Empirical Macroeconomics

Non Fundamental shocks

Francesco Franco

Nova SBE

April 20, 2020

Fundamental shocks: invertible MA

A difference equation interpretation

Consider an invertible $MA(1)$

$$Y_t = \mu + \epsilon_t + \theta \epsilon_{t-1}$$

you are solving for ϵ_t

$$\epsilon_t = -\theta \epsilon_{t-1} + (Y_t - \mu)$$

that you can solve backwards provided $|\theta| < 1$

$$\epsilon_t = (1 + \theta L)^{-1} (Y_t - \mu)$$

past values of Y_t contains ϵ_t.

Non-Fundamental shocks: non-Invertible MA

A difference equation interpretation

Consider a non invertible $MA(1)$

$$\epsilon_t = -\theta\epsilon_{t-1} + (Y_t - \mu)$$

provided $|\theta| \geq 1$ .Forward one period

$$\epsilon_{t+1} = -\theta\epsilon_t + (Y_{t+1} - \mu)$$

and solve for ϵ_t

$$\epsilon_t = -\frac{1}{\theta}\epsilon_{t+1} + \frac{1}{\theta}(Y_{t+1} - \mu)$$

$$\epsilon_t = \left(1 + \frac{1}{\theta}F\right)^{-1}\frac{1}{\theta}(FY_t - \mu)$$

where $Fx_t = x_{t+1}$. You need future values of Y_t to recover ϵ_t.

Invertible Moving Average

MA and VAR

- More: for any non-invertible $MA(1)$ we have found a non-invertible $MA(1)$ and vice-versa.
- Take the above invertible $MA(1)$

$$Y_t = \mu + \epsilon_t + \theta \epsilon_{t-1}$$

- The moments are $E\left(Y_t\right) = \mu$, $E\left(Y_t - \mu\right)^2 = \left(1 + \theta^2\right)\sigma^2$ and $E\left(Y_t - \mu\right)\left(Y_{t-1} - \mu\right) = \theta\sigma^2$.

Non-Invertible Moving Average

MA and VAR

- Consider a seemingly different non invertible $MA(1)$

$$Y_t = \mu + \tilde{\epsilon}_t + \tilde{\theta}\tilde{\epsilon}_{t-1}$$

- The moments are $E(Y_t) = \mu$, $E(Y_t - \mu)^2 = \left(1 + \tilde{\theta}^2\right)\tilde{\sigma}^2$ and $E(Y_t - \mu)(Y_{t-1} - \mu) = \tilde{\theta}\tilde{\sigma}^2$.
- Then

$$\theta = \tilde{\theta}^{-1}$$

$$\sigma^2 = \tilde{\theta}^2\tilde{\sigma}^2$$

Non-Invertible Vector Moving Average

VMA

The VMA $Y_t = C(L)\epsilon_t$ is invertible if and only if the roots of the polynomial $|C(z)| = 0$ lie outside the unit circle
Example

$$\begin{pmatrix} Y_{1t} \\ Y_{2t} \end{pmatrix} = \begin{pmatrix} 1 & L \\ 0 & \theta - L \end{pmatrix} \begin{pmatrix} \epsilon_{1t} \\ \epsilon_{2t} \end{pmatrix}$$

the $det\,(C(z)) = \theta - z$ is zero for $\theta = z$. Therefore for invertibility $|z| > 1$ which is satisfied if $|\theta| > 1$.

Invertible and Non-invertible

Recovering the shocks

- When we estimate a VAR we always recover the innovations of the invertible MA, or the Wold decomposition innovations.
- What if our model/theory implies non-fundamental shocks?
- ϵ_t can be non fundamental for Y_t. But suppose it is fundamental for another variable ?
- Fundamentalness is therefore an information issue.

An example

Non-fundamentalness and News Shocks (Fonr et al.)

Consider a simple asset price model. We assume the total factor productivity a_t follows

$$a_t = a_{t-1} + \epsilon_{t-2} + v_t$$

Where ϵ_t is a news shock and v_t is a shock affecting TFP on impact. The agents observe the shock ϵ_t at time t and react to it immediately while the shock will affect TFP only at time $t + 2$.

An example

Non-fundamentalness and News Shocks (Forni et al.)

The RA maximizes

$$E_t \sum_{t=0}^{\infty} \beta^t c_t$$

s.t.

$$c_t + p_t n_{t+1} = (p_t + a_t)\, n_t$$

FOC

$$\lambda_t = 1$$

$$\lambda_t p_t = \beta E_t \lambda_{t+1} \left[p_{t+1} + a_{t+1} \right]$$

Combining we get

$$p_t = \sum_{j=1}^{\infty} \beta^j E_t \left[a_{t+j} \right]$$

An example

Non-fundamentalness and News Shocks (Forni et al.)

Now $p_t = \sum_{j=1}^{\infty} \beta^j E_t \left[a_{t+j}\right]$ is using our DGP on productivity $a_t = a_{t-1} + \epsilon_{t-2} + v_t$

$$E_t a_{t+1} = a_t + \epsilon_{t-1}$$

$$E_t a_{t+2} = E_t a_{t+1} + \epsilon_t = a_t + \epsilon_{t-1} + \epsilon_t$$

$$E_t a_{t+3} = E_t a_{t+2} = a_t + \epsilon_{t-1} + \epsilon_t$$

and so on. Using it in the price $p_t = \beta \sum_{j=1}^{\infty} \beta^{j-1} E_t \left[a_{t+j}\right]$ we get

$$p_t = \frac{\beta}{1-\beta} a_t + \frac{\beta}{1-\beta} \left(\beta \epsilon_t + \epsilon_{t-1}\right)$$

An example

Non-fundamentalness and News Shocks (Forni et al.)

Now assume we observe p_t and a_t. Take the first difference and obtain the VMA:

$$\begin{pmatrix} \Delta a_t \\ \Delta p_t \end{pmatrix} = \begin{pmatrix} L^2 & 1 \\ \frac{\beta^2}{1-\beta} + \beta L & \frac{\beta}{1-\beta} \end{pmatrix} \begin{pmatrix} \epsilon_t \\ v_t \end{pmatrix}$$

the polynomial to be solved is determinant is $\frac{\beta}{1-\beta} z^2 - \frac{\beta^2}{1-\beta} - \beta z = 0$ and the roots are $z = 1$ and $z = -\beta$. Given $|\beta| < 1$ the shocks are non-fundamental for the variables Δa_t and Δp_t.

State Space Representation

Economic Models as state space models

Most Dynamic macro models can be casted in a *VMA* but also into a State Space representation:

$$S_{t+1} = AS_t + B\eta_{t+1}$$

$$X_t = \tilde{C}S_t$$

where S_t are the states and X_t the observations. We can re-write the SSM as

$$S_t = AS_{t-1} + B\eta_t$$

$$X_t = CS_{t-1} + D\eta_t$$

where S_t is a vector of states, X_t is a vector of observables and η_t are shocks.

State Space Representation

Our example

The previous example $S_t = AS_{t-1} + B\eta_t$ is

$$\begin{pmatrix} a_t \\ \epsilon_t \\ \epsilon_{t-1} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} a_{t-1} \\ \epsilon_{t-1} \\ \epsilon_{t-2} \end{pmatrix} + \begin{pmatrix} 0 & 1 \\ 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \epsilon_t \\ \eta_t \end{pmatrix}$$

and $X_t = CS_{t-1} + D\eta_t$ is

$$\begin{pmatrix} a_t \\ p_t \end{pmatrix} = \begin{pmatrix} 1 & 0 & 1 \\ \frac{\beta}{1-\beta} & \frac{\beta}{1-\beta} & \frac{\beta}{1-\beta} \end{pmatrix} \begin{pmatrix} a_{t-1} \\ \epsilon_{t-1} \\ \epsilon_{t-2} \end{pmatrix} + \begin{pmatrix} 0 & 1 \\ \frac{\beta^2}{1-\beta} & \frac{\beta}{1-\beta} \end{pmatrix} \begin{pmatrix} \epsilon_t \\ \eta_t \end{pmatrix}$$

Non Fundamentnalness in the SS model

An information issue

Consider D to be square: number of observations is equal to number of structural shocks

$$\eta_t = D^{-1}\left(X_t - CS_{t-1}\right)$$

Non Fundamentnalness in the SS model

An information issue

Plugging ϵ into the state equation:

$$S_t = AS_{t-1} + BD^{-1}\left(X_t - CS_{t-1}\right)$$

$$S_t = \left(A - BD^{-1}C\right)S_{t-1} + BD^{-1}X_t$$

solve backwards

$$\left(I - \left(A - BD^{-1}C\right)L\right)S_t = BD^{-1}X_t$$

$$S_t = \left(I - \left(A - BD^{-1}C\right)L\right)^{-1}BD^{-1}X_t$$

provided the eigenvalues of$A - BD^{-1}C$ all be strictly less than one in modulus. Then the past of X_t reveals S_t. The states are fundamental for X_t.

Non Fundamentnalness in the SS model

An information issue: shocks are non-fundamental wrt to observed variables

In our example $A - BD^{-1}C =$

$$\begin{pmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} - \begin{pmatrix} 0 & 1 \\ 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} -\frac{1}{\beta} & \frac{1-\beta}{\beta^2} \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 1 \\ \frac{\beta}{1-\beta} & \frac{\beta}{1-\beta} & \frac{\beta}{1-\beta} \end{pmatrix}$$

the eigenvalues are $(-\frac{1}{\beta}, 1)$ not respected as expected, therefore the pass X_t do not reveal the S_t.

Non Fundamentnalness in the SS model

An information issue

Therefore the condition (ABCD condition) that $\left(A - BD^{-1}C\right)$:

$$X_t = CS_{t-1} + D\eta_t$$

$$X_t = C\left(I - \left(A - BD^{-1}C\right)L\right)^{-1} BD^{-1}X_{t-1} + u_t$$

$$X_t = \sum_{j=1}^{\infty} \Phi_j X_{t-j} + u_t$$

is a sufficient condition for a VAR on observables to have innovations that map directly back into structural shocks in population. The variance of the residuals $\Sigma_u = D\Sigma_\eta D'$ and you can apply the identification restrictions if you have them.

DFM-FAVAR

Solving the Information problem using factors?

What if the ABCD condition is not respected? The idea is to have sufficient information to reveal the states. Use the state equation to obtain the shocks:

$$S_t = AS_{t-1} + B\eta_t$$

and notice that the structural shocks are always fundamental to the states. If you could observe the states you would simply run a VAR in the states and recover the shocks. Suppose B^{-1} exists (almost always, here left inverse)

$$\eta_t = B^{-1}S_t - B^{-1}AS_{t-1}$$

DFM-FAVAR

Solving the Information problem using factors?

Plug into the observation the shocks you recover from the state equation

$$X_t = CS_{t-1} + D\eta_t$$

$$X_t = CS_{t-1} + D(B^{-1}S_t - B^{-1}AS_{t-1})$$

$$X_t = \left(DB^{-1} + \left(C - DB^{-1}A\right)L\right)S_t$$

Write the previous equation as

$$X_t = \Lambda F_t$$

where $\Lambda = \left[DB^{-1}\ C - DB^{-1}A\right]$ and $F_t = (S_t'\ S_{t-1}')'$. Which is to underline the connection between the states and what are called factors. The idea of the FAVAR is to measure the unobserved states with factors and run a VAR with factors and observed variables.

DFM-FAVAR

Static Form

NIn the BBE paper we assume that the variables $\mathcal{X}_t$ are a function of a small number of states unobservables states. In the DF literature the observables are usually mesured with noise e_t and the states are factors F_t. So the two notations in parallel:

$$\mathcal{X}_t = \Lambda F_t + e_t \qquad\qquad X_t = \Lambda F_t$$

$$F_t = \Psi(L) F_{t-1} + G\eta_t \qquad\qquad S_t = AS_{t-1} + B\eta_t$$

Λ is usually called the dynamic factor loading. Assume for simplicity that the e_t to be $i.i.d.$ uncorrelated across series.