + \mathbb{E}\left(\int_{T-1}^{T} \Big(\frac{P_t}{P_{T-1}} - 1\Big)\frac{\theta_t}{\bar{\theta}_T} dt\right)^2 \\
&= \sigma^2_\theta + \mathbb{E}\left(\int_{T-1}^{T}\int_{T-1}^{T} \Big(\frac{P_t}{P_{T-1}} - 1\Big)\Big(\frac{P_s}{P_{T-1}} - 1\Big)\frac{\theta_t\theta_s}{\bar{\theta}^2_T} dtds\right) \\
&= \sigma^2_\theta + \int_{T-1}^{T}\int_{T-1}^{T} \mathbb{E}\left(\Big(\frac{P_{\min(t,s)}}{P_{T-1}}\Big)^2 \frac{P_{\max(t,s)}}{P_{\min(t,s)}} - \frac{P_t}{P_{T-1}} - \frac{P_s}{P_{T-1}} - 1\right)\mathbb{E}\left(\frac{\theta_t\theta_s}{\bar{\theta}^2_T}\right) dtds \\
&= \sigma^2_\theta + \int_{T-1}^{T}\int_{T-1}^{T} \mathrm{Var}\left(\frac{P_{\min(t,s)}}{P_{T-1}}\right)\mathbb{E}\left(\frac{\theta_t\theta_s}{\bar{\theta}^2_T}\right) dtds \\
&\approx \sigma^2_\theta + \sigma^2_P \int_{T-1}^{T}\int_{T-1}^{T} \min(t,s)\mathbb{E}\left(\frac{\theta_t\theta_s}{\bar{\theta}^2_T}\right) dtds \\
&= \sigma^2_\theta + \sigma^2_P \int_{T-1}^{T}\int_{T-1}^{T} \min(t,s)\mathbb{E}\left(\Big(1 + \frac{\theta_t - \bar{\theta}_T}{\bar{\theta}_T}\Big)\Big(1 + \frac{\theta_s - \bar{\theta}_T}{\bar{\theta}_T}\Big)\right) dtds \\
&= \sigma^2_\theta + \sigma^2_P \int_{T-1}^{T}\int_{T-1}^{T} \min(t,s)\left(1 + \mathbb{E}\Big(\hat{\theta}_{t,T}\Big) + \mathbb{E}\Big(\hat{\theta}_{s,T}\Big) + \mathbb{E}\Big(\hat{\theta}_{t,T}\hat{\theta}_{s,T}\Big)\right) dtds
\end{aligned}$$

where $\hat{\theta}_{t,T} = \frac{\theta_t - \bar{\theta}_T}{\bar{\theta}_T}$. Continuing:

$$\begin{aligned}
\mathrm{Var}\big(\mathcal{Q}^1_{T,N}\big) \approx \sigma^2_\theta + \sigma^2_P \int_{T-1}^{T}\int_{T-1}^{T} \min(t,s)dtds \\
+ \sigma^2_P \underbrace{\int_{T-1}^{T}\int_{T-1}^{T} \min(t,s)\bigg(\mathbb{E}\Big(\hat{\theta}_{t,T}\Big) + \mathbb{E}\Big(\hat{\theta}_{s,T}\Big) + \mathbb{E}\Big(\hat{\theta}_{t,T}\hat{\theta}_{s,T}\Big)\bigg)dtds}_{\approx 0} \\
= \sigma^2_\theta + \sigma^2_P \int_{T-1}^{T}\left(\int_{T-1}^{s} tdt + \int_{s}^{T} sdt\right)ds \\
= \sigma^2_\theta + \frac{1}{3}\sigma^2_P
\end{aligned}$$

A very similar calculation shows that:

$$\mathrm{Var}\big(\mathcal{Q}^2_{T,N}\big) \approx \sigma^2_\theta + \frac{1}{3}\sigma^2_P$$

So we get that:

$$\begin{aligned}
\mathrm{Var}\Big(\Delta^N \log(\bar{y}_T)\Big) &= \mathrm{Var}\big(\mathcal{P}_{T,N}\big) + \mathrm{Var}\big(\mathcal{Q}^1_{T,N}\big) + \mathrm{Var}\big(\mathcal{Q}^2_{T,N}\big) \\
&\approx (N-1)\sigma^2_P + (\sigma^2_\theta + \frac{1}{3}\sigma^2_P) + (\sigma^2_\theta + \frac{1}{3}\sigma^2_P) \\
&= (N - \frac{1}{3})\sigma^2_P + 2\sigma^2_\theta
\end{aligned}$$

Now we turn to consumption. Consumption responds to permanent income with elasticity ϕ, while the impulse response to a transitory shock is given by some function $g : \mathbb{R}^+ \to \mathbb{R}$ with $g(s) = 0$ for $s > 2$. Total consumption *flow* is then given by:

$$C_t = C^P_t C^\theta_t$$

where

$$C_t^P = e^{\phi p_t - t\nu_{p_c}}$$

$$C_t^\theta = e^{\int_{t-2}^t g(t-s)dq_s - \nu_{q_c}}$$

and ν_{p_c} and ν_{q_c} are defined such that $\mathbb{E}\Big(\frac{C_t^P}{C_s^P}\Big) = \mathbb{E}(C_t^\theta) = 1$ for all $t \geq s$. Analogous to the case with log income growth over N years (equation 2.10) we get:

$$\Delta^N \log(\bar{c}_T) = \log(\frac{C_{T-1}^P}{C_{T-N}^P}) + \log\left(\int_{T-1}^T \frac{C_t^P}{C_{T-1}^P} C_t^\theta dt\right) - \log\left(\int_{T-N-1}^{T-N} \frac{C_t^P}{C_{T-N}^P} C_t^\theta dt\right) \tag{2.11}$$

Defining $\mathcal{C}_{T,N}^P$, $\mathcal{C}_{T,N}^1$ and $\mathcal{C}_{T,N}^2$ to be the three parts of the sum in equation 2.11 respectively, we have:

$$\begin{aligned}
\mathcal{C}_{T,N}^P &= \log(\frac{C_{T-1}^P}{C_{T-N}^P}) \\
&= \phi \log(\frac{P_{T-1}}{P_{T-N}}) - (N-1)(\nu_{p_c} - \phi\nu_p) \\
\Rightarrow \mathrm{Cov}(\mathcal{P}_{T,N}, \mathcal{C}_{T,N}^P) &= (N-1)\phi \mathrm{Var}\Big(\log(\frac{P_T}{P_{T-1}})\Big) \\
&= (N-1)\phi\sigma_P^2
\end{aligned}$$

and that:

$$\begin{aligned}
\mathcal{C}_{T,N}^1 &= \log\left(\int_{T-1}^T \frac{C_t^P}{C_{T-1}^P} C_t^\theta dt\right) \\
&= \log\left(\int_{T-1}^T \Big(\frac{P_t}{P_{T-1}}\Big)^\phi e^{-(t-(T-1))(\nu_{p_c} - \phi\nu_p)} C_t^\theta dt\right) \\
&\approx \log\left(\bar{C_T^\theta}\right) + \int_{T-1}^T \left(\Big(\frac{P_t}{P_{T-1}}\Big)^\phi e^{-(t-(T-1))(\nu_{p_c} - \phi\nu_p)} - 1\right) \frac{C_t^\theta}{\bar{C_T^\theta}} dt
\end{aligned}$$

where the steps taken in the approximation are the same as we did in the case of income.

$$
\begin{aligned}
\mathrm{Cov}\Big(\mathcal{Q}^1_{T,N}, \mathcal{C}^1_{T,N}\Big) &= \mathrm{Cov}\Big(\log\Big(\bar{\theta}_T\Big), \log\Big(\bar{C}^\theta_T\Big)\Big) \\
&\quad + \mathbb{E}\Bigg(\int_{T-1}^{T}\int_{T-1}^{T}\Big(\frac{P_t}{P_{T-1}} - 1\Big)\Big(\Big(\frac{P_s}{P_{T-1}}\Big)^{\phi} e^{-(s-(T-1))(\nu_{p_c}-\phi\nu_p)} - 1\Big)\frac{\theta_t}{\bar{\theta}_T}\frac{C^\theta_s}{\bar{C}^\theta_T} dtds\Bigg) \\
&= \mathrm{Cov}\Big(\log\Big(\bar{\theta}_T\Big), \log\Big(\bar{C}^\theta_T\Big)\Big) \\
&\quad + \mathbb{E}\Bigg(\int_{T-1}^{T}\int_{T-1}^{T}\Big(\Big(\frac{P_{\min(t,s)}}{P_{T-1}}\Big)^{1+\phi} e^{-(\min(t,s)-(T-1))(\nu_{p_c}-\phi\nu_p)} - 1\Big)\frac{\theta_t}{\bar{\theta}_T}\frac{C^\theta_s}{\bar{C}^\theta_T} dtds\Bigg) \\
&= \mathrm{Cov}\Big(\log\Big(\bar{\theta}_T\Big), \log\Big(\bar{C}^\theta_T\Big)\Big) \\
&\quad + \int_{T-1}^{T}\int_{T-1}^{T}\mathbb{E}\Big(\Big(\frac{P_{\min(t,s)}}{P_{T-1}}\Big)^{1+\phi} e^{-(\min(t,s)-(T-1))(\nu_{p_c}-\phi\nu_p)} - 1\Big) dtds \\
&\approx 0 \begin{cases} \quad + \int_{T-1}^{T}\int_{T-1}^{T}\mathbb{E}\Bigg(\Big(\Big(\frac{P_{\min(t,s)}}{P_{T-1}}\Big)^{1+\phi} e^{-(\min(t,s)-(T-1))(\nu_{p_c}-\phi\nu_p)} - 1\Big) \\ \qquad \times\Big(\mathbb{E}\Big(\hat{\theta}_t\Big) + \mathbb{E}\Big(\hat{C}^\theta_s\Big) + \mathbb{E}\Big(\hat{\theta}_t\hat{C}^\theta_s\Big)\Big)\Bigg) dtds \end{cases} \\
&= \mathrm{Cov}\Big(\log\Big(\bar{\theta}_T\Big), \log\Big(\bar{C}^\theta_T\Big)\Big) \\
&\quad + \int_{0}^{1}\int_{0}^{1}\mathbb{E}\Big(P^{1+\phi}_{\min(t,s)} e^{-\min(t,s)(\nu_{p_c}-\phi\nu_p)} - 1\Big) dtds
\end{aligned}
$$

where $\hat{C}^\theta_{t,T} = \frac{C^\theta_t - \bar{C}^\theta_T}{\bar{C}^\theta_T}$. We now assume that p_t has no jumps, and is therefore a Brownian motion. With this assumption, $\nu_p = \frac{1}{2}\sigma^2_P$ and $\nu_{p_c} = \frac{1}{2}\phi^2\sigma^2_P$ and $\mathbb{E}(P^{1+\phi}_t) =$

$e^{\frac{1}{2}t(1+\phi)^2\sigma_P^2-\frac{1}{2}t(1+\phi)\sigma_P^2}$ so that:

$$
\begin{aligned}
\mathrm{Cov}\Big(\mathcal{Q}^1_{T,N},\mathcal{C}^1_{T,N}\Big) &= \mathrm{Cov}\Big(\log\Big(\bar{\theta}_T\Big),\log\Big(\bar{C}^\theta_T\Big)\Big)\\
&\quad+\int_0^1\int_0^1\Big(e^{\frac{1}{2}\min(s,t)\sigma_P^2((1+\phi)^2-(1+\phi)-\phi^2+\phi)}-1\Big)dtds\\
&= \mathrm{Cov}\Big(\log\Big(\bar{\theta}_T\Big),\log\Big(\bar{C}^\theta_T\Big)\Big)\\
&\quad+\int_0^1\int_0^1\Big(e^{\min(s,t)\phi\sigma_P^2}-1\Big)dtds\\
&\approx \mathrm{Cov}\Big(\log\Big(\bar{\theta}_T\Big),\log\Big(\bar{C}^\theta_T\Big)\Big)+\phi\sigma_P^2\int_0^1\int_0^1\min(s,t)dtds\\
&= \mathrm{Cov}\Big(\log\Big(\bar{\theta}_T\Big),\log\Big(\bar{C}^\theta_T\Big)\Big)+\frac{1}{3}\phi\sigma_P^2
\end{aligned}
$$

Similarly

$$
\mathrm{Cov}\Big(\mathcal{Q}^2_{T,N},\mathcal{C}^2_{T,N}\Big)\approx\mathrm{Cov}\Big(\log\Big(\bar{\theta}_T\Big),\log\Big(\bar{C}^\theta_T\Big)\Big)+\frac{1}{3}\phi\sigma_P^2
$$

so that the covariance of income growth with consumption growth over N years is:

$$
\mathrm{Cov}\Big(\Delta^N\log(\bar{y}_T),\Delta^N\log(\bar{c}_T)\Big)=(N-\frac{1}{3})\phi\sigma_P^2+2\mathrm{Cov}(\tilde{y},\tilde{c})
$$

where $\tilde{y}=\log\Big(\bar{\theta}_T\Big)$ and $\tilde{c}=\log\Big(\bar{C}^\theta_T\Big)$

2.11.2 Sample Selection

We choose to look at households whose head is between the age of 30 and 55 in 2008. This is driven by the desire to remove households for whom the assumption that most of the income growth is unexpected is not likely to be fulfilled. For the old and the young it is likely that individual households will have a lot of information about their income path that is not available to the econometrician (for example the year in which they plan to retire, or the fact that they are on a specific career track with set expectations of promotion and pay raises). We also want to remove households whose income volatility is increasing or decreasing sharply. Figures 2.14 and 2.15 show how our estimates of both income variance and MPX vary with age. The dots represent the point estimate for each age while the lines are the centered moving averages over the five nearest age groups. The solid black line shows the total variance of income growth over 1 year. It should not be surprising that income growth for households with heads in their 20's is highly volatile. This volatility plateaus around the age of 35 and stays at a constant level until the point of retirement at which point it temporarily grows before falling to an even lower level. We can see that while both transitory and permanent shocks to income are high early in life, permanent income shocks are particularly high while individuals find their place in the workforce. From the age of 30 to 55 both transitory and permanent shocks are approximately the same size and remarkably stable. At retirement shocks to permanent income rise, not surprisingly as retirement itself will be seen in the model as a shock, even as transitory income variance declines.

As the model assumes the variance to permanent and transitory shocks to be constant in the observed period, interpretation of the numbers outside of the 30-55 age group needs to be treated with care. However, the figure clearly shows that within this age group the assumption of constant variance appears to be a reasonable one.

The dotted black line shows the variance of Δy assuming no persistence in the transitory component. The fact that this line is slightly above the empirical variance of Δy is consistent with some persistence in the transitory component of income, justifying our decision to exclude growth over 1 and 2 years in our identification.

The level of both permanent and transitory shock variance for households aged 30 to 55 is approximately 0.0035, reflecting a standard deviation of 6%. Estimates using US data are significantly higher, especially for the transitory shock variance (for example Carroll and Samwick (1997) estimate 0.02 for permanent and 0.04 for transitory). This difference may be due to lower income inequality in Denmark, more progressive taxation and more generous unemployment insurance. The lower transitory variance will also be due to significantly reduced measurement error relative to the survey based US data.

2.11.3 The Danish Mortgage Market

Mortgage loans in Denmark are issued by specialized mortgage banks, which fully finance loans by issuing bonds. Interest rates are directly determined by sales prices at the bond market. That is, borrowers only pay the bond market interest rate plus a supplementary fee for the mortgage bank. Most loans are issued as 20 or 30-year loans, and households can only obtain loans from mortgage banks for up to 80 per cent of the value at loan origination of properties used as permanent residences. The remaining (more insecure) part of the funding may be provided by commercial banks.

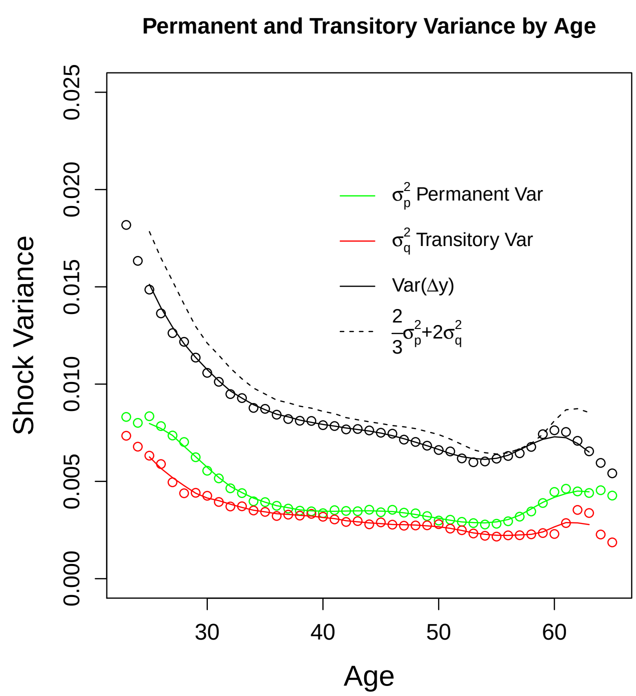


Figure 2.14: Permanent and Transitory Shock Variance by Age

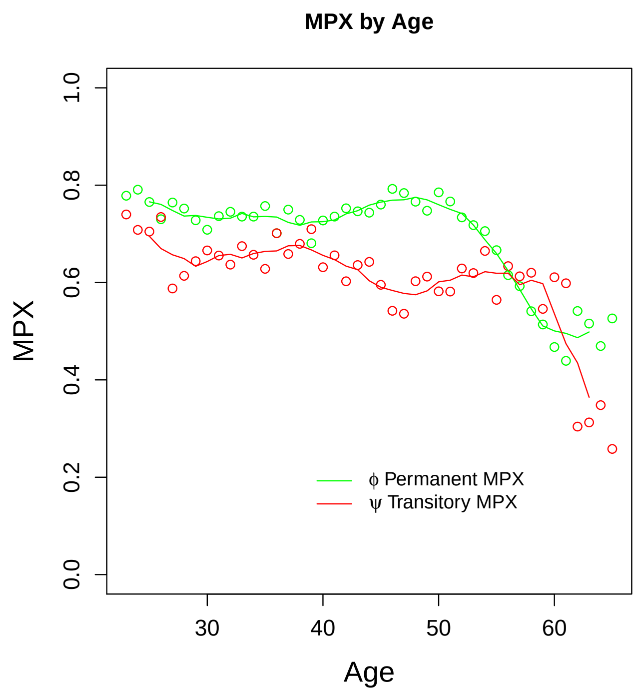


Figure 2.15: MPX by Age

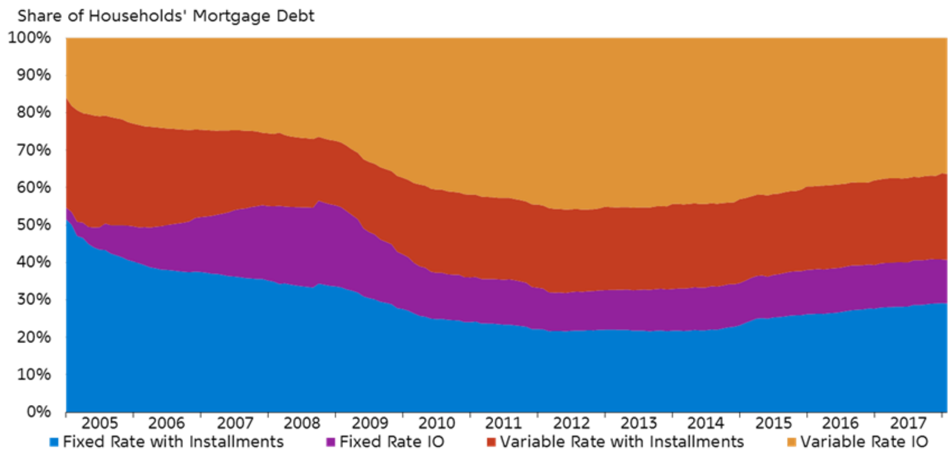


Figure 2.16: Mortgage Debt by Type (All Households)

Source: Danmarks Nationalbank

The close link between loans and bonds, as well as fixed loan-to-value ratios, fast foreclosure procedures, full recourse, etc., mean that mortgage banks do not assume significant market risks. The status of Danish covered mortgage bonds as a safe asset class (AAA-rated by e.g. S&P) implies that borrowers have access to very cheap real estate funding.

The Danish mortgage system has been functioning for two centuries, but significant liberalization has taken place over the past 20 years. Variable interest loans were (re-)introduced in 1996 while interest only loans were introduced in 2003. These new loan characteristics are by now very popular, see figure 2.16. In contrast to the US, where most mortgage debt is fixed rate, 40% of mortgage debt in Denmark is variable rate, with interest fixation periods mostly between 6 months and 5 years. Fixed rate loans come with an option for early redemption. This implies that in

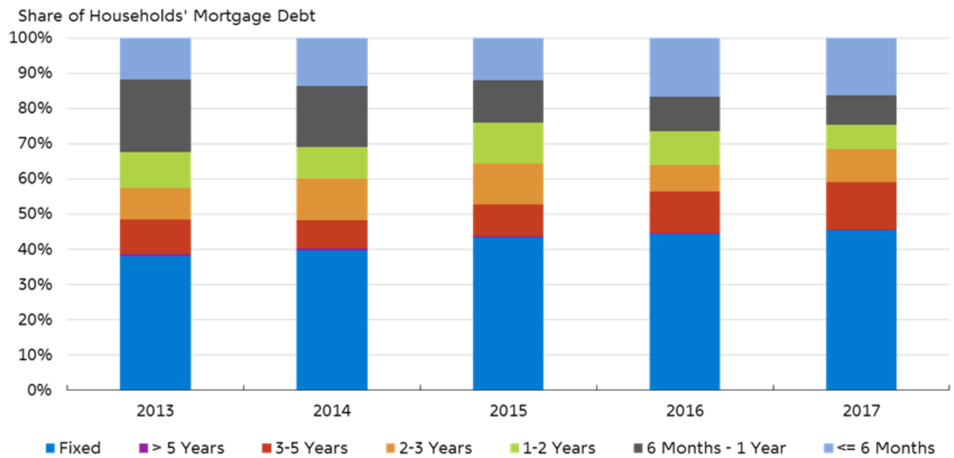


Figure 2.17: Mortgage Debt by Maturity (All Households Excluding Self-employed)

Source: Danmarks Nationalbank

practice, refinancing of fixed rate mortgages often takes place, both when interest rates decrease and increase. The latter may be attractive because borrowers have the option to repay their loan by purchasing the corresponding amount of bonds. When interest rates increase, the bond value decreases, so the option to repay the loan by purchasing the corresponding amount of bonds in essence acts as an equity insurance.

Around one fourth of the total loan balance is due to have interest rates reset over a 12 month period (see figure 2.17). This figure only comprises loans that automatically will have a new interest rate, and not active decisions to refinance or extract equity.

2.11.4 Details on the Calculation of NNP and URE

The Net Nominal Position (NNP) and Unhedged Interest Rate Exposure (URE) for the various sectors in the Danish economy are calculated from our household level dataset as well as the financial accounts from the national accounts statistics. All calculations are based on average values over the years 2009-2015, deflated by the consumer price index.

2.11.4.1 NNP and URE for Households

The NNP for households is calculated as financial assets minus liabilities. As financial assets, we include bank deposits as well as the market value of securities (excluding shares). Liabilities include all debt to financial institutions (including credit card debt) as well as publicly administered student debt, tax debt and other debt to government bodies. This data is reported to the tax authorities by financial institutions on behalf of the households.

URE is calculated as annual savings (i.e. after-tax income minus expenditure) plus maturing assets minus maturing liabilities. As maturing assets, we include all bank deposits, thereby assuming that they are floating rate. We assume a maturity of 5 years for securities held by households, and therefore include 20% of the value of securities. Regarding liabilites, we assume that all bank debt is floating rate. According to the interest rate statistics collected by Danmarks Nationalbank since 2013, on average 95% of bank debt from households is floating rate. Most of this is tied either to a market reference rate or to the Danmarks Nationalbank rate on certificates of deposit, with immediate adjustment. For mortgage debt, we have detailed information allowing us to calculate the stock of debt which is due to have interest rates reset over the coming 12 months. Voluntary refinancing of mortage loans, with or without extraction of additional equity, takes place to a large extent in Denmark. Our measure of maturing liabilities only includes the loans which contractually are due to have their interest rates reset, and we do not attempt to estimate the amount of additional refinancing. For remaining liabilities, which constitute very small amounts, we have no information regarding maturity, so we assume 5 years.

2.11.4.2 Other Sectors

NNP for the other sectors in the economy is obtained from the financial accounts statistics compiled by Danmarks Nationalbank. To most closely resemble the definition used in the household level data, we define NNP as net assets (i.e. assets minus liabilities) in the following categories: "Currency and deposits", "Securities other than shares", "Loans", and "Trade credits and other accounts receivable/payable".

NNP for the whole economy should in principle sum to 0. However, the household level microdata on bank deposits that we have access to is exclusive of certain types of savings (specialized children's savings accounts as well as some forms of pension savings accounts administered by banks) which are included in the financial accounts statistics. For the age group included in our sample, these types of accounts can be assumed to be largely illiquid. We therefore group those deposits (33 billion USD) together with the assets of pension funds (see table 2.7.3).[54]

URE for non-households is also based on the financial accounts. We do not in the national accounts observe the maturity of different asset and liability classes. We hold household URE fixed at the values from the micro-level data and take advantage of the identity that total URE in the economy must be 0 to calibrate the maturity for the remaining sectors of the economy. This results in a maturity of assets and liabilities for non-households of 3.65 years.

2.11.5 Persistent Consumption Response

2.11.5.1 Details on Section 2.9.3 Simulations

For the simulations in section 2.9.3 we divided each year into 20 sub-intervals. Both permanent and transitory shocks occur each period, and the transitory shocks have no persistence. At an annual frequency the variance of permanent and transitory shocks are equal. Households spend their permanent income each period, along with their consumption response to the history of transitory shocks. For the exponential

[54] In practice, this amount is calculated as a residual, which may also reflect other minor differences between the household level data and the national accounts statistics. For example, holdings of banknotes and coins are not observed in the microdata but allocated based on certain assumptions in the financial accounts. For our exercise, the impact of such other differences is likely to be very small.

decay model this is:

$$c_t = p_t + (1-\rho)\sum_{n=0}^{\infty}\rho^n\varepsilon_{t-n}$$

In Fagereng, Holm, and Natvik (2016) the T year MPC is estimated as a function:

$$\mathrm{MPC}_T = \theta_1 T^{\theta_2}$$

where θ_1 controls the size of the response and θ_2 the speed of decay. We vary θ_1 and choose $\theta_2 = 0.2142$ according to their estimate. In this model consumption in period t (measured in sub-intervals) is:

$$c_t = p_t + \theta_1\sum_{n=0}^{\infty}\Big((\frac{n+1}{20})^{\theta_2} - (\frac{n}{20})^{\theta_2}\Big)\varepsilon_{t-n}$$

We then time-aggregate both income and consumption over each 20 sub-interval period, choose a sample of 13 years, and run our estimation procedure with $N = 3, 4, 5$. The transitory MPC estimates are shown in figure 2.13, the permanent estimates are shown here in figure 2.18. The bias in permanent estimates is small across the range of transitory MPCs.

2.11.5.2 Persistent Consumption in the Preference Shock Model

Using a model we are able to calculate precisely the partial derivative of expenditure with respect to transitory income. To be comparable to the time period of our

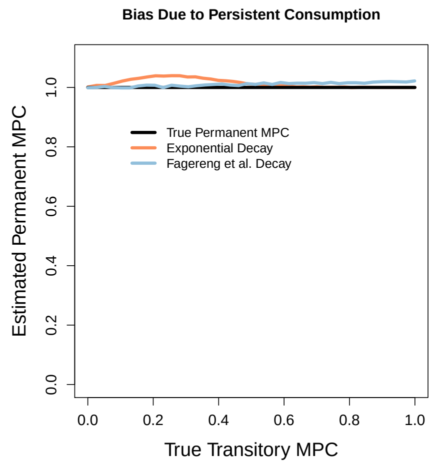


Figure 2.18: Bias from Persistent Consumption

empirical MPX we take the mean MPX over 1, 2, 3 and 4 quarters:[55]

$$\mathrm{MPX_{model}} = 1 - \frac{1}{4}\sum_{i=1}^{4}(1 - \mathrm{MPX}_q)^i$$

where MPX_q is the partial derivative in the quarterly model.

Figure 2.19 shows how the empirical method performs on data simulated from the preference shock model of section 2.8. The method works well when the MPX is high, but overestimates the MPX when it is low. This is a direct result of the assumption that the consumption response to a transitory shocks decays within a 2 year period. The consumption response in the model is very close to the exponential decay model simulated above, so it is not surprising that the bias is large for low values of the MPX. As above, empirical evidence suggests that, even for households

[55] Remember our empirical method measures the covariance of income with expenditure in the same calendar year. If the shock happens in the first quarter, then we will count expenditure over the next four quarters. If the shocks happen in the final quarter, then only one quarter of expenditure will be captured.

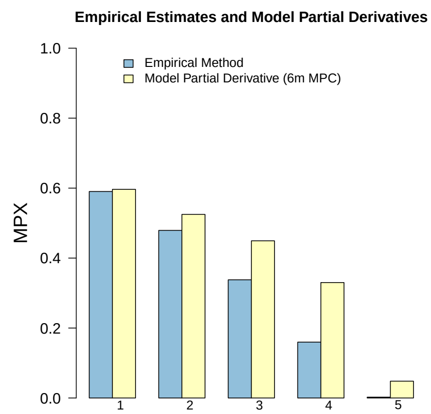


Figure 2.19: Empirical Method on Simulated Data versus Partial Derivative

with low MPX, the initial decay of the consumption response occurs much faster than exponential decay would suggest.

2.11.5.3 Estimates Using Different Values of N

Table 2.6: ψ Estimates Using Different N

			n_2				
		1	2	3	4	5	6
	1		0.58	0.59	0.59	0.60	0.60
	2			0.62	0.62	0.62	0.62
n_1	3				0.62	0.62	0.63
	4					0.62	0.64
	5						0.68
	6						

Table 2.6 shows the estimates of the transitory MPX that we recover from our estimation sample when we just use $N = n_1, n_2$ in our identification equations 2.4 and 2.6. Remember in our main results we used GMM with $N = 3, 4, 5$ and we high circled $N = 3, 5$ to highlight where we get identification from in the paper. The purpose of this exercise is to show that the estimation results are not very sensitive to the values of N chosen. This also provides more evidence that the assumption we made that the transitory consumption response lasts less than 2 years is not biasing our results significantly. In fact the results are not changed dramatically even when $N = 1, 2$, which suggests the majority of the transitory consumption response is very short-lived.

2.11.6 Labor Elasticity Model

Here we detail the model and simulation results summarized in section 2.9.2. The model extends the standard incomplete markets model from section 2.8, incorporating both preference shocks, so that households have some years when their utility of consumption is greater than others, and labor elasticity, so that households can adjust their income based on the marginal utility of consumption. The household's problem is to maximize expected lifetime utility:

$$\mathbb{E}_t \sum_{n=t}^{\infty} \beta^n \left(\mathcal{X}_n \frac{\mathbf{c}_n^{1-\rho}}{1-\rho} - \frac{\boldsymbol{\ell}_n^{1+\frac{1}{\xi}}}{1+\frac{1}{\xi}} \right)$$

subject to the constraints:

$$\mathbf{a}_t = \mathbf{m}_t - \mathbf{c}_t$$

$$\mathbf{b}_t = R\mathbf{a}_t$$

$$\mathbf{y}_t = l_t w_t$$

$$\boldsymbol{\ell}_t = l_t \mathbf{p}_t^{\frac{1-\rho}{1+\frac{1}{\xi}}}$$

$$w_t = \theta_t \mathbf{p}_t$$

$$\mathbf{p}_t = \Psi_t \mathbf{p}_{t-1}$$

$$\mathbf{m}_t = \mathbf{b}_t + \mathbf{y}_t$$

The normalization of labor ($\boldsymbol{\ell}_t = l_t(\mathbf{p}_t^{\frac{1-\rho}{1+\frac{1}{\xi}}})$) is set up to allow labor supply to move elastically with transitory income, but the long run supply of labor does not depend on permanent income (as observed in the consistency of hours worked over long time periods and across countries). The key additional features of this model are i) the preference shock factor and ii) the elasticity of labor.

Labor elasticity is controlled by the Frisch elasticity ξ. When the wage (relative to permanent income) increases by $x\%$, hours worked increase by $\xi\%$. We will examine values of the Frisch elasticity between 0 and 0.5 to cover the range of estimates from microeconomic studies.

β	Frisch Elasticity					σ_q	Frisch Elasticity				
	0.00	0.13	0.25	0.38	0.50		0.00	0.13	0.25	0.38	0.50
0.00	0.99	0.99	0.99	0.99	0.99	0.00	0.06	0.06	0.05	0.05	0.04
0.10	0.99	0.99	0.99	0.99	0.99	0.10	0.06	0.06	0.05	0.05	0.04
0.20	0.99	0.99	0.99	0.99	0.99	0.20	0.06	0.06	0.05	0.05	0.04
0.30	0.99	0.99	0.99	0.99	0.99	0.30	0.06	0.06	0.05	0.05	0.04
0.40	0.98	0.98	0.98	0.98	0.99	0.40	0.06	0.06	0.05	0.05	0.04

Table 2.7: Fitted Discount Factors and Transitory Shock Standard Deviation

ϕ	Frisch Elasticity					σ_c	Frisch Elasticity				
	0.00	0.13	0.25	0.38	0.50		0.00	0.13	0.25	0.38	0.50
0.00	0.98	0.98	0.98	0.98	0.98	0.00	0.05	0.05	0.05	0.05	0.05
0.10	0.98	0.98	0.98	0.99	0.98	0.10	0.08	0.08	0.08	0.08	0.08
0.20	0.98	0.98	0.98	0.97	0.97	0.20	0.14	0.14	0.14	0.14	0.14
0.30	0.98	0.97	0.97	0.96	0.96	0.30	0.20	0.20	0.20	0.20	0.20
0.40	0.98	0.96	0.95	0.95	0.94	0.40	0.26	0.26	0.26	0.26	0.26

Table 2.8: Simulation Estimates of ϕ and Consumption Growth Standard Deviation

MPC	Frisch Elasticity				
	0.00	0.13	0.25	0.38	0.50
0.00	0.06	0.05	0.04	0.04	0.03
0.10	0.07	0.06	0.05	0.07	0.04
0.20	0.09	0.08	0.07	0.06	0.05
0.30	0.13	0.10	0.09	0.07	0.06
0.40	0.16	0.13	0.11	0.10	0.08

ψ	Frisch Elasticity				
	0.00	0.13	0.25	0.38	0.50
0.00	0.01	0.01	0.01	0.00	0.00
0.10	0.01	0.01	0.01	0.02	0.01
0.20	0.01	0.02	0.02	0.03	0.03
0.30	0.02	0.04	0.06	0.08	0.10
0.40	0.04	0.09	0.14	0.20	0.25

Table 2.9: Simulation Estimates of MPC and ψ

In tables 2.7, 2.8 and 2.9 we have varied the size of the Frisch elasticity and annualized preference shock. In each cell we have kept constant the overall annualized income growth variance and the median liquid asset to annual income ratio.[56] To achieve this we vary the discount factor and the variance of transitory wage shocks.

Table 2.7 shows how the discount factor, β, and the annualized transitory shock standard deviation vary. As the size of the preference shocks increase, so does the precautionary motive for households. As we have fixed the median amount of precautionary savings, the discount factor drops slightly to compensate. The right-hand panel shows that the standard deviation of transitory shocks required to match the overall level of income growth variance goes down as labor supply elasticity increases. This is as expected - when the transitory wage is low households will work fewer hours. This amplifies the variance of the transitory income shock relative to the wage

[56]We calibrate to an annualized growth variance of 0.01 and a median liquid asset to annual income ratio of 0.5 to approximately match the upper quintile of the liquid wealth distribution.

shock. The size of the preference shocks has little effect on the imputed size of the transitory shocks.

The left-hand panel of table 2.8 shows the estimate of ϕ (the MPX out of permanent shocks) is close to 1 for variations of preference shocks and labor elasticities. This is unsurprising as labor does not respond to a change in permanent income. The right-hand panel shows a very significant increase in the standard deviation of consumption growth as the size of the preference shocks increases. With no preference shocks, the standard deviation of consumption growth (0.05) is about half of the standard deviation of income (0.1). As the size of preference shocks increases, so does consumption growth variance, with the standard deviation growing to 0.26 for large preference shocks. This is still much smaller than 0.37, which comes directly from the data, although this high number from the data is likely to be contaminated with measurement error in assets. A further consideration is that much of the observed variance in expenditure growth will be due to durable items, such as home improvements and vehicles. We analyzed the effect of durables on our estimates in section 2.9.1, but to the extent that these goods can be financed, our model with no borrowing may overestimate both the expenditure variance and the labor supply response to preference shocks.

Table 2.9 compares the actual mean MPC in the model with our empirical method for estimating the transitory expenditure elasticity. The left-hand panel shows that both preference shocks and labor elasticity, often both missing in consumption models for simplicity, have quantitatively significant impacts on the implied MPC. Increasing the Frisch elasticity from 0 to 0.5 (the full range of micro-estimates) decreases the 6 month MPC from 6% to just 3%. This is because households now have an extra tool with which to insure against low consumption. When they receive a negative transitory shock to their wealth, they will consume less, which in turn will increase their marginal utility of consumption and induce them to work more hours. Therefore their actual income loss will be lower than the shock to their wealth and they will reduce their consumption by less than if they were unable to adjust their labor supply. In contrast, increasing the size of the preference shocks greatly increases the MPC. This is a result of the higher precautionary savings motive and consequently lower discount factor, even while median savings are unchanged.

The right-hand panel of table 2.9 shows the effect of preference shocks and labor elasticity on our empirical estimates of ψ, the transitory MPX. The top row shows that our estimate is lower than the MPC (due to the fact that at these low levels of MPC, more than 2 years is required for the transitory effect to decay away). It does however follow the same pattern as the MPC and falls in magnitude as the ability of households to adjust labor supply increases. Similarly, going down the first row shows that the estimated MPX increases with the preference shock. However, the similarity to the MPX table ends when we increase *both* labor elasticity *and* size of the preference shocks. Our estimate can grow large, up to a value of 0.25, when extreme values are chosen: a Frisch elasticity of 0.5 and a preference shock standard deviation of 0.4. This measured transitory MPX now bears little relation to the MPX (which is 0.08). Instead it is being driven by reverse causality, whereby preference shocks are driving consumption along with the decision to increase labor. The observed 'shocks' to income are therefore highly correlated with consumption, but they are not causing the consumption dynamics exogenously.

This exercise suggests the bias due to reverse causality is likely to be small, but further investigation may be worthwhile.

2.11.7 RIP or HIP Income Process?

There has been a long-standing and unresolved quest in the literature to find a parsimonious representation of the labor income process. Two competing candidates are Restricted Income Profile (RIP) and Heterogeneous Income Profile (HIP) processes. Both can be described by the equations:

$$y_h^i = \beta^i h + z_h^i + \varepsilon_i^h$$

$$z_h^i = \rho z_{h-1]}^i + \eta_i^h$$

where i indexes the worker and h the years of experience. ε_i^h represents a transitory shock to income, while η_i^h is persistent. β^i represents an idiosyncratic persistent growth factor.

In the RIP model, $\beta^i = 0$ and ρ is usually estimated to be very close to 1 (in this paper we assumed $\rho = 1$). In the HIP model β^i has a cross sectional variance $\sigma_\beta^2 > 0$

and ρ is normally estimated to be significantly lower than 1, around 0.8. The reason these are difficult to tell apart is because the theory gives not strong indication in which model the cross-sectional variance of income growth over N years should grow faster. In the RIP model with $\rho = 1$, the cross-sectional variance of income growth increases linearly with N. In the RIP model with $\rho \approx 0.8$ the growth in the cross-sectional variance of income growth will decrease due to the low ρ, but increase due to the idiosyncratic β^i.

Figure 2.20 shows the empirical values for income growth variance and the covariance of income and expenditure growth over N years. We have also plotted the fitted values for these statistics that are implied by our model when fitted to $N = 3, 4, 5$ as we do in the paper. We see the empirical variance and covariance decline slightly below the model fitted line as N becomes large. This fits with the finding that ρ in the RIP model is usually slightly below 1.0, around 0.98 or 0.99. We also note that around the region where we achieve our identification ($N = 3, 4, 5$), there is very little curvature in the empirical statistics and the increase in both variance and covariance is close to linear.

While this linearity around $N = 3, 4, 5$ cannot help us distinguish between the RIP and HIP process, it does imply that our empirical methodology may be somewhat robust to misspecification along this dimension. If we assume that the expenditure response to a change in z^i_h and to the increase from the persistent idiosyncratic growth

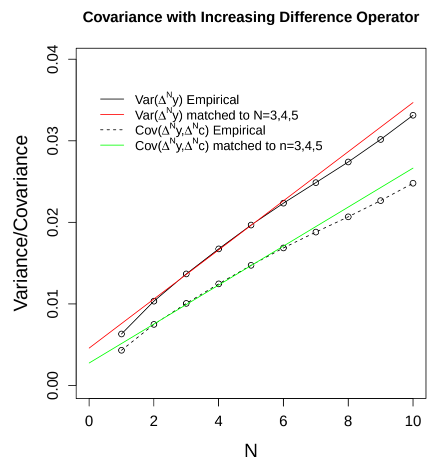


Figure 2.20: Variance and Covariance with Years of Growth

are equal to ϕ, and the response to a transitory shock is ψ, that is:

$$\Delta^N c_h^i \approx \phi \Delta^N (\beta^i h + z_h^i) + \psi \Delta^N \varepsilon_i^h$$

Then the fact that $\mathrm{Var}\big(\Delta^N(\beta^i h + z_h^i)\big)$ grows approximately linearly with N means that our empirical method will correctly identify ϕ and ψ.

A full investigation of the implications of different income processes is beyond the scope of this paper but would be a very useful exercise for future research.

2.11.8 Time Varying Risk

Figure 2.21 shows how the standard deviation of income growth has changed over the sample period. From trough to peak the standard deviation increases approximately 10%. In the simulation referred to in section 2.9.6 we assume that both

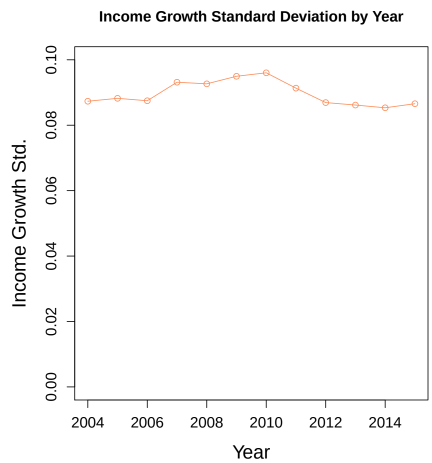


Figure 2.21: Standard Deviation of Income Growth

transitory income and transitory consumption response have no persistence. We divide each year into 20 sub-periods, choose the variance of permanent shocks to be 0.003 and allow time varying transitory shocks to match the pattern in figure 2.21. We choose values of $\phi = 1$ and $\psi = 0.5$ and apply our estimation procedure (that assumes constant variance) to the simulated data. We recover estimated values of ϕ and ψ to be 1.006 and 0.499 respectively.

2.11.9 Robustness

As would be clear from the main text, we have made a number of choices regarding both data and variable definitions as well as more methodological issues. In a series of graphs this appendix presents a number of robustness checks aimed at assessing the extent to which our results are sensitive to the specific choices.

We begin with a number of robustness checks regarding our imputed expenditure measure, which may suffer from measurement error. In figure 2.22, we compare our baseline estimates of the MPX to estimates based on different sample selection procedures. First, we exclude all households that own stocks corresponding to more than 10,000 USD (10% of households in our sample). Second, we do not remove households which have negative imputed expenditure. We remove those households in our baseline sample since negative expenditure is clearly not a good estimate of actual expenditure. However, for example in the event that negative expenditure arises because of classical measurement error, removal of negative estimates may be asymmetric and introduce an upward bias in average imputed expenditure. Third, to check that large outliers do not drive our results, we remove observations in the top and bottom 2.5% in terms of level and change of income and expenditure. In the baseline calculations, we use only a 1% cut-off. Our results are qualitatively unchanged when using these alternative approaches to take account of measurement error. In terms of magnitudes of the estimated MPXs, the largest difference to the baseline results seems to be found when we include negative expenditure estimates. As expected, this makes the largest difference among the wealthier households. The specification of outliers also matters somewhat for the point estimates of MPX in certain groups of households, but differences are not large.

Another robustness check consists of specifying consumption and income in logs rather than in levels. The fundamental difference is that the log specification yields an

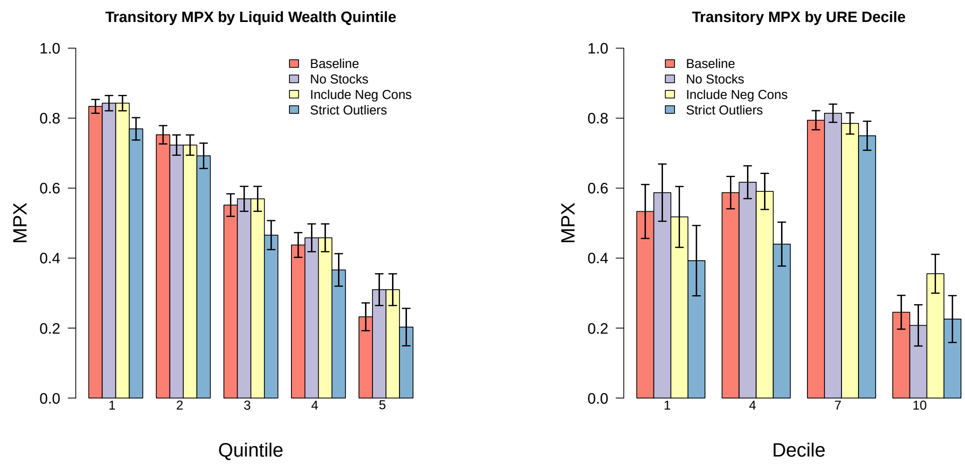


Figure 2.22: Robustness of Liquid Wealth and URE Distributions

elasticity rather than an MPX. Hence, some difference between level and log results must be expected for households which only spend a fraction of their annual income (typically wealthier households). Indeed, as expected figure 2.23 demonstrates that results hold qualitatively when specifying income and expenditure in logs rather than in levels, whereas estimated elasticities are higher than the MPXs for the more wealthy households and those with high URE. Time varying income risk may also potentially contribute to differences between results based on levels and logs. However, as shown in section 2.9.6 this is not likely to be important in our setting.

As discussed in section 2.5.1 we use labor income of the head of the household as our prime measure of income in line with previous literature. Various mechanisms, e.g. intra-household income insurance, may give rise to differences between results based on income of the head of household and total household income. However, figure 2.24

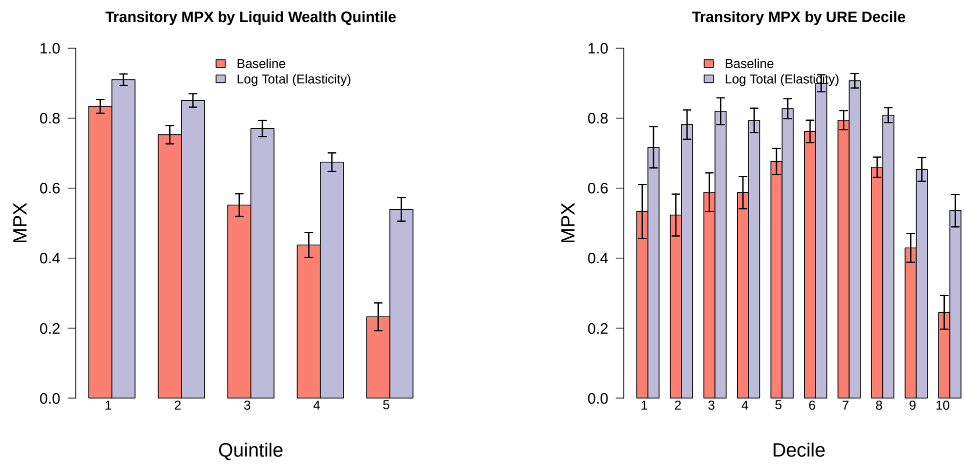


Figure 2.23: Results Using Log Income and Expenditure

demonstrates that there is virtually no difference in our results between using total household income and only the household head's income. Appendix 2.11.11 briefly discusses the potential role that intra-household insurance may play, which we leave as an area for future research.

Finally, figure 2.6 shows the distribution of MPX by quintile of liquid wealth. It might be argued that the relevant level of liquid wealth is relative to income rather than in absolute terms. Figure 2.25 demonstrates that results based on quintiles of liquid wealth divided by permanent income are similar. Also, results (not shown here) where deciles are based on a broader definition of liquid wealth, i.e. including stock and bond holdings, are similar to our baseline results.

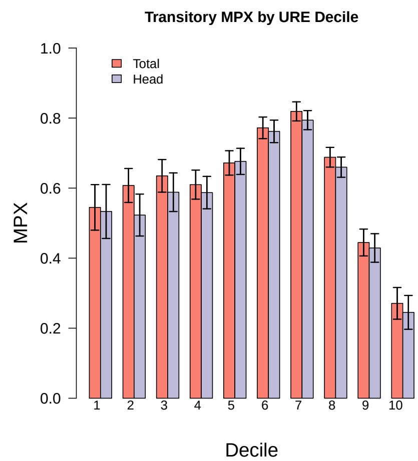


Figure 2.24: Results Using Total Labor Income and Head Labor Income

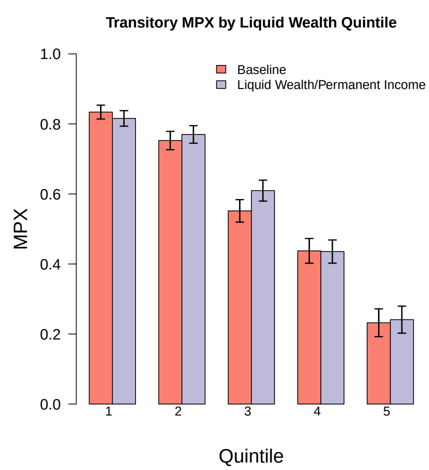


Figure 2.25: Results Using Quintiles of Liquid Wealth over Permanent Income vs Liquid Wealth

2.11.10 Distribution of Permanent MPX by NNP, URE and Income

Figure 2.26 shows the distribution of both transitory and permanent MPX by NNP, URE and income decile. The transitory numbers are a repeat of figure 2.8.

2.11.11 Intra-household Income Insurance

As discussed in section 2.5.1 we use labor income of the head of the household as our prime measure of income in line with previous literature. Figure 2.24 demonstrates that results based on total household income and income of the head of household are similar. However, MPXs from transitory shocks to the income of the spouse are lower than MPXs from shocks to total income, in particular for the less wealthy households, as demonstrated in figure 2.27. This indicates heterogeneity in the role that intra-household income insurance plays across different groups of households. We leave this interesting topic for future research.

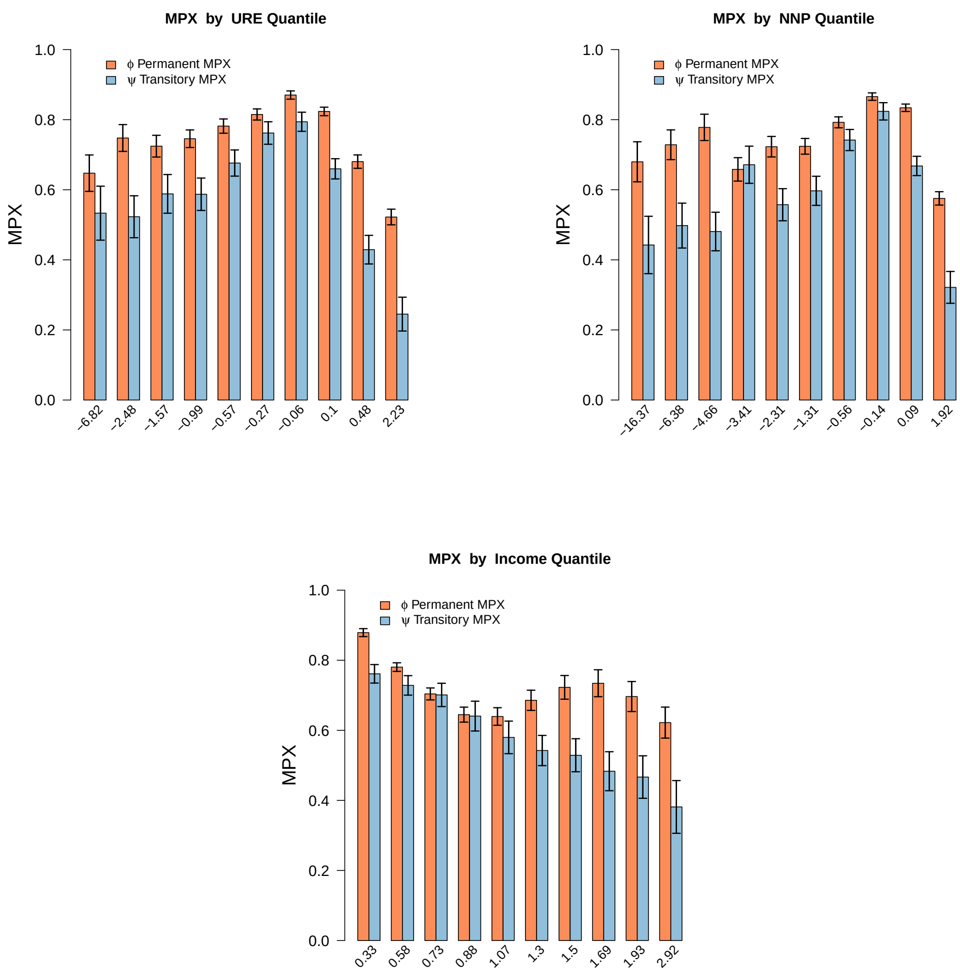


Figure 2.26: MPX Distribution by URE, NNP and Income

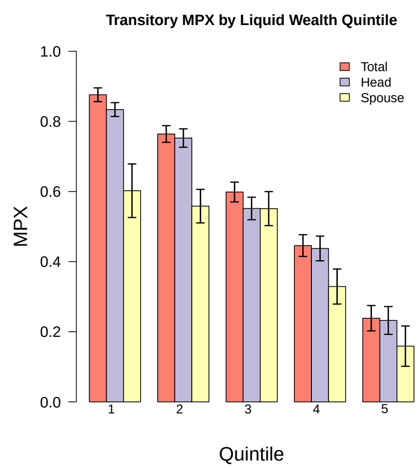


Figure 2.27: Results Using Total, Head and Spouse Labor Income

Chapter 3

Monetary Policy Transmission with Many Agents

3.1 Abstract

We analyze the transmission mechanism of monetary policy to consumption in New Keynesian models with heterogeneous agents. We show that in these models the countercyclical nature of profits, empirically false, plays a large role in amplifying the intertemporal substitution channel. On the other hand the interest rate exposure channel, empirically large, plays a small role. Our analysis makes use of the partial equilibrium decomposition of Auclert (2017) which we show to perform well even in models where the assumptions do not hold. We suggest expanding the role of the interest rate exposure channel, while dampening the amplification effect

of countercyclical profits, is of primary quantitative importance in future work.

3.2 Introduction

What is the mechanism via which a monetary policy shock affects consumption? In standard representative agent New Keynesian (RANK) models this is through the intertemporal substitution channel: when real interest rates decline, the price of consumption today drops relative to the price in the future, so households choose to consume more today.

However, recent evidence from microdata has brought this mechanism into question.[1] A number of new models claim to match both the micro and macro data better.

This paper uses the lens of the monetary policy decomposition presented in Auclert (2017) to analyze some standard modeling approaches. The advantage of Auclert's decomposition is that it can be very closely tied to the microdata. However, strictly it requires assumptions that do not hold up in many models. Our paper also analyzes how useful the decomposition is in these cases. In particular the method assumes that a transitory monetary policy shock has no persistent effects. This is clearly the case in any model with no predetermined variables: transitory shocks cannot be propagated

[1] In particular household marginal propensity to consume appear to be much higher than RANK models would suggest (e.g. Parker, Souleles, Johnson, and McClelland (2013) among many others) and the elasticity of intertermporal substitution is likely small (e.g. Best, Cloyne, Ilzetzki, and Kleven (2018))

into the future because there are no state variables that can carry information with them. Such models include the standard RANK model without capital as well as the beseline two agent New Keynesian (TANK) model we consider in this paper. We break this assumption in two ways. First, we add capital, a predetermined variable, to our TANK model which results in persistence of a monetary policy shock due to the slow movement of the capital stock. Second, in our heterogeneous agent New Keynesian (HANK) model, the entire distribution of assets is a predetermined state variable with potentially important dynamics.

3.2.1 Findings

We begin our analysis with a standard TANK model in which a proportion of households live hand-to-mouth, have no debt, and earn only labor income.[2] As there is no debt neither the interest rate exposure not the Fisher channel play a role. We find instead a large role being played by the earnings heterogeneity channel: firm profits are countercyclical in the New Keynesian model, so the poor households see significantly more income variation over the business cycle than the wealthy. This is an important finding and draws into question some of the key results in the HANK literature so far. That profits are countercyclical is not empirically true, so this transmission mechanism does not fit the evidence.[3]

Without a large earnings heterogeneity channel, the standard TANK model would

[2]We closely follow Debortoli and Gali (2018) here.

[3]Broer, Hansen, Krusell, and Oberg (2018) come to a similar conclusion.

continue to lean very heavily on the intertemporal substitution channel. Our next iteration of the model shows a potential for a very different transmission mechanism, one that fits the microdata but that current models do not quantitatively capture. We allow the hand-to-mouth households in our TANK model to maintain a debt up to some fraction of their steady state income. When interest rates are low they will be able borrow a little more, and when they are high a little less, thus providing an interest rate exposure channel through which monetary policy acts.

We find the income channels (both aggregate and heterogeneous) act as a multiplier for both the intertemporal substitution and the interest rate exposure channels. As we decrease the elasticity of intertemporal substitution, the intertemporal substitution channels diminishes along with the income multipliers of it. The interest rate exposure channel, on the other hand, remains the same size. This suggests it may be possible to create a monetary policy model in which intertemporal substitution plays no role at all, but which nonetheless fits the macrodata. This is something we plan to tackle in future work.

The rest of the paper investigates how useful Auclert's decomposition is in models with predetermined variables. Our first such model extends the TANK model with capital. We find the decomposition fails when we have no convex capital adjustment costs, but that with standard parameterizations of these costs the decomposition accounts for over 95 percent of the change in consumption.

Finally we consider a one-asset HANK model.[4] This breaks with the assumption required for Auclert's decomposition in two ways. First, as with other HANK models the entire distribution of assets is a predetermined state variable, allowing for potential persistence following a monetary policy shock. Second, its use of Greenwood-Hercowitz-Huffman (GHH) non-separable preferences introduces a sixth transmission mechanism relating to the Hicksian elasticity of consumption with hours worked. We show that although the distribution of assets is predetermined, a monetary policy shock has little persistence. However, the strong link between consumption and hours worked in the GHH preference specification acts as a large transmission mechanism, and one for which there is little empirical support. We conclude that GHH preferences should not be used in quantitative models of this kind.[5]

Overall we believe significant progress has been made in understanding the transmission mechanism of monetary policy recently, both empirically and theoretically. However, we find there is still a great deal of divergence between the two and believe going forward models should target the interest rate exposure and aggregate income channels, and give the intertemporal substitution and earnings heterogeneity channel a smaller role.

[4]Our model closely relates to the two asset model presented in Bayer and Luetticke (2018).

[5]For a related detailed criticism of GHH preferences, see Auclert and Rognlie (2017).

3.3 Transmission Channels

We will make heavy use of the monetary policy partial equilibrium decomposition described in Auclert (2017). He makes the assumption that for an individual a one time shock to nominal interest rates looks like i) a transitory change in income, ii) a one off change in the price level, iii) a change in the real interest rate. Here we provide a brief description of each of the five channels he identifies and then give some indication as to the conditions under which they sum to the aggregate change in consumption. For more detail please refer to Auclert's paper.

3.3.1 Aggregate Income Channel

The aggregate income channel measures how much consumption changes due to the change in aggregate income, under the assumption that all incomes move proportionally. The size of this channel is given by:

$$AggInc = \mathbb{E}_i \left(\mathrm{MPC}_i Y_i \right) \frac{dY}{Y} \tag{3.1}$$

where MPC_i is the marginal propensity to consume of household i and the expectation is taken over all households.[6] That is the aggregate income channel is the income weighted marginal propensity to consume multiplied by the change in aggregate income.

[6] Strictly this is the marginal propensity to consume out of income *after* accounting for labor response. In our models hours are either rationed or do not depend on wealth, so this definition of MPC coincides with the more standard definition.

3.3.2 Earnings Heterogeneity Channel

A monetary policy shock may not change the income of every household proportionally. If households with high MPCs see relatively larger income changes than households with low MPCs, then overall the channel through which monetary policy affects consumption through income will be larger than measured by the aggregate income channel. The total income channel is simply the expectation of each household's MPC multiplied by their own change in income, $\mathbb{E}_i\left(\mathrm{MPC}_i dY_i\right)$. The earnings heterogeneity channel is measured as the residual of the total income channel after taking away the aggregate income channel:

$$EarnHet = \mathbb{E}_i\left(\mathrm{MPC}_i dY_i\right) - \mathbb{E}_i\left(\mathrm{MPC}_i Y_i\right)\frac{dY}{Y} \tag{3.2}$$

3.3.3 Interest Rate Exposure Channel

The interest rate exposure channel measures how much households change their consumption due to unhedged interest rate exposure (URE). Unhedged interest rate exposure is defined as the difference between all maturing assets (including income) and maturing liabilities (including planned consumption), and is therefore the quantity of saving that is planned to be invested at this periods interest rate. When this period's real interest rate goes up, this effectively increases the budget constraint of those households who have positive unhedged interest rate exposure. Under certain conditions these households will increase their consumption by their MPC multiplied by the change in their budget constraint. That is:

$$IRE = \mathbb{E}_i \left(\mathrm{MPC}_i \mathrm{URE}_i\right) \frac{dR}{R} \tag{3.3}$$

where R is the real interest rate.

3.3.4 Fisher Channel

Inflation has the effect of changing the real value of nominal assets and debts. The Fisher channel measures how this affects aggregate consumption, making the assumption that households individual MPCs apply to this change in wealth. The key household level measure here is the net nominal position (NNP), that is the sum of all nominal assets net of nominal debts for each household. The size of the channel is then:

$$Fisher = -\mathbb{E}_i \left(\mathrm{MPC}_i \mathrm{NNP}_i\right) \frac{dP}{P} \tag{3.4}$$

where P is the price level.

3.3.5 Intertemporal Substitution Channel

Finally the intertemporal substitution channel measures how much households will shift their consumption between time periods due to the change in the real interest rate.

$$IntSubs = \mathbb{E}_i \left(\frac{1}{\sigma_i}(1 - \mathrm{MPC}_i)\frac{C_i}{C}\right) \frac{dR}{R} \tag{3.5}$$

where $\frac{1}{\sigma_i}$ is the household's elasticity of intertemporal substitution, C_i is their consumption this period and C is the mean level of consumption this period.

3.3.6 Aggregation

Auclert (2017) shows the conditions under which these five channels sum exactly to the aggregate change in consumption following a monetary policy shock. First, preferences must be separable. Second, a monetary policy shock has a purely transitory effect, changing income and the real interest rate for one period only, while effecting a one time change in the price level. For New Keynesian models with no predetermined variables, such as the standard consumption New Keynesian model or the baseline two agent New Keynesian model presented below, this is the case. Models with capital, or where the distribution of wealth persists into the next period such as the HANK model presented below, do not fit this decomposition. We will measure the error as the difference between the sum of the five channels and the actual change in consumption.

3.4 A TANK Model in which the Decomposition Works Exactly

3.4.1 Model Overview

We begin our analysis with a baseline two agent New Keynesian (TANK) model. Our baseline TANK model is composed of two types of agents, Ricardian and Keynesian, along with a continuum of intermediate goods firms, a perfectly competitive final goods firm, and a monetary policy authority. The model is closely related to the standard New Keynesian model with Calvo pricing frictions, the main difference being the addition of the Keynesian households. A key addition in our model, compared with other TANK models, is to allow for the Keynesian households to hold a non-zero quantity of short term nominal debt (owed to the Ricardian households) so that we have non-trivial levels for households' unhedged interest rate exposure (URE) and net nominal positions (NNP).

The advantage of starting our analysis with this model is that it contains no predetermined variables, and therefore the conditions for our partial equilibrium decomposition hold exactly. As well as being a useful starting point to build upon, it also highlights how the transmission mechanism works in TANK models (and HANK models more generally), showing just how important the earnings heterogeneity channel is in these models.

3.4.2 Households

A proportion λ of households, which we shall call Keynesian, live hand-to-mouth, consuming all their income in each period. The remaining $(1-\lambda)$, which we shall call Ricardian, are unconstrained optimizing agents. Following Debortoli and Gali (2018), and in order to keep the supply side as simple as possible, we assume the markup on wages (see below) is high enough that households supply as much labor as demanded by the firms.

3.4.2.1 Ricardian Households

Each period Ricardian households choose how much to consume, C_t^R, in order to maximize their life time (separable) utility:

$$\mathbb{E}\sum_{t=0}^{\infty}\beta^t\left(\frac{\left(C_t^R\right)^{1-\sigma}}{1-\sigma}-\frac{\left(N_t^R\right)^{1+\psi}}{1+\psi}\right)$$

where N_t^R is their hours worked. They are subject to the budget constraint:

$$P_tC_t^R+I_t^{-1}B_{t+1}=N_t^RW_t+P_tD_t+B_t$$

where P_t is the price level in period t, I_t is the gross nominal interest rate between t and $t+1$, B_t is the quantity of bonds bought at time $t-1$ paying one unit of nominal currency in period t, W_t is the nominal wage per unit of labor in period t and D_t is the real dividend payed by firms in period t. All firm profit goes to the Ricardian households and this is shared equally between them.

The Euler equation for these Ricardian households IS:

$$\left(C_t^R\right)^{-\sigma} = \beta\mathbb{E}\left(I_t \frac{P_t}{P_{t+1}} \left(C_{t+1}^R\right)^{-\sigma}\right) \tag{3.6}$$

3.4.2.2 Keynesian Households

Keynesian households are more impatient that the Ricardian households and as a result are up against their borrowing limit. They can borrow nominal bonds up to the point where their expected *real* payment in the next period is equal to a fixed fraction Ω of their steady state income. Each period they optimize their period utility:

$$\frac{\left(C_t^K\right)^{1-\sigma}}{1-\sigma} - \frac{\left(N_t^K\right)^{1+\psi}}{1+\psi}$$

subject to their budget constraint:

$$C_t^K \leq N_t^K \frac{W_t}{P_t} + \left(I_t^{-1} \frac{\mathbb{E}_t P_{t+1}}{P_t} - \frac{\mathbb{E}_{t-1} P_t}{P_t}\right) \Omega \bar{N}_K \overline{W/P} \tag{3.7}$$

where $\overline{W/P}$ and $\bar{N}_K$ are the steady state real wage and hours worked by Keynesian households.

3.4.2.3 Household Aggregation and Wage Schedule

With the Keynesian proportion of households equal to λ, total consumption is:

$$C_t = \lambda C_t^K + (1-\lambda) C_t^R \tag{3.8}$$

Hours are equally rationed between both types of household such that:

$$N_t = N_t^K = N_t^R \tag{3.9}$$

The real wage is set according to the demand schedule:

$$\frac{W_t}{P_t} = \mathcal{M}^{\omega} \left(C_t\right)^{\sigma} \left(N_t\right)^{\psi} \tag{3.10}$$

where $\mathcal{M}^{\omega} > 1$ can be interpreted as the gross average markup of wages. We assume $\frac{W_t}{P_t} \geq \left(C_t^R\right)^{\sigma} \left(N_t^R\right)^{\psi} \geq \left(C_t^K\right)^{\sigma} \left(N_t^K\right)^{\psi}$ so that households always provide the hours demanded by the firms.[7]

3.4.3 Firms

The production side of the economy follows the standard New Keynesian model with Calvo price adjustment. The firm side of the economy is identical to that presented in Gali (2008) except for the fact that firms choose both labor and capital (and thus their production function has constant returns to scale) each period. This simplifies the analysis a little, as all firms share the same marginal cost. In our base model we hold the aggregate quantity of capital constant, but including it here allows for easy extension to the model with investment.

[7] This demand schedule follows Debortoli and Gali (2018) and is close to the wages a union representing both types of household would set. We also tried allowing wages to be set by the market. This results in counter-factual results such as Keynesian and Ricardian households moving their hours worked in opposite directions during a recession.

3.4.3.1 Final Goods Firm

The final goods firm produces a final consumption good, Y_t, from intermediated inputs, $X_t(j)$ for $j \in [0,1]$ using the technology:

$$Y_t = \left(\int_0^1 X_t(j)^{1-\frac{1}{\varepsilon}} dj \right)^{\frac{\varepsilon}{\varepsilon-1}}$$

Profit maximization yields the demand schedule $X_t(j) = \left(\frac{P_t(j)}{P_t} \right)^{-\varepsilon}$ where P_t is the price of the final good. Competition also imposes a zero profit condition that yields $P_t = \left(\int_0^1 P_t^{1-\varepsilon} \right)^{\frac{1}{1-\varepsilon}}$.

3.4.3.2 Intermediate Goods Firm

There is a continuum of intermediate goods firms, indexed by $j \in [0,1]$ each of which uses both labor and capital each period according to the production function:

$$X_t(j) = AK_t(j)^{\alpha} N_t(j)^{1-\alpha}$$

As our focus is on monetary policy shocks, we assume the technology level (A) to be constant. Constant returns to scale results in the marginal cost being equal for all firms.

The probability that a firm is able to adjust its price in any period is equal to $1-\theta$. A firm that is able to adjust its price in period t will choose a price P^* to maximize the current market value of profits it will make while the price remains effective. That is firm j solves the problem:

$$\max_{P^*} \sum_{k=0}^{\infty} \theta^k \mathbb{E}_t \{\Lambda_{t,t+k} X_{t+k}(j)(P_t^* - MC_{t+k}P_{t+k})\} \tag{3.11}$$

subject to the demand constraints:

$$X_t(j) = \left(\frac{P_t^*}{P_{t+k}}\right)^{-\varepsilon} \tag{3.12}$$

where $\Lambda_{t,t+k} = \beta^k \left(\frac{c_{t+k}^R}{c_t^R}\right)^{-\sigma} \left(\frac{P_t}{P_{t+k}}\right)$ is the stochastic discount factor for nominal payoffs, for the Ricardian households who own the profits from the firms.

The first order condition arising from 3.11 and 3.12 is:

$$\sum_{k=0}^{\infty} \theta^k \mathbb{E}_t \Big\{ \Lambda_{t,t+k} X_{t+k}(j) \left(P_t^* - \frac{\varepsilon}{\varepsilon - 1} MC_{t+k} P_{t+k} \right) \Big\} = 0 \tag{3.13}$$

Finally, with only a fraction $1 - \theta$ of firms changing their prices in any given period, the aggregate price level moves according to:

$$P_t = \left(\theta P_{t-1}^{1-\varepsilon} + (1-\theta)(P_t^*)^{1-\varepsilon}\right)^{\frac{1}{1-\varepsilon}}$$

3.4.4 Monetary Policy

We assume the central bank follows a simple log-linear Taylor rule with weight on inflation only:

$$i_t = \phi_\pi \pi_t + \nu_t \tag{3.14}$$

where i_t and π_t are the log deviations from the nominal steady-state interest rate and inflation rate respectively. In line with the transitory nature of the experiment we are running, we assume no persistence in ν_t.

3.4.5 Equilibrium

As our baseline model has no investment, the goods market clearing condition is:

$$Y_t = C_t \tag{3.15}$$

and the total capital and labor used must equal that available, $\int_0^1 K_t(j)dj = \bar{K}$ and $\int_0^1 N_t(j)dj = N_t$.

3.4.6 Steady State

We will study small fluctuations around the zero inflation steady-state. As hours are allocated evenly between the two types of households we have that the share of hours worked by Keynesians is $\overline{n}_K = \lambda$, and that by Ricardians is $\overline{n}_R = 1 - \lambda$. The steady state consumption shares ($\overline{c}_K = \lambda\overline{C^K}/\overline{Y}$ and $\overline{c}_R = (1-\lambda)\overline{C^R}/\overline{Y}$) are less simple, both because Ricardians earn all the income from the firms and they get paid interest from the Keynesian households' debt. In steady-state the markup over marginal cost is equal to $\frac{\varepsilon}{\varepsilon-1}$, and the real wage is equal to the marginal productivity of labor adjusted down by this markup, $(1-\alpha)\frac{\varepsilon-1}{\varepsilon}\frac{\overline{Y}}{\overline{N}}$.

Using this steady-state wage, along with the Keynesian budget constraint (3.7) we can identify the steady-state proportion of Keynesian consumption:

$$\overline{c}_K = \lambda\left(1 - \Omega(1-\beta)\right)\frac{\varepsilon - 1}{\varepsilon}(1-\alpha) \tag{3.16}$$

3.4.7 Log-linearized Model

We use small letters to indicate percentage changes from steady-state values and then linearize around the steady-state. We begin with the basic building blocks of the New Keynesian model. First the Euler equation for Ricardian households, linearized from equation 3.6, becomes:

$$c_t^R = \mathbb{E}_t c_{t+1}^R - \frac{1}{\sigma}(i_t - \mathbb{E}_t \pi_{t+1}) \tag{3.17}$$

The New Keynesian Phillips curve, derived from the pricing equation 3.13, is:

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \frac{(1-\theta)(1-\beta\theta)}{\theta}\left(\sigma + \frac{\psi + \alpha}{1-\alpha}\right)\tilde{y}_t \tag{3.18}$$

where the output gap, $\tilde{y}_t$, in this case with fixed technology and capital is just the percentage deviation of output from steady-state output.

The monetary policy rule is already linearized and we take it directly from equation 3.14.

Unlike the standard New Keynesian model, these three are not enough to pin down the model as the Euler equation (3.17) is determined by Ricardian households, while total consumption and production involves the Keynesians too. We have the aggregation conditions from equations 3.8, 3.9 and 3.15:

$$c_t = \overline{c}_K c_t^K + \overline{c}_R c_t^R \tag{3.19}$$

$$n_t = \overline{n}_K n_t^K + \overline{n}_R n_t^R \tag{3.20}$$

$$\tilde{y}_t = c_t \tag{3.21}$$

and the Keynesian budget condition from equation 3.7:

$$(1-\Omega(1-\beta))c_t^K = w_t + n_t^K + \Omega\left(\pi_t - \mathbb{E}_{t-1}\pi_t\right) - \beta\Omega(i_t - \mathbb{E}_t\pi_{t+1}) \tag{3.22}$$

where w_t is the real wage in period t. Note $\pi_t - \mathbb{E}_{t-1}\pi_t$ represents unexpected inflation between $t-1$ and t and relates to the net nominal position of the Keynesian households. The expected return on nominal bonds, $r_t = i_t - \mathbb{E}_t\pi_{t+1}$, would be the real interest between t and $t+1$ if such a market existed and relates to the unhedged interest rate exposure of the Keynesian households. In the case where there is no debt ($\Omega = 0$), both these components of the budget constraint disappear. Further note that in this model $\mathbb{E}_{t-1}\pi_t$ will always be equal to zero, so the model has no predetermined variables.

The first order condition for hours worked, equation 3.10, along with the equal allocation of hours, give:

$$w_t = \sigma c_t + \psi n_t \tag{3.23}$$

$$n_t^R = n_t^K \tag{3.24}$$

Finally the connection between hours worked and the output gap is given by:

$$\tilde{y}_t = (1-\alpha)n_t \tag{3.25}$$

Note capital does not appear in the linearized production function because of the fixed capital assumption.

The final baseline model consists of the Taylor rule, equation 3.14, along with the equations 3.17 through 3.25 and the identity $r_t = i_t - \mathbb{E}_t\pi_{t+1}$.

Table 3.1: Baseline Calibration

σ	1.0	Inverse EIS
ψ	1.0	Inverse Frisch Elasticity
ϕ_π	1.5	Taylor Rule Coefficient
θ	0.667	Calvo stickiness parameter
β	1.0	Discount Factor
α	0.33	Capital Share
ε	6.0	Elasticity of sub. between goods
λ	0.2	Share of Keynesian Households
Ω	0.0	Keynesian Debt as Share of Income
δ	0.1	Depreciation (capital model only)

3.4.8 Calibration

We calibrate to standard parameters based on annual periods. Baseline parameters are shown in table 3.1. We will vary some of these to see how the size of the different transmission mechanisms changes with them.

3.5 Results from the Baseline TANK Model

As there are no predetermined variables in our baseline TANK model the decomposition of transmission mechanisms described in section 3.3 works exactly. Here we look at how monetary policy divides into the five different channels according to the proportion of Keynesian households, as well as the extent to which they are able to take on debt.

3.5.1 Model with No Debt

To start we consider the model in which the Keynesian households cannot hold any debt, as is standard in TANK models.[8] The transitory nature of the shock means that expected inflation next period is zero, and hence a one percent decrease in the nominal rate translates exactly into a one percent decrease in the real rate (if it were to trade).

Figure 3.1 shows the size of each transmission channel following a one percentage point decrease in the nominal interest rate, where the proportion of Keynesian households is on the x-axis. Note that both the interest rate exposure channel and the Fisher channel are absent in this model as there is no debt between the two types of agent. The left side of the graph shows the transmission channels when there are very few Keynesian households. As has been well documented[9] the intertemporal

[8]For examples see Debortoli and Gali (2018), Gali, Lopez-Salido, and Valles (2007) and Broer, Hansen, Krusell, and Oberg (2018).

[9]e.g. Kaplan, Moll, and Violante (2016)

substitution model dominates in this case, seen here in the division of transmission channels along the y-axis corresponding to the RANK model. We have set the elasticity of intertemporal substitution equal to one, so a one percentage point decrease in the real interest rate increases consumption of the Ricardian households by exactly one percent. This is seen as the intercept with the y-axis, divided into a large intertemporal substitution channel of size β, and a small aggregate income channel of size $1 - \beta$.

Moving along the x-axis increases the proportion of Keynesian households. The size of the intertemporal substitution channel decreases in line with the consumption share of consumption by Ricardian households. As Ricardian households own all the capital as well as the profits from the firms, their consumption share falls more slowly that their share of households. As we introduce Keynesian households the size of the aggregate income channel increases, as the average MPC across households grows. While this aggregate income channel is substantial, it ends up being dominated by the earnings heterogeneity channel. This channel is both less intuitive and economically more questionable. It arises because during a boom, the extra income is not distributed equally between the Keynesian and Ricardian households, but instead goes predominantly to the Keynesian households. This is due to the fact that when the output gap is positive, markups above marginal cost are small, so workers get paid closer to their marginal product while the profits of the firms are reduced. When the proportion of Keynesian households reaches 0.3 this earnings heterogeneity channel

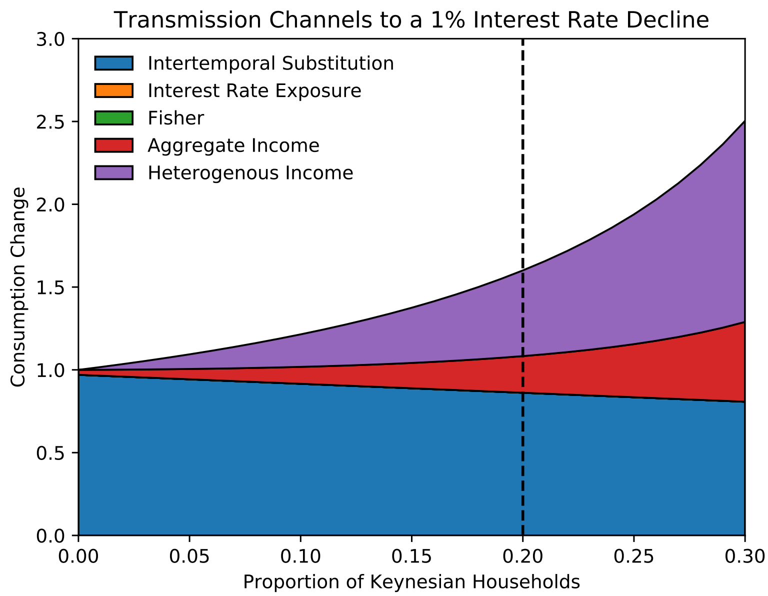


Figure 3.1: Changing the Proportion of Keynesian Households, $\sigma = 1$

actually dominates both of the other channels.

This feature of the standard New Keynesian model, that markups are low during a boom and high during a recession, is not backed by empirical evidence and has led some away from price frictions and toward nominal wage frictions.[10]. While we are sympathetic to this approach, for this paper we maintain the sticky price assumption to stay close to the existing HANK literature. One way to remove this earnings heterogeneity channel completely would be to divide the income from capital and profits proportionally between the Keynesian and Ricardian households. In that model, the total consumption change would remain unchanged as the number of Keynesian households increased, with the intertemporal substitution channel decreases propor-

[10]This point is emphasized in Broer, Hansen, Krusell, and Oberg (2018) and motivates modeling choices in Auclert and Rognlie (2018)

tional to the share of Ricardian's in the economy and the aggregate income channel making up the remainder. While the channels would be different, in this model the aggregate dynamics would be *identical* to the RANK model.

3.5.2 Introducing Debt

In this section we analyze what happens when the Keynesian households are allowed to take on debt equal to some fraction of their steady state income. For the remainder of this section we will fix the proportion of Keynesian households at 0.2, giving an economy-wide MPC of just over 20 percent. This number is chosen both because it is close to a number of the current theoretical HANK models, and a larger number causes indeterminacy problems for some parameterizations.[11] However, we accept 0.2 is on the low end of empirical estimates.[12] In figure 3.1 from the previous section, there is a dotted line drawn where the proportion of Keynesian households equals 0.2. This shows the size of the transmission channels for this section when there is no debt.

Figure 3.2 shows how the size of the transmission channels change with the level of debt held by the Keynesian households, with the three panels showing this for decreasing elasticity of substitution.[13] Starting with the top panel, we consider how

[11] See Gali, Lopez-Salido, and Valles (2004) for a detailed discussion on determinacy of TANK models.

[12] A large literature aims to estimate MPCS. See Johnson, Parker, and Souleles (2006), Parker, Souleles, Johnson, and McClelland (2013), Fagereng, Holm, and Natvik (2016) and Crawley and Kuchler (2018) for a small selection of examples.

[13] The elasticity of substitution is equal to $1/\sigma$, so the three panels in figure 3.2 represent an

the model behaves with an elasticity of substitution equal to one. The intercepts with the y-axis exactly correspond with the intercepts with the dotted line from figure 3.1. This is the size of the transmission channels when a proportion 0.2 of households are Keynesian and these households have no debt. As in the previous section, the intertemporal substitution channel is slightly below one, while the income channels also play a significant role due to presence of Keynesian households. However, with no debt at the intersection with the y-axis both the interest rate exposure and Fisher channels are zero.

As the quantity of debt that the Keynesian households can take on increases, both the interest rate exposure and Fisher channel start to become quantitatively important. Still looking at the top panel of figure 3.2, we see both of these channels growing, but they are still dominated by the intertemporal substitution channel. The two income channels grow in exact proportion to the other three channels, acting as a constant multiplier of the other three channels, no matter the quantity of debt. It may be useful to think of the transmission of monetary policy acting in stages. First aggregate demand is directly affected by the intertemporal substitution and interest rate exposure channels. The size of these channels depends only on the change in the interest rate, and is not changed as output and inflation change. The size of the Fisher channel is proportional to the amount of nominal debt, multiplied by the size of the overall change in income.[14] Finally the income channels are each a constant

elasticity of substitution of 1, 0.5 and 0.33 respectively.

[14]This is because inflation is proportional to the output gap in this model.

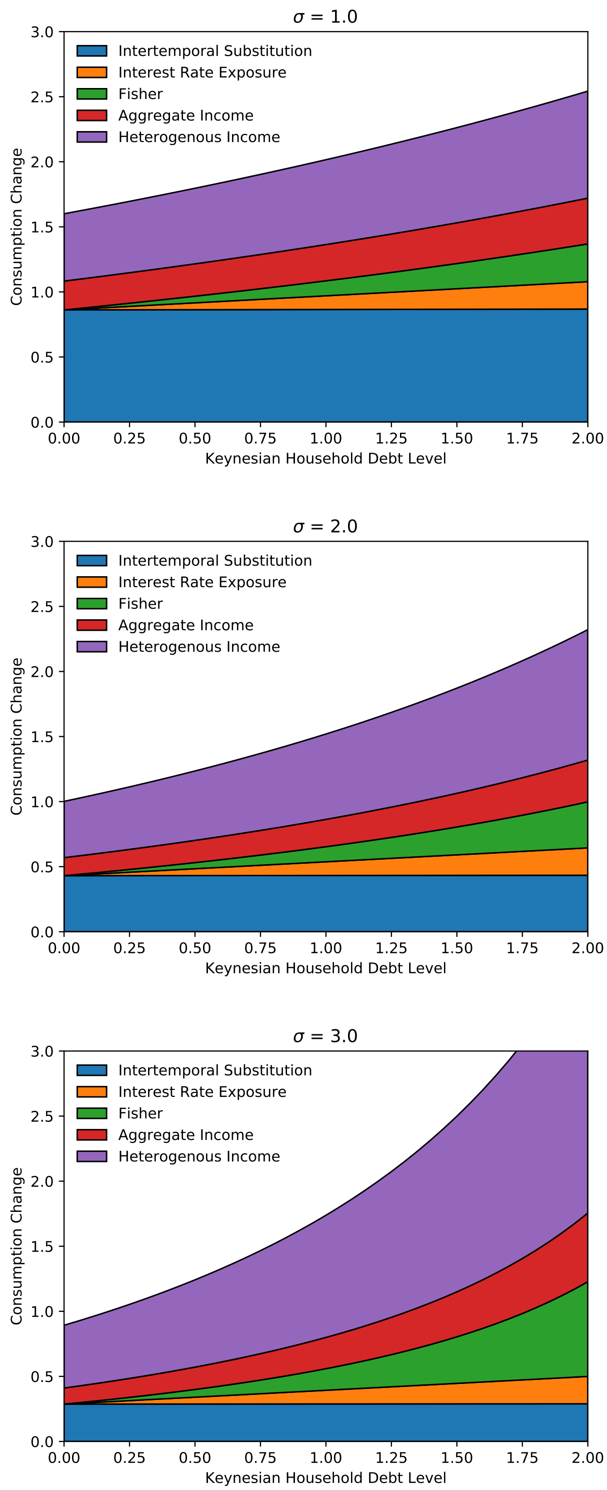


Figure 3.2: Changing the Debt of Keynesian Households

proportion of the total income change. We can think of intertemporal substitution and interest rate exposure as providing the initial 'kick', which is then augmented by the Fisher and income channels.

The center and bottom panels of figure 3.2 show the same channels, but when the elasticity of substitution is 0.5 and 0.33 respectively. The size of the intertemporal substitution channel is reduced in the same proportion, by 0.5 and 0.33 as the Ricardian households are now less happy to shift consumption between periods. However, the interest rate exposure channel remains exactly the same size as before. It is determined by the change in the borrowing cost along with the size of the debt, both of which are unchanged. The aggregate income channel is also exactly the same multiple of the other channels in all three panels, as is the Fisher channel.[15] The earnings heterogeneity multiplier grows significantly with σ. This is because the markup, and hence firm profits, become *more* countercyclical with higher σ. Again, this feature of the standard New Keynesian model is undesirable and leads us here to be unable analyze the model under low elasticities of substitution that we believe to be more empirically reasonable.[16]

This brings us to a broader point: the calibration of the elasticity of intertemporal substitution (EIS) in the standard New Keynsian model has been chosen to match aggregate data, despite the little micro evidence we have suggesting a much lower

[15]The aggregate income multiplier is constant across both debt levels and intertemporal elasticity. The Fisher multiplier varies by debt level, but for any particular debt level it does not vary with intertemporal elasticity

[16]See Havranek (2015) for a meta-study for EIS estimates.

level. Figure 3.2 shows why, in the absence of debt, a large EIS is required: with no debt the intertemporal substitution channel is the only 'kick' to aggregate demand, so if this is small we need very large multipliers to get a sizable consumption response to monetary policy. If we make the EIS small, we need something else to take its place. Interest rate exposure is another 'kick', that empirical evidence has shown could be large,[17]. By introducing interest rate exposure, we allow our models to use more micro-founded estimates of the EIS while still generating the kinds of aggregate responses estimated in the macro data.

3.6 Relaxing the Fixed Capital Assumption

We now relax the assumption of fixed capital and allow for investment. If there are no costs to investment, then households will invest until the new capital stock gives rise to the changed interest rate, which will result in a very persistent change in the interest rate. We will need convex investment adjustment costs to avoid this persistence, and hope to show that reasonable calibrations result in little change in the capital stock and hence low interest rate persistence.

[17]See Auclert (2017) and Crawley and Kuchler (2018).

3.6.1 The Model

The model is identical to the baseline model, except for the fact that the Ricardian households are now able to invest in capital as well as nominal bonds. Aggregate investment at time t, Inv_t, along with the level of capital at time t, K_t, together determine the capital level at time $t+1$:

$$\text{Inv}_t = \Phi\left(\frac{K_{t+1}}{K_t}\right) K_t \tag{3.26}$$

where $\Phi(1) = \delta$ is the per period depreciation, $\Phi'(1) = 1$ and $\Phi''(1) = \psi_K > 0$ represents convex capital adjustment costs. It is the fact that capital in period $t+1$ is *predetermined* in period t that differentiates this model from the baseline model in terms of breaking the assumptions required for Auclert's decomposition to hold. In steady state the investment share of income is $\overline{inv} = \frac{\varepsilon-1}{\varepsilon}\frac{\delta\alpha}{1/\beta-(1-\delta)}$.[18]

3.6.2 Changes Relative to the Linear Baseline Model

Given nominal interest rate and inflation expectations, the individual optimization problems for both Ricardian and Keynesian households, as well as firms, remains identical to the baseline model. That results in equations 3.17, 3.22, and 3.23 remaining unchanged. Differences occur in aggregation.

[18] This comes from equating the steady-state return from investment with $1/\beta$, the steady-state real interest rate, and using the fact that in equilibrium the total income allocated to capital is equal to $\frac{\alpha}{1-\alpha}$ times the total income allocated to labor. For other steady-state ratios, equation 3.16 remains the same, but now $\bar{c}_R = 1 - \overline{inv} - \bar{c}_K$, taking account of the fact that investment now takes a chunk out of output which is no longer equal to aggregate consumption.

As the natural level of output (output that would occur with flexible prices) is no longer constant, the output gap, $\tilde{y}$, is no longer equal to output. The model needs equations to define the natural level output and the output gap, along with an adjusted New Keynesian Phillips curve:[19]

$$y^n = \frac{\alpha(1+\psi)}{\frac{\sigma(1-\alpha)}{\bar{c}_K+\bar{c}_R}+\psi+\alpha}k_t + \frac{(1-\alpha)\sigma\frac{\overline{inv}}{\bar{c}_K+\bar{c}_R}}{\frac{\sigma(1-\alpha)}{\bar{c}_K+\bar{c}_R}+\psi+\alpha}inv_t \tag{3.27}$$

$$\tilde{y}_t = y_t - y^n \tag{3.28}$$

$$\pi_t = \beta\mathbb{E}_t\pi_{t+1} + \frac{(1-\theta)(1-\beta\theta)}{\theta}\left(\frac{\sigma}{\bar{c}_K+\bar{c}_R}+\frac{\psi+\alpha}{1-\alpha}\right)\tilde{y}_t \tag{3.29}$$

Furthermore, the aggregate production function, equation 3.25, now includes capital:

$$y = \alpha k_t + (1-\alpha)n_t \tag{3.30}$$

Aggregation of output now includes the investment share, so equation 3.19 is replaced by:

$$y_t = \bar{c}_K c_t^K + \bar{c}_R c_t^R + \overline{inv}\ inv_t \tag{3.31}$$

The law of motion for capital is introduced to the model:

$$\delta\ inv_t = k_{t+1} - (1-\delta)k_t \tag{3.32}$$

As is the equation for the shadow price of capital, q_t, determined by the convexity of adjustment costs:

$$q_t = \psi_K(k_{t+1} - k_t) \tag{3.33}$$

[19]Natural output is derived from the fact that under flexible prices, the markup over marginal cost will be constant ($\frac{\varepsilon}{\varepsilon-1}$). Investment is taken as given.

Finally we require an equation to equate the expected return on investment with the expected real return on nominal bonds:

$$r_t + q_t = \beta(1-\delta)\mathbb{E}_t q_{t+1} + (1-\beta(1-\delta))(\mathbb{E}_t(w_{t+1}+n_{t+1}) - k_{t+1}) \tag{3.34}$$

3.6.3 Results from the Model with Investment

For our partial equilibrium decomposition to approximate the aggregate consumption change, the shock to income, interest rates and inflation must be close to transitory. This poses a serious challenge for a model with capital, which is a slow moving variable. Figure 3.3 shows the problem. The figure displays the path of capital following a one percentage point negative shock to the nominal interest rate for different levels of capital adjustment convexity. Immediately we can see capital is a very persistent variable, with more than half of the increase in capital still present after six years. When there is no convexity in the capital adjustment costs ($\psi_c = 0$), the one percentage point decrease in the nominal rate results in a *large* positive increase in the quantity of capital. For typical values of ψ_c, often between one and three, the change is an order of magnitude smaller, while unsurprisingly capital remains unchanged in the case of infinite capital adjustment costs. This suggests the case of infinite adjustment costs will be similar to the fixed capital model. As we will see later this is mostly true, but the presence of positive investment makes the measurement of URE more subtle.

As figure 3.3 makes clear, capital is highly persistent. However, we can see in

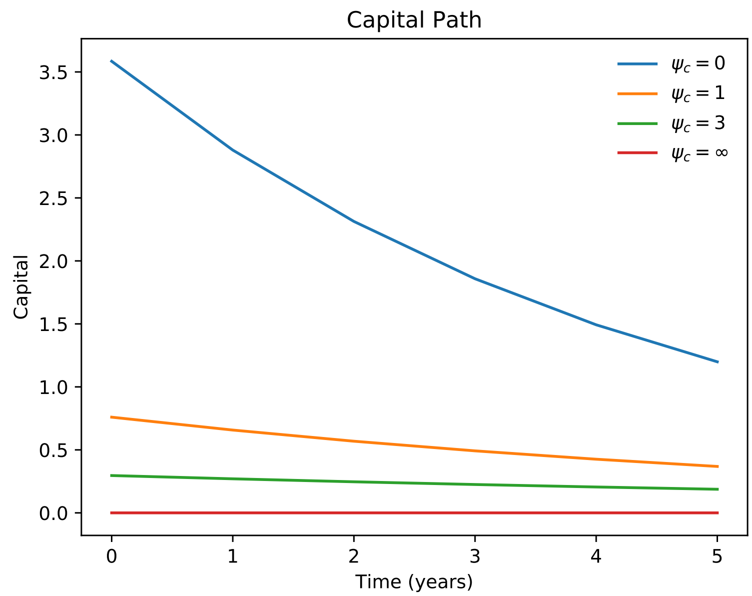


Figure 3.3: Path of Capital following a 1% nominal interest rate shock

figure 3.4 that in the cases where $\psi_c \geq 1$ the nominal interest rate path along with the consumption path for both Keynesian and Ricardian households is close to transitory. First consider the paths from figure 3.4 in which there are no convex adjustment costs to capital. In this case a one percent decrease in the nominal rate is highly persistent, as the level of capital fully adjusts such that the marginal product of capital is lower by the change in interest rate and this change in persists. All this extra investment in the first period dramatically (and unrealistically) increases wages, which the Keynesian households consume that period. Ricardian households invest most of their extra income, which allows them to maintain a higher path of consumption going forward.

When convex adjustment costs are introduced things look very different. Ricardian households can no longer increase capital to maintain consumption going forward, because the adjustment costs kick in. As a result the change in nominal interest rate is almost entirely transitory (in the case where $\psi_c = \infty$ this is exactly true), as is the change in consumption behavior of both types of household. Again, due to the countercyclical behavior of profits in the standard New Keynesian model, Keynesians react much more to the change than Ricardians.

In order to quantify how large of a deviation the model with capital is from the assumptions needed for our decomposition to work exactly, table 3.2 shows the percent difference between the true consumption change and that estimated using the partial equilibrium decomposition. The table shows the error for total consumption, as well as the error individually calculated for both the Ricardian and Keynesian households. First note that the method correctly estimates the consumption changes for the Keynesian households. This is because their behavior only depends on current income and the persistence can only affect consumption through the intertemporal substitution channel and wealth effects, which in their case are always zero. The error for Ricardian households (and overall consumption change) is unsurprisingly large when there are no convex adjustment costs, but this quickly comes down for standard calibrations if ψ_c between 1 and 3. Furthermore, as the value of σ rises, and the intertemporal substitution channel gets relatively smaller, this error diminishes.

The existence of capital raises the question of whether we should be including a wealth effect as a separate channel through which monetary policy operates. With convex capital adjustment costs, a decrease in the interest rate will increase the value

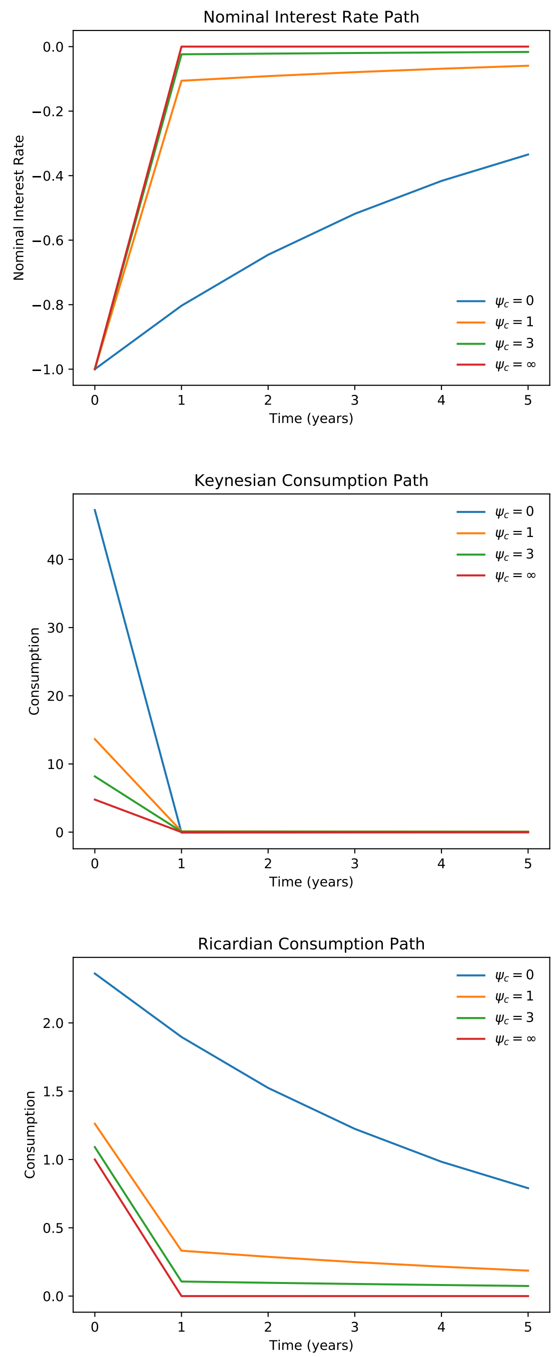


Figure 3.4: Paths following a 1% nominal interest rate decrease

of existing capital and hence the wealth of capital holders. Indeed this is also the case in the baseline fixed capital model. In that model the stream of income to the Ricardians from the capital is offset by the stream of consumption generated by it. While the Ricardians increase their wealth when the price of capital increases, this is exactly offset by the increase in the value of their planned consumption. That is the increase in wealth does not allow them in increase their consumption in every period, it is instead an artifact of the fact that with a lower interest rate consumption today is relatively cheaper, that is the wealth effect is entirely subsumed in the intertemporal substitution effect.

The model with capital does not allow such an easy interpretation of the change in wealth, even in the case with infinite capital adjustment costs. This is because the Ricardians are consistently investing to offset depreciation. In our decomposition this saving counts as unhedged interest rate exposure because their return on investment will be equal to the real interest rate. However, investments that were already planned will not be subject to this higher price - it is the marginal investments that suffer from the convex adjustment costs. If we change our definition of unhedged interest rate exposure to exclude planned investment, then partial equilibrium decomposition gives no error for the model with infinite adjustment costs.

The change in the value of existing assets is shown in figure 3.5. With no adjustment costs the value of assets remains constant over time as the consumption asset is freely exchangeable with capital next period. Similarly, with infinite adjustment

Table 3.2: Percentage Error of Decomposition

ψ_c	Total Consumption	Ricardian Consumption	Keynesian Consumption
0	-18.3 %	-57.3 %	0.0 %
1	-8.8 %	-18.5 %	0.0 %
3	-4.0 %	-7.0 %	0.0 %
∞	-0.6 %	-0.8 %	0.0 %

costs the price of a unit of capital next period moves one for one with the interest rate. For values in between the price of assets jumps up in period one followed by a persistent period in which capital adjusts back down to the steady state, and hence assets prices are low due to convex adjustment costs.

Overall the partial equilibrium decomposition works reasonably well with the addition of capital in the TANK model. However, in our model firms are risk neutral and it is clear that models in which firms are also able to have unhedged exposures to inflation and interest rates could complicate the transmission mechanism in quantitatively important ways. This may be especially true with the introduction of the banking sector, which has been shown emprirically to hold large unhedged interest rate exposures.[20]

[20] See Landier, Sraer, and Thesmar (2013). While empirical evidence suggests households are negatively exposed to interest rate hikes, the financial sector seems to be positively exposed. This suggests the transmission of monetary policy may be very different in times when the banking sector

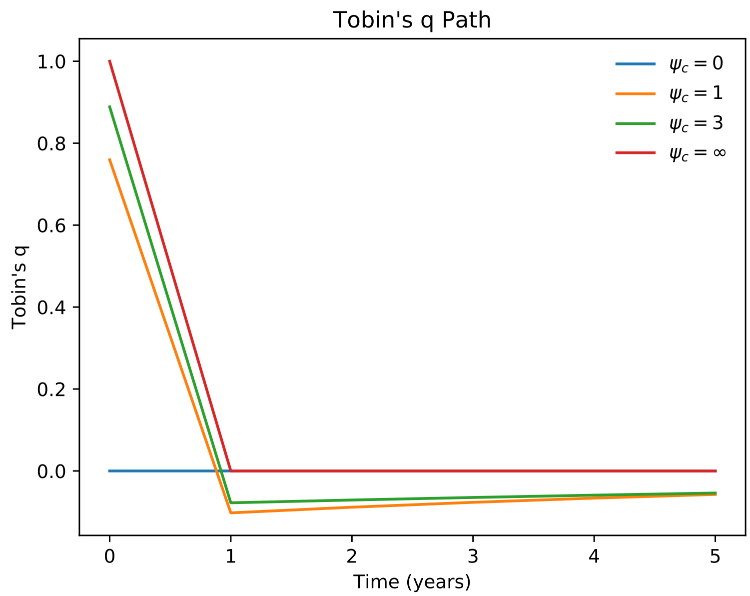


Figure 3.5: Path of Tobin's q following a 1% nominal interest rate shock

3.7 A Simple HANK Model

3.7.1 The Model

The model we study is a one asset version of the HANK model presented in Bayer and Luetticke (2018). We also follow the solution method presented in that paper.

3.7.1.1 Households

In a given period household i has labor productivity h_{it}, chooses their consumption c_{it} and hours worked n_{it}. Households have Greenwood, Hercowitz and Huffman

is working well to those when the banking sector is in crisis (when interest rate declines may not be as effective).

(GHH) preferences and act in order to maximize their expected utility:[21]

$$\mathbb{E}\sum_{t=0}^{\infty}\beta^t u(c_{it}-h_{it}\nu(n_{it}))$$

where $u(x)=\frac{x^{1-\sigma}}{1-\sigma}$ and $\nu(n)=\frac{n^{1-\psi}}{1-\psi}$.

Households consume a consumption bundle formed according to a Dixit-Stiglitz aggregator:

$$c_{it}=\left(\int_{j=0}^{1} c_{ijt}^{\frac{\varepsilon-1}{\varepsilon}}\,dj\right)^{\frac{\varepsilon}{\varepsilon-1}}$$

The price of each good is p_{jt} resulting in the aggregate price level $P_t=\left(\int_{j=0}^{1} p_{jt}^{1-\varepsilon}dj\right)^{\frac{1}{1-\varepsilon}}$ with demand for each good:

$$c_{ijt}=\left(\frac{p_{jt}}{P_t}\right)^{-\varepsilon} c_{it}$$

Household labor productivity evolves according to a $\log - AR(1)$ process, with a fixed probability that the household becomes an entrepreneur, receives no labor income, but instead collects a share of the firm profits:

$$h_{it}=\begin{cases}\exp(\rho_h h_{it-1}+\epsilon_{it}^{h}) & \text{with prob } 1-\zeta \text{ if } h_{it-1}\neq 0\\ 0 & \text{with prob } \iota \text{ if } h_{it-1}=0\\ 1 & \text{otherwise}\end{cases}$$

That is a non-entrepreneur switches to an entrepreneur state with probability ζ, while an entrepreneur switches to a non-entrepreneur with unit labor productivity

[21] The use of GHH preferences is primarily motivated to significantly simplify the solution method. Auclert and Rognlie (2017) show GHH preferences can lead to unrealistically large fiscal multipliers, while separable preferences lead to other counter factual results.

with probability ι. All households choose the same number of hours, due to GHH preferences. Total productive hours worked, $\int_{i=0}^{\infty} h_{it} n_{it} di = N(\omega_t)$, therefore depend only on the real wage, ω_t.

In the entrepreneur state the household receives a fixed share of the economic profits of the firms, Π_t, and these rents are not tradeable.

Households must pay a tax τ on all their rent and labor income.

3.7.1.2 Price Setting

Prices are set by risk-neutral managers who form a group of measure zero.[22] We assume Rotemberg (1982) pricing frictions, leading to a New Keynesian Phillips curve:

$$\log\left(\frac{P_t}{P_{t-1}}\right) = \beta \mathbb{E}_t \left(\log\left(\frac{P_{t+1}}{P_t}\right)\frac{Y_{t+1}}{Y_t}\right) + \kappa\left(MC_t - \frac{\varepsilon - 1}{\varepsilon}\right)$$

where Y_t is total output in period t, MC_t is the real marginal cost and κ measures the size of the Rotemberg price frictions. In equilibrium all goods will have the same price.

3.7.1.3 Fiscal Policy

Our model assumes the government will always owe a constant proportion of GDP in the following period. It does not make any real purchases, but services its debt through lumps sum taxes.

[22] Assuming the price setters are risk neutral makes the optimal price setting problem tractable without taking away from the important economics of the model.

3.7.1.4 Monetary Policy

As in the TANK model, we assume the central bank follows the Taylor rule given in equation 3.14.

3.7.2 Results from the HANK model

3.7.2.1 Effect of GHH Preferences

Our HANK model uses non-separable GHH preferences. This results in a strong link between consumption and hours worked that is not accounted for in our partial equilibrium decomposition. To take account of this we need to add a sixth 'GHH' transmission mechanism:

$$\text{GHHchannel} = \mathbb{E}\left((1 - MPC_i)h_i\right)\frac{\bar{N}}{\psi}d\omega$$

The intuition here is that households smooth $c_{it} - h_{it}\nu(n_{it})$ over time periods, so if a household works more in one period they will need to consume more in that period to achieve the same marginal utility of consumption. As we shall see, this channel dominates all the other five transmission channels.

3.7.2.2 Is a Monetary Policy Shock Transitory in the HANK Model?

While the aggregate level of government debt at the start of each period will only change in the initial period, the distribution of wealth may propagate through time leading to errors in our decomposition. Figure 3.6 shows the impulse response of a one percentage point decline in the nominal interest rate. It is clear the the aggregate consumption response, along with the wage response, is transitory in nature. However, both the real interest rate and the inflation impulse response functions show some persistence, possibly due to the fact that the monetary policy decline acts as a wealth transfer from the wealthy to the less wealthy, so higher interest rates are required to dampen demand back down to the steady state level. The aggregate size of the response is also noticeably large. The one percentage point decline in nominal interest rates leads to almost 12 percentage point increase in consumption. Much of this is due to the unusual nature of GHH preferences as will become clear in the next section.

3.7.2.3 Which Transmission Channels Are Important?

Table 3.3 shows how the period 1 consumption response divides into the six channels we have identified, as well as a (small) error term that subsumes the persistent behavior. It is clear from the table that the GHH channel dominates the transmis-

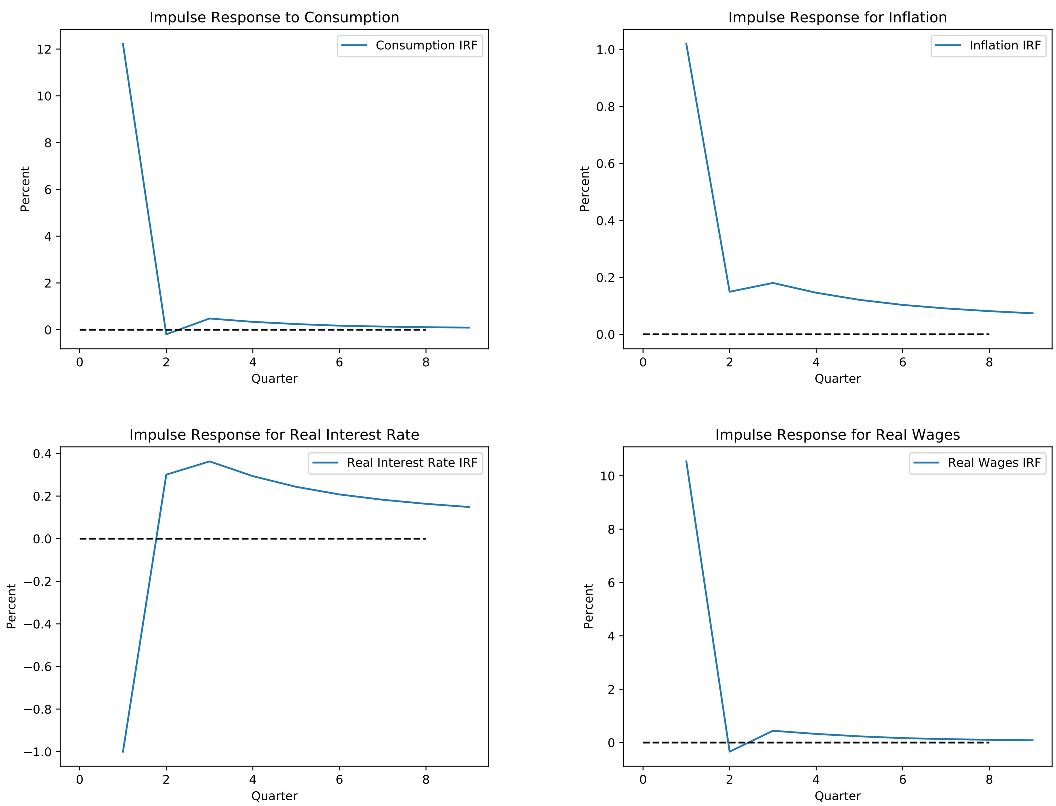


Figure 3.6: IRFs following a 1% Nominal Interest Rate Decline for the HANK Model

Table 3.3: Transmission Channel Importance

Aggregate Income	7.3%
Earnings Heterogeneity	6.1%
Interest Rate Exposure	4.4%
Fisher	3.9%
Intertemporal Substitution	2.3%
GHH Channel	75.2%
Error	0.9%

sion mechanism in this model. The complementarity between increased hours and increased consumption in this model with GHH preferences causes consumption to increase greatly in the first period when wages rise. Such a result has little empirical backing and strongly suggests GHH preferences are not suitable for this kind of work.

Beyond the GHH channel, we see that the aggregate and heterogeneous income channels are both large. Even though the MPC in this model is relatively small, these act as multipliers to the GHH channel and therefore become quantitatively important. The intertemporal substitution channel, along with the interest rate exposure and Fisher channels play a small role in the monetary policy transmission mechanism for this model.

3.8 Conclusion

Our paper shows that the transmission mechanism of monetary policy can look very different in a model with heterogeneity in household behavior. Our view of the empirical evidence is that the interest rate exposure channel is likely to be of primary quantitative importance. However, we have shown in this paper that such a mechanism is of limited importance in the standard TANK and HANK models in use today.

We believe much progress has been made recently in understanding the role of consumption behavior in macroeconomic models. While there are clear gaps in our understanding, a path forward bridging both empirical results and theory is within sight. At present the dynamics of inflation, in our paper taken from the New Keynesian Phillips curve, remains a separate area of research to which we have little to add.

We believe future research should focus on reducing the countercyclical profits of New Keynesian models which leads to a large earnings heterogeneity channel in our models. Furthermore, finding models with small intertemporal substitution channels, while still maintaining determinacy, is of primary importance.

Bibliography

Abildgren, K., A. Kuchler, A. S. L. Rasmussen, and H. S. Sorensen (2018): "Consistency between household-level consumption data from registers and surveys," Paper presented at the 35th IARIW General Conference, Copenhagen, August 2018.

Agarwal, S., and W. Qian (2014): "Consumption and Debt Response to Unanticipated Income Shocks: Evidence from a Natural Experiment in Singapore," *American Economic Review*, 104(12), 4205–4230.

Altonji, J., and L. M. Segal (1996): "Small-Sample Bias in GMM Estimation of Covariance Structures," *Journal of Business & Economic Statistics*, 14(3), 353–66.

Altonji, J. G., and A. Siow (1987): "Testing the Response of Consumption to Income Changes with (Noisy) Panel Data," *The Quarterly Journal of Economics*, 102(2), 293–328.

Ampudia, M., D. Georgarakos, J. Slacalek, O. Tristani, P. Vermeulen, AND G. VIOLANTE (2018): "Monetary Policy and Household Inequality," SSRN Scholarly Paper ID 3223542, Social Science Research Network, Rochester, NY.

ARELLANO, M., R. BLUNDELL, AND S. BONHOMME (2017): "Earnings and Consumption Dynamics: A Nonlinear Panel Data Framework," *Econometrica*, 85(3), 693–734.

AUCLERT, A. (2017): "Monetary Policy and the Redistribution Channel," NBER Working Paper 23451, National Bureau of Economic Research.

AUCLERT, A., AND M. ROGNLIE (2017): "A note on multipliers in NK models with GHH preferences," working paper.

——— (2018): "Inequality and Aggregate Demand," Working Paper 24280, National Bureau of Economic Research.

BAKER, S. R. (2015): "Debt and the Consumption Response to Household Income Shocks," SSRN Scholarly Paper ID 2541142, Social Science Research Network, Rochester, NY.

BAKER, S. R., L. KUENG, M. PAGEL, AND S. MEYER (2018): "Measurement Error in Imputed Consumption," SSRN Scholarly Paper ID 3179117, Social Science Research Network, Rochester, NY.

BAYER, C., AND R. LUETTICKE (2018): "Solving heterogeneous agent models in discrete time with many idiosyncratic states by perturbation methods," working paper.

Best, M. C., J. Cloyne, E. Ilzetzki, and H. Kleven (2018): "Estimating the Elasticity of Intertemporal Substitution Using Mortgage Notches," NBER Working Paper 24948, National Bureau of Economic Research.

Blanchard, O. J., F. Lopez-de Silanes, and A. Shleifer (1994): "What do firms do with cash windfalls?," *Journal of Financial Economics*, 36(3), 337–360.

Blundell, R., L. Pistaferri, and I. Preston (2008): "Consumption Inequality and Partial Insurance," *American Economic Review*, 98(5), 1887–1921.

Broer, T., N.-J. H. Hansen, P. Krusell, and E. Oberg (2018): "The New Keynesian Transmission Mechanism: A Heterogeneous-Agent Perspective," *The Review of Economic Studies*.

Browning, M., and M. D. Collado (2001): "The Response of Expenditures to Anticipated Income Changes: Panel Data Estimates," *American Economic Review*, 91(3), 681–692.

Browning, M., and S. Leth-Petersen (2003): "Imputing consumption from income and wealth information," *Economic Journal*, 113(488), F282–F301.

Bunn, P., J. Le Roux, K. Reinold, and P. Surico (2018): "The consumption response to positive and negative income shocks," *Journal of Monetary Economics*, 96, 1–15.

Campbell, J. Y., and N. G. Mankiw (1989): "Consumption, Income and Interest Rates: Reinterpreting the Time Series Evidence," NBER Chapters, National Bureau of Economic Research.

Carroll, C., and A. Samwick (1997): "The nature of precautionary wealth," *Journal of Monetary Economics*, 40(1), 41–71.

Carroll, C., J. Slacalek, K. Tokuoka, and M. N. White (2017): "The distribution of wealth and the marginal propensity to consume," *Quantitative Economics*, 8(3), 977–1020.

Carroll, C. D. (1997): "Buffer Stock Saving and the Life Cycle/Permanent Income Hypothesis," *Quarterly Journal of Economics*, CXII(1), 1–56.

Carroll, C. D. (2001): "Death to the Log-Linearized Consumption Euler Equation! (And Very Poor Health to the Second-Order Approximation)," *Advances in Macroeconomics*, 1(1).

Carroll, C. D., E. Crawley, J. Slacalek, K. Tokuoka, and M. N. White (2018): "Sticky Expectations and Consumption Dynamics," NBER Working Paper 24377, National Bureau of Economic Research.

Cesarini, D., E. Lindqvist, M. J. Notowidigdo, and R. Ostling (2017):

"The Effect of Wealth on Individual and Household Labor Supply: Evidence from Swedish Lotteries," *American Economic Review*, 107(12), 3917–3946.

Cloyne, J., C. Ferreira, and P. Surico (2016): "Monetary policy when households have debt: new evidence on the transmission mechanism," Bank of England Working Paper 589, Bank of England.

Commault, J. (2017): "How Does Consumption Respond to a Transitory Income Shock? Reconciling Natural Experiments and Structural Estimations," Working Paper.

Coronado, J. L., J. P. Lupton, and L. M. Sheiner (2005): "The Household Spending Response to the 2003 Tax Cut: Evidence from Survey Data," FEDS Discussion Paper 32, Federal Reserve Board.

Crawley, E. (2018): "Time Aggregation in Panel Data on Income and Consumption," Mimeo, Department of Economics, Johns Hopkins University.

Crawley, E., and A. Kuchler (2018): "Consumption Heterogeneity: Micro Drivers and Macro Implications," working paper 129, Danmarks Nationalbank.

David, M., K. H. Marquis, J. C. Moore, L. L. Stinson, and E. J. Welniak (1997): "Income Measurement Error in Surveys: A Review - Semantic Scholar," .

Debortoli, D., and J. Gali (2018): "Monetary Policy with Heterogeneous Agents: Insights from TANK models," mimeo, CREI.

Doepke, M., and M. Schneider (2006): "Inflation and the Redistribution of Nominal Wealth," *Journal of Political Economy*, 114(6), 1069–1097.

Fagereng, A., L. Guiso, D. Malacrino, and L. Pistaferri (2016): "Heterogeneity and Persistence in Returns to Wealth," NBER Working Paper 22822, National Bureau of Economic Research.

Fagereng, A., and E. Halvorsen (2015): "Imputing consumption from Norwegian income and wealth registry data," Discussion Paper, Statistics Norway, Research Department.

Fagereng, A., M. B. Holm, and G. J. Natvik (2016): "MPC Heterogeneity and Household Balance Sheets," Discussion Paper, Statistics Norway, Research Department.

Fuhrer, J. C. (2000): "Habit Formation in Consumption and Its Implications for Monetary-Policy Models," *American Economic Review*, 90(3), 367–390.

Fuster, A., G. Kaplan, and B. Zafar (2018): "What Would You Do With \$500? Spending Responses to Gains, Losses, News and Loans," NBER Working Paper 24386, National Bureau of Economic Research.

Gali, J. (2008): *Monetary Policy, Inflation, and the Business Cycle*. Princeton University Press.

Gali, J., D. Lopez-Salido, and J. Valles (2007): "Understanding the Effects of Government Spending on Consumption," *Journal of the European Economic Association*, 5(1), 227–270.

Gali, J., J. D. Lopez-Salido, and J. Valles (2004): "Rule-of-Thumb Consumers and the Design of Interest Rate Rules," Discussion Paper 10392, National Bureau of Economic Research, Inc.

Ganong, P., and P. Noel (2017): "Consumer Spending During Unemployment: Positive and Normative Implications," Working Paper.

Gelman, M. (2016): "What Drives Heterogeneity in the Marginal Propensity to Consume? Temporary Shocks vs Persistent Characteristics," Working Paper.

Gelman, M., Y. Gorodnichenko, S. Kariv, D. Koustas, M. D. Shapiro, D. Silverman, and S. Tadelis (2016): "The Response of Consumer Spending to Changes in Gasoline Prices," NBER Working Paper 22969, National Bureau of Economic Research.

Gelman, M., S. Kariv, M. D. Shapiro, D. Silverman, and S. Tadelis (2014): "Harnessing naturally occurring data to measure the response of spending to income," *Science*, 345(6193), 212–215.

Gorea, D., and V. Midrigan (2017): "Liquidity Constraints in the U.S. Housing Market," NBER Working Paper 23345, National Bureau of Economic Research.

Guvenen, F. (2007): "Learning Your Earning: Are Labor Income Shocks Really Very Persistent?," *American Economic Review*, 97(3), 687–712.

——— (2009): "An empirical investigation of labor income processes," *Review of Economic Dynamics*, 12(1), 58–79.

Hall, R., and F. Mishkin (1982): "The Sensitivity of Consumption to Transitory Income: Estimates from Panel Data on Households," *Econometrica*, 50(2), 461–81.

Hausman, J. K. (2012): "Fiscal Policy and Economic Recovery: The Case of the 1936 Veterans' Bonus," mimeo, University of California, Berkeley.

Havranek, T. (2015): "Measuring Intertemporal Substitution: The Importance of Method Choices and Selective Reporting," *Journal of the European Economic Association*, 13(6), 1180–1204.

Heathcote, J., F. Perri, and G. L. Violante (2010): "Unequal we stand: An empirical analysis of economic inequality in the United States, 1967-2006," *Review of Economic Dynamics*, 13(1), 15–51.

Hryshko, D., and I. Manovskii (2018): "Income Dynamics and Consumption Insurance," Working Paper.

Hsieh, C.-T. (2003): "Do Consumers React to Anticipated Income Changes? Evidence from the Alaska Permanent Fund," *American Economic Review*, 99, 397–405.

JAPPELLI, T., AND L. PISTAFERRI (2010): "The Consumption Response to Income Changes," *Annual Review of Economics*, 2(1), 479–506.

——— (2014): "Fiscal Policy and MPC Heterogeneity," *American Economic Journal: Macroeconomics*, 6(4), 107–136.

JOHNSON, D. S., J. A. PARKER, AND N. S. SOULELES (2006): "Household Expenditure and the Income Tax Rebates of 2001," *American Economic Review*, 96(5), 1589–1610.

——— (2009): "The Response of Consumer Spending to Rebates During an Expansion: Evidence from the 2003 Child Tax Credit," Working Paper, The Wharton School.

KAPLAN, G., B. MOLL, AND G. L. VIOLANTE (2016): "Monetary Policy According to HANK," NBER Working Paper 21897, National Bureau of Economic Research.

KAPLAN, G., AND G. L. VIOLANTE (2010): "How Much Consumption Insurance beyond Self-Insurance?," *American Economic Journal: Macroeconomics*, 2(4), 53–87.

KAPLAN, G., AND G. L. VIOLANTE (2018): "Microeconomic Heterogeneity and Macroeconomic Shocks," NBER Working Paper 24734, National Bureau of Economic Research.

Koijen, R., S. V. Nieuwerburgh, and R. Vestman (2014): “Judging the Quality of Survey Data by Comparison with ”Truth” as Measured by Administrative Records: Evidence From Sweden,” in *NBER Chapters: Improving the Measurement of Consumer Expenditures*, pp. 308–346. National Bureau of Economic Research.

Krueger, D., K. Mitman, and F. Perri (2016): “Macroeconomics and Household Heterogeneity,” NBER Working Paper 22319, National Bureau of Economic Research.

Krusell, P., and A. Smith (1998): “Income and Wealth Heterogeneity in the Macroeconomy,” *Journal of Political Economy*, 106(5), 867–896.

Landier, A., D. Sraer, and D. Thesmar (2013): “Banks’ Exposure to Interest Rate Risk and The Transmission of Monetary Policy,” NBER Working Paper 18857, National Bureau of Economic Research.

Low, H., C. Meghir, and L. Pistaferri (2010): “Wage Risk and Employment Risk over the Life Cycle,” *American Economic Review*, 100(4), 1432–1467.

Lusardi, A. (1996): “Permanent Income, Current Income, and Consumption: Evidence from Two Panel Data Sets,” *Journal of Business and Economic Statistics*, 14(1), 81–90.

Maggio, M. D., A. Kermani, and K. Majlesi (2018): “Stock Market Returns and Consumption," NBER Working Paper 24262, National Bureau of Economic Research.

Mankiw, N. G., and R. Reis (2002): "Sticky Information versus Sticky Prices: A Proposal to Replace the New Keynesian Phillips Curve," *The Quarterly Journal of Economics*, 117(4), 1295–1328.

Manovskii, I., and D. Hryshko (2017): "How Much Consumption Insurance in the U.S.?," Working Paper 1584, Society for Economic Dynamics.

Meghir, C., and L. Pistaferri (2004): "Income Variance Dynamics and Heterogeneity," *Econometrica*, 72(1), 1–32.

Moffitt, R. A., and P. Gottschalk (2012): "Trends in the Transitory Variance of Male Earnings Methods and Evidence," *Journal of Human Resources*, 47(1), 204–236.

Moffitt, R. A., and S. Zhang (2018): "Income Volatility and the PSID: Past Research and New Results," NBER Working Paper 24390, National Bureau of Economic Research.

Nielsen, H. S., and A. Vissing-jorgensen (2004): "The Impact of Labor Income Risk on Educational Choices: Estimates and Implied Risk Aversion," Working Paper, University of Aarhus.

Parker, J. A. (1999): "The Reaction of Household Consumption to Predictable Changes in Social Security Taxes," *American Economic Review*, 89(4), 959–973.

Parker, J. A., N. S. Souleles, D. S. Johnson, and R. McClelland (2013): "Consumer spending and the economic stimulus payments of 2008," *American Economic Review*, 103(6), 2530–2553.

Peterman, W. B. (2016): "Reconciling Micro and Macro Estimates of the Frisch Labor Supply Elasticity," *Economic Inquiry*, 54(1), 100–120.

Rotemberg, J. J. (1982): "Sticky Prices in the United States," *Journal of Political Economy*, 90(6), 1187–1211.

Sahm, C. R., M. D. Shapiro, and J. B. Slemrod (2010): "Household Response to the 2008 Tax Rebate: Survey Evidence and Aggregate Implications," *Tax Policy and the Economy*, 24, 69–110.

Shapiro, M. D., and J. Slemrod (2009): "Did the 2008 Tax Rebates Stimulate Spending?," *American Economic Review*, 99(2), 374–79.

Souleles, N. S. (1999): "The Response of Household Consumption to Income Tax Refunds," *American Economic Review*, 89(4), 947–958.

——— (2002): "Consumer Response to the Reagan Tax Cuts," *Journal of Public Economics*, 85, 99–120.

Straub, L. (2018): "Consumption, Savings, and the Distribution of Permanent Income," Working Paper.

Vardardottir, A., and M. Pagel (2016): "The Liquid Hand-to-Mouth: Evidence from a Personal Finance Management Software," 2016 Meeting Paper 789, Society for Economic Dynamics.

Violante, G., G. Kaplan, and J. Weidner (2014): "The Wealthy Hand-to-Mouth," 2014 Meeting Paper 192, Society for Economic Dynamics.

Wong, A. (2016): "Population aging and the transmission of monetary policy to consumption," 2016 Meeting Paper 716, Society for Economic Dynamics.

Working, H. (1960): "Note on the Correlation of First Differences of Averages in a Random Chain," *Econometrica*, 28(4), 916–918.