Empirical Macroeconomics

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Growth and Fluctuations

Supply and Demand

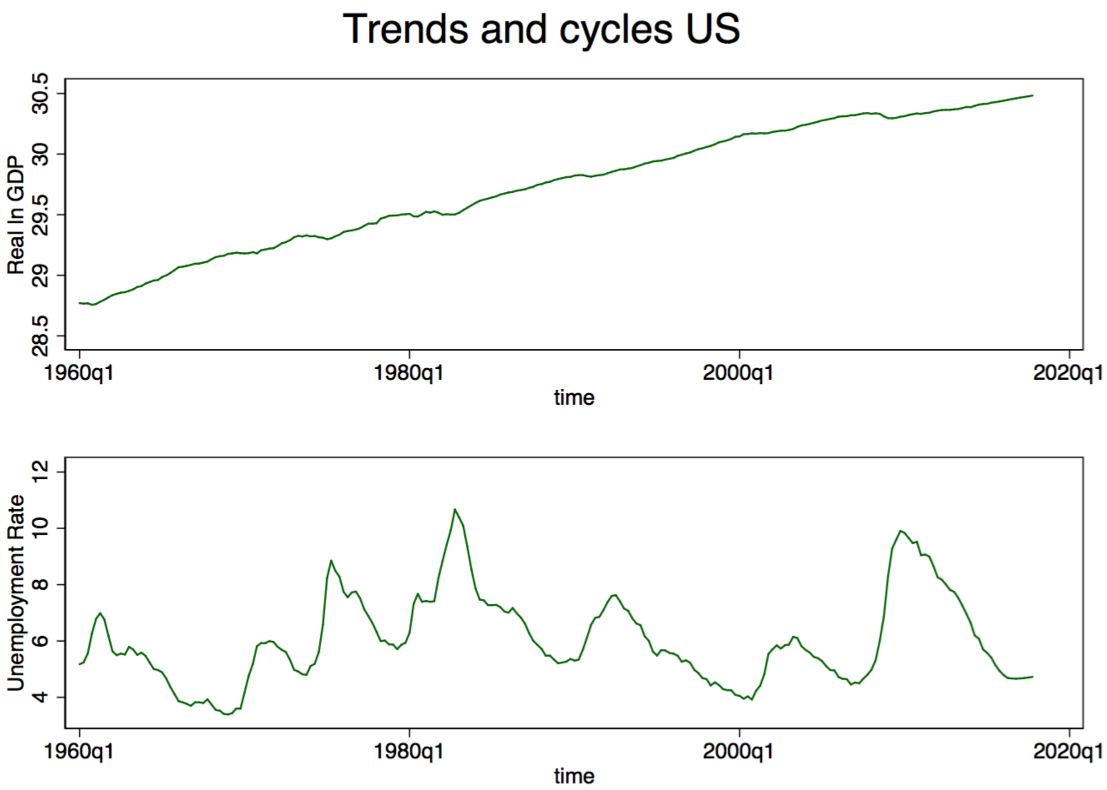

Figure: US dynamics

Growth and Fluctuations

Supply and Demand

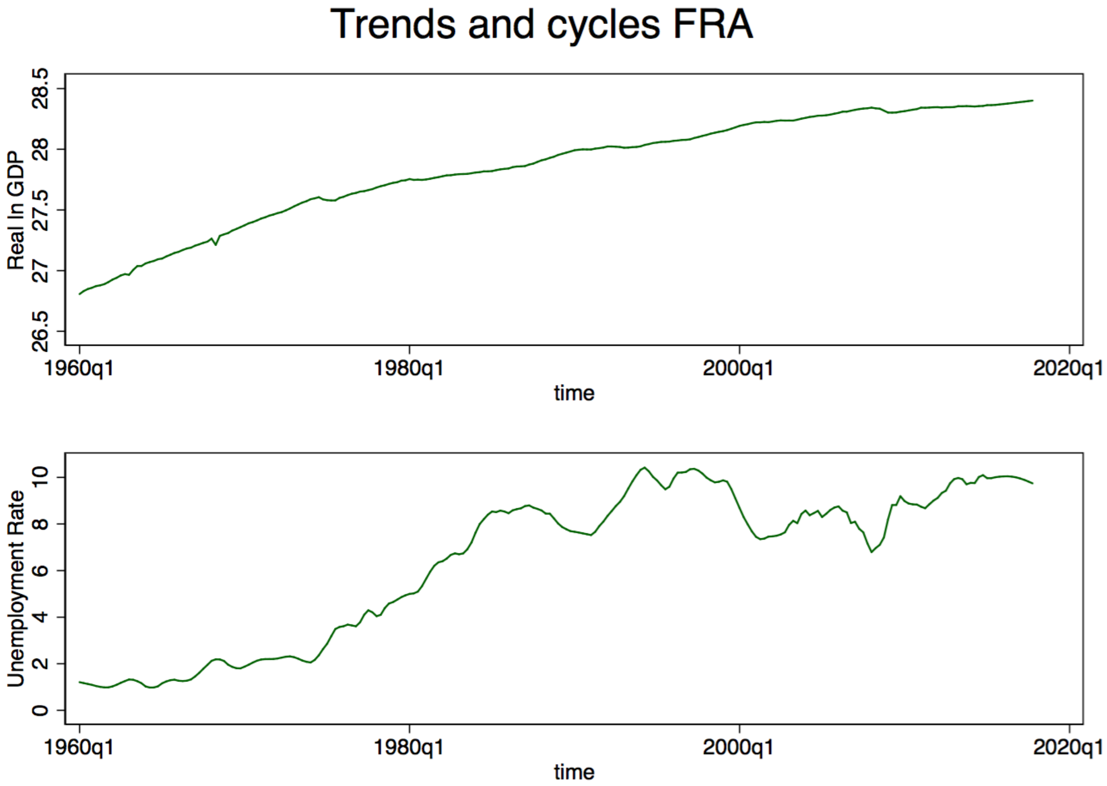

Figure: France dynamics

Growth and Fluctuations

Supply and Demand

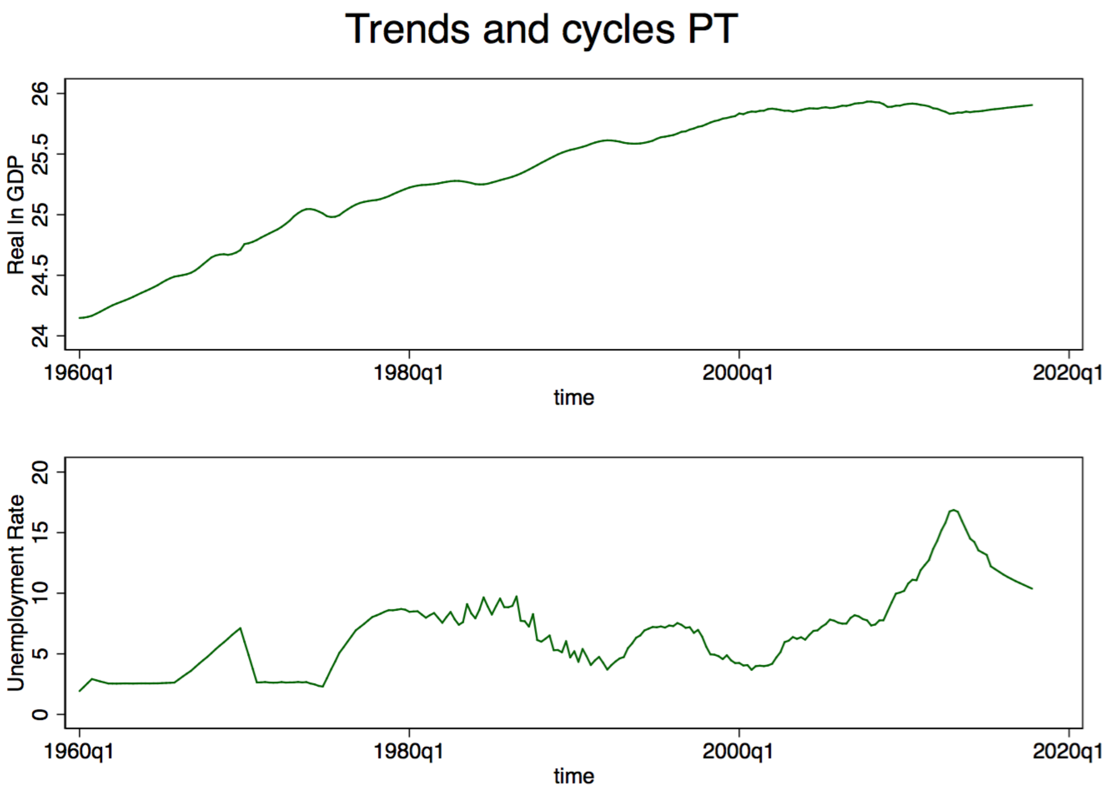

Figure: Portugal dynamics

SVAR

Recap: What for

1. study the expected response of the model variables to a given one-time structural shock: in our case permanent versus temporary.
2. allow the construction of forecast error variance decompositions that quantify the average contribution of a given structural shock to the variability of the data.
3. used to provide historical decompositions that measure the cumulative contribution of each structural shock to the evolution of each variable over time.
4. allow the construction of forecast scenarios conditional on hypothetical sequences of future structural shocks.

Var to SVAR

Recap: What for

- There is a large body of literature on the specification and estimation of reduced-form VAR models
- The success of such VAR models as descriptive tools and to some extent as forecasting tools is well established.
- Svar: after decomposing forecast errors into structural shocks that are mutually uncorrelated and have an economic interpretation we can assess the causal effects of these shocks on the model variables.

The Economic Model

MA

In the paper we find an example of a model that is a MA. You can also build a New Keynesian model with the same implications. This is fairly general. Let Y and U denote the logarithm of GNP and the level of the unemployment rate, e_d and e_S be the two disturbances. $X = (\Delta Y, U)$, and let $\epsilon = (e_d, e_s)$. The vector moving average (VMA) representation of the model :

$$X_t = A(0)\varepsilon_t + A(1)\varepsilon_{t-1} + ...$$

- with the LR: identifying restriction $\sum_{j=1}^{\infty} a_{11}(j) = 0$
- $Var(\epsilon) = I$, orthogonality is identifying restriction, unit variance is normalization

Wold representation

Fundamental

Since X_t is stationary, it has a Wold-moving average representation

$$X_t = v_t + C(1)v_{t-1} + .. + C(k-1)v_{t-k} + ...$$

with $Var(v) = \Omega$

From Wold to Model

Identification

Notice model is

$$X_t = A(0)\varepsilon_t + A(1)\varepsilon_{t-1} + ...$$

Wold estimated from data is

$$X_t = v_t + C(1)v_{t-1} + .. + C(k-1)v_{t-k} + ...$$

now for every t :

$$v_t = A(0)\epsilon_t$$

which means $C(1)v_{t-1} = A(1)\epsilon_{t-1} = C(1)A(0)\epsilon_{t-1}$. So finding A_0 allows you to find all the original A_j (remember you estimate all the C_k)

Identification

Write the relation beween reduced form and structural for residuals in matrix form

$$v = A(0)\epsilon$$

$$\begin{pmatrix} v_t^{\Delta Y} \\ v_t^{U} \end{pmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} \epsilon_t^d \\ \epsilon_t^s \end{bmatrix}$$

first row: $v_t^{\Delta Y} = a_{11}\epsilon_t^d + a_{12}\epsilon_t^s$
second row: $v_t^{U} = a_{21}\epsilon_t^d + a_{22}\epsilon_t^s$

Identification

Information comes from the variance of the data

$$\Omega = A(0)A(0)'$$

three restrictions

$$\begin{pmatrix} v_t^{\Delta Y} \\ v_t^{U} \end{pmatrix} \begin{pmatrix} v_t^{\Delta Y} \\ v_t^{U} \end{pmatrix}' = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} \epsilon_t^d \\ \epsilon_t^s \end{bmatrix} \begin{bmatrix} \epsilon_t^d \\ \epsilon_t^s \end{bmatrix}' \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}'$$

where now LHS is 3 distincts elements $\omega_{11}, \omega_{12}, \omega_{22}$ and RHS is 4 distincts elements (remember to use $\begin{bmatrix} \epsilon_t^d \\ \epsilon_t^s \end{bmatrix} \begin{bmatrix} \epsilon_t^d \\ \epsilon_t^s \end{bmatrix}' = I$). With one more restriction we can solve the system of 4 equation in 4 unknowns.

Identification

In levels, we have for the first row of the Wold we have:

$$Y_t = v_{1t} + (I_1 + C_1(1))v_{1t-1} + .. + (I_1 + C_1(1) + .. + C_1(k))v_{t-k} + ...$$

which means that the Long Run effects are:

$$LR = (I + C(1) + C(2) + ...)A(0) = (I - C)^{-1}A(0)$$

and restriction is $LR_{11} = 0$.

Issues

Limitations:

- require an accurate estimate of the impulse responses at the infinite horizon. Similar issues that testing unit-root.
- require structural shocks to be fundamental (true for any identification strategy)

Estimation

To recover the Wold we first estimating and then inverting the vector autoregressive representation of X (Not Always feasible...).
Estimate:

$$X_t = B(1)X_{t-1} + B(2)X_{t-2} + ... + v_t$$

then invert

$$X_t = C(L)^{-1}v_t$$
$$X_t = v_t + C(1)v_{t-1} + ...$$

Canonical Forms

Any VAR(p) can be casted in VAR(1)

It is convenient in the code to work with canonical forms:

$$X_t = B(1)X_{t-1} + B(2)X_{t-2} + ... + B(p)X_{t-p} + v_t$$

$$\begin{bmatrix} X_t \\ X_{t-1} \\ ... \\ X_{t-p} \end{bmatrix} = \begin{bmatrix} B(1) & B(2) & ... & B(p) \\ I_n & 0 & ... & 0 \\ 0 & I_n & ... & 0 \\ 0 & 0 & ... & 0 \end{bmatrix} \begin{bmatrix} X_{t-1} \\ X_{t-2} \\ ... \\ X_{t-p-1} \end{bmatrix} + \begin{bmatrix} I_n \\ 0 \\ 0 \\ 0 \end{bmatrix} v_t$$

$$X_t^c = B^c X_{t-1}^c + G^c v_t$$

Canonical Forms

Then it is easy to find the MA form

$$X_t^c = B^c X_{t-1}^c + G^c v_t$$

$$X_t^c = (I - B^c L)^{-1} G^c v_t$$

$$X_t^c = C^c(L) G^c v_t$$

then just work with the first n rows. $X_t = C(L)v_t$ with $C(0) = I_n$ and $G^c = I_n$.

IRF and VDEC

After identification we can find the IRF

$$\hat{X}_{t+s} = \sum_{i=0}^{s-1} C(i)A(0)\epsilon_{t+s-i}$$

and the variance decomposition

$$FEV^D = E_{t_0}\left[\left(X_{t_0+s} - \widehat{X}_{t_0+s}\right)\left(X_{t_0+s} - \widehat{X}_{t_0+s}\right)'\right] = \sum_{t=0}^{s-1} C_t \Omega C_t'$$

$$\Omega = A(0)A'(0) = \sum_{j=1}^{n_{vars}} a_j a_j'$$

Notes on estimation

The estimation of the VAR where T is sample, K is number of variables and p is number of lags, $\mathcal{L}$ is log likelihood and t_p is the number of parmeters.
Pre estimation number of lags selected through FPE, Akaike,Hannan and Quinn

$$FPE = |\Omega| \left(\frac{T + Kp + 1}{T + Kp - 1} \right)^K$$

$$AIC = -2 \left(\frac{\mathcal{L}}{T} \right) + 2 \frac{t_p}{T}$$

$$SBIC = -2 \left(\frac{\mathcal{L}}{T} \right) + 2 \frac{ln(T)}{T} t_p$$

$$HQIC = -2 \left(\frac{\mathcal{L}}{T} \right) + \frac{2ln(ln(T))}{T} t_p$$

Notes on estimation

The VAR residuals must be well-behaved:Normal: non serially correlated.

Unit root

Non stationarity

Widely disputed are the hypothesis that log GNP is reasonably characterized as a unit root process or a trend stationary process.

$$y_t = y_{t-1} + \epsilon_t$$

or

$$y_t = \beta t + \epsilon_t$$

Our results make clear that uncritical repetition of the "we don't know, and we don't care" mantra is just as scientifically irresponsible as blind adoption of the view that "all macroeconomic series are difference-stationary," or the view that "all macroeconomic series are trend- stationary." There is simply no substitute for serious, case-by-case, analysis. Debiold and Senhadji. 1996

Unit root

Non stationarity

In the OLS estimation of an AR(1)

$$y_t = \rho y_{t-1} + \epsilon_t$$

with ϵ_t iid $N\left(0, \sigma^2\right)$ and $y_0 = 0$ the OLS estimate of ρ is given by

$$\hat{\rho} = \frac{\sum_{t=1}^{n} y_{t-1} y_t}{\sum_{t=1}^{n} y_{t-1}^2}$$

Unit root

Non stationarity

If $|\rho| < 1$, then

$$\sqrt{n}\left(\hat{\rho}_n - \rho\right) \rightarrow N\left(0, 1 - \rho^2\right)$$

But if this result was valid when $\rho = 1$ then the distribution would have variance zero. (Theory in Hamilton chap 17). We need to find a suitable non degenarate distribution to test hypotesis $H_0 : \rho = 1$

Unit root

Augmented Dickey-Fuller

we fit

$$\Delta y_t = \alpha + \beta y_{t-1} + \delta t + \sum_{j=1}^{k} \zeta_j \Delta y_{t-j} + e_t$$

via OLS. the test statistic for $H_0 : \beta = 0$ is $Z_t = \hat{\beta}/\hat{\sigma}_\beta$. Critical values.

Autocorrelation

Test

Definition

ε_t is not iid since it is correlated with some ε_{t-s}.

Anything that causes correlation between the residuals and the regressor will make LS inconsistent. For instance, a model with a lagged dependent variables as regressor and autocorrelated shocks.

1. Estimate $\rho = corr(\hat{\varepsilon}_t, \hat{\varepsilon}_{t-1})$ and use a t-test on ρ
2. Durbin-Watson: $DW \approx 2 - 2\rho$. Reject H_0 if $DW \leq 1.5$.

Heteroskedasticity

Phillips Perron

PP correct both autocorrelation and heteroskedasticity with Newy-West but fits: $y_t = \alpha + \rho y_{t-1} + \delta t + \epsilon_t$. See code.