Empirical Macroeconomics

Non Fundamental shocks

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Motivation

Dynamic Factors Models (DFM)

- small number of unobserved common dynamic factors that produce the observed comovements of economic time series.
- common dynamic factors are driven by the common structural economic shock
- hundreds of economic time series variables that potentially contain information about these underlying shocks.

DFM

Premise

- high dimensional times series $\mathcal{X}_t$ of dimension N ($i = 1, ...N$)
- few latent dynamic factors f_t, say q driven by the dynamic factor shocks η_t

$$f_t = \lambda(L) f_{t-1} + \eta_t$$

where the q factors can enter with p lags. Rewrite the factors in Canonical form

$$F_t = \Psi(L) F_{t-1} + G\eta_t$$

where F is of dimension $r \geq q$. (Example with 2)

DFM

Static Form

We assume that the variables $\mathcal{X}_t$ are a function of a small number of states unobservables states. In the DF literature the observables are usually mesured with noise e_t and the states are factors F_t.

$$\mathcal{X}_t = \Lambda F_t + e_t$$

$$F_t = \Psi(L)F_{t-1} + G\eta_t$$

Λ is usually called the dynamic factor loading. Assume for simplicity that the e_t to be $i.i.d.$ uncorrelated across series.

Principal Components

Extracting the factors

How do you estimate the factors and loadings? Assume one factor F_t. The the cross-sectional average

$$\frac{1}{N}\sum_{i=1}^{N}\mathcal{X}_{it} = \bar{\mathcal{X}}_t = \bar{\Lambda}F_t + \bar{e}_t$$

given our assumption on e_{it} we have $\bar{e}_t \to 0$ which implies that $\bar{\mathcal{X}}_t = \bar{\Lambda}F_t$. In other words if $\bar{\Lambda} \neq 0$ $\mathcal{X}_t$ estimates F_t up to a scale.

Principal Components

Extracting the factors

Up to a scale: the variance of the data is

$$\Sigma_{\mathcal{X}} = \Lambda \Sigma_F \Lambda'$$

where you have to remember that the dimension of $\mathcal{X}_t$ is N and of F is $r \ll N$. Notice also that Λ and F are not separately indentified. Inserting a nonsingular matrix H and replacing Λ by ΛH^{-1} and F by HF you obtain the same variance of the data. With on factor the variance is a single number, but you cannot identify the variance from the loading. This ambiguity is handled by adopting an arbitrary statistical normalization which (implicitly) imposes an arbitrary H.

Principal Components

The principal components normalization

Under this normalization the columns of Λ are orthogonal and scaled to have unit norm

$$N^{-1}\Lambda'\Lambda = I_r$$

and

$$E\left[F_t F_t'\right] \text{ is diagonal}$$

using this weighted average (instead of the unweighted average)

$$N^{-1}\Lambda'\mathcal{X}_t = N^{-1}\Lambda'\Lambda F_t$$

you get the factors. But how to find the loadings?

Principal Components

Estimation of Factors: principal components

- Take the filtered data (standardized to have sample mean zero and unit standard deviation)
- The estimators of F_t and Λ solve the minimization problem

$$\min_{\{F_i\}_{i=1}^{T},\Lambda} (NT)^{-1} \sum_{t=1}^{T} [\mathcal{X}_t - \Lambda F_t]' [\mathcal{X}_t - \Lambda F_t]$$

subject to $N^{-1}\Lambda'\Lambda = I_r$.

Under our simplified assumptions

$$\hat{F}_t = N^{-1}\hat{\Lambda}'\mathcal{X}_t$$

and $\hat{\Lambda}$is the matrix of eigenvectors of the sample variance matrix of $\mathcal{X}_t$. Otherwise need to solve the minimization.

FAVAR

SDFM and FAVAR

Take the DFM

$$\mathcal{X}_t = \Lambda F_t + e_t$$

$$F_t = \Psi(L) F_{t-1} + G\eta_t$$

Consider the case where you observe one factor without error (the interest rate in BBE):

$$\mathcal{X}_t = \Lambda \begin{bmatrix} Y_t \\ F_t \end{bmatrix} + \begin{bmatrix} 0 \\ e_t \end{bmatrix}$$

then

$$\begin{bmatrix} Y_t \\ F_t \end{bmatrix} = \Phi(L) \begin{bmatrix} Y_{t-1} \\ F_{t-1} \end{bmatrix} + G\eta_t$$

which is a reduced form VAR. The structural identification follows the SVAR literature by looking for a matrix A such that $\eta_t = A\epsilon_t$ where $\epsilon_t$are the structural shocks.

BBE

Motivating the FAVAR Structure: An Example

Backward-looking model as an example (Supply and Demand):

$$
\begin{aligned}
\pi_t &= \delta\pi_{t-1} + \kappa\left(y_{t-1} - y^n_{t-1}\right) + s_t \\
y_t &= \phi y_{t-1} - \psi\left(R_{t-1} - \pi_{t-1}\right) + d_t \\
y^n_t &= \rho y^n_{t-1} + \eta_t \\
s_t &= \alpha s_{t-1} + \nu_t
\end{aligned}
$$

SVAR/FAVAR

Motivating the FAVAR Structure: An Example

The monetary policy authority follows the Taylor Rule

$$R_t = \beta\pi_t + \gamma\left(y_t - y_t^n\right) + \epsilon_t^r$$

Monetary policy has two dimensions:

1. systematic: rule
2. unexpected ϵ_t

SVAR/FAVAR

Motivating the FAVAR Structure: An Example

The model can be casted in State Space Form:

$$\begin{bmatrix} F_t \\ Y_t \end{bmatrix} = \Phi(L) \begin{bmatrix} F_{t-1} \\ Y_{t-1} \end{bmatrix} + \epsilon_t$$

where $\Phi = \begin{bmatrix} \rho & 0 & 0 & 0 & 0 \\ 0 & \alpha & 0 & 0 & 0 \\ -\kappa & \alpha & \delta & \kappa & 0 \\ 0 & 0 & \psi & \phi & -\psi \\ \gamma\psi & \beta\alpha & (\beta\delta + \gamma\psi) & (\beta\kappa + \gamma\phi) & -(\beta\kappa + \gamma\rho) \end{bmatrix}$

and $\epsilon_t = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\gamma & \beta & \gamma & 1 \end{bmatrix} \begin{bmatrix} \eta_t \\ v_t \\ d_t \\ \epsilon_t^r \end{bmatrix}$ and

$$(F_t' \; Y_t')' = (y_t^n \; s_t \; \pi_t \; y_t \; R_t)'$$

SVAR/FAVAR

Motivating the FAVAR Structure: An Example

The division between what you observe and what you do not observe determine if the model has unobserved states. If yes we know there are potential information issues (discuss η shock) and we treat the unobserved states as factors. Assume the large data set is related to the variables in our model as

$$\mathcal{X}_t = \Lambda \left(y_t^n \; s_t \; \pi_t \; y_t \; R_t \right)' + e_t$$

and treat

$$\begin{aligned} F_t &= \left(y_t^n \; s_t \; \pi_t \; y_t \; \right)' \\ Y_t &= \left(R_t \right)' \end{aligned}$$

SVAR/FAVAR

Motivating the FAVAR Structure: An Example

In the paper BBE have a very large dataset. In their slow-R-fast scheme, monetary policy shocks or news/financial shocks are assumed not to affect slow-moving variables like output, employment, and price indexes within a period, monetary policy responds within a period to shocks to slow-moving variables but not to news or financial shocks, and fast-moving variables (like asset prices) respond to all shocks, including news/financial shocks that are reflected only in those variables. Let s denote slow moving variables and f fast moving variables.

SVAR/FAVAR

slow-R-fast scheme

Then the Bernanke, Boivin, and Eliasz (2005) implementation of the slow-R-fast identification scheme is

$$\begin{bmatrix} \mathcal{X}_t^s \\ \mathcal{X}_t^f \end{bmatrix} = \begin{bmatrix} \Lambda_{ss} & 0 & 0 \\ \Lambda_{fs} & \Lambda_{fr} & \Lambda_{ff} \end{bmatrix} \begin{bmatrix} F_t^s \\ R_t \\ F_t^f \end{bmatrix} + e_t$$

$$\begin{bmatrix} F_t^s \\ R_t \\ F_t^f \end{bmatrix} = \Phi(L) \begin{bmatrix} F_{t-1}^s \\ R_{t-1} \\ F_{t-1}^f \end{bmatrix} + \begin{bmatrix} \eta_t^s \\ \eta_t^r \\ \eta_t^f \end{bmatrix}$$

and

$$\begin{bmatrix} \eta_t^s \\ \eta_t^r \\ \eta_t^f \end{bmatrix} = \begin{bmatrix} A_{ss} & 0 & 0 \\ A_{rs} & 1 & 0 \\ A_{fs} & A_{fr} & A_{ff} \end{bmatrix} \begin{bmatrix} \epsilon_t^s \\ \epsilon_t^r \\ \epsilon_t^f \end{bmatrix}$$

SVAR/FAVAR

Empirical Implementation

Let us go to the code.

Appendix*

Autocorrelation

Tipycally the noise can be serially correlated

$$e_t = D(L)e_{t-1} + \zeta_t$$

where each element of ζ_t, ζ_{it} is i.i.d. $N\left(0, \sigma^2_{\zeta_i}\right)$, i=1...N with $E\zeta_{it}\zeta_{js}$ for all s if $j \neq i$. You can rewrite our DFM as

$$\mathcal{X}_t = \tilde{\Lambda}F_t + D(L)\mathcal{X}_{t-1} + \zeta_t$$

Appendix

The general DFM model

To sumarize the DFM is

$$\mathcal{X}_t = \tilde{\Lambda} F_t + D(L)\mathcal{X}_{t-1} + \zeta_t$$

$$F_t = \Psi(L) F_{t-1} + G\eta_t$$

with $E\eta_t\zeta_{t-k} = 0$ for all $k \geq 0$

Appendix
Extracting the factors

The variance of the data is

$$\Sigma_{\tilde{\mathcal{X}}} = \tilde{\Lambda}\Sigma_F\tilde{\Lambda}' + \Sigma_\zeta$$

where you have to remember that the dimension of $\mathcal{X}_t$ is N and of F is $r \ll N$. Notice also that $\tilde{\Lambda}$ and F are not separately indentfied. Inserting a nonsingular matrix H and replacing $\tilde{\Lambda}$ by $\tilde{\Lambda}H^{-1}$ and F by HF you obtain the same variance of the data. This ambiguity is handled by adopting an arbitrary statistical normalization which (implicitly) imposes an arbitrary H.

Appendix

Estimation of Factors: Kalman

- The unknown coefficients of the DFM (with additional lag length and normalization restrictions) can be estimated by Gaussian maximum likelihood using the Kalman Filter.
- When n is very large, however, this method is computationally burdensome.

Appendix

Estimation of Factors: principal components

- Take the filtered data (standardized to have sample mean zero and unit standard deviation)
- The estimators of F_t and Λ solve the minimization problem

$$\min_{\{F_i\}_{i=1}^{T},\Lambda,D(L)} T^{-1}\sum_{t=1}^{T}\left[(I-D(L)L)\mathcal{X}_t-\Lambda F_t\right]'\left[(I-D(L)L)\mathcal{X}_t-\Lambda F_t'\right]$$

  which is solve iteratively (pca in Stata) and get $\hat{F}_t$ and $\hat{\Lambda}$ and $\hat{D}$.
- Once you have F_t you treat first q principal components as the dynamic factors

Appendix

The DFM and Reduced-Form VARs

Take the DFM

$$\mathcal{X}_t = \tilde{\Lambda} F_t + D(L)\mathcal{X}_{t-1} + \zeta_t$$

$$F_t = \Psi(L) F_{t-1} + G\eta_t$$

Insert the second into the first

$$\mathcal{X}_t = \tilde{\Lambda}\Psi(L) F_{t-1} + D(L)\mathcal{X}_{t-1} + \tilde{\Lambda} G\eta_t + \zeta_t$$

aand you obtain the FAVAR (with the exclusion restrictions)

$$\begin{bmatrix} F_t \\ \mathcal{X}_t \end{bmatrix} = \begin{bmatrix} \Psi(L) & 0 \\ \tilde{\Lambda}\Psi(L) & D(L) \end{bmatrix} \begin{bmatrix} F_{t-1} \\ X_{t-1} \end{bmatrix} + \begin{bmatrix} G & 0 \\ \tilde{\Lambda} G & I \end{bmatrix} \begin{bmatrix} \eta_t \\ \zeta_t \end{bmatrix}$$

In Bernanke, Boivin, Esraz:

$$\begin{bmatrix} F_t \\ \mathcal{X}_t \end{bmatrix} = \Phi(L) \begin{bmatrix} F_{t-1} \\ X_{t-1} \end{bmatrix} + \begin{bmatrix} \epsilon_{Ft} \\ \epsilon_{\mathcal{X}t} \end{bmatrix}$$

Appendix

The DFM and Reduced-Form VARs

The innovations and the shocks are related as follows

$$\begin{bmatrix} \epsilon_{Ft} \\ \epsilon_{\mathcal{X}t} \end{bmatrix} = \begin{bmatrix} G & 0 \\ \tilde{\Lambda} G & I \end{bmatrix} \begin{bmatrix} \eta_t \\ \zeta_t \end{bmatrix}$$

therefore the variance of the data and the model are related as follows:

$$\Sigma_\epsilon = \begin{bmatrix} G\Sigma_\eta G' & G\Sigma_\eta G'\tilde{\Lambda}' \\ \tilde{\Lambda} G\Sigma_\eta G' & \tilde{\Lambda} G\Sigma_\eta G'\tilde{\Lambda}' + \Sigma_\zeta \end{bmatrix}$$

Appendix
IRFs and FEVD

The IRF are recovered from the MA

$$\mathcal{X}_t = C(L)\eta_t + u_t$$

where $C(L) = [I - D(L)L]^{-1}\,\tilde{\Lambda}\,[I - \Psi(L)L]^{-1}\,G$ and $u_t = [I - D(L)L]^{-1}\,\zeta_t$which shows you can revover the response to all variables of the dataset if you identify η_t

Appendix

Identification

Analogously to structural VAR analysis, the dynamic factor structural shocks are assumed to be linearly related to the reduced form dynamic factor innovations η_t

$$\eta_t^s = H\eta_t$$

therefore

$$\mathcal{X}_t = C(L)H^{-1}\eta_t^s + u_t$$

where $C(L) = [I - D(L)L]^{-1}\,\tilde{\Lambda}\,[I - \Psi(L)L]^{-1}\,G$ as above.