第五章

1.

由书 P153 知,$Re_{crit}=2100$

$$
\begin{aligned}
\upsilon_{crit} &= \frac{Re\upsilon}{D} = \frac{2100\times 4.41\times 10^{-6}}{0.1524} \\
&= 0.0608\,(m/s) \\
\upsilon_{crit} &= \frac{Re\upsilon}{D} = \frac{2100\times 1.13\times 10^{-6}}{0.1524} \\
&= 0.0156\,(m/s)
\end{aligned}
$$

2.

$$Re = \frac{VD}{\upsilon} = \frac{1.607\times 0.305}{1.13\times 10^{-6}} = 2.88\times 10^5 \gg 2100$$

$\therefore$ 为湍流

$$Re = \frac{VD}{\upsilon} = \frac{1.607\times 0.305}{205\times 10^{-6}} = 1.59\times 10^5 < 2100$$

$\therefore$ 为层流

3.

$Re_{crit}=2100$

$$V = \frac{Q}{S} = \frac{Q}{\frac{\pi}{4}D^2} = \frac{4Q}{\pi D^2}$$

代入：

$$Re = \frac{VD}{\upsilon} = \frac{4QD}{\pi D^2\upsilon} = \frac{4Q}{\pi D\upsilon}$$

$$\therefore D_{min} = \frac{4Q}{Re\pi\upsilon} = \frac{4\times 5.67\times 10^{-3}}{2100\pi\times 6.08\times 10^{-6}} = 0.565(m)$$

$\therefore D\geqslant 0.565m$

4.

公式见书 P332

$$
\begin{aligned}
\therefore Le &= 0.06Re_D\times D \\
&= 0.06\times 1500\times 0.01 \\
&= 0.9(m)
\end{aligned}
$$

5.

$$V = \frac{4Q}{\pi D^2}\text{(推导见第 3 题)}$$

$$
\begin{aligned}
\therefore Re &= \frac{VD}{\upsilon} = \frac{4Q}{\pi D\upsilon} \\
&= \frac{4\times 0.8\times 10^{-3}}{\pi\times 0.1\times\upsilon} = \frac{0.01019}{\upsilon}
\end{aligned}
$$

当$Re>2100$时，为湍流

此时:$\upsilon < \frac{0.01019}{Re} = \frac{0.01019}{2100} = 4.85\times 10^{-6}$

查表 (书 P534-535)：$\upsilon = 1.51\times 10^{-5}(m^2/s)$

$\upsilon = 1.004\times 10^{-6}(m^2/s)$

$\upsilon = \frac{1.49}{1260} = 1.18\times 10^{-3}(m^2/s)$

$\therefore$(1)(2)(5)(6) 为层流，(3)(4) 为湍流。

6.

（此题默认球直径为 0.1m）

$$
\begin{aligned}
\text{空气：}V_{crit} = \frac{\upsilon Re}{D} &= \frac{1.51\times 10^{-5}\times 250000}{0.1} = 37.75(m/s) \\
\text{水：}V_{crit} &= \frac{1.004\times 10^{-6}\times 250000}{0.1} = 2.51(m/s) \\
\text{氢气：}V_{crit} &= \frac{\frac{0.9\times 10^{-5}}{0.0839}\times 250000}{0.1} = 268.2(m/s)
\end{aligned}
$$

7.

$$
\begin{aligned}
&\because \nabla\cdot V = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0. \therefore \text{为不可压流动} \\
&\tau_{xx} = 2\mu\frac{\partial u}{\partial x} = 0.\text{同理}\tau_{yy} = \tau_{zz} = 0 \\
&\tau_{xy} = \mu\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) = \mu(zt+zt) = 2\mu zt = 0.02zt \\
&\tau_{xz} = \mu\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right) = \mu yt = 0.01yt \\
&\tau_{yz} = \mu\left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right) = \mu xt = 0.01xt
\end{aligned}
$$

8.

$$
\begin{aligned}
&\because \nabla\cdot V = 0. \therefore \text{为不可压流动} \\
&\tau_{xx} = \tau_{yy} = \tau_{zz} = 0 \\
&\tau_{xy} = \mu\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) = \mu(2+1) = 0.024Pa\cdot s \\
&\tau_{xz} = \mu\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right) = \mu(3+2) = 0.040Pa\cdot s \\
&\tau_{yz} = \mu\left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right) = \mu(3+4) = 0.056Pa\cdot s
\end{aligned}
$$

9.

$\because$ 层流. $\therefore v=0$(分层流动，y 方向速度为 0)

$\because \nabla\cdot V = 0. \therefore$ 为不可压流动

$$
\begin{aligned}
\therefore f_{sx} &= \mu\nabla^2 u = \mu\frac{\partial^2 u}{\partial y^2} \\
&= \mu u_{max}\left[-2\left(\frac{1}{n}\right)\frac{1}{n}\right] = -\frac{2\mu u_{max}}{h^2} \\
&= -3.76(N/m^3)
\end{aligned}
$$

10.11.12 三题大体思路一致，将速度场代入 N-S 方程，得到关于 p^* 的方程。以 p^* 有解为条件，可得到关于 n 或 r 的微分方程，求解即得到结果。由于过程过于复杂，笔者认为理解思路即可，此处不再赘述。

13.

已知条件：$v=0,\dfrac{\partial p^*}{\partial x}=0.$

由连续方程：$\nabla\cdot V=\dfrac{\partial u}{\partial x}+0=0,\therefore\dfrac{\partial u}{\partial x}=0.$

x轴方向的 $N-S$ 方程：$0=-\dfrac{\partial^2 u}{\partial y^2}$

$\therefore u=Ay+B.$代入$u\mid_{y=0}=U_1,u\mid_{y=b}=U_2,$

$\therefore u=\dfrac{u_2-u_1}{b}y+u_1$

14.

(1) 由书 P143 页，有：

$$\frac{\mathrm{d}^2u}{\mathrm{d}y^2}=\frac{1}{\mu}\frac{\mathrm{d}p^*}{\mathrm{d}x},\quad\therefore\frac{\mathrm{d}u}{\mathrm{d}y}=\frac{1}{\mu}\frac{\mathrm{d}p^*}{\mathrm{d}x}y+c_1.$$

由书 P149：$\dfrac{\mathrm{d}u}{\mathrm{d}y}\mid_{y=h}=0$

$\therefore c_1=-\dfrac{h}{\mu}\dfrac{\mathrm{d}p^*}{\mathrm{d}x},\quad\dfrac{\mathrm{d}u}{\mathrm{d}y}=\dfrac{1}{\mu}\dfrac{\mathrm{d}p^*}{\mathrm{d}x}(y-h)$

再做积分：$u=\dfrac{y}{\mu}\dfrac{\mathrm{d}p^*}{\mathrm{d}x}(\dfrac{y}{2}-h)\qquad(1)$

而$\vec{g}=g(\sin\theta\vec{i}-\cos\theta\vec{j})$

$\therefore p^*=p+\rho g(\cos\theta y-\sin\theta x)$

$\because$ 上表面为自由液面 $\therefore\dfrac{\mathrm{d}p}{\mathrm{d}x}=0$(见书 P148)

$\therefore\dfrac{\mathrm{d}p^*}{\mathrm{d}x}=-\rho g\sin\theta$

代入 (1)，得$u=\dfrac{\rho g}{\mu}y\left(h-\dfrac{y}{2}\right)\sin\theta$

(2)

$$\begin{aligned}\dot{Q}&=\int_{y=0}^{y=h}u\mathrm{d}S=\int_{y=0}^{y=h}u(\mathrm{d}y\times 1)(\text{单位宽度为 1})\\&=\int_0^h\frac{\rho g}{\mu}\sin\theta y\left(h-\frac{y}{2}\right)\mathrm{d}y\\&=\frac{\rho g\sin\theta}{\mu}\left(\frac{1}{2}hy^2-\frac{1}{6}y^3\right)\Big|_0^h\\&=\frac{\rho g\sin\theta h^3}{3\mu}\end{aligned}$$

(3)

$$\bar{u}=\frac{\dot{Q}}{S}=\frac{\dot{Q}}{h\times 1}=\frac{\rho g\sin\theta h^2}{3\mu}$$

代入速度分布式，得：$y\left(h-\dfrac{y}{2}\right)=\dfrac{h^2}{3}$

解得：$y=\dfrac{3-\sqrt{3}}{3}$(另一根超过 h，舍去)

15.

由 z 轴方向 N-S 方程：

$0=-\dfrac{\partial p}{\partial z}+g\sin 30^\circ+\upsilon\dfrac{1}{r}\dfrac{\mathrm{d}}{\mathrm{d}r}\left(r\dfrac{\mathrm{d}V_z}{\mathrm{d}r}\right)$

由于压强为常数，$\dfrac{\partial p}{\partial z}=0\quad\therefore-\dfrac{g}{2\upsilon}=\dfrac{1}{r}\dfrac{\mathrm{d}}{\mathrm{d}r}\left(r\dfrac{\mathrm{d}V_z}{\mathrm{d}r}\right)$

解得：$V_z=-\dfrac{gr^2}{8\upsilon}+c_1\ln r+c_2$

代入边界条件：$V=\dfrac{\rho g}{8\mu}(r_0^2-r^2)$

$$\begin{aligned}\therefore\dot{Q}=SV&=\int_0^{r_0}\frac{\rho g}{8\mu}(r_0^2-r^2)\times 2\pi r\mathrm{d}r\\&=\frac{\rho g}{8\mu}\pi\left(r_0^2r^2-\frac{1}{2}r^4\right)\\&=\frac{\rho g\pi}{16\mu}r_0^4\end{aligned}$$

16.

以 $r=37.5mm+0.025mm$ 处为 $y=0$ 面，$r=37.5mm$ 处为 $y=h$ 面，问题化为平面库埃特流动。

$$U=\frac{100}{60}\pi d=\frac{100\pi\times 0.075}{60}=\frac{\pi}{8}$$

$\tau_w=-\mu\dfrac{U}{h}=-0.2\times\dfrac{pi}{8\times 0.025\times 10^{-3}}=-1000\pi$

而$S=\pi dh=\pi\times 0.075\times 0.16=0.03770(m^2)$

$M=F\times r=\tau Sr=-1000\pi\times 0.03770\times\dfrac{0.075}{2}=-4.44(N\cdot m$

17.

参考书 P151 下方对圆柱体流体微元的分析：

$$\tau=\frac{r}{2}\frac{\Delta p^*}{L}=K\left(\frac{\mathrm{d}u}{\mathrm{d}y}\right)^n\qquad(2)$$

重力在水平管道流动方向分量为 0，$\therefore\Delta p^*=\Delta p$

由几何意义，$|dy|=|dr|\therefore|\dfrac{\mathrm{d}u}{\mathrm{d}r}|=|\dfrac{\mathrm{d}u}{\mathrm{d}y}|$

考虑到 r 增大时，u 变小，$\dfrac{\mathrm{d}u}{\mathrm{d}r}<0$

由 (2)：$\dfrac{\mathrm{d}u}{\mathrm{d}r}=\pm\sqrt[n]{\dfrac{r\Delta p}{2KL}}$因此应取负号

两边积分，$u=-\sqrt[n]{\dfrac{\Delta p}{2KL}}\dfrac{n}{n+1}r^{\frac{n+1}{n}}+c$

由边界条件$r=R,u=0$

$\therefore u=\sqrt[n]{\dfrac{\Delta p}{2KL}}\dfrac{n}{n+1}\left[R^{\frac{n+1}{n}}-r^{\frac{n+1}{n}}\right]$

即 $=\sqrt[n]{\dfrac{R\Delta p}{2KL}}\dfrac{nR}{n+1}\left[1-\dfrac{r}{R}^{\frac{n+1}{n}}\right]$

18.

可参考书 P153 例 5.6

$$V_z=\frac{1}{4\mu}\frac{\mathrm{d}p^*}{\mathrm{d}z}r^2+c_1\ln r+c_2$$

由于压强为常数，重力在水平管道流动方向分量为 0，$\dfrac{\partial p^*}{\partial z}=\dfrac{\partial p}{\partial z}=0$

代入边界条件$r=r_0, V=0; r=R, V=V_0$：

解得：$V=\dfrac{V_0\ln\frac{r}{r_0}}{\ln\frac{R}{r_0}}$

$$\tau_w=-\mu\frac{\mathrm{d}V}{\mathrm{d}r}=\frac{\mu V_0}{\ln\frac{R}{r_0}}\frac{\frac{1}{r_0}}{\frac{r}{r_0}}$$

$$=\frac{\mu V_0}{R\ln\frac{R}{r_0}}$$

$$\therefore F=\tau S=\frac{\mu V_0}{R\ln\frac{R}{r_0}}\times(2\pi R\times 1)=\frac{2\pi\mu V_0}{\ln\frac{R}{r_0}}$$