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CUSUM control charts for monitoring optimal portfolio weights

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ABSTRACT

A portfolio investor requires statistical tools for the timely detection of changes in the optimal portfolio composition. Several multivariate cumulative sum (CUSUM) control charts are proposed for the purpose of monitoring optimal portfolio weights. The ability of the CUSUM schemes to detect important types of changes in the optimal portfolio weights is analyzed in an extensive Monte Carlo simulation study. The empirical application of control charts shows that the proposed methodology can provide a significant reduction of the portfolio volatility.

1. Introduction

The mean-variance portfolio approach of Markowitz (1952) provides a possibility to determine optimal portfolio weights relying on the assumption of known means and covariances of the risky asset returns. Since these quantities are unknown in practice they have to be estimated from the historical data. An important benchmark in portfolio selection is the global minimum variance portfolio (GMVP), which exhibits the lowest attainable variance. Moreover, this portfolio is the starting point of the mean-variance efficient frontier. The GMVP weights depend solely on the covariance matrix of the asset returns, but not on the hardly predictable mean asset returns. The latter often cause large estimation errors in the optimal portfolio proportions (Best and Grauer, 1991). Since the covariance matrix can usually be estimated and forecasted much better, the GMVP is useful for investment decisions from both theoretical and practical viewpoints.

Since the covariances change over time, the GMVP weights alter over time as well. Assume that the GMVP weights remain unchanged over some historical period of time. Then the investor needs statistical instruments to check, whether the former GMVP composition remains optimal. That means, the investor should decide, whether the previous (null) hypothesis about the GMVP weights is still valid in the current period. Since these decisions should be made at every new point in time, this problem is of sequential nature.

Statistical process control (SPC) suggests control charts as an appropriate instrument for dealing with sequential decision problems (see, e.g. Montgomery, 2005). In SPC terminology, the investor should timely detect a non-random unknown change point, where the actual parameters of the weight process deviate from their targets, e.g. historically presumed values. A control chart consists of a control statistic and a rejection area. The chart provides a signal if the control statistic gets into the rejection area for the first time after the monitoring has started. A signal indicates that there is possibly a change in the monitored parameters, so that the null hypothesis does not hold any more. Alternatively, there could be a false signal, whereas the process parameters remain unchanged. A good chart provides quickly a signal after a change and no signal for

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a long time if no change occurs. The investor should reconsider his opinion about the current GMVP proportions after each signal.

The most popular control charts refer to exponentially weighted moving average (EWMA) (Roberts, 1959) and cumulated sum (CUSUM) (Page, 1954) families. Multivariate charts are required for monitoring the vector of the GMVP weights. Golosnoy and Schmid (2007) and Golosnoy et al. (2010) propose several multivariate EWMA control charts (cf. Lowry et al., 1992) for monitoring the GMVP composition. This paper elaborates further tools for the surveillance of the GMVP weights and contributes to the current literature in four main aspects. First, several multivariate CUSUM control charts are developed for sequentially monitoring the GMVP weights. Second, the detecting ability of the suggested CUSUM schemes is investigated within an extensive Monte Carlo simulation study. Third, we consider the possibility of combining different charts in order to exploit their individual advantages for the GMVP surveillance. Fourth, an empirical study illustrates how the information obtained by the control chart signals can be used for portfolio volatility reduction.

In the current paper we develop several CUSUM charts for monitoring changes in the GMVP composition. The monitored processes are the estimated GMVP weights $\hat{\mathbf{w}}_{n,t}$ as well as the auxiliary characteristic $\mathbf{q}_t$, which is related to the estimated GMVP weights but exhibits more appealing stochastic properties (Golosnoy et al., 2010). Thus we extend the family of EWMA schemes for the GMVP surveillance by suggesting multivariate CUSUM-w and CUSUM-q charts, respectively. The popular MCUSUM1 and MCUSUM2 charts of Pignatiello and Runger (1990) as well as the projection pursuit (PPCUSUM) scheme of Ngai and Zhang (2001) are adapted for our purposes and analyzed within a Monte Carlo study. The optimal chart design is identified for different types of changes in the covariance matrix of asset returns, leading to the alterations in the GMVP composition. Comparing the performance of the CUSUM schemes to those of the EWMA charts, we find that the CUSUM schemes exhibit a similar behavior to the EWMA charts for many types of changes in the GMVP weights. Remarkably, the CUSUM charts outperform the EWMA counterparts in detection of large shifts in the GMVP composition. Since there is no single control chart which outperforms alternative schemes for all considered shifts in the GMVP proportions, we consider the idea to combine control charts in order to exploit the advantages of the different schemes simultaneously in case of uncertainty about coming changes.

The empirical application illustrates the usefulness of control chart signals for minimizing the ex ante variance of portfolio returns. We suggest to interpret the obtained signals as a failure of the target GMVP weights and propose to close risky positions for one period in these situations. If there is another signal in the coming period, the positions stay closed, otherwise the investor selects the target portfolio again. Such a strategy leads to a significant reduction of the out-of-sample portfolio variance compared to benchmark portfolio approaches which neglect signal information. A strategy using simultaneous signals appears to be especially successful in our study.

The rest of the paper is organized as follows. Section 2 describes the portfolio problem and formulates the surveillance task for the GMVP weights. Section 3 introduces the CUSUM control charts for monitoring the processes of the GMVP weights and the auxiliary characteristics. Section 4 investigates the properties of the CUSUM charts by extensive Monte Carlo simulations. Section 5 provides an empirical application of our monitoring methodology for portfolio volatility reduction. Section 6 concludes, while the proofs are provided in the Appendix.

2. Portfolio problem

2.1. Global minimum variance portfolio

Suppose that there are k risky assets available for the portfolio composition. Assume that the vector of asset returns $\mathbf{X}_t$ follows a k-dimensional normal distribution, i.e. $\mathbf{X}_t \sim \mathcal{N}_k(\boldsymbol{\mu}_t, \boldsymbol{\Sigma}_t)$ for $t \in \mathbb{N}$. This is a common assumption for a broad class of financial models (cf. Tsay, 2005). Moreover, the asset returns are assumed to be not autocorrelated for all t, which is in line with the efficient market hypothesis. A single investment period lasts from t till $t+1$. Following Markowitz (1952), the optimal portfolio weight $\boldsymbol{w}_t$ at time t should depend on the unknown moments $\boldsymbol{\mu}_{t+1}$ and $\boldsymbol{\Sigma}_{t+1}$. In practice, these quantities have to be estimated based on the information set available at time t.

There are considerable difficulties in estimating and predicting the mean vector $\boldsymbol{\mu}_{t+1}$ (Merton, 1980). The errors arising by the mean estimation have a damaging impact on the portfolio performance (Best and Grauer, 1991). The covariance matrix $\boldsymbol{\Sigma}_{t+1}$ exhibits much more predictability because of (persistent) volatility clusters which indicate a disagreement of the market participants about future cash flows on risky assets (cf. Tsay, 2005). The volatility clusters cause pronounced positive autocorrelations in the absolute and squared series of the daily asset returns. These are the reasons for selecting the global minimum variance portfolio (GMVP), which provides the weights leading to the smallest attainable portfolio variance under restriction that the sum of the weights is equal to 1. The GMVP weights are given by

$$\mathbf{w}_t = \frac{\boldsymbol{\Sigma}_{t+1}^{-1}\mathbf{1}}{\mathbf{1}'\boldsymbol{\Sigma}_{t+1}^{-1}\mathbf{1}}, \tag{1}$$

where $\mathbf{1}$ is a k-dimensional vector whose components are equal to one. The GMVP weights do not require a knowledge of the mean vector $\boldsymbol{\mu}_{t+1}$, so that the estimation risk caused by an unknown mean returns is eliminated. Note that both negative elements as well as weights greater than 1 are permitted in the vector $\mathbf{w}_t$.

There are various approaches in financial applications for modeling the development of the covariance matrix $\boldsymbol{\Sigma}_t$ over time. This paper assumes that there exist some (local) time periods where the covariance matrix remains constant, i.e. $\boldsymbol{\Sigma}_t = \boldsymbol{\Sigma}$. Then, however, at some change point $t = \tau$, there is a change in the covariance matrix such that $\boldsymbol{\Sigma}_t = \boldsymbol{\Sigma}_1 \neq \boldsymbol{\Sigma}$. Then the new covariance $\boldsymbol{\Sigma}_1$ remains constant until the next change point occurs, etc. Such an approach corresponds to the locally constant volatility modeling of Hsu et al. (1974) which has been investigated in numerous studies like, e.g., Chen and Gupta (1997), Kim and Kon (1999), Härdle et al. (2003), Chen et al. (2010), Fried (in press). This model presumes that changes in the covariance matrix do not happen often and that the matrix stays constant between two subsequent change points. Since the number and the positions of the change points are not exactly specified, the locally constant volatility model is flexible enough to mimic the majority of stylized facts frequently found in volatility time series.

The locally constant volatility model implies that the GMVP weights are constant over some period of time. The GMVP investor is interested only in those changes in $\boldsymbol{\Sigma}_t$, which alter the GMVP composition $\mathbf{w}_t$. Thus, it is sufficient to consider the k-dimensional vector of weights $\mathbf{w}_t$ instead of the covariance matrix $\boldsymbol{\Sigma}_t$ with $k(k+1)/2$ parameters. Assume that for $t \leq 0$ the GMVP weights remain constant $\mathbf{w}_t = \mathbf{w}$, where the target vector $\mathbf{w}$ is presumed to be known. The time until a fixed, but unknown change point $\tau \in \mathbb{N}$ is called the in-control phase. The time period after a change in $\boldsymbol{\Sigma}_t$ leading to a change in the GMVP weights is the out-of-control phase. The latter starts from $t \geq \tau$ and is characterized by $\boldsymbol{\Sigma}_t = \boldsymbol{\Sigma}_1 \neq \boldsymbol{\Sigma}$, so that $\mathbf{w}_t = \mathbf{w}_1 \neq \mathbf{w}$ with

$$\mathbf{w}_t = \mathbf{w} = \frac{\boldsymbol{\Sigma}^{-1}\mathbf{1}}{\mathbf{1}'\boldsymbol{\Sigma}^{-1}\mathbf{1}} \quad \text{for } t < \tau, \quad \text{and} \quad \mathbf{w}_t = \mathbf{w}_1 = \frac{\boldsymbol{\Sigma}_1^{-1}\mathbf{1}}{\mathbf{1}'\boldsymbol{\Sigma}_1^{-1}\mathbf{1}} \quad \text{for } t \geq \tau.$$

In order to detect a change from the in-control to the out-of-control phase, the investor should differentiate between the null hypothesis $H_{0,t} : \mathbf{w}_t = \mathbf{w}$ and the alternative $H_{1,t} : \mathbf{w}_t \neq \mathbf{w}$ at every new time point $t \geq 1$. Since it is unclear how many periods elapse until a change occurs or the null hypothesis $H_{0,t}$ is rejected for the first time, the number of consecutive decisions to make is a random variable. Thus, the monitoring problem is of sequential nature. Consequently, a usual fixed sample test is not appropriate in such a setting due to difficulties with controlling the overall size of the test (Woodall, 2000).

2.2. Sequential monitoring of the GMVP weights

SPC suggests control charts in order to deal with sequential decision problems. In the meantime control charts are one of the most important statistical tools. A one-sided control chart consists of a control statistic Z_t and a control limit h, which determines the rejection area. If the control statistic falls into the rejection area for the first time, a signal is given and the null hypothesis is rejected. The run length $L \geq 0$ of the control chart is the time until the first signal occurs. The run length is a random variable, which is defined for a one-sided control chart as

$$L = \inf\{t \in \mathbb{N} : Z_t > h\}. \tag{2}$$

Note that L depends on h, i.e. $L = L(h)$. The distribution of the run length L is the most important quantity for the characterization of a control chart. A good chart gives no signal for a long time if there is no actual change, i.e. the process is always in control with $\tau = \infty$. In this case the run length L should be large. Alternatively, a chart should also provide a quick signal after the change point, e.g. for $t \geq \tau$. Thus, the distance $L - \tau$ given $L \geq \tau$ should be as small as possible in this case. The average run length (ARL) is denoted by $E_\tau(L)$ for a given change point τ. Assuming that $\tau = \infty$ the in-control ARL $E(L)$ denotes the average time until the first false signal happens. For the out-of-control ARL $E_1(L)$, however, it is assumed that a change happens already at $\tau = 1$, so that it equals the average time until the first (correct) signal.

The control limit h is chosen with respect to the in-control distribution of L for each given control chart. In order to determine h, each chart is calibrated with respect to its in-control distribution. More precisely, the control limit h is the solution of the equation

$$E(L(h)) \stackrel{!}{=} \xi. \tag{3}$$

The quantity ξ is a given value which equals the average number of periods before the first false signal occurs. It is often taken as $\xi = 500$ in engineering applications (Montgomery, 2005), while in financial applications the value of ξ is chosen smaller for daily decisions, e.g. about 100–150 days or 1/2 year of daily observations (Golosnoy et al., 2008).

The performance of competing charts is usually compared via the out-of-control ARL $E_1(L)$. Since the assumption of a change already at $\tau = 1$ is rather restrictive, further measures have been introduced. The comprehensive overview of the goodness measure for sequential problems is provided by Frisén (2003). The worst case conditional expected delay (WED) measure of Pollak and Siegmund (1985), denoted by $\mathcal{D}_{PS}$, is given as

$$\mathcal{D}_{PS} = \sup_m \mathcal{D}_m, \qquad \mathcal{D}_m = E_m(L - m + 1 | L \geq m), \quad m \in \mathbb{N}. \tag{4}$$

This measure is frequently applied for performance evaluation of multivariate charts (Frisén et al., 2010). Since the change point τ may arise at any position, the quantity m takes values within $\mathbb{N}$. The practical calculation of the WED is computationally demanding because it can only be evaluated at a finite number of possible positions. In our simulation study we consider the values $m \in \{1, 2, \ldots, 30\}$ as in Golosnoy et al. (2010).

2.3. Process characteristics

The control statistic Z_t is determined by means of the observations from the process of daily asset returns $\{\mathbf{X}_t\}$. These returns are used for the estimation of the GMVP weights $\hat{\mathbf{w}}_{n,t}$. For this purpose the unknown covariance matrix is replaced by its sample estimator $\hat{\mathbf{\Sigma}}_{n,t}$, which is based on n previously observed daily returns

$$\hat{\mathbf{\Sigma}}_{n,t} = \frac{1}{n-1}\sum_{i=t-n+1}^{t}(\mathbf{X}_i - \bar{\mathbf{X}}_t)(\mathbf{X}_i - \bar{\mathbf{X}}_t)', \quad \bar{\mathbf{X}}_t = \frac{1}{n}\sum_{i=t-n+1}^{t}\mathbf{X}_i. \tag{5}$$

The corresponding estimator of the GMVP weights is given by

$$\hat{\mathbf{w}}_{n,t} = \frac{\hat{\mathbf{\Sigma}}_{n,t}^{-1}\mathbf{1}}{\mathbf{1}'\hat{\mathbf{\Sigma}}_{n,t}^{-1}\mathbf{1}}, \tag{6}$$

so that the GMVP weights are exclusively a function of the covariance matrix estimator $\hat{\mathbf{\Sigma}}_{n,t}$. The estimated GMVP weights should be recalculated at any new point in time. Assuming independent and normally distributed asset returns, Okhrin and Schmid (2006) show that

$$E(\hat{\mathbf{w}}_{n,t}) = \mathbf{w}, \qquad Cov(\hat{\mathbf{w}}_{n,t}) = \frac{1}{n-k-1}\frac{\mathbf{Q}}{\mathbf{1}'\mathbf{\Sigma}^{-1}\mathbf{1}}, \quad \mathbf{Q} = \mathbf{\Sigma}^{-1} - \frac{\mathbf{\Sigma}^{-1}\mathbf{1}\mathbf{1}'\mathbf{\Sigma}^{-1}}{\mathbf{1}'\mathbf{\Sigma}^{-1}\mathbf{1}}. \tag{7}$$

Using these expressions Golosnoy and Schmid (2007) develop and analyze several EWMA procedures based on the process $\hat{\mathbf{w}}_{n,t}$ which are denoted in the following as EWMA-w control charts.

However, monitoring the process of weights $\{\hat{\mathbf{w}}_{n,t}\}$ is not the only possibility to detect alterations from the GMVP composition. The estimated GMVP weights exhibit some statistical properties which are undesirable from the SPC point of view. In particular, the weights are strongly autocorrelated due to their construction via the overlapping estimation windows used for the calculation of the covariance matrix in (5). Not accounting for autocorrelation can lead to undesired consequences for the control chart performance (cf. Knoth and Schmid, 2004). The strong positive autocorrelation in the process of the estimated weights is the reason for the unsatisfactory performance of the monitoring schemes for detecting certain types of changes (Golosnoy and Schmid, 2007). In order to improve the detection ability, Golosnoy et al. (2010) suggest to monitor another process characteristic, further denoted as $\mathbf{q}_t$, which is related to the estimated GMVP weights but has more appealing stochastic properties from the SPC viewpoint. The auxiliary process $\{\mathbf{q}_t\}$ is defined as

$$\mathbf{q}_t = -\mathbf{Q}((\mathbf{X}_t - \boldsymbol{\mu})(\mathbf{X}_t - \boldsymbol{\mu})' - \mathbf{\Sigma})\mathbf{w} = -\mathbf{Q}(\mathbf{X}_t - \boldsymbol{\mu})(\mathbf{X}_t - \boldsymbol{\mu})'\mathbf{w}.$$

The stochastic properties of the process $\mathbf{q}_t$ are derived by Golosnoy et al. (2010). The first two moments of $\mathbf{q}_t$ in the in-control state are given by

$$E(\mathbf{q}_t) = \mathbf{0}, \qquad Cov(\mathbf{q}_t) = \frac{\mathbf{Q}}{\mathbf{1}'\mathbf{\Sigma}^{-1}\mathbf{1}}. \tag{8}$$

It holds for the out-of-control situation that

$$E_1(\mathbf{q}_t) = -\mathbf{Q}\,\mathbf{\Sigma}_1\mathbf{w}, \qquad Cov_1(\mathbf{q}_t) = \mathbf{Q}(\mathbf{\Sigma}_1\mathbf{w}\mathbf{w}'\mathbf{\Sigma}_1 + (\mathbf{w}'\mathbf{\Sigma}_1\mathbf{w})\mathbf{\Sigma}_1)\mathbf{Q}.$$

The variables $\{\mathbf{q}_t\}$ are not autocorrelated, moreover, they are independent assuming normally distributed asset returns. These properties are advantageous for the application of control charts. Contrary to the GMVP weights $\hat{\mathbf{w}}_{n,t}$, the auxiliary vector $\mathbf{q}_t$ does not depend on the window size n, which is another advantage for the practical implementation. In order to detect a change in the weight process the mean behavior of the characteristic quantity $\{\mathbf{q}_t\}$ is monitored. Note that a change in the mean of $\{\mathbf{q}_t\}$ implies that a change in the optimal weights has occurred as well. Thus $\mathbf{q}_t$ provides a suitable measure for our purposes.

Since both vectors $\hat{\mathbf{w}}_{n,t}$ and $\mathbf{q}_t$ sum up to one, their covariance matrices do not have the full rank. Further analysis is conducted based on the $k-1$ dimensional vectors $\hat{\mathbf{w}}^*_{n,t}$ and $\mathbf{q}^*_t$, which are obtained by deleting the k-th element of the vectors $\hat{\mathbf{w}}_{n,t}$ and $\mathbf{q}_t$. Then the corresponding covariance matrices of $\hat{\mathbf{w}}^*_{n,t}$ and $\mathbf{q}^*_t$ are positive definite. The EWMA-w and EWMA-q control charts for the processes $\{\hat{\mathbf{w}}^*_{n,t}\}$ and $\{\mathbf{q}^*_t\}$ are investigated in detail in Golosnoy et al. (2010). Although EMWA schemes are popular in practice due to their simplicity, CUSUM schemes are of importance as well. CUSUM control charts originate from Wald's sequential probability ratio test and show a set of appealing optimality properties for detecting changes (Moustakides, 1986, 2004). In this paper we introduce several multivariate CUSUM-w and CUSUM-q control schemes for the GMVP surveillance, which are based on the demeaned sums of w- and q-characteristics. The required in-control expectations of these sums are zeros:

$$E\left(\sum_{i=t-b+1}^{t}(\hat{\mathbf{w}}_{n,i} - \mathbf{w})\right) = \mathbf{0}, \qquad E\left(\sum_{i=t-b+1}^{t}\mathbf{q}_i\right) = \mathbf{0}, \quad b \geq 1,\ b \in \mathbb{N}.$$

3. CUSUM charts for portfolio surveillance

Now we introduce several CUSUM control charts based on $\{\hat{\mathbf{w}}^*_{n,t}\}$ and $\{\mathbf{q}^*_t\}$ processes for monitoring the GMVP weights. Since a change in the GMVP weights can arise in any direction, only directionally invariant multivariate CUSUM charts are considered here. The MCUSUM1 and MCUSUM2 procedures of Pignatiello and Runger (1990) and the PPCUSUM scheme of Ngai and Zhang (2001) are applied as CUSUM-w and CUSUM-q charts. Note that all charts start at $t = 1$.

3.1. CUSUM-w control charts

The MCUSUM1 chart for the weights resembles the MCUSUM1 chart of Pignatiello and Runger and uses the sum vector $\mathbf{s}_{t-b_t,t} = \sum_{i=t-b_t+1}^{t}(\hat{\mathbf{w}}^*_{n,i} - \mathbf{w}^*)$. Its is given by

$$wMC1_t = \max\left\{\sqrt{b_t}\sqrt{\mathbf{s}'_{t-b_t,t}Cov(\mathbf{s}_{t-b_t,t})^{-1}\mathbf{s}_{t-b_t,t}} - \frac{1}{2}gb_t, 0\right\}, \quad t \geq 1.$$

The reference quantity $g \geq 0$ is related to the expected shift size. Its choice is discussed later in Section 4.1. The scheme gives a signal if $wMC1_t > h_{wMC1}$. The positive integer b_t equals the number of periods since the most recent renewal of the vector $\mathbf{s}_{t-b_t,t}$, $t \geq 1$:

$$b_t = \begin{cases} b_{t-1} + 1 & \text{if } wMC1_{t-1} > 0, \\ 1 & \text{otherwise.} \end{cases}$$

Moreover, the starting values are $b_0 = 0$ and $wMC1_0 = 0$.

The MCUSUM2 chart for the weights is based on a further control scheme of Pignatiello and Runger. Let the quantity

$$wD^2_t = (\hat{\mathbf{w}}^*_{n,t} - \mathbf{w}^*)'Cov(\hat{\mathbf{w}}^*_{n,t})^{-1}(\hat{\mathbf{w}}^*_{n,t} - \mathbf{w}^*)$$

measure the distance between the estimated weights and the in-control GMVP weights. The chart is based on the control statistic

$$wMC2_t = \max\left\{0, wMC2_{t-1} + wD^2_t - \frac{1}{2}g^2 - (k-1)\right\}, \qquad wMC2_0 = 0.$$

A signal is given if $wMC2_t > h_{wMC2}$. It holds that $g \geq 0$.

The PPCUSUM chart for the weights makes use of the projected pursuit CUSUM procedure of Ngai and Zhang. Denoting $\mathbf{s}_{t-b,t} = \sum_{i=t-b+1}^{t}(\hat{\mathbf{w}}^*_{n,i} - \mathbf{w}^*)$ for $b = 0, 1, \ldots, t-1$, define the norm

$$\|\mathbf{s}_{t-b,t}\| = \sqrt{b}\sqrt{\mathbf{s}'_{t-b,t}Cov(\mathbf{s}_{t-b,t})^{-1}\mathbf{s}_{t-b,t}}.$$

Then the PPCUSUM statistic is given by

$$wPP_t = \max\{0, \|\mathbf{s}_{t-1,t}\| - g/2, \|\mathbf{s}_{t-2,t}\| - 2g/2, \ldots, \|\mathbf{s}_{0,t}\| - tg/2\}, \quad g \geq 0.$$

A signal is given at time point t if $wPP_t > h_{wPP}$.

3.2. CUSUM-q control charts

The MCUSUM1 chart for the process $\{\mathbf{q}^*_t\}$ is based on the statistic

$$qMC1_t = \max\left\{\sqrt{\mathbf{v}'_{t-b_t,t}Cov(\mathbf{v}_{t-b_t,t})^{-1}\mathbf{v}_{t-b_t,t}} - \frac{1}{2}gb_t, 0\right\}, \quad \mathbf{v}_{t-b_t,t} = \sum_{i=t-b_t+1}^{t}\mathbf{q}^*_i,$$

with the starting value $qMC1_0 = 0$. The number of periods after the last renewal b_t is given by

$$b_t = \begin{cases} b_{t-1} + 1 & \text{if } qMC1_{t-1} > 0, \\ 1 & \text{otherwise.} \end{cases}$$

A signal is given if $qMC1_t > h_{qMC1}$.

The MCUSUM2 chart for the process $\{\mathbf{q}^*_t\}$ makes use of the quadratic form

$$qD^2_t = \mathbf{q}^{*\prime}_t Cov(\mathbf{q}^*_t)^{-1}\mathbf{q}^*_t.$$

The control statistic is defined as

$$qMC2_t = \max\left\{0, qMC2_t + qD^2_t - \frac{1}{2}g^2 - k + 1\right\}, \qquad qMC2_0 = 0.$$

A signal occurs if $qMC2_t > h_{qMC2}$.

The PPCUSUM chart for $\{\mathbf{q}^*_t\}$ is based on the PPCUSUM statistic

$$qPP_t = \max\{0, \|\mathbf{v}_{t-1,t}\| - g/2, \ldots, \|\mathbf{v}_{0,t}\| - tg/2\}, \qquad \mathbf{v}_{t-b,t} = \sum_{i=t-b+1}^{t}\mathbf{q}^*_i.$$

A signal is triggered at time point t if $qPP_t > h_{qPP}$.

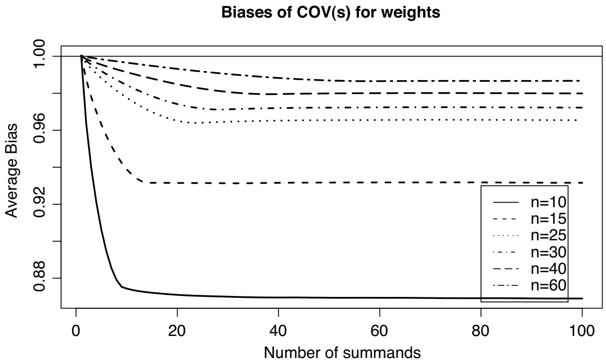

Fig. 1. Finite sample bias $B(b)$ for the approximated covariance matrix $Cov^a(\mathbf{s}_{t-b,t})$ as a function of the number of summands b, calculated based on 10^7 replications.

3.3. Derivation of the in-control covariance matrices

Statements about the in-control covariance matrices of the sums $\mathbf{s}_{l,u} = \sum_{i=l+1}^{u}(\hat{\mathbf{w}}^*_{n,i} - \mathbf{w}^*)$ and $\mathbf{v}_{l,u} = \sum_{i=l+1}^{u}\mathbf{q}^*_i$ are required for calculating the design of the w- and q-CUSUM control charts.

Assuming the process $\{\mathbf{X}_t\}$ to be strictly stationary, it holds that the distribution function of $\{\hat{\mathbf{w}}^*_{n,t}\}$ is also strictly stationary for fixed n, so that $Cov(\hat{\mathbf{w}}^*_{n,t}, \hat{\mathbf{w}}^*_{n,t-s})$ does not depend on t but only on the time lag s. Since $E(\mathbf{s}_t) = \mathbf{0}$ we obtain for $1 \le l \le u$ that

$$Cov(\mathbf{s}_{l,u}) = (u-l)Cov(\hat{\mathbf{w}}^*_{n,t}) + 2\sum_{i=1}^{u-l-1}(u-l-i)Cov(\hat{\mathbf{w}}^*_{n,t}, \hat{\mathbf{w}}^*_{n,t-i}).$$

Due to $Cov(\hat{\mathbf{w}}^*_{n,t}, \hat{\mathbf{w}}^*_{n,t-i}) = 0$ for $i \ge n$, it follows that

$$Cov(\mathbf{s}_{l,u}) = (u-l)Cov(\hat{\mathbf{w}}^*_{n,t}) + 2\sum_{i=1}^{min\{n-1,u-l-1\}}(u-l-i)Cov(\hat{\mathbf{w}}^*_{n,t}, \hat{\mathbf{w}}^*_{n,t-i}). \tag{9}$$

Assuming the process $\{\mathbf{X}_t\}$ be a sequence of independent random vectors with $\mathbf{X}_t \sim \mathcal{N}_k(\boldsymbol{\mu}, \boldsymbol{\Sigma})$, Golosnoy and Schmid (2007) show that for $s < n$ the covariance matrix $Cov(\hat{\mathbf{w}}^*_{n,t}, \hat{\mathbf{w}}^*_{n,t-s})$ can be suitably approximated by the matrix $Cov^a(\hat{\mathbf{w}}^*_{n,t}, \hat{\mathbf{w}}^*_{n,t-s}) = (n-s-1)Cov(\hat{\mathbf{w}}^*_{n,t})/(n-1)$, because these matrices are asymptotically equal for s fixed and $n \to \infty$. Furthermore, they show within a simulation study that this approximation works well even in the case of sufficiently large finite n values. Replacing $Cov(\hat{\mathbf{w}}^*_{n,t}, \hat{\mathbf{w}}^*_{n,t-s})$ by $Cov^a(\hat{\mathbf{w}}^*_{n,t}, \hat{\mathbf{w}}^*_{n,t-s})$ in (9), we get the following approximation to the covariance matrix of $\mathbf{s}_{t-b,t} = \sum_{i=t-b+1}^{t}(\hat{\mathbf{w}}^*_{n,i} - \mathbf{w}^*)$

$$Cov^a(\mathbf{s}_{t-b,t}) = \begin{cases} Cov(\hat{\mathbf{w}}^*_{n,t})(b - b^3 + 3b^2n - 3b^2)/(3n-3) & \text{for } b < n, \\ Cov(\hat{\mathbf{w}}^*_{n,t})\,(b(n-1) - n(n-2)/3) & \text{for } b \ge n. \end{cases} \tag{10}$$

The derivation of this result is given in the Appendix.

Since we are primarily interested in small estimation periods with $n \le 60$, the finite sample bias in the covariance matrix $Cov^a(\mathbf{s}_{t-b,t})$ should be evaluated. The bias is calculated as

$$B(b) = \mathbf{1}'(\widehat{Cov}(\mathbf{s}_{t-b,t}) \div Cov^a(\mathbf{s}_{t-b,t}))\mathbf{1}/k^2,$$

where the operator $\div$ stands for the element-by-element matrix division. The matrix $\widehat{Cov}(\mathbf{s}_{t-b,t})$ is the sample estimator of the exact covariance matrix obtained within a simulation study with a very high number of replications. Fig. 1 plots the average finite sample bias for the parameters provided in Section 4.1. The bias in Fig. 1 is less than 5% for $n \ge 25$ but increases for smaller values of n. Alternatively, the exact in-control covariance matrix could be directly calculated within a Monte Carlo simulation study or via a nonparametric bootstrap procedure based on historical asset returns. Such an approach would allow us to weaken the distributional assumptions on the asset return distribution. The exact in-control results for $Cov(\mathbf{v}_{l,u})$ can be easier obtained. Assuming that $\{\mathbf{X}_t\}$ is a sequence of independent and identically distributed variables it follows that the variables $\mathbf{q}^*_t$ are independent as well, so that $Cov(\mathbf{v}_{l,u}) = (u-l+1)\ Cov(\mathbf{q}^*_t)$.

3.4. EWMA control charts

The multivariate EWMA-w and EWMA-q control charts are devised by Golosnoy and Schmid (2007) and Golosnoy et al. (2010). Here we provide the control statistics and stopping rules for these schemes, whereas further details can be found in the original papers. Let $\mathbf{c}_t = \hat{\mathbf{w}}^*_{n,t}$ for the EWMA-w charts and let $\mathbf{c}_t = \mathbf{q}^*_t$ for the EWMA-q charts.

The Mahalanobis EWMA charts are constructed by transforming the vector of interest $\mathbf{c}_t$ to a univariate quantity by taking the Mahalanobis distance between $\mathbf{c}_t$ and $E(\mathbf{c}_t)$, namely $T_t = (\mathbf{c}_t - E(\mathbf{c}_t))' Cov(\mathbf{c}_t)^{-1}(\mathbf{c}_t - E(\mathbf{c}_t))$. Then a univariate EWMA recursion is applied to this quantity. For $t \geq 1$ the control statistic is given by

$$Z_t = (1-\lambda)Z_{t-1} + \lambda T_t, \tag{11}$$

with $Z_0 = E(T_t) = k - 1$. The smoothing factor λ takes values within the interval $(0, 1]$. The control charts give a signal at time $t \geq 1$ if $Z_t > h$.

The MEWMA charts are based on the idea to apply a multivariate EWMA recursion directly to the components of the characteristic $\mathbf{c}_t$. The advantage of this approach is that each element obtains its own smoothing factor, allowing more flexibility compared to the univariate EWMA. The MEWMA recursion is given as

$$\mathbf{Z}_t = (\mathbf{I} - \mathbf{R})\mathbf{Z}_{t-1} + \mathbf{R}\mathbf{c}_t, \qquad \mathbf{Z}_0 = E(\mathbf{c}_t). \tag{12}$$

The symbol $\mathbf{I}$ stands for the $(k-1)\times(k-1)$ identity matrix and $\mathbf{R} = diag(r_1, \ldots, r_{k-1})$ is a diagonal matrix with elements $0 < r_i \leq 1$ for $i \in \{1, \ldots, k-1\}$. The vector $\mathbf{Z}_t$ can be written as

$$\mathbf{Z}_t = (\mathbf{I} - \mathbf{R})^t \mathbf{Z}_0 + \mathbf{R}\sum_{v=0}^{t-1}(\mathbf{I} - \mathbf{R})^v \mathbf{c}_{t-v}. \tag{13}$$

If $E(\mathbf{c}_t)$ is constant over time it holds that $E(\mathbf{Z}_t) = E(\mathbf{c}_t)$. A signal is given if

$$(\mathbf{Z}_t - E(\mathbf{Z}_t))' \; Cov(\mathbf{Z}_t)^{-1} \; (\mathbf{Z}_t - E(\mathbf{Z}_t)) \; > h. \tag{14}$$

The in-control covariance matrices $Cov(\mathbf{Z}_t)$ are derived by Golosnoy and Schmid (2007) for the w-process and by Golosnoy et al. (2010) for the q-process.

4. Simulation study

The properties of the CUSUM schemes introduced in Section 3 are analyzed within an extensive Monte Carlo simulation study, where the detecting ability of the CUSUM schemes is compared with those in the EWMA charts for various types of changes. First, the control limits for all charts are determined such that the charts provide the same in-control ARLs. Then, using the obtained control limits, the out-of-control performance is evaluated by computing the out-of-control ARL and WED measures. Since no explicit formulas are available for the run length distribution of these control schemes, these quantities are estimated within a simulation study.

4.1. Design of the study

The design of the study is chosen similar to Golosnoy et al. (2010) for simplifying the comparison of the EWMA and CUSUM control procedures. Consider a portfolio with $k = 4$ risky assets. The asset returns are assumed to be independent over time and to follow a multivariate normal distribution $\mathbf{X}_t \sim \mathcal{N}_k(\boldsymbol{\mu}, \boldsymbol{\Sigma})$. The annualized in-control asset return distribution parameters are the mean vector $\boldsymbol{\mu} = (0.07, 0.085, 0.11, 0.125)'$ and the covariance matrix

$$\boldsymbol{\Sigma} = \mathbf{S}\boldsymbol{\Upsilon}\mathbf{S} = \begin{pmatrix} 0.135 & 0 & 0 & 0 \\ 0 & 0.15 & 0 & 0 \\ 0 & 0 & 0.17 & 0 \\ 0 & 0 & 0 & 0.19 \end{pmatrix} \begin{pmatrix} 1 & 0.4 & 0.35 & 0.3 \\ 0.4 & 1 & 0.45 & 0.4 \\ 0.35 & 0.45 & 1 & 0.5 \\ 0.3 & 0.4 & 0.5 & 1 \end{pmatrix} \begin{pmatrix} 0.135 & 0 & 0 & 0 \\ 0 & 0.15 & 0 & 0 \\ 0 & 0 & 0.17 & 0 \\ 0 & 0 & 0 & 0.19 \end{pmatrix},$$

where the diagonal matrix $\mathbf{S}$ contains the standard deviations, while $\boldsymbol{\Upsilon}$ is the correlation matrix. These values are close to the long term estimates for the major stock market indices. The daily parameters are obtained by dividing the distribution parameters by a factor 250, i.e. $\boldsymbol{\mu}/250$ and $\boldsymbol{\Sigma}/250$, where 250 roughly corresponds to the number of trading days per year. The corresponding true vector of the GMVP weights is $\mathbf{w} = (0.4766, 0.2735, 0.1469, 0.1030)'$.

The EWMA control charts do not require a specification of the out-of-control state for their implementation. On the contrary, there is a requirement to know the expected shifts in order to establish the optimality of CUSUM charts (cf. Montgomery, 2005). The parameter g of the CUSUM charts (Pignatiello and Runger, 1990; Ngai and Zhang, 2001) is related to the expected shift size in the GMVP weights. Since the true shifts are usually unknown, the smallest out-of-control ARL and WED are identified with respect to various choices of g for all considered CUSUM schemes. All control charts are calibrated to provide the same in-control ARL $\xi = 120$ days for all pairs of the parameter values, namely (n, λ) for the EWMA and (n, g) for the CUSUM schemes. The length of the estimation window n is set equal to $n \in \{10, 15, 25, 30, 40, 60\}$. The EWMA smoothing parameter λ takes values from the set $\{0.1, 0.2, \ldots, 1.0\}$, while the CUSUM reference values g are taken from the set $g \in \{0.0, 0.2, \ldots, 4.0\}$. The values of g are motivated by the shift sizes in our study.

The in-control covariance matrix $\boldsymbol{\Sigma}$ alters to the new matrix $\boldsymbol{\Sigma}_1$, which determines the out-of-control GMVP weights $\mathbf{w}_1$. Following Golosnoy et al. (2010), our study concentrates on two types of changes in the covariance matrix. Study I considers only changes in the variances of the asset returns. In particular, we change the first and the fourth asset variances. The

Table 1
Absolute distance (AD) and squared Mahalanobis distance (MD^2) between the in- and out-of-control GMVP weights.

Study I	$d_1 = 0.5$	$d_1 = 1.0$	$d_1 = 2.0$	$d_1 = 3.0$
$d_4 = 0.5$	0.996 1.337	0.967 1.775	0.953 1.761	0.914 1.621
$d_4 = 1.0$	1.217 3.843	AD = 0 $\text{MD}^2 = 0$	0.352 0.318	0.349 0.327
$d_4 = 2.0$	1.479 6.376	0.909 1.567	1.231 2.109	1.234 2.118
$d_4 = 3.0$	1.469 6.454	1.054 2.107	1.379 2.728	1.382 2.721

Study II $(d_1, \ldots, d_4)$	$\zeta = 0.5$	$\zeta = 1.0$	$\zeta = 1.5$
(1.0, 1.0, 1.0, 1.0)	0.038 0.061	AD = 0 $\text{MD}^2 = 0$	0.041 0.111
(1.1, 1.2, 1.3, 1.4)	0.018 0.005	0.069 0.149	0.128 0.611
(1.2, 1.4, 1.6, 1.8)	0.046 0.067	0.098 0.369	0.155 1.029
(1.3, 1.6, 1.9, 2.2)	0.062 0.155	0.112 0.566	0.163 1.334
(1.4, 1.8, 2.2, 2.6)	0.072 0.243	0.120 0.730	0.165 1.561

out-of-control covariance matrix equals $\mathbf{\Sigma}_1 = \mathbf{D\Sigma D}$ with $\mathbf{D} = diag(d_1, 1, 1, d_4)$. The values for the change parameters are $d_1 \in \{0.5, 1, 2, 3\}$ and $d_4 \in \{0.5, 1, 2, 3\}$. Altogether, there are 15 different out-of-control situations in study I.

Study II considers both changes in the variances and the correlations. They are modeled by using the matrices $\mathbf{D}$ and $\boldsymbol{\Upsilon}_1$ with

$$\mathbf{D} = \begin{pmatrix} d_1 & 0 & 0 & 0 \\ 0 & d_2 & 0 & 0 \\ 0 & 0 & d_3 & 0 \\ 0 & 0 & 0 & d_4 \end{pmatrix}, \qquad \boldsymbol{\Upsilon}_1 = \begin{pmatrix} 1 & 0.4\zeta & 0.35\zeta & 0.3\zeta \\ 0.4\zeta & 1 & 0.45\zeta & 0.4\zeta \\ 0.35\zeta & 0.45\zeta & 1 & 0.5\zeta \\ 0.3\zeta & 0.4\zeta & 0.5\zeta & 1 \end{pmatrix}.$$

The out-of-control matrix is given by $\mathbf{\Sigma}_1 = \mathbf{DS}\boldsymbol{\Upsilon}_1\mathbf{SD}$. Moreover, we select $d_i = 1 + d \cdot i/10$ and choose $d \in \{0, 1, 2, 4\}$, $\zeta \in \{0.5, 1, 1.5\}$. Thus the choice of the pair of parameters (d, ζ) governs this type of change. There are 14 distinct out-of-control cases in study II.

The impact of these changes on the GMVP weights is illustrated by the measures between the true in- and out-of-control GMVP weights $\mathbf{w}$ and $\mathbf{w}_1$. The first measure AD=$|\mathbf{w}_1 - \mathbf{w}|'\mathbf{1}$ is based on the sum of the absolute distances between the weights. The second measure is the weighted distance between the vectors $\mathbf{w}^*$ and $\mathbf{w}_1^*$:

$$\text{MD}^2 = (\mathbf{w}_1^* - \mathbf{w}^*)' Cov(\hat{\mathbf{w}}_{n,0}^*)^{-1}(\mathbf{w}_1^* - \mathbf{w}^*)/(n - k - 1),$$

whereas the divisor $n - k - 1$ provides independence on n. The parameter g of multivariate CUSUM charts should be of the same order as the MD distance. The MD and the AD distance for both studies I and II are presented in Table 1.

Table 1 shows that the changes of study I have a larger impact on the GMVP weights than the changes considered in study II. Thus, study I discusses the detection of comparatively large changes in the GMVP weights, whereas study II deals with signaling rather small changes. Of course, even the in-control process parameters are in general unknown. However, the covariance matrix could be estimated quite precisely, so that in the following we neglect the risks arising due to the in-control parameter estimation. The impact of estimation risk on the control chart behavior is analyzed by Jensen et al. (2006), Champ and Jones-Farmer (2007). In general, the number of false alarms increases if the covariance matrix estimates are used instead of the true values. However, the out-of-control ARLs are only marginally affected in the case of widening the control limits in order to account for additional estimation risk. In our study the estimation risk in $\mathbf{\Sigma}$ can be considered as negligible due to the comparatively small number of assets (here: $k = 4$) in the portfolio, but it would be more of importance for larger values of k.

The performance of the control charts is evaluated with the out-of-control ARL and the worst-case conditional expected delay (WED) measures. The out-of-control ARL is calculated under the restrictive assumption that the change happens immediately at $\tau = 1$. The WED measure is calculated for different changes points $m \in \{1, \ldots, 30\}$. Note that the calculation of the WED is of order 10^2 more consuming in terms of computational time than of time for the ARL calculation.

4.2. Simulation results

The results of study I, where the GMVP weights alter only due to changes in the asset variances, are reported in Table 2 for the out-of-control ARL and Table 3 for the WED measures. The smallest values are in bold, while the second smallest ones are typed in italic. The RL standard deviations and the optimal control chart parameters are reported in parentheses

Table 2
Sample estimators of the out-of-control ARLs for study I – changes asset variances only – based on 10^6 repetitions. The empirical standard deviations of the RLs, the best parameters g for CUSUM and λ for EWMA and the best n are given in parentheses.

$d_4 \cdot^{\cdot^{\cdot}} d_1$	0.5		1.0		2.0		3.0	
	26.92	(14.48, 2.0, 40)	26.74	(16.57, 0.6, 40)	26.48	(16.45, 0.6, 40)	27.18	(15.87, 3.0, 60)
	26.10	(13.55, 2.6, 40)	26.49	(12.87, 2.8, 40)	26.05	(12.40, 2.6, 40)	27.06	(15.57, 2.4, 60)
	26.13	(13.78, 1.8, 40)	*26.48*	(12.80, 1.8, 40)	26.08	(12.64, 1.8, 40)	27.11	(15.68, 2.6, 60)
	32.58	(12.81, 0.2)	33.02	(13.47, 0.2)	*10.20*	(7.49, 0.8)	*4.90*	(3.76, 1.2)
	369.39	(369.10, 4.0)	246.68	(245.99, 4.0)	16.82	(15.55, 1.8)	5.95	(5.21, 2.4)
0.5	41.42	(14.43, 0.2)	42.28	(15.17, 0.2)	11.31	(7.81, 0.8)	5.15	(3.89, 1.4)
	–	–	–	–	–	–	–	–
	26.14	(13.35, 0.5, 40)	26.52	(12.71, 0.7, 40)	26.07	(12.01, 0.4, 40)	27.04	(14.93, 0.3, 60)
	25.48	(15.02, 0.1, 30)	**25.18**	(14.26, 0.1, 25)	24.80	(13.00, 0.1, 30)	25.64	(16.57, 0.1, 60)
	347.62	(330.28, 1.0)	237.67	(235.07, 1.0)	15.96	(15.06, 0.1)	5.69	(4.92, 0.1)
	72.97	(67.21, 0.1)	66.87	(63.08, 0.1)	**8.81**	(8.03, 0.1)	**4.04**	(3.57, 0.1)
	19.84	(10.91, 0.8, 25)	*MCUSUM1-w*		44.50	(29.12, 0.2, 60)	36.47	(21.91, 0.2, 60)
	19.17	(8.67, 3.0, 25)	*MCUSUM 2-w*		47.32	(35.59, 0.4, 60)	41.66	(30.58, 0.0, 60)
	19.06	(8.75, 3.0, 25)	*PPCUSUM-w*		47.67	(36.96, 0.4, 60)	42.31	(33.65, 0.4, 60)
	35.98	(15.97, 0.2)	*MCUSUM 1-q*		*10.38*	(8.86, 1.0)	*4.76*	(3.91, 1.4)
	155.53	(155.01, 4.0)	*MCUSUM 2-q*		11.93	(10.34, 1.6)	5.12	(4.36, 2.4)
1.0	46.30	(17.91, 0.2)	*PPCUSUM-q*		10.51	(8.66, 1.2)	4.78	(3.86, 1.8)
	–	–	–	–	–	–	–	–
	19.08	(8.76, 1.0, 25)	*MahEWMA-w*		47.28	(40.85, 0.1, 60)	43.36	(39.43, 0.1, 60)
	19.07	(8.89, 0.7, 25)	*MEWMA-w*		44.21	(41.87, 0.1, 60)	40.38	(39.40, 0.1, 60)
	152.73	(152.21, 1.0)	*MahEWMA-q*		11.08	(9.93, 0.1)	4.87	(4.07, 0.1)
	64.94	(64.21, 0.1)	*MEWMA-q*		**8.32**	(8.53, 0.1)	**3.81**	(3.58, 0.1)
	13.18	(8.28, 3.0, 60)	15.47	(10.74, 3.0, 60)	13.93	(9.49, 3.0, 60)	12.46	(8.59, 3.0, 60)
	12.54	(8.10, 2.6, 60)	14.86	(10.65, 2.6, 60)	13.27	(9.38, 2.6, 60)	11.81	(8.51, 2.6, 60)
	12.48	(8.10, 2.2, 60)	14.82	(10.63, 3.0, 60)	13.22	(9.36, 2.8, 60)	11.75	(8.51, 2.8, 60)
	5.97	(4.48, 1.0)	*6.25*	(4.96, 1.0)	4.51	(3.77, 1.4)	3.18	(2.56, 1.6)
	7.47	(6.47, 2.0)	7.06	(6.02, 2.0)	4.49	(3.61, 2.2)	3.13	(2.42, 2.6)
2.0	6.38	(4.94, 1.4)	6.49	(5.33, 1.6)	4.44	(3.57, 2.0)	3.11	(2.42, 2.6)
	–	–	–	–	–	–	–	–
	12.53	(8.10, 1.0, 60)	14.87	(10.68, 1.0, 60)	13.25	(9.38, 1.0, 60)	11.79	(8.51, 1.0, 60)
	12.51	(8.08, 1.0, 60)	14.87	(10.69, 1.0, 60)	13.26	(9.39, 1.0, 60)	11.84	(8.52, 1.0, 60)
	7.0	2 (6.10, 0.1)	6.59	(5.63, 0.1)	*4.23*	(3.35, 0.1,)	*3.00*	(2.26, 0.1)
	4.77	(4.33, 0.1)	**4.86**	(4.66, 0.1)	**3.32**	(3.14, 0.1)	**2.42**	(2.09, 0.1)
	7.79	(5.10, 3.0, 60)	8.14	(5.55, 3.0, 60)	7.91	(5.46, 3.0, 60)	7.40	(5.06, 3.0, 60)
	6.96	(4.82, 2.6, 60)	7.33	(5.30, 2.6, 60)	7.06	(5.12, 2.6, 60)	6.56	(4.74, 2.6, 60)
	6.94	(4.81, 3.0, 60)	7.29	(5.29, 3.0, 60)	7.04	(5.12, 2.2, 60)	6.52	(4.72, 2.2, 60)
	2.99	(2.21, 1.6)	*2.99*	(2.25, 1.6)	2.61	(1.98, 1.8)	2.21	(1.61, 2.0)
	3.25	(2.61, 3.0)	3.17	(2.54, 3.0)	2.64	(1.99, 3.0)	2.20	(1.56, 3.2)
3.0	3.08	(2.32, 2.2)	3.06	(2.38, 2.6)	2.62	(1.95, 2.6)	2.18	(1.54, 3.0)
	–	–	–	–	–	–	–	-
	6.95	(4.82, 1.0, 60)	7.32	(5.31, 1.0, 60)	7.06	(5.12, 1.0, 60)	6.55	(4.74, 1.0, 60)
	6.94	(4.82, 1.0, 60)	7.33	(5.32, 1.0, 60)	7.04	(5.09, 1.0, 60)	6.53	(4.72, 1.0, 60)
	3.16	(2.47, 0.1)	3.08	(2.38, 0.1)	*2.56*	(1.86, 0.1)	*2.15*	(1.48, 0.1)
	2.46	(1.96, 0.1)	**2.44**	(1.99, 0.1)	**2.08**	(1.63, 0.1)	**1.79**	(1.30, 0.1)

for each given type of change. The standard deviations of the performance measures could be roughly obtained by dividing the RL standard deviations by the square root of the number of simulation runs, i.e. by $\sqrt{10^6} = 1000$ for the ARLs and by $\sqrt{10^5} \approx 316.2$ for the WEDs.

The smallest ARL and WED values obtained for the CUSUM charts are comparable with those of the EWMA charts. The best CUSUM procedures seem to be at least not worse than the EWMA schemes for the ARL and better for the WED criterion. In particular, the smallest WED is attained for the CUSUM charts in 11 cases out of 15, whereas the EWMA approach leads only in 4 cases to the smallest WEDs. The overall best performance for the changes analyzed in study I can be observed for the MCUSUM1-q and the MCUSUM2-w charts.

Golosnoy et al. (2010) find out that EWMA-q charts are more appropriate for detecting changes in GMVP weights due to increases in asset variances, while EWMA-w charts provide better results for detecting changes caused by asset variance decreases. The current study shows that these findings are still valid for the considered CUSUM charts. The comparison of the CUSUM-q charts with the CUSUM-w charts shows that similar to the EWMA approaches the q-schemes provide better results if the variances of the asset returns are increasing while the w-charts dominate if the variances are decreasing. Note that the majority of q-charts are not able to detect changes in the GMVP weights caused by variance reductions. A similar result is obtained by Golosnoy et al. (2010). It motivates the simultaneous use of both w-charts and q-charts to ensure a timely detection of different types of changes. The best CUSUM-q chart is the MCUSUM1-q scheme, as in Bodnar and Schmid (2007).

Table 3
Sample estimators of the WEDs for study I – changes in asset variances only – based on 10^5 repetitions. The empirical standard deviations of the RLs, the worst value of m, the best parameters g for CUSUM and λ for EWMA, and the best n are given in parentheses.

$d_4 \diagdown d_1$	0.5		1.0		2.0		3.0	
	27.25	(18.68, 0.6, 27, 40)	**26.88**	(16.80, 0.6, 27, 40)	26.59	(16.61, 0.6, 27, 40)	26.81	(19.11, 0.4, 9, 60)
	26.60	(12.81, 2.0, 29, 40)	*27.18*	(12.24, 2.0, 23, 40)	26.50	(11.81, 2.0, 23, 40)	27.76	(12.61, 2.0, 23, 40)
	27.97	(12.46, 1.6, 23, 40)	27.66	(12.50, 0.6, 15, 25)	27.64	(11.02, 1.4, 25, 40)	29.06	(12.24, 1.6, 21, 40)
	34.43	(18.19, 0.4, 11)	35.09	(19.52, 0.4, 11)	**10.38**	(7.62, 0.8, 25)	**5.02**	(3.83, 1.2, 25)
	372.81	(371.93, 4.0, 3)	248.22	(247.43, 4.0, 5)	16.80	(15.78, 2.0, 23)	5.94	(5.19, 2.4, 1)
0.5	42.17	(19.93, 0.4, 15)	43.10	(21.37, 0.4, 3)	11.34	(7.81, 0.8, 23)	5.19	(4.08, 1.6, 23)
	–	–	–	–	–	–	–	–
	27.88	(13.64, 0.1, 29, 30)	27.48	(12.77, 0.2, 21, 30)	27.45	(10.81, 0.3, 28, 40)	28.72	(11.42, 0.2, 25, 40)
	28.43	(12.25, 0.6, 3, 40)	27.80	(12.09, 0.2, 8, 30)	27.68	(11.88, 0.2, 9, 30)	29.17	(11.44, 0.3, 9, 40)
	349.26	(329.94, 1.0, 15)	239.21	(236.21, 1.0, 1)	16.50	(15.37, 0.1, 30)	5.89	(5.18, 0.2, 16)
	76.70	(67.04, 0.1, 15)	72.37	(63.52, 0.1, 27)	*11.05*	(8.14, 0.1, 23)	*5.16*	(4.00, 0.2, 24)
	20.05	(10.34, 0.8, 25, 25)	*MCUSUM 1-w*		44.73	(29.72, 0.2, 5, 60)	37.00	(22.57, 0.2, 5, 60)
	19.69	(8.41, 2.6, 25, 25)	*MCUSUM 2-w*		47.19	(35.51, 0.4, 1, 60)	41.54	(30.38, 0.0, 1, 60)
	19.83	(8.29, 3.0, 21, 25)	*PPCUSUM-w*		48.70	(26.24, 0.2, 1, 60)	42.32	(20.88, 0.2, 1, 60)
	39.60	(24.92, 0.4, 3)	*MCUSUM 1-q*		**10.59**	(8.71, 0.8, 17)	*4.87*	(4.02, 1.6, 3)
	155.85	(155.80, 4.0, 21)	*MCUSUM 2-q*		11.90	(10.27, 1.6, 1)	5.12	(4.37, 2.4, 1)
1.0	47.76	(26.22, 0.4, 25)	*PPCUSUM-q*		*10.61*	(8.78, 1.2, 15)	**4.81**	(3.94, 2.0, 13)
	–	–	–	–	–	–	–	–
	19.88	(8.11, 0.9, 23, 25)	*MahEWMA-w*		51.82	(40.47, 0.1, 29, 60)	47.43	(39.24, 0.1, 29, 60)
	19.99	(8.07, 1.0, 2, 25)	*MEWMA-w*		54.67	(40.01, 0.1, 30, 60)	50.28	(38.07, 0.1, 29, 60)
	153.96	(153.82, 1.0, 7)	*MahEWMA-q*		11.50	(10.18, 0.1, 27)	5.05	(4.31, 0.2, 30)
	71.33	(64.18, 0.1, 17)	*MEWMA-q*		10.83	(9.44, 0.2, 12)	4.89	(4.08, 0.3, 10)
	13.67	(7.79, 0.8, 27, 25)	17.14	(14.82, 0.6, 29, 60)	15.30	(12.46, 0.6, 29, 60)	13.77	(11.29, 0.6, 29, 60)
	13.15	(5.94, 2.8, 25, 25)	16.22	(10.25, 2.0, 29, 60)	14.55	(8.95, 2.2, 25, 60)	12.91	(8.18, 2.6, 29, 60)
	13.25	(5.94, 2.4, 23, 25)	16.43	(10.24, 2.4, 25, 60)	14.54	(8.93, 2.8, 29, 60)	12.88	(8.20, 2.6, 29, 60)
	6.13	(4.39, 0.8, 21)	**6.37**	(5.02, 1.0, 26)	4.62	(3.76, 1.2, 28)	3.27	(2.62, 1.6, 9)
	7.47	(6.46, 2.0, 1)	7.09	(6.07, 2.0, 1)	4.48	(3.64, 2.4, 1)	*3.12*	(2.46, 2.8, 19)
2.0	6.42	(4.78, 1.2, 13)	6.52	(5.31, 1.6, 21)	*4.47*	(3.63, 2.0, 17)	3.13	(2.47, 2.8, 15)
	–	–	–	–	–	–	–	–
	13.23	(5.78, 0.6, 30, 25)	16.48	(10.21, 1.0, 30, 60)	14.56	(8.94, 1.0, 30, 60)	12.91	(8.22, 1.0, 30, 60)
	13.33	(5.81, 0.8, 4, 25)	16.63	(10.25, 1.0, 17, 60)	14.75	(8.90, 1.0, 6, 60)	13.07	(8.11, 1.0, 18, 60)
	7.31	(6.29, 0.1, 29)	6.85	(5.73, 0.1, 20)	**4.40**	(3.47, 0.1, 23)	**3.09**	(2.38, 0.2, 18)
	6.32	(4.96, 0.2, 25)	*6.49*	(5.27, 0.2, 23)	4.52	(3.73, 0.3, 19)	3.18	(2.53, 0.4, 24)
	8.33	(6.22, 0.6, 27, 60)	8.86	(6.94, 0.6, 29, 60)	8.63	(6.60, 0.6, 27, 60)	8.16	(6.08, 0.6, 25, 60)
	7.58	(4.65, 2.6, 29, 60)	8.00	(5.15, 2.6, 29, 60)	7.69	(4.98, 2.6, 29, 60)	7.09	(4.63, 2.6, 23, 60)
	7.57	(4.65, 2.2, 29, 60)	7.97	(5.16, 3.0, 29, 60)	7.66	(4.96, 3.0, 29, 60)	7.09	(4.63, 2.2, 27, 60)
	3.05	(2.21, 1.4, 20)	**3.06**	(2.27, 1.4, 7)	2.68	(2.00, 1.6, 20)	2.27	(1.64, 1.8, 12)
	3.25	(2.64, 3.2, 1)	3.17	(2.52, 2.8, 25)	2.64	(1.99, 2.8, 21)	*2.20*	(1.57, 3.2, 15)
3.0	3.10	(2.31, 2.0, 7)	*3.08*	(2.37, 2.4, 19)	*2.63*	(1.97, 2.8, 11)	2.20	(1.58, 3.6, 9)
	–	–	–	–	–	–	–	–
	7.57	(4.66, 1.0, 29, 60)	7.98	(5.15, 1.0, 30, 60)	7.65	(4.98, 1.0, 29, 60)	7.07	(4.62, 1.0, 24, 60)
	7.68	(4.64, 1.0, 18, 60)	8.08	(5.15, 1.0, 19, 60)	7.76	(4.97, 1.0, 17, 60)	7.19	(4.66, 1.0, 27, 60)
	3.22	(2.56, 0.2, 14)	3.15	(2.49, 0.2, 11)	**2.63**	(1.95, 0.2, 15)	**2.19**	(1.55, 0.3, 7)
	3.08	(2.31, 0.3, 29)	3.08	(2.34, 0.3, 19)	2.65	(1.99, 0.4, 14)	2.22	(1.61, 0.5, 17)

As expected, the optimal values of the parameter g increase if the distance between the in- and out-of-control GMVP weight vector increases. The EWMA schemes show optimality primarily for the largest changes, where they reduce to the no-memory Shewhart-type chart with $\lambda = 1$. The optimal value of n for all w-charts is mostly $n = 60$.

Now we consider the evidence for detection of changes in study II. The results for study II, where both variances and correlations are changed, are provided in Tables 4 and 5. The CUSUM-q charts show reasonable performance for detecting changes in the variances and the correlations. Both the ARL and WED criteria favor the MCUSUM2-q chart as the best among the CUSUM-q schemes. The CUSUM-w charts seem to be the most suitable ones for detecting changes due to increasing correlation. Note that all charts, with the exception of MCUSUM1-q scheme, have difficulties to detect GMVP changes due to decreasing correlations only. Again, the CUSUM and EWMA exhibit similar detecting behavior. In study II the MEWMA-q scheme provides the overall best results for changes in both variances and correlations. The MCUSUM1-q chart remains, however, a close competitor of the MEWMA-q chart. It is difficult to make recommendations about the choice of optimal design parameters in study II, because the changes considered here are fairly small as shown in Table 1.

4.3. Combined charts

Similar to the results of Golosnoy et al. (2010) for EWMA charts, there is no single CUSUM scheme dominating the others for all considered types of changes. In particular, the w-charts are more appropriate to detect changes caused by

Table 4
Sample estimators of the out-of-control ARLs for study II – changes both in variances and correlations – based on 10^6 repetitions. The empirical standard deviations of the RLs, the best parameters g for CUSUM and λ for EWMA, and the best n are given in parentheses.

$(d_1, \ldots, d_4)(std)$	0.5		1.0		1.5	
$\ddots\ \zeta\,(corr)$						
	121.04	(72.35, 0.0, 60)	*MCUSUM 1-w*		35.64	(29.31, 3.0, 25)
	208.99	(222.32, 0.0, 60)	*MCUSUM 2-w*		34.32	(27.93, 3.0, 25)
	125.57	(65.16, 0.0, 60)	*PPCUSUM-w*		**33.66**	(27.60, 2.6, 25)
	81.95	(67.52, 0.2)	*MCUSUM 1-q*		103.53	(58.95, 0.0)
	101.64	(92.60, 0.4)	*MCUSUM 2-q*		193.55	(193.15, 4.0)
1.0, 1.0, 1.0, 1.0	*88.84*	(59.17, 0.2)	*PPCUSUM-q*		110.90	(51.53, 0.0)
	–	–	–	–	–	–
	222.14	(245.25, 0.1, 60)	*MahEWMA-w*		33.81	(27.54, 1.0, 25)
	204.29	(242.15, 0.1, 60)	*MEWMA-w*		33.86	(27.67, 1.0, 25)
	101.72	(101.91, 0.1)	*MahEWMA-q*		189.64	(188.84, 1.0)
	89.74	(98.23, 0.1)	*MEWMA-q*		160.60	(170.21, 0.1)
	184.88	(135.31, 0.0, 60)	77.00	(69.62, 0.2, 60)	30.67	(24.06, 3.0, 40)
	590.61	(668.53, 0.0, 60)	80.63	(74.77, 0.4, 60)	29.94	(23.74, 3.0, 40)
	184.14	(107.48, 0.0, 60)	79.58	(60.07, 0.2, 60)	29.81	(23.68, 2.4, 40)
	21.97	(21.67, 1.6)	20.26	(19.67, 1.2)	19.47	(17.40, 0.8)
	15.78	(12.65, 1.2)	15.55	(12.51, 1.2)	19.65	(16.55, 1.2)
1.1, 1.2, 1.3, 1.4	18.43	(17.01, 1.6)	17.63	(15.96, 1.4)	19.18	(16.25, 1.0)
	–	–	–	–	–	–
	553.63	(517.38, 0.1, 60)	79.78	(79.61, 0.1, 60)	29.88	(23.70, 1.0, 40)
	500.70	(518.52, 0.1, 60)	75.92	(82.22, 0.1, 60)	29.90	(23.74, 0.9, 40)
	14.55	(12.93, 0.1)	**14.33**	(12.74, 0.1)	18.34	(16.85, 0.1)
	16.45	(20.02, 0.1)	*14.84*	(17.53, 0.1)	**15.39**	(16.84, 0.1)
	120.52	(68.08, 0.0, 60)	48.52	(47.43, 1.6, 60)	21.96	(17.36, 3.0, 60)
	272.32	(294.20, 0.0, 60)	48.52	(48.98, 2.4, 60)	21.31	(17.29, 2.6, 60)
	126.61	(63.18, 0.0, 60)	48.47	(48.09, 1.4, 60)	21.26	(17.24, 3.0, 60)
	8.75	(8.50, 1.8)	8.00	(7.55, 1.4)	8.11	(7.28, 1.2)
	6.78	(5.39, 1.8)	6.57	(5.23, 1.8)	7.49	(6.14, 1.8)
1.2, 1.4, 1.6, 1.8	7.42	(6.45, 2.0)	7.04	(6.01, 1.8)	7.68	(6.49, 1.6)
	–	–	–	–	–	–
	291.35	(318.51, 0.1, 60)	48.35	(44.12, 0.1, 60)	21.34	(17.32, 1.0, 60)
	255.82	(305.86, 0.1, 60)	44.98	(44.93, 0.1, 60)	21.34	(17.35, 1.0, 60)
	6.20	(5.02, 0.1)	*6.03*	(4.89, 0.1)	*6.91*	(5.76, 0.1)
	5.84	(6.96, 0.1)	**5.40**	(6.15, 0.1)	**5.80**	(6.26, 0.1)
	89.23	(42.47, 0.0, 60)	33.18	(33.25, 3.0, 60)	16.59	(13.47, 3.0, 60)
	138.45	(140.74, 0.0, 60)	32.84	(33.24, 2.4, 60)	15.84	(13.40, 2.6, 60)
	94.65	(41.10, 0.0, 60)	32.70	(33.75, 2.4, 60)	15.77	(13.40, 2.8, 60)
	4.93	(4.62, 1.8)	4.61	(4.22, 1.8)	4.80	(4.26, 1.6)
	4.13	(3.21, 2.2)	4.01	(3.11, 2.2)	4.43	(3.52, 2.2)
1.3, 1.6, 1.9, 2.2	4.35	(3.63, 2.6)	4.18	(3.43, 2.2)	4.53	(3.80, 2.6)
	–	–	–	–	–	–
	147.03	(161.25, 0.1, 60)	32.97	(33.96, 1.0, 60)	15.85	(13.45, 1.0, 60)
	127.52	(151.94, 0.1, 60)	31.67	(30.56, 0.1, 60)	15.84	(13.37, 1.0, 60)
	3.87	(2.95, 0.1)	*3.77*	(2.87, 0.1)	*4.15*	(3.25, 0.1)
	3.28	(3.49, 0.1)	**3.14**	(3.21, 0.1)	**3.41**	(3.42, 0.1)
	67.70	(52.78, 0.2, 60)	24.26	(25.13, 3.0, 60)	13.12	(10.92, 3.0, 60)
	88.14	(85.41, 0.0, 60)	23.99	(25.59, 2.6, 60)	12.28	(10.792.6, 60)
	72.54	(49.62, 0.2, 60)	23.86	(25.27, 2.0, 60)	12.23	(10.82, 2.8, 60)
	3.38	(3.00, 2.0)	3.23	(2.80, 2.0)	3.39	(2.89, 1.8)
	2.99	(2.24, 2.6)	2.92	(2.18, 2.6)	3.16	(2.42, 2.6)
1.4, 1.8, 2.2, 2.6	3.06	(2.42, 3.2)	2.99	(2.36, 3.4)	3.21	(2.53, 2.6)
	–	–	–	–	–	–
	90.85	(97.42, 0.1, 60)	23.92	(25.53, 1.0, 60)	12.31	(10.83, 1.0, 60)
	78.72	(92.22, 0.1, 60)	24.02	(25.59, 1.0, 60)	12.28	(10.88, 1.0, 60)
	2.85	(2.06, 0.1)	*2.80*	(2.03, 0.1)	*3.02*	(2.24, 0.1)
	2.35	(2.17, 0.1)	*2.29*	(2.07, 0.1)	**2.46**	(2.24, 0.1)

a volatility reduction, while the q-charts are more suitable to detect changes caused by a volatility increase. Since in practice there is usually no *a priori* information available about the direction and the magnitude of a coming change, there is uncertainty about which chart should be used. A natural idea is to combine the advantages of the w- and q-schemes by using simultaneous control charts (see, e.g., Woodall and Ncube, 1985).

A simultaneous chart provides a signal if any of the combined single charts gives a signal. The required control limits for the single charts are calculated with respect to the run length distribution of the simultaneous chart. Assuming two control

Table 5
Sample estimators of the WEDs for study II – changes both in variances and correlations – based on 10^5 repetitions. The empirical standard deviations of the RLs, the worst value of m, the best parameters g for CUSUM and λ for EWMA, and the best n are given in parentheses.

$(d_1, \ldots, d_4)(std)$ $\ddots$ $\zeta\,(corr)$	0.5		1.0		1.5	
	120.94	(72.39, 0.0, 1, 60)	$MCUSUM$ 1-w		38.00	(29.10, 3.0, 23, 25)
	209.76	(222.79, 0.0, 1, 60)	$MCUSUM$ 2-w		35.48	(27.91, 3.0, 25, 25)
	125.96	(64.95, 0.0, 1, 60)	$PPCUSUM$-w		**35.19**	(27.39, 3.0, 23, 25)
	86.54	(80.23, 0.4, 11)	$MCUSUM$ 1-q		103.56	(58.80, 0.0, 1)
	101.53	(93.48, 0.4, 3)	$MCUSUM$ 2-q		194.27	(193.34, 4.0, 13)
1.0, 1.0, 1.0, 1.0	*89.47*	(73.31, 0.4, 5)	$PPCUSUM$-q		111.52	(51.95, 0.0, 11)
	–	–	—		–	–
	243.96	(246.24, 0.1, 29, 60)	$MahEWMA$-w		*35.39*	(27.36, 1.0, 19, 25)
	250.87	(249.96, 0.1, 25, 60)	$MEWMA$-w		35.82	(27.14, 1.0, 2, 25)
	103.55	(102.32, 0.1, 22)	$MahEWMA$-q		190.87	(189.28, 1.0, 30)
	101.25	(98.79, 0.1, 25)	$MEWMA$-q		174.70	(172.05, 0.1, 19)
	185.27	(135.92, 0.0, 1, 60)	77.24	(70.36, 0.2, 23, 60)	33.52	(23.39, 3.0, 25, 40)
	588.63	(672.74, 0.0, 11, 60)	80.48	(74.19, 0.4, 1, 60)	31.28	(23.35, 2.0, 21, 40)
	184.61	(108.57, 0.0, 1, 60)	79.70	(60.00, 0.2, 1, 60)	32.03	(25.49, 3.0, 15, 25)
	22.16	(21.71, 1.6, 5)	20.46	(19.78, 1.2, 17)	19.71	(17.52, 0.8, 19)
	15.74	(12.57, 1.2, 1)	15.65	(12.55, 1.2, 1)	19.58	(16.49, 1.2, 1)
1.1, 1.2, 1.3, 1.4	18.49	(17.06, 1.6, 25)	17.72	(16.27, 1.6, 5)	19.31	(16.95, 1.2, 7)
	–	–	– –		–	–
	601.53	(512.98, 0.1, 30, 60)	87.24	(80.04, 0.1, 29, 60)	31.94	(23.33, 0.1, 21, 25)
	621.18	(510.27, 0.1, 16, 60)	93.86	(89.01, 0.7, 17, 60)	32.48	(24.68, 1.0, 4, 30)
	15.16	(13.14, 0.1, 28)	**14.91**	(12.94, 0.1, 30)	**18.99**	(17.07, 0.1, 25)
	20.32	(19.75, 0.4, 15)	19.13	(18.25, 0.3, 5)	20.02	(17.42, 0.1, 25)
	120.60	(68.66, 0.0, 9, 60)	54.85	(47.41, 1.2, 27, 60)	24.84	(16.65, 3.0, 29, 60)
	270.58	(293.84, 0.0, 1, 60)	48.71	(45.16, 1.6, 1, 60)	23.09	(16.79, 2.0, 29, 60)
	126.37	(63.09, 0.0, 1, 60)	53.86	(49.19, 1.8, 27, 60)	23.69	(16.83, 2.6, 27, 60)
	8.89	(8.49, 4.0, 21)	8.20	(7.64, 1.2, 3)	8.27	(7.40, 1.2, 7)
	6.79	(5.48, 2.0, 1)	*6.59*	(5.13, 1.6, 1)	*7.48*	(6.01, 1.6, 1)
1.2, 1.4, 1.6, 1.8	7.43	(6.45, 2.0, 25)	7.07	(6.11, 2.0, 17)	7.70	(6.51, 1.6, 11)
	–	–	–	–	–	–
	314.22	(322.16, 0.1, 29, 60)	52.46	(44.02, 0.1, 27, 60)	23.74	(16.89, 1.0, 30, 60)
	317.88	(316.69, 0.1, 19, 60)	55.68	(48.69, 0.5, 6, 60)	24.22	(16.89, 1.0, 4, 60)
	6.52	(5.20, 0.1, 22)	**6.32**	(5.03, 0.1, 30)	**7.22**	(5.93, 0.1, 24)
	7.94	(7.27, 0.4, 23)	7.47	(6.78, 0.4, 20)	8.00	(7.16, 0.3, 24)
	89.33	(42.94, 0.0, 9, 60)	37.28	(33.39, 3.0, 27, 60)	18.65	(13.09, 3.0, 25, 60)
	138.06	(139.86, 0.0, 1, 60)	34.02	(31.51, 1.8, 29, 60)	17.44	(13.05, 2.0, 23, 60)
	94.18	(40.64, 0.0, 1, 60)	36.28	(33.37, 1.8, 29, 60)	17.48	(13.20, 2.8, 29, 60)
	5.06	(4.64, 4.0, 3)	4.74	(4.27, 1.6, 21)	4.94	(4.31, 1.6, 13)
	4.12	(3.25, 2.4, 1)	*4.02*	(3.13, 2.4, 1)	*4.42*	(3.58, 2.4, 1)
1.3, 1.6, 1.9, 2.2	4.37	(3.63, 2.4, 7)	4.20	(3.46, 2.4, 9)	4.55	(3.82, 2.4, 19)
	–	–	–	–	–	–
	157.45	(161.55, 0.1, 30, 60)	36.14	(32.65, 0.5, 28, 60)	17.46	(13.22, 1.0, 27, 60)
	159.57	(157.23, 0.1, 27, 60)	37.58	(35.24, 1.0, 9, 60)	18.06	(13.39, 1.0, 12, 60)
	4.04	(3.07, 0.1, 29)	**3.94**	(3.01, 0.1, 27)	**4.33**	(3.38, 0.1, 24)
	4.54	(3.99, 0.6, 13)	4.35	(3.76, 0.5, 19)	4.68	(4.00, 0.4, 26)
	68.09	(53.03, 0.2, 11, 60)	27.49	(25.29, 3.0, 27, 60)	14.69	(10.61, 3.0, 29, 60)
	88.04	(85.10, 0.0, 1, 60)	25.59	(23.74, 1.8, 29, 60)	13.54	(10.76, 2.6, 27, 60)
	72.76	(49.42, 0.2, 1, 60)	26.38	(25.58, 2.0, 29, 60)	13.47	(10.77, 2.8, 23, 60)
	3.48	(3.04, 4.0, 13)	3.33	(2.86, 2.0, 3)	3.49	(2.96, 2.0, 27)
	2.99	(2.27, 2.8, 1)	*2.92*	(2.16, 2.4, 9)	*3.17*	(2.39, 2.4, 1)
1.4, 1.8, 2.2, 2.6	3.09	(2.46, 3.2, 21)	3.01	(2.36, 2.8, 7)	3.23	(2.57, 2.8, 21)
	–	–	–	–	–	–
	96.81	(98.16, 0.1, 29, 60)	26.46	(25.69, 0.9, 29, 60)	13.53	(10.83, 1.0, 28, 60)
	101.00	(93.26, 0.1, 26, 60)	27.06	(26.08, 1.0, 8, 60)	13.90	(10.84, 1.0, 25, 60)
	2.95	(2.19, 0.2, 27)	**2.89**	(2.15, 0.2, 17)	**3.12**	(2.39, 0.2, 14)
	3.17	(2.62, 0.7, 29)	3.08	(2.51, 0.6, 17)	3.30	(2.70, 0.5, 16)

charts for independent processes and run lengths L_1 and L_2, the in-control ARL $E(L^*)$ of the joint scheme is given by

$$E(L^*) = \left(1 - \left(1 - \frac{1}{E(L_1)}\right)\left(1 - \frac{1}{E(L_2)}\right)\right)^{-1}. \tag{15}$$

In our case it holds that $\hat{\mathbf{w}}_{n,t}$ and $\mathbf{q}_t$ are asymptotically uncorrelated

$$\lim_{n\to\infty} Cov(\hat{\mathbf{w}}_{n,t}, \mathbf{q}_t) = \lim_{n\to\infty} E(\hat{\mathbf{w}}_{n,t}\, \mathbf{q}_t') = \mathbf{0}_{k\times k}. \tag{16}$$

Table 6
Control design of various combined charts for a fixed in-control ARL of 120.

	g	Control limit
*MCUSUM*2-w for $\hat{\mathbf{w}}_{60,t}$	0.0	93.8
*MCUSUM*2-w for $\hat{\mathbf{w}}_{60,t}$	2.0	24.5
*MCUSUM*1-q	1.0	7.0
*MCUSUM*1-q	2.0	5.5

Table 7
Sample estimators of the out-of-control ARLs in study I for combinations of the MCUSUM2-w ($n = 60$) chart with the MCUSUM1-q scheme based on 10^5 simulations. The empirical standard deviations of the RLs are given in parentheses.

$d_4 \ddots d_1$	0.5	1.0	2.0	3.0
0.5	38.68(15.61)	39.37(15.55)	11.96(8.71)	5.50(3.98)
	40.81(14.85)	40.97(14.85)	14.27(11.56)	5.63(4.58)
	34.40(15.73)	35.09(15.68)	11.64(8.52)	5.43(3.92)
	35.93(15.45)	36.30(15.37)	13.66(11.13)	5.60(4.55)
1.0	36.36(14.29)	$gmc2w = 0.0, gmc1q = 1.0$	12.19(10.10)	5.41(4.22)
	37.15(13.98)	$gmc2w = 0.0, gmc1q = 2.0$	13.35(12.07)	5.35(4.50)
	32.23(14.60)	$gmc2w = 2.0, gmc1q = 1.0$	11.94(9.97)	5.38(4.18)
	32.75(14.49)	$gmc2w = 2.0, gmc1q = 2.0$	12.96(11.94)	5.32(4.44)
2.0	6.82(4.87)	7.20(5.49)	5.29(4.10)	3.71(2.84)
	7.31(5.94)	7.55(6.43)	5.32(4.60)	3.57(2.95)
	6.65(4.74)	7.07(5.38)	5.22(4.04)	3.69(2.79)
	7.07(5.74)	7.35(6.29)	5.25(4.51)	3.57(2.93)
3.0	3.32(2.29)	3.35(2.37)	2.96(2.11)	2.52(1.76)
	3.27(2.47)	3.28(2.49)	2.87(2.20)	2.40(1.77)
	3.30(2.27)	3.35(2.36)	2.96(2.08)	2.51(1.75)
	3.23(2.40)	3.26(2.47)	2.86(2.18)	2.40(1.76)

Since the finite sample correlation is of order $1/n$, it appears to be negligibly small in our study with $n \geq 30$. Assuming equal in-control ARLs for both charts $E(L_1) = E(L_2) = E(L)$, solving the Eq. (15) yields

$$E(L) = E(L^*) + \sqrt{E(L^*)^2 - E(L^*)}. \tag{17}$$

Thus, we determine the control limits $h_i, i = 1, 2$ for the single charts by setting the ARL for the joint chart $E(L^*) = \xi$, where ξ is the fixed in-control ARL of the joint scheme. Then the control limits h_1 and h_2 are obtained by solving the equations $E(L_i(h_i)) = \xi + \sqrt{\xi^2 - \xi}$ for each scheme separately. Here we focus on the simultaneous chart consisting of the MCUSUM2-w and the MCUSUM1-q scheme. Setting the in-control ARL equal to $E(L^*) = \xi = 120$, the control limits of the single charts are calculated to provide $E(L(h_i)) \approx 239.5$. Table 6 reports the control limits of the joint scheme for four different choices of the reference values.

The performance of the joint schemes is analyzed using Monte Carlo simulations. The out-of-control ARLs for study I are presented in Table 7 and those for study II in Table 8. The CED values with $m = 30$ are not reported here because they are very similar to the ARL results.

Table 7 indicates that, although the combined charts exhibit larger out-of-control ARLs compared to the best single charts for study I, they provide on average more stable detecting behavior. For example, the choice of the best parameters for the MCUSUM2-w chart in order to detect the change $d_1 = 0.5, d_4 = 3.0$ would result in the out-of-control ARL = 27.06 (cf. Table 2), whereas all considered combined charts provide the out-of-control ARLs less than 6. These findings are supported by evidence obtained from study II, see Table 8. All considered combined charts show a sound detecting ability which insures against inappropriate choices of the chart parameters. They are more robust in the presence of uncertainty about the type of change than the single schemes. Note that we do not try to identify the best possible control chart combination, but show that combining schemes allows us to exclude undesired large ARLs due to possible problems with the design of monitoring procedures.

5. Empirical study

The practical importance of the suggested control charts for portfolio selection is demonstrated by applying them to a real-time portfolio policy with the aim to minimize the ex ante portfolio variance. The empirical study is organized as follows. First, we describe the data and the benchmark portfolio strategies, which do not exploit control charts for the task of portfolio variance minimization. After that, we propose portfolio strategies which exploit signal information. Finally, we discuss the possibility to combine the information on the signals from various control charts and present empirical insights about the performance of this attempt. Note that we focus here not on dating changes in financial series (cf. Zeileis et al., 2010), but on exploiting signal information for investment policies.

Table 8
Sample estimators of the out-of-control ARLs in study II for combinations of the MCUSUM2-w ($n = 60$) chart with the MCUSUM1-q scheme based on 10^5 simulations. The empirical standard deviations of the RLs are given in parentheses.

$(d_1, \ldots, d_4)(std)$ ⋱ $\zeta(corr)$	0.5	1.0	1.5
1.0, 1.0, 1.0, 1.0	144.14(144.98)	$gmc2w = 0.0, gmc1q = 1.0$	68.07(47.42)
	147.12(147.39)	$gmc2w = 0.0, gmc1q = 2.0$	67.35(46.54)
	142.66(148.69)	$gmc2w = 2.0, gmc1q = 1.0$	62.55(49.49)
	146.43(152.87)	$gmc2w = 2.0, gmc1q = 2.0$	61.94(48.38)
1.1, 1.2, 1.3, 1.4	33.43(32.79)	27.96(26.49)	23.43(19.44)
	32.57(32.56)	28.59(27.60)	25.26(20.92)
	32.30(32.33)	26.95(26.43)	22.38(19.33)
	31.58(32.22)	27.56(27.44)	23.66(20.60)
1.2, 1.4, 1.6, 1.8	12.02(11.39)	10.40(9.50)	9.85(8.40)
	11.63(11.30)	10.34(9.84)	10.21(9.30)
	11.79(11.30)	10.16(9.33)	9.58(8.26)
	11.44(11.29)	10.08(9.71)	9.87(9.01)
1.3, 1.6, 1.9, 2.2	6.43(5.88)	5.83(5.08)	5.82(4.85)
	6.12(5.81)	5.58(5.16)	5.70(5.09)
	6.36(5.86)	5.73(4.99)	5.75(4.79)
	6.06(5.76)	5.53(5.14)	5.63(5.02)
1.4, 1.8, 2.2, 2.6	4.24(3.67)	3.97(3.32)	4.08(3.29)
	4.01(3.63)	3.75(3.32)	3.92(3.40)
	4.23(3.66)	3.94(3.28)	4.03(3.24)
	3.99(3.64)	3.73(3.28)	3.88(3.32)

5.1. Data and benchmark portfolio strategies

We consider a portfolio with $k = 4$ broad and well-diversified MSCI country indices of risky assets. Such a number of assets is typical for portfolio selection problems, where the Markowitz procedure is directly applicable (cf. Michaud, 1998). On the contrary, asset allocation problems with $k \geq 10$ suffer from the estimation risk due to the unknown parameters of the asset return distribution. Factor models or other approaches should be used in such situations in order to mitigate the estimation risk (cf. DeMiguel et al., 2009). Moreover, daily returns on broad indices (averages) are closer to conditional normality compared to returns on risky assets. These MSCI price indices characterize the most important financial markets, namely France, Germany, UK, and USA (MSCI-PI-FRA, MSCI-PI-GER, MSCI-PI-UK, MSCI-PI-USA). The data is taken from `DataStream` for the time period from 01.01.1980 to 31.12.2007. The log-returns are calculated on a daily basis. The first 250 return observations in 1980 are used for fitting the target process. Therefore, our dataset for the out-of-sample evaluation is a period of 26 years which consists of 7054 daily returns.

The investor aims to minimize the out-of-sample one-day-ahead portfolio variance. We distinguish between several basic portfolio rules which do not use a control chart approach:

(i) An investment in only one of the four risky assets. This method completely neglects the benefits of diversification and relies on a single financial market.
(ii) Equal weights portfolio with $1/k$ proportions for all assets (DeMiguel et al., 2009).
(iii) Overall GMVP weights. The non-feasible static strategy with the weights, estimated with the whole sample of 7304 observations: $\hat{\mathbf{w}}_{7304} = (0.123, 0.079, 0.275, 0.524)'$.
(iv) Static GMVP. The weights $\hat{\mathbf{w}}_{n,0}$ are calculated for $n \in \{25, 60, 90, 120, 250\}$ at $t = 0$ and retained until the end of the sample $t = 7054$.
(v) Dynamic GMVP. The weights $\hat{\mathbf{w}}_{n,t}$ are reestimated as in (6) at every new day t with $n \in \{25, 60, 90, 120, 250\}$.

These basic rules are used as the benchmarks in the further analysis. The portfolio weights according to rules (i) to (iv) remain constant over time. The time series of the estimated GMVP weights in rule (v) over the entire period of 26 years are shown in Fig. 2 for $n = 60, 250$. The variability of GMVP weights reduces if n increases. Note that the strategies (ii) to (v) may require portfolio rebalancing back to the target weights at any point in time. The performance of all considered portfolio strategies is evaluated based on the moments of the realized daily portfolio returns, which are calculated as $x_{p,t} = \hat{\mathbf{w}}'_{n,t-1}\mathbf{X}_t$. Table 9 reports the means $E(x_{p,t})$, the standard deviations $V^{1/2}(x_{p,t})$ as well as the information ratio $I\!R = E(x_{p,t})/V^{1/2}(x_{p,t})$ (Grinold and Kan, 2000) for 7054 daily portfolio returns of strategies (i) to (iv). The performance measures in Table 9 are annualized, i.e. recalculated on a yearly basis.

The standard errors of the moment estimates are required in order to assess the statistical significance of the differences between various strategies in Table 9. The standard errors of the reported moments are obtained via the simple bootstrap procedure (cf. Shao and Tu, 1996) for the portfolio returns from approaches (ii), (iii), and (v). Bootstrap samples of 7054 portfolio returns are generated 10^4 times. The results of the bootstrap exercise are reported in Table 10.

The bootstrapped means of the portfolio return characteristics coincide with the corresponding sample values in Table 9. Special attention should be drawn to the bootstrapped standard deviations, which are given in parentheses. The standard

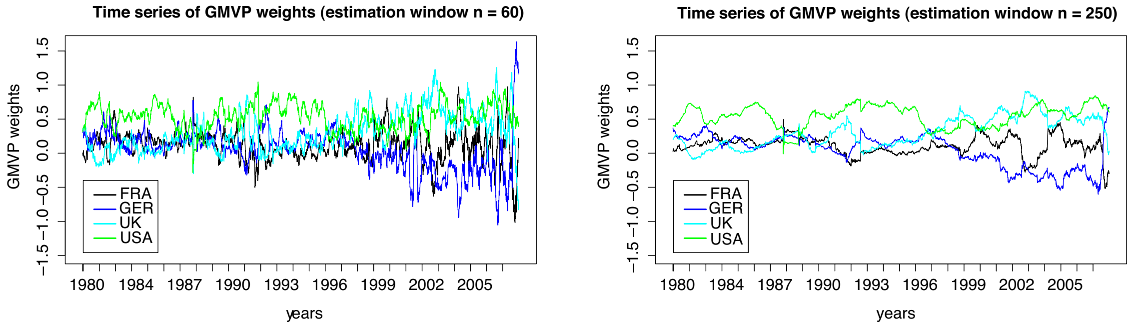

Fig. 2. The time evolution of the empirical GMVP weights $\hat{\mathbf{w}}_{n,t}$ based on $n = 60$ and $n = 250$ most recent observations.

Table 9
Annualized means, standard deviations, and *IR*s of portfolio returns.

	Strategies	Weight vector	Mean (x_p), %	Std. of (x_p), %	*IR*
(i)	France	$\mathbf{w} = (1, 0, 0, 0)'$	9.78	19.85	0.493
	Germany	$\mathbf{w} = (0, 1, 0, 0)'$	10.01	21.21	0.472
	UK	$\mathbf{w} = (0, 0, 1, 0)'$	7.89	17.99	0.439
	USA	$\mathbf{w} = (0, 0, 0, 1)'$	8.56	16.04	0.534
(ii)	Equal weights	$\mathbf{w} = \mathbf{1}/k$	9.06	14.67	0.618
(iii)	Overall GMVP	$\hat{\mathbf{w}}_{7304}$	8.63	13.45	0.641
		$\hat{\mathbf{w}}_{25,t=0}$	9.10	16.87	0.539
		$\hat{\mathbf{w}}_{60,t=0}$	8.80	14.38	0.612
(iv)	Static GMVP	$\hat{\mathbf{w}}_{90,t=0}$	8.92	14.45	0.617
		$\hat{\mathbf{w}}_{120,t=0}$	8.92	14.23	0.627
		$\hat{\mathbf{w}}_{250,t=0}$	8.98	14.30	0.628
		$\hat{\mathbf{w}}_{25,t}$	8.06	13.77	0.586
		$\hat{\mathbf{w}}_{60,t}$	7.47	13.11	0.570
(v)	Dynamic GMVP	$\hat{\mathbf{w}}_{90,t}$	7.63	13.06	0.584
		$\hat{\mathbf{w}}_{120,t}$	7.80	12.97	0.602
		$\hat{\mathbf{w}}_{250,t}$	7.63	12.89	0.592

Table 10
Bootstrapped annualized moments and their standard deviations (in parentheses) of the characteristics of portfolio returns.

	Strategy	Mean, %	Std. %	*IR*
(ii)	Equal weights	9.0748 (2.7419)	14.6600 (0.3065)	0.6199 (0.1895)
(iii)	Overall GMVP $\hat{\mathbf{w}}_{7304}$	8.6363 (2.5235)	13.4441 (0.4170)	0.6443 (0.1927)
(iv)	Dynamic GMVP $\hat{\mathbf{w}}_{250,t}$	7.6724 (2.4179)	12.8850 (0.3348)	0.5968 (0.1914)

deviations of the portfolio means and the *IR*s in Table 10 are very large. There is no statistical difference between the means and the *IR*s of rules (i) to (v) in Table 9. On the contrary, the standard errors of the portfolio return standard deviations $V^{1/2}(x_{p,t})$ are small, so there are significant differences in the portfolio return variances in Table 9. Thus, it is reasonable to rank the portfolio strategies here using solely their variances and to choose the GMVP as an appropriate investment opportunity.

The performance of the equal weight strategy (ii) compared to the single-asset strategies (i) clearly illustrates the importance of diversification for volatility reduction. The infeasible full sample GMVP based on $n = 7304$ returns (iii) exhibits a significantly lower variance compared to the equally weighted portfolio (ii) and the static GMVP (iv). The implementation of the dynamic GMVP approaches (v) provides even a smaller portfolio volatility, especially for $n \geq 60$. The smallest out-of-sample portfolio variance is achieved for $n = 250$. A further increase of n does not provide any additional benefits for the variance minimizing investor.

5.2. Portfolio strategies based on control charts

Now we investigate whether the use of the signal information obtained by the control charts permits a further reduction of the out-of-sample portfolio variance compared to the basic benchmark rules (i)–(v). The simulation results show that the q-CUSUM and q-EWMA charts provide a sound detecting ability for various types of changes in the GMVP weights. In the empirical study we exploit 4 types of q-control charts, namely the MCUSUM1, MCUSUM2, PPCUSUM, and the Mahalanobis EWMA schemes. The control limits are chosen to provide an in-control ARL of 120 days. Thus we can expect on average at least one signal within six months for all schemes. Note that the control limits are determined using Monte Carlo simulations

Table 11
Performance of portfolio strategies based on **q**-charts. Annualized means, standard deviations, and *IR*s: upper part – equal weights, lower part – dynamic weights $\hat{\mathbf{w}}_{250,t}$.

Target	Strategies	Parameter	# alarms	Mean (x_p), %	Std. (x_p), %	*IR*
$\mathbf{1}/k$	*MCUSUM* 1-*q*	1.0	492 (7.0%)	11.12	12.27	0.907
$\mathbf{1}/k$	*MCUSUM* 1-*q*	2.0	438 (6.2%)	12.30	12.16	1.012
$\mathbf{1}/k$	*MCUSUM* 2-*q*	1.0	391 (5.5%)	12.26	12.39	0.989
$\mathbf{1}/k$	*MCUSUM* 2-*q*	3.0	400 (5.7%)	12.79	12.28	1.042
$\mathbf{1}/k$	*PPCUSUM*-*q*	2.0	499 (7.1%)	11.38	12.24	0.929
$\mathbf{1}/k$	*PPCUSUM*-*q*	3.0	451 (6.4%)	11.59	12.40	0.934
$\mathbf{1}/k$	*MahEWMA*-*q*	0.1	264 (3.7%)	12.27	12.77	0.961
$\mathbf{1}/k$	*MahEWMA*-*q*	1.0	254 (3.6%)	11.45	12.88	0.889
$\hat{\mathbf{w}}_{250,t}$	*MCUSUM* 1-*q*	1.0	251 (3.6%)	10.48	11.32	0.926
$\hat{\mathbf{w}}_{250,t}$	*MCUSUM* 1-*q*	2.0	226 (3.2%)	11.28	11.37	0.993
$\hat{\mathbf{w}}_{250,t}$	*MCUSUM* 2-*q*	1.0	211 (3.0%)	10.91	11.33	0.963
$\hat{\mathbf{w}}_{250,t}$	*MCUSUM* 2-*q*	3.0	219 (3.1%)	10.60	11.32	0.936
$\hat{\mathbf{w}}_{250,t}$	*PPCUSUM*-*q*	2.0	224 (3.2%)	10.37	11.48	0.904
$\hat{\mathbf{w}}_{250,t}$	*PPCUSUM*-*q*	2.0	224 (3.2%)	10.37	11.48	0.904
$\hat{\mathbf{w}}_{250,t}$	*MahEWMA*-*q*	0.1	143 (2.0%)	10.67	11.62	0.918
$\hat{\mathbf{w}}_{250,t}$	*MahEWMA*-*q*	1.0	135 (1.9%)	10.75	11.67	0.921

based on the covariance matrix from Section 4.1. It turns out that they are not very sensitive with respect to other plausible choices of the covariance matrix of the asset returns.

The knowledge of the in-control parameters is required for starting the control charts. The mean asset return $\boldsymbol{\mu}$ is necessary for calculating the process $\{\mathbf{q}_t\}$. Assuming a simple diffusion process for the asset prices, the precision of the mean estimation can be improved only by taking for estimation purposes rather long historical intervals (cf. Merton, 1980). Moreover, the estimation risk due to imprecise mean estimates has a damaging impact on the portfolio performance (cf. Best and Grauer, 1991). Thus, the estimates of the local means can hardly be used in portfolio analysis for determining the optimal proportions. The null hypothesis about the equal mean returns cannot be rejected in our full dataset, see Table 10. Since the mean returns for all assets are around 8% per year or $\bar{\mu} = 0.08/250$ per day over the whole sample for all considered assets, as reported in Table 9, we take this value as given in order to escape from the problems with the mean estimation. Moreover, the control charts require knowledge of the covariance matrix of the underlying target process. Both covariance matrices $Cov(\hat{\mathbf{w}}_{n,t})$ and $Cov(\mathbf{q}_t)$ depend solely on the covariance matrix of the asset returns $\boldsymbol{\Sigma}_t$, as in (7) and (8). We estimate the matrix $\boldsymbol{\Sigma}_t$ at each time point t using the RiskMetrics methodology with a decay parameter $\delta = 0.96$ by

$$\tilde{\boldsymbol{\Sigma}}_t = (1-\delta)(\mathbf{X}_{t-1} - \bar{\mu}\mathbf{1})(\mathbf{X}_{t-1} - \bar{\mu}\mathbf{1})' + \delta\tilde{\boldsymbol{\Sigma}}_{t-1}. \tag{18}$$

Then the matrices $Cov(\hat{\mathbf{w}}_{n,t})$ and $Cov(\mathbf{q}_t)$ are calculated using $\tilde{\boldsymbol{\Sigma}}_t$ as if they were the non-stochastic parameters. The sample covariance matrix estimator $\hat{\boldsymbol{\Sigma}}_{n,t}$ in (5) is used for calculating $\hat{\mathbf{w}}_{n,t}$. Note that the singularity problem of the covariance matrix does not occur in our case because the number of assets k is small. The control charts for the singular covariance matrix are considered by Bodnar et al. (2009). Alternatively, dimension reduction techniques (cf. El Karoui, 2010) can be of interest for a large number of assets in the portfolio.

Each obtained signal must be carefully checked on possible causes and consequences. In practice, a financial analyst has much more information about the investment decisions in such situations. Another important problem is how to react to the obtained signals, or how to exploit the signal information in the portfolio strategy. In the case of no signals the investor should use the target weights, which are (a) equal weights; (b) the dynamic GMVP weights based on $\hat{\mathbf{w}}_{n,t}$ with $n = 250$. Each signal from the control charts is interpreted as evidence that the target portfolio weights have changed. Then there is a problem how to react on signals in the real-time portfolio policy. In industrial applications control charts are running until a signal occurs, afterwards the process is stopped and the machine is maintained. In finance, however, we cannot stop the process and eliminate the reasons of the change which are frequently unknown. Golosnoy (2007) use the signal information for reducing or expanding the length of the estimation windows in order to calculate the GMVP weights. This paper presents another possibility to utilize signals in portfolio selection.

Any obtained signal indicates that something may be wrong with the monitoring process, so that the investor has no reliable information about the proper GMVP composition. In such situations we propose to close all risky positions for one period if a signal is given. Such an approach corresponds to the idea to leave the market in turbulent times. Thus, the investment rules using the signals are formulated as:

(a) Invest in equal weights $1/k$ and start the monitoring procedure. If there is signal close all risky positions for one period. After that the strategy continues with equal weights, etc.
(b) Invest in the dynamic GMVP weights $\hat{\mathbf{w}}_{250,t}$ and start the monitoring procedure. If there is a signal all risky positions are closed for one period. After that the strategy continues.

Table 11 provides the annualized portfolio returns, the standard deviations, and the information ratios for both strategies (a) and (b) and various control charts.

Table 12
Performance of portfolio strategies based on a combination of the *MCUSUM* 1-w with $n = 60$ and *MCUSUM* 1-*q* charts with dynamic target values $\hat{\mathbf{w}}_{250,t}$ – annualized means, standard deviations, and *IR*s.

Strategies	Parameter	# alarms	Mean (x_p), %	Std. (x_p), %	*IR*
MCUSUM 1-w	0.0				
+*MCUSUM* 1-*q*	1.0	401 (5.7%)	7.80	12.17	0.641
+*MCUSUM* 1-*q*	2.0	433 (6.1%)	7.58	12.18	0.623
+*MahEWMA-q*	0.1	116 (1.6%)	9.22	12.37	0.746
+*MahEWMA-q*	1.0	103 (1.5%)	9.93	12.27	0.810
MCUSUM $1 - w$	1.0				
+*MCUSUM* 1-*q*	1.0	557 (7.9%)	11.63	10.94	1.063
+*MCUSUM* 1-*q*	2.0	639 (9.1%)	10.20	10.71	0.952
+*MahEWMA-q*	0.1	134 (1.9%)	9.51	11.78	0.807
+*MahEWMA-q*	1.0	118 (1.7%)	10.20	11.84	0.861
MCUSUM 1-w	2.0				
+*MCUSUM* 1-*q*	1.0	545 (7.7%)	11.62	11.03	1.054
+*MCUSUM* 1-*q*	2.0	633 (9.0%)	9.39	10.82	0.868
+*MahEWMA-q*	0.1	137 (1.9%)	9.83	11.81	0.832
+*MahEWMA-q*	1.0	112 (1.6%)	10.06	11.84	0.850
MCUSUM 1-w	3.0				
+*MCUSUM* 1-*q*	1.0	540 (7.7%)	9.03	11.19	0.807
+*MCUSUM* 1-*q*	2.0	622 (8.8%)	8.31	10.97	0.757
+*MahEWMA-q*	0.1	131 (1.9%)	9.53	11.88	0.802
+*MahEWMA-q*	1.0	101 (1.4%)	9.87	11.96	0.825

The upper part of Table 11 deals with the portfolio with equal weights, while the lower part reports the results for the dynamic GMVP portfolio. The portfolio strategies using control charts are compared with the benchmark approaches (i) to (v). The portfolio variance is used as a performance measure. The strategies described in (a) and (b) lead to a volatility reduction in comparison with the basic benchmarks (i) to (v). The best results are obtained for the dynamic GMVP approach using the signals from the *q*-charts. However, the portfolio with equal weights outperforms the basic benchmarks as well. Our results illustrate the great importance of the choice of the target portfolio weights for the investment decisions. Of course, it is also of importance which control chart is applied. However, there is no unique conclusion about the best control procedure. Note that our approach provides an increase in the portfolio means and *IR*s, pointing out that signals could indicate on turmoil times on financial markets.

5.3. Portfolio strategies based on simultaneous charts

The empirical results show that the total number of signals differs for the competing charts and processes to monitor. Moreover, since the financial process cannot be stopped for the retrospective analysis, there exist no good methods to differentiate between correct and false signals. The results of the simulation study in Section 4 report that the performance of different charts depends on the type and the size of the change.

Another reason to use both w- and q-charts is to differentiate between single outliers and long term changes. A daily outlier caused by some extraordinary events is quickly transferred into the asset prices. Volatility changes are usually of a different nature. Volatility clusters reflect (persistent) divergences in opinions about the further development of the financial markets. Both w- and q-charts should provide a signal in days with outliers and with permanent changes. However, the q-charts would not give a signal at the next day in the case of a one-day outlier, because the information is already incorporated in asset prices. This is a clear advantage of the q-charts. The w-charts, however, would signal this outlier in the subsequent periods as well. Thus, combining charts is a method to make the procedure more robust against an unknown change type.

In general, there is a possibility to combine signals from any number of control schemes. This study is restricted to combining the signals from two charts with the following decision rule. If a signal is given by both schemes simultaneously, then the target portfolio composition is rejected, the investor sells all risky assets and leaves the market. Otherwise, the investment is done according to the assumed target vector. The procedure continues next period with the reestimated target weights $\hat{\mathbf{w}}_{250,t}$. Note that this approach differs from the combined charts analyzed in Section 4.3, where the run length is determined by a quickest (but not simultaneous) signal from two schemes.

The dynamic GMVP with $\hat{\mathbf{w}}_{250,t}$ weight vector is chosen as a target. The first chart is the MCUSUM1 scheme for the process of weights $\{\hat{\mathbf{w}}_{60,t}\}$, while the second chart is always based on the auxiliary process $\{\mathbf{q}\}$. In particular, we apply the MCUSUM1-q or the Mahalanobis EWMA-q schemes. The results for this approach are provided in Table 12.

Table 12 illustrates that the portfolio strategy reacting only on simultaneous signals allows further remarkable reductions of the out-of-sample portfolio variance. The volatility reduction for the simultaneous charts given the dynamic GMVP target $\hat{\mathbf{w}}_{250,t}$ and zero weight alternative is statistically significant compared to the single chart results, see Table 11. The overall best results are achieved by combining MCUSUM1-w with $g \geq 1.0$ and MCUSUM1-q charts. This evidence shows that the rules using signal combinations are of interest for minimizing portfolio variance.

6. Summary

This paper elaborates multivariate cumulated sum (CUSUM) control charts for sequential monitoring the global minimum variance portfolio (GMVP) weights. The GMVP leads to the smallest attainable portfolio variance and thus remains an important benchmark both in financial theory and empirical applications. A variance minimizing investor requires timely detection of changes in the GMVP composition. The suggested CUSUM charts are compared to the EWMA procedures of Golosnoy et al. (2010) in an extensive Monte Carlo simulation study. Although there are no overall best schemes, we are able to give recommendations about the choice of the control designs. In particular, we identify schemes which behave quite robust with respect to different types of changes in the GMVP weights.

The empirical study suggests the methodology of applying the information from the control charts for the real-time portfolio policy. The signals obtained by the charts are used in order to decide whether the target weights are still valid or not any more. Furthermore, the signal information obtained from control chart application is exploited for portfolio variance minimization. The achieved reduction of ex ante portfolio variance is statistically significant compared to the benchmark portfolio composition receipts.

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Appendix

Proof of the approximation in (10). The approximation is given by

$$(n-1)\ Cov^a(\mathbf{s}_{t-b,t}) = \left(b + 2\sum_{i=1}^{min\{n-1,b-1\}} (b-i)(n-i-1)\right) Cov(\hat{\mathbf{w}}^*_{n,t}). \tag{19}$$

If $n > b$ we get that (cf. Bronstein and Semendjajev, 1997)

$$\sum_{j=1}^{b-1}(b-j)(n-j-1) = \sum_{j=1}^{b-1} b(n-j-1) - \sum_{j=1}^{b-1} j(n-1) + \sum_{j=1}^{b-1} j^2 = \frac{b(b-1)(3n-b-4)}{6}$$

and thus obtain that

$$Cov^a(\mathbf{s}_{t-b,t}) = \frac{b - b^3 + 3b^2 n - 3b^2}{3(n-1)}\ Cov(\hat{\mathbf{w}}^*_{n,t}).$$

For $b \geq n$ it follows that

$$\sum_{j=1}^{n-2}(b-j)(n-j-1) = \sum_{j=1}^{n-2} b(n-j-1) - \sum_{j=1}^{n-2} j(n-1) + \sum_{j=1}^{n-2} j^2 = \frac{(3b-n)(n^2-3n+2)}{6}.$$

This leads to

$$Cov^a(\mathbf{s}_{t-b,t}) = \left(b(n-1) - \frac{n(n-2)}{3}\right)\ Cov(\hat{\mathbf{w}}^*_{n,t}).$$

This completes the proof. □

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