Homework 1

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Homework 1

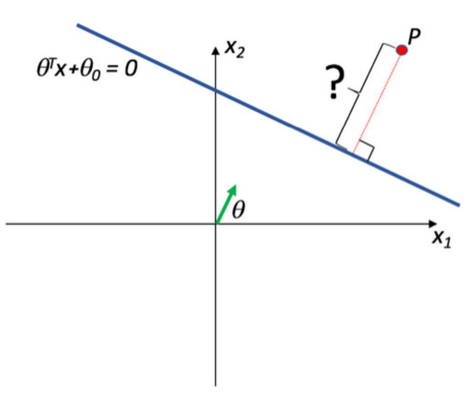
(0.8571428571428571 / 1.0 points)

1) Numpy procedures for hyperplanes and separators

Relevant material on linear classifiers in the notes
Helpful numpy explanations at the bottom of the page.

1.1) General hyperplane, distance to point

Let p be an arbitrary point in R^d. Give a formula for the **signed** perpendicular distance from the hyperplane specified by θ, θ_0 to this point p.



Enter your answer as a Python expression. Use `theta` for θ, `theta_0` for θ_0, `p` for the point p, `transpose(x)` for transpose of an array, `norm(x)` for the length (L2-norm) of a vector, and `x@y` to indicate a matrix product of two arrays.

Formula for signed distance: (transpose(theta)@ p + theta_0) / norm(theta)

Check Syntax | Submit | View Answer | Ask for Help 100.00%

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Your entry was parsed as:

$$\frac{(\theta)^T \mathbf{p} + \theta_0}{\|\theta\|}$$

1.2) Code for signed distance!

Write a Python function using numpy operations (no loops!) that takes column vectors (d by 1) `x` and `th` (of the same dimension) and scalar `th0` and returns the signed perpendicular distance (as a 1 by 1 array) from the hyperplane encoded by `(th, th0)` to `x`. Note that you are allowed to use the "length" function defined in previous coding questions (includig week 1 exercises).

```
import numpy as np
def signed_dist(x, th, th0):
    return ((th.T @ x) + th0) / length(th)

```

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1.3) Code for side of hyperplane

Write a Python function that takes as input

- a column vector `x`
- a column vector `th` that is of the same dimension as `x`
- a scalar `th0`

and returns

- `+1` if `x` is on the positive side of the hyperplane encoded by `(th, th0)`
- `0` if on the hyperplane
- `-1` otherwise.

The answer should be a 2D array (a 1 by 1). Look at the *sign* function. Note that you are allowed to use any functions defined in week 1's exercises.

```
import numpy as np
def positive(x, th, th0):
    res = (th.T @ x + th0)[0][0]
    if res > 0:
        return [[1]]
    elif res == 0:
        return [[0]]
    else:
        return [[-1]]

```

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Here is the solution we wrote:

```
import numpy as np
# x is dimension d by 1
# th is dimension d by 1
# th0 is dimension 1 by 1
# return 1 by 1 matrix of +1, 0, -1
def positive(x, th, th0):
   return np.sign(np.dot(np.transpose(th), x) + th0)
```

Explanation:

First, recall the formula for how we determine which side of the hyperplane defined by θ, θ_0 a point x lies on:

$$\text{sign}(\theta^T x + \theta_0)$$

The expression inside the sign() function can be coded the same way we did in the previous problem, leading to our desired solution: np.sign(np.dot(np.transpose(θ), x) + θ_0).

Another clever way to solve this problem uses the signed_distance function from the previous problem. Note that the expression inside the sign() function above is equal to $||\theta||$ times the signed perpendicular distance from the previous problem. Thus, we could write our solution as np.sign(signed_dist(x, θ, θ_0)*length(θ)). However, length(θ) is a positive scalar, so it doesn't actually affect which side of the hyperplane x lies on, so we can remove that term entirely, leading to our solution np.sign(signed_dist(x, θ, θ_0)).

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Now, given a hyperplane and a set of data points, we can think about which points are on which side of the hyperplane. This is something we do in many machine-learning algorithms, as we will explore soon. It is also a chance to begin using numpy on larger chunks of data.

1.4) Expressions operating on data

We define `data` to be a 2 by 5 array (two rows, five columns) of scalars. It represents 5 data points in two dimensions. We also define `labels` to be a 1 by 5 array (1 row, five columns) of 1 and -1 values.

```
data = np.transpose(np.array([[1, 2], [1, 3], [2, 1], [1, -1], [2, -1]]))
labels = rv([-1, -1, +1, +1, +1])
```

For each subproblem, provide a Python expression that sets `A` to the quantity specified. Note that `A` should always be a 2D numpy array. Only one relatively short expression is needed for each one. No loops!

You can use (our version) of the `length` and `positive` functions; they are already defined, don't paste in your definitions. Those functions if written purely as matrix operations should work with a 2D data array, not just a single column vector as the first argument, with no change.

1. `A` should be a 1 by 5 array of values, either +1, 0 or -1, indicating, for each point in `data`, whether it is on the positive side of the hyperplane defined by `th, th0`. **Use `data, th, th0` as variables in your submission.**

```
import numpy as np
A = 'write your expression here'
A = np.sign((np.transpose(data) @ th + th0).T)
```

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2. `A` should be a 1 by 5 array of boolean values, either `True` or `False`, indicating for each point in `data` and corresponding label in `labels` whether it is correctly classified by hyperplane `th = [1, 1], th0 = -2` . That is, return `True` when the side of the hyperplane (specified by θ, θ_0) that the point is on agrees with the specified label.

```
import numpy as np
A = 'write your expression here'
A = (np.sign((np.transpose(data) @ th + th0).T) == labels)
```

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1.5) Score

Write a procedure that takes as input

- `data`: a `d` by `n` array of floats (representing `n` data points in `d` dimensions)
- `labels`: a `1` by `n` array of elements in `(+1, -1)`, representing target labels
- `th`: a `d` by `1` array of floats that together with
- `th0`: a single scalar or 1 by 1 array, represents a hyperplane

and returns the number of points for which the label is equal to the output of the `positive` function on the point.

Since `numpy` treats `False` as 0 and `True` as 1, you can take the sum of a collection of Boolean values directly.

```
import numpy as np
def score(data, labels, th, th0):
    return np.sum((np.sign((np.transpose(data) @ th + th0).T) == labels))

```

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1.6) Best separator

Now assume that we have some "candidate" classifiers that we want to pick the best one out of. Assume you have `ths`, a d by m array of m candidate θ's (each θ has dimension d by 1), and `th0s`, a 1 by m array of the corresponding m candidate θ_0's. Each of the θ, θ_0 pair represents a hyperplane that characterizes a binary classifier.

Write a procedure that takes as input

- `data`: a `d` by `n` array of floats (representing `n` data points in `d` dimensions)
- `labels`: a `1` by `n` array of elements in `(+1, -1)`, representing target labels
- `ths`: a `d` by `m` array of floats representing m candidate θ's (each θ has dimension d by 1)
- `th0s`: a `1` by `m` array of the corresponding m candidate θ_0's.

and finds the hyperplane with the highest score on the data and labels. In case of a tie, return the first hyperplane with the highest score, in the form of

- a tuple of a `d` by 1 array and an offset in the form of 1 by 1 array.

The function `score` that you wrote above was for a single hyperplane separator. Think about how to generalize it to multiple hyperplanes and include this modified (if necessary) definition of `score` in the answer box.

Note: Look below the answer box for useful numpy functions!

```
import numpy as np

def best_separator(data, labels, ths, th0s):
    bool_mat = (np.sign((np.transpose(data) @ ths + th0s)) == labels.T)
    max_id = np.argmax(np.sum(bool_mat, axis = 0))
    return (np.array([(ths.T)[max_id]]).T, np.array([[th0s[0][max_id]]]))

```

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Reference Material: Handy Numpy Functions and Their Usage

In order to avoid using for loops, you will need to use the following numpy functions. (So that you replace for loops with matrix operations)

A. `np.sum` with axis

`np.sum` can take an optional argument `axis`. Axis 0 is row and 1 is column in a 2D numpy array. **The way to understand the "axis" of numpy sum is that it sums(collapses) the given matrix in the direction of the specified axis. So when it collapses the axis 0 (row), it becomes just one row and column-wise sum.** Let's look at examples.

```
>>> np.sum(np.array([[1,1,1],[2,2,2]]), axis=1)
array([3, 6])
>>> np.sum(np.array([[1,1,1],[2,2,2]]), axis=0)
array([3, 3, 3])
```

Note that `axis=1` (column) will "squash" (or collapse) sum `np.array([[1,1,1],[2,2,2]])` in the column direction. On the other hand, `axis=0` (row) will collapse-sum `np.array([[1,1,1],[2,2,2]])` in the row direction.

B. Comparing matrices of different dimensions / advanced np.sum

Note that two matrices `A, B` below have same number of columns but different row dimensions.

```
>>> A = np.array([[1,1,1],[2,2,2],[3,3,3]])
>>> B = np.array([[1,2,3]])
>>> A==B
array([[ True, False, False],
       [False,  True, False],
       [False, False,  True]])
```

The operation `A==B` copies `B` three times row-wise so that it matches the dimension of `A` and then element-wise compaires `A` and `B`.

We can apply `A==B` to np.sum like below.

```
>>> A = np.array([[1,1,1],[2,2,2],[3,3,3]])
>>> B = np.array([[1,0,0],[2,2,0],[3,3,3]])
>>> np.sum(A==B, axis=1)
array([1, 2, 3])
```

C. np.sign

`np.sign`, given a numpy array as input, outputs a numpy array of the same dimension such that its element is the sign of each element of the input. Let's look at an example.

```
>>> np.sign(np.array([-3,0,5]))
array([-1,  0,  1])
```

D. np.argmax

`np.argmax`, given a numpy array as input, outputs the index of the maximum element of the input. Let's look at an example.

```
>>> np.argmax(np.array([[1,2,3],[4,5,6]]))
5
```

Note that the argmax index is given assuming the input array is flattened. So in our case, with 6 being the maximum element, 5 was returned instead of something like (1,2).

E. np.reshape

For a np array `A`, you can call `A.reshape((dim1_size,dim2_size,...))` in order to change the shape of the array.

```
>>> A = np.array([[1,2,3],[4,5,6]])
>>> A.reshape((3,2))
array([[1, 2],
       [3, 4],
       [5, 6]])
```

Note, the new shape has to have the same number of elements as the original.