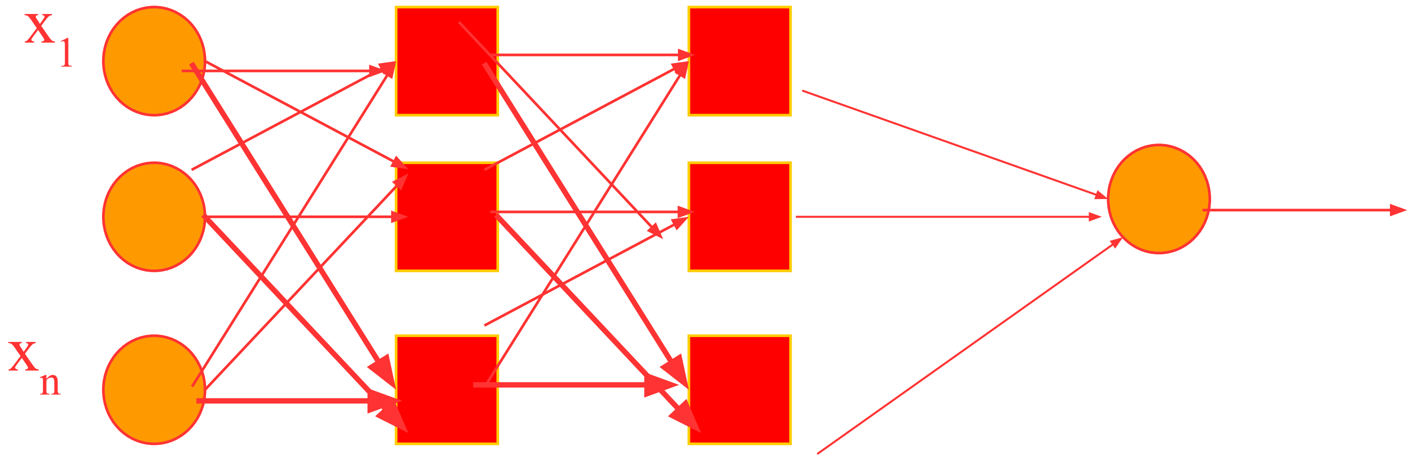
Multi Layer Perceptron
(MLP)

Multi Layer Networks



What is MLP?

A **Multilayer Perceptron (MLP)** is a type of artificial neural network that consists of **multiple layers** of nodes (or neurons), typically organized into three layers:

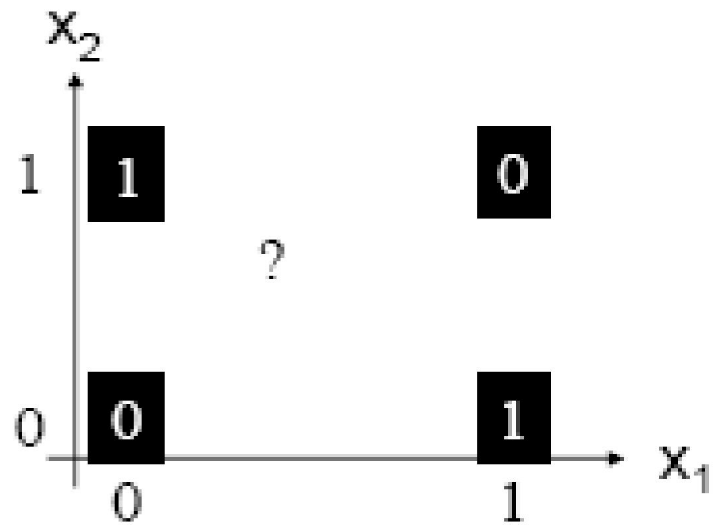
1. **Input Layer**: Takes in the features of the data.
2. **Hidden Layers**: One or more layers where the data undergoes transformations through weighted connections and activation functions. This is where the model "learns" complex patterns.
3. **Output Layer**: Produces the final output, which could represent class probabilities (in classification tasks) or continuous values (in regression tasks).

The XOR Problem

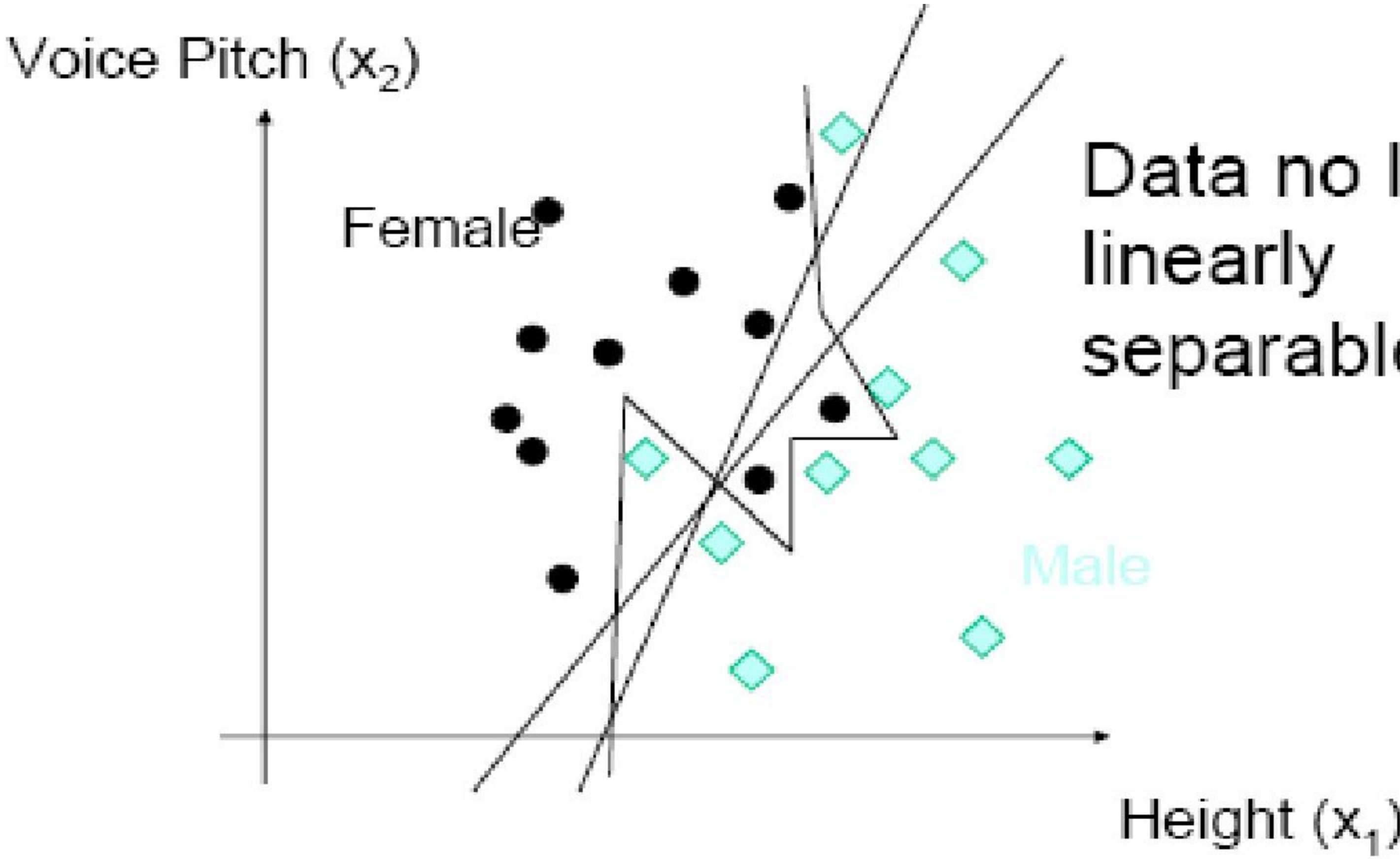
A Perceptron cannot represent Exclusive OR since it is not linearly separable.

XOR function

x_1	x_2	y
0	0	0
0	1	1
1	0	1
1	1	0



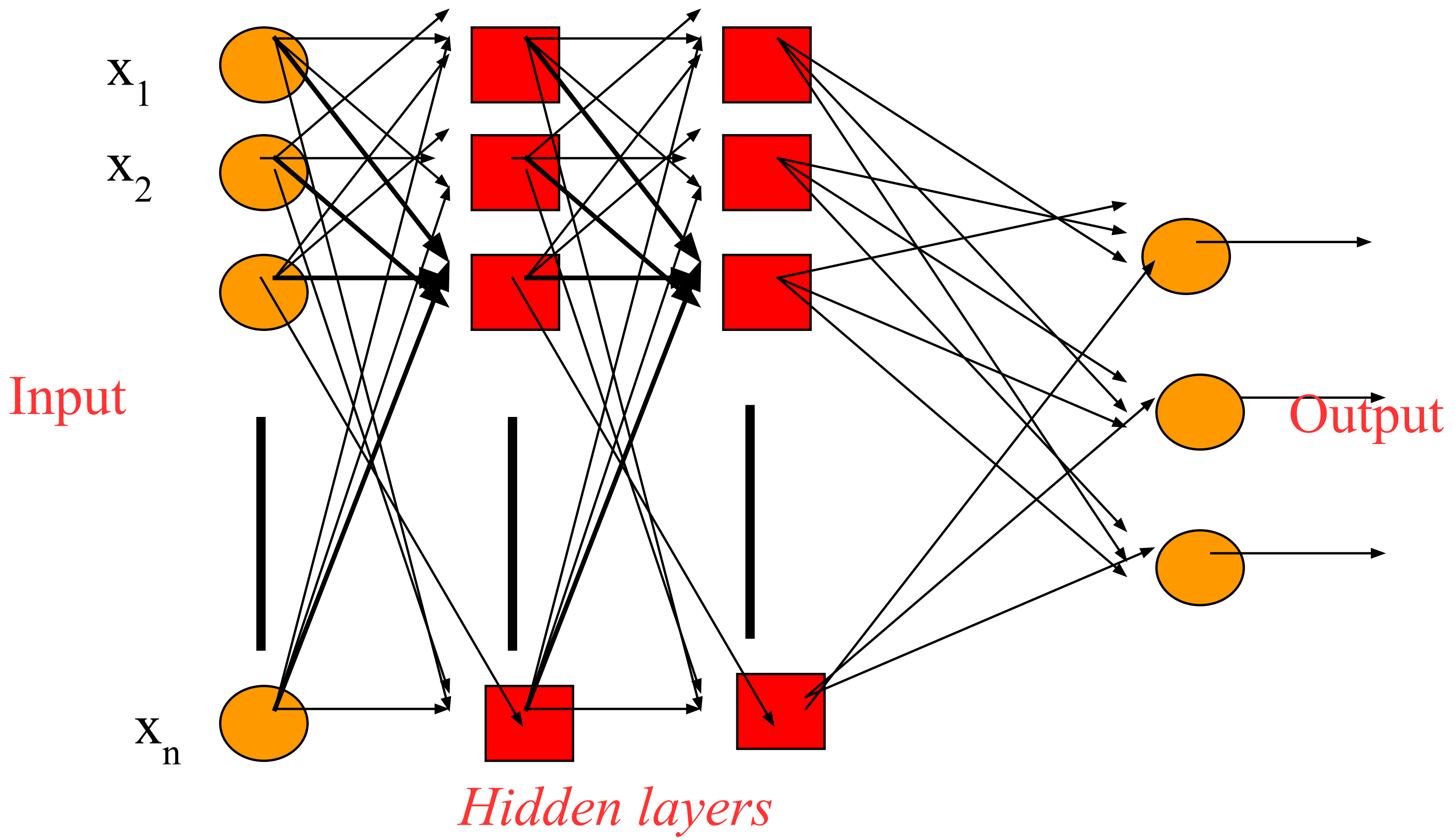
Non-Linear Data



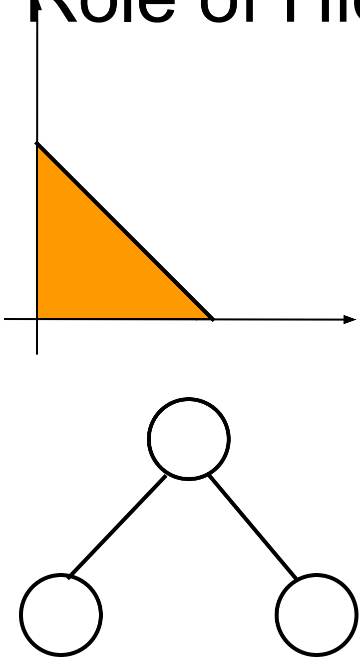
Data no longer linearly separable

What is a good decision boundary ?

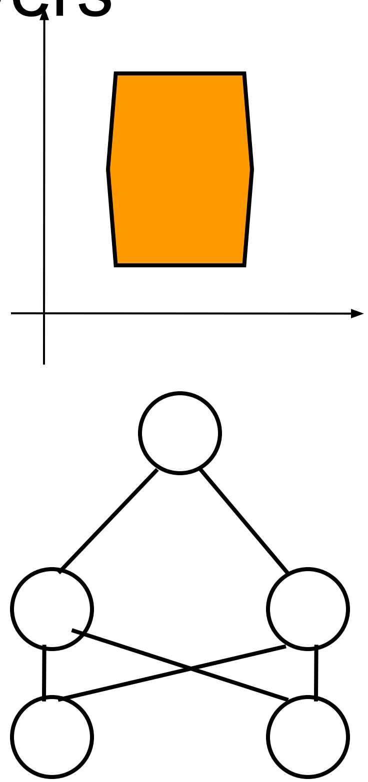
Layers in MLP



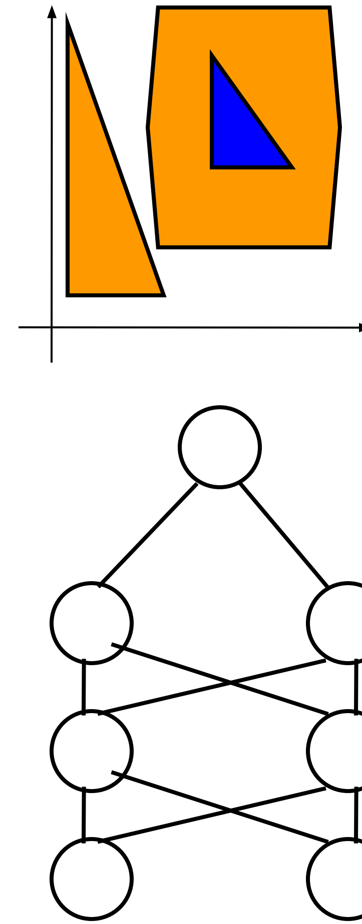
Role of Hidden Layers



1st layer draws linear boundaries



2nd layer combines the boundaries



3rd layer can generate arbitrarily complex boundaries

Backpropagation

Backpropagation has two phases:

- Forward pass phase: computes 'functional signal', feed forward propagation of input pattern signals through network

- Backward pass phase: computes 'error signal', *propagates* the error *backwards* through network starting at output units (where the error is the difference between actual and desired output values)

Steps in MLP

1. Forward pass
2. Backpropagation

Forward Propagation of Activity

- **Step 1**: Initialise weights at random, choose a learning rate η
- Until network is trained:
- For each training example i.e. input pattern and target output(s):
- **Step 2**: Do forward pass through net (with fixed weights) to produce output(s)
  - i.e., in Forward Direction, layer by layer:
    - Inputs applied
    - Multiplied by weights
    - Summed
    - ‘Squashed’ by sigmoid activation function
    - Output passed to each neuron in next layer
  - Repeat above until network output(s) produced

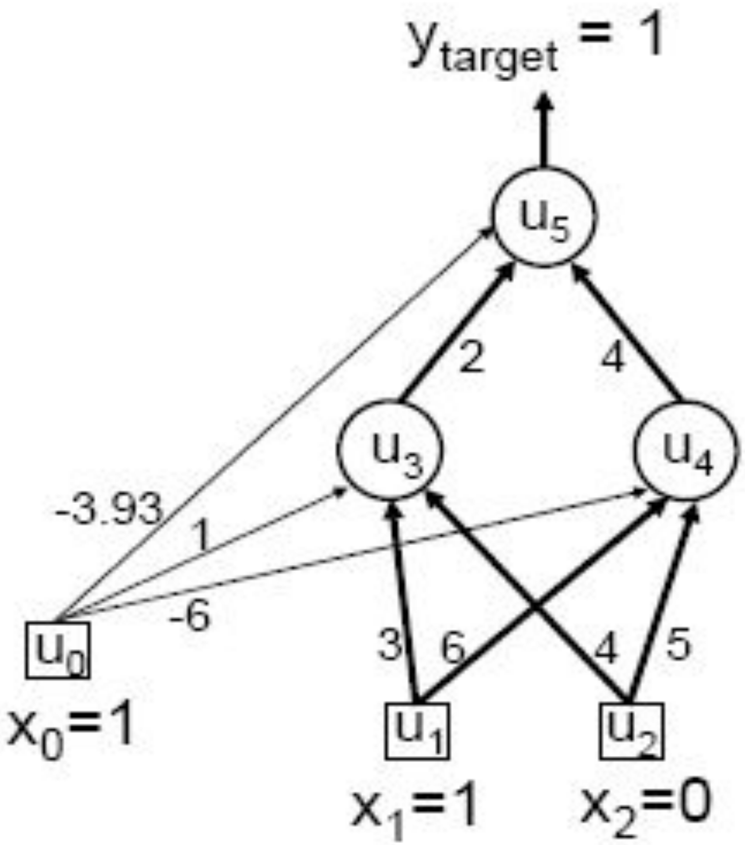
Step 3. Back-propagation of error

- Compute error (delta or local gradient) for each output unit $\delta\ k$
- Layer-by-layer, compute error (delta or localgradient) for each hidden unit $\delta\ j$ by backpropagating errors

Step 4: Next, update all the weights Δwij *b*y gradient descent, and go back to Step 2

- The overall MLP learning algorithm, involving forward pass and backpropagation of error (until the network training completion), is known as the Back Propagation (BP) algorithm

MLP/BP Example



Current state:

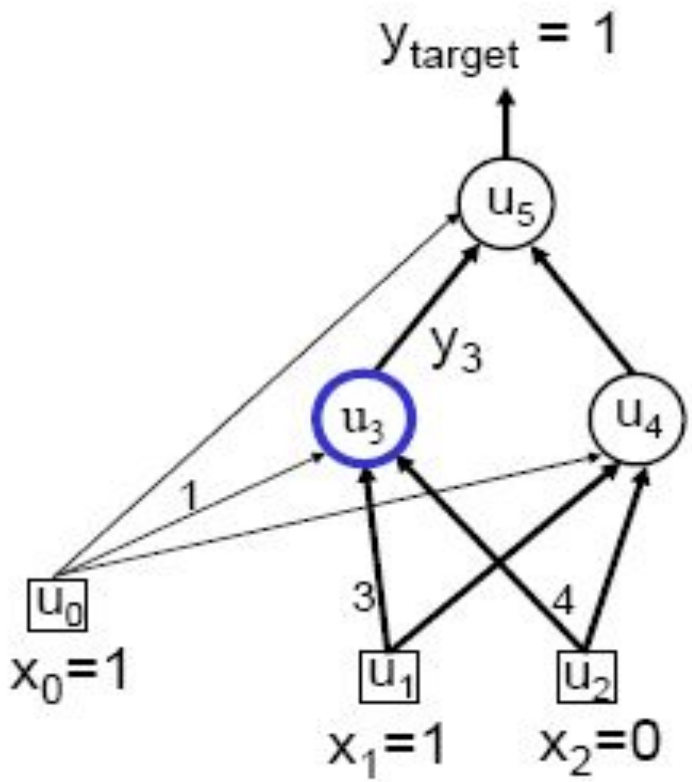
- Weights on arrows e.g. $w_{13} = 3$, $w_{35} = 2$, $w_{24} = 5$
- Bias weights, e.g.

bias for unit 4 (u_4) is w_{04}= -6

Training example (e.g. for logical OR problem):

- Input pattern is x_1=1, x_2=0
- Target output is y_{target}=1

Example: Forward Pass



Output for any neuron/unit j can be calculated from:

$$a_j = \sum_i w_{ij} x_i$$

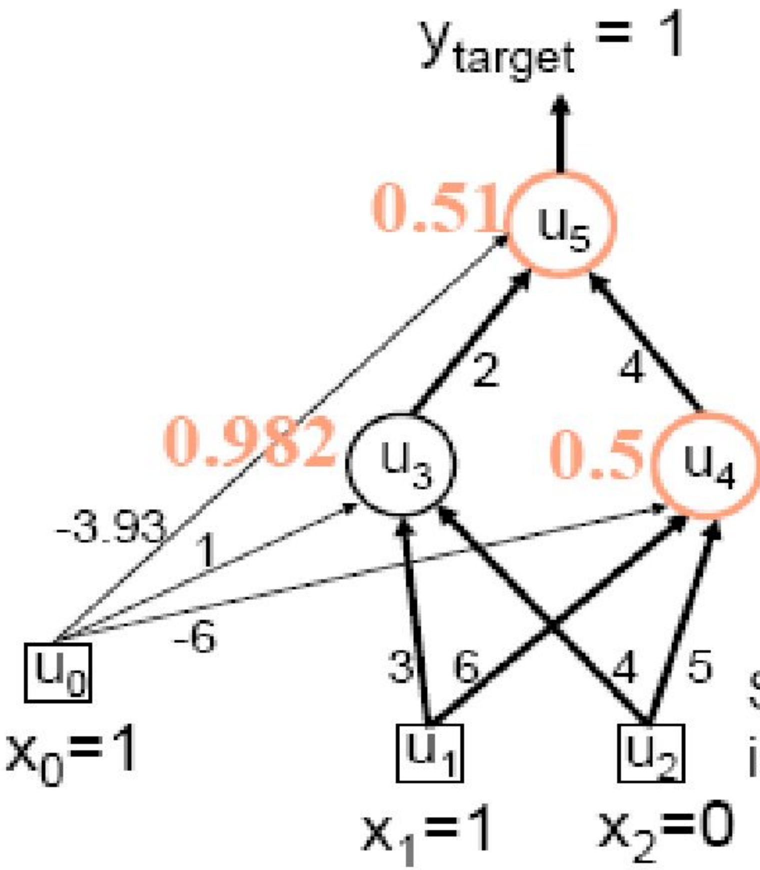
$$y_j = f(a_j) = \frac{1}{1+e^{-a_j}}$$

e.g Calculating output for Neuron/unit 3 in hidden layer:

$$a_3 = 1*1 + 3*1 + 4*0 = 4$$

$$y_3 = f(4) = \frac{1}{1+e^{-4}} = 0.982$$

Example: Forward Pass

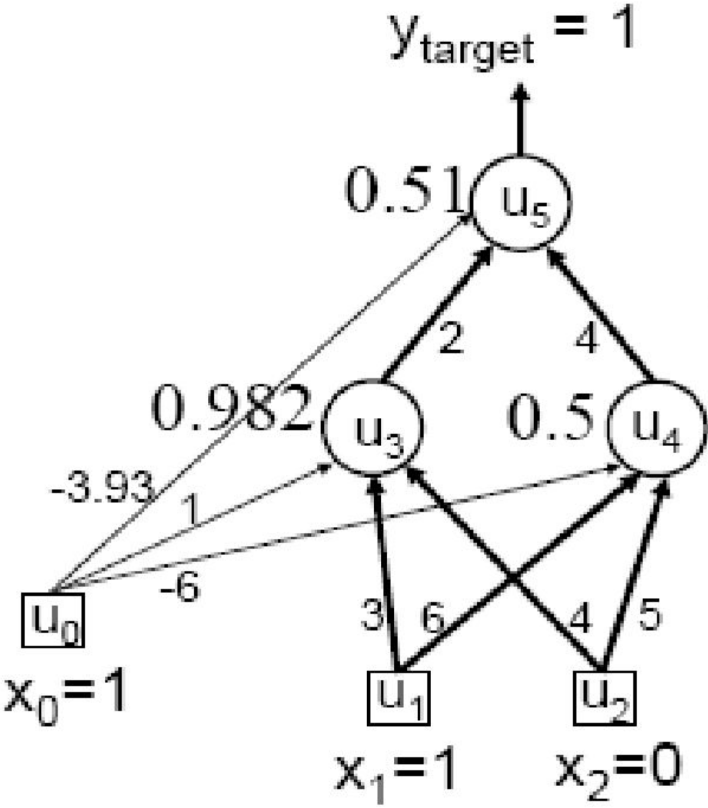


Unit	activation	output
	a_j	y_j
u_3	4.00	0.982
u_4	0.00	0.500
u_5	0.04	**0.510**

(network output)

So the error for this training example is: $(y_{target} - y_5) = (1 - 0.510) = 0.490$

Example: Backward Pass



Now compute delta values starting at the output:

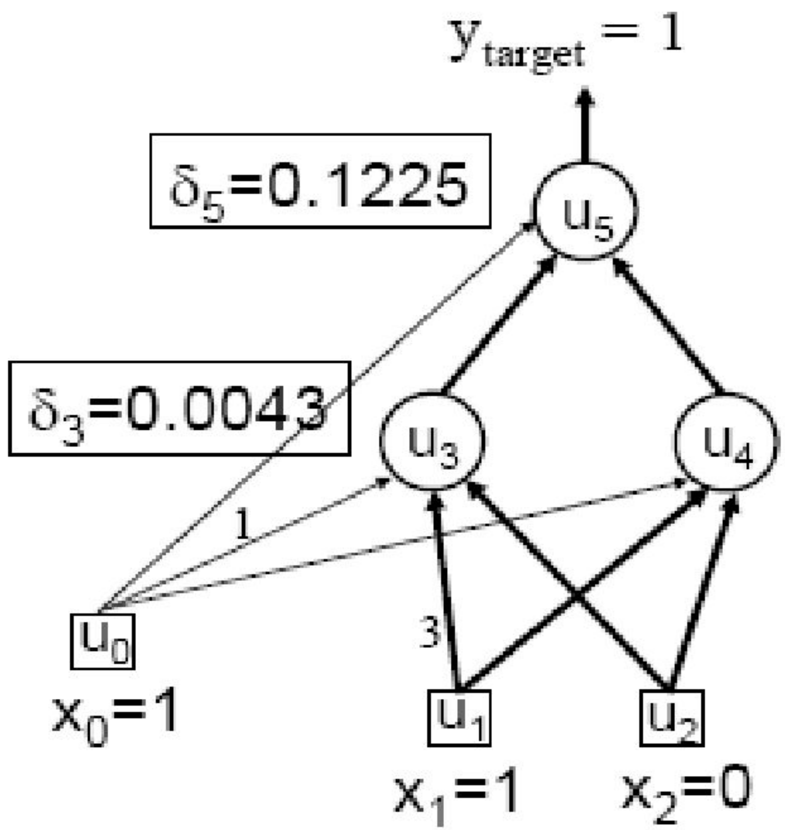
$$\delta_5 = y_5(1 - y_5)(y_{target} - y_5)$$
$$= 0.51(1 - 0.51) \times 0.49$$
$$= \mathbf{0.1225}$$

Then for hidden units:

$$\delta_4 = y_4(1 - y_4)\, w_{45}\, \delta_5$$
$$= 0.5(1 - 0.5) \times 4 \times 0.1225$$
$$= \mathbf{0.1225}$$

$$\delta_3 = y_3(1 - y_3)\, w_{35}\, \delta_5$$
$$= 0.982(1-0.982) \times 2 \times 0.1225$$
$$= \mathbf{0.0043}$$

Example: Update Weights Using Generalized Delta Rule (BP)



- Set learning rate η = 0.1

  Change weights by:

  $$\Delta w_{ij} = \eta \delta_j y_i$$

- e.g.bias weight on u_3:

Δw_{03} = $\eta \delta_3 x_0$

=0.1*0.0043*1

= 0.0004

So, new w_{03} ⌫

w_{03}(old)+Δw_{03}

=1+0.0004=1.0004

- and likewise:

  w_{13} ⌫ 3 + 0.0004

  = 3.0004

Similarly for the all weights wij:

i	j	w_{ij}	δ_j	y_i	Updated w_{ij}
0	3	**1**	0.0043	1.0	**1.0004**
1	3	**3**	0.0043	1.0	**3.0004**
2	3	**4**	0.0043	0.0	**4.0000**
0	4	**-6**	0.1225	1.0	**-5.9878**
1	4	**6**	0.1225	1.0	**6.0123**
2	4	**5**	0.1225	0.0	**5.0000**
0	5	**-3.92**	0.1225	1.0	**-3.9078**
3	5	**2**	0.1225	0.9820	**2.0120**
4	5	**4**	0.1225	0.5	**4.0061**