Measuring Heterogeneity in Consumption Behavior: Micro Drivers and Macro Implications

July 25, 2018

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Abstract

This paper uses a novel econometric method to estimate households' consumption responses to permanent and transitory shocks to income. Our method builds upon Blundell, Pistaferri, and Preston (2008), using techniques from Crawley (2018) to correct for the time aggregation problem. Applying our method to a confidential panel dataset covering the entire Danish population, we find heterogeneity in consumption behavior across dimensions of particular importance for monetary policy. Households who stand to lose from an interest rate hike have significantly larger consumption responses than those who stand to gain. Overall we estimate this interest rate exposure channel to be of an order of magnitude larger than the intertemporal substitution channel that dominates in representative agent New Keynesian models.

Keywords Uncertainty, Consumption Dynamics, MPC

JEL codes D12, D31, D91, E21

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Thanks to Danmarks Nationalbank for providing financial support as well as invaluable access to their data.

1 Introduction

This paper makes two contributions to the literature on household consumption behavior and its implications for the macroeconomy. First, we develop a new methodology to estimate the consumption responses to income shocks using covariance restrictions on household level panel data. Second, using administrative data covering the entire Danish population, we use the method to quantify the importance of heterogeneity in monetary policy transmission. For example, we estimate the interest rate exposure channel, which in mute in representative agent models, to be of an order of magnitude larger than the intertemporal substitution channel that typically dominates in New Keynesian models.

Our method is closely related to Blundell, Pistaferri, and Preston (2008) (henceforth BPP) who estimate 'insurance' parameters to transitory and permanent shocks to income using covariance restrictions on a panel of household income and consumption. The BPP methodology (or those closely related to it) has become standard in the literature[1] despite the fact that the estimate of the consumption response to transitory income shocks using this method is often an order of magnitude lower than that implied by the natural experiment literature. Crawley (2018) shows one way in which this methodology is seriously flawed: a failure to account for the time aggregation problem of Working (1960). He shows that in a simulated dataset where households follow the permanent income hypothesis, that is they respond one-for-one to shocks to permanent income but not at all to transitory income shocks, the estimate for the consumption response to transitory income shocks using the BPP methodology is -0.6 (a 10% transitory increase to income is estimated to result in a 6% decrease in consumption, where the true change is 0%). Our methodology not only correctly accounts for the time aggregation problem but is also robust to generic short term dynamics in both income and consumption. Our empirical results are also broadly in line with the long literature that uses natural experiments to measure marginal propensities to consume out of transitory income. The popularity of the BPP methodology, despite its flaws, shows the high demand for a method of extracting consumption behavior from panel data on income and consumption. We believe our method, or slight variants of it, provides a robust alternative to BPP and has a wide array of potential applications, particularly when applied to the growing number of large panels such as the Danish administrative data used in this paper, or those available from financial aggregation platforms.[2]

The need for better methods and data to measure consumption behavior at the household level has grown with the increasing recognition that household heterogeneity may play a key role in macroeconomic dynamics. Kaplan and Violante (2018) provide a nice overview of the theoretical literature incorporating household heterogeneity into models of economic fluctuations. Computational and methodological limitations, along with early work by Krusell and Smith (1998) showing the aggregate dynamics of a TFP

[1] See for example Violante, Kaplan, and Weidner (2014), Auclert (2017) and Manovskii and Hryshko (2017). Recent work by Arellano, Blundell, and Bonhomme (2017) uses a quantile-based approach to allow for non-linear dynamics, but the time aggregation problem is not addressed.

[2] While access is very restricted, panel data from financial aggregation platforms has been highly informative about consumption behavior. In the US examples include Gelman, Kariv, Shapiro, Silverman, and Tadelis (2014), Ganong and Noel (2017) and Baker (2015), while Vardardottir and Pagel (2016) uses data from Iceland.

shock were not much changed in a heterogeneous agent model, have resulted in a slow start for this literature. However, recent advances have allowed for a new generation of Heterogeneous Agent New Keynesian (HANK) models that, as their name suggests, combine elements from both the heterogeneous agent and New Keynesian literature. These models not only match the growing evidence on micro consumption behavior, but also imply very different aggregate dynamics and/or propagation mechanisms following macroeconomic shocks, compared to their representative agent equivalents. In particular the transmission mechanism of monetary policy can look very different in a HANK model. For example, in the model of Kaplan, Moll, and Violante (2016) the intertemporal substitution channel is dwarfed by indirect general equilibrium effects, in stark contrast to a representative agent model.

While these HANK models make clear the potential importance of heterogeneity in economic fluctuations, and particularly for monetary policy, their quantitative results hinge on previously unimportant assumptions such as the tenure of debt instruments and the government's fiscal rule. Thus far the ability of these models to help us distinguish transmission channels empirically has been limited. Auclert (2017), in contrast to the fully structural HANK models, takes a simplified approach to aggregate dynamics, and one that we will follow in this paper, in order to derive a set of sufficient statistics, directly measurable from a suitable dataset, that is highly informative about the relative size of different monetary policy transmission channels. His methodology benefits from being transparent and closely tied to the data. However, with the small sample sizes of the publically available panel surveys he uses, along with the methods used to impute transitory MPCs (including BPP), he is in the end unable to attain convincing quantitative estimates for the sufficient statistics he identifies.[3] Our new methodology for estimating consumption behavior, combined with the fact that our panel covers the entire Danish population so that we can reconcile aggregate balance sheet positions of households with that of the government, firms and foreigners in the national accounts, provides and almost ideal setting in which to estimate Auclert's sufficient statistics. We find heterogeneity in households' interest rate exposure to be very relevant to monetary policy transmission. Households that stand to lose from an interest rate hike (typically those with a large variable rate mortgage) are likely to cut back their consumption far more than those who stand to gain (typically those with large liquid asset holdings). We estimate this interest rate exposure channel to be of an order of magnitude larger than the intertemporal substitution channel.

1.1 Empirical Evidence on Heterogeneity in Consumption Behavior

Most micro empirical evidence on consumption behavior comes in the form of an estimate of the marginal propensity to consume from a one-off source of income over the following three months to one year. Table 1 shows a selection of the population average estimates from the literature. In representative agent models of the macroeconomy, the agent is usually found to have an annual MPC out of transitory shocks of around 3-5%, about an

[3] Fagereng, Holm, and Natvik (2016) also estimate these sufficient statistics, imputing MPCs from lottery winnings in Norway, but the are also limited by sample size.

order or magnitude lower than most of the empirical estimates in this table. The inability of these macro models to match even this most basic fact about micro consumption behavior undermines their claim to be micro-founded and raises questions about the reliability of their macro implications. Ad-hoc fixes, such as the addition of the hand-to-mouth consumers in Campbell and Mankiw (1989), also bear little relation to the micro data. The new set of heterogeneous agent models come with testable implications for the distribution of MPCs across dimensions such as net or liquid wealth (see Carroll, Slacalek, Tokuoka, and White (2016) and Violante, Kaplan, and Weidner (2014)). Most of the studies in table 1 do not have the power to say much if anything along these dimensions. Three methods are used to empirically determine the marginal propensity to consume. The first is to identify a natural experiment and then measure the consumption response to it. Often this is done using the Consumer Expenditure Survey in the US. For example Johnson, Parker, and Souleles (2006) use randomly assigned timing of 2001 tax rebates and specially included questions in the Consumer Expenditure Survey to identify a three month aggregate marginal propensity to consume of 0.2-0.4. This method is generally considered the most reliable, but estimates vary and there is no strong consensus. Identification issues arise as to when exactly households learn about the payment versus when it is received and it is unclear the extent to which external validity extends from these natural experiments to the kinds of transitory shocks found in heterogeneous agent models. As most of these studies rely on consumer survey data they tend to lack power due to high measurement error and low sample sizes. As a result they have produced very little evidence of how the MPC varies among different groups in the economy. A recent paper by Fagereng, Holm, and Natvik (2016) overcomes some of these problems. By using lottery data, the shock to income is truly random.[4] They use registry data from Norway similar to the data we use from Denmark and have a sample of over 30,000 lottery winners over 10 years. As a result they can identify the MPC for households with differing liquid wealth, as well as by the size of the lottery win. They find that households in the lowest quartile of liquid wealth have an MPC of approximately 0.61 over a 6 month period, as opposed to 0.45 for households in the highest quartile of liquid wealth. Using data from a financial aggregator, Gelman (2016) has enough power to identify large differences in the impulse response to a tax rebate at a monthly frequency between household quintiles of cash-on-hand.

The second method is simply to ask individuals how much of a transitory income change they would consume, as was done in the Italian Survey of Household Income and Wealth in 2010 and the NY Fed's Survey of Consumer Expectations in 2016-2017. Jappelli and Pistaferri (2014) find an aggregate MPC of 0.48 using this Italian data and are able to identify clear differences across levels of liquid wealth. Fuster, Kaplan, and Zafar (2018) find a lower aggregate MPC in the NY Fed's survey, but find heterogeneity by both size a sign of the shock. While this method holds great promise, it is clearly limited by the accuracy of households' own response to the question.

[4] We should note that even lottery winnings have some problems. First the results hold for winners of the lottery who may not be representative of the wider population. Second the consumption response to a lottery win may be very different to other income shocks. For example you may spend a significant portion of a small lottery win just celebrating the fact.

Table 1 Estimates of the Marginal Propensity to Consume from Income Shocks

	Consumption Measure				
Permanent Shocks	Nondurables	Total PCE	Horizon	Method	Event/Sample
Blundell, Pistaferri, and Preston (2008)*	0.65		∼	1	Estimation Sample: 1980–92
Gelman, Gorodnichenko, Kariv, Koustas, Shapiro, Silverman, and Tadelis (2016)		1.0	∼	3	Gasoline Price Shock
Transitory Shocks					
Agarwal and Qian (2014)		0.90	10m	1	Growth Dividend Program Singapore 2011
Blundell, Pistaferri, and Preston (2008)*	0.05			3	Estimation Sample: 1980–92
Browning and Collado (2001)		∼ 0		1	Spanish ECPF Data, 1985–95
Coronado, Lupton, and Sheiner (2005)		0.36	1y	1	2003 Tax Cut
Fuster, Kaplan, and Zafar (2018)		0.08–0.31	3m	2	NY Fed Survey Cons. Expectations
Gelman (2016)		0.13	3m	1	Tax refunds 2013-2016
Hausman (2012)		0.6–0.75	1y	1	1936 Veterans' Bonus
Hsieh (2003)*	∼ 0	0.6–0.75		1	CEX, 1980–2001
Jappelli and Pistaferri (2014)	0.48			2	Italy, 2010
Johnson, Parker, and Souleles (2009)	∼ 0.25		3m	1	2003 Child Tax Credit
Lusardi (1996)*	0.2–0.5			3	Estimation Sample: 1980–87
Parker (1999)	0.2		3m	1	Estimation Sample: 1980–93
Parker, Souleles, Johnson, and McClelland (2013)	0.12–0.30	0.50–0.90	3m	1	2008 Economic Stimulus
Sahm, Shapiro, and Slemrod (2010)		∼ 1/3	1y	1	2008 Economic Stimulus
Shapiro and Slemrod (2009)		∼ 1/3	1y	1	2008 Economic Stimulus
Souleles (1999)	0.045–0.09	0.34–0.64	3m	1	Estimation Sample: 1980–91
Souleles (2002)	0.6–0.9		1y	1	The Reagan Tax Cuts of the Early 1980s

* Elasticity. Methods: 1) Natural Experiment 2) Survey question 3) Covariance restrictions

The third method is to impose covariance restrictions on panel data of income and consumption and use these to identify the consumption response to income shocks of differing persistence. The most well known of these is the paper by Blundell, Pistaferri, and Preston (2008) which uses imputed non-durable consumption data based on food expenditure reported in PSID data. They estimate a consumption elasticity (closely related to an MPC if households consume most of their income) and find very little consumption response to transitory shocks, but as discussed previously this estimate is strongly downward biased.

This paper also adds to the limited literature on consumption responses to permanent shocks to income. Natural experiments for permanent shocks are very hard to come by. Gelman, Gorodnichenko, Kariv, Koustas, Shapiro, Silverman, and Tadelis (2016) use shocks to gasoline prices as a proxy for a permanent shock to income and find an MPC close to 1 across the population. BPP find a consumption elasticity to permanent shocks to income around 0.65 (the permanent shock elasticity is less affected by the time aggregation problem). For a more complete overview of the literature on consumption responses to income changes, see Jappelli and Pistaferri (2010).

2 Empirical Strategy

2.1 Methodology Intuition

Our empirical strategy will be to impose some covariance restrictions on our data and apply GMM in order to estimate parameters of interest. While this strategy allows us to precisely estimate these quantities, in some ways it obscures from the key features of the data that are driving the result. In this section we present some very simple regressions of expenditure growth on income growth and compare them with what we would expect in some very well understood baseline models.

We will look at the estimate of β^N in the model

$$\Delta^N c_{it} = \alpha^N + \beta \Delta^N y_{it} + \varepsilon_{it}$$

where N, the number of years over which growth is measured, varies from 1 to 10. Our identification comes from the fact that transitory income shocks make up a relatively large proportion of the variance of income growth over a short period, and that permanent income shocks dominate the variance of income growth over a long period. Figure 1 shows what we would expect to see under three well known models, as well as what we actually observe the data. In a complete markets model in which all idiosyncratic shocks to income are insured against there is no relation between income growth and consumption growth, as represented by the blue horizontal line at zero. In the Solow growth model, and also some old Keynesian models, households' expenditure is a constant proportion of income that period, regardless of transitory shocks to income. The green horizontal line around 0.75 shows what we would see in a model of this type where households spend 75% of their income each period. The red line shows the results for a typical buffer-stock saving model. In this model the regression of consumption

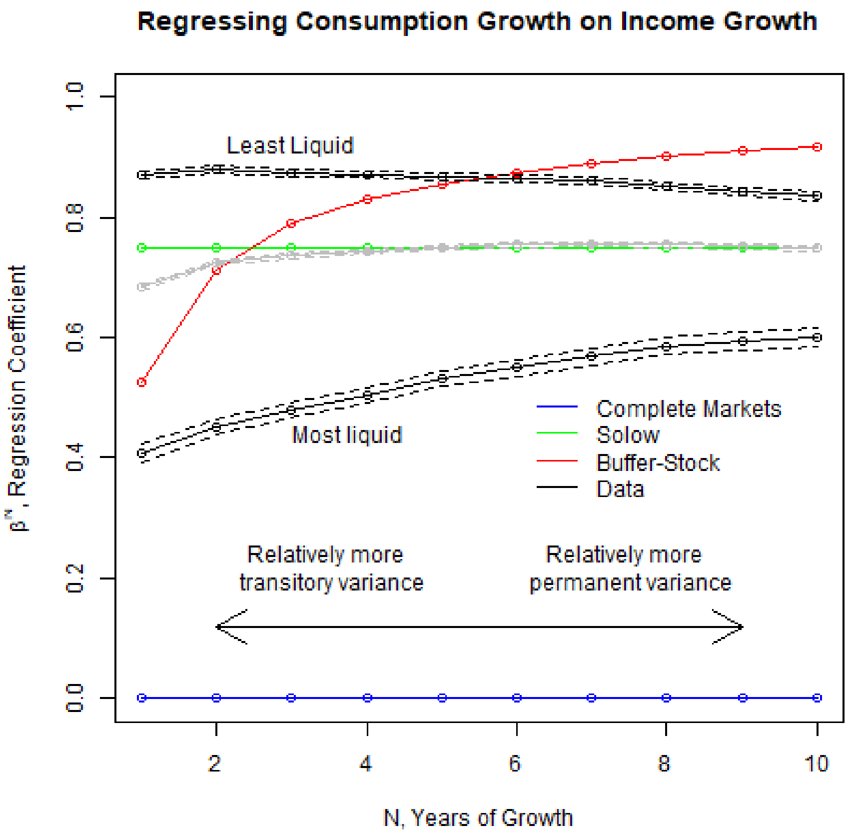

Figure 1 Regression coefficients of consumption growth on income growth

growth on income growth over one year yields a relatively small number as households are able to self-insure against the transitory shocks that dominate at this frequency. As the time period over which income growth is measured increases the observed income growth is proportionally more permanent and self-insurance is not possible. The red line asymptotes towards 1.0 as N gets large.

The gray line shows the results of these regressions using all households in the Danish sample. It is striking that the data appears to be closest to the Solow model, with only a small decrease in the regression coefficient over short periods. However, aggregating all households in this way hides a large degree of heterogeneity, particularly across households with different levels of liquid wealth. The two black lines shows the regression coefficients where the sample is restricted to households in the lowest and highest quintiles of liquid wealth (averaged over the observed period) respectively. For households in the lowest quintile there is no evidence of consumption smoothing. As the regression coefficient is relatively stable for this group over N, the result that the marginal propensity to consume out of both transitory and permanent shocks are similar and very high is robust to a large degree of model misspecification in the next section. Households in the top quintile of liquid wealth show a clear upward slope in figure 1, indicating a substantial degree of consumption smoothing. The fact that the regression coefficient for this group appears to asymptote well below 1 also suggests, in contrast to standard buffer-stock models, that the MPC out of permanent shocks for liquid households is significantly lower than 1.

2.2 Aside: Why Not BPP? A Brief Introduction to the Time Aggregation Problem

As explained above, our identification is going to come from the shape of income and consumption covariance over increasing periods of time. An obvious question is why we have chosen not to use the well known methodology of Blundell, Pistaferri, and Preston (2008) who achieve identification of transitory shocks from the fact that a transitory shock in period t will mean-revert in period $t+1$.[5] Unfortunately the method is not robust to the time aggregation problem of Working (1960). While macro-economists have long been aware of the importance of time aggregation in time series regressions (see Campbell and Mankiw (1989) for a well known example), the problem appears to have escaped the attention of the household finance and labor economics literature. We will therefore briefly describe the problem here. For a more detailed account with particular attention to BPP, see Crawley (2018).

Figure 2 shows how the problem arises. The solid 'Income flow' line shows the true income flow of a household who receives zero income throughout year 1, zero income for the first half of year two, and then a constant income flow of 1.0 per year during the second half of the second year and in the third year. The dashed line shows the observed *total* income of the household in years 1, 2 and 3: zero in year 1, 0.5 in year 2 and 1.0 in year 3. The important thing to note is that despite there only being one 'shock' to the income flow over the three year period, the naïve observer would assume there had been two shocks, one between years 1 and 2 and another between years 2 and 3. This effect is of particular importance to econometric techniques that make use of the auto-covariance structure of data processes. For example the first difference of a random walk in discrete time has no autocorrelation, but the first difference of a time-aggregated random walk in continuous time has an autocorrelation equal to $1/4$. BPP uses time aggregated income data and achieves identification of transitory variance precisely though the auto-covariance structure. This is why, as explained in the introduction, the problem is particularly pervasive for this methodology.

While it would be possible to stick very closely to the original BPP model and adjust the covariance restrictions to take account of the time aggregation problem,[6] we have found that in practice the underlying assumptions made by BPP (in particular that

[5] Kaplan and Violante (2010) show in discrete time simulations that the methodology works reasonably well for standard calibrations of buffer-stock models and end up concluding "The BPP insurance coefficients should become central in quantitative macroeconomics". However, some recent papers such as Commault (2017) and Hryshko and Manovskii (2018) have pointed to other potential problems of the methodology.

[6] Crawley (2018) takes this more straightforward approach using the same PSID data as used in BPP.

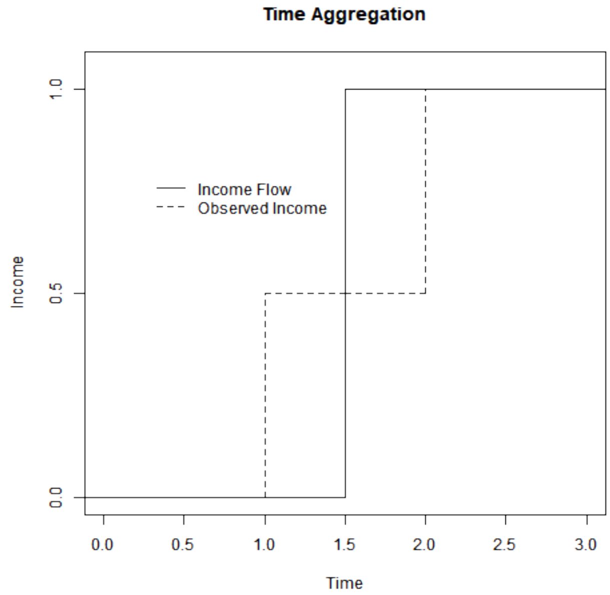

Figure 2 The Time Aggregation Problem

consumption follows a random walk) do not fit with the data.[7] Therefore we have chosen to attain identification in a manner similar to Carroll and Samwick (1997) which allows us to be agnostic about the exact short term dynamics of income and consumption.

2.3 Covariance Restrictions

2.3.1 Income Dynamics: Carroll and Samwick (1997)

Our identification of permanent and transitory income variance will follow the methodology of Carroll and Samwick (1997) closely. Our method will correctly account for time aggregation, but due to identification coming from income growth over 3, 4 and 5 years, rather than the covariance of income growth at time $t+1$ with time t as in BPP, time aggregation only introduces a small bias in the estimates of Carroll and Samwick (1997). We will first describe the method without time aggregation and then show how the estimates need to be adjusted.

Carroll and Samwick (1997) assume that income is composed of a permanent component that follows a random walk and a transitory MA(2) component. That is:

$$y_t = p_t + \varepsilon_t + \theta_1 \varepsilon_{t-1} + \theta_2 \varepsilon_{t-2}$$
$$p_t = p_{t-1} + \zeta_t$$

[7]Kaplan and Violante (2010) show that without time aggregation, the BPP method correctly identifies the transitory consumption response in the period of the income shock regardless of the consumption dynamics going forward. This fact is again not robust to the time aggregation problem. With time aggregation taken into account the estimates are highly sensitive to assumptions about short term consumption dynamics.

where ζ_t and ε_t are mean zero random variables, independent of each other and of themselves over time. Each has constant variance, σ_ζ and σ_ε respectively. For $N \geq 3$ we have:

$$\begin{aligned}\Delta^N y_t &= \zeta_t + \zeta_{t-1} + ... + \zeta_{t-N+1} \\ &\qquad + \varepsilon_t + \theta_1\varepsilon_{t-1} + \theta_2\varepsilon_{t-2} - (\varepsilon_{t-N} + \theta_1\varepsilon_{t-1-N} + \theta_2\varepsilon_{t-2-N}) \\ \Rightarrow \mathrm{Var}(\Delta^N y_t) &= N\sigma_\zeta + 2\underbrace{(1+\theta_1^2+\theta_2^2)\sigma_\varepsilon}_{\text{“Total” transitory variance}} \qquad \text{for } N \geq 3 \end{aligned} \tag{1}$$

Equation 1 shows that the variance of income growth grows linearly with the number of years of growth beyond 3. The transitory component adds variance at the beginning and end of the growth period, but any transitory shock to income that occurs in the middle of the period does not affect income growth as it will have died out by the end. Carroll and Samwick (1997) use this to identify the variance of permanent shocks ($\mathrm{Var}(\zeta)$) and the “total” transitory variance ($(1+\theta_1^2+\theta_2^2)\mathrm{Var}(\varepsilon)$). While similar to BPP it is important to note that BPP attempts to identify the variance of initial impact of the transitory shock, $\mathrm{Var}(\varepsilon)$, rather than the “total” transitory variance. While the notion of “total” transitory variance will carry over naturally into the time aggregated case, the variance of the initial impact does not have a natural interpretation.

The administrative data we use in this paper is at an annual frequency and measures the sum of income over the observed year. If shocks to income occurred only on 1st January every year then we could use equation 1 to identify permanent and transitory variance. It is important to distinguish between a model in which shocks happen about once a year (for example) but can occur at any point in the year, versus a model in which shocks to income happen on a timetable. The former can be modeled in continuous time with jumps occurring as a Poisson process approximately once a year. The later is best modeled as a discrete time model. In this paper we will take the former approach. While some types of jobs may have a regular schedule on which pay appraisals take place, many of the larger permanent shocks to income occur when a worker changes job which can occur at any point in the year. We (along with the literature) lack a clear understanding of what makes up the bulk of the transitory shocks to income and the Danish data is a potentially rich source for further research in this area.[8] Furthermore, even if each individual household experienced shocks on a pre-set timetable, if the timetable itself varies across the year for different households, our approach would yield unbiased results. While there is a big change in going from an underlying annual process to a quarterly process, the further change from quarterly to continuous time is much smaller.[9]. As the exposition is much simpler in continuous time, we will therefore chose to present our own method in continuous time.

To write the equivalent model in continuous time we define two underlying martingale processes (possibly with jumps), P_t and Q_t where P_t will represent the *flow* of permanent income at time t and the change in Q_t provides the transitory *impulses* that generate

[8]While we use annual income data in this paper in order to match with our expenditure data, monthly labor income data is collected, along with employer-employee matching.

[9]See Crawley (2018)

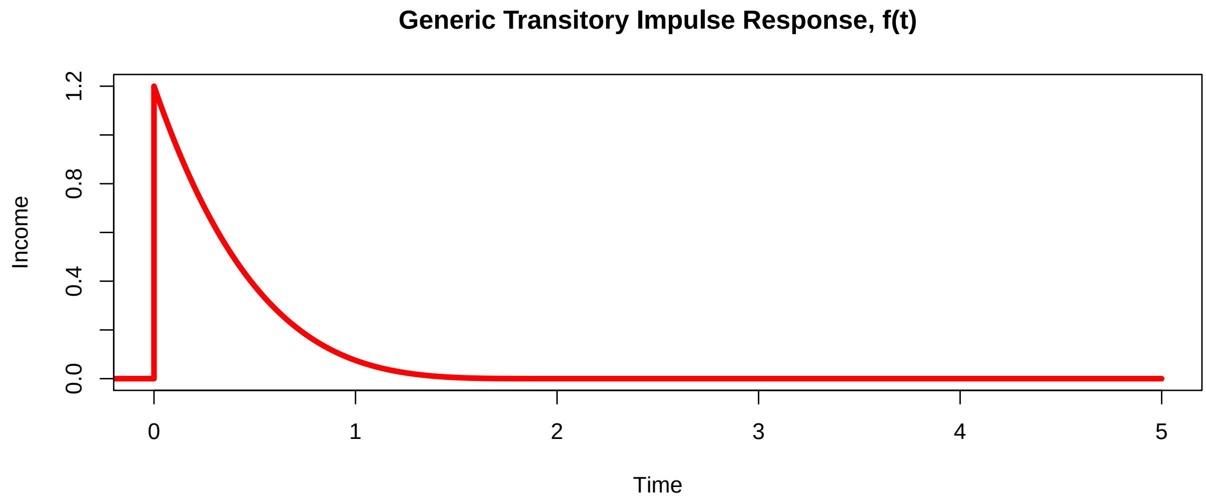

Figure 3 Generic Transitory Shock Impulse Response

the transitory income. We assume that for all $s_1 > s_2 > s_3 > s_4 > 0$:

$$\begin{aligned}
\mathrm{Var}(P_{s_1} - P_{s_2}) &= (s_1 - s_2)\sigma_P^2 \\
\mathrm{Cov}(P_{s_1} - P_{s_2}, P_{s_3} - P_{s_4}) &= 0 \\
P_s = 0 &\qquad \text{if } s < 0
\end{aligned}$$

and similarly for Q_t. That is these martingales have independent increments. As a useful benchmark, two Brownian motions satisfy these criteria.

The natural generalization of the MA(2) transitory income process from Carroll and Samwick (1997) is to allow for a generically shaped transitory income shock that decays to zero in under 2 years. Figure 3 shows an example of such a transitory income shape $f(t)$, but the model also allows for completely transitory shocks in which case $f(t)$ would be a delta function with all the income from the transitory shocks arriving as a mass at the time of the shock. In this model the *flow* of income arriving at time t is given by the flow of permanent income and the sum of income arising from any transitory shocks to income that have occurred in the previous two years:

$$y_t = P_t + \int_{t-2}^{t} f(t-s)dQ_s$$

We do not observe y_t directly but instead $\bar{y}_T$, the time aggregated income over each one year period.

$$\bar{y}_T = \int_{T-1}^{T} y_t dt \text{ for } T \in \{1, 2, 3...\} \tag{2}$$

Taking the N^{th} difference for $N \geq 3$ we get:

$$\Delta^N \bar{y}_T = \int_{T-1}^{T} y_t dt - \int_{T-N-1}^{T-N} y_t dt$$

$$= \int_{T-1}^{T} (T-s)dP_s + (P_{T-1} - P_{T-N}) + \int_{T-N-1}^{T-N} (s-(T-2))dP_s$$
$$+ \Big(\int_{T-1}^{T} \int_{t-2}^{t} f(t-s)dQ_t - \int_{T-N-1}^{T-N} \int_{t-2}^{t} f(t-s)dQ_t \Big) \quad (3)$$

The variance of time aggregated income of an N year period is therefore:[10]

$$\text{Var}(\Delta^N \bar{y}_T) = (N - \frac{1}{3})\sigma_P^2 + 2\text{Var}(\tilde{y}) \text{ for } n \geq 3 \quad (4)$$

This is similar to the non time aggregated case (equation 1) except the coefficient on permanent variance is $N - \frac{1}{3}$. This error, though having less serious consequences than for BPP, has nevertheless been overlooked by the large literature that studies income dynamics using panel data.[11] As with the MA(2) case the transitory variance identified is the variance of "total" transitory income received in the year, $\tilde{y}$, where this is defined as:

$$\tilde{y_T} = \int_{T-1}^{T} \int_{t-2}^{t} f(t-s)dQ_s dt \quad (5)$$

We will use equation 4 with $N \in \{3, 4, 5\}$.

2.3.2 Consumption Dynamics

Our approach will be to extend the identification of income variance by using growth over 3, 4 and 5 years to also identify the covariance of income and consumption. In contrast to BPP, which assumes consumption follows a random walk, we will instead assume that the impulse response to a transitory shock follows a generic path, $g(t)$, that like the transitory income shock has fallen to zero two years after the news of the shock. Figure 4 shows possible paths for both income and consumption, along with the alternative random walk impulse response of BPP. The best evidence for the speed at which the consumption response decays comes from Gelman (2016) and Fagereng, Holm, and Natvik (2016), both of which show the response has entirely or almost entirely decayed two years after the shock. In section **??** we will show how this assumption may potentially biases the transitory consumption response up but that this bias is small, especially for all but the most liquid households. We will maintain the assumption from BPP that the consumption response to a permanent shock to income follows a random walk proportional to the permanent shock. Under these assumptions the instantaneous flow of consumption is given by:

$$c_t = \phi P_s + \int_{t-2}^{t} g(t-s)dQ_s$$

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[10] See appendix A for full details of this derivation, including how we can approximate a log income process with levels.

[11] For examples see Moffitt and Gottschalk (2012), Meghir and Pistaferri (2004), Nielsen and Vissing-jorgensen (2004), Heathcote, Perri, and Violante (2010) and more recent quantile regression approaches such as Arellano, Blundell, and Bonhomme (2017).

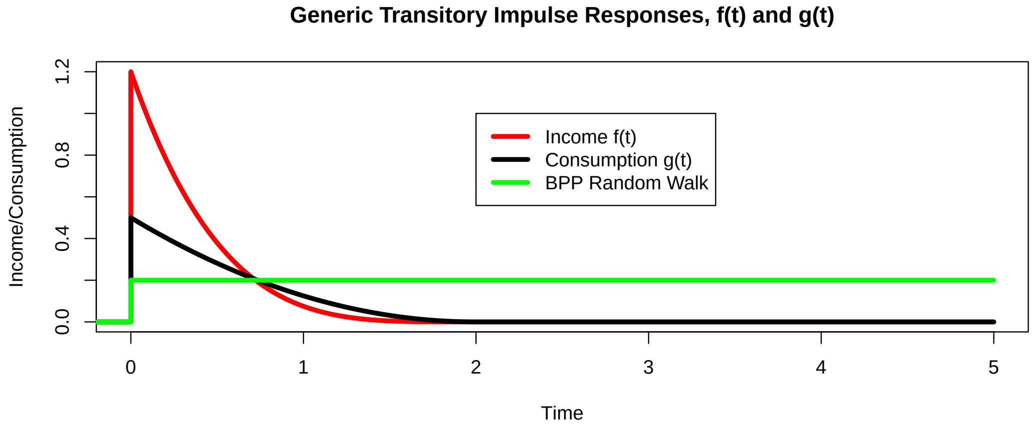

Figure 4 Generic Transitory Shock Impulse Response

and the covariance of time aggregated income and consumption growth over $N \geq 3$ years is given by

$$\mathrm{Cov}(\Delta^N \bar{c_T}, \Delta^N \bar{y_T}) = \phi(N - \frac{1}{3})\sigma_p^2 + 2\mathrm{Cov}(\tilde{c}, \tilde{y}) \text{ for } N \geq 3 \tag{6}$$

where total transitory income, $\tilde{y}$ is given by equation 5 and total transitory consumption, $\tilde{c}$, is defined by:

$$\tilde{c_T} = \int_{T-1}^{T} \int_{t-2}^{t} g(t-s) dQ_s dt \tag{7}$$

Using the equations for variance (4) and covariance (6) of income and consumption growth over N years for at least 2 different values of N, we are able to identify the 4 unknowns we are interested in:

1. σ_p^2 Variance of permanent shocks
2. $\sigma_{\tilde{q}}^2 = \mathrm{Var}(\tilde{y})$ Variance of transitory income received in a year
3. ϕ Marginal Propensity to eXpend (MPX) w.r.t permanent income
4. $\psi = \frac{\mathrm{Cov}(\tilde{c},\tilde{y})}{\mathrm{Var}(\tilde{y})}$ Regression coefficient of transitory consumption w.r.t transitory income over a year (MPX out of transitory income)

Our panel data covers 13 years and we choose to use growth over 3, 4 and 5 years to balance greater identification (longer growth periods give more power) with three identification problems that grow with N. The first is the fact that many households drop out of the sample if we demand they have reliable data for the too many consecutive years. The second is that if the permanent shock in fact decays slowly over time (e.g. is in fact AR(1)), the bias this introduces will be larger for large N. Finally, the validity of running the regressions in levels (rather than logs) is reduced over large N when the potential for the variance of income to change significantly from the start to end of the sample is high. In section **??** we test the importance of these issues. Using growth over 3 different time periods means the system is over identified with 6 equations with which to estimate these 4 unknowns. We follow BPP and use diagonally weighted minimum distance estimation, although our results are not significantly changed by using other popular weighting methods.[12]

As a lot of our analysis will focus on the parameter ψ is it worth describing exactly what this is and why we have labeled it the marginal propensity to expend out of transitory income. If we were able to exactly observe transitory income and consumption resulting from transitory income then ψ would be the regression coefficient of on this transitory consumption on transitory income. If transitory income shocks have no persistence this is approximately a six month MPX (on average the shock will happen six months into the year so that the regression will pick up the change in consumption in the following six months. If transitory income shocks have a little persistence (appendix B shows evidence of a small amount of transitory income persistence) ψ can only loosely be interpreted as the MPX to an income shock and the reader should bear in mind that the true interpretation is, "if income is higher by one unit this year due to transitory factors, then consumption this year will be expected to be higher by ψ units".

3 Data

Our panel data on income and expenditure comes from Danish registry data from 2003-2015. This data has a number of advantages over survey based measures. First, the sample contains millions of households rather than thousands. Second, households are required by law to report their data so there is much less risk of selection bias through drop outs. Third, measurement error in income data is largely eradicated, as employees' income data is third party reported by their employer, compared to survey data where self reported income has been shown to be particularly unreliable for irregular income.[13]

3.1 Income

We are interested in income and consumption decisions at the household level. We define a household as having either one or two adult members. Two adults are considered to be in the same household if they are living together and a) are married to each other or have entered into a registered partnership, b) have at least one common child registered in the Civil Registration System or, c) are of opposite sex and have an age difference of 15 years or less, are not closely related and live in a household with no other adults.[14]

[12]In general our results may be subject to misspecification problems, but the sample size of our data means that standard errors are small.

[13]See David, Marquis, Moore, Stinson, and Welniak (1997) for a survey of income measurement error issues in survey data.

[14]Adults living at the same address but not meeting one of the three criteria are regarded as separate families. Children living with their parents are regarded as members of their parents' family if they are under 25 years old, have never been married or entered into a registered partnership and do not themselves have children. A family meeting these criteria can consist of only two generations. If three or more generations live at the same address, the two younger generations are considered one family, while the members of the eldest generation constitute a separate family.

In the panel data, an individual's household will change if he or she gets married or divorced. This leads to some selection bias given that we require households to survive for at least 5 years. Following the literature our baseline results will be reported using the labor income of the head of household.[15] We will use after tax and transfer income as we are interested in the consumption response to these changes in income, although the method could be used to measure the extent of consumption insurance provided by the tax and transfer system. Our date comes from the adminstrative records from the tax authority. The tax reporting system in Denmark is highly automated and individuals bear little of the reporting burden. For employees income is reported by their employers and is thought to be highly accurate. The underground economy in Denmark is small. We remove business owners from the sample as their income may be less accurately reported, but more importantly, because the expenditure imputation method does not work well for them (see section 3.2).

We work with the residual of income after controlling for observable characteristics of households that may affect their income and consumption. To start we remove households in the top and bottom 1% of the income distribution. We then normalize by average household income over the observed period, and regress income on dummys for age, year, highest level of education, marital status, homeowner status and number of children along with interaction of age with education, marital status and homeowner status. We take the change in the residuals of this regression to be the unexpected income change for a household from one year to the next and remove households in the top and bottom 1% of the unexpected income *change* distribution.

3.2 Imputed Expenditure

Our expenditure data comes from imputing expenditure from income and wealth. Along with other Scandinavian countries, Denmark is unusual in that tax reporting includes information about wealth along with income, a legacy from the wealth tax that was phased out between 1989 and 1997. Following the methodology from Browning and Leth-Petersen (2003) and Fagereng and Halvorsen (2015) we impute expenditure using the identity:

$$\bar{C}_t \equiv \bar{Y}_t - \bar{S}_t = \bar{Y}_t - P_t - \Delta NW_t$$

where $\bar{C}_t$, $\bar{Y}_t$ and $\bar{S}_t$ are the sum of expenditure, income and savings over the year t respectively. P_t is contributions to privately administered pension schemes, for which we have very accurate data due to tax deductibility ΔNW_t is the change in (non-pension, non-housing) net worth measured at the end of years t and $t-1$. Banks and brokers are required to report the value of their clients' accounts on 31st December each year, and the the tax reporting year runs from 1st January to 31st December, so the data for income and wealth reported in the tax returns matches with that required to use this identity to impute consumption.

[15] See Moffitt and Zhang (2018) for an overview of the literature on income volatility in the PSID. In contrast the PSID literature we define the head of household as the highest earner over the 13 year period in our sample. We believe this definition better fits the social structure in Denmark.

The method works well for households with simple financial lives. One of the biggest problems with the method is its inability to handle capital gains well. The income used in the imputation includes all labor income and capital income, however it excludes capital gains. The value of assets will vary both due to savings from reported income but also due to capital gains and losses. We handle this in a number of ways. First, we completely exclude housing wealth from our measures of net worth and saving, treating housing as an off balance sheet asset. The problem with treating housing in this way is that we must exclude households in years in which they are involved in a housing transaction. For the self employed, it is also difficult to distinguish between expenditure and investment in their business, so we exclude all households who receive more than a trivial amount of their income from business ventures. Finally, households that hold significant equity investments are likely to see sizable capital gains and losses. We make a naive adjustment by making the assumption that they hold a diversified index of stocks. While this will likely lead to significant measurement error for these individuals, the concern is mitigated first by the fact that stock holding is much more unusual in Denmark than in the US for example. Only around 10% of households hold any stocks, and for many of those stocks make up only a small proportion of their total wealth. Furthermore, as we will explain in section **??**, measurement error in consumption is not a concern unless it is correlated with income. This seems unlikely to be the case, except for households that hold significant equity in the firms in which they work. Another concern with the imputation method is transfers of wealth, say between family members or friends. Indeed imputed expenditure is negative for approximately 3% of households and this may explain a proportion of that. We throw out both income and expenditure data for households in years in which their expenditure is negative. In section **??** we test the robustness of our results to sample selection bias problems that these issues may give rise to.

As with income, we work with the residual of expenditure after normalizing by mean household income and controlling for the same observable features as income. We follow exactly the same steps as described in section 3.1.

In evaluating how much we can learn from such a measure, it should be compared to the best alternatives available to economists. In the original BPP paper the authors only have access to food expenditures from the PSID data and impute total non-durable consumption by comparison with the Consumer Expenditure Survey. Self reported consumption is also notoriously poor quality even in comparison to self reported income. Furthermore, in the PSID data the questions in food expenditure are ambiguous as to which period exactly the question is referring to. A recent paper by Abildgren, Kuchler, Solange, and Sorensen (2018) shows that the mean levels of expenditure from this imputation method are close to those from the national accounts (see figure 5). They find large differences at the household level between the consumer survey and imputed expenditure although it is not clear that this is a problem with the imputation method as opposed to the survey measure. Indeed for car purchases, for which we have highly accurate register data, the consumer survey shows significant underreporting, consistent with Koijen, Nieuwerburgh, and Vestman (2014) who find 30% underreporting of car purchases in the Swedish consumer survey. We believe that, with the exception of

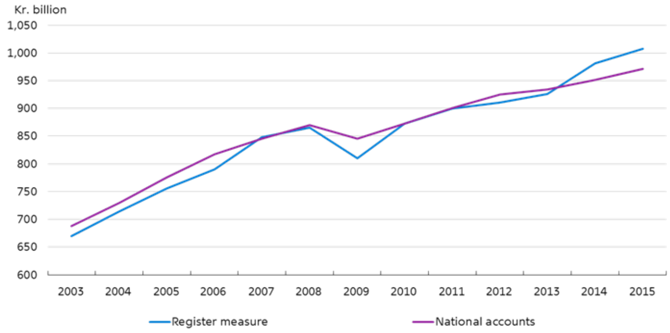

Figure 5 Imputed Register Measure and National Account Measure of Expenditure (from Abildgren, Kuchler, Solange, and Sorensen (2018))

transaction level data reported by financial aggregation applications, the imputation method we use is some of the highest quality expenditure data available to researches for the types of questions we are addressing.

3.3 Sample Selection

As our methodology requires income uncertainty to be relatively constant through the observed period[16] and the young and old are likely to have significant predictable income trends unobservable to the econometrician, we limit the sample to households headed by an individual between the age of 30 and 55 in 2008. Our final sample contains 7.7 million observations over 2004-2015 from an age group population total of 18.1 million. The selection criteria that reduces the sample size the most is the requirement that a household does not make a housing transaction for a period of 5 years. Table 3.3 shows summary statistics for all Danish households whose head fits into this age group as a whole as well as the sample we use in estimation. It is reassuring that both the mean and median values for after tax income and consumption are similar in the estimation sample and the population. Our estimation sample has much lower standard deviations as a mechanical result of excluding the top and bottom 1% of the income and consumption distributions which contains extreme values. Sample selection shows up in homeownership and car ownership as we exclude those households buy a house at the end of a five year period but who otherwise would be counted as renters. This also results in our sample being on average one year older than the population. Unhedged Interest Rate Exposure (URE) and Net Nominal Position (NNP) will be discussed in section 5, but again the significant differences here are due to the housing transaction criteria.

[16] Appendix B shows the assumption holds for this age group.

	Estimation Sample			Population (Age 30-55)		
	Mean	Median	Std Dev	Mean	Median	Std Dev
After Tax Income	59,261	57,804	28,819	58,312	53,304	68,799
Consumption	52,680	48,344	28,581	51,427	45,222	43,312
Liquid Assets	18,438	6,856	33,016	23,331	6,578	81,473
Net Worth	74,937	19,115	157,295	85,799	12,952	564,404
Homeowner	0.57	1.00	0.50	0.50	1.00	0.50
Car Owner	0.66	1.00	0.47	0.55	1.00	0.50
Higher Education	0.31	0.00	0.46	0.33	0.00	0.47
Age	43.5	44.0	7.1	42.5	42.0	7.3
URE	-28,052	-12,627	108,382	-44,134	-18,395	245,655
NNP	-109,685	-65,810	156,523	-158,321	-85,207	542,498
No. household-year obs		7,664,360			18,050,340	

Notes: Values are 2015 USD. Age refers to the age in 2008 of the main income earner in the household. For the purposes of calculation of consumption in the population, top and bottom 1% in terms of consumption have been excluded. URE and NNP can only be calculated in the period 2009-2015 due to mortgage information being insufficiently detailed in the previous years.

Table 2 Summary Statistics

4 Income and Consumption Characteristics by Household Wealth

Using our entire estimation sample we find a mean MPX out of transitory shocks of 0.50 and a mean MPX out of permanent shocks of 0.72. However, these averaged results hide a significant amount of heterogeneity. From the standpoint of consumption theory it is the ability of households to self-insure with their own wealth that most determines how much they smooth their consumption over shocks. We divide our estimation sample into quintiles according to both liquid wealth (which we define as bank deposits[17]) and net wealth. In each case wealth is measured as the mean household wealth holdings over the entire sample period.

Figure 6 shows the estimated income variances and MPX's for households in each quintile of liquid wealth. Looking at the left hand variance panel first, it is noticeable that income uncertainty, and particularly permanent income uncertainty, is highest for households in the lowest quintile of liquid wealth. This is some evidence towards the idea that heterogeneous tastes (e.g. discount factors of risk aversion) may be more important that income risk in determining wealth held for precautionary saving. For households in the top three quintiles of liquid wealth it is remarkable how similar their level of income risk is. Note that in contrast to standard estimates of the US income process, permanent income variance in Denmark is slightly higher than transitory variance, likely due to the high levels of social insurance available in Denmark. The variance level, at just over

[17]The results are little changed using any other definition of liquid wealth as long as housing and debts are excluded. See section ??.

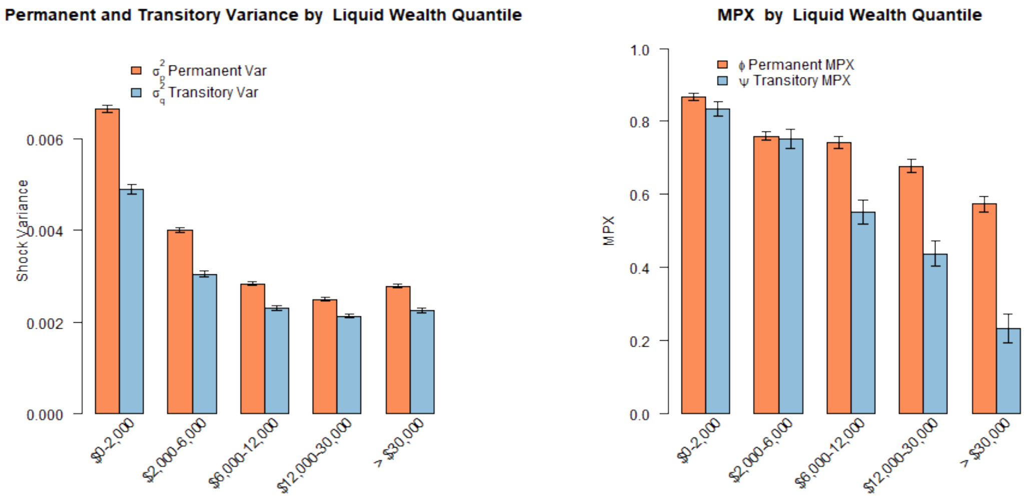

Figure 6 Variance and MPX by Liquid Wealth Quintile

0.002 for these top three quintiles, represents a standard deviation of just below 5% of permanent income per year.

Note that the estimates of income variance we obtain are highly sensitive to our treatment of outliers, but our MPX estimates do not change.[18]

The right hand panel of figure 6 shows our estimates for the MPX out of permanent and transitory shocks by liquid wealth quintile. The lowest wealth quintile, who hold less than \$2,000 in bank deposits on average over the sample period, look somewhat like hand-to-mouth consumers. They respond almost equally to permanent and transitory shocks, spending over 80% of income shocks in the year that it arrives. However, the fact that both permanent and transitory MPX's are very similar and significantly less than 1 suggests these households may be more accurately modeled as saving in an illiquid asset such as housing or a pension following a rule of thumb (say 20% of income) and then living hand to mouth on the remainder. As the quintile of liquid wealth increases, the MPX out of both transitory and permanent income decreases. In the top quintile, formed of households that maintained a mean bank balance above \$30,000, the MPX out of permanent shocks is 0.57 and out of transitory shocks 0.23. From the point of view of theory the responsiveness of spending our of permanent shocks in this quintile is low, while that of transitory shocks is high. A more thorough discussion of how these results compare to a standard model calibrated to Danish characteristics will wait until section 6.

Figure 7 shows the estimates for households grouped by quintiles fo net wealth. Here the pattern is slightly different. The quintile with the highest MPX out of both transitory and permanent income is the second lowest, the quintile that contains zero net worth. Households in the lowest quintile, those with over \$20,000 in net debt, do not seem

[18] See section ?? for evidence of this as well as a discussion of why this is the case.

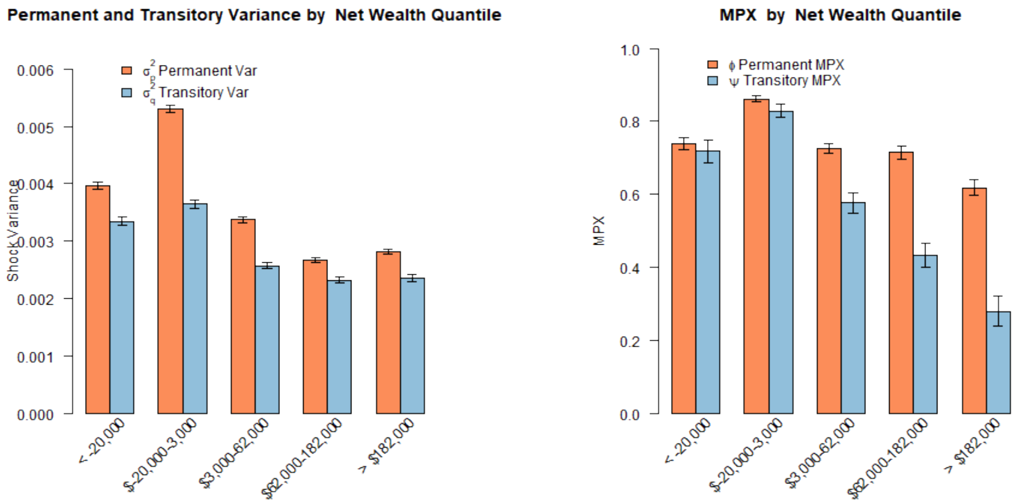

Figure 7 Variance and MPX by Net Wealth Quintile

to distinguish between permanent and transitory income shocks in their consumption responses, but their MPX for both is about 10 percentage points lower than the quintile with close to zero net wealth. The pattern for quintiles 3 to 5 looks similar to that for liquid wealth: the MPX out of transitory shocks falls sharply to around 0.28, while that out of permanent shocks also falls but more slowly to 0.62.

These results are broadly in line with the literature. The population mean of 0.5 for transitory MPX is a little higher than most estimates from table 1, but bearing in mind our estimate includes durables and is best compared to a six month MPC, it is certainly not an outlier. The MPX out of permanent shocks of 0.72 is also between the BPP estimate of 0.65[19] and the estimate of 1.0 from Gelman, Gorodnichenko, Kariv, Koustas, Shapiro, Silverman, and Tadelis (2016). The strength of the relationship between liquid wealth and MPC is similar to that found in Gelman (2016) and stronger than in Fagereng, Holm, and Natvik (2016).

5 Application: Monetary Policy and the Redistribution Channel

Auclert (2017) lays out a clear and intuitive theory as to how heterogeneity in the marginal propensity to consume out of transitory shocks affects the transmission mechanism of monetary policy. He identifies five channels through which monetary policy can act, three of which are mute in the absence of heterogeneity. He then uses this theory to identify a small set of sufficient statistics that help distinguish which of these

[19]The permanent ‘insurance’ coefficient estimated by BPP does not suffer as much from the time aggregation problem as the transitory coefficient.

channels are of quantitative importance. While these statistics in theory are highly informative about the transmission mechanism of monetary policy, in his paper he has neither the data nor the methods to be able to estimate them convincingly. He states, “As administrative quality household surveys become available and more sophisticated identification methods for MPCs arise, a priority for future work is to refine the estimates I provide here”. Given we have administrative data, along with a new method to estimate MPCs, a natural application of our work is to estimate Auclert’s sufficient statistics. Our data has two significant advantages over previous efforts at this.[20] First, our sample size is very large, containing a large percentage of all households in Denmark. Second, we have detailed balance sheet information for not only households within our sample, but also for those excluded from our sample. Furthermore, we are able to identify interest rate risk and nominal positions held by firms, foreigners and government so that the aggregate position is zero, as required in equilibrium. This allows us to avoid some of the more egregious assumptions used in aggregating household data.

5.1 Distribution of MPX across NNP, URE and Income

The redistribution effects of monetary policy depend crucially on two household characteristics, their Net Nominal Position (henceforth NNP) and Unhedged Interest Rate Exposure (henceforth URE).

- NNP is the net value of a household’s nominal assets and liabilities. It’s relevance for analyzing the redistributive effects of monetary policy comes from the fact that an unexpected rise in the price level will decrease the wealth of households with positive nominal assets, redistributing it to those with negative NNP (who now have less real debt). In administrative data we are able to observe directly held nominal positions at the household level, including bank deposits and loans, bond holdings and mortgages. In aggregate the directly held aggregate NNP position of the household sector is negative, which from the national accounts we will see is balanced by the financial sector as well as foreigners.

- URE measures the total amount that a household plans to save at the going interest rate that period. It is the difference between all maturing assets (including income) and liabilities (including planned consumption). For example, a household with a large variable rate mortgage will likely have very negative URE. For them the entire value of their mortgage will be adjusted to the new rate. When the interest rate rises for one period they will see their disposable income (after mortgage payments) go down, and hence if they have a high MPX their spending will also decrease. To calculate URE we assume all bank deposits and bank debt have a variable rate that changes instantaneously. For mortgage debt we directly observe the amount resetting over the following year. In Denmark mortgages tend to reset only in January and July,[21] so we assume that the new rate will only apply for half

[20] As well as Auclert (2017), a new version of Fagereng, Holm, and Natvik (2016) also attempts to estimate these statistics.

[21] See appendix C for more details on the Danish mortgage market.

the year. For all other assets and liabilities we assume a maturity of five years. As with NNP we find households on aggregate have a negative URE position in our data and this is counterbalanced by the interest rate position of the financial sector. See appendix ******** for more detail on how we calculate NNP and URE positions.

Figure 8 show how the MPX varies across household values for URE, NNP and Income. In each case the value on the x-axis has been divided by the mean level of consumption. As mortgages in Denmark are a mixture of fixed and variable rates (see appendix C for details on the Danish mortgage market), we can think of a typical household with negative URE or NNP as having a large mortgage, while those with positive URE or NNP are wealthy households with lots of liquid wealth. A clear pattern emerges that is not evident in previous attempts to measure the distribution of MPX across these dimensions. First, and most importantly for the theory, the average MPX for those with negative URE and NNP positions is significantly greater than those with positive URE or NNP. This confirms the intuition that households who owe a lot of floating rate debt have higher MPXs than those who own this debt, and leads to an interest rate exposure channel in which lowering interest rates increases expenditure. Second, the highest MPXs don't belong to houses with large debts but instead are a characteristic of households with close to zero position in URE and NNP. These households can be characterized as being poor hand-to-mouth as they have few if any assets and their consumption behavior is not affected by changes interest rates directly.[22] Note the mean levels of both URE and NNP are negative for the households in our estimation sample, so even a constant (positive) MPX would result in interest rate hikes reducing their expenditure if not balanced by indirectly held exposures.[23] While the distribution of MPX by income will not be central to our analysis, figure 8 shows a clear downward trend. If the income of lower income households decreases more than that of high income households during a monetary policy contraction, then expenditure will go down by more than the mean income weighted MPX that would be the result of a representative agent model. For comparison the distribution of MPX out of permanent income shocks across these dimensions can be found in appendix D.

5.2 Theoretical Setup and Sufficient Statistics

Auclert's method is to consider individual households' consumption response to a monetary policy shock in which i) the real rate of interest changes for one period by dR, ii) the price level makes a one time change of dP and then remains at the new level, and iii) aggregate income makes a transitory change of dY. While the dynamics here are clearly stylized, and in particular lack any lag in the economy's response, we believe such a simple experiment is highly informative as to the relative sizes of each transmission channel.

[22] Neither the interest rate exposure channel nor the intertemporal substitution channel will have much impact on their consumption. Monetary policy will impact their expenditure strongly through income effects.

[23] In contrast, Auclert finds a mean positive URE across households. We believe the difference is partly due to the prevalence of fixed rate mortgages in the US, but also due to underreporting of expenditures, especially in the PSID data.

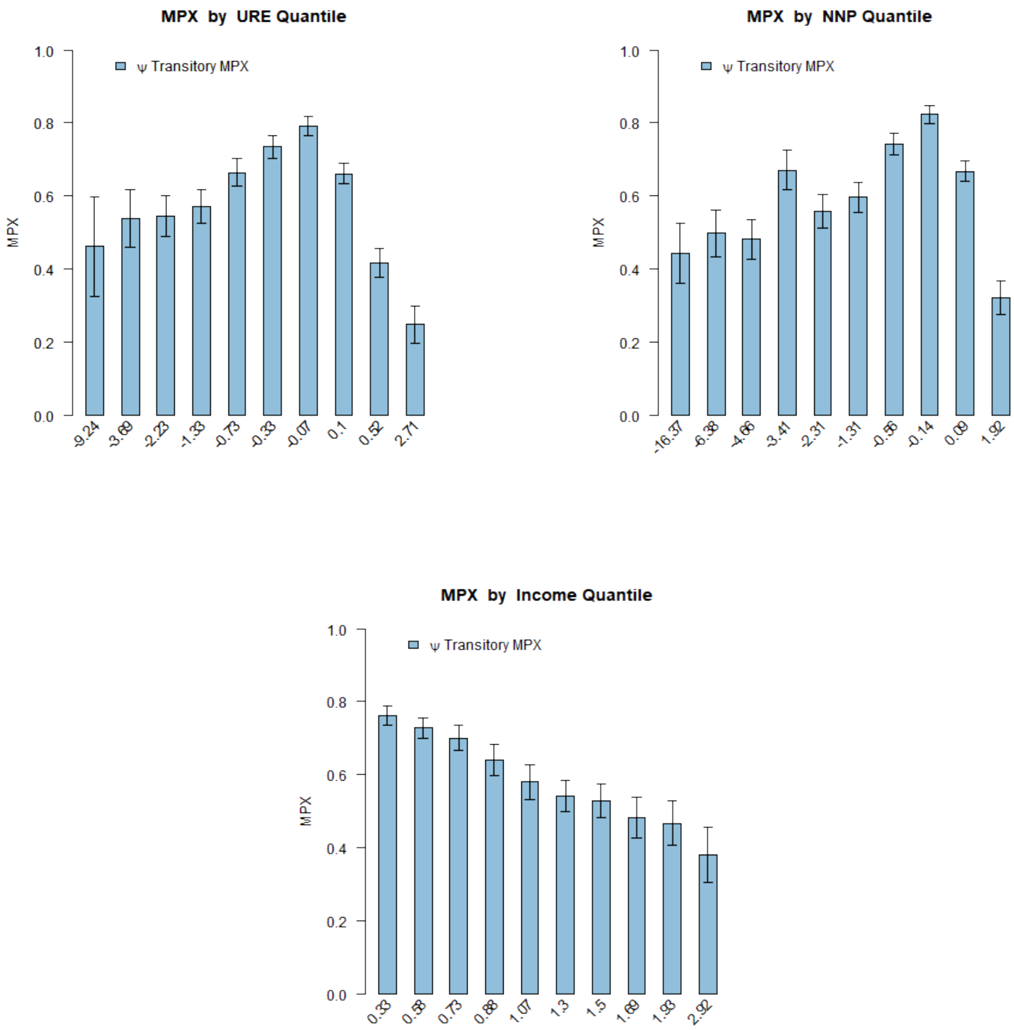

Figure 8 MPX Distribution by URE, NNP and Income

Auclert divides the effect of monetary on aggregate consumption into five distinct channels:

$$\frac{dC}{C} = \overbrace{\mathcal{M}\frac{dY}{Y}}^{\text{Aggregate Income Channel}} \overbrace{+\gamma\mathcal{E}_Y\frac{dY}{Y}}^{\text{Earning's Heterogeity Channel}} \overbrace{-\mathcal{E}_P\frac{dP}{P}}^{\text{Fisher Channel}} \underbrace{+\mathcal{E}_R\frac{dR}{R}}_{\text{Interest Rate Exposure Channel}} \underbrace{-\sigma\mathcal{S}\frac{dR}{R}}_{\text{Intertemporal Substitution Channel}} \tag{8}$$

where σ is the intertemporal elasticity of substitution, γ is the elasticity of relative income to aggregate income[24] and the five sufficient statistics, $\mathcal{M}$, $\mathcal{E}_Y$, $\mathcal{E}_P$, $\mathcal{E}_R$ and $\mathcal{S}$ are measurable in the data and defined in table 3. We chose to define these statistics to include the consumption effects coming from exposures not directly held by households. We allocate the aggregate URE and NNP exposure from our estimation sample into seven bins so that the total exposure across the economy is zero. These bins include households with (i) young (<30) and (ii) old (>55) heads, and exposures held by households indirectly through (iii) pensions funds, (iv) government, (v) non-financial corporates, (vi) financials and (vii) exposures held by the rest of the world. Within each of these bins we assume no heterogeneity so that the MPX with respect to these exposures is constant. This is a conservative assumption, likely to underestimate the size of the heterogeneous agent channels. Our assumptions on the level of these MPX's can be seen in table 4.

We define $\mathcal{E}_R$ as:

$$\mathcal{E}_R = \frac{1}{C}\left[\sum_{i \in \text{URE deciles}} \text{MPX}_i\text{URE}_i + \sum_{j \in \text{bins}} \text{MPX}_j\text{URE}_j\right] \tag{9}$$

where i sums over the ten deciles of URE, j over the seven bins defined above and C is aggregate household expenditure in the economy. This method of dealing with the fact that aggregate exposure does not equal zero in the estimation sample is different to the approach taken by Auclert. He assumes the residual exposure is distributed equally across households in the sample. By making use of the national accounts we believe we are able to get a better handle on the likely MPX's to attach to this residual exposure. Table 3 shows the definitions we use for each of the five measurable statistics in equation 8.

[24] Here we are making the simplifying assumptions that these quantities are common for all households, see Auclert (2017) for a discussion.

Table 3 Sufficient Statistics Definitions

Statistic	Definition	Description
$\mathcal{M}$	$\frac{1}{C}\left[\sum_{i\in\text{Income deciles}} \text{MPX}_i Y_i + \sum_{j\in\{\text{young,old}\}} \text{MPX}_j Y_j\right]$	Income-weighted MPX
$\mathcal{E}_Y$	$\mathcal{M} - \overline{\text{MPX}}\frac{Y}{C}$	Redistribution elasticity for Y
$\mathcal{E}_P$	$\frac{1}{C}\left[\sum_{i\in\text{NNP deciles}} \text{MPX}_i \text{NNP}_i + \sum_{j\in\text{bins}} \text{MPX}_j \text{NNP}_j\right]$	Redistribution elasticity for P
$\mathcal{E}_R$	$\frac{1}{C}\left[\sum_{i\in\text{URE deciles}} \text{MPX}_i \text{URE}_i + \sum_{j\in\text{bins}} \text{MPX}_j \text{URE}_j\right]$	Redistribution elasticity for R
$\mathcal{S}$	$1 - \frac{1}{C}\left[\sum_{i\in\text{Consumption deciles}} \text{MPX}_i C_i + \sum_{j\in\{\text{young,old}\}} \text{MPX}_j C_j\right]$	Hicksian scaling factor

Note: $\overline{\text{MPX}}$ is the mean MPX over all households in the economy. Y and C are aggregate household income and consumption respectively. bins refers to the seven categories for which we have allocated URE and NNP exposures outside our estimation sample. {young,old} are the two bins that contain young and old households (the other five bins only relevant for URE and NNP exposures as Y and C measure *household* income and consumption).

5.3 Out of Sample MPX

The assumptions we make about the MPX of the young and the old, as well as out of indirectly held URE and NNP exposures are shown in table 4. In each case we believe we have made conservative choices that will underestimate the size of the heterogeneous agent channels of monetary policy. For the young we choose an MPX of 0.5, in line with the rest of the population. As the young have aggregate negative exposures, choosing an MPX on the low side is conservative. Similarly for the old we choose an MPX of 0.2, on the low side, although the aggregate exposure for this group is close to zero. The assumption that there is no heterogeneity in MPX within these groups is also a very conservative assumption.

Much of the URE and NNP exposure is not held directly on the balance sheet of households, but instead indirectly through pension funds, corporates and the government. There is significant evidence that the MPX out of shocks to the value of pension wealth, stocks or the government balance sheet is substantially lower than the MPX from income. We choose to use the estimate from Maggio, Kermani, and Majlesi (2018) that households' MPX from changes in stock market wealth is about 10%. This choice is the most quantitatively important as the bin containing the most exposure is the financial sector, which is positively exposed to interest rate increases. This may seems surprising as banks are typically thought of having long term assets and short term debt that would result in negative URE exposure. However, our findings are in line with Landier, Sraer, and Thesmar (2013) who find the aggregate financial sector benefits from interest rate hikes, although there is a large amount of heterogeneity between different banks. An important caveat is due here: we focus on the MPX out of changes in the assets indirectly held by households through the financial sector and do not assume any spending or lending response at the bank level. While this may be a reasonable assumption in good times when banks are not credit constrained, it is especially not the case during a banking crisis. This could possibly result in monetary policy being much less effective during a banking crisis as the interest rate exposure channel to household spending is counterbalanced by a channel from bank balance sheet interest rate exposure to lending.

We choose an MPX of zero for government and the rest of the world. There is no evidence that households respond in any significant way to changes in the government's balance sheet, and furthermore a low MPX is a conservative assumption for the size of the heterogeneous agent channels. As Denmark is a very small part of the world economy we assume foreigners spend a negligible proportion of their wealth there.

Table 4 Aggregating Redistribution Elasticities

	MPX	NNP	URE	$\mathcal{E}_P$ component	$\mathcal{E}_R$ component
Estimation Sample	**See Distribution**	**-1,395**	**-338**	**-0.74**	**-0.35**
Young	0.5	-104	-38	-0.05	-0.02
Old	0.2	-295	-6	-0.06	-0.00
Pension Funds	0.1	716	143	0.08	0.02
Government	0.0	-598	-120	0.00	0.00
Non-financial Corp.	0.1	-334	-67	-0.04	-0.01
Financial Sector	0.1	1,533	380	0.16	0.04
Rest of World	0.0	227	45	0.00	0.00
Total		**-251**	**0**	**-0.66**	**-0.33**

Notes: NNP and URE numbers are in billions of 2015 Kr.

5.4 Results

Our estimates of the five sufficient statistics are shown in table 5. The aggregate income channel is summarized by $\mathcal{M}$ that we estimate to be 0.46. This means that if income for all households in the economy increased by 1%, aggregate consumption would increase by 46 basis points. This is broadly in line with calibrations of saver-spender models designed to fit evidence from Campbell and Mankiw (1989). We find no role for the redistribution effect of income, $\mathcal{E}_Y$, but this result is strongly influenced by our modeling choices for the old and the young. From figure 8 we can see a clear negative correlation between income and MPX. Using only our estimation sample yields an estimate for $\mathcal{E}_Y$ of -0.08. Either of these numbers suggest the earning's heterogeneity channel likely plays only a small role in the transmission mechanism. $\mathcal{S}$, the Hicksian scaling factor, is 0.56, which reduces the size of the intertemporal substitution channel by close to a half.

The two statistics of most interest are $\mathcal{E}_P$ and $\mathcal{E}_R$, both of which act through redistribution from households with low MPX to those with high MPX. $\mathcal{E}_P$ is estimated to be 0.66 suggesting that a one time increase in the price level of 1% increases aggregate consumption by 66 basis points due to redistribution from those with large nominal assets to those with large nominal debts. This Fisher channel of monetary policy is emphasized in Doepke and Schneider (2006). The interest rate exposure channel is also large. We estimate $\mathcal{E}_R$ to be 0.33 suggesting that a 1% increase in the interest rate increases expenditure by 33 basis points. For both of these channels, but particularly the interest rate exposure channel, it is informative to compare to the size of the intertemporal substitution channel. An increase in the real interest rate reduces aggregate consumption today by $\sigma\mathcal{S}$ multiplied by the percent change in the rate where σ is the intertemporal elasticity of substitution. Reliable estimates of σ have been elusive to the economics profession, but there is very little evidence of a large positive number. Reasonable estimates range from 0.1 to 0.5, resulting in the size of the intertemporal substitution channel being 0.05 to 0.26, the lower end of which is almost an order of magnitude smaller than our estimate of $\mathcal{E}_R$.

Table 5 Sufficient Statistics

$\mathcal{M}$	$\mathcal{E}_Y$	$\mathcal{E}_P$	$\mathcal{E}_R$	$\mathcal{S}$
0.46	-0.00	-0.66	-0.33	0.56

A long outstanding question in monetary economics is why monetary policy acts with a lag. Two competing theories are habits models such as Fuhrer (2000) and sticky information models such as Mankiw and Reis (2002). A recent paper by Carroll, Crawley, Slacalek, Tokuoka, and White (2018) finds evidence towards the idea that households react fast to their own idiosyncratic income shocks but news about macroeconomic shocks takes time to be absorbed. A possible third alternative to both of these is that households respond strongly to their realized income today, but not to income anticipated in the future. As it takes time for variable rate mortgages to reset (typically 6 months in Denmark), this would result in the interest rate exposure channel acting with a delay. Indeed the literature on consumption responses to transitory income shocks has generally found little difference between anticipated and unanticipated responses. Many of the estimates in table 1 use anticipated shocks (such as tax rebates) as an instrument and find large MPCs, suggesting households do not necessarily pay attention to anticipated cash flows until they arrive. A recent paper by Ganong and Noel (2017) shows this very clearly: there is a sharp consumption drop in the month that unemployment benefits expire, an entirely anticipated event. A model which takes these results seriously, along with a large role for the interest rate exposure channel of monetary policy, could be a fruitful area of future research.

6 Benchmark Model and Taste Shock Extension

In this section we calibrate a standard incomplete markets model to Danish characteristics, including the liquid wealth distribution in Denmark, and use it to see if i) we can match the consumption responses we measure in the data and ii) test if our methodology for extracting the MPX out of transitory and permanent shocks works well in this setting.[25] We show that our econometric method is downward biased for the

[25]This exercise is similar to that performed in Kaplan and Violante (2010) for the BPP methodology.

transitory MPX and upward biased for the permanent MPX, but that these biases are small when the MPX is large, as we see in the data.

Motivated by the fact that the standard model results in lower transitory MPX numbers than we find in the data, we make a simple extension to the model to account for potentially large preference shocks. We propose that such shocks, which have generally played a much smaller role in the literature than income shocks, are perhaps quantitatively more important for precautionary savings behavior.

6.1 Benchmark Model Calibrated to Danish Data

Our baseline model is the now very familiar buffer-stock saving model of Carroll (1997). Given market resources ($\mathbf{m}_t$), households in this model maximize expected utility:

$$\mathbb{E}_t \sum_{i=t}^{\infty} \beta^i u(\mathbf{c}_i)$$

subject to the constraints:

$$\begin{aligned}
\mathbf{a}_t &= \mathbf{m}_t - \mathbf{c}_t \\
\mathbf{b}_t &= R\mathbf{a}_t \\
\mathbf{y}_t &= \theta_t \mathbf{p}_t \\
\mathbf{p}_t &= \Psi_t \mathbf{p}_{t-1} \\
\mathbf{m}_t &= \mathbf{b}_t + \mathbf{y}_t
\end{aligned}$$

Where the felicity function, $u(\mathbf{c})$ is CRRA. We calibrate our model to match both the income uncertainty (as measured using our methodology) and the liquid wealth distribution in Denmark. To be able to match the liquid wealth distribution, especially at the low end, we follow Krusell and Smith (1998) and Carroll, Slacalek, Tokuoka, and White (2016) and allow for ex-ante heterogeneity in the discount factor β. Specifically an agent i has a discount factor β_i where β_i is i.i.d across agents and follows a uniform distribution between β_{low} and β_{high}. These two parameters allow us to match the fact that while the mean level of liquid assets is high, about half of all households have close to zero liquid assets. Matching the lower part of this distribution is critical to generate transitory consumption elasticities substantially above zero. The Lorenz curve for liquid assets, both in the data and in the model, is shown in figure 9

6.2 Model with Preference Shocks

The baseline model exhibits two features in tension with the data. First, the marginal propensity to consume out of transitory income shocks, while exhibiting the right shape relative to the liquid wealth, is too low relative to the data. Second, as would be expected in a consumption smoothing model like this, the path of expenditure is significantly less volatile than income. This is strongly at odds with the data which show the standard deviation of changes in expenditure to be around 0.37, compared to 0.12 for income. There is very little evidence on the true size of expenditure shocks, partly because of

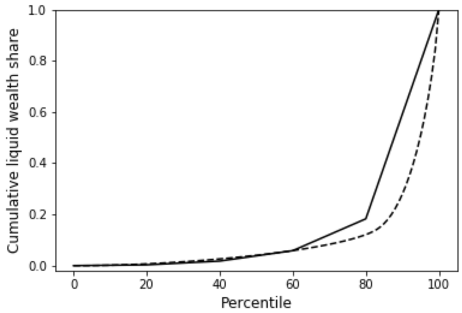

Figure 9 Lorenz Curve for Danish Liquid Assets

large measurement errors known to be present in consumption survey data. While we believe the 0.37 number from our data also contains measurement error, as well as large expected expenditures such as new cars for which financial may be readily available, it seems likely that the expenditure shocks could be large. Indeed typical financial advice to maintain a buffer stock will mention unexpected costs such as medical bills or a leaky roof before income shocks.[26] A simple tweak to the baseline model can help the model fit the data along both these dimensions. To achieve this we add a preference shock to expected utility:

$$\mathbb{E}_t \sum_{i=t}^{\infty} \beta^i \mathcal{X}_i u(\mathbf{c}_i)$$

where $\mathcal{X}_i$ is i.i.d. and calibrated such that the average MPX for the entire sample matches the data.

6.3 MPX by Liquid Wealth

The top panel of figure 10 shows how the transitory MPX of the two models compares with the data. While the fact that the MPX decreases with liquid wealth quintile is robust in both models and in the data, there are two features worth noting. First, large preference shocks are required to push the transitory MPX close to the levels we see in the data (in our example preference shocks have an annual standard deviation of 0.3 *****NEED TO CHECK THIS****). Second, neither of the two models is able to explain the high MPX out of transitory shocks that we observe for the top quintile of liquid assets. These households, who hold a mean balance above $30,000, appear very responsive to transitory shocks despite their large buffer stock they could potentially use to smooth income shocks.

The bottom panel of figure 10 shows another failure of both these two simple models, neither is able to capture the fact that the consumption response to permanent shocks is substantially below 1, even for middle and low quintiles of liquid wealth. A more

[26]For example Forbes magazine in 2016 suggests "you could find yourself thrown off by a chipped tooth or fender bender. So having an emergency fund padded with nine months of the highest earner's net income may help give you a bit more peace of mind that you could weather a financial storm."

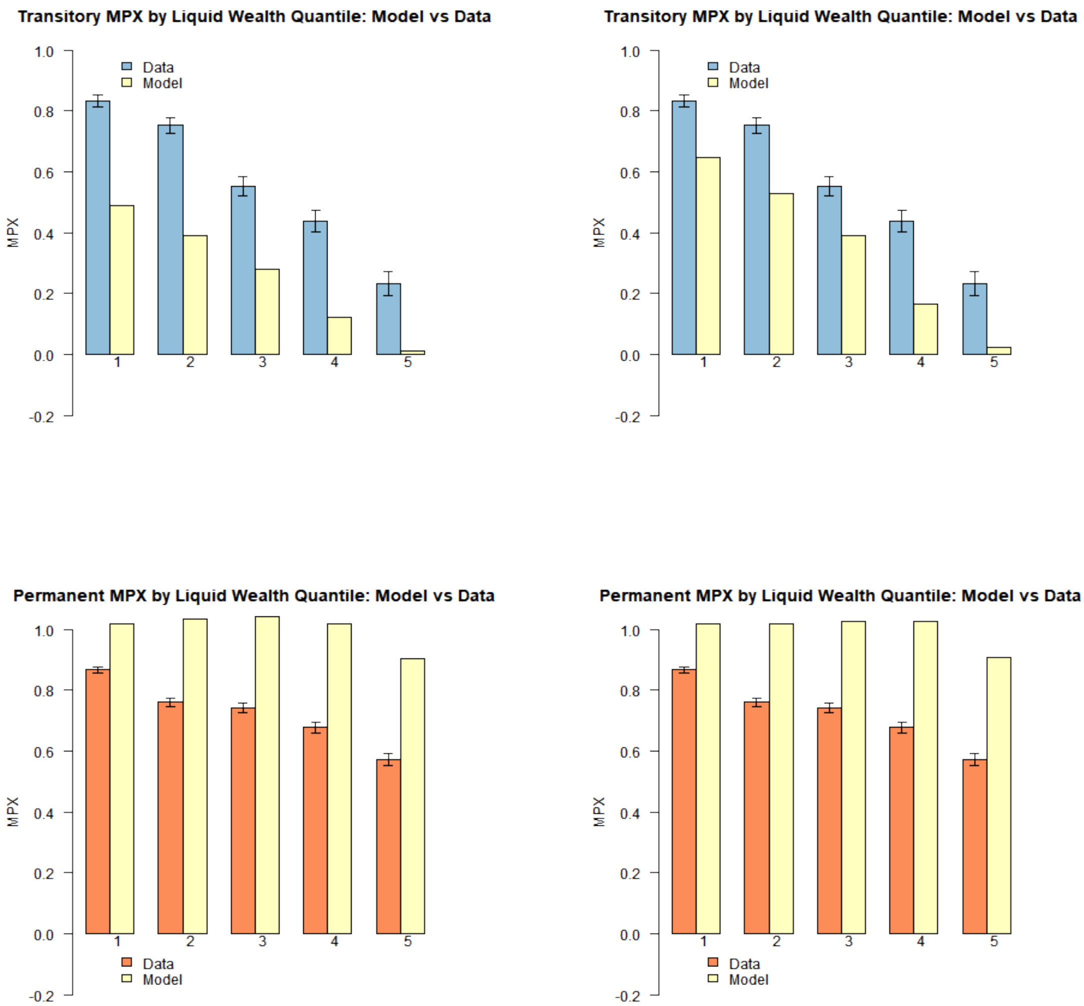

Figure 10 Baseline model (LHS) and Preference Shock Model (RHS) with the Data

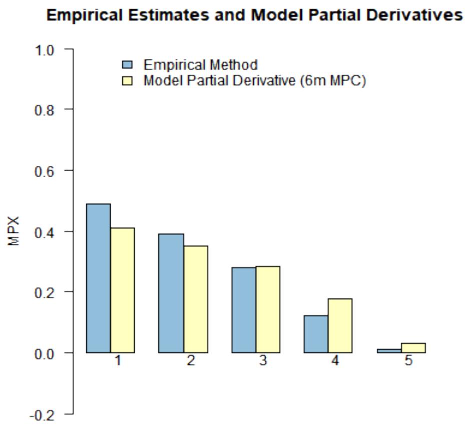

Figure 11 Empirical Method on Simulated Data versus Partial Derivative

sophisticated model which contains both an illiquid asset and a lifecycle component may do better along this front.

6.4 Performance of the Empirical Method

Using the model we are able to calculate precisely the partial derivative of expenditure with respect to transitory income. To be comparable to the time period of our empirical MPX we take the mean MPX over 1,2,3 and 4 quarters:[27]

$$\mathrm{MPX}_{\mathrm{model}} = \frac{1}{4}\sum_{i=0}^{3}(1-\mathrm{MPX}_q)^i \mathrm{MPX}_q$$

where MPX_q is the partial derivative in the quarterly model.

Figure 11 shows that the method performs well when the MPX is high, but overestimates the MPX when it is low. This is a direct result of the assumption that the consumption response to a transitory shocks decays within a two year period. This is a close approximation to the truth when the MPX is high, but the consumption response, which decays as an AR(1) in this model, can last longer when the MPX is low. While this is a problem for our method at the high end of the wealth distribution in these models, Fagereng, Holm, and Natvik (2016) shows strong evidence that the consumption response for the wealthy decays a lot faster than an AR(1) process would suggest. Their impulse response graphs, identified using lottery winnings, show that despite high initial consumption responses of the wealthy, after two years their spending has returned close to where it would have been had they not won a lottery prize. Such

[27] Remember our empirical method measures the covariance of income with expenditure in the same calendar year. If the shock happens in the first quarter, then we will count expenditure over the next four quarters. If the shocks happens in the final quarter, then only one quarter of expenditure will be captured.

behavior among the wealthy would suggest our empirical method works better for these groups in the data than it does in this simulated setting.

7 Threats to Identification

7.1 Durables

A critique of our empirical methodology is that it does not take account of durable goods, while our data includes all spending (except on real estate) and therefore includes large and durable goods such as cars and home improvements. The empirical model assumes that in response to a transitive income shock, expenditure increases temporarily for up to two years. This is entirely consistent with a model that includes durable goods. However, the model assumes that in response to a permanent shock to income, expenditure increases once to a new permanent level. A model that included permanent goods would instead imply a large one off expenditure on durable goods to get the household up to their desired stream of durable good services, followed by a decrease back to a permanent level of spending that accounts for replenishing the higher level of depreciating durable goods.

To make this idea more explicit, it will help to write down a simple model. The model will show that our empirical methodology continues to estimate the consumption response to permanent and transitory shocks, but that these need to be interpreted carefully. The model uses the same income process as section 2.3. Remembering the income process is made up of two martingale processes, P_t and Q_t, which may have jumps. Instantaneous income is given by:

$$dy_t = \Big(\int_0^t dP_s\Big)dt + dQ_t$$

while instantaneous expenditure now has both a durable and non-durable component:

$$dc_t = \phi_{nd}\Big(\int_0^t dP_s\Big)dt + \phi_d dP_t + \psi dQ_s$$

Here we have assumed that the expenditure response to transitory shocks is instantaneous, but it would not change things to assume as before that the response decays to zero after two years. However, it is important that the durable component of the expenditure response to permanent shocks occurs instantaneously with the shock (or very soon after). Aggregating income and consumption annually gives:

$$\begin{aligned}
\Delta^N \bar{y}_T = &\Big(\int_{T-N-1}^{T-N}(s-(T-N-1))dP_s + \int_{T-N}^{T-1} dP_s + \int_{T-1}^{T}(T-s)dP_s\Big) \\
&+ \Big(\int_{T-1}^{T} dQ_t - \int_{T-N-1}^{T-N} dQ_t\Big) \\
\Delta^N \bar{c}_T = \phi_{nd}&\Big(\int_{T-N-1}^{T-N}(s-(T-N-1))dP_s + \int_{T-N}^{T-1} dP_s + \int_{T-1}^{T}(T-s)dP_s\Big)
\end{aligned}$$

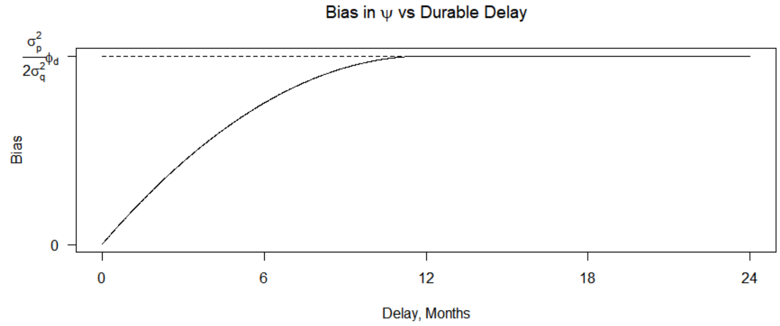

Figure 12 Bias in Transitory MPX with Delay in Durable Goods Purchase

$$+\phi_d\Big(\int_{T-1}^{T} dP_t - \int_{T-N-1}^{T-N} dP_t\Big)$$
$$+\psi\Big(\int_{T-1}^{T} dQ_t - \int_{T-N-1}^{T-N} dQ_t\Big)$$

From this we can calculate the covariance:

$$\begin{aligned}\text{Cov}(\Delta^n \bar{c_T}, \Delta^n \bar{y_T}) &= \phi_{nd}\text{Var}(\Delta^n \bar{y_T}) \\ &\quad + \phi_d\left(\int_{T-1}^{T}(T-s)\sigma_P^2 dt - \int_{T-N-1}^{T-N}(s-(T-N-1))\sigma_P^2 dt\right) \\ &\quad + \psi\left(\int_{T-1}^{T}\sigma_Q^2 dt + \int_{T-N-1}^{T-N}\sigma_Q^2 dt\right) \\ &= \phi_{nd}(n-\frac{1}{3})\sigma_P^2 + 0 + 2\psi\sigma_Q^2\end{aligned}$$

So the durable component of the covariance cancels out and our identification method correctly identifies ϕ_{nd} and ψ, but is unable to identify ϕ_d.

However, if there is some delay between the household receiving the permanent income shock and purchasing the durable goods, then this introduces an upward bias into the estimate of transitory MPX. The size of the bias grows with the number of months delay between the permanent income shock and the durable goods purchase, plateauing after twelve months at a level of $\frac{\sigma_p^2}{2\sigma_q^2}\phi_d$. Figure 12 shows how this bias increases with the delay.

In order to quantify how large this bias may be in practice we make use of the car registry data available in Denmark. Using data on the current value of cars owned by a household, we perform the same residual calculation to find the change in car value that is unpredictable with the household characteristics we are able to observe. We then construct two new expenditure panels, one in which we remove expenditures on cars and a second in which we make a proxy for non-durable consumption by removing expenditures on cars multiplied by $\frac{1}{0.421}$ (Car purchases make up 42.1% of durable

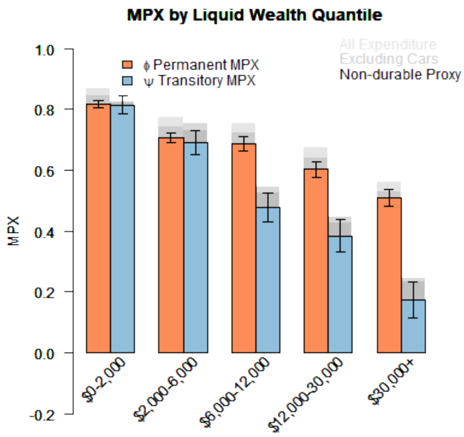

Figure 13 MPX Removing Cars and Using the Non-durable Proxy Panel

expenditure in Denmark).

$$C_T^{\text{nocar}} = C_T - \Delta\text{CarValue}$$
$$C_T^{\text{nondurable}} = C_T - \frac{1}{0.421}\Delta\text{CarValue}$$

The second, non-durable proxy consumption panel, can be modeled as the true non-durable consumption panel with classical measurement error added. This classical measurement error does not bias our estimates, so we can use this non-durable proxy panel to estimate an unbiased MPC out of transitory shocks, where the MPC does not include durable expenditures.

The results of this exercise can be seen in figure 13. Even without bias, we would expect the non-durable proxy estimates to be lower than those including all expenditures as the definition of transitory MPX changes over the three panels to exclude cars and then all durable goods. For the lower quintiles of liquid wealth it therefore looks as though the bias is likely very small, as non-durable goods make up 10% of spending and the MPX estimates are smaller by an amount in this region. For the top quintile of liquid wealth there looks to be some bias, with the estimate of MPX for all expenditures decreasing from 25% to an MPC for non-durable goods of 17%.

While there is some evidence that our results may be biased up for those in the top quintiles of liquid assets, this bias will only have a small effect on our overall conclusions. As the relevant number for the monetary policy exercise is the MPX rather than the MPC, we have chosen not to adjust our baseline results using this method and accept that a small bias may exist in our data. It should be noted that such a bias will cause the heterogeneous channels of monetary policy to appear smaller than they actually are.

7.2 Labor Elasticity

The empirical results of this paper suggest that the consumption response to a transitory shock to income lies at the high end of the existing literature. In this section we try to reconcile the results we get with the existing literature by asking if we can build a model in which our empirical method would estimate much larger MPXs than more traditional approaches to measuring the marginal propensity to consume out of transitory shocks. The intuition we build upon is that the households may wish to work longer hours, and hence earn more, in years when their expenditure is particularly high. If this were the case the income process would not be well modeled as being exogenous and our method would have a reverse causality problem embedded in it. Here we build such a model and see how much reverse causality can plausibly contribute to the results we obtain. The model extends the standard incomplete markets model from section 6, incorporating both preference shocks, so that households have some years when their utility of consumption is greater than others, and labor elasticity, so that households can adjust their income based on the marginal utility of consumption. The household's problem is to maximize expected lifetime utility:

$$\mathbb{E}_t \sum_{n=t}^{\infty} \beta^n \left(\mathcal{X}_n \frac{\mathbf{c}_n^{1-\rho}}{1-\rho} - \frac{\boldsymbol{\ell}_n^{1+\frac{1}{\xi}}}{1+\frac{1}{\xi}} \right)$$

subject to the constraints:

$$\begin{aligned}
\mathbf{a}_t &= \mathbf{m}_t - \mathbf{c}_t \\
\mathbf{b}_t &= R\mathbf{a}_t \\
\mathbf{y}_t &= l_t w_t \\
\boldsymbol{\ell}_t &= l_t \mathbf{p}_t^{\frac{1-\rho}{1+\frac{1}{\xi}}} \\
w_t &= \theta_t \mathbf{p}_t \\
\mathbf{p}_t &= \Psi_t \mathbf{p}_{t-1} \\
\mathbf{m}_t &= \mathbf{b}_t + \mathbf{y}_t
\end{aligned}$$

The normalization of labor ($\boldsymbol{\ell}_t = l_t \mathbf{p}_t^{\frac{1-\rho}{1+\frac{1}{\xi}}}$) is set up to allow labor supply to move elastically with transitory income, but the long run supply of labor does not depend on permanent income (as observed in the consistency of hours worked over long time periods and across countries). The key additional features of this model are i) the preference shock factor and ii) the elasticity of labor.

Labor elasticity is controlled by the Frisch elasticity ξ. When the wage (relative to permanent income) increases by x%, hours worked increase by ξ%. Estimates of the Frisch elasticity in micro-data studies range from 0 to 0.5, while macroeconomic studies generally find a much larger elasticity of between 2 and 4 (see Peterman (2016)). We will study a range of elasticities between 0 and 1, easily covering the microeconomic estimate range. We will not consider estimates of the Frisch elasticity in the macroeconomic range as it seems likely to us that these estimates are high due to labor market frictions over the business cycle, rather than genuine labor supply choices of households.

	β	Frisch Elasticity				
		0.00	0.13	0.25	0.38	0.50
	0.00	0.99	0.99	0.99	0.99	0.99
Preference shock	0.10	0.99	0.99	0.99	0.99	0.99
	0.20	0.98	0.98	0.98	0.98	0.98
	0.30	0.97	0.98	0.98	0.98	0.98
	0.40	0.96	0.96	0.97	0.97	0.98

	σ_q	Frisch Elasticity				
		0.00	0.13	0.25	0.38	0.50
	0.00	0.07	0.07	0.06	0.05	0.05
Preference shock	0.10	0.07	0.07	0.06	0.06	0.05
	0.20	0.07	0.07	0.06	0.06	0.05
	0.30	0.07	0.07	0.06	0.05	0.05
	0.40	0.07	0.07	0.06	0.05	0.04

Table 6 Fitted discount factors and transitory shock standard deviation

	ϕ	Frisch Elasticity				
		0.00	0.13	0.25	0.38	0.50
	0.00	1.00	1.00	0.99	0.99	0.99
Preference shock	0.10	1.00	1.00	1.00	1.00	0.99
	0.20	1.02	1.01	1.01	1.00	1.00
	0.30	1.03	1.02	1.01	1.00	1.00
	0.40	1.03	1.02	1.01	0.99	0.98

	Std($\Delta \log c$)	Frisch Elasticity				
		0.00	0.13	0.25	0.38	0.50
	0.00	0.05	0.05	0.05	0.05	0.05
Preference shock	0.10	0.08	0.08	0.08	0.08	0.08
	0.20	0.13	0.13	0.13	0.13	0.13
	0.30	0.18	0.18	0.18	0.18	0.19
	0.40	0.23	0.23	0.24	0.24	0.24

Table 7 Simulation estimates of ϕ and consumption growth standard deviation

	MPC	Frisch Elasticity				
		0.00	0.13	0.25	0.38	0.50
	0.00	0.17	0.13	0.11	0.09	0.08
Preference shock	0.10	0.19	0.15	0.13	0.11	0.09
	0.20	0.25	0.20	0.16	0.13	0.11
	0.30	0.32	0.26	0.21	0.17	0.14
	0.40	0.38	0.31	0.25	0.20	0.16

	ψ	Frisch Elasticity				
		0.00	0.13	0.25	0.38	0.50
	0.00	0.07	0.05	0.04	0.04	0.03
Preference shock	0.10	0.09	0.07	0.06	0.05	0.05
	0.20	0.14	0.12	0.11	0.12	0.13
	0.30	0.20	0.20	0.23	0.28	0.32
	0.40	0.27	0.33	0.43	0.54	0.64

Table 8 Simulation estimates of 6 month MPC and ψ

In tables 6, 7 and 8 we have varied the size of the Frisch elasticity and annualized preference shock. In each cell we have kept constant the overall annualized income growth variance and the median liquid asset to annual income ratio (equal to 0.2 in the data). To achieve this we vary the discount factor and the variance of transitory wage shocks.

Table 6 shows how the discount factor, β and the annualized transitory shock standard deviation vary. As the size of the preference shocks increase, so does the precautionary motive for households. As we have fixed the median amount of precautionary savings, the discount factor drops significantly to compensate. This effect is most prominent when there is no labor elasticity. As labor elasticity increases households are able to insure themselves against preference shocks with their labor supply, so the change in discount rate is less pronounced. The right hand panel shows the standard deviation of transitory shocks required to match the overall level of income growth variance goes down as labor supply elasticity increases. This is as expected - when the transitory wage is low households will work fewer hours. This amplifies the variance of the transitory income shock relative to the wage shock. The size of the preference shocks have little effect on the imputed size of the transitory shocks, except when both the preference shock and the labor elasticity are large. At this point the preference shocks themselves induce significant changes in hours worked and hence income, requiring less exogenous variance in income to match the total income variance target.

The left hand panel of table 7 shows the estimate of ϕ (the MPX out of permanent shocks) is close to 1 for variations of preference shocks and labor elasticities. This is unsurprising as labor does not respond to a change in permanent income. The right hand panel shows a very significant increase in the standard deviation of consumption growth as the size of the preference shocks increases. With no preference shocks, the standard deviation of consumption growth (0.05) is about half of the standard deviation of income (0.12). As the size of preference shocks increases, so does consumption growth variance, with the standard deviation growing to 0.23 for large preference shocks. This is still much smaller than 0.37, which comes directly from the data, although this high number from the data is likely to be contaminated with measurement error in assets. A further consideration is that much of the observed variance in expenditure growth will be due to durable items, such as home improvements and vehicles. We analyze the effect of durables on our estimates in section 7.1, but to the extent that these goods can be financed, our model with no borrowing may overestimate both the expenditure variance and the labor supply response to preference shocks.

Table 8 compares the actual mean six month MPC in the model with our empirical method for estimating the transitory expenditure elasticity. While these concepts are clearly not equivalent, comparing the two gives a good understanding as to what might be going on. The left hand panel shows that both preference shocks and labor elasticity, often both missing in consumption models for simplicity, have quantitatively significant impacts on the implied marginal propensity to consume. Increasing the Frisch elasticity from 0 to 0.5 (the full range of micro-estimates) decreases the six month MPC from 17% to just 8%. This is because households now have an extra tool with which to insure against low consumption. When they receive a negative transitory shock to their wealth, they will consume less, which in turn will increase their marginal utility of consumption and induce them to work more hours. Therefore their actual income loss will be lower than the shock to their wealth and they will reduce their consumption by less than if they were unable to adjust their labor supply. In contrast, increasing the size of the preference shocks greatly increases the marginal propensity to consume. This is a result of the higher precautionary savings motive and consequently lower discount factor, even while median savings are unchanged. Households with a large positive preference shock will have very little motive to save, as with such low level of savings they will soon get close to the natural borrowing constraint with an MPC close to unity. Many recent papers, such as Krueger, Mitman, and Perri (2016), have attempted to carefully quantify the macroeconomic dynamic consequences of a serious heterogeneous agent model, but thus far have not included significant preference shocks in their calibrations. The evidence here suggests that such shocks may have a quantitatively important role to play, especially in increasing the marginal propensity to consume. To the extent that the precautionary motive is driven by preference shocks as opposed to income shocks, social insurance for unemployment will not reduce precautionary savings as much as these models presently suggest.

The right hand panel of table 8 shows the effect of preference shocks and labor elasticity on our empirical estimates of ψ, the transitory MPX. The top row shows that our estimate is lower than the six month MPC (due to the fact that at these low levels of

MPC, more than two years is required for the transitory effect to decay away). It does however follow the same pattern as the MPC and falls in magnitude as the ability of households to adjust labor supply increases. Similarly, going down the first row shows that the estimated MPX increases with the preference shock. However, the similarity to the MPX table ends when we increase *both* labor elasticity *and* the size of the preference shocks. Our estimate can grow large, up to a value of 0.64, getting close to our empirical estimates, when the Frisch elasticity is 0.5 and the preference shock standard deviation is 0.4. This measured transitory MPX now bears little relation to the six month MPX (which is 0.16). Instead it is being driven by reverse causality, whereby preference shocks are driving consumption along with the decision to increase labor. The observed 'shocks' to income are therefore highly correlated with consumption, but they are not causing the consumption dynamics exogenously.

7.3 Semi-Permanent Shocks

SHOW THAT AN AR(1) PROCESS FOR PERMANENT SHOCKS DOESN"T BIAS RESULTS TOO MUCH One reason to choose N=3,4,5 and not a higher N

7.4 Persistent Consumption Response

SHOW THAT ESTIMATES ARE RELATIVELY STABLE TO CHOICES OF N= 2,3,4,5,6

7.5 Measurement Error

INCOME MEASUREMENT ERROR A POTENTIALLY LARGE PROBLEM, but we think it is well measured

8 Robustness

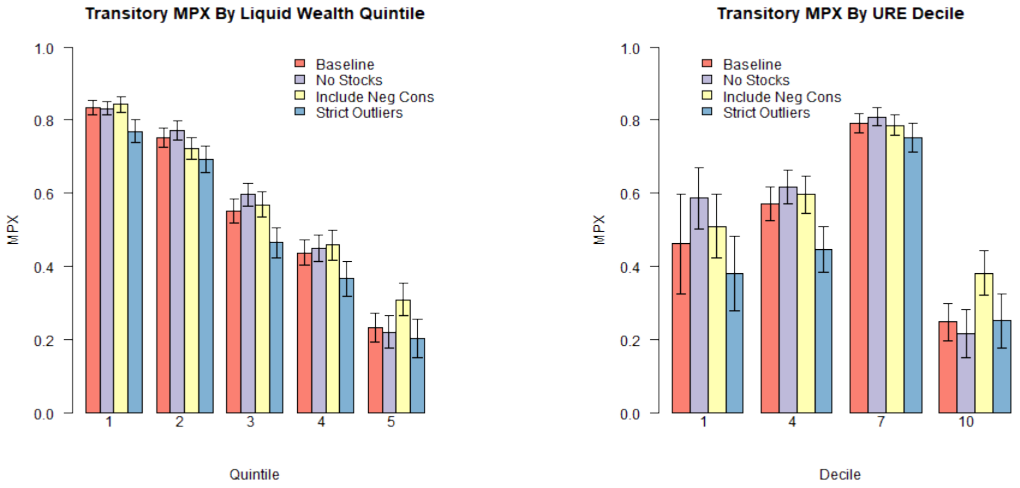

Figure 14 Robustness of Liquid Wealth and URE Distributions

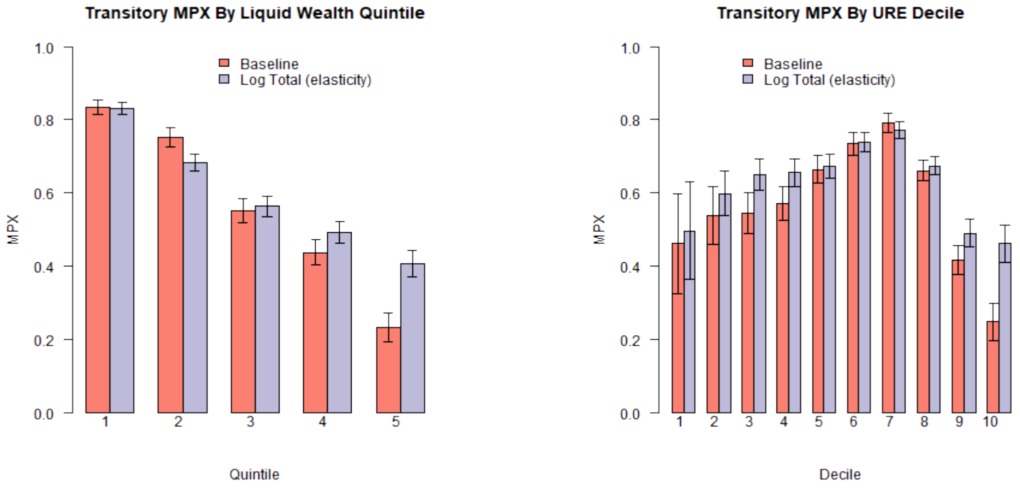

Figure 15 Results Using Log Income and Expenditure

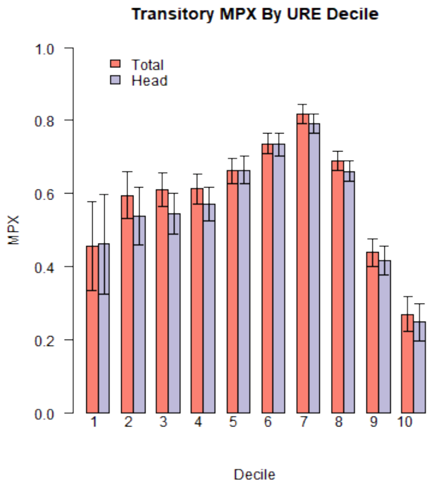

Figure 16 Results Using Total Labor Income and Head Labor Income

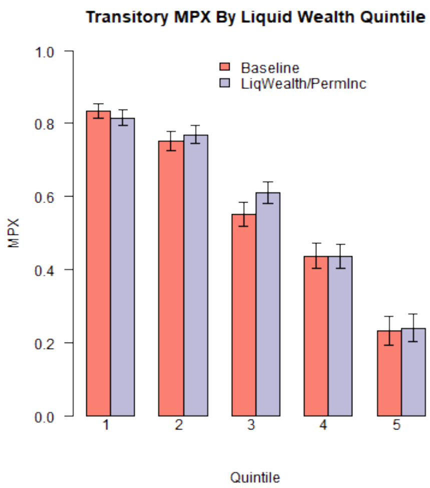

Figure 17 Results Using Quintiles of Liquid Wealth over Permanent Income vs Liquid Wealth

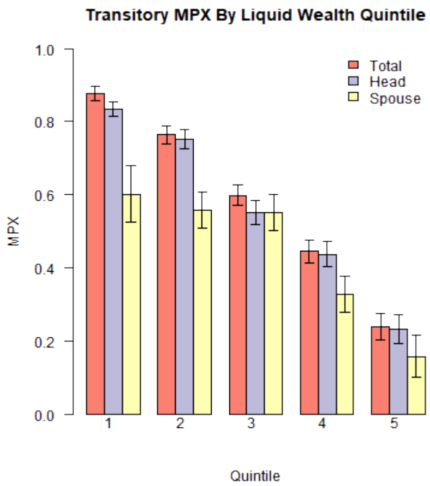

Figure 18 Results Using Total, Head and Spouse Labor Income

9 Conclusion

In this paper we have developed a novel method for calculating consumption response to both permanent and transitory shocks to income. We see a wide variety of potential applications for this methodology, particularly when applied to registry data such as the Danish data we make use of here, or financial aggregation platform data which would allow for more detailed analysis of the type of spending taking place. We present what we think is a particularly important application: identifying the size of different channels of monetary policy. Our results show a large role for heterogeneity in monetary policy transmission that is completely missed in standard New Keynesian models......

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Appendix

A Identification with Time Aggregation

In this section we formalize the continuous time model and calculate the relevant variance and covariances. We begin by defining permanent income. Let p_t for $t \in \mathbb{R}^+$ be a martingale process (possibly with jumps) with independent stationary increments and ν_p be such that $\mathbb{E}(e^{p_t - p_{t-1}}) = e^{\nu_p}$. Define the permanent component of income as:

$$P_t = e^{p_t - t\nu_p}$$

Note that $\mathbb{E}\left(\frac{P_{t+s}}{P_t}\right) = 1$ for all $s \geq 0$.

Next we define transitory income. Let q_t on $t \in \mathbb{R}^+$ also be a martingale process, independent of p_t, with independent stationary increments. Let $f : \mathbb{R}^+ \to \mathbb{R}$ be the impulse response of income to changes in q_t. We will assume that the impulse response to a transitory shock to income is over after two years, that is $f(s) = 0$ for $s > 2$. The transitory component of income is then defined as:

$$\theta_t = e^{\int_{t-2}^{t} f(t-s)dq_s - \nu_q}$$

where $e^{\nu_q} = \mathbb{E}e^{\int_{t-2}^{t} f(t-s)dq_s}$ so that $\mathbb{E}\theta_t = 1$.

We are now in a position to talk about total income. Total income *flow* at time t is given by:

$$\begin{aligned} Y_t &= P_t\theta_t \\ &= e^{p_t - t\nu_p + \int_{t-2}^{t} f(t-s)dq_s - \nu_q} \end{aligned}$$

Observable income is the sum of income *flow* over a one year period, that is:

$$\bar{Y}_T = \int_{T-1}^{T} P_t\theta_t dt$$

We will be focused on the log of observable income growth over N years:

$$\begin{aligned} \Delta^N \log(\bar{y}_T) &= \log\Big(\int_{T-1}^{T} P_t\theta_t dt\Big) - \log\Big(\int_{T-N-1}^{T-N} P_t\theta_t dt\Big) \\ &= \log(\frac{P_{T-1}}{P_{T-N}}) + \log\left(\int_{T-1}^{T} \frac{P_t}{P_{T-1}}\theta_t dt\right) - \log\left(\int_{T-N-1}^{T-N} \frac{P_t}{P_{T-N}}\theta_t dt\right) \end{aligned} \tag{10}$$

Note that if $N \geq 3$ each of the three components of equation 10 are mutually independent because both p_t and q_t have independent increments, and θ_t is independent of q_s for $s < t-2$ and $s > t$. Defining $\mathcal{P}_{T,N}$, $\mathcal{Q}^1_{T,N}$ and $\mathcal{Q}^2_{T,N}$ to be the three parts of the sum in equation 10 respectively, we have:

$$\begin{aligned} \mathcal{P}_{T,N} &= \log(\frac{P_{T-1}}{P_{T-N}}) \\ \Rightarrow \text{Var}(\mathcal{P}_{T,N}) &= (N-1)\text{Var}\Big(\log(\frac{P_T}{P_{T-1}})\Big) \end{aligned}$$

$$= (N-1)\sigma_P^2$$

where σ_P^2 is defined to be $\mathrm{Var}\Big(\log(\frac{P_T}{P_{T-1}})\Big)$, which does not depend on T because p_t has independent increments. Moving on to the components that contain a mix of both permanent and transitory income, and defining $\bar{\theta}_T = \int_{T-1}^{T} \theta_t dt$, we have

$$\begin{aligned}
\mathcal{Q}^1_{T,N} &= \log\left(\int_{T-1}^{T} \frac{P_t}{P_{T-1}}\theta_t dt\right) \\
&= \log\left(\int_{T-1}^{T} \theta_t dt + \int_{T-1}^{T} \Big(\frac{P_t}{P_{T-1}} - 1\Big)\theta_t dt\right) \\
&= \log\left(\bar{\theta}_T\right) + \log\left(1 + \int_{T-1}^{T} \Big(\frac{P_t}{P_{T-1}} - 1\Big)\frac{\theta_t}{\bar{\theta}_T} dt\right) \\
&\approx \log\left(\bar{\theta}_T\right) + \int_{T-1}^{T} \Big(\frac{P_t}{P_{T-1}} - 1\Big)\frac{\theta_t}{\bar{\theta}_T} dt
\end{aligned}$$

Where the approximation holds so long as $\frac{P_t}{P_{T-1}}$ is close to 1 for $T-1 \leq t \leq T$, that is the permanent shock does not move a lot in the course of one year. Define:

$$\sigma_\theta^2 = \mathrm{Var}\left(\log\left(\bar{\theta}_T\right)\right)$$

So that

$$\begin{aligned}
\mathrm{Var}\big(\mathcal{Q}^1_{T,N}\big) &\approx \sigma_\theta^2 + \mathbb{E}\left(\int_{T-1}^{T} \Big(\frac{P_t}{P_{T-1}} - 1\Big)\frac{\theta_t}{\bar{\theta}_T} dt\right)^2 \\
&= \sigma_\theta^2 + \mathbb{E}\left(\int_{T-1}^{T}\int_{T-1}^{T} \Big(\frac{P_t}{P_{T-1}} - 1\Big)\Big(\frac{P_s}{P_{T-1}} - 1\Big)\frac{\theta_t\theta_s}{\bar{\theta}_T^2} dtds\right) \\
&= \sigma_\theta^2 + \int_{T-1}^{T}\int_{T-1}^{T} \mathbb{E}\left(\Big(\frac{P_{\min(t,s)}}{P_{T-1}}\Big)^2 \frac{P_{\max(t,s)}}{P_{\min(t,s)}} - \frac{P_t}{P_{T-1}} - \frac{P_s}{P_{T-1}} - 1\right)\mathbb{E}\left(\frac{\theta_t\theta_s}{\bar{\theta}_T^2}\right) dtds \\
&= \sigma_\theta^2 + \int_{T-1}^{T}\int_{T-1}^{T} \mathrm{Var}\left(\frac{P_{\min(t,s)}}{P_{T-1}}\right)\mathbb{E}\left(\frac{\theta_t\theta_s}{\bar{\theta}_T^2}\right) dtds \\
&\approx \sigma_\theta^2 + \sigma_P^2\int_{T-1}^{T}\int_{T-1}^{T} \min(t,s)\mathbb{E}\left(\frac{\theta_t\theta_s}{\bar{\theta}_T^2}\right) dtds \\
&= \sigma_\theta^2 + \sigma_P^2\int_{T-1}^{T}\int_{T-1}^{T} \min(t,s)\mathbb{E}\left(\Big(1 + \frac{\theta_t - \bar{\theta}_T}{\bar{\theta}_T}\Big)\Big(1 + \frac{\theta_s - \bar{\theta}_T}{\bar{\theta}_T}\Big)\right) dtds \\
&= \sigma_\theta^2 + \sigma_P^2\int_{T-1}^{T}\int_{T-1}^{T} \min(t,s)\left(1 + \mathbb{E}\Big(\hat{\theta}_{t,T}\Big) + \mathbb{E}\Big(\hat{\theta}_{s,T}\Big) + \mathbb{E}\Big(\hat{\theta}_{t,T}\hat{\theta}_{s,T}\Big)\right) dtds
\end{aligned}$$

where $\hat{\theta}_{t,T} = \frac{\theta_t - \bar{\theta}_T}{\bar{\theta}_T}$. Continuing:

$$\begin{aligned}
\mathrm{Var}\Big(\mathcal{Q}^1_{T,N}\Big) &\approx \sigma_\theta^2 + \sigma_P^2 \int_{T-1}^{T}\int_{T-1}^{T} \min(t,s)dtds \\
&\quad + \underbrace{\sigma_P^2 \int_{T-1}^{T}\int_{T-1}^{T} \min(t,s)\Bigg(\mathbb{E}\Big(\hat{\theta}_{t,T}\Big) + \mathbb{E}\Big(\hat{\theta}_{s,T}\Big) + \mathbb{E}\Big(\hat{\theta}_{t,T}\hat{\theta}_{s,T}\Big)\Bigg)dtds}_{\approx 0} \\
&= \sigma_\theta^2 + \sigma_P^2 \int_{T-1}^{T}\Bigg(\int_{T-1}^{s} tdt + \int_{s}^{T} sdt\Bigg)ds \\
&= \sigma_\theta^2 + \frac{1}{3}\sigma_P^2
\end{aligned}$$

A very similar calculation shows that:

$$\mathrm{Var}\Big(\mathcal{Q}^2_{T,N}\Big) \approx \sigma_\theta^2 + \frac{1}{3}\sigma_P^2$$

So we get that:

$$\begin{aligned}
\mathrm{Var}\Big(\Delta^N \log(\bar{y}_T)\Big) &= \mathrm{Var}\Big(\mathcal{P}_{T,N}\Big) + \mathrm{Var}\Big(\mathcal{Q}^1_{T,N}\Big) + \mathrm{Var}\Big(\mathcal{Q}^2_{T,N}\Big) \\
&\approx (N-1)\sigma_P^2 + (\sigma_\theta^2 + \frac{1}{3}\sigma_P^2) + (\sigma_\theta^2 + \frac{1}{3}\sigma_P^2) \\
&= (N - \frac{1}{3})\sigma_P^2 + 2\sigma_\theta^2
\end{aligned}$$

Now we turn to consumption. Consumption responds to permanent income with elasticity ϕ, while the impulse response to a transitory shock is given by some function $g : \mathbb{R}^+ \to \mathbb{R}$ with $g(s) = 0$ for $s > 2$. Total consumption *flow* is then given by:

$$C_t = C_t^P C_t^\theta$$

where

$$\begin{aligned}
C_t^P &= e^{\phi p_t - t\nu_{p_c}} \\
C_t^\theta &= e^{\int_{t-2}^{t} g(t-s)dq_s - \nu_{q_c}}
\end{aligned}$$

and ν_{p_c} and ν_{q_c} are defined such that $\mathbb{E}\Big(\frac{C_t^P}{C_s^P}\Big) = \mathbb{E}(C_t^\theta) = 1$ for all $t \geq s$. Analogous to the case with log income growth over N years (equation 10) we get:

$$\Delta^N \log(\bar{c}_T) = \log(\frac{C^P_{T-1}}{C^P_{T-N}}) + \log\Bigg(\int_{T-1}^{T} \frac{C_t^P}{C^P_{T-1}} C_t^\theta dt\Bigg) - \log\Bigg(\int_{T-N-1}^{T-N} \frac{C_t^P}{C^P_{T-N}} C_t^\theta dt\Bigg) \quad (11)$$

Defining $\mathcal{C}^P_{T,N}$, $\mathcal{C}^1_{T,N}$ and $\mathcal{C}^2_{T,N}$ to be the three parts of the sum in equation 11 respectively, we have:

$$\mathcal{C}^P_{T,N} = \log(\frac{C^P_{T-1}}{C^P_{T-N}})$$

$$
\begin{aligned}
&= \phi \log(\frac{P_{T-1}}{P_{T-N}}) - (N-1)(\nu_{p_c} - \phi\nu_p) \\
\Rightarrow \mathrm{Cov}(\mathcal{P}_{T,N}, \mathcal{C}^P_{T,N}) &= (N-1)\phi \mathrm{Var}\Big(\log(\frac{P_T}{P_{T-1}})\Big) \\
&= (N-1)\phi\sigma_P^2
\end{aligned}
$$

and that:

$$
\begin{aligned}
\mathcal{C}^1_{T,N} &= \log\left(\int_{T-1}^{T} \frac{C_t^P}{C_{T-1}^P} C_t^\theta dt \right) \\
&= \log\left(\int_{T-1}^{T} \Big(\frac{P_t}{P_{T-1}}\Big)^{\phi} e^{-(t-(T-1))(\nu_{pc}-\phi\nu_p)} C_t^\theta dt \right) \\
&\approx \log\left(\bar{C_T^\theta}\right) + \int_{T-1}^{T} \Big(\Big(\frac{P_t}{P_{T-1}}\Big)^{\phi} e^{-(t-(T-1))(\nu_{p_c}-\phi\nu_p)} - 1\Big)\frac{C_t^\theta}{\bar{C_T^\theta}} dt
\end{aligned}
$$

where the steps taken in the approximation are the same as we did in the case of income.

$$
\begin{aligned}
\mathrm{Cov}\Big(\mathcal{Q}^1_{T,N}, \mathcal{C}^1_{T,N}\Big) &= \mathrm{Cov}\Big(\log\left(\bar{\theta}_T\right), \log\left(\bar{C_T^\theta}\right)\Big) \\
&\quad + \mathbb{E}\Bigg(\int_{T-1}^{T}\int_{T-1}^{T} \Big(\frac{P_t}{P_{T-1}} - 1\Big)\Big(\Big(\frac{P_s}{P_{T-1}}\Big)^{\phi} e^{-(s-(T-1))(\nu_{p_c}-\phi\nu_p)} - 1\Big)\frac{\theta_t}{\bar{\theta}_T}\frac{C_s^\theta}{\bar{C_T^\theta}} dtds\Bigg) \\
&= \mathrm{Cov}\Big(\log\left(\bar{\theta}_T\right), \log\left(\bar{C_T^\theta}\right)\Big) \\
&\quad + \mathbb{E}\Bigg(\int_{T-1}^{T}\int_{T-1}^{T} \Big(\Big(\frac{P_{\min(t,s)}}{P_{T-1}}\Big)^{1+\phi} e^{-(\min(t,s)-(T-1))(\nu_{p_c}-\phi\nu_p)} - 1\Big)\frac{\theta_t}{\bar{\theta}_T}\frac{C_s^\theta}{\bar{C_T^\theta}} dtds\Bigg) \\
&= \mathrm{Cov}\Big(\log\left(\bar{\theta}_T\right), \log\left(\bar{C_T^\theta}\right)\Big) \\
&\quad + \int_{T-1}^{T}\int_{T-1}^{T} \mathbb{E}\Big(\Big(\frac{P_{\min(t,s)}}{P_{T-1}}\Big)^{1+\phi} e^{-(\min(t,s)-(T-1))(\nu_{p_c}-\phi\nu_p)} - 1\Big) dtds \\
&\approx 0 \left\{ \begin{aligned} &+ \textstyle\int_{T-1}^{T}\int_{T-1}^{T} \mathbb{E}\Bigg(\Big(\Big(\frac{P_{\min(t,s)}}{P_{T-1}}\Big)^{1+\phi} e^{-(\min(t,s)-(T-1))(\nu_{p_c}-\phi\nu_p)} - 1\Big) \\ &\qquad\qquad \times\Big(\mathbb{E}\Big(\hat{\theta}_t\Big) + \mathbb{E}\Big(\hat{C}_s^\theta\Big) + \mathbb{E}\Big(\hat{\theta}_t\hat{C}_s^\theta\Big)\Big)\Bigg) dtds \end{aligned}\right. \\
&= \mathrm{Cov}\Big(\log\left(\bar{\theta}_T\right), \log\left(\bar{C_T^\theta}\right)\Big) \\
&\quad + \int_0^1\int_0^1 \mathbb{E}\Big(P_{\min(t,s)}^{1+\phi} e^{-\min(t,s)(\nu_{p_c}-\phi\nu_p)} - 1\Big) dtds
\end{aligned}
$$

where $\hat{C}_{t,T}^\theta = \frac{C_t^\theta - \bar{C}_T^\theta}{\bar{C}_T^\theta}$. We now assume that p_t has no jumps, and is therefore a Brownian motion. With this assumption, $\nu_p = \frac{1}{2}\sigma_P^2$ and $\nu_{p_c} = \frac{1}{2}\phi^2\sigma_P^2$ and $\mathbb{E}(P_t^{1+\phi}) =$

$e^{\frac{1}{2}t(1+\phi)^2\sigma_P^2 - \frac{1}{2}t(1+\phi)\sigma_P^2}$ so that:

$$\begin{aligned}
\mathrm{Cov}\Big(\mathcal{Q}^1_{T,N}, \mathcal{C}^1_{T,N}\Big) &= \mathrm{Cov}\Big(\log\left(\bar{\theta}_T\right), \log\left(\bar{C^\theta_T}\right)\Big) \\
&\quad + \int_0^1 \int_0^1 \left(e^{\frac{1}{2}\min(s,t)\sigma_P^2((1+\phi)^2-(1+\phi)-\phi^2+\phi)} - 1\right) dt ds \\
&= \mathrm{Cov}\Big(\log\left(\bar{\theta}_T\right), \log\left(\bar{C^\theta_T}\right)\Big) \\
&\quad + \int_0^1 \int_0^1 \left(e^{\min(s,t)\phi\sigma_P^2} - 1\right) dt ds \\
&\approx \mathrm{Cov}\Big(\log\left(\bar{\theta}_T\right), \log\left(\bar{C^\theta_T}\right)\Big) + \phi\sigma_P^2 \int_0^1 \int_0^1 \min(s,t) dt ds \\
&= \mathrm{Cov}\Big(\log\left(\bar{\theta}_T\right), \log\left(\bar{C^\theta_T}\right)\Big) + \frac{1}{3}\phi\sigma_P^2
\end{aligned}$$

Similarly

$$\mathrm{Cov}\Big(\mathcal{Q}^2_{T,N}, \mathcal{C}^2_{T,N}\Big) \approx \mathrm{Cov}\Big(\log\left(\bar{\theta}_T\right), \log\left(\bar{C^\theta_T}\right)\Big) + \frac{1}{3}\phi\sigma_P^2$$

So that the covariance of income growth with consumption growth over N years is:

$$\mathrm{Cov}\Big(\Delta^N \log(\bar{y}_T), \Delta^N \log(\bar{c}_T)\Big) = (N - \frac{1}{3})\phi\sigma_P^2 + 2\mathrm{Cov}(\tilde{y}, \tilde{c})$$

where $\tilde{y} = \log\left(\bar{\theta}_T\right)$ and $\tilde{c} = \log\left(\bar{C^\theta_T}\right)$

B Sample Selection

We choose to look at households whose head is between the ages of 30 and 55 in 2008. This is driven by the desire to remove households for who the assumption that most of the income growth is unexpected. For the old and the young it is likely that individual households will have a lot of information about their income path that is not available to the econometrician (for example the year in which they plan to retire, or the fact that they are on a specific career track with set expectations of promotion and pay raises). We also want to remove households whose income volatility is increasing or decreasing sharply. Figures 19 and 20 show how our estimates of both income variance and MPX vary with age. The dots represent the point estimate for each age while the lines are the centered moving averages over the five nearest age groups. The solid black line shows the total variance of income growth over 1 year. It should not be surprising that income growth for households with heads in their 20's is highly volatile. This volatility plateaus around the age of 35 and stays at a constant level until the point of retirement at which point it temporarily grows before falling to an even lower level. We can see that while both transitory and permanent shocks to income are high early in life, permanent income shocks are particularly large while individuals find their place in the workforce. From the age of 30 to 55 both transitory and permanent shocks are approximately the same size and remarkably stable. At retirement shocks to permanent income rise, not

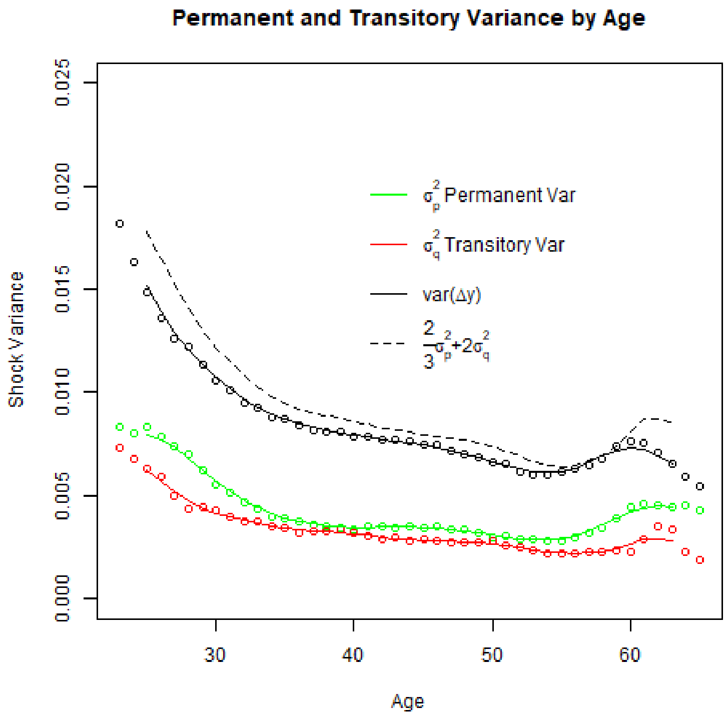

Figure 19 Permanent and Transitory Shock Variance by Age

surprisingly as retirement itself will be seen in the model as a shock, even as transitory income variance declines.

As the model assumes the variance to permanent and transitory shocks is constant in the observed period, interpretation of the numbers outside of the 30-55 age group needs to be treated with care. However, the figure clearly shows that within this age group the assumption of constant variance appears to be a reasonable one.

The dotted black line shows the variance of Δy assuming no persistence in the transitory component. The fact that this line is slightly above the empirical variance of Δy is consistent with some persistence in the transitory component of income, justifying our decision exclude growth over one and two years in our identification.

The level of both permanent and transitory shock variance for households aged 30 to 55 is approximately 0.0035, reflecting a standard deviation of 6%. Estimates using US data are significantly higher, especially for the transitory shock variance (for example Carroll and Samwick (1997) estimate 0.02 for permanent and 0.04 for transitory). This difference may be due to lower income inequality in Denmark, more progressive taxation and more generous unemployment insurance. The lower transitory variance will also be due to significantly reduced measurement error relative to the survey based US data.

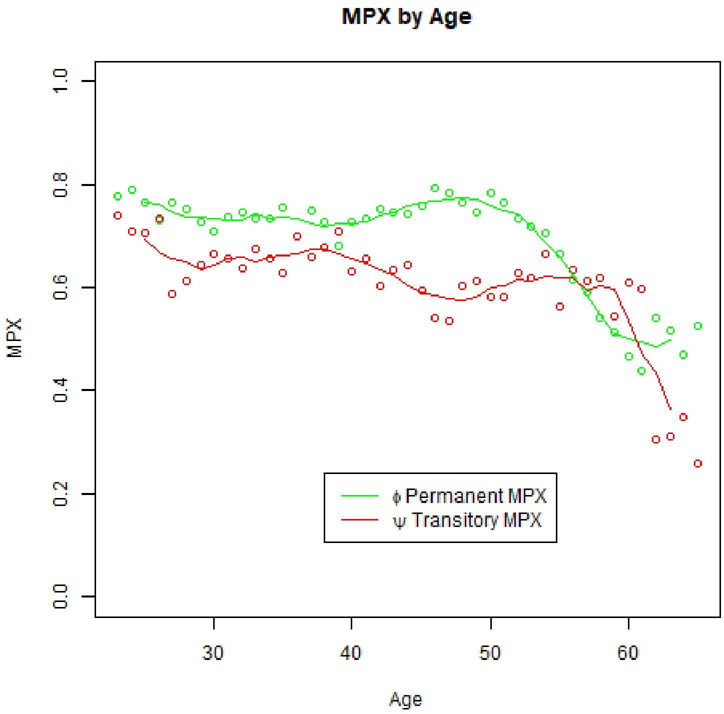

Figure 20 MPX by Age

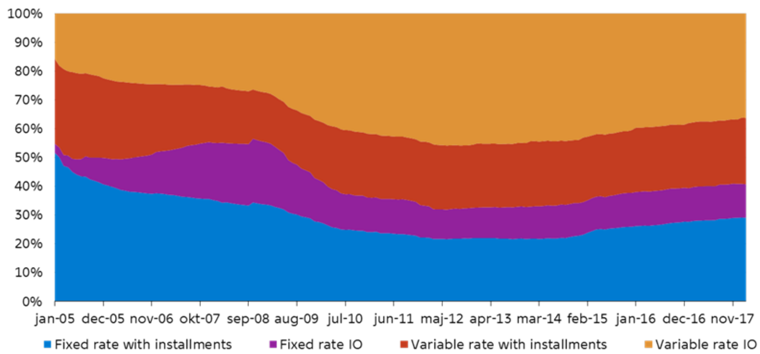

Figure 21 Mortgage Debt by Type

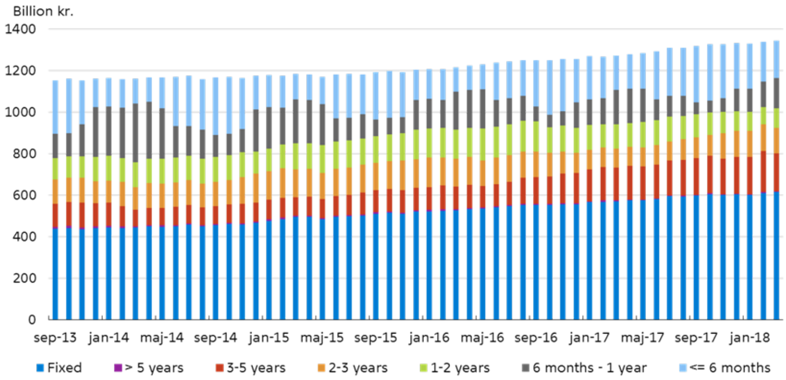

Figure 22 Mortgage Debt by Maturity

C Danish Mortgage Market

D Distribution of Permanent and Transitory MPX by URE, NNP and Income

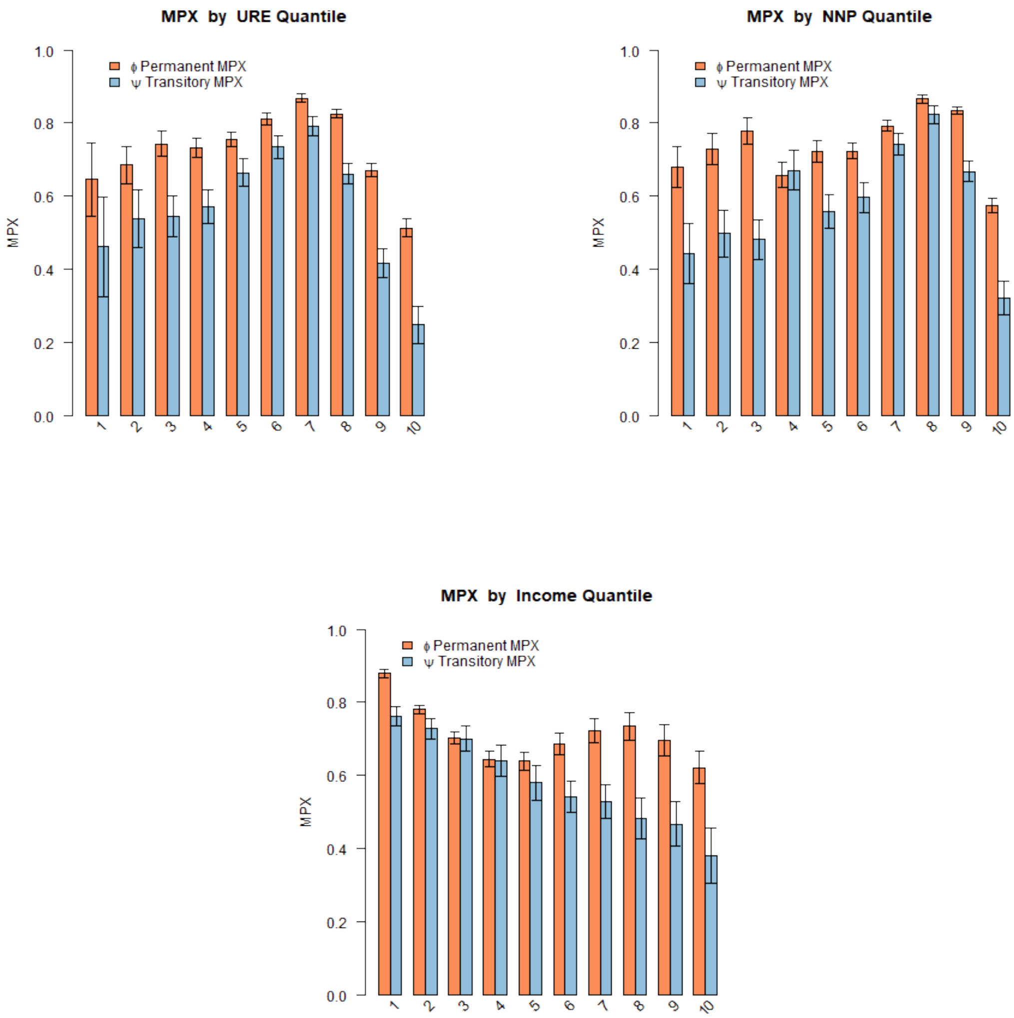

Figure 23 MPX Distribution by URE, NNP and Income